Quality Competition, Market Structure and Endogenous Growth*

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Discussion Paper
No EI/25
July 2000

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* I am grateful to John Sutton, to whom I am intellectually most indebted to in conducting this research project. I am also grateful to Chenggang Xu for encouragement and helpful comments. I thank Alan Manning, Michele Piccione, Max Steuer, Leopoldo Yanes, other seminar participants at LSE, Danny Quah and Peter Davis for helpful comments and discussions. Financial support from K. C. Wong scholarship is gratefully acknowledged.
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Abstract

I study the role of quality competition in endogenous growth and institutional factors which can affect growth through affecting quality competition. The R&D-based growth literature as it stands attributes the incentives for innovations to monopolist market structure, and regards the driving force of growth being the 'rent-pull'. This paper presents a 'competition-push' theory of growth by considering an environment where firms can coexist and compete in quality within the same markets. Quality competition takes the form of vertical product differentiation or cost-reducing process innovation, which requires endogenous fixed R&D cost. Due to the nonrival and excludable features of 'quality' and consequent nonconvexity, market concentration naturally occurs in a manner such that R&D intensity and market structure are determined simultaneously in equilibrium. The main conclusions are that quality competition suffices to provide incentives for innovation at industry level, and through knowledge spillovers it also drives aggregate technical progress, that institutional restriction on free entry into quality competition may be desirable to some degree, but monopolization is usually not optimal, that credit constraint which limits quality competition is detrimental to growth.

JEL Classification: L13, L16, O31, O41
Keywords: endogenous growth, quality competition, vertical differentiation, endogenous sunk cost, competition-push, rent-pull, institutional barriers to entry, credit constraint

1 Introduction

Is market competition good or bad for growth? This question can trace back to Schumpeter when he in 1942 argued that perfect competition was unsuitable for encouraging innovation and suggested the need for imperfect competition taking the role. Economists have taken a major interest in this issue ever since. Some followers of Schumpeter regarded monopolist rents as the driving force of innovations and believed in a one-way causal relationship between market concentration and R&D intensity. For about four decades related research has mainly been industry level empirical investigations (See Cohen and Levin 1989 for a comprehensive survey). It turned out that conclusive evidence supporting a clear-cut one-way causal relationship failed to emerge.

Related theoretical analysis appeared much later. It remained for the application of game theory to help with explaining why the presumed one-way causal relationship between market concentration and R&D may not be the case. (Dasgupta and Stiglitz 1980, Sutton 1998). Formalizing the issue at aggregate level appeared even later, until the techniques of handling general equilibrium with imperfect competition ready, with the emergence of the R&D-based growth theories (Romer 1990, Grossman and Helpman 1992, Aghion and Howitt 1992). Following Romer's 1990 argument that the nonrival and excludable characteristics of ideas can cause nonconvexity and make price-taking behavior unstable, the R&D-based growth models have brought the Schumpeterian view about imperfect competition and innovation into the main stream of economics.

As for whether market competition is good for growth, the R&D-based growth literature, as it stands, follows an extreme interpretation of the Schumpeterian argument by attributing the incentives for innovations to monopolist rents, and by implying that there is a clear-cut one-way causal relationship between market concentration and R&D. Competition in the existing models is marginalized by being confined to monopolistic competition. Logically, since monopolist rent is regarded as the key to growth, any direct competition within a single market would hurt monopolist rent therefore must be bad for growth.[1]

The problem with the monopolistic specification of imperfect competition is that it only captures one extreme possibility. To see this, one needs to realize that nonconvexity does not warrant a monopolist market structure. Dasgupta and Stiglitz (1980) and Sutton (1991), both with game-theoretic approach, show that natural monopoly is not a necessary consequence of nonconvexity. Multiple firms coexisting within a single market and competing in 'quality' can be a stable situation. So to justify monopolist market structure as a generic feature in innovative industries, one needs to appeal to something else, say, patents or other market exclusion devices.

[1] One way to break this logic is to take agency problem into consideration. As shown in Aghion-Howitt (1998), competition may help to alleviate agency problem and therefore contribute to growth.

A seemingly straightforward argument used by many authors about the link between a patent and a monopoly is that a patent, by definition, gives an exclusive right to the inventor for producing and selling that product, therefore generates monopoly. The problem with this argument is that the coverage of the granted exclusive right may be so narrow that it does not prevent a close substitute being produced and sold by a competitor. Any empirically sensible definition of a market must treat sufficiently close substitutes as a single product. In this sense a patent does not necessarily lead to a monopoly in a 'single' market. This view is consistent with the empirical evidence, provided by Levin et al. (1987), which suggests that the effectiveness of patents to prevent competition is in general very limited and the primary reason for this is due to the ability for competitors to 'invent around' the patents.

While patents and other technology exclusion devices, secrecy and systematic incompatibility, to mention a few, don't necessarily grant monopolist positions in markets, they certainly tend to make the costly and profitable ideas at least partially excludable beyond the investing firms. As a result one more dimension is added to the rivalry between firms, that is quality competition. Quality competition takes the form of vertical product differentiation or cost-reducing process innovation, which requires a fixed cost of R&D. When firms have to compete in consumers' perceived quality or cost effectiveness, they will choose their R&D intensities in a strategic environment. Since the endogenous R&D cost is a barrier to entry, then the market structure is also determined in the strategic interaction.

This study combines the literature of quality competition (Dasgupta and Stiglitz 1980, Sutton 1991, Ch.3, Sutton 1998, Ch.3 & 4) and the literature of R&D-based aggregate growth. A key insight from the quality competition literature is that in industries where quality competition plays an important role, market structure and R&D intensities are determined simultaneously in equilibrium. In this sense quality-competitive industries have their natural structures, usually of natural oligopolies. In these industries, incentives for R&D are provided primarily by quality competition. Through knowledge spillovers, the quality competition-pushed innovations may well contribute to the technical progress at the aggregate level and play a prime role in endogenous economic growth.

This view is a kind of 'competition-push' theory of endogenous growth, which contrasts with the view commonly presented in the existing R&D-based growth models, that may deserve being called 'rent-pull' theory of endogenous growth. The distinction logically origins from the different specifications of entry into direct competition. The quality competition model allows free entry, which dries up rents that are considered the source of incentives for innovations by the existing R&D-based growth literature. The sharp contrast between the present analysis and the literature regarding the role of rents in promoting growth naturally raise two questions. The first is a positive one: whether rents are feasible if quality competition prevails? The second question is normative: whether rents are necessary for endogenous growth? This paper provides then a comprehensive analytical framework, which is beyond the model itself, for comparing the model with the literature, and addressing the above two questions.

A new insight emerges from the model when it is extended to the analysis of the impact of finance on growth. It is found that imperfection in capital market can significantly affect endogenous growth through affecting quality competition. Escalation in quality competition requires large amount of long term investment in illiquid assets. Therefore credit constraint which limits investing in quality can jeopardize the quality competition and growth.

The existing literature on the relation between competition and endogenous growth also includes Boldrin and Levine (2000) and Hellwig and Irmen (1999), both of which study possibilities of endogenous growth under perfect competition. Boldrin and Levine (2000) show that when there are overall constant returns to scale and technical progress depends only on endogenous adoption of exogenously given technologies, perfect competition can generate endogenous growth. Hellwig and Irmen (1999) show, that by defying the usual replication argument convexity and pricing taking behavior are maintained while intra-marginal profit can still be available and can cover R&D cost. These models both point out that when convexity is reserved, competitive equilibrium possibly suffices to generate endogenous growth. The analysis presented in this paper agrees with these two papers on that rents are not always necessary in providing incentives for innovations, but does, as they don't, agree with linchpin argument of the R&D-based growth theories, that the nonrival feature of ideas generates nonconvexity.

The remainder of paper is organized as follows. Section 2 lays out the model. Section 3 shows how quality competition suffices to promote innovation and growth. Section 4 provides an analytical comparison of this model with some in the literature, it discusses the impacts of institutional barriers to entry on growth. Section 5 extends the model to examine the impact of credit constraint on competition and growth. Section 6 concludes. Algebraic details of the derivation and proofs are placed in the Appendices.

2 The Model

2.1 Basic Setup

2.1.1 Two sectors

The model represents an economy which consists of two sectors: a final good sector and an intermediate good sector with multiple industries producing different varieties. For convenience, in what follows, the term 'industry' is reserved for an industry in the intermediate good sector. The final good can be used in consumption and can also be used as input to produce intermediate goods. The variety of intermediate goods can grow as a result of innovations. In the final good sector there is constant returns to scale and perfect competition. In each industry firms compete in 'quality' as well as in price. Due to the nonrival and excludable features of 'quality' and the consequent nonconvexity, the firms are not price takers and the market structure is endogenous.

2.1.2 Industry Level Technology, Nonrival and Excludable 'qualities'

The technology in an industry can be captured by an endogenous cost structure. The variable cost is all capital cost of final good and has constant returns to scale. The 'quality' represents either consumers' 'perceived quality' measure by their willingness to pay, or productivity level. In abstract mathematical terms these become equivalent, and both can be measured by the amount of effective units produced by unit input of final good. So in the rest of paper, quality and productivity are regarded as interchangeable terms. Quality can be improved by R&D which requires a fixed cost. The fixed cost is all the innovative labor cost, and is a convex increasing function of quality index. An intermediate good producer can trade off between fixed cost and variable cost. There are overall increasing returns to scale due to the fact that the quality is non-rival, and can be applied to all units of output with zero marginal cost. The quality is excludable through exclusion devices such as patent or secrecy, so it is only accessible by the firm which invests in it.

2.1.3 Aggregate Level Technology and Externalities

The emergence of new varieties of intermediated goods and therefore new markets in each period, $\dot{A}$, is a function of innovative efforts made by firms in industries, measured by L_2, and the spillover from knowledge embedded in existing intermediated goods, A, i.e., $\dot{A} = \delta L_2 A$. The spillover from A to $\dot{A}$ has been well recognized by the R&D-based growth literature. The spillover from quality creation by firms to aggregate growth in knowledge needs a bit more comments. In this model, The innovative labor L_2 are hired by firms in industries to create qualities. In so doing, they also contribute to generic progress in knowledge and generate prototype designs of new varieties of intermediated goods as by-products. So this kind of spillover is flowing from one type of innovation, i.e., quality improvement, towards another type of innovation, i.e., new prototype creation. The emergence of the prototype design of a new variety predicts a new emerging market which can soon be entered and competed in.

2.1.4 3-stage Game in Each Industry

Whenever a new industry emerges, a three-stage game is played in emerging industry. In the first stage, potential entrants simultaneously choose whether to enter the new industry. In the second stage, those firms which have entered the market choose their 'quality' levels (u_i), which will be achieved by R&D and incurring costs of hiring innovative labor. In the third stage, firms with their achieved 'quality' level compete in quantity in a Cournot manner[2].

2.1.5 Consumption

The are constant L identical consumers who live infinitely and each has one unit of labor per period. The labor can be supplied either as simple labor used in final good sector, or innovative labor used in industries to do R&D. Each consumer has a discounted, constant elasticity preference:

$$\int_0^\infty \frac{c^{1-\varepsilon}}{1-\varepsilon} e^{-\rho t} dt \text{ for } \varepsilon > 0.$$

The Euler equation for intertemporal optimization for given interest rate r is that $\frac{\dot{c}}{c} = \frac{r-\rho}{\varepsilon}$.

2.1.6 Final Good Sector

The final good can be treated as the numeraire. Its production function is

$$Y = L_1^{1-\alpha} \int_0^A x_i^\alpha di,$$

where Y is the total output of final good, L_1 is labor input, x_i is the input of intermediate good i, A is the number of intermediate goods, therefore also the number of different intermediate goods markets, $0 < \alpha < 1$.

There is perfect competition in final good market due to constant returns to scale, so equating marginal product of labor the wage rate gives us the inverse labor demand function

$$w = \frac{(1-\alpha) Y}{L_1}. \tag{1}$$

Similarly by equating the marginal product of input i with its price gives us the inverse demand function for intermediate good i,

$$p_i = \alpha L_1^{1-\alpha} x_i^{\alpha-1}. \tag{2}$$

2.2 The Game of Quality Competition with Free Entry

In each emerging industry a 3-stage game is played. The solution concept adopted in this paper is subgame perfect Nash equilibrium, and it can be solved by backward induction. The third stage subgame is a Cournot game of quantity competition, a Cournot-Nash equilibrium exists and is unique for a given set of quality

[2]The specification of toughness of price competition throughout this paper is fixed to Cournot quantity competition. Tougher price competition as captured by a Bertrand price competition, or softer price competitions which may result in collusions are beyond the scope of this paper.

levels of all firms and the demand condition of the economy. The subgame Nash equilibrium determines for each firm a reduced form profit function as follows

$$\pi_i\left(u_i \mid u_{-i}\right) = \frac{\alpha^{\frac{1}{1-\alpha}} L_1}{1-\alpha} \left(\frac{N-1+\alpha}{\sum_{j=1}^{N} \frac{1}{u_j}}\right)^{\frac{\alpha}{1-\alpha}} \left(1 - \frac{N-1+\alpha}{\sum_{j=1}^{N} \frac{u_i}{u_j}}\right)^2, \tag{*}$$

where u_i is the quality level of firm of i, u_{-i} is a $(N-1)$-tuple of quality levels of other firm except firm i, for $i = 1, 2, \cdots, N$; N is the number of firms in the industry and L_1 is the amount of labor employed in the final sector.

Interested readers can find the derivation of this reduced form profit function in Appendix A. The key feature of it is that a firm's profit increases with its relative quality level against its rivals', i.e., $\sum_{j=1}^{N} \frac{u_i}{u_j}$. This function tells how the strategic environment responds to vertical differentiations of firms. It captures the Darwinian selection pressure embedded in the environment constituted by customers and rivals. This environment provides firms incentives to outperform their rivals in R&D and quality. When they vie in quality, they are constrained by the R&D cost, which is a function of quality level as show below:

$$F\left(u_i; w, A\right) = w\frac{\mu}{A}u_i^{\beta}, \tag{**}$$

where $\frac{\mu}{A}u_i^{\beta}$ $(\beta > 1)$ is the input of labor to generate quality level u_i, w is the market wage rate and μ is a constant. The fixed cost is therefore all labor cost. It is worth to emphasize that the increase in A, i.e., the aggregate level stock of knowledge, can shift the fixed cost curve downward as shown in Figure 1. The intuition is that knowledge spillover from aggregate technical progress enhances the efficiency of R&D by each new firm. This positive trend may be offset by the negative trend caused by increase in wage rate, also shown in Figure 1.

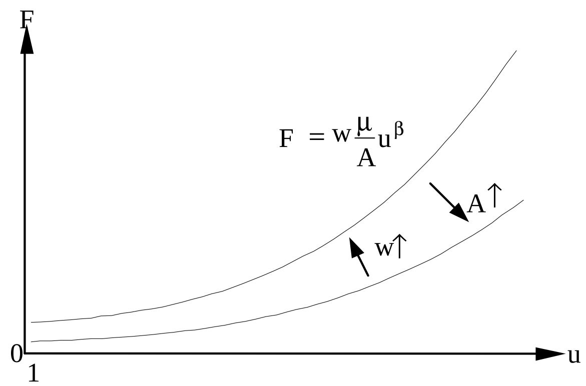

Figure 1: Fixed cost function

Therefore each firm's objective in second stage subgame is to maximize its net profit, in the present value sense, by choosing own quality level given others' quality levels,

$$\max_{u_i} \left\{ \frac{\alpha^{\frac{1}{1-\alpha}} L_1}{r(1-\alpha)} \left(\frac{N-1+\alpha}{\sum_{j=1}^{N} \frac{1}{u_j}} \right)^{\frac{\alpha}{1-\alpha}} \left(1 - \frac{N-1+\alpha}{\sum_{j=1}^{N} \frac{u_i}{u_j}} \right)^2 - \frac{w\mu}{A} u_i^{\beta} \right\}, \tag{3}$$

where r is the interest rate. If we take the reduced form profit function and the fixed cost function as given, then the game in its essence is a game of quality competition with free entry.

3 Equilibrium

3.1 Market Structure and R&D in Each Industry

Equilibrium analysis is focused on a symmetric subgame perfect Nash equilibrium, where each firm equates the marginal benefit of increasing quality and the marginal cost. The implied best-reply function for each firm in a symmetric outcome is that

$$F = \frac{w\mu}{A} u^{\beta} = \frac{S}{r} \frac{1}{\beta} \left(\frac{\alpha}{N^3} + 2(N-1) \frac{N-1+\alpha}{N^3} \right), \tag{4}$$

where $S \equiv \alpha^{\frac{1}{1-\alpha}} L_1 \left(\frac{N-1+\alpha}{N} u \right)^{\frac{\alpha}{1-\alpha}}$ is the industry's total revenue per period.

It is assumed that any intermediate good industry has free-entry. This results in the following zero-profit condition, which equates the fixed cost of the firm $\frac{w\mu}{A} u^{\beta}$ with the present value of profit flows $\frac{S}{r} \frac{1-\alpha}{N^2}$,

$$F = \frac{w\mu}{A} u^{\beta} = \frac{S}{r} \frac{1-\alpha}{N^2}. \tag{5}$$

Figure 2 presents the above best-reply condition and zero-profit condition as two curves. The vertical axis is the industry R&D intensity and the horizontal axis is the number of firms in each industry. The graph shows that the Best-reply curve is upward sloping and the zero profit curve is downward slopping and they cross once so that it implies that an identical symmetric Nash equilibrium exists, where R&D level and market structure are simultaneously determined. This suggests that there is a two-way causality between market concentration and R&D intensity. This result defies the view which portraits a simple causal relationship between the two variables. The graph also shows that when the parameter β decreases, it shifts the Best-reply curve upward therefore increases both market concentration $1/N$ and industry R&D intensity (from **a** to **b**). The intuition for this result is as follows. A lower β means a higher degree of overall

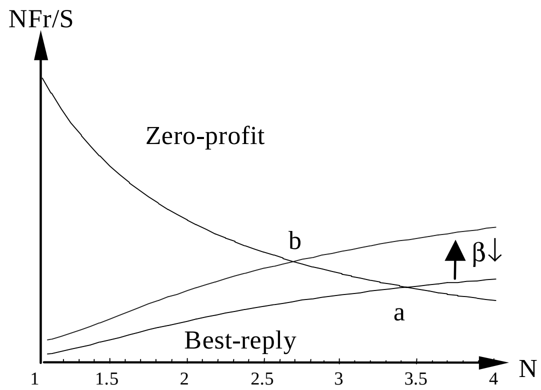

Figure 2: Market structure and industry R&D intensity

increasing returns to scale at firm level[3]. The market equilibrium finds a way to exploit this by having more concentrated market structure in each intermediate good industry. These findings are formally stated in the following

Proposition 1 [4]*The market structure and the R&D intensity of each intermediate good industry are simultaneously determined in the symmetric equilibrium. The equilibrium number of firms in each industry is* $N_f = n(\beta, \alpha) \equiv \frac{n_0 + \sqrt{n_0^2 - 2(2-\alpha)}}{2}$*, where* $n_0 \equiv \frac{(2+\beta)(1-\alpha)}{2} + 1$*. Furthermore* N_f *is increasing in* β*.*

Proof. See Appendix B.

3.2 Balanced Growth Equilibrium

Following the conventional strategy to characterize a dynamic general equilibrium model, the balanced growth equilibrium is considered, where $\frac{\dot{Y}}{Y} = \frac{\dot{c}}{c} = \frac{\dot{A}}{A} = \frac{\dot{w}}{w} = g$ is satisfied and L_1, L_2 are constant over time. Immediate implications of these include that $g = \frac{\dot{c}}{c} = \frac{r-\rho}{\varepsilon}$, that $g = \frac{\dot{A}}{A} = \delta L_2$, that $\frac{Y}{A}$ and $\frac{w}{A}$ are constant over time. A further implication is that the demand condition: $p = \alpha L_1^{1-\alpha} x^{\alpha-1}$ and the fixed cost function: $F(u; w, A) = w\frac{\mu}{A}u^{\beta}$, therefore the 3-stage game played

[3]The overall production function of each intermediate good producer is $x_i = u_i y_i = \left(\frac{A}{\mu}L_{2i}\right)^{\frac{1}{\beta}} y_i$, which is homogeneous of degree $1 + \frac{1}{\beta}$ in (L_{2i}, y_i). To see this in detail, note that L_{2i} is the input of innovative labor to achieve u_i, which is the productivity level, $L_{2i} = \frac{\mu}{A}u_i^{\beta}$ $\Leftrightarrow u_i = \left(\frac{A}{\mu}L_{2i}\right)^{\frac{1}{\beta}}$, and y_i is the final good used as input to produce the intermediate good.

[4]The results here are consistent with those found in Dasgupta and Stiglitz (1980) and Sutton (1991). The setup of the game in Dasgupta and Stiglitz (1980) is a simultaneous game, unlike the sequential game setting in this paper. Sutton (1991) differs slightly in that α is fixed to 0. With a general α in the current setting, it is found that N_f is decreasing in α.

in each industry are identical overtime. Then it is reasonable to focus attention only on symmetric outcomes.

In the symmetric balanced growth equilibrium, the total revenue in each industry per period is identically

$$S = px = \frac{\alpha Y}{A}, \tag{6}$$

where α is the cost share of all intermediate goods in final good sector, therefore αY is the total revenue of all industries per period and $\frac{\alpha Y}{A}$ is the total revenue of each industry. Given $\frac{Y}{A}$ is constant, S should also be constant.

The symmetric balanced growth equilibrium also implies that u^{β} is identical for all industries. Then the demand for innovative labor L_2 can be calculated as follows,

$$L_2 = \dot{A} N_f \frac{\mu}{A} u^{\beta}, \tag{7}$$

where $\dot{A}$ is number of emerging industries per period, N_f is the number of firms in each industry and $\frac{\mu}{A} u^{\beta}$ is the demand for innovative labor by each new firm. This condition combined with the aggregate growth equation

$$\dot{A} = \delta L_2 A. \tag{8}$$

pins down the quality level in each industry as follows

$$u^{\beta} = \frac{1}{\mu N_f \delta}. \tag{9}$$

To close the model, it remains to add the labor market clearing condition $L = L_1 + L_2$. Given the zero-profit condition $\frac{w\mu}{A} u^{\beta} = \frac{S}{r} \frac{1-\alpha}{N^2}$, a few manipulations with eq. (1), (9) and (6) to eliminate w, u^{β}, S put in place the final brick of the model as

$$r = \frac{\alpha \delta L_1}{N_f}. \tag{10}$$

The system is finally boiled down to four equations: $\begin{cases} r = \frac{\alpha \delta L_1}{N_f} \\ g = \frac{r-\rho}{\varepsilon} \\ g = \delta L_2 \\ L = L_1 + L_2 \end{cases}$, which imply[5] the balanced growth rate:

$$g = \frac{\alpha \delta L - \rho N_f}{\alpha + N_f \varepsilon}. \tag{11}$$

The expression of the growth rate has the following features captured by

[5] Other results are $L_1 = N_f \frac{\varepsilon \delta L + \rho}{\delta(\alpha + N_f \varepsilon)}$, $r = \alpha \frac{\varepsilon \delta L + \rho}{\alpha + N_f \varepsilon}$, $L_2 = \frac{\alpha \delta L - \rho N_f}{\delta(\alpha + N_f \varepsilon)}$.

Proposition 2 *The growth rate g is decreasing in the equilibrium number of firms in each industry, N_f, and it is decreasing in β.*

Proof. $\frac{\partial g}{\partial N_f} = -\frac{\alpha(\rho+\varepsilon\delta L)}{\left(\alpha+N_f\varepsilon\right)^2} < 0$. Given $\frac{\partial N_f}{\partial \beta} > 0$, $\frac{\partial g}{\partial \beta} = \frac{\partial g}{\partial N_f}\frac{\partial N_f}{\partial \beta} < 0$. ■

It provides some new insights about R&D based growth. The equilibrium market structure of each industry: N_f enters the expression and has a negative impact on growth rate. But it would be a misunderstanding if one inferred that growth could be enhanced by making industrial structures more concentrated, because the market structure appearing in the expression is endogenously determined by equilibrium. Figure 3 can help to demonstrate that change in market structure happens only as a response to the change in a parameter underlying the quality competition game, such as β. For example, a decrease in β implies an increase in the degree of scale economy. The equilibrium of the quality competition game will respond through more concentrated market structures, which will also suffice and necessitate more R&D conducted in each industry. Through spillovers from the industrial R&D the whole economy benefits from the increased degree of scale economy, in the sense of a higher growth rate.

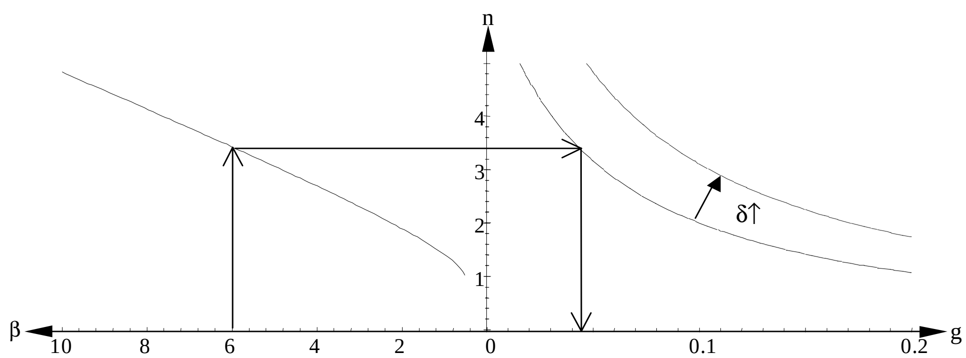

Figure 3: The Causal relationship between β, n and g: $\left.\begin{array}{cc}\beta\rightarrow & n\\ & \delta\end{array}\right\}\rightarrow g$

3.3 Discussions

3.3.1 Free Competition without Price-taking

When quality competition prevails and nonconvexity results from the nonrival feature of 'quality', the endogenous fixed cost will form a natural barrier to entry. Consequently, though without any regulation on entry, profitability will dictate free but limited entry. In this environment price-taking behavior can not be sustained but the market should nevertheless be regarded very competitive due to free entry. In this sense, a competitive market without perfect (price) competition is conceivable and nonconvexity is compatible with (quality-)competitive markets. The quality competition model stands in between the two extreme cases most studied in the literature, i.e., monopoly and perfect competition, presenting a picture of natural oligopolies.

3.3.2 Variety Expansion or Cohort Replacement

The above analysis of quality competition-based growth is presented in the context of a variety-expansion economy. It is assumed that the emergence of new grounds of quality competition relies on the expansion of varieties in a Romer (1990) manner. However, the major findings from the analysis are more general than this particular setting. In another paper of the author (Tong, 2000)[6], which presents a different version of the quality competition-base growth model, the same key results have been achieved from the context of a cohort-replacement economy. There the emergence of new grounds of quality competition is assumed to follow the replacement of earlier cohorts of technologies by successive ones in a manner akin to the Aghion-Howitt (1992) specification of 'creative destruction'. The departure from the Aghion-Howitt (1992) model is that the Tong (2000) model allows multiple firms of the same cohort to compete in quality within a single market.

Given that the variety expansion and cohort replacement are two major modes of fundamental innovations, showing that quality competition-pushed innovation can complement either variety-expansion or cohort-replacement in generating endogenous growth assures the robustness of the analysis.

3.3.3 Rents: Feasibility and Desirability

A major implication of this analysis reduces the significance of rents as a source of the incentives for innovations as compared with the literature, and calls into question the desirability of monopolization. It remains to be discussed whether this implication relies crucially on some strong assumptions used in the model. In the model, rents are dried up completely by free entry and the kind of activities which must be compensated by rents are assumed unnecessary for generating endogenous growth. These features stand on the opposite ends of the spectrums against the usual assumptions used in the literature, such as entry is completely restricted to a monopoly and the activities attracted by so generated monopolist rents are the engine of technical progress.

In the spirit of some models in the literature, it is possible that the availability of various exclusion devices may put some restrictions on free entry into quality competition. Binding restrictions of such kind generate rents, which are harvested by those who intentionally search for, create and preempt new grounds for quality

[6] The paper is available from the author at request.

competitions. If this kind of activity is a necessary contribution to aggregate technical progress, then restriction on free entry into quality competition to some extent is desirable.

In order to compare with the literature more rigorously, the next section provides an analytical framework which can accommodate this model and some others in the literature as special cases, and allows the aforementioned possibility being represented. Within this framework the robustness of some claims are scrutinized.

4 An Analytical Comparison with the Literature

4.1 A Game of Quality Competition with Exogenous Market Structure

To achieve a more comprehensive framework for the comparison with the literature, the assumption of free entry into quality competition needs to be relaxed. This modifies the game played in each industry to a 2-stage game, i.e., a game of quality competition with exogenous market structure. It is assumed that the number of firms in each industry is identically N, for $1 \leq N \leq N_f$. The equilibrium of the modified game then is characterized by the exogenous N and the best reply condition (4).

4.1.1 Rents vs. Fixed Costs

Figure 4 shows that when $N < N_f$ the zero-profit condition (5) does not hold, as it does at point **a** when $N = N_f$. Instead, there are positive net profits, i.e., rents, residual after fixed costs are covered.

The present value of the rents therefore is a function of β, α, N as follows,

$$V(\beta, \alpha, N) = \frac{S}{r}\frac{1-\alpha}{N^2} - F,$$

where $\frac{S}{r}\frac{1-\alpha}{N^2}$ is the present value of future profits flows and
$F = \frac{S}{r}\frac{1}{\beta}\left(\frac{\alpha}{N^3} + 2(N-1)\frac{N-1+\alpha}{N^3}\right)$ is the fixed cost by eq. (4).

To simplify discussion, rent and fixed cost can be denominated by the present value of industry total revenue S/r. Define $\Pi(\beta, \alpha, N) \equiv \frac{V(\beta,\alpha,N)}{S/r} = \frac{(1-\alpha)\beta N - \alpha - 2(N-1)(N-1+\alpha)}{\beta N^3}$ and $F_0(\beta, \alpha, N) \equiv \frac{F}{S/r} = \frac{1}{\beta}\left(\frac{\alpha}{N^3} + 2(N-1)\frac{N-1+\alpha}{N^3}\right)$. It is easy to see that $\Pi(\beta, \alpha, N) > 0$ for $N < N_f$, that $\Pi(\beta, \alpha, N) = 0$ for $N = N_f$.

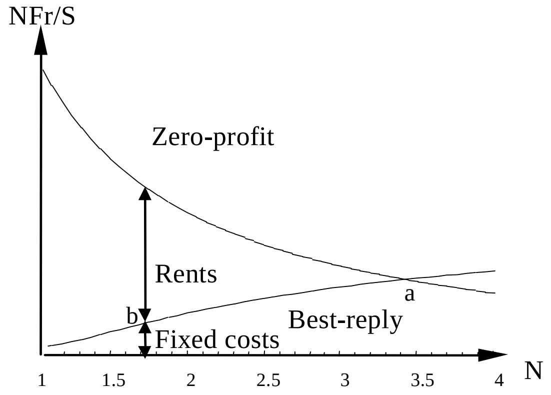

Figure 4: Market structure and spilt of profit

4.1.2 Market Structure, Profitability and Split of Profits

Figure 4 also shows that when $1 < N \leq N_f$, decreasing N will increase the share of rents and decrease the share of fixed costs. The intuition is that when the restriction on free entry is tightened the split of profits will shift in favor of rents against fixed costs. This result is formally stated by the following

Remark 1 *If* $1 < N \leq N_f$ *then* $\frac{\partial(N\Pi(\beta,\alpha,N)+NF_0)}{\partial N} < 0$, $\frac{\partial(N\Pi(\beta,\alpha,N))}{\partial N} < 0$ *and* $\frac{\partial(NF_0)}{\partial N} > 0$.

Proof. See appendix C.

4.2 Aggregate Technology, 'Competition-Push' and 'Rent-Pull'

The aggregate growth equation (8) used in the model implies that any activities which must be compensated by rents are irrelevant to generating technical progress. This needs being contrasted with the usual specification in the literature. To represent the commonly used aggregate growth equations in the literature as well as the one used in the previous model, the following formulation can be considered,

$$\dot{A} = \delta \min\left(\frac{L_2}{\theta}, \frac{L_3}{1-\theta}\right) A, \text{ for } 0 \leq \theta \leq 1, \qquad \text{(aggregate technology)}$$

where L_2 is the hired innovative labor, L_3 is the kind of innovative labor which can only be compensated by rents rather than wages. L_3 can be thought as the entrepreneurial efforts involved in searching, creating and preempting new grounds of quality competition. In the context of a variety expansion economy, entrepreneurial efforts may be needed for the emergence of new industries. The entrepreneurs are the founders of industrial firms and their efforts are compensated by residual incomes of the firms, i.e., the rents. The parameter θ determines to what extent L_2 or L_3 is a scarce resource in the economy. For example, if $\theta = 1$, then the equation is reduced to $\dot{A} = \delta L_2 A$, which implies that L_3 is not scarce at all. This is actually assumed in the model presented above. On the contrary, the existing literature usually assumes that $\theta = 0$, which implies that L_2 is not scarce at all, and $\dot{A} = \delta L_3 A$. In the middle between these two extreme cases, $\dot{A} = \delta \frac{L_2}{\theta} A$ if $\frac{L_2}{\theta} < \frac{L_3}{1-\theta}$ holds in equilibrium and $\dot{A} = \delta \frac{L_3}{1-\theta} A$ if $\frac{L_2}{\theta} \geq \frac{L_3}{1-\theta}$ holds in equilibrium. For a given value of θ, the optimal ratio between L_2 and L_3 is $\frac{L_2}{L_3} = \frac{\theta}{1-\theta}$.

Because of perfect mobility of labor between L_2 and L_3, each entrepreneur's income should equate the income of a hired innovation professional in equilibrium. Since L_2's total income equals to total fixed costs and L_3's total income equals to total rents, as a result the ratio between L_2 and L_3 should equal to the fixed costs-rents ratio, i.e.,

$$\frac{L_2}{L_3} = \frac{F_0}{\Pi}. \qquad \text{(entrepreneurial arbitrage)}$$

Therefore there exists an optimal split of profit such that $\frac{F_0}{\Pi} = \frac{\theta}{1-\theta}$ or $\frac{NF_0}{N(F_0+\Pi)} = \theta$, which imply[7] $N = N^* \equiv \frac{n_1 + \sqrt{n_1^2 - 2(2-\alpha)}}{2}$, where $n_1 \equiv \frac{(\beta\theta+2)(1-\alpha)}{2} + 1$.

Figure 5 shows the optimal split of profit: $\frac{NF_0}{N(F_0+\Pi)} = \theta$ as the dashed curve for a given value of θ. The Best-reply curve between **a** and **b** is the actual split of profit. It can be seen in the graph that point **e** represents the unique optimal market structure for growth. For a point between **a** and **e** the share of rents is suboptimally low. On the other hand, for a point between **b** and **e** the share of rents is suboptimally too high and market is too concentrated. So once quality competition is recognized as an important dimension of competition, and due to the significance of competition-push, rent-pull is no longer the sole driving force of growth, it is easy to see that market concentration is not always good for growth. This is made even clearer when the balanced growth rate is solved explicitly.

4.3 Institutional Barriers to Entry and Growth

To obtain the closed form growth function within this more comprehensive framework, one can refer to the derivations within the previous model, but need to notice that the zero profit condition (eq. (5)) is replaced by the (12) condition, that the (12) replaces eq. (8) and the new labor market clearing condition is changed to $L = L_1 + L_2 + L_3$.

[7]It implies the following quadratic equation $N^2 - \left(\frac{(\beta\theta+2)(1-\alpha)}{2} + 1\right) N + \frac{2-\alpha}{2} = 0$.

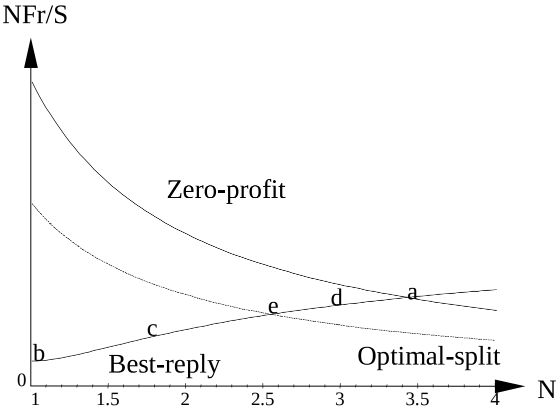

Figure 5: Optimal split of profit

Then the implied balanced growth rate is

$$g = \frac{\frac{\alpha\delta N}{1-\alpha}\min\left(\frac{F_0}{\theta}, \frac{\Pi}{1-\theta}\right)L - \rho}{\varepsilon + \frac{\alpha}{N}}.$$

Remark 2 *N^* is the optimal market structure for growth. N^* is increasing in θ. When $\theta = 1$, $N^* = N_f$. Furthermore when $N < N^*$, $\frac{\partial g}{\partial N} > 0$; when $N \geq N^*$, $\frac{\partial g}{\partial N} < 0$.*

Proof. See Appendix D.

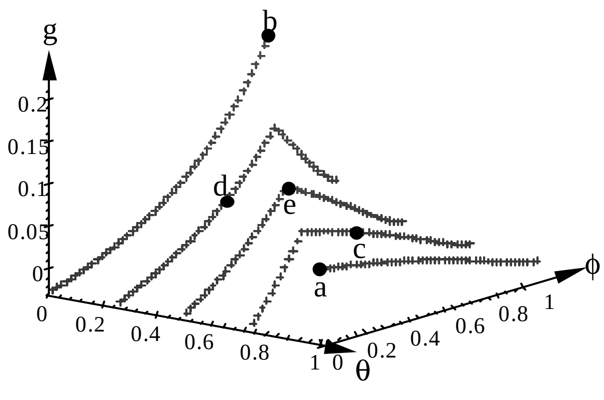

Figure 6: The relationship between ϕ and g for given θ

As expected, once some extreme assumptions, such as $\theta = 1$ or $\theta = 0$, are relaxed, the normative question about restrictions on free entry becomes very subtle. This can be seen from Figure 6, where $\phi \equiv \frac{N_f - N}{N_f - 1}$, which is a nice linear transformation of N such that when $N = N_f$, $\phi = 0$; when $N = 1$, $\phi = 1$. That $\theta = 1$ is the assumption implied by the model of the current paper, under which the equilibrium with free entry (point **a**: $\theta = 1$, $\phi = 0$) is optimal. On the other hand, that $\theta = 0$ is the standard assumption implied by the existing literature, under which the monopolist structure (point **b**: $\theta = 0$, $\phi = 1$) is optimal. On the three curves between the two extreme cases, the relationship between ϕ and g is non-monotone. As for whether a given ϕ is too big (as at point **c**), too small (as at point **d**) or optimal (as at point **e**), it all subtly depends on the parameter θ. When θ decreases, restrictions on free entry become more likely to be desirable. However, it is quite safe to claim that monopolization is not in general optimal given the belief that θ should not be too close to 0 generically. This is formally stated by the following

Remark 3 *When* $\theta > \frac{\alpha}{\beta(1-\alpha)}$, $N^* > 1$, *i.e., monopolization is not optimal for growth.*

Proof. $\theta > \frac{\alpha}{\beta(1-\alpha)} \Rightarrow n_1 > 2 - \frac{1}{2}\alpha \Rightarrow N^* > 1$. ■

Consider a numerical example: $\alpha = 1/3$, $\beta = 5$, $N_f = 3.0611$, $\frac{\alpha}{\beta(1-\alpha)} = 0.1$. It is quite reasonable to assume that $\theta > 0.1$, therefore monopolization should not be optimal in this case.

This analytical comparison with the literature supports the idea that to enhance growth direct quality competition should be allowed at least to a certain extent.

5 Extending the Model: Credit Constraint

Now the discussion returns from the comparison with the literature to the model itself, to see how it can be extended to the analysis of impact of finance on growth. As has been argued in previous sections, quality competition provides incentives for intentional investment in R&D, which is crucial for economic growth. On the other hand, quality competition-pushed investment in R&D is usually large in quantity and illiquid in the form of assets, therefore requires external finance. This renders capital market to play an important role in endogenous growth through financing R&D. In the model as presented in section 2, it has been implicitly assumed that capital market is perfect, so what happens there should be trivial. However the assumption itself is not trivial as far as economic growth is concerned, therefore deserves some reconsideration.

When capital market is not perfect due to reasons[8] which are beyond the scope of this paper, firms may have difficulties in financing R&D up to their preferred

[8] For example, Kiyotaki and Moore (1997) attribute the reason why creditors may set credit limits to that the debtors can not credibly commit not to strategically default on debt repayments.

levels. The simplest way to capture this problem is to assume that there are exogenously imposed credit limits on all firms' fixed cost expenditures. Since in the symmetric balanced growth equilibrium there is a one-to-one mapping between fixed cost level F and quality level u, credit constraints end up imposing constraints on quality levels. Therefore the analysis can be simplified by the assumption that there are exogenously imposed constraints on quality levels. Consequently the unconstrained maximization problem (3) should be modified to the following constrained maximization problem,

$$\max_{u_i} \frac{\alpha^{\frac{1}{1-\alpha}} L_1}{r(1-\alpha)} \left(\frac{N_c - 1 + \alpha}{\sum_{j=1}^{N_c} \frac{1}{u_j}}\right)^{\frac{\alpha}{1-\alpha}} \left(1 - \frac{N_c - 1 + \alpha}{\sum_{j=1}^{N_c} \frac{u_i}{u_j}}\right)^2 - \frac{w\mu}{A} u_i^\beta,$$
$$s.t. \quad : \quad u_i^\beta \leq \eta u^\beta \tag{12}$$

where $u^\beta = \frac{1}{\mu N_f \delta}$ is the unconstrained maximum point in the balanced growth equilibrium (See eq. (9)) and η is the index of credit constraint, $0 < \eta < 1$.

5.1 Corner Solution and Fragmented Market Structure

The definition of η implies that when $\eta < 1$ the unconstrained equilibrium quality level is no long feasible, therefore the credit constraint must be binding and it results in the following corner solution to the maximization problem,

$$u_i^\beta = u_c^\beta = \eta \frac{1}{\mu N_f \delta}, \tag{13}$$

where u_c is the constrained maximum point for each firm.

By eq. (9), in the balanced growth equilibrium each firm's quality level is pinned down by

$$u_c^\beta = \frac{1}{\mu N_c \delta},$$

where N_c is the number of firms in each industry. Comparing the above two equations immediately reveals the relationship between N_c and N_f as follows,

$$N_c = \frac{N_f}{\eta}, \text{ for } 0 < \eta < 1, \tag{14}$$

which then implies

Proposition 3 $\frac{\partial N_c}{\partial \eta} < 0$, *i.e., tighter credit constraint leads to more fragmented market structure.*

Figure 7 shows that when there is credit constraint, the Best-reply condition no longer holds. In stead of point **a** point **b** becomes the equilibrium, where the industry fixed cost level is lower and the market structure is more fragmented.

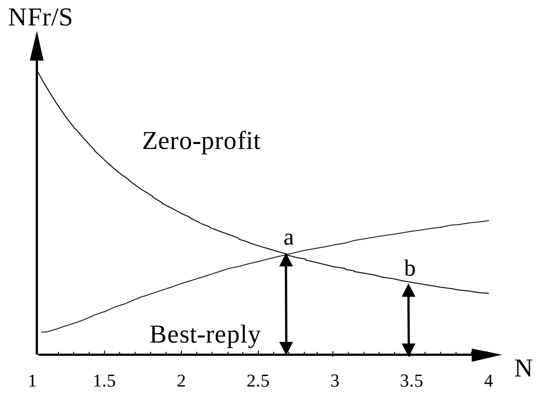

Figure 7: Credit constraint and market structure

5.2 Credit Constrained Growth

In this extension of the model, growth function (8) is replaced by

$$g = \frac{\alpha\delta L - \rho N_c}{\alpha + N_c \varepsilon} = \frac{\alpha\delta L - \rho\frac{n(\beta,\alpha)}{\eta}}{\alpha + \frac{n(\beta,\alpha)}{\eta}\varepsilon}. \tag{15}$$

Figure 8 shows the causal relationship between η, N_c and g. Tighter credit constraint, i.e., smaller η, while reducing R&D intensity, also makes market structure more fragmented. Since equilibrium market concentration and growth rate are positively correlated, tighter credit constraint will reduce balanced growth rate, i.e.,

$$\frac{\partial g}{\partial \eta} > 0. \tag{16}$$

Formally this is stated in the following

Proposition 4 *If there is binding credit constraint in capital market, i.e., $\eta < 1$, then g is increasing in η, i.e., ceteris paribus, the less credit-constrained economy has higher growth rate.*

This stark result is consistent with the empirical finding that there is a positive correlation between financial development and growth performance across countries. (King and Levine, 1993, Rajan and Zingales, 1996). The new insight gained from this analysis is to identify a new channel through which finance can affect growth. This analysis emphasizes that finance can affect competitive pattern. When firms are financially constrained in competing in quality, they are forced to compete in price in more fragmented markets. So this analysis predicts that low growth rate is likely to associate with market fragmentation, conditional on there is no institutional barriers to entry. This prediction is consistent with the evidence provided by Kumar, Rajan and Zingales (1999) showing that firm size in external-finance-dependent industries is positively correlated with financial development.

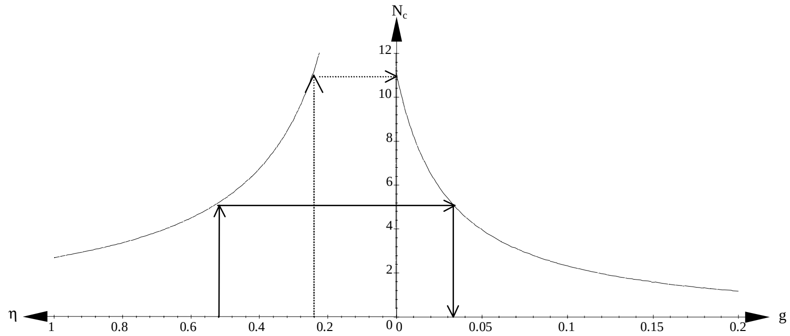

Figure 8: The Causal relationship between η, N_c and g: $\eta \rightarrow N_c \rightarrow g$

5.3 Poverty Trap

Figure 8 also shows that there is threshold of η, below which[9] the sustained positive balanced growth rate is impossible. This result can be inferred from equation (15) as follows,

$$\eta \leq \frac{\rho n(\beta, \alpha)}{\alpha \delta L} \Rightarrow g \leq 0, \tag{17}$$

which suggests

Proposition 5 *There exists a lower bound of* η, $\eta_{low} \equiv \frac{\rho n(\beta,\alpha)}{\alpha \delta L}$, *such that if* $\eta \leq \eta_{low}$, *then* $g = 0$, *i.e., growth can not sustain in an economy which has too tight credit constraints in capital market.*[10]

The implication is that quality competition as a driving force of economic growth, can not function properly due to imperfection in capital market. Severe financial underdevelopment may fail quality competition-pushed endogenous growth at all.

It is worth mentioning that the credit constraint discussed above is not necessarily due to unavailability of fund within an economy. It may be mainly due to that the infrastructure, especially institutions in the economy, is incapable to solve various informational problems or contract enforcement problems. And it is suspected this is a major reason underlying underdevelopment and poverty.

[9]It is indicated by the end of the vertical dotted arrow.

[10]Due to irreversibility of innovation, g is bounded from below by 0 in our model, but zero growth is possible.

Furthermore, although this analysis only focuses on the impact of credit constraint on quality competition and growth, the logic of the analysis can well be extended to studying the impact of constraints in human capital market, such as employees' adoption and investment in learning and improving new technologies.

As for whether market competition is good for growth, a very interesting result stems out of this extended model. That is when credit constraint is tightened, competition shifts to more in price and less in quality, consequently growth is lower. The implication is obvious, while quality competition is good for growth, the impact of price competition may be ambiguous.

6 Conclusion

It is suggested that quality competition is good for growth in the first order effect. Quality competition creates demand for innovations within individual industries, and more importantly, it creates demand for hired innovative professionals. The very existence of hired innovative professionals contributes to keeping growth opportunities continuously emerging in the economy.

Due the significance of quality competition to promoting growth, credit constraints which limit investments in quality can harm growth. Therefore financial development can affect economic growth through affecting quality competition.

In an analytical comparison with the existing R&D-based growth literature, subtlety regarding restrictions on free entry is recognized. It is found that restrictions on free entry into quality competition to a certain degree may be growth-enhancing. The is due to the fact that entrepreneurial innovative efforts may be necessary for searching and creating new opportunities of quality competition. Entrepreneurial efforts need compensation in the form of rents rather than wages. However, monopolization is not in general optimal even if it is feasible. To promote growth, direct quality competition should be reserved at least to a certain extent.

7 Appendices

A Derivation of the reduced form profit function

To ease notation, the subscript which identifies industries is dropped. Then the inverse demand function (2) faced by a representative industry becomes

$$p = \alpha L_1^{1-\alpha} x^{\alpha-1},$$

where[11] x is the industry's output, $x = \sum_{j=1}^{N} x^j$, x^j is the output of firm j and N is the number of firms. It can rewritten as

$$p = \left(\frac{\alpha^{\frac{1}{1-\alpha}} L_1}{\sum_{j=1}^{N} x^j} \right)^{1-\alpha}.$$

Consider the Cournot competition in quantity: for given (x^j), $j \neq i$, firm i chooses x^i to maximize its profit:

$$\max_{x^i} \left(\left(\frac{\alpha^{\frac{1}{1-\alpha}} L_1}{\sum_{j=1}^{N} x^j} \right)^{1-\alpha} - \frac{1}{u_i} \right) x^i,$$

where $\frac{1}{u_i}$ is the cost of producing one unit of effective quantity. u_i is the 'quality' (productivity[12]). The FOC of the problem is,

$$-(1-\alpha) \frac{\left(\frac{\alpha^{\frac{1}{1-\alpha}} L_1}{\sum_{j=1}^{N} x^j} \right)^{1-\alpha}}{\sum_{j=1}^{N} x^j} x^i + \left(\frac{\alpha^{\frac{1}{1-\alpha}} L_1}{\sum_{j=1}^{N} x^j} \right)^{1-\alpha} - \frac{1}{u_i} = 0.$$

Substituting $\left(\frac{\alpha^{\frac{1}{1-\alpha}} L_1}{\sum_{j=1}^{N} x^j} \right)^{1-\alpha}$ with p and solving for x^i gives the following equation:

$$x^i = \frac{\left(p - \frac{1}{u_i} \right) x}{(1-\alpha)\, p}.$$

The market share of firm i then is

$$\frac{x^i}{x} = \frac{1}{1-\alpha} \frac{p - \frac{1}{u_i}}{p}.$$

Summing up the market shares of all firm to eliminate x^i and x: $\sum_{i=1}^{N} \frac{x^i}{x} = 1 = \frac{1}{1-\alpha} \frac{Np - \sum_{i=1}^{N} \frac{1}{u_i}}{p}$ and solving for p leads to

$$p = \frac{\sum_{j=1}^{N} \frac{1}{u_j}}{N - 1 + \alpha}.$$

By the above equations the market share of firm i can be solved as $\frac{x^i}{x} = \frac{1}{1-\alpha} \left(1 - \frac{N-1+\alpha}{\sum_{j=1}^{N} \frac{u_i}{u_j}} \right)$.

[11] If x is interpreted as effective quantity, then this demand function can be applied to both homogeneous good or vertically differentiated good.

[12] In a vertical product differentiation story, u_i can interpreted as the consumers' perceived quality index.

The mark-up ratio of firm i is $\frac{p-\frac{1}{u_i}}{p} = \left(1 - \frac{N-1+\alpha}{\sum_{j=1}^{N} \frac{u_i}{u_j}}\right)$.

The industry output in equilibrium then is

$$x = \alpha^{\frac{1}{1-\alpha}} L_1 \left(\frac{N-1+\alpha}{\sum_{j=1}^{N} \frac{1}{u_j}} \right)^{\frac{1}{1-\alpha}}.$$

The total revenue of the industry turns out to be

$$S \equiv px = \alpha^{\frac{1}{1-\alpha}} L_1 \left(\frac{N-1+\alpha}{\sum_{j=1}^{N} \frac{1}{u_j}} \right)^{\frac{\alpha}{1-\alpha}}.$$

Finally the reduced form profit function is derived as

$$\pi_i = \frac{\alpha^{\frac{1}{1-\alpha}} L_1}{1-\alpha} \left(\frac{N-1+\alpha}{\sum_{j=1}^{N} \frac{1}{u_j}} \right)^{\frac{\alpha}{1-\alpha}} \left(1 - \frac{N-1+\alpha}{\sum_{j=1}^{N} \frac{u_i}{u_j}} \right)^2.$$

B Proof of Proposition 1

By the best reply condition eq. (4) and the zero profit condition eq. (5) the following quadratic equation of N_f can be derived: $N_f^2 - N_f \left(\frac{(2+\beta)(1-\alpha)}{2} + 1 \right) + \frac{2-\alpha}{2} = 0$, the largest positive solution of which is $N_f = n(\beta, \alpha) \equiv \frac{n_0 + \sqrt{n_0^2 - 2(2-\alpha)}}{2}$,where $n_0 \equiv \frac{(2+\beta)(1-\alpha)}{2} + 1$. It is easy to see that $\frac{\partial N_f}{\partial \beta} = \frac{\partial N_f}{\partial n_0} \frac{\partial n_0}{\partial \beta} > 0$. ■

C Proof of Remark 1

By Proposition 1, $N_f = \frac{n_0 + \sqrt{n_0^2 - 2(2-\alpha)}}{2}$,where $n_0 \equiv \frac{(2+\beta)(1-\alpha)}{2} + 1$, so $N_f > 1$ $\Rightarrow \sqrt{n_0^2 - 2(2-\alpha)} > 2 - n_0 \Rightarrow \begin{cases} n_0^2 - 2(2-\alpha) > (2-n_0)^2 & \text{if } n_0 \leq 2 \\ n_0^2 - 2(2-\alpha) > 0 & \text{if } n_0 > 2 \end{cases} \Rightarrow$

$\begin{cases} n_0 > (2-\alpha) + \frac{1}{2}\alpha & \text{if } n_0 \leq 2 \\ n_0 > 2 & \text{otherwise} \end{cases} \Rightarrow n_0 > (2-\alpha) + \frac{1}{2}\alpha > 2 - \alpha.$

$\frac{\partial (N\Pi(\beta,\alpha,N))}{\partial N} = \frac{-2(Nn_0 - (2-\alpha))}{\beta N^3} < \frac{-2((N-1)(2-\alpha))}{\beta N^3} < 0,$

$\frac{\partial (NF_0)}{\partial N} = \frac{2(N-1)(2-\alpha)}{\beta N^3} > 0$ when $N > 1$. ■

D Proof of Remark 2

Given $g = \frac{\frac{\alpha\delta N}{1-\alpha} \min\left(\frac{F_0}{\theta}, \frac{\Pi}{1-\theta}\right) L - \rho}{\varepsilon + \frac{\alpha}{N}}$.

Note that $N > N^*$ and $g > 0 \Rightarrow \frac{F_0}{\theta} > \frac{\Pi}{1-\theta}$ and $g = \frac{\frac{\alpha\delta N}{1-\alpha} \frac{\Pi}{1-\theta} L - \rho}{\varepsilon + \frac{\alpha}{N}} > 0 \Rightarrow \frac{\alpha\delta N\Pi}{(1-\alpha)(1-\theta)} L > \rho$,

$$\frac{\partial g}{\partial N}=\frac{\partial}{\partial N}\left(\frac{\frac{\alpha\delta LN^2\Pi}{(1-\alpha)(1-\theta)}-\rho N}{N\varepsilon+\alpha}\right)$$
$$=\frac{\partial}{\partial N}\left(\frac{\alpha\delta L}{(1-\alpha)(1-\theta)}\frac{(1-\alpha)\beta N-\alpha-2(N-1)(N-1+\alpha)}{\beta N}-\rho N\right)\frac{1}{N\varepsilon+\alpha}+\frac{\partial}{\partial N}\left(\frac{1}{N\varepsilon+\alpha}\right)\left(\frac{\alpha\delta LN\Pi}{(1-\alpha)(1-\theta)}-\rho\right)N$$
$$=-\left(\alpha\delta L\underset{(+)}{\frac{2\left(N^2-1\right)+\alpha}{(1-\alpha)(1-\theta)\beta N^2}}+\rho\right)\frac{1}{N\varepsilon+\alpha}-\frac{\varepsilon}{(\varepsilon N+\alpha)^2}\underset{(+)}{\left(\frac{\alpha\delta LN\Pi}{(1-\alpha)(1-\theta)}-\rho\right)}N<0.$$

Note that $N\leq N^*$ and $g>0\Rightarrow\frac{F_0}{\theta}\leq\frac{\Pi}{1-\theta}$ and $g=\frac{\frac{\alpha\delta N}{1-\alpha}\frac{F_0}{\theta}L-\rho}{\varepsilon+\frac{\alpha}{N}}>0\Rightarrow\frac{\alpha\delta LNF_0}{(1-\alpha)\theta}>\rho$

$$\frac{\partial g}{\partial N}=\frac{\partial}{\partial N}\left(\frac{\frac{\alpha\delta LN^2F_0}{(1-\alpha)\theta}-\rho N}{N\varepsilon+\alpha}\right)$$
$$=\frac{\partial}{\partial N}\left(\frac{\alpha\delta L}{(1-\alpha)\theta}N^2F_0-\rho N\right)\frac{1}{N\varepsilon+\alpha}-\frac{\alpha\delta LNF_0-\rho}{(1-\alpha)\theta}N\frac{\partial}{\partial N}\left(\frac{1}{N\varepsilon+\alpha}\right)$$
$$=\left(\frac{\alpha\delta L}{(1-\alpha)\theta}\frac{\partial}{\partial N}\left(\frac{\alpha+2(N-1)(N-1+\alpha)}{\beta N}\right)-\rho\right)\frac{1}{N\varepsilon+\alpha}-\frac{\alpha\delta LNF_0-\rho}{(1-\alpha)\theta}N\frac{\partial}{\partial N}\left(\frac{1}{N\varepsilon+\alpha}\right)$$
$$=\left(\frac{\alpha\delta L}{(1-\alpha)\theta}\frac{2N^2+\alpha-2}{\beta N^2}-\rho\right)\frac{1}{N\varepsilon+\alpha}+\frac{\alpha\delta LNF_0-\rho}{(1-\alpha)\theta}N\frac{\varepsilon}{(N\varepsilon+\alpha)^2}$$
$$<\left(\frac{\alpha\delta L}{(1-\alpha)\theta}\frac{2N^2+\alpha-2}{\beta N^2}-\frac{\alpha\delta LNF_0}{(1-\alpha)\theta}\right)\frac{1}{N\varepsilon+\alpha}+\frac{\alpha\delta LNF_0-\rho}{(1-\alpha)\theta}N\frac{\varepsilon}{(N\varepsilon+\alpha)^2}$$
$$=\left(\frac{\alpha\delta L}{(1-\alpha)\theta}\frac{2N^2+\alpha-2}{\beta N^2}-\frac{\alpha\delta L}{(1-\alpha)\theta}\frac{\alpha+2(N-1)(N-1+\alpha)}{\beta N^2}\right)\frac{1}{N\varepsilon+\alpha}+\frac{\alpha\delta LNF_0-\rho}{(1-\alpha)\theta}N\frac{\varepsilon}{(N\varepsilon+\alpha)^2}$$
$$=\frac{\alpha\delta L}{(1-\alpha)\theta}\underset{(+)}{\frac{2(2-\alpha)(N-1)}{\beta N^2}}\frac{1}{N\varepsilon+\alpha}+\underset{(+)}{\frac{\alpha\delta LNF_0-\rho}{(1-\alpha)\theta}}N\frac{\varepsilon}{(N\varepsilon+\alpha)^2}>0.\ \blacksquare$$

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