Abstract

We study a model of non-cooperative bargaining between two buyers and a seller in the presence of externalities and coordination between buyers. We find that bargaining can break buyer coordination and that the seller’s payoff can be decreasing in its own bargaining power.

1 Introduction

Terms and conditions of a contract are typically the outcome of negotiations among the parties involved. In these negotiations bargaining power is essential in determining whether or not trade occurs and how the value created is distributed. Typically a greater level of bargaining power is associated with better terms of trade. To put it another way: a party with greater contractual force is generally able to appropriate a greater share of surplus. In particular, an extreme form of bargaining power is when one party can make a take-it-or-leave-it offer to the other party. In this case, not only would we expect the party making the offer to appropriate the entire surplus but also we would guess that this party couldn’t gain by offering its rival the opportunity to propose the terms of trade.

However, we show in this paper that this common intuition does not need to hold in a setting in which there is bilateral contracting between more than two parties and multilateral externalities are present. Indeed, in this environment we demonstrate how a party can increase its payoff by offering bargaining power to its rivals.

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To illustrate this result, we consider a simple duopoly where two identical Cournot competitors (the buyers) produce using a constant-marginal-cost production technology. One upstream firm (the seller) can provide a technology that reduces the marginal cost of production for the buyers. An example of this situation can be a firm providing new software to the Cournot competitors or new cost-reducing equipment. The adoption of this equipment creates a negative externality among buyers: profits of a firm are greater if the competitor does not adopt it. Nevertheless, despite reducing buyers' marginal costs, this technology is not efficient from a production point of view. More precisely, the increase in profits that buyers derive from the adoption of new equipment is not enough to compensate the cost that the seller sustains to produce it. To put it another way: total surplus of the vertical structure is not maximized.

The questions we want to answer are the following: can this seller sell its product to the duopolists? And if it can, what is his profit going to be?

A "Chicago School" argument would answer the first question in a negative way: rational buyers would never buy the piece of equipment because their profits would be reduced. Nevertheless the presence of externalities among buyers constitutes an incentive to adopt the inefficient technology and the seller may exploit these externalities to trade with them.

We will show that by giving the buyers all the bargaining power, i.e. letting them make a take-it-or-leave-it offer to the seller about the terms of the contract, there is no trade in equilibrium and the Chicago School predictions are realized. On the other hand, if we endow the seller with the entire contractual power, as in all models frequently cited to criticize the Chicago claim, inefficient trade arises.

In particular, in the case in which the seller has the entire bargaining power not only he can implement trade as Nash equilibrium but also he can implement it as a unique Nash equilibrium. Indeed, also if buyers coordinate on their preferred equilibrium the seller can contract with both of them using a *divide and conquer* strategy: he must offer at least one buyer a contract that induces him to trade even if he expects the other buyer not to and simultaneously he can offer the other buyer a contract that he will accept if he expects the first buyer to be contracted.

From this exercise it clearly appears that the allocation of bargaining power is fundamental in determining whether or not inefficient trade may emerge as an equilibrium outcome. The purpose of this paper is to depart from these extreme forms of bargaining power to study more intermediate environments. To do so we introduce a dynamic model of non-cooperative bargaining à la Rubinstein. We show that the equilibrium outcome of this bargaining game can differ remarkably from those of the extreme cases described above. In particular, we found that the bargaining procedure can break coordination between buyers and trigger a sort of competition among them.

To have the intuition of why this happens consider a simple case in which there are only two periods and no discounting. At T=1 both buyers simultaneously and
non-cooperatively propose a transfer to the seller to obtain the technology. Having observed these offers the seller can accept both of them, only one, or none. The buyer whose offer has been accepted receives the technology and at T=2 the seller is going to propose a transfer to the uncontracted buyers.

Clearly if in T=2 both buyers have not been contracted, the seller will divide and conquer them asking for a low transfer to one of them, say buyer A and a high transfer to B. But then in T=1, B, predicting this future discrimination, is going to ask for a transfer slightly lower than the one is going to be offered to A next period in order to be the one contracted at better terms. This triggers a kind of Bertrand competition between the two buyers allowing the seller not only to trade with both of them but also to obtain very high transfers. Therefore, quite counter-intuitively, the seller can obtain a higher profit offering bargaining power to the buyers.

Extending the bargaining game to an infinite horizon, we show that the degree of inefficiency of trade determines the equilibrium outcome. If the loss of efficiency involved by the new technology is not too large, trade occurs as a unique Subgame-Perfect Nash Equilibrium. If instead the inefficiency exceeds a precise threshold, the game displays a multiplicity of equilibria and some of them sustain the efficient outcome. We show that in the case in which the unique Subgame Perfect Nash Equilibrium of the infinite horizon game involves trade, the counter-intuitive result still holds: if the discount factor is large enough it is more profitable for the seller to engage in an infinite-horizon bargaining process than to do a take-it-or-leave-it offer.

Our analysis not only illustrates some new aspects of vertical structures but also offers some novel insights into the literature on multi-principal multi-agent games. Various papers have studied the theoretical problem of bilateral contracting between one principal and a multiplicity of agents. A survey of this literature is presented in Section 2. In particular two approaches have been developed to analyze the problem: the offer game and the bidding game. In the offer game the principal proposes the contract to the agents whereas in the bidding game agents make offers to the principal. Our model is indeed a multi-agent game and the two approaches correspond to the two extreme distributions of bargaining power in which either the principal (the seller) or the agents (the buyers) make a take-it-or-leave-it offer to the counterpart. From a theoretical point of view our contribution is twofold. First, we identify a class of multi-agent games that when solved using the offer game approach have an inefficient outcome whereas when solved using the bidding game approach display efficiency. Second, we combine these two approaches in a dynamic framework to study a model of non-cooperative bargaining.

The paper is organized as follows. Section 2 discusses the related literature. In Section 3 we present the model and we discuss its static implications. In Section 4 we study the bargaining problem considering both a finite and an infinite horizon. Section 5 examines the main assumptions. Section 6 concludes.
2 Related Literature

Two are the approaches appeared in the literature to study bilateral contracting between one principal and N agents. Segal and Whinston (2003) define these approaches as the offer game and the bidding game.

In the offer game the principal makes simultaneous offers to the agents. A first theoretical treatment of such games has been proposed in Segal (1999) which studies the general nature of arising inefficiencies. In particular it is shown that when the principal commits to publicly observed bilateral contracts, inefficiencies arise due to externalities on agents’ reservation utility. The sequential version of this model has been formalized by Möller (2003). He shows that when externalities on agents’ reservation utility are absent but externalities on contracted agents are present sequential contracting is inefficient whereas simultaneous contracting is efficient.

In a subsequent paper Segal (2003) examines the effects of prohibiting the principal from coordinating agents on his preferred equilibrium and from making different contracts available to different agents. Inefficiencies under different contracting regimes are linked to the sign of the relevant externalities and are shown to be typically reduced by both restrictions.

A dynamic model of contracting with externalities is presented by Genicot and Ray (2003). In their model the reservation utilities of the agents depend positively on the number of agents who have not been contracted. They study the best strategy for the principal to approach agents if agents coordinate their actions. This strategy is split up into two phases. In phase one, simultaneous offers at good terms are made to a number of agents. In phase two, offers must be made sequentially and they are close to the minimum of all the outside options.

In the bidding game agents make simultaneous offers to the principal. Bernheim and Whinston (1986) present a model of an auction where bidders are submitting a menu of offers to the auctioneer. They prove the existence of an interesting subset of Nash Equilibria they called truthful, provide arguments in defence of this refinement and characterize the set of equilibrium payoffs.

Laussel and Le Breton (2001) present a set of theoretical results combining the approach of Bernheim and Whinston (1986) with cooperative game theory. Martimort and Stole (2003) analyze a model where each bidder can only observe and contract upon the auctioneer’s action that he directly cares about and not on the whole array of actions as in Bernheim and Whinston (1986). Dynamic versions of these models have been proposed by Prat and Rusitichini (1998) and Bergermann and Valimaki (2003).

Marx and Shaffer (2002) study bargaining power in sequential contracting in a model in which a buyer negotiates sequentially with two sellers. They describe bargaining power of a seller as the probability this seller has to make a take-it-or-leave-it offer to the buyer. Clearly if sellers are identical the buyer chooses to trade with the seller endowed with less bargaining power. Therefore, also in this model a seller pay-
off can be decreasing in its own bargaining power. This result, despite being similar to ours, relies on a different way of modeling bargaining power and it does not consider coordination among sellers.

Our model is also related to the literature on exclusive dealing. In particular, two major theories have appeared in this literature. On one side, members of the Chicago School claim that exclusive dealing should be regarded as an efficient contractual form. Their argument is that if exclusive dealing arises in equilibrium it is because it is efficient to sell only one product in the market. To put it another way: exclusive dealing occurs only if it maximizes the profits of the vertical structure as a whole. More recently, other scholars have claimed that the Chicago argument is not completely accurate. Focusing on externalities among buyers arising because of the exclusionary agreements these studies demonstrate how exclusive dealing need not to be efficient. Indeed, in these alternative models exclusive dealing occurs in equilibrium but it does not maximize the surplus of the vertical structure.

The Chicago School view on exclusionary agreement is summarized by the influential works of Posner (1976) and Bork (1978). A formal microeconomic model describing this theory appears in Bernheim and Whinston (1998). One of the first critiques to this approach comes from Rasmusen et al. (1991). They argue that an inefficient incumbent may exclude rivals profitably exploiting buyers' lack of coordination. Their results have been subsequently corrected by Segal and Whinston (2000) using a more appropriate equilibrium concept. Also in this paper inefficient exclusive dealing can be sustained as an equilibrium.

3 The Model

We consider a market where two Cournot oligopolists (A and B, henceforth buyers) produce facing a constant marginal cost \( c \).

Market demand can be represented by the (twice continuously differentiable) inverse demand function \( P(Q) \) with \( P'(Q) < 0 \) where \( Q = q_A + q_B \) is the total industry output. To exclude the trivial case in which production is not viable we assume \( P(0) > c \). We put the following, rather standard, restriction on the shape of the demand curve:

\[ P'(z) + zP''(z) < 0 \quad \text{for all } \quad z > 0. \]  

(A1)

In this simple setting, each buyer \( i \in \{A, B\} \) solves the following problem:

\[
\max_{q_i \geq 0} (P(Q) - c)q_i.
\]

Novshek (1985) shows that condition (A1) is necessary to have the existence of a Cournot equilibrium. Moreover, combined with constant marginal cost it guarantees uniqueness. We indicate the quantities produced in equilibrium as \( q^C \) and the corresponding profits of each of the two firms as \( \pi^C \).
A firm $M$, (henceforth seller) is willing to sell a piece of equipment to $A$ and $B$. The product sold by $M$ eliminates the production cost for the buyers adopting it. Nevertheless to produce this equipment is costly: it costs $C(1)$ to serve one firm and $C(2)$ to serve both of them.

Therefore if neither of the duopolist buys from $M$ both of them obtain $\pi^C$. If, instead, $M$ sells to one buyer the second stage game becomes a Cournot duopoly with different costs and both Dixit (1986) and Shapiro (1989) show that in comparison to the previous case, profits for the contracted buyer ($\pi^E$) increase whereas those of the uncontracted buyer ($\pi^{NE}$) decrease: $\pi^E > \pi^C > \pi^{NE}$. We indicate the bilateral surplus of the buyer and the seller as $F(1) \equiv \pi^E - C(1)$. Finally, if both downstream firms buy from $M$ they both have zero marginal production cost and the total surplus of the vertical structure is $F(2) \equiv 2\pi^0 - C(2)$ where $\pi^0$ is the profit of a buyer if both of them produce at zero marginal cost.

Depending on the specific functional forms of $C(\cdot)$ and $P(\cdot)$, we may observe a value for $F(2)$ greater, lower or equal to $2F(1)$. In the following, for simplicity, we will focus on the case in which $F(2) = 2F(1) \equiv 2F$ and we postpone the discussion of the cases in which $F(2)$ exceeds or is less than $2F(1)$ to Section 5.

We assume that a contract between $M$ and a buyer specifies the fraction of surplus that the buyer is going to appropriate. Therefore, after that a contract has been signed, the seller appropriates the entire surplus from the relation and transfers part of it to the buyer\(^1\). For instance, if $A$ accepts a contract with transfer $x_A$ and $B$ accepts a contract with transfer $x_B$ then the payoff of the seller is going to be $2F - x_A - x_B$ whereas $x_A$ and $x_B$ are going to be the payoffs for the two buyers.

Finally we assume that the following condition is satisfied:

$$\pi^C > 2F - \pi^C > \pi^{NE}. \tag{A2}$$

Assumptions A2 is fundamental to describe the environment we want to study. In our model trade is appealing both for the seller and the buyers: the former obtains positive profits, the latter have their production costs reduced. Nevertheless this assumptions guarantees that trade is not efficient from a production point of view: it does not maximize the surplus of the vertical structure. Indeed the efficient outcome for the vertical structure (the outcome maximizing the total surplus of the three parties) is not to buy the piece of equipment from the seller. Conversely the outcome that maximizes consumer surplus is to have both firms purchasing from $M^2$.

### 3.1 The offer and the bidding game

Solutions to the game described vary with the contracting procedure chosen.

\(^1\)We could have equally chosen to model transfers from the buyer to the seller. Nevertheless our modelling strategy renders easier the description of the bargaining game in the following.

\(^2\)Kimmel (1992) presents a general analysis of effects of cost changes on oligopolists’ profits and consumer welfare. In particular, for the case of linear demand: $2q^0 > q^E + q^{NE} > 2q^C$. 

6
Let us start with the offer game approach. Segal (1999) studies the case in which the seller publicly commits to a vector of transfers \( t = (t_A, t_B) \) and each buyer either accepts or rejects the offer. The equilibrium he describes is one in which \( M \) offers transfers that render each buyer indifferent between accepting and rejecting the contract as long as the other buyer accepts. More specifically Segal considers a game between buyers described by the following bimatrix:

<table>
<thead>
<tr>
<th>Buyer A</th>
<th>Accept</th>
<th>Reject</th>
</tr>
</thead>
<tbody>
<tr>
<td>Accept</td>
<td>( t_A, t_B )</td>
<td>( t_A, \pi^{NE} )</td>
</tr>
<tr>
<td>Reject</td>
<td>( \pi^{NE}, t_B )</td>
<td>( \pi^C, \pi^C )</td>
</tr>
</tbody>
</table>

If \( t = (\pi^{NE}, \pi^{NE}) \) then (Accept, Accept) is a Nash equilibrium of the game between buyers. Moreover this vector of transfer is the cheapest way for \( M \) to implement (Accept, Accept) as a Nash Equilibrium. It is important to notice that because of increasing externalities on non-traders this equilibrium is not unique. Segal’s analysis focuses only on this equilibrium point because he assumes that the seller can coordinate buyers on his preferred equilibrium. This is equivalent to impose coordination failure between the two buyers.

(Reject, Reject) is another Nash Equilibrium of the game. A comparison of the two shows that only the second is coalition proof in the sense of Bernheim, Peleg and Whinston (1987) and Bernheim and Whinston (1987). The definition of Coalition-Proof Nash Equilibrium is recursive: “an agreement is coalition proof if it is efficient within the class of self-enforcing agreements, where self-enforceability requires that no coalition can benefit by deviating in a self-enforcing way” [Bernheim et al. (1987)]. In our simple model it just requires that the Nash Equilibrium is not Pareto dominated by another Nash Equilibrium.

Segal (2001) deals with this coordination problem. To implement (Accept, Accept) as a unique Nash Equilibrium the seller must offer at least one buyer, say \( A \), a transfer that induces him to accept even if he expects the other buyer not to. Simultaneously it can offer \( B \) a transfer that induces him to trade if he expects \( A \) to be contracted. Segal (2001) calls this strategy “divide and conquer”. In our model it implies the vector of transfers \( t = (\pi^C, \pi^{NE}) \). Notice that it is profitable for \( M \) to offer such transfers because (A2) implies \( 2F - \pi^C - \pi^{NE} > 0 \).

We now turn to the bidding game in which both buyers propose take it or leave it offers to the seller. To analyze this case we follow Martimort and Stole (2003) and we consider a game in which each buyer can only observe and contract upon the seller’s action that it directly cares about. More precisely we assume that the contractual space of this game is exactly the same as the one in the offer game described above: each buyer proposes a transfer (a share of the surplus) in order to obtain \( M \)’s product. This bidding game has two types of Nash Equilibria. In one of these equilibria both buyers ask for \( F \) and the seller trades with both of them. The other one is the set of
equilibria in which both buyers ask for a transfer greater than $F$ and trade does not occur. This second set of Nash Equilibria is coalition proof. Thus we can say that in the bidding game, all coalition proof Nash Equilibria are efficient.

Summing up, the game proposed has a continuum of efficient coalition proof Nash Equilibria if it is solved using the bidding game approach whereas it has a unique inefficient coalition proof Nash Equilibrium with transfers $t = (\pi^C, \pi^{NE})$ if solved using the offer game procedure\(^3\). Therefore, the predictions on trade of our model change with the solution concept chosen.

4 Bargaining

As we have seen in the previous section, just changing our assumptions on bargaining power, our model can be used to sustain both a "Chicago" and a "post-Chicago" argument on inefficient trade. The identity of the party offering the contract is therefore fundamental. This observation leads us to analyze more intermediate allocations of bargaining power.

Given the take-it-or-leave-it nature of offers in both models we find natural to explore the issue using a noncooperative bargaining game of alternating offers. In the next section we present a model adapting Rubinstein’s bargaining procedure to the vertical structure.

In the following analysis we impose a kind of lexicographic preference for the seller: whenever he has to divide and conquer he is going to offer the largest transfer to $A$\(^4\).

4.1 A Two Period Bargaining Game

We turn now to the analysis of a two stage bargaining game. To do this we need to move from a static model to a dynamic framework. In particular we adapt to our model the dynamic game proposed by Genicot and Ray (2003).

The time horizon is infinite and actions take place at times $T=1,2,3,\ldots$. We consider $F$, $\pi^C$ and $\pi^{NE}$ as per period payoffs. We assume a common discount factor $\delta$.

As Genicot and Ray (2003) we impose throughout the following assumption:

Buyer Coordination: Only perfect equilibria satisfying the following restriction are considered: there is no date and no subset of buyers who have received offers at that date, who can change their responses and all be strictly better off, with the additional property that the changed responses are also individual best responses, given the equilibrium continuation from that point on.

\(^3\)It is easy to see that both outcomes do not differ if we consider the sequential version of the two games.

\(^4\)This simplifies both analysis and notation. In Section 5.3 we discuss and relax this assumption.
This assumption may be viewed as coalition-proofness requirement applied to the set of buyers.

Let us analyze, in this dynamic setting, a take it or leave it offer by the seller in the first period. If \( M \) offers a vector of transfers \( t = (t_A, t_B) \) and both buyers reject their contract they keep producing with marginal cost \( c \) for an infinite horizon and they obtain a payoff of \( \pi^C/(1 - \delta) \). If only one of the two rejects the contract he obtains \( \pi^{NE}/(1 - \delta) \). Therefore the divide and conquer strategy for \( M \) implies the following transfers:

\[
  t_A = \frac{\pi^C}{1 - \delta} \quad \text{and} \quad t_B = \frac{\pi^{NE}}{1 - \delta}.
\]

Normalizing \(^5\) lifetime payoffs by multiplying them by \((1 - \delta)\) we can notice how the result is exactly equivalent to the one of the corresponding static game. The same applies to the case in which buyers make a take-it-or-leave-it offer: each of them asks a (normalized) transfer greater than \( F \) and trade does not occur.

Consider now the following two period bargaining model. At \( T=1 \) each buyer simultaneously and non-cooperatively proposes a transfer to the seller. Having observed these offers \( M \) can accept both of them, only one or none. The buyer whose offer has been accepted receives the transfer, whereas the one whose offer has been rejected receives the outside option \((\pi^C \text{ or } \pi^{NE} \text{ depending on the trade between the other two parties})\) for one period. At \( T=2 \) the seller is going to propose a transfer to the uncontracted buyers. The single buyer can either accept the offer or obtain the outside option for an infinite horizon.

Solving the game we obtain the following result:

**Proposition 1** There exists a \( \hat{\delta} \in [0,1] \) such that if \( \delta \geq \hat{\delta} \) there exists a unique subgame-perfect Nash equilibrium in which both buyers trade in \( T=1 \). As \( \delta \) tends to one the payoffs of the buyers tend to \( \pi^{NE} \).

**Proof.** See Appendix. □

The intuition for the result can be obtained from Figure 1. The behavior of the seller in the first period depends on the transfers proposed by the buyers. If both buyers ask for a transfer inside area 1 the best reply for \( M \) is to accept both of them. If one of the two transfers is inside area 2, independently of the amount of the other, the optimal reply for the seller is to contract in the first period the buyer proposing the lower transfer and to offer \( \pi^{NE} \) in the second period to the buyer remained uncontracted. If both buyers ask for a transfer in area 3 or above then the best reply for \( M \) is to reject both transfers and to offer \( \pi^C \) to \( A \) and \( \pi^{NE} \) to \( B \) in the second period.

\(^5\)This permits us to study the effect of a change in the discount factor in the following propositions. Moreover it allows us to consider transfers as per-period transfers from \( M \) to the buyers.
Let us consider now the optimal strategy for the buyers. We start studying the case in which both buyers ask for a very large transfer in period 1. If this happens, they are going to receive $\pi^C$ for one period whereas in the second period $A$ is going to receive $\pi^C$ and $B$ is going to be offered $\pi^{NE}$. It is therefore clear that $B$ can be the one having an incentive to deviate given his lower payoff in the second period. The maximum transfer that $B$ can obtain in the first period given that $A$ asks for an amount above area 2 is given by $F(1 - \delta) + \delta \pi^C$ that lies on the border of the triangle 2. Therefore if

$$F(1 - \delta) + \delta \pi^C > \pi^C (1 - \delta) + \delta \pi^{NE}$$

$B$ has an incentive to deviate. This implies that there can be an equilibrium in which both buyers are contracted in the second period only if

$$\delta \leq \tilde{\delta} = \frac{\pi^C - F}{\pi^C - F + \pi^C - \pi^{NE}}.$$  

Conversely for $\delta > \tilde{\delta}$ this equilibrium cannot exist. Inside this region buyer $B$ has an incentive to deviate. It is important to notice how this profitable deviation triggers competition between buyers. Indeed $A$, knowing that $B$ is going to deviate, has an incentive to propose a transfer lower than the one announced by $B$ in order to be the one contracted first. This Bertrand-style competition implies that the only

\[\text{Figure 1: Equilibrium in a two stage game.}\]
possible equilibrium for $\delta > \tilde{\delta}$ involves transfers that are both accepted immediately by the seller, namely:

$$t_A = t_B = F(1 - \delta) + \delta \pi^{NE}.$$  

Buyers’ equilibrium payoffs are indicated by thick lines in Figure 1. As we can notice the striking feature of the two period bargaining game is that, despite coalition proofness, for $\delta$ close to one the unique subgame perfect Nash equilibrium payoff tends to the \textit{coordination failure} payoff of the one shot game. This result is interesting because it corresponds to a payoff outside the convex hull of the payoffs obtained in the two one shot games.

In particular, the bargaining procedure is source of competition between the two buyers: for values of $\delta$ large enough $B$ prefers being contracted in the first period rather than in the second and this breaks coordination. Therefore we conclude that the seller is better off offering some form of bargaining power to the buyers rather than making a take it or leave it offer. In fact, in the one shot game his total payoff is $2F - \pi^C - \pi^{NE}$ whereas, for $\delta$ close to one the payoff in the two period game tends to $2F - 2\pi^{NE}$. The seller is therefore exploiting a \textit{second mover advantage}.

What if buyers move second? Using backward induction we know that if both buyers are uncontracted in the second period then they can guarantee for themselves a payoff of $\pi^C$. If only one of them is uncontracted he can guarantee for himself a payoff of $F$ since this is the maximum amount that renders $M$ indifferent between contracting the free buyer or not. Therefore in the first period the seller must offer a transfer equal to $\pi^C$ to $A$ and equal to $\pi^{NE}(1 - \delta) + \delta F$ to $B$. Notice that this contract is profitable for the seller if and only if:

$$2F - \pi^C - \pi^{NE}(1 - \delta) - \delta F > 0$$

that is

$$0 \leq \delta \leq \delta' = \frac{2F - \pi^C - \pi^{NE}}{F - \pi^{NE}}.$$  

In this case, for large values of $\delta$, the outcome of the bargaining procedure does not differ from the one of the one shot game in which the buyers propose a transfer to the seller. Indeed we have efficiency. For $\delta \leq \delta'$ trade is going to occur and at the limit, $\delta = 0$, the outcome exactly corresponds to the one of the one shot game in which $M$ makes a take it or leave it offer. These results are summarized in Figure 2 where thick lines indicate buyers’ payoffs. Moreover we can see that also in this case the second movers have an advantage respect to the first mover.
4.2 General Bargaining Games

Having described the class of games we intend to study we can now generalize the results introduced in the previous section to various time horizons. The following two propositions describes outcomes for finite horizons bargaining games.

**Proposition 2** For any bargaining game of finite length, \( T > 2 \), in which the seller offers in the last period there exists a \( \delta \in [0, 1] \) such that if \( \delta \geq \delta' \) in the unique Subgame Perfect Nash Equilibrium both buyers trade in the first period. Moreover:

\[
\lim_{\delta \to 1} t_A(\delta) = \lim_{\delta \to 1} t_B(\delta) = \pi^{NE}.
\]

**Proof.** See Appendix. ■

The intuition behind this result is quite simple. As \( \delta \) gets larger the cost for the seller to wait one period becomes negligible and therefore he can threaten the buyers to wait until period T-2 when he can contract both of them with transfers arbitrarily close to \( \pi^{NE} \).

The same extension is possible for games in which buyers are last movers as next proposition describes.
Proposition 3 For any bargaining game of finite length, $T > 2$, in which the buyers offer in the last period and $\delta$ is large enough the unique Subgame Perfect Nash Equilibrium outcome is going to be efficient.

Proof. Suppose that this is not the case and that both buyers are contracted with transfers $t_A(\delta)$ and $t_B(\delta)$. Then as $\delta$ gets close to one we need that $t_A(\delta) \to \pi^C$ and that $t_B(\delta) \to F$ because these are the payoffs that each buyer can guarantee to himself at time $T$ and waiting is almost costless. But by assumption (A2) we have that $2F - \pi^C - F < 0$ which implies that trade is not profitable for $M$ as $\delta$ gets large. Therefore we have a contradiction.

From the previous propositions we conclude that in a finite horizon bargaining game with a discount factor close to one, the identity of the last mover drives the outcome of the game. Indeed, trade occurs if the seller is last mover and it does not occur if the buyers offer in the last period. This result is not surprising since also in Rubinstein(1982) two-player bargaining game if the horizon if finite and $\delta = 1$ the last mover obtains the entire surplus. What is striking different from Rubinstein(1982) is that in our model the payoff obtained by $M$ in a bargaining game in which he is the last mover differs from what he gets in the one shot game where he makes a take it or leave it offer. More specifically the bargaining procedure allows him to break the coordination between the two buyers obtaining a transfer that corresponds to the coordination failure outcome of the one shot game.

We now turn to the analysis of the infinite horizon game. In this case the last mover advantage arising because of the finite length of the game no longer exists. To compute the Subgame Perfect Nash Equilibrium, we use a procedure proposed in Sutton (1986): we derive the supremum and the infimum of the set of subgame perfect Nash equilibria and we compare them. The following proposition shows that if trade is not too inefficient then there is a unique SPNE in which trade occurs.

Proposition 4 Consider an infinite horizon game in which the following condition is satisfied:

$$2F - \pi^C - \frac{\pi^{NE} + F}{2} > 0$$

(1)

If the seller offers first, for $\delta$ large enough there is a unique Subgame Perfect Nash Equilibrium in which trade occurs and transfers are:

$$t_A(\delta) = \pi^C(1 - \delta) + \delta F(1 - \delta) + \delta^2 \left( \frac{\pi^{NE} + \delta F}{1 + \delta} \right)$$

$$t_B(\delta) = \frac{\pi^{NE} + \delta F}{1 + \delta}.$$

If buyers offer first the unique SPNE implies:

$$t_A(\delta) = t_B(\delta) = \frac{F + \delta \pi^{NE}}{1 + \delta}.$$
Proof. See Appendix. □

To discuss our result we recall the one-stage deviation principle for infinite horizon games (Fudenberg and Tirole, 1998): \textit{in an infinite-horizon multi stage game with observed actions that is continuous at infinity\footnote{This condition is satisfied if overall payoffs are discounted sum of per period payoffs and per period payoffs are bounded.}, profile $s$ is subgame perfect if and only if there is no player $i$ and strategy $\widehat{s}_i$ that agrees with $s_i$ except at a single date $t$ and history $h^t$, and such that $\widehat{s}_i$ is a better response to $s_{-i}$ than $s_i$ conditional on history $h^t$ being reached.}

Using this definition we can easily see that the transfers described in Proposition 4 satisfy the one-deviation principle. Consider for example the case in which $M$ is the player offering in period zero. If $A$ rejects the offer he can obtain $\pi^C$ for one period (given coordination between buyers) but he cannot obtain more than $(F + \delta \pi^{NE})/(1 + \delta)$ as a transfer in future periods and therefore he has no incentive to deviate (the precise strategies supporting these payoffs are described in the Appendix).

It is important to notice that as the discount factor gets large, equilibrium transfers do not tend to the coordination failure transfers any more. At the limit they now tend to $(F + \pi^{NE})/2$ that is the average between the coordination failure transfer of the offer game and $F$. More specifically transfers now tend to the average between transfers of the dominated equilibria of the two one-shot games.

Surprisingly also in the infinite horizon game it is cheaper for the seller to trade with both buyers. Indeed at the limit the total transfer that $M$ has to pay is given by $F + \pi^{NE}$ that is definitely less than the total transfer that has to be paid in the one-shot game: $\pi^C + \pi^{NE}$. Therefore the counter-intuitive conclusion remains: offering bargaining power the seller is better off.

If (1) does not hold we do not have a unique Subgame Perfect Nash Equilibrium any more. There are now multiple equilibria but we can nevertheless characterize the supremum and the infimum of their set. In particular, as next proposition highlights, the supremum of the SPNE payoffs is $(\pi^C, \pi^C)$.

**Proposition 5** Consider an infinite horizon game in which the following condition is satisfied:

\[
2F - \pi^C - \frac{\pi^{NE} + F}{2} < 0.
\]

Then, independently of the order of moves, for $\delta$ large enough, there is a multiplicity of equilibria. Some of them are efficient.

Proof. See Appendix. □

Our analysis has shown how conditions for efficiency differ in the finite horizon bargaining game respect to the infinite horizon model. In the finite horizon case
the identity of the last mover is fundamental. If buyers make the last proposal the equilibrium is efficient, if the seller makes the last offer then trade is going to occur. In the infinite horizon model the results are quite different. In this case, what drives the result is the degree of inefficiency of trade. If condition (1) is satisfied the unique SPNE involves trade. However condition (1) places a upper bound to the level of inefficiency. Not all the projects implementable with a divide and conquer strategy in the one shot game can be implemented in the infinite horizon bargaining game.

To see this more clearly we can rewrite condition (A2) as

\[ F > F_{\text{NE}} + \frac{\pi^C}{2} \]

and condition (1) as

\[ F > F_{\infty} > \frac{\pi^N + 2\pi^C}{3}. \]

Since \( F_{\infty} > F \) we can see that there can be values for \( F \) such that inefficient trade is the only equilibrium in a finite bargaining game whereas it is not in the infinite horizon case. More precisely let \( I_S = 2\pi^C - 2F \) be the maximum level of inefficiency that may arise in the model presented by Segal (2003) and let \( I_{\infty} = 2\pi^C - 2F_{\infty} \) be the maximal inefficiency that may arise in the infinite horizon version of our game. It is easy to see that \( I_S > I_{\infty} \). More precisely \( I_{\infty} = 0.75I_S \). We can now obtain some new insights on the efficiency debate. Clearly the Chicago School argument does not work in our model and inefficient trade can be sustained in equilibrium. Nevertheless the predictions of Segal (2003) appear too pessimistic as well. The maximum level of inefficiency that his model predicts appears here quite extreme because associated to absence of contractual power for buyers. As we remove this assumption and we consider an environment in which the contractual power is more evenly distributed the level of inefficiency that can arise in equilibrium is reduced.

### 4.3 Cooperative Bargaining

As noted above, the results obtained for the infinite horizon bargaining problem can be considered as a less extreme distribution of contractual power across players. In this Section we want to show formally that when the discount factor tends to one the bargaining power becomes equally split between the seller and the two buyers. More precisely we support this intuition using a cooperative game approach.

Let us start defining a 3-player weighted Nash bargaining problem\(^7\) as a triple \( \{X, d, w\} \) where \( X \) represents the total surplus to share, \( d \) is a vector describing the disagreement payoffs and \( w \) is the vector denoting the bargaining power of the three players with \( \sum_{i=1}^{3} w_i = 1 \).

---

\(^7\)For an overview and axiomatization of the \( n \)-players Nash bargaining solution see de Clippel (2004).
As the following proposition shows the cooperative problem in which weights are given by $1/2$ for the seller and $1/4$ for each buyer, disagreement payoffs are given by $d = (0, \pi^{NE}, \pi^{NE})$ and the surplus to share is $2F$, displays exactly the same outcome as the non-cooperative game of Proposition 4.

**Proposition 6** Consider an infinite horizon bargaining game in which the following condition is satisfied: $2F - \pi^{C} - \frac{\pi^{NE} + F}{2} > 0$. Then as $\delta$ tends to 1 the unique subgame perfect Nash equilibrium tends to the weighted Nash bargaining solution of

$$
\left\{ 2F; (0, \pi^{NE}, \pi^{NE}); \left( \frac{1}{2}, \frac{1}{4}, \frac{1}{4} \right) \right\}.
$$

**Proof.** Kalai (1977) shows that the weighted Nash bargaining solution solves

$$
\max_{t_A, t_B} (2F - t_A - t_B)^\frac{1}{2} \left( t_A - \pi^{NE} \right)^\frac{1}{2} \left( t_B - \pi^{NE} \right)^\frac{1}{2}.
$$

Differentiating and solving yields

$$
t_A = t_B = \frac{F + \pi^{NE}}{2}.
$$

What this proposition tells us is that it is quite legitimate to consider the infinite horizon version of our bargaining model to describe equilibria of an environment where the contractual power is equally spread not only between the seller and the two buyers but also between buyers.

## 5 Discussion

In this Section we are going to examine some features of the results obtained in Section 4. In particular we are going to analyze the advantage that the seller has when he offers for second. Moreover we are going to discuss and to relax two assumptions of the model: constant returns to scale and the lexicographic preferences for the seller.

### 5.1 The Second Mover Advantage

From the two stage bargaining game we have observed how, for values of $\delta$ close to 1, M prefers offering second rather than offering first. In some sense this result is similar to what we observe in Rubinstein’s bargaining model if we restrict time horizon to two periods. In that framework the second mover obtains the discounted value of the entire surplus and the payoff of the first mover tends to zero as the discount factor
gets close to one. At the limit (\(\delta = 1\)) the second mover obtains exactly the same payoff he would obtain if he had the entire bargaining power and he did a take it or leave it offer to the other player.

In our two period model a different result arises. By being second mover the seller obtains a payoff greater than the one he would obtain by having the entire contractual power. In particular, the bargaining game breaks coordination between the two buyers. The second mover advantage is therefore much stronger in our game than in Rubinstein’s one. More specifically, if we give the seller the option to choose between a take it or leave it offer and a two period bargaining game in which he is the second mover he will choose the second option. This result is quite counter-intuitive: 

**offering bargaining power M is better off.**

As next proposition shows, also in the infinite horizon game the seller prefers being second mover for \(\delta\) large enough.

**Proposition 7**  There exists a \(\bar{\delta}\) such that for \(\tilde{\delta} < \delta < 1\) if the bargaining game has infinite horizon and trade occurs, the seller prefers being second mover rather than being first mover.

**Proof.** The payoff of \(M\) being first mover is:

\[
\pi^{1st} = 2F - \pi^C(1 - \delta) - \pi^{NE}(1 - \delta) - 2\delta \frac{F + \delta \pi^{NE}}{1 + \delta}
\]

being second is

\[
\pi^{2nd} = 2F - \frac{F + \delta \pi^{NE}}{1 + \delta}.
\]

It is easy to see that

\[
\pi^{2nd} > \pi^{1st} \iff \delta > \tilde{\delta} = \frac{2F - \pi^C - \pi^{NE}}{\pi^C - \pi^{NE}}.
\]

Notice that condition (A2) implies \(0 \leq \tilde{\delta} < 1\).}

This result is in striking difference with Rubinstein’s conclusion. In his infinite horizon model each player prefers being first mover for any \(\delta<1\). In our game there exists a range of values for \(\delta\) in which \(M\) is strictly better off offering second. This happens because competition between buyers occurs in periods in which they offer. This implies that if \(M\) is second mover he can exploit competition from the beginning. Anticipating the divide and conquer strategy of the seller in next period, buyers undercut each other bids granting \(M\) a high payoff for period 1. On the other hand, if the seller is first mover buyers can still coordinate to remain uncontracted in the first period. In this case the seller has to divide and conquer them to extract some surplus in the first period, but this is costly for him.

Therefore being second mover grants \(M\) a higher payoff in the first period 1. Clearly as \(\delta\) tends to 1 the weight given to the first period payoff becomes negligible and at the limit this advantage disappears.
5.2 Returns to Scale

In Section 4 we have focused our analysis on the special case in which the bilateral surplus generated by new equipment displayed a kind of constant returns to scale (CRS), i.e. the surplus generated from trading with two buyers was exactly double of the one obtained dealing with only one of them. Indeed we imposed $F(2) = 2F(1)$. Let us now consider what happens if we remove this assumption.

As before we want to focus on the case in which the seller is inefficient and therefore we want to keep the following assumption satisfied:

$$\pi^c > F(2) - \pi^c > \pi^{NE}.$$  \hspace{1cm} (A1')

We start now examining the case of decreasing returns to scale (DRS), namely $F(2) < 2F(1)$. Clearly as long as (A1') is satisfied, the outcome of the one shot offer game does not change. Nevertheless, the solution of the bidding game is slightly different. As in the CRS case the game has two Nash Equilibria. In one of them both buyers ask for a high transfer and remain uncontracted in the other they both ask for $F(2) - F(1)$. Clearly the first equilibrium is selected using a coalition proof refinement. If we consider the two period bargaining game in which the seller offers second the results are very similar to those of the CRS. More precisely, the equilibrium transfers are now:

$$t_A = t_B = (F(2) - F(1))(1 - \delta) + \delta \pi^{NE}.$$  \hspace{1cm} (3)

Therefore the only difference with CRS is that now the buyers appropriate of smaller proportion of the value of the surplus (because $2[F(2) - F(1)] < F(2)$) and they leave a higher rent to the seller. It is therefore clear that once we have substituted $F$ with $F(2) - F(1)$, all the results obtained in the previous section for finite and infinite horizon can be extended to DRS.

A similar analysis can be done for the increasing returns to scale (IRS). Also in this case the outcome of the offer game does not change. The outcome of the one period bidding game is different. In this case the only Nash Equilibrium is the efficient one. Nevertheless it can be shown that the two stage bargaining game, for $\delta$ large enough, has a unique equilibrium whose transfers correspond to (3). Also in this case we can extend all the results of Section 4 to the case of IRS as well just substituting $F$ with $F(2) - F(1)$ in the expression of the transfers. It is easy to see that in this case, for values of $\delta$ close to one, buyers appropriate a larger proportion of surplus than in the CRS framework.

5.3 Discrimination between Buyers

Our previous analysis has focused on the special case in which the seller, if he was to divide and conquer, offered the higher transfer to A and if he was to accept only one
of two equal transfers, chose A as well. The model can be easily generalized to the case in which A is preferred to B only with probability $p$.

In the game with only two periods (and M offering in the second one) the expected payoff of A if both buyers remain uncontracted in period 1 is given by $(1 - \delta)\pi^C + \delta [\pi^{NE}(1 - p) + p\pi^C]$ and conversely the expected payoff for B is given by $\pi^C(1 - \delta) + \delta [\pi^{NE}p + (1 - p)\pi^C]$. It is easy to see that the two expected payoff coincide in the case in which $p = 1/2$ and the expected payoff of A is greater if $p > 0.5$. To this difference in expected payoff it corresponds a difference in the threshold value for the discount factor triggering a deviation. In particular the threshold discount factors for A and B are given by:

$$
\delta_A = \frac{\pi^C - F}{\pi^C - F + \pi^C - [\pi^Cp + \pi^{NE}(1 - p)]},
$$

$$
\delta_B = \frac{\pi^C - F}{\pi^C - F + \pi^C - [\pi^{NE}p + \pi^C(1 - p)]}.
$$

Trivially the two thresholds coincide only if $p = 0.5$. If $p > 0.5$ then the threshold for B is strictly lower than the one for A and at the limit ($p = 1$) the threshold is the lowest as possible and only B has an incentive to deviate. It is easy to see that if the discount factor has a value for which one of the two buyers has an incentive to deviate, then the competition among buyers implies that the transfer offered tends to $F(1 - \delta) + \delta\pi^{NE}$ as in the case discussed in Section 3. Therefore the uncertainty on the identity of the preferred buyer is not affecting the equilibrium transfers for $\delta$ large enough. The only impact of the uncertainty is on the threshold discount factor triggering deviation that gets greater as $p$ tends to 0.5.

As we can see from the previous example, if $\delta$ is close to one the equilibrium payoffs that the two buyers obtain in the first period are independent from the randomization of the seller in the second period. Therefore adding two periods to the game the only randomization that the single buyer considers is the one in period two because the payoffs in period 3 are independent from the randomization of period 4. But for the same argument developed above for large values of $\delta$ even this randomization does not matter. We can conclude that for $\delta$ close to one our results for bargaining games with finite horizon are robust to randomization between buyers by the seller in his divide and conquer strategy.

Moreover, considering an infinite horizon framework the results are robust as well. It is sufficient to reformulate the proof of Proposition 4 and Proposition 5 assuming the existence of two possible values for the suprema of the continuation payoffs and considering A obtaining the largest with probability $p$. 

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6 Conclusions

In this paper we have identified a particular class of games that solved using the offer game approach (Segal (2001)) have an inefficient outcome whereas solved using the bidding game approach (Martimort and Stole (2003)) display efficiency. Using this framework, we have combined these two approaches in a model of noncooperative bargaining a la Rubinstein. We have shown that the outcome of the bargaining procedure differs remarkably from those of the one shot games for values of the discount factor close to one. In particular the principal can induce coordination failure among agents and increase his payoff offering contractual power. Moreover, extending the bargaining game to an infinite horizon we find that if the equilibrium of the offer game is not too inefficient then the infinite horizon extension has a unique Subgame Perfect Nash Equilibrium that displays a second mover advantage.

It is interesting to compare our model with the one proposed by Genicot and Ray (2003). In their model they study how a principal approaches contract provision to agents when coordination failure among the latter group is explicitly ruled out. Two variants are studied. In both of them the principal has the entire bargaining power and can choose either to contract simultaneously or sequentially. In the first variant the principal cannot re-approach agents if they reject the contract. In this case there is a unique equilibrium in which simultaneous and relatively attractive offers are made to a number of agents in a first phase and offers are made sequentially to the remaining agents and they are close to the very lowest of all the outside option in a second phase. Applying this model to our game this outcome corresponds exactly to the divide and conquer strategy that the manufacturer uses in the one shot game. In the second version of their model the principal can repeatedly approach the same agent and there is a multiplicity of equilibria. In some of these agents have the power to force delay. In the worst equilibrium for the agents their payoffs tend to the one of coordination failure as the discount factor tends to one. This result is quite different from ours. In their paper coordination failure is achieved as a (not unique) subgame perfect Nash equilibrium increasing the bargaining power of the principal compared the one shot game. The principal can not only choose between contracting simultaneously or sequentially but also can re-approach each agent as many times he likes. Conversely, in our model coordination failure is achieved reducing the bargaining power of the principal (the seller) by allowing agents (the buyers) to bid.

7 Appendix

Proof of Proposition 1

We solve the game using backward induction. At T=2 if both buyers are not contracted the seller offers \( t = (\pi^C, \pi^{NE}) \) and trades with both. If only one is free he offers him \( \pi^{NE} \) and the buyer accepts.
First of all we show that it does not exist an equilibrium in which a buyer trades at $T=1$ and the other at $T=2$. Suppose there exists a pair of transfers $(t_A, t_B)$ for which the only buyer served at $T=1$ is buyer $B$. The profit for $M$ accepting the offer of $B$ and rejecting the one of $A$ has to be larger than the one he gets rejecting both of them. More formally:

$$(F - t_B)(1 - \delta) + \delta \left(2F - t_B - \pi^{NE}\right) \geq \delta(2F - \pi^{NE} - \pi^C)$$

or

$$t_B \leq F(1 - \delta) + \delta \pi^C. \quad (4)$$

Moreover rejecting $A$’s offer has to be better than accepting his offer:

$$(F - t_B)(1 - \delta) + \delta \left(2F - t_B - \pi^{NE}\right) \geq 2F - t_A - t_B$$

or

$$t_A \geq F(1 - \delta) + \delta \pi^{NE}. \quad (5)$$

To have $M$ choosing $B$ instead of $A$ we need:

$$t_A \geq t_B. \quad (6)$$

In addition, we need to show that no buyer has an incentive to deviate from these transfers. It is easy to see that the payoff of $A$ in this equilibrium is simply $\pi^{NE}$. This implies that for any $t_B > F(1 - \delta) + \delta \pi^{NE}$ $A$ has an incentive to slightly undercut $B$’s offer to trade first. So the only possible candidate is $t_B = F(1 - \delta) + \delta \pi^{NE}$ and $t_A > F(1 - \delta) + \delta \pi^{NE}$. But now $A$ can deviate offering $t_A = F(1 - \delta) + \delta \pi^{NE}$ that is going to be accepted by $M$ and it guarantees to $A$ a larger payoff.

Using the same argument we can show that it does not exist a pair of transfers for which the only agent served at $T=1$ is $A$.

We now show that for any value of $\delta$ there exists an equilibrium where both buyers are served in $T=1$. A first condition that has to be satisfied is that to accept both transfers has to be better than to reject all of them, more specifically:

$$2F - t_A - t_B \geq \delta \left(2F - \pi^{NE} - \pi^C\right)$$

or

$$t_A + t_B \leq 2F(1 - \delta) + \delta \left(\pi^{NE} + \pi^C\right). \quad (7)$$

It is also necessary that $M$ cannot deviate serving one buyer only and obtaining a larger payoff:

$$2F - t_A - t_B \geq (F - t_A)(1 - \delta) + \delta \left(2F - t_A - \pi^{NE}\right)$$
and

$$2F - t_A - t_B \geq (F - t_B)(1 - \delta) + \delta (2F - t_B - \pi^{NE})$$

which imply

$$t_A \leq F(1 - \delta) + \delta \pi^{NE} \tag{8}$$

and

$$t_B \leq F(1 - \delta) + \delta \pi^{NE}. \tag{9}$$

Notice that (8) and (9) $\Rightarrow$ (7) therefore the natural candidate Nash Equilibrium is $t_A = t_B = t^* = F(1 - \delta) + \delta \pi^{NE}$. It is easy to see that no deviation is profitable for the buyers. Asking a $t < t^*$ implies a payoff of $\tilde{t}$ and this is not optimal. Asking a $t > t^*$ implies a payoff of $\pi^{NE}$ which is less than $t^*$. Notice that the equilibrium payoffs of the two buyers tend to $\pi^{NE}$ as $\delta$ goes to 1.

Let us show that there exists an equilibrium in which both buyers trade in $T=2$ only if $\delta \leq \tilde{\delta}$. The seller prefers not to trade in $T=1$ rather than trading with both, this implies

$$2F - t_A - t_B \leq \delta (2F - \pi^C - \pi^{NE})$$

or

$$t_A + t_B \geq 2F(1 - \delta) + \delta (\pi^{NE} + \pi^C). \tag{10}$$

Moreover the seller must not prefer to trade with only one buyer:

$$\delta (2F - \pi^C - \pi^{NE}) \geq (F - t_A)(1 - \delta) + \delta (2F - t_A - \pi^{NE})$$

and

$$\delta (2F - \pi^C - \pi^{NE}) \geq (F - t_B)(1 - \delta) + \delta (2F - t_B - \pi^{NE})$$

or

$$t_A \geq F(1 - \delta) + \delta \pi^C \tag{11}$$

and

$$t_B \geq F(1 - \delta) + \delta \pi^C. \tag{12}$$

Notice that (11) and (12) $\Rightarrow$ (10) therefore a candidate Nash equilibrium is given by any $t^* > F(1 - \delta) + \delta \pi^C$. Let us see if there is a possible deviation for the buyers. If they ask for an higher transfer their payoffs do not change. If they ask for a lower transfer their payoffs change as long as the transfer is lower than $\tilde{t} = F(1 - \delta) + \delta \pi^C$.

For A is never optimal to ask $\tilde{t} \leq \pi^C$. For B asking $\tilde{t}$ is not optimal as long as:

$$\tilde{t} = F(1 - \delta) + \delta \pi^C \leq \pi^C (1 - \delta) + \delta \pi^{NE}$$
or

\[ \delta \leq \hat{\delta} = \frac{\pi^C - F}{\pi^C - F + \pi^C - \pi^{NE}}. \]

It is immediate to see that as long as both Nash Equilibria exist, the one in which there is delay to trade is coalition proof. This concludes the proof of the proposition.

**Proof of Proposition 2**

For any even number \( T \geq 2 \) we define:

\[ \ell^T(\delta) = \begin{cases} 
F(1 - \delta) + \delta \pi^{NE} & \text{if } T = 2 \\
(1 - \delta) \sum_{t=0}^{T-2} \delta^{2t} F + (1 - \delta) \sum_{t=0}^{T-2} \delta^{2t+1} \pi^{NE} + \delta^{T-1} \pi^{NE} & \text{if } T > 2.
\end{cases} \]

Notice that for a given \( T \), \( \ell^T(\delta) \) is continuous in \( \delta \) and converges to \( \pi^{NE} \) as \( \delta \to 1 \).

We want to show that for any game of length \( T > 2 \) in which \( M \) makes the last offer the equilibrium transfers are:

\[ \begin{align*}
\hat{t}_A^T &= \pi^C(1 - \delta) + \delta \ell^{T-1}(\delta) \\
\hat{t}_B^T &= \pi^{NE}(1 - \delta) + \delta \ell^{T-1}(\delta)
\end{align*} \]  

if \( T \) is odd and

\[ \begin{align*}
\hat{t}_A^T &= \hat{t}_B^T = \ell^T(\delta)
\end{align*} \]  

if \( T \) is even.

From Proposition 1 we know that if the game has only two periods and \( \delta \geq \hat{\delta} \) trade is going to occur with transfers:

\[ t_A = t_B = \ell^2(\delta) = F(1 - \delta) + \delta \pi^{NE}. \]

Using this result we show that the transfers in (13), (14) and (15) characterize the subgame perfect Nash Equilibrium of any bargaining game with length \( T = t + 2 \) for any \( t \in \{1, 2, 3, \ldots\} \). The proof proceeds by induction. Consider \( t = 1 \) so \( T = 3 \). \( M \) can trade with both buyers in period 3 implementing a divide and conquer strategy:

\[ \begin{align*}
\hat{t}_A^3 &= \pi^C(1 - \delta) + \delta \ell^2(\delta) \\
\hat{t}_B^3 &= \pi^{NE}(1 - \delta) + \delta \ell^2(\delta)
\end{align*} \]

and making each buyer indifferent between accepting and rejecting the offer. Moreover it is easy to see that \( M \) has no incentive to delay trade or to contract only one of the buyers.

We now assume that the equilibrium transfers are those specified for a general \( t \) and we show that (13), (14) and (15) still hold for \( t + 1 \). To assume that the property
is true for a general $t$ is equivalent to assume that it holds for a general $T > 2$. Therefore we need to show that it holds for games of length $T + 1$.

If $T$ is even $M$ can contract both buyers in $T + 1$ offering

$$
\hat{t}_{A}^{T+1} = \pi^{C}(1 - \delta) + \delta \pi_{-}^{T}(\delta)
$$

$$
\hat{t}_{B}^{T+1} = \pi^{NE}(1 - \delta) + \delta \pi_{-}^{T}(\delta)
$$

that make each buyer indifferent between accepting and rejecting the offer. Moreover $M$ has no incentive to delay trade or to contract only one of the buyers.

If $T$ is odd $B$ has an incentive to be contracted in $T + 1$ if

$$
F(1 - \delta) + \delta \hat{t}_{A}^{T} > \pi^{C}(1 - \delta) + \delta \hat{t}_{B}^{T}.
$$

Moreover, given that for every $T$ odd

$$
\hat{t}_{A}^{T} - \hat{t}_{B}^{T} = (\pi^{C} - \pi^{NE})(1 - \delta)
$$

we obtain the condition

$$
\delta > \hat{\delta} = \frac{\pi^{C} - F}{\pi^{C} - \pi^{NE}}.
$$

This deviation triggers Bertrand competition between buyers that induces transfers:

$$
\hat{t}_{A}^{T+1} = \hat{t}_{B}^{T+1} = F(1 - \delta) + \delta \pi_{-}^{T}(\delta) = \pi_{-}^{T+1}(\delta).
$$

Furthermore $M$ is going to accept the contract if:

$$
2F - 2\pi_{-}^{T+1}(\delta) > F(1 - \delta) + \delta(1 - \delta)\pi^{C} - \delta(1 - \delta)\pi^{NE} - 2\delta^{2}\pi_{-}^{T-1}(\delta)
$$

or

$$
\delta 2F - \delta(1 - \delta)\pi^{C} - \delta(1 - \delta)\pi^{NE} - 2\delta^{2}\pi_{-}^{T-1}(\delta)
$$

$$
< 2F - 2F(1 - \delta) - 2\delta \pi^{C}(1 - \delta) - 2\delta^{2}\pi_{-}^{T-1}(\delta)
$$

that is true if

$$
\delta(1 - \delta)\pi^{NE} < \delta(1 - \delta)\pi^{C}
$$

that is satisfied for any value of $\delta$.

We can therefore conclude that if $\delta \in [\hat{\delta}, 1]$ trade occurs immediately with transfers $(\hat{t}_{A})$ and $(\hat{t}_{B})$ tending to $\pi^{NE}$ as $\delta \to 1$. This proves Proposition 2.
Proof of Proposition 4.

We start our analysis giving some basic results. Suppose that \( M \) is offering in \( t = 0 \). In this situation the outside option payoff for a retailer not contracted, given that the other has accepted the contract at \( t = 0 \) corresponds to the following transfer:

\[
\tau(\delta) = \lim_{T \to \infty} \left[ (1 - \delta) \sum_{t=0}^{T-1} \delta^{2t+1} F + (1 - \delta) \sum_{t=0}^{T-1} \delta^{2t} \pi^{NE} \right] = \\
= \lim_{T \to \infty} \left[ (1 - \delta) F \sum_{t=0}^{T-1} \delta^{2t} + (1 - \delta) \pi^{NE} \sum_{t=0}^{T-1} \delta^{2t} \right] = \\
= (1 - \delta) F \lim_{T \to \infty} \sum_{t=0}^{T-1} \delta^{2t} + (1 - \delta) \pi^{NE} \lim_{T \to \infty} \sum_{t=0}^{T-1} \delta^{2t} = \\
= \frac{(1 - \delta) \left( \delta F + \pi^{NE} \right)}{1 - \delta^2} = \frac{\pi^{NE} + \delta F}{1 + \delta}.
\]

Moreover, as stated in the proposition, we are going to consider the case in which the following condition holds:

\[
2F - \pi^C - \frac{\pi^{NE} + F}{2} > 0. \quad (16)
\]

The proof of the following lemma is immediate and follows directly from Rubinstein (1982).

**Lemma A1** Consider a subgame played by \( M \) and one buyer starting in the period after an agreement between \( M \) and the other buyer has been reached. In this case the agreement is going to be reached immediately and the transfer is going to be \( \tau(\delta) \) if the seller offers and \( F(1 - \delta) + \delta \tau(\delta) \) if the buyer offers the contract.

It is indeed easy to see that, once one of the two buyers has reached an agreement, our game becomes identical to a Rubinstein (1982) bargaining model with outside option of \( \pi^{NE} \). This result allows us to narrow down the possible range of equilibrium payoff.

**CASE 1: M First Mover**

We will now characterize the outcome if \( M \) is the first mover. Indeed, we can now adopt an argument similar to the one used by Sutton (1986) and Jun (1989) and classify the equilibria according to when and who accepts the offer for the first time. There are five possible types of equilibria:
1. Both buyers accept in $t = 2n$ \( n = 0, 1, 2, \ldots \)
2. Only one buyer accepts in $t = 2n$ \( n = 0, 1, 2, \ldots \)
3. M accepts both offers in $t = 2n + 1$ \( n = 0, 1, 2, \ldots \)
4. M accepts only one offer in $t = 2n + 1$ \( n = 0, 1, 2, \ldots \)
5. No offer is ever accepted.

Note that equilibria of type 2 and 4 are fully specified because of Lemma A1. With the following lemma we show not only that there exist equilibria of type 1 but also that the payoff of such equilibria is unique.

**Lemma A2** For Equilibria of type 1 there is a unique possible SPNE payoff in which

\[
\begin{align*}
t_A &= \pi^C(1 - \delta) + \delta F(1 - \delta) + \delta^2 \left( \frac{\pi^{NE} + \delta F}{1 + \delta} \right) \quad (17) \\
\text{and } t_B &= \frac{\pi^{NE} + \delta F}{1 + \delta}. \quad (18)
\end{align*}
\]

**Proof.** Consider equilibria of type 1 and define the suprema payoffs that the buyers can achieve from the offer of the manufacturer as $\bar{v}_A$ and $\bar{v}_B$. Let us focus on the subgame starting at time 2. Since this subgame has the same structure as the original game, apart from a rescaling of payoffs, the set of suprema has to be the same. Suppose without loss of generality that $\bar{v}_A \geq \bar{v}_B$. Let us consider first the case of equality: $\bar{v}_A = \bar{v}_B = \bar{v}$. For $\delta$ large enough we have that both buyers in $t = 1$ are going to ask a large transfer obtaining a payoff of

\[
\pi^C(1 - \delta) + \delta \bar{v}
\]

and this implies that the greater payoff that A can obtain in $t = 0$ is going to be:

\[
\bar{v} = (1 - \delta)\pi^C + \delta \pi^C(1 - \delta) + \delta^2 \bar{v}
\]

which implies $\bar{v} = \pi^C$. But then if M offers $\pi^C$ to A he is going to accept and this allows M to contract immediately B offering $r(\delta)$ to him and contradicting the assumed equality [and he can do this for values of $\delta$ large enough because of (16) !]. Suppose now that $\bar{v}_A > \bar{v}_B$. Notice that this can happen if and only if both buyers are contracted at some point in time. It is easy to see that there is an incentive for B to deviate in $t = 2$ if $F(1 - \delta) + \delta \bar{v}_A > (1 - \delta)\pi^C + \delta \bar{v}_B$ that is satisfied for $\delta$ large enough and implies that the supremum payoffs in $t = 0$ are:

\[
\begin{align*}
\bar{v}_A &= (1 - \delta)\pi^C + \delta F(1 - \delta) + \delta^2 \bar{v}_B \\
\bar{v}_B &= (1 - \delta)\pi^{NE} + \delta F(1 - \delta) + \delta^2 \bar{v}_B.
\end{align*}
\]
Solving we obtain:

\[
\bar{v}_A = (1 - \delta)\pi^C + \delta F(1 - \delta) + \delta^2 \frac{\pi^{NE} + \delta F}{1 + \delta}
\]

\[
\bar{v}_B = \frac{\pi^{NE} + \delta F}{1 + \delta}.
\]

It is possible to repeat the same exercise for the infimum rather than the supremum obtaining the same result. Therefore there exists a unique SPNE payoff and it corresponds to trading with the transfers specified in (17) and (18). Moreover there is no loss of generality in considering time 2. We can start from any even period greater than 2 and obtain the same result. ■

Having shown that under condition (16) there exists a unique SPNE of type one\(^8\) we now show that equilibria of type 2 and equilibria of type 4 cannot exist.

**Lemma A3** There cannot be an Equilibrium of type 2.

**Proof.** To have such an equilibrium we need that:

\[
(F - t_A)(1 - \delta) + \delta[F - F(1 - \delta) - \delta r(\delta)] \geq 2F - t_A - r(\delta)
\]

\[
F(1 - \delta) + \delta[F - F + \delta(F - r(\delta))] \geq 2F - r(\delta)
\]

\[
\delta^2[F - r(\delta)] \geq F - r(\delta)
\]

that is a contradiction. ■

**Lemma A4** There cannot be Equilibria of type 4.

**Proof.** The conditions to have such an equilibrium are:

\[
(F - t_A)(1 - \delta) + \delta(2F - t_A - r(\delta)) \geq 2F - t_A - t_B
\]

\[t_B \geq F(1 - \delta) + \delta r(\delta)\]

and \(t_B > t_A\). In this case the payoff of B is going to be \(\pi^{NE}(1 - \delta) + \delta r(\delta)\) but, because of the same argument used in the proof of Lemma A2, he can profitably deviate asking for \(F(1 - \delta) + \delta r(\delta)\) and being contracted. ■

Let us now turn to equilibria of type 3. The following lemma shows, using a procedure similar to the one used to study equilibria of type 1, that if both buyers are not contracted, they will reach an agreement offering to M the same transfer.

**Lemma A5** If both retailers are free and \(t\) is odd an agreement is reached with \(t_A = t_B = \frac{F + \delta \pi^{NE}}{1 + \delta}\).

---

\(^8\)We specify the corresponding strategies at the end of this Section.
**Proof.** Suppose there exist equilibria in period $t+2$ in which the offers of both buyers are accepted and their supremum payoffs are $\bar{v}_A$ and $\bar{v}_B$. The maximum payoffs they can achieve in period $t+1$ are $(1-\delta)\pi^C + \delta \bar{v}_A$ and $(1-\delta)\pi^C + \delta \bar{v}_B$ and they are obtained rejecting any offer of the seller. Therefore the corresponding suprema of period $t$ are $\bar{v}_A = \bar{v}_B = \pi^C$. But if these are the suprema then in period $t+1$ the seller can profitably deviate offering $\pi^C$ to $A$ and $r(\delta)$ to $B$ and these transfers are going to be accepted. Therefore the two suprema to consider in period $t+1$ are $\pi^C$ and $r(\delta)$. This discrimination is source of competition in period $t$ and implies equilibrium transfers $t_A = t_B = F(1-\delta) + \delta r(\delta)$ that can be rewritten as these specified in the lemma. ■

Given that it is possible to reach an agreement both if M offers first and if buyers make an offer we will now show that if M is first mover he is not going to wait one period.

**Lemma A6** If M is the first to offer he prefers to contract immediately rather than to postpone to next period.

**Proof.** We need to compare the payoffs of M contracting and waiting:

$$\delta^t \left[ 2F - \pi^C(1-\delta) - \pi^{NE}(1-\delta) - 2\delta \frac{F + \delta \pi^{NE}}{1+\delta} \right] > \delta^{t+1} \left[ 2F - 2 \frac{F + \delta \pi^{NE}}{1+\delta} \right]$$

$$2F - \pi^C - \pi^{NE} > 0$$

that is true by assumption. ■

Notice that this lemma implies that M is going to contract the two buyers immediately. Therefore nonexistence of equilibria of type 5 can be seen as a corollary of this lemma. We specify now strategies that implement this equilibrium.

**Seller’s Strategy.**

Offers:

- offer (17) and (18) when $t = 2n, n = 0, 1, 2$...and both buyers are not contracted
- offer (18) if only one buyer is not contracted.

Acceptance:

- accept both offers if both $t_A$ and $t_B \leq \frac{F + \delta \pi^{NE}}{1+\delta}$;
- accept $\min \{t_A, t_B\}$ if $t_A$ or $t_B \geq \frac{F + \delta \pi^{NE}}{1+\delta}$,

and $\min \{t_A, t_B\} \leq \tilde{t} = F(1-\delta) + \delta \left[ \pi^C(1-\delta) + \delta F(1-\delta) + \delta^2 \left( \frac{\pi^{NE} + \delta F}{1+\delta} \right) \right]$, accept offer of A if $t_A = t_B$;
reject both offers if \( \min \{ t_A, t_B \} > \tilde{t} \);

- if there is only one buyer uncontracted accept iff \( t_i \leq \frac{F + \delta \pi^{NE}}{1 + \delta} \) with \( i \in \{ A, B \} \).

**Buyers' Strategies**

- A accepts if both buyers are free and \( t_A \geq \pi^C(1-\delta)+\delta F(1-\delta)+\delta^2 \left( \frac{\pi^{NE} + \delta F}{1 + \delta} \right) \)

- B accepts if offered \( t_B \geq \frac{\pi^{NE} + \delta F}{1 + \delta} \) and A does not reject. Rejects otherwise.

- If they are both free they offer

\[
 t_A = \begin{cases} 
 \pi^C & \text{if } t_B > \tilde{t} \\
 t_B & \frac{F + \delta \pi^{NE}}{1 + \delta} < t_B \leq \tilde{t} \\
 \frac{F + \delta \pi^{NE}}{1 + \delta} & \text{if } \frac{F + \delta \pi^{NE}}{1 + \delta} \leq t_B 
\end{cases}
\]

\[
 t_B = \begin{cases} 
 \tilde{t} & \text{if } t_A > u_0 \\
 t_A - \varepsilon & \frac{F + \delta \pi^{NE}}{1 + \delta} < t_A \leq \tilde{t} \\
 \frac{F + \delta \pi^{NE}}{1 + \delta} & \frac{F + \delta \pi^{NE}}{1 + \delta} \leq t_A 
\end{cases}
\]

- If only one buyer is not contracted he is going to accept (18) and offer \( \frac{F + \delta \pi^{NE}}{1 + \delta} \).

**CASE 2: Buyers first movers**

The analysis of the case in which both buyers move first is very similar to the previous case. Also in this case we have 5 possible equilibrium outcomes:

1. Both buyers accept in \( t = 2n + 1 \) \( n = 0, 1, 2, \ldots \)
2. Only one buyer accepts in \( t = 2n + 1 \) \( n = 0, 1, 2, \ldots \)
3. M accepts both offers in \( t = 2n \) \( n = 0, 1, 2, \ldots \)
4. M accepts only one offer in \( t = 2n \) \( n = 0, 1, 2, \ldots \)
5. No offer is ever accepted.

Using the results of case 1 we know that equilibria 2, 4 and 5 cannot exist. What we need to show is that M prefers to contract both retailers immediately rather than to wait for one period, i.e. equilibria of type 3 are preferred to those of type 1. This is proved in the following lemma.

**Lemma A7** The seller prefers contracting both buyers in $t = 0$ rather than waiting one period.

**Proof.** We need to show that

$$
\delta^{t+1} \left[ 2F - \pi^C(1 - \delta) - \pi^{NE}(1 - \delta) - 2\delta \frac{F + \delta \pi^{NE}}{1 + \delta} \right] < \delta^t \left[ 2F - 2 \frac{F + \delta \pi^{NE}}{1 + \delta} \right]
$$

that rewrites as

$$
2F(1 - \delta) - \delta \pi^C(1 - \delta) - \delta \pi^{NE}(1 - \delta) < (1 - \delta^2) 2 \frac{F + \delta \pi^{NE}}{1 + \delta}
$$

$$
2F - \delta \pi^C - \delta \pi^{NE} < 2 \left[ F + \delta \pi^{NE} \right]
$$

Therefore, if condition (16) holds, there will be immediate agreement on the transfers specified in the proposition. The strategies supporting this equilibrium are exactly the same as those described in case 1.

**Proof of Proposition 5**

Let us use the results obtained in the proof of proposition 4. We have shown that there are 5 possible candidate equilibria if M offers first. Let us analyze an equilibrium of type 1 and define the supremum that the buyers can achieve from the offer of M as $\overline{v}_A$ and $\overline{v}_B$.

We focus on the subgame starting at time 2. Since this subgame has the same structure as the original game, apart from a rescaling of payoffs, the set of suprema has to be the same. Suppose without loss of generality that $\overline{v}_A \geq \overline{v}_B$. Let us consider first the case of equality: $\overline{v}_A = \overline{v}_B = \overline{v}$. For $\delta$ large enough we have that both buyers in $t = 1$ are going to ask for

$$
\pi^C(1 - \delta) + \delta \overline{v}
$$

and this implies that the equilibrium offer in $t = 0$ for A is going to be:

$$
\overline{v} = (1 - \delta) \pi^C + \delta \pi^C(1 - \delta) + \delta^2 \overline{v}
$$
which implies $\nu = \pi^C$. Give that condition (16) does not hold the seller cannot contract both A offering $\pi^C$ and B offering $\nu(\delta)$. Therefore both A and B can guarantee a payoff of $\pi^C$. This is indeed the the supremum value of the SPNE payoffs.

If we consider the infimum, just assuming $t_A \neq t_B$, we will obtain that the corresponding payoff is exactly the one obtained in (17) and (18).

References


