Hierarchic contracting

Miguel A. Sanchez & Rafael Hortala-Vallve*

London School of Economics and Political Science

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The Toyota Centre
Suntory and Toyota International Centres for Economics
and Related Disciplines
London School of Economics
Houghton Street
London WC2A 2A

Telephone: (+44 020) 7955 6674

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Abstract

We analyze the contracting structure in a moral hazard setting with several agents where output is produced jointly and is the only contractible variable. Since the salary of each agent is a function of all agents efforts, a positive externality arises between them. This externality is not internalised by a centralised structure where the principal contracts directly with each agent. Instead, we find that a hierarchic structure (i.e. the delegation of "contracting rights" from the principal to the agents) internalises the externality by making agents "residual claimants". Consequently, the second best situation can be improved upon just by changing the contracting structure of the principal-agents relationship. The analysis is relevant to the literature on decentralisation, outsourcing, subcontracting and intra-firm organization.

JEL Classification: D80, J30, L22, M12

Keywords: Principal-multi-agent relationships, moral hazard, team production, decentralisation, hierarchies, contract design

Contents

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Telephone: UK+20 7955 6674
Fax: UK+20 7955 6951
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1 Introduction

1.1 Motivation

Probably one of the most important issues regarding every organization is how to structure its operation. The solution to this question encompasses so many factors that a thorough answer is almost impossible. In the present paper we focus on just one aspect of it, namely, the contracting structure.

Certainly the allocation of contracting rights is important both in the public sector (e.g. decentralization of public services) and private sectors, within and between firms (e.g. team production, producing within the firm or outsourcing, franchising). Our goal is to highlight how the contracting structure affects the efficiency of the organizations.

Our model consists of a principal who hires a group of agents in order to jointly produce an output in a situation where this output is the only contractible variable. The combination of these two elements generates the essential ingredient in our analysis: a positive externality between agents.

Since output is jointly produced and wages are contingent on it, the effort of any agent increases not only her own wage but also every other agent's wages. However, each player's decision takes into account only her *private* benefits (wage) and costs (risk plus disutility of effort), and these are, due to the positive externality, strictly lower than the *social* benefits. As a consequence, each agent exerts too little effort compared to the efficient situation.

The model departs from Holmstrom's *team production* model (Holmstrom (1982)) due to our assumption that output is not deterministic, given the effort exerted by each member of the team. Hence the principal cannot offer a *budget-breaking contract* to achieve first best. Instead, the principal faces the classical trade-off between incentives and insurance. When the risk is low, the principal is able to provide close-to-first-best incentives to the agents (i.e., incentives such that private benefits are close to the social ones) but as the riskiness of the project increases, the principal can no longer provide such incentives to the agents (it is too costly) and hence the misalignment of incentives becomes larger. Consequently, whatever way the principal has to internalize such externality, it becomes relatively more efficient the riskier the project is.

Given this situation, we find that a decentralized or hierarchic structure (as opposed to a centralized one where the principal retains for herself the contracting rights and contracts directly with each agent) mitigates the inefficiency generated by the randomness of the project by internalizing the positive externality each agent has on the team. This happens because the agent who is assigned contracting rights becomes residual claimant (whatever is not paid to her subordinates accrues to herself) and thus she realizes the positive externality that her effort has on them. That is, higher effort increases the expected output and provides extra insurance for her employees. This way she internalizes the effect of her effort choice on the other agents' participation constraints: more of her own effort not only increases her own wage, but also the other agents' wages.

1.2 Related Literature

Without trying to provide an exhaustive review of the extensive literature on team production and asymmetric information, we survey the most directly related works and we refer to the bibliographies therein for further information.

A large branch of the literature on hierarchies - for example Melumad, Mookherjee and Reichelstein (1995), among others - concentrates on the effects of hidden information (i.e. adverse selection). Their driving force is that agents are heterogeneous, have different productivities or marginal costs, and the principal needs to design contracts taking into account that agents will use their private information for their own benefit. The issue of decentralization of the contracting structure is mostly irrelevant in that context given the revelation principle. Hence the attention focuses on how to assign the different tasks according to the information that agents are willing to provide (e.g. Prendergast (1995)) and on analysing the effects of monitoring (e.g. Baliga and Sjolstrom (1998), Faure-Grimaud *et al* (2003)) or project choice (Choe and Park (2003)).

The seminal reference on moral hazard in teams is Holmstrom (1982). As we said above, our analysis departs from it because of the assumption that the technology is stochastic instead of deterministic. Most related to our work is Itoh (1991), though in his model different agents produce different non-deterministic outputs and the issue is whether the principal is better off by providing incentives for them to cooperate.[1] Note that the fact that in our setting output is jointly produced precludes "relative performance" incentive schemes, the implementation of tournaments (as in Lazear and Rosen (1981)) or collusion among agents.

The closest references to our study are those of Macho-Stadler and Perez-Castrillo (1998) and Jelovac and Macho-Stadler (2002). Their analyses consist of comparing different contracting structures in a binary effort model but their study is based on the different timing of events rather than the effect of the externality between agents. Moreover their results diverge from ours. They find that the hierarchic structure does not always perform better than the centralized one. A detailed comparison of our results to theirs is provided in Section 4.

The rest of the article is organized as follows: In Section 2 we present the model and we solve it under the different contracting structures: centralized and decentralized. In Section 3 we compare both structures and discuss the higher performance of the hierarchic one. Section 4 considers the robustness of the results to alternative specifications and explores some extensions. Finally, Section 5 concludes.

2 The Model

The model presented below is based on Holmstrom and Milgrom (1987).

[1] Alternatively, Itoh (1994) assumes joint production but considers whether it is best for the principal to delegate the tasks partially or completely. In the latter case he also compares the cases where tasks are undertaken by the same individual or by different ones. Hence, he studies the effect of the allocation of tasks in the presence of externalities and different degrees of costs substitutabilities rather than the contracting structure.

A principal hires two identical agents ($i = 1, 2$) to undertake the production of a joint output, x. Agents are assumed identical in order to concentrate solely on the effects of different contracting structures. Output is assumed to be normally distributed with mean μ and variance $\sigma^2 > 0$. The expected output of the project increases with the effort exerted by the agents. For simplicity we assume that expected output is simply the sum of both agents' efforts. (In Section 4 we discuss the consequences of relaxing this assumption).

Given that efforts are *non-verifiable,* contracts $(w_i(x), i = 1, 2)$ can only be contingent on realized output. We constrain such contracts to be linear in output.[2]

Contract offers are assumed to be public, and so observable by every party. We assume the principal can credibly commit to her pledged policy, thus avoiding the issue of renegotiation.

The previous assumptions are summarized by:

$$\begin{cases} \cdot \quad x \sim N(\mu, \sigma^2) \\ \\ \cdot \quad \mu(e_1, e_2) = e_1 + e_2 \\ \\ \cdot \quad w_i(x) = a_i + b_i x, \quad i = 1, 2 \end{cases}$$

Hereafter, capital letters denote aggregate variables. Thus $E := e_1 + e_2$, $W(x) := w_1(x) + w_2(x)$, etc.

We assume the principal to be risk neutral, though her risk-neutrality is not fundamental for the analysis. Hence she maximizes the expected profits, π (i.e. expected output minus total wages). Agents are assumed to be risk averse with *constant absolute risk aversion* (CARA) utility and index of risk aversion $r > 0$. Later in the text we show that our results are robust to the introduction of risk aversion heterogeneity. The disutility of effort $\phi(e_i)$ is assumed to be quadratic on $e_i \in \mathbb{R}_+$. Hence the utility of

[2]Holmstrom and Milgrom (1987) provide a rationalization for this assumption in contexts where the contract is repeatedly offered over time. Note that considering contracts to be linear in output is equivalent to the more realistic assumption of considering them linear in profits.

the principal and the agents are, respectively:

$$\begin{cases} \cdot \quad V(x) = x - W(x) \\ \\ \cdot \quad U(w_i(x), e_i) = -re^{-r\left(w_i(x) - \frac{1}{2}e_i^2\right)} \end{cases}$$

2.1 First Best

As a benchmark case we solve for the first best solution where efforts are contractible. The principal's program reads as follows:

$$\max_{\{w_1(x), e_1, w_2(x), e_2\}} E_x \{V(x)\}$$

$$\text{s.t.} \begin{cases} \cdot \; E_x \{U(w_1(x), e_1)\} \geq U_0 \\ \\ \cdot \; E_x \{U(w_2(x), e_2)\} \geq U_0 \end{cases}$$

That is, the principal maximizes the expected profit subject to offering at least the reservation utility to each agent (hereafter, participation constraints are denoted $PC1$ and $PC2$ respectively).

Given the CARA utility functions and normally distributed profits, we can transform the agents expected utility functions into their certainty equivalent. $E\{U(\cdot)\} \rightarrow E(\cdot) - \frac{r}{2}Var(\cdot)$. Hence the expected utility of agent i is:

$$E_x \{U(w_i(x), e_i)\} \rightarrow a_i + b_i \cdot E - \frac{\gamma}{2}b_i^2 - \frac{1}{2}e_i^2$$

where $\gamma := r\sigma^2 > 0$. The expression above has a very clear interpretation. It is equal to the agent's expected wage minus the costs associated to the risk she is bearing and minus the cost of effort.

Without loss of generality we assume that the reservation utility after the certainty equivalence transformation is 0.

The first best solution is:

$$\begin{cases} \cdot \; e_1^* = e_2^* = 1 \\ \\ \cdot \; w_i^*(x) = \frac{1}{2}, i = 1, 2 \end{cases}$$

That is, the risk neutral principal absorbs all the risk and fully insures the risk averse agents by offering them a constant wage.

Notice also that both agents are treated equally, their participation constraints are binding and the expected profit of the principal at the optimum ($\Pi^* = 1$) is twice that of the single agent first best problem.

2.2 Centralized Second Best

A standard assumption in moral hazard settings is that the principal is not able to directly contract on effort. This is typically assumed to be the consequence of a deficient or too costly monitoring technology. Whichever the rationale, when we move from the first best setting where contracts can be contingent on effort, the principal faces the customary trade-off between incentives and insurance.

We first consider the *standard second best* situation, where the principal contracts directly with both agents. In the sequel we call that situation the *centralized second best.* Then we analyse the *hierarchic second best* situation, where the principal contracts with one of the agents, and this agent subsequently contracts with the remaining one.

Due to the non-contractibility of effort, the principal can offer contracts contingent only on output. Hence, she must design the contracts in such a way that they provide the right incentives for the agents to exert effort. Technically, this implies adding to the first best program two new constraints, the incentive compatibility constraints (denoted $IC1$ and $IC2$ hereafter).

The program of the principal reads now as follows:

$$\max_{\{w_1(x), e_1, w_2(x), e_2\}} E_x \{V(x)\}$$

$$\text{s.t.} \begin{cases} \cdot \ E_x \{U(w_1(x), e_1)\} \geq U_0 \\ \cdot \ E_x \{U(w_2(x), e_2)\} \geq U_0 \\ \cdot \ e_1 \in \arg\max_{\dot{e}_1} E_x \{U(w_1(x), \dot{e}_1)\} \\ \cdot \ e_2 \in \arg\max_{\dot{e}_2} E_x \{U(w_2(x), \dot{e}_2)\} \end{cases}$$

The unique solution to the previous program is:

$$\begin{cases} \cdot\ e_1^{**} = e_2^{**} = \frac{1}{1+\gamma} \\ \\ \cdot\ w_i^{**}(x) = a^{**} + \frac{1}{1+\gamma} \cdot x; i = 1, 2 \\ \\ \cdot\ a^{**} \text{ is such that the PCs are binding.} \end{cases}$$

The trade-off between incentives and insurance leads to a sub-optimal effort ($e_i^{**} < e_i^*$, $i = 1, 2$) and transfers some risk to the agents ($b_i^{**} > b_i^* = 0$, $i = 1, 2$), compared to the first best case. Consequently, the profits of the principal (and so total welfare) are lower, too: $\Pi^{**} = \frac{1}{1+\gamma} < \Pi^*$. As usual, the higher the parameter γ is (the higher the risk aversion of the agents or the higher the variance of the project) the higher the inefficiency of the second best solution with respect to the first best one.

The incentive power of the wage schedule is captured by the slope of the contract. Note that the chosen effort by each individual depends only on the power of the incentive scheme of her own contract ($e_i = b_i$). This is a particular feature of our model (the robustness of our results when this condition is not met is discussed in Section 4.2).

Once again, agents are treated symmetrically: the principal offers exactly the same contract to both of them. This seems intuitive since agents are homogeneous and identical to each other.[3]

As we observed in the first best case, the expected profits are twice the single agent second best profits. Nevertheless, from the agent's point of view the fixed payment of the contract is now

[3] The fact that agents are treated symmetrically reinforces the idea that collusion through side contracting cannot occur. We could also imagine agents colluding through forming a cartel and deciding jointly how much effort to exert. In this way, they will completely internalize the positive externality and the only source of inefficiency will be the standard moral hazard one. Given that agents are risk averse and that costs are convex, the situation is analogous to a single agent situation where the cost of total effort E is $\frac{1}{4}E^2$ and the total risk born by the agents is $\frac{\gamma}{4}B^2$.

Itoh (1993) provides a rigorous analysis of this situation allowing agents to side contract on the observed efforts: the principal is always better off because she can now provide the same incentives at a lower risk level given that agents are monitoring their partners efforts.

lower given that the expected output of the project is twice that in the single agent case.

This highlights the essential element that is incorporated into the problem when we move from the single-agent to the multi-agent case: there is now a positive externality between the agents. Since output is produced jointly and is the only contractible variable, *the wage of each agent is a function of the efforts of all the agents involved in production*, not only her own. When an agent increases her effort, she increases the expected wages of *all* the agents.

However, the principal is unable to internalize this externality when contracting in a centralized manner.

2.3 Hierarchic Second Best

Nevertheless we can improve upon the second-best solution simply by modifying the contracting structure of the model. The main intuition is that by transferring contracting rights to, say, agent 1 (hence establishing a hierarchy between the agents), she internalizes the positive externality her effort has on agent 2. This is so because she realizes that an increase in her own effort does not only increase her expected salary but also increases the salary of her subordinate, thus relaxing $PC2$. We can clearly see that the private benefits of agent 1 have now increased with respect to the ones she had under the centralized structure. Consequently, she is willing to exert more effort at no extra cost for the principal. Ultimately, the principal extracts all this extra surplus from agent 1 and hence the change in the contracting structure results in a higher expected surplus for the principal.[4]

Under the hierarchic structure, the principal contracts with only one agent, who then subcontracts with the remaining one. The timing of the game is as follows: the principal offers a contract $W(x) = A + Bx$ to agent 1 and subsequently agent 1 offers a contract $w_2(x) = a_2 + b_2x$ (s.t. $a_2 \leq A$ and $b_2 \leq B$) to agent 2; then the agents decide their optimal effort (IC) and, if their PCs

[4] Note that the hierarchic structure internalizes the positive externality that agent 1 has on agent 2 (hence equating her private benefits to her social benefits) but obviously does not affect agent 2's private benefits.

are met, they exert effort. Finally, output is realized and payments are made.

Notice that there is a subtle change in the game played by the agents when we move from the centralized to the hierarchic case. In the centralized case it is implicitly assumed that agents decide their levels of effort independently and simultaneously (i.e. they play a Cournot-type of game). However, in the hierarchic structure agent 1 acts as a Stackelberg leader and agent 2 as a follower (i.e. players are no longer playing mutual best responses). In other words, agent 2 acts knowing what the optimal choice of effort of the other agent is.

Under our particular modelling assumptions this does not play any role, given that the optimal choice of effort depends only on the slope of the contract and not on the effort chosen by the other player. In Section 4 we show under which circumstances our results remain true.

We solve the program of the principal by backwards induction. Hence, we first analyse the program of agent 1 once she is offered the contract $W(x) = A + Bx$ subject to the optimal behaviour of agent 2 (i.e. $PC2$ and $IC2$):

$$\max_{\{e_1, w_2(x), e_2\}} E_x \{U(W(x) - w_2(x), e_1)\}$$

$$\text{s.t.} \begin{cases} \cdot\ E_x \{U(w_2(x), e_2)\} \geq U_0 \\ \\ \cdot\ e_2 \in \arg\max_{\dot{e}_2} E_x \{U(w_2(x), \dot{e}_2)\} \end{cases}$$

Its unique solution is:

$$\begin{cases} \cdot\ e_1^h = B \\ \\ \cdot\ e_2^h = \frac{1+\gamma}{1+2\gamma} B \\ \\ \cdot\ w_2^h(x) = a^h + \frac{1+\gamma}{1+2\gamma} B \cdot x \\ \\ \cdot\ a^h \text{ is such that } PC2 \text{ is binding.} \end{cases}$$

As we observed in the second best case, each agent chooses her own optimal level of effort such that it is equal to the incentive power of her contract ($e_1 = B$ and $e_2 = b_2$). However, in this case agent 1 exerts a strictly higher effort than agent 2 ($B > \frac{1+\gamma}{1+2\gamma}B$). Note that at a first glance this does not seem optimal from the principal's perspective: given the convex disutility of effort the optimal allocation should, *a priori*, involve both agents exerting exactly the same level of effort.

The derivative of the Lagrangian of the previous program with respect to e_1 shows clearly how agent 1 internalizes the positive externality she exerts on agent 2 through its participation constraint. The Lagrangian of the previous program is:

$$\mathcal{L} = (A - a_2) + (B - b_2) \cdot E - \frac{\gamma}{2}(B - b_2)^2 - \frac{1}{2}e_1^2 + \\ + \lambda \left(a_2 + b_2 \cdot E - \frac{\gamma}{2}b_2^2 - \frac{1}{2}e_2^2 \right) + \mu (e_2 - b_2)$$

and its derivative with respect to e_1 should be equal to zero at the optimum:

$$\frac{\partial \mathcal{L}}{\partial e_1} = (B - b_2) - e_1 + \lambda \cdot b_2 = 0$$

The above condition differs from the $IC1$ under the centralized structure due to the fact that it includes the term $(\lambda \cdot b_2)$. That term captures precisely the positive externality that agent 1 has on player 2.

Given agent 1's optimal behaviour we can move one stage backwards and consider the principal's problem. Her program reads as follows:

$$\max_{\{w_1(x), e_1, w_2(x), e_2\}} E_x \{V(x)\}$$

$$\text{s.t.} \begin{cases} \cdot \; E_x \{U(w_1(x), e_1)\} \geq U_0 \\ \cdot \; e_1 = B \\ \cdot \; e_2 = b_2 = \frac{1+\gamma}{1+2\gamma}B \\ \cdot \; a = a^h(B) \end{cases}$$

where the last three conditions come directly from agent 1's program, thus ensuring that the solution is a subgame perfect equilibrium.

Its unique solution is:

$$\begin{cases} \cdot\ e_1^h = \frac{2+3\gamma}{(1+\gamma)(2+\gamma)} \\ \cdot\ e_2^h = \frac{3\gamma+2}{(\gamma+2)(2\gamma+1)} \\ \cdot\ W^h(x) = A^h + \frac{2+3\gamma}{(1+\gamma)(2+\gamma)} \cdot x \\ \cdot\ w_2^h(x) = a^h + \frac{3\gamma+2}{(\gamma+2)(2\gamma+1)} \cdot x \\ \cdot\ a^h \text{ is such that } PC2 \text{ is binding} \\ \cdot\ A^h \text{ is such that } PC1 \text{ is binding} \end{cases}$$

The profits of the principal (and so total welfare) are now:

$$\Pi^h = \frac{1}{2}\frac{(2+3\gamma)^2}{(1+2\gamma)(2+3\gamma+\gamma^2)} < \Pi^*$$

Before we discuss the differences between the hierarchic and centralized structures in the following section it is worth pointing out that the principal cannot do better than the hierarchic allocation through centralized contracting precisely because, as long as the principal is the only one that contracts with the agents, both agents fail to internalize the externality.[5]

3 Results

Two comparisons are relevant between the centralized and hierarchic structures. First, from the point of view of the principal, which one yields higher profits? And second, from the point of view of the agents, how do the allocations of effort and risk change when we move from one contracting structure to the other?

[5] The fact that the principal cannot do better through centralized contracting shows, in particular, that she cannot do better by contracting sequentially either (once again, the agents fail to internalize the externality).

Proposition 1 *The hierarchic structure always yields a higher expected profit to the principal than the centralized one. The relative gains are higher the higher the risk of the project and/or the index of risk aversion of the agents.*

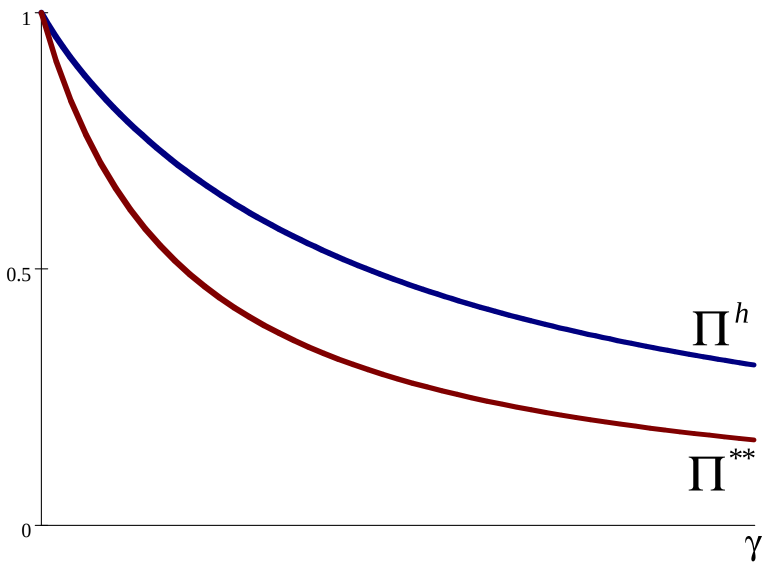

Figure 1: Expected profits

The proof is immediate from comparing:

$$\Pi^{**} = \frac{1}{1+\gamma}$$

and

$$\Pi^h = \frac{1}{2}\frac{(2+3\gamma)^2}{(1+2\gamma)(2+3\gamma+\gamma^2)}$$

and showing that

$$\Pi^h/\Pi^{**}$$

is increasing on γ. Figure 1 shows both expected profits in terms of γ.

Proposition 2 *Under the hierarchic structure, agent 1 exerts more effort than agent 2. Moreover, both agents exert more effort than under the centralized structure, i.e.* $e_1^h > e_2^h > e_1^{**} = e_2^{**}\ \forall \gamma > 0$.

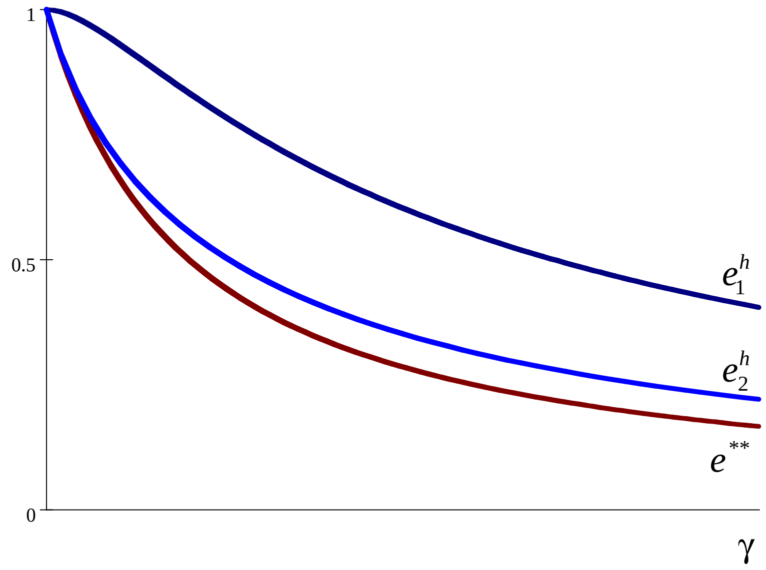

Figure 2: Efforts

The proof of $e_2^h > e_1^{**} = e_2^{**}$ is obtained from comparing

$$e_2^h = \frac{3\gamma + 2}{(\gamma + 2)(2\gamma + 1)}$$

and $e_1^{**} = e_2^{**} = \frac{1}{1+\gamma}$. The result that $e_1^h > e_2^h$, meanwhile, is immediate from the fact that

$$b^h = \frac{1+\gamma}{1+2\gamma} B^h$$

(and $e_2^h = b^h; e_1^h = B^h$).

Also from agent 1's program we find that:

$$\frac{\partial b_2^h}{\partial B^h} = \frac{1+\gamma}{1+2\gamma} \in \left[\frac{1}{2}, 1\right]$$

This derivative shows that whenever the principal induces a higher effort on agent 1 (remember that $e_1^h = B^h$), this one also provides more incentives to agent 2 (since $e_2^h = b^h$). From the principal's perspective this generates a more than interesting *trickle* *down effect* that multiplies the initial first-order effect of an increase in B^h by increasing also b_2^h. However, this second-order effect is always lower than one and decreases with γ. That is, whenever we increase γ agent 1 is less willing to induce higher incentives on agent 2 (it becomes more costly). Figure 2 shows the efforts of both agents under both contracting structures as functions of the parameter γ. In particular, we can see that the risk borne by the agents under the hierarchic structure tends to zero as $\gamma \to \infty$, which is consistent with the derivative above.

It is also easy to see that a direct consequence of Proposition 2 is that the expected output under the hierarchic structure is greater than under the centralized one.

Notice that the slopes of the wage contracts, besides providing information on the optimally invested efforts, indicate also the risk borne by the agents. Given the certainty equivalent, the risk borne by agent i is equal to $\frac{r}{2}Var(w_i(x)) = \frac{\gamma}{2}b_i^2$. Thus, agent 2's risk is $\frac{\gamma}{2}\left(b_2^h\right)^2$, and agent 1's is $\frac{\gamma}{2}\left(B^h - b_2^h\right)^2$. Note that agent 1's risk depends on her *net* wage ($W(x) - w_2(x) = A - a_2 + (B - b_2)x$). However, her invested effort depends on the gross wage given that she internalizes her positive externality on agent 2.

Proposition 3 *Under the hierarchic structure, agent 1 bears less risk than agent 2. Moreover, the latter bears more risk under the hierarchic structure than under the centralized one.*

Formally, the proposition can be re-written as follows:

$$\begin{cases} \cdot\ b^H > B^H - b^H, \forall\gamma \geq 0 \\ \\ \cdot\ b^H > b^{**}, \forall\gamma > 0 \end{cases}$$

and is depicted in Figure 3.[6]

[6] A question that becomes relevant once we allow for heterogeneous agents on the level of risk aversion is whether the principal should contract with the most or the least risk averse agent under the hierarchic structure. Given Proposition 3 the answer is immediate: the least risk averse should end up at the bottom given that the agent at the top is assuming the lower level of risk. It is easy to prove that in this case the principal is still better off under the hierarchic structure than under the centralised one.

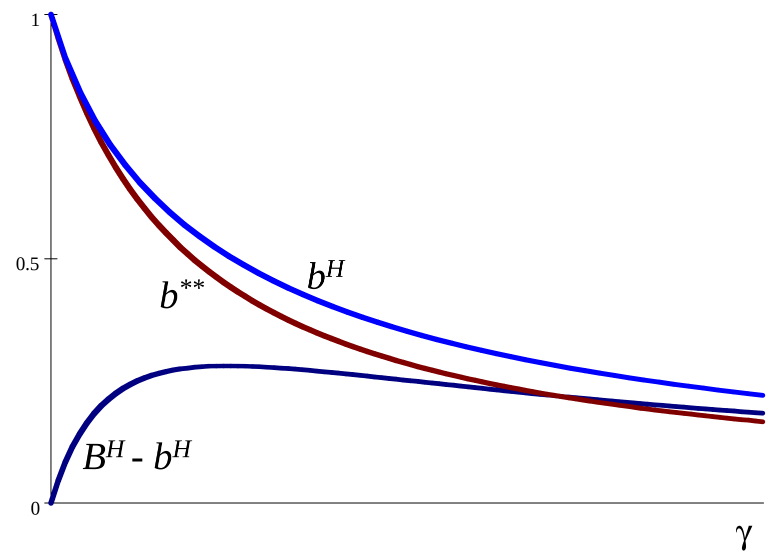

Figure 3: Risks

Linking this result to the derivative above, we can see that agent 1 is able to transfer part of her risk to agent 2. This transfer is almost complete when $\gamma \simeq 0$ (agents are almost risk neutral) and decreases as γ increases since it becomes more and more costly to insure a more and more risk averse agent 2.

At first sight, the fact that the agent higher in the hierarchy bears less risk than her subordinates may seem puzzling. However, when risk is interpreted not only as the variability of pecuniary payments, it is indeed the case that outsourcing, subcontracting or decentralization are ways to pass some risk to those at the lower levels of the hierarchy. That is precisely one of the main criticisms of subcontracting in the construction industry, outsourcing in manufacturing or the privatization of public services. Namely, they generate precarious and deregulated working conditions.

Proposition 3 highlights the fact that the hierarchic structure cannot be replicated by a centralized one. It shows that although agent 1 is exerting more effort than agent 2 (Proposition 2), she is bearing less risk. This seems to be at odds with the basic moral hazard intuition that more incentives (higher effort) are associated with more risk. Once again, the reason is that the hierarchic structure provides extra incentives to agent 1 through the internalization of the positive externality she has on the remaining agent.

What are the mechanisms that make hierarchies more efficient than centralized structures?

Imagine, for illustrative purposes, that the principal offers to agent 1, under the hierarchic structure, the optimal centralized aggregate wage contract ($A = 2 \cdot a^{**}$ and $B = 2 \cdot b^{**}$). Agent 1's optimal effort is $e_1 = B$ and she can offer agent 2 the centralized second best contract. In that case agent 2 invests the same effort as in the centralized case but now $PC2$ is not binding given that agent 1 is exerting twice the effort of that in the centralized case. Hence, agent 1 optimally adjusts the contract offered to agent 2 until $PC2$ is binding (i.e. increasing agent 2's incentives and decreasing her own risk), thus relaxing her own participation constraint. Ultimately, the principal extracts all the extra surplus obtained by agent 1 by making $PC1$ binding again. Hence we see that agent 1 exerts an effort higher than e_1^{**} since she internalizes the positive externality she has on agent 2. As a result of that, we also observe a second order effect on agent 2's effort (the derivative): given that her participation constraint is not binding any more, she is offered a slightly modified contract where more incentives and less insurance are provided.

At the hierarchic optimum the principal does *even better* and offers a different contract from the aggregate centralized one.

4 Discussion

In spite of the convexity of the disutility of effort and the risk aversion of agents the hierarchic structure distributes asymmetrically the risk and the effort among the agents. But, regardless of the inefficiency inherent in that, the internalization of the externality makes the principal better off overall.

4.1 The role of hierarchy

The hierarchic structure introduces extra incentive constraints in the principal's program given that the offered contract to the second agent needs now to be optimal from agent 1's perspective. This negative aspect is outweighed by the fact that agent 1 now internalizes the positive externality she has on the remaining agent. All gains come from handing over contracting rights to agent 1, thus making her the residual claimant of agent 2's activities. Moreover, agent 1's incentives with respect to agent 2 are very much aligned with the principal's ones given that any increase in the former's utility function is appropriated by the latter. From an egalitarian point of view the hierarchic situation may be seen as worse than the centralized one, given that identical agents are treated differently and the allocation of effort and risk between agents is asymmetric.[7]

The obvious question, given the results presented above, is why we do not see hierarchies everywhere since they seem to outperform centralized structures. First of all, for the results to hold true, we need the two basic assumptions for the existence of the positive externality, that is, team production and output being the only contractible variable. Then, there is a long list of elements that we deliberately ignored so that we could isolate the contracting structure effects from aspects such as the presence of different tasks, monitoring, heterogeneity of agents, complementarity or substitutability of efforts, etc. Our point is precisely that, even in the presence of any of those elements, the contracting structure still matters.

Without departing from our model we think that it is worth analysing how a limited liability constraint ($w_i(x) \geq 0, \forall x$) may affect the comparison between the agents under the hierarchic structure. In order to capture the intuition of what could happen if we introduced such a constraint under the hierarchic and centralized structures we can check which is the maximum value of x for which the contract offered to the agents is negative (i.e. the point where

[7] The non egalitarian outcome, given identical agents, leads us to view the introduction of behavioural assumptions as an interesting line for future research on the topic of hierarchies.

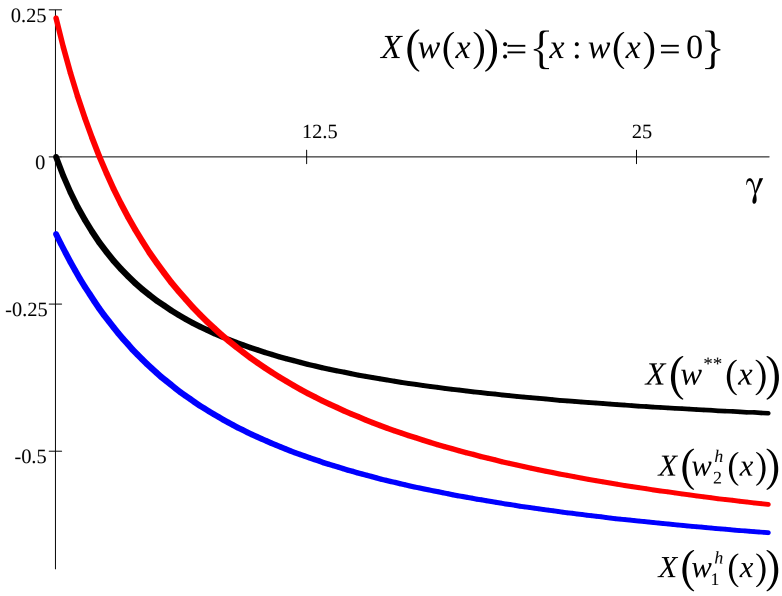

Figure 4: Limited liability

the limited liability constraint bites). In Figure 4 we have depicted these thresholds in terms of the parameter γ. Note that the curve that corresponds to the hierarchic net contract of the first agent is always lower than the two remaining curves. That is, the limited liability constraint of agent 1 under the hierarchic structure bites at a lower value of realized output than the one of agent 2. This indicates that agent 1 is able to ensure that she gets a negative payment less often than agent 2 (that is consistent with Proposition 3 - agent 1 bears less risk - and with the fact that the principal is able to induce a high level of effort on agent 1 at a low level of risk). When comparing the centralized and hierarchic structures, we can see that for low levels of γ the hierarchic structure is more affected by the limited liability constraint (in particular, agent 2's realization of wages is negative for a larger range of output realizations). However, for large values of γ, limited liability constrains the centralized structure more.[8]

[8] The heuristical argument that limited liability constraints bite less under the hierarchic structure for high levels of γ, together with the fact that the

Obviously the analysis is a partial one and the problem of how the principal optimally adjusts the offered contracts in the presence of limited liability constraints is left for future research.

Another simplifying assumption that may have strong effects in a real world application of our model is the existence of contracting costs. The fact that the hierarchic structure requires the writing of as many different contracts as levels existing in the hierarchy may decrease the benefits in overall welfare for the principal and affects the number of hierarchies we observe in the real world.

Finally, there is also a case for allowing agents to communicate with the principal and hence act as monitoring devices of the other agent's activities. This is the focus of Baliga and Sjolstrom (1998), Faure-Grimaud et al (2003), and is beyond the scope of our analysis.

4.2 Robustness To Different Specifications

The main objective at this point is to show which modelling assumptions are essential for our results to hold true. As a result of assuming constant risk aversion utilities, normally distributed profits and linear contracts, we managed to find a closed form solution. There are two main consequences that are essential for our results.

First, the incentive constraints of each agent depend only on her own wage and are independent of the effort invested by the remaining agents (i.e. there are no strategic complementarities or substitutabilities).[9] Consequently, an agent affects the other agent only by increasing her expected utility or, in other words, only by relaxing her participation constraint. That is precisely the driving force of all our analysis.

Second, under the hierarchic structure the first agent is not

relative gains of the hierarchic structure are strictly increasing in γ (tending to outperform 125% better than the centralised structure as γ tends to ∞), should lead us to observe more hierarchic structures in situations where agents are very risk averse or where the randomness of the project is greater.

[9] Strategic complementarities or substitutabilities can arise, for instance, from dropping the condition that the expected output is the sum of the exerted efforts.

only internalizing the externality but is also modifying the wage offered to her subordinate with respect to the second best situation. Nevertheless, her optimal choice of effort does not depend on the distribution of the wages and depends only on the total wages, given that she internalizes completely her externality on agent 2 through her participation constraint.

When we consider a more general setting the analysis becomes ambiguous precisely because the previous two observations no longer hold. That implies that when we analyse the first agent program under the hierarchic structure other effects may outweigh the positive effects of internalizing the externality. Suppose first that the new setting is still such that there are no strategic complementarities or substitutabilities. In that case, agent 1 may switch the contracting structure in such a way that she induces a different risk structure on agent 2 with respect to the one that the principal would offer under a centralized structure. This would not matter if the internalization of the externality made agent 1's optimal effort only contingent on the total wages ($W(x)$) but in general that is not true. One could imagine a situation where the incentives of agent 1 on setting agent 2's contract may make the overall effect of the hierarchic structure worse for the principal.

The presence of strategic complementarities seems to reinforce the pre-eminence of the hierarchic structure over the centralized one. But strategic substitutabilities work against the incentives of agent 1 to invest more effort under the hierarchic structure and hence may outweigh in some cases the positive effects that result from internalizing the externality.

Lastly, the root of the benefits of the hierarchic structure over the centralized one is that agent 1 internalizes the externality. For that to be the case we need to assume that agent 1 is able to have a first mover advantage. Otherwise, she would choose her effort according to her own net wage regardless of the positive externality she has on agent 2 (technically, she does not choose her own effort subject to the participation constraint of agent 2 and thus she does not internalize the positive effect that her own effort has on the participation constraint of agent 2; the key is that agent 1 can take advantage of the interaction of her two choice-variables (e_1 and $w_2(x)$). We believe that this is the most natural situation: the leader of a team or the individual who occupies a place at the top of the hierarchy not only proposes a contract to the subordinates but, at the same time, also announces her plans for the future (her effort decision). It would be boundedly rational for her not to take into account the strategic effects of both her decisions.

Precisely, Macho-Stadler *et al* (1998 and 2002) define a hierarchic structure where agent 1 decides her own effort after offering the contract to agent 2. Hence, agent 1 does not take into account the essential effect of our analysis (i.e. the positive externality agent 1 has on relaxing the participation constraint of agent 2). But instead the authors contemplate the two elements we avoided in our analysis. Their goal is to determine whether the strategic complementarity between efforts is larger than the negative effect of introducing an extra incentive constraint on the wage offered to agent 2. In a binary outcome scenario with a Cobb-Douglas effort production function they find that the centralized structure does better whenever the productivity parameters of both agents are small enough (i.e. whenever the strategic complementarity is unimportant).

5 Conclusion

We have presented a case where the allocation of contracting rights plays a crucial role in an organization's efficiency.

Whenever a principal needs to hire a group of agents to produce a joint output and is only able to write contracts conditional on this output, a positive externality arises between the agents. We have shown that a hierarchic structure, unlike the centralized one, can partially internalize this externality since, by reallocating contracting rights, it makes the agents at the top of the hierarchy "residual claimants" of the workers below them. This way, the agents at the top realize the effect that their effort choices have on their subordinates' participation constraints, thus internalizing the externality. This allows the principal to induce higher incentives to the agents at the top of the hierarchy without increasing insurance costs. The overall effect is an increase in the expected profits. Moreover, agents higher in the hierarchy exert more effort and bear less risk than those in lower tiers.

The crucial factor that explains the pre-eminence of a decentralized structure over the centralized one is that the first one gives agents more decisive power (contracting rights). This results in the internalization of the externality by agent 1 and, due to the alignment of the latter's and the principal's incentives, in higher profits for the principal. No structure where the principal retains all contracting rights can, therefore, outperform (or even match the performance of) the hierarchic one: the externality would not be internalised. Moreover, these results hold *in spite of* assuming homogeneous agents.

Summing up, the above analysis means that hierarchic structures should not only matter for supervising and efficiently allocating tasks but may introduce gains simply because of the change in the contracting structure. Its relevance seems to arise in several distinct building blocks of economic life, and so the present analysis can be easily applied to issues such as decentralization within the government, privatization of public services, hierarchies and delegation within the firm, subcontracting, outsourcing, franchising and sharecropping, among others.

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