A Quantitative Analysis of the Retail Market for Illicit Drugs*

Manolis Galenianos[¶] Alessandro Gavazza[§]

December 2014

Abstract

We develop a theoretical framework to study illicit drugs markets and we estimate it using data on purchases of crack cocaine. Buyers are searching for high-quality drugs, but they determine drugs' quality (i.e., their *purity*) only after consuming them. Hence, sellers can rip off first-time buyers or can offer higher-quality drugs to induce buyers to purchase from them again. In equilibrium, a distribution of qualities persists. The estimated model implies that sellers' moral hazard reduces the average and increases the dispersion of drug purity. Moreover, increasing penalties may increase the purity and affordability of the drugs traded because doing so increases sellers' relative profitability of targeting loyal buyers versus first-time buyers.

*We are grateful to Nicola Pavoni and many seminar audiences for useful comments and discussions. Mi Luo and Simon Mongey provided excellent research assistance.

¶Department of Economics, Royal Holloway, University of London. Egham Hill, Egham TW20 0EX, United Kingdom. Email: manolis.galenianos@gmail.com.

§Department of Economics, London School of Economics. Houghton Street, London WC2A 2AE, United Kingdom. Email: a.gavazza@lse.ac.uk.

1 Introduction

How do markets for illicit commodities, such as narcotics, differ from regular markets? What would happen to the consumption and prices of narcotics if their trade were legalized? How do changes in the intensity of enforcement affect them?

We study these questions by building and estimating a model that focuses on pervasive moral hazard as the distinguishing characteristic of the retail market for illicit drugs—i.e., the "cutting" of drugs.[1] We quantify the effects of sellers' moral hazard on the quality and consumption of drugs, providing potential insights into how market outcomes might differ under legalization. We note that the presence of moral hazard leads to counter-intuitive implications for policing.

We model a market with unobserved quality, search frictions and repeated trade. Buyers with heterogeneous taste for drugs search for sellers with heterogeneous costs of supplying drugs. Buyers meet new sellers randomly and cannot observe drug purity before consuming it—i.e., illicit drugs are *experience* goods. Following the key insight of Galenianos *et al.* (2012), quality is non-contractible, creating a moral hazard problem; this is the key way in which the model captures an illegal market, in which there are no institutions to enforce quality standards.[2] Buyers sample sellers until they find one who offers a sufficiently-high level of quality, at which point they return to this preferred seller. The buyer-seller relationship persists until either the buyer meets a new seller who offers higher-quality drugs or an exogenous shock (e.g., policing) severs it.

Buyers' inability to ascertain drug quality creates a trade-off for sellers. On the one hand, moral hazard: sellers can offer low-quality drugs, thereby maximizing instantaneous profits. On the other hand, long-term relationships: sellers can offer high-quality drugs that induce buyers to purchase from them again, thereby increasing their customer base. Hence, long-term relationships are the principal counterweight to the moral hazard problem inherent in illegal markets. In equilibrium, a distribution of quality levels persists: high-cost sellers rip their (first-time) buyers off by offering zero-purity drugs, whereas low-cost sellers offer

[1] In Section 3 we document that approximately 15 percent of crack-cocaine purchases involve zero purity level—i.e., they are complete rip-offs. It is hard to find a legal market with comparable levels of outright fraud. Moreover, the cutting of drugs mostly happens at the retail level since wholesale operations (i.e., transportation) are more efficient if drugs are pure.

[2] We study the market *given* that trade is illegal, abstracting from *why* it is illegal, which may be because of paternalism or because of externalities that market participants impose on non-market participants (for an analysis of some of the externalities due to crack cocaine, see Fryer *et al.*, 2013). Hence, we do not perform a welfare analysis in the counterfactuals of Section 5.4, since, for example, externalities could be of first-order importance in aggregate welfare. Of course, some of these externalities arise exactly because trade is illegal.

high-quality drugs to attract long-term customers.

In our quantitative analysis, we estimate the model combining two distinct datasets that provide key pieces of information on the crack cocaine market in the U.S.: 1) the distribution of drug qualities offered in the market; 2) the proportion of buyers who purchase from their regular sellers; and 3) buyers' frequency of drug purchases, including the difference in purchase frequency between those who buy from regular sellers and those who buy from new sellers. The model fits the data well, and the estimation reveals that the key elements of our model play an important role in the crack-cocaine market. Specifically, search frictions are non-trivial: a buyer meets a new seller, on average, approximately every 25 days. Relationships between buyers and sellers are quite short-lived: they exogenously end, on average, every 40 days. Nonetheless, regular buyers are valuable to sellers, as they consume very frequently—on average, approximately 19 times per month—confirming that frequent users account for the vast majority of crack cocaine purchases (Kilmer *et al.*, 2014). Overall, search and information frictions imply a large dispersion of drug qualities. Finally, the estimates imply that sellers' profits are extremely skewed, with very few (low-cost) sellers reaping substantial profits and most sellers earning less than the minimum wage, consistent with the descriptive evidence reported by Levitt and Venkatesh (2000).

We use our parameter estimates to perform two counterfactual analyses. First, we quantify the importance of imperfect observability of drug quality and moral hazard for market outcomes. Since we posit that pervasive moral hazard is the defining characteristic of illegal markets, this counterfactual could provide some insights on how outcomes would differ if drugs markets were legal.[3] We modify our baseline model by assuming that buyers observe drug quality before making a purchase, thereby eliminating moral hazard. In the equilibrium of this counterfactual market, zero-purity drugs disappear, but quality dispersion persists due to search frictions. Quantitatively, average quality increases by 20 percent. Interestingly, the standard deviation of purity decreases by approximately 75 percent relative to the baseline case, implying that imperfect observability, rather than search frictions, mainly accounts for the dispersion of quality.[4] Overall, this change in the distribution of purity triggers an increase in the aggregate consumption of crack cocaine by approximately nine percent.

[3] Of course, legalization would affect several other features of the market. For example, search frictions would decline.

[4] A reduction in quality dispersion may bring health benefits: Caulkins (2007) argues that greater variability in purity predicts overdoses, because users inadvertently consume more pure drug than they intend to when they purchase drugs that are more pure than is typical.

Our second counterfactual studies how penalties on buyers and sellers affect market outcomes. The motivation is that in the past 30 years the U.S. has markedly increased the enforcement and severity of drug laws—the so-called "war on drugs"—which has resulted in the tripling of arrests for drug-related offenses.[5] Interestingly, during the same period, drugs have become dramatically *cheaper* and *purer*. In our model, a lower enforcement may lead to lower drug quality. This counterintuitive result is the outcome of the interaction between moral hazard and long-term relationships. Lower enforcement on sellers increases their number. Hence, the rate at which buyers meet with new sellers increases and the expected duration of a buyer-seller relationships falls. Therefore, the profitability of cheating relative to that of establishing long-term relationships increases, which leads to more cheating and lower average quality. Quantitatively, we find that a 15-percent decrease in sellers' penalties leads to a 3.5-percent decrease in average quality (and similar results obtain when we reduce buyers' penalties). Hence, our analysis suggests that increasing penalties may have contributed to the observed increased purity and affordability of retail drugs in the U.S. Of course, the market for drugs has changed in many ways over time (among others, through economies of scale in the transportation of drugs to the U.S.); nonetheless, we find it interesting that our model is consistent with observed trends in the time-series.

The paper proceeds as follows. Section 2 reviews the literature. Section 3 introduces the data. Section 4 presents the theoretical model. Section 5 presents our quantitative analysis: we estimate the model, illustrate its main implications, and perform counterfactual analyses. Section 6 concludes. Appendices A and B collect all proofs and report the derivation of the density of buyers' preferences in the ADAM dataset, respectively. Appendix C further presents the details of the solution of the model with observable quality.

2 Related Literature

The paper contributes to several strands of the literature. The first is the literature on illegal markets. Most theoretical models of illegal markets build on the traditional economic assumptions of perfect information and/or a centralized market: see, e.g., Becker *et al.* (2006) for an influential analysis and Bushway and Reuter (2008) for a review of this literature.[6] This framework, however, abstracts from two defining features of illicit markets on

[5] At the same time the number of arrests for non-drug related offenses has barely changed (Kuziemko and Levitt, 2004).

[6] Notable exceptions are the discussion of information issues in drug markets in Reuter and Caulkins (2004), and the analysis of limited information about drug accessibility in Jacobi and Sovinsky (2013).

which we focus: moral hazard and search frictions. Hence, they cannot explain the extensive amount of cheating that we observe in the data and the implications it has for policy.

The second is the literature that studies firms' quality decisions when quality is not observable (or not contractible). To our knowledge, papers in this strand of literature examine markets for legal goods. Among the theoretical contributions, Gale and Rosenthal (1994) present a model in which buyers have to pay a cost before finding a high-quality seller. Many empirical contributions have analyzed the role of quality certification and consumers' and suppliers' responses to it; for a thorough survey, see Dranove and Jin (2010). Empirical analyses of moral hazard have focused mainly on the behavior of intermediaries; see, for example, Iizuka (2007, 2012) for the case of physicians and prescription drugs. Our paper examines the importance of moral hazard in the context of an illegal market, where search and matching frictions are pervasive, and innovates on previous descriptive empirical work by estimating a dynamic equilibrium model that allows a quantification of these effects on market outcomes.

Finally, this paper is also related to the empirical literature on the structural estimation of search models. Almost all empirical applications to product markets use static models: recent contributions include Hortaçsu and Syverson (2004), Hong and Shum (2006), Wildenbeest (2011), and Allen *et al.* (2014). Notable exceptions are Cebul *et al.* (2011), who use a dynamic search model to understand the role of buyers' turnover in health insurance pricing, and Gavazza (2013), who estimates a dynamic model of a decentralized asset market to study the effects of intermediaries on asset allocations and asset prices. Instead, we build a dynamic model that highlights the role of long-term buyer-seller relationships.[7] Most applications of dynamic search models focus on labor markets; Eckstein and Van den Berg (2007) provide an insightful survey of this literature and, within the labor-market context, Bontemps *et al.* (1999) is the closest empirical paper. We innovate on all previous empirical contributions by quantifying the effects of an additional friction—i.e., the imperfect observability of product quality at the time of the transaction—and of its implications for how penalties affect market outcomes.

[7] Hence, the paper also touches upon some of the issues that the literature on customer markets highlights. Most empirical investigations focus on markets for services, whereas we focus on a product market. See, for example, Boot (2000) for a survey of the literature on relationship banking.

3 Data

The available data dictate some of the modeling choices of this paper. For this reason we describe the data before presenting the model. This description also introduces some of the identification issues that we discuss in more detail in Section 5.2.

3.1 Data Sources

We combine two distinct datasets. The first is an extensive database on drug purchases. The second is a survey that collects information about drug use among arrestees. We now describe each dataset in more detail.

STRIDE—The System to Retrieve Information from Drug Evidence (STRIDE) is a database of drug exhibits sent to Drug Enforcement Administration (DEA) laboratories for analysis. Exhibits in the database are from the DEA, other federal agencies, and local law enforcement agencies. The data contain records of acquisitions of illegal drugs by undercover agents and DEA informants. Economic analyses of markets for illegal drugs have widely used STRIDE, although it is not a representative sample of the drugs available in the United States.[8]

The entire dataset has approximately 915,000 observations for the period 1982-2007 for a number of different drugs and acquisition methods. We focus on crack cocaine and keep the observations acquired through purchases (i.e., we drop seizures) and clean the data of missing values and other unreliable observations, as Arkes *et al.* (2008) suggest. Our quantitative analysis of Section 5 uses data for the years 2001-2003 because of the time limitations of our other data source, as described below. Moreover, since the focus of our model is on retail transactions, we include in our estimation sample purchases with a value of less than $200 in real 2003 dollars.

ADAM—The Arrestee Drug Abuse Monitoring (ADAM) data set is a quarterly survey of persons arrested or booked on local and state charges within the past 48 hours in various metropolitan areas in the United States.[9] The survey asks questions about the use of drugs and alcohol. The arrestees participate in the survey voluntarily under full confidentiality:

[8]Horowitz (2001) notes that the time series of drug prices in Washington, D.C. differ depending on which agency collected the data. However, Arkes *et al.* (2008) show that these inconsistencies disappear almost entirely by controlling for the size of the transaction (above or below five grams). Hence, we restrict our analysis to transactions with a value below 200 constant 2003 dollars.

[9]The number of these areas changes from years to year based on the availability of the data. From 2001 to 2003, it has been 33, 36 and 39, respectively.

Table 1: Summary Statistics

Panel A: STRIDE (N=2,310)	Mean	Std. Dev.	Min	Max
Price (2003 dollars)	95.174	48.163	1.052	200.000
Weight (Grams)	1.103	1.086	0.004	9.500
Purity (%)	56.194	28.042	0.000	98.000
Pure Quantity	0.643	0.682	0.000	4.422
Pure Grams per $100	2.086	1.714	0.000	9.980
Panel B: ADAM (N=64,462)				
Obtained Drug in Last 30 Days (%)	16.899	37.474	0.000	100.000
Purchased from Regular Dealer (%)	52.481	49.941	0.000	100.000
Purchases in Past 30 Days, Matched	16.331	11.124	1.000	30.000
Purchases in Past 30 Days, Unmatched	11.548	10.419	1.000	30.000

Notes—This table provides summary statistics of the variables used in the empirical analysis. Panel A presents summary statistics of the variables obtained from the STRIDE dataset; Panel B presents summary statistics of the variables obtained from the ADAM dataset. Drug prices have been deflated using the GDP Implicit Price Deflator, with 2003 as the base year.

only about ten percent of the arrestees reject the interview request (Dave, 2008). In addition to the interviews, participants provide urine samples, which are analyzed for validation of arrestees' self-reported drug use. Since 2000, the survey has included a drug market procurement module and collects information on the arrestee's most recent drug purchase for those who report having used drugs in the previous 30 days. Information collected includes the number of drug purchases in the past 30 days and whether arrestees last purchased drugs from their regular dealer. We have data from the 2001-2003 surveys.

3.2 Data Description

Table 1 provides summary statistics of the main variables used in the quantitative analysis. Panel A refers to the STRIDE dataset and Panel B to the ADAM dataset.

Panel A reports some interesting patterns. Because most transactions happen at round dollar values (i.e., 50 or 100 dollars), the heterogeneity in Price is small relative to the heterogeneity in Pure Quantity (i.e., the product of Weight and Purity): the coefficient of variation of Pure Quantity is 60-percent larger than that of Price.[10] We take the ratio of Pure Quantity and Price to construct the variable Pure Grams per $100;

[10] Some variation in Price across years is due to the fact that we deflate prices.

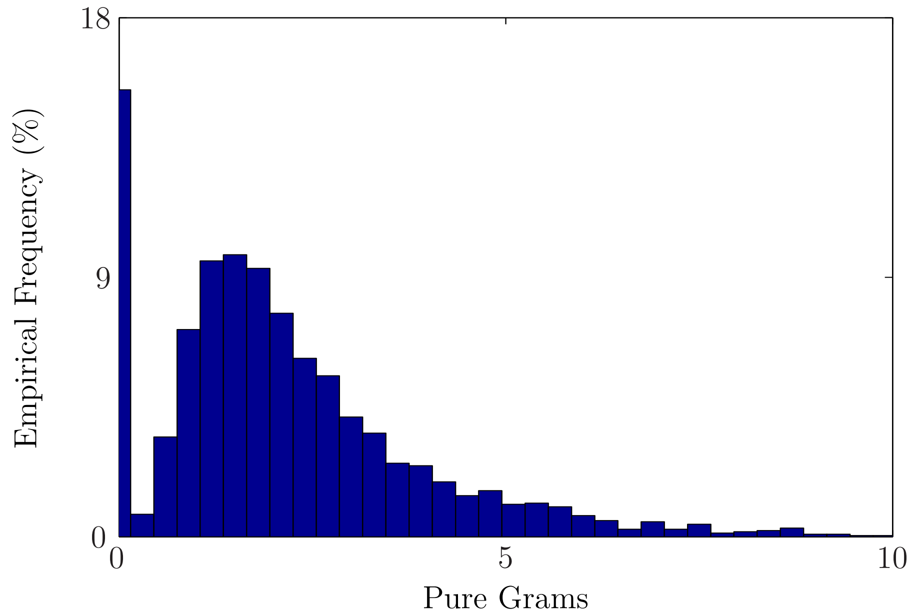

FIG. 1: Histogram of pure grams of crack cocaine per \$100.

Figure 1 displays its empirical distribution, which displays substantial variation, with 15.4 percent of the observations having a value of zero—i.e., complete rip-offs.[11] Moreover, there are almost no observations with a positive, but very small amount of cocaine, suggesting that the distribution features a gap between the mass point at zero and the density of positive qualities (Galenianos *et al.*, 2012).

Panel B reports that approximately 17 percent of all arrestees purchased crack cocaine in the past 30 days. Among that group, the average number of PURCHASES IN PAST 30 DAYS equals 12.98 (thus, the unconditional average of PURCHASES IN PAST 30 DAYS is 9.6), thereby confirming that frequent users account for the vast majority of drug expenditures (Kilmer *et al.*, 2014). Of those who purchased crack cocaine, 52.5 percent report buying from their regular source. Interestingly, individuals purchasing from their regular dealers report an average of 16.3 PURCHASES IN PAST 30 DAYS, whereas individuals purchasing either from an occasional source or from a new source have an average of 11.5 PURCHASES IN PAST 30 DAYS. The model will interpret this difference as different consumption rates between buyers who are currently matched to a seller and buyers who are currently not matched, taking into account that buyers will choose to match with a seller depending on their preferences for drugs.

Overall, these two datasets provide a rich description of the retail crack-cocaine market

[11] Galenianos *et al.* (2012) report that geographic and temporal variation is not large, contributing approximately 20 percent to the overall variation.

and are well-suited for investigating the role of search frictions, imperfect observability, and buyer-seller relationships in this market. Specifically, our model interprets the dispersion of Pure Grams per $100 as originating from both search frictions and imperfect observability, thereby allowing us to calculate the contribution of each to the overall dispersion. Moreover, the ADAM dataset is useful in measuring the frequency of buyer-seller long-term relationships and buyers' consumption rates. Since ADAM likely oversamples drug users, we will explicitly account for sample selection in our quantitative analysis.

With all their advantages, however, the datasets pose some challenges. Most importantly, both datasets are cross-sectional and, therefore, we do not observe buyers' and sellers' behavior over time. Specifically, we do not observe sellers transacting with several buyers or the dynamics of the terms of trade within individual buyer-seller relationships. These limitations imply that a model in which sellers discriminate between different buyers, while theoretically feasible, would be difficult to identify with the available data. Similarly, we do not observe whether sellers vary the quality of their offerings during their relationships with buyers, and theory argues that quality could either increase or decrease over time (Board and Meyer-ter Vehn, 2013). Hence, in the absence of more-detailed measurement, our model will (successfully) match the available data by focusing on heterogeneity across sellers. We will discuss data limitations and their implications for our modeling assumptions further in Section 6.

4 The Model

4.1 Environment

Time runs continuously, the horizon is infinite and the future is discounted at rate r.

There is a continuum of potential buyers of measure $\bar{B}$ who are heterogeneous with respect to their preferences for consuming drugs: buyers' marginal utility z of consuming drugs is distributed according to a continuous, connected and log-concave distribution $\bar{M}(\cdot)$ with support $[0, \bar{z}]$. Buyer heterogeneity may arise because of differences in innate preferences or the strength of addiction, as in Becker and Murphy (1988).[12] Each buyer decides whether to participate in the market. If he does not participate, his payoff is zero. If he participates, he pays a flow cost K_B and trades with sellers. Buyers' participation cost, like sellers' cost below, represents the reduced convenience, ethical constraints, and criminal punishments

[12] However, we do not model addiction, over and above what buyers' entry decision captures.

imposed on traders (Becker *et al.*, 2006). We denote the measure of participating buyers by B. Buyers maximize their expected discounted utility.

There is a continuum of potential sellers of measure $\bar{S}$ who are heterogeneous with respect to their cost of providing quality drugs: seller's marginal cost c of providing quality is distributed according a continuous and connected distribution $\bar{D}(\cdot)$ with support $(0, \infty)$. Seller heterogeneity might arise because of skill differentials or differences in connections with upstream suppliers. Each seller decides whether to participate in the market, in which case he pays flow cost K_S. We denote the measure of participating sellers by S. Sellers maximize their steady-state profits.

The buyers and sellers who participate in the market meet and trade with each other. At any point in time, a buyer is either matched with a seller (his *regular* seller) or he is unmatched. There are two types of meetings: *new* meetings, where a buyer and a seller meet for the first time, and *repeat* meetings, where a buyer meets with his regular seller. At a meeting, a transaction takes place, and, after a new meeting, the buyer decides whether or not to form a match with the new seller (see below).

In a transaction, the buyer pays a fixed price p and receives quality q, which corresponds to the pure quantity of drugs in the trade. A type-c seller has costs cq to supply drugs of quality q. At the time of the transaction, the buyer and seller both observe p, but the buyer does not observe quality q. After the transaction, the buyer consumes his purchase and perfectly learns q. The instantaneous utility that a type-z buyer receives from consuming quality q equals zq.

The main assumption about sellers' behavior is that, once they decide on the quality level to offer, they commit to that choice. That is, a seller supplies the same quality at all times, and, as a result, a buyer knows the quality that he will receive from a particular seller in the future after the first purchase from him.[13]

After transacting with a new seller, the buyer decides whether to *match* with that seller. Specifically, if the buyer is unmatched, he chooses whether to match with the seller or to remain unmatched; if the buyer is already matched with a different seller, he chooses whether to match with the new seller and sever the earlier match, or to return to his previous regular seller. Therefore, a match dissolves endogenously when a matched buyer participates in a new meeting and chooses to switch to the new seller. Additionally, a match dissolves at the exogenous rate δ, and the buyer becomes unmatched.

The meeting function $m(B, S)$ determines the flow of new meetings. The meeting function

[13] See Section 3 for a discussion of the difficulties of identifying a model with alternative assumptions. See Galenianos *et al.* (2012) for a theoretical justification of this assumption.

has constant returns to scale, is increasing and concave in both arguments, and satisfies $m(0, S) = m(B, 0) = 0$ and the Inada conditions. Let $\theta = \frac{B}{S}$ denote the buyer-seller ratio in the market, and let $\alpha_B(\theta) = \frac{m(B,S)}{B}$ be the rate at which a buyer meets with a new seller and $\alpha_S(\theta) = \frac{m(B,S)}{S}$ the rate at which a seller meets with a new buyer. Our assumptions imply that $\alpha_B(\cdot)$ is strictly decreasing and $\alpha_S(\cdot)$ is strictly increasing in θ.

The flow of repeat meetings equals γ, which is the rate at which a matched buyer contacts his regular seller. In these meetings there is no asymmetric information about q since the seller always offers the same quality.

To summarize, all buyers (whether matched or unmatched) meet new sellers at the same rate $\alpha_B(\theta)$, and matched buyers additionally meet their regular sellers at rate γ.

A potential buyer of type z decides whether to participate in the market and, if so, his reservation quality for becoming matched with a new seller (the reservation quality of a matched buyer is, trivially, the quality that he receives from his regular seller). Let R_z be the reservation quality of an unmatched type-z buyer and $\bar{V}_z$ be his value of being unmatched in the market. A type-z buyer participates in the market if and only if

$$r\bar{V}_z \geq K_B.$$

The outcome of buyers' actions is the measure of participating buyers B and the distribution $H(\cdot)$ of their reservation qualities.

A potential seller of type c decides whether to participate in the market and, if so, the quality that he offers. Steady state profits of a type-c seller offering quality q are:

$$\pi_c(q) = t(q)(p - cq),$$

where $t(q)$ denotes the steady state flow of transactions when offering q and $p - cq$ is the margin per transaction. A type-c seller enters the market if and only if

$$\pi_c(q^*(c)) \quad \geq \quad K_S,$$

where $q^*(c)$ denotes his profit-maximizing quality.

The outcome of sellers' actions is the measure of participating sellers S and the distribution $F(\cdot)$ of offered qualities.

Definition 1 *An equilibrium consists of the actions of buyers* $\{B, H(\cdot)\}$ *and the actions* *of sellers $\{S, F(\cdot)\}$ such that entry, reservation qualities and offered qualities are chosen optimally and the market is in steady state.*

4.2 The Buyers

We derive buyers' optimal actions, taking as given the distribution of offered qualities $F(\cdot)$ and the number of participating sellers S.

Consider a buyer of type z who takes the behavior of sellers and other buyers as given. When unmatched, the buyer meets a new seller at rate $\alpha_B(\theta)$. At the meeting he pays price p and receives quality x, randomly drawn from $F(\cdot)$. If the quality level x exceeds his reservation R_z, the buyer matches with the seller, thereby obtaining capital gain $V_z(x) - \bar{V}_z$, where $V_z(x)$ is the value of being matched with a seller offering x. The value of an unmatched type-z buyer satisfies:

$$r\bar{V}_z = \alpha_B(\theta)\Big(z\int_0^{\bar{q}} x dF(x) + \int_{R_z}^{\bar{q}} (V_z(x) - \bar{V}_z) dF(x) - p\Big). \tag{1}$$

When matched with a seller who offers quality q, three events might occur: (1) at rate γ, the buyer meets his regular seller, pays p and receives q; (2) at rate δ, an exogenous shock destroys the match and the buyer becomes unmatched obtaining a (negative) capital gain $\bar{V}_z - V_z(q)$; and (3) at rate $\alpha_B(\theta)$, he meets a new seller, pays p and receives quality x, drawn from the quality distribution $F(\cdot)$. If the new seller's quality exceeds q, then the buyer matches with the new seller and leaves his current seller, thereby obtaining capital gain $V_z(x) - V_z(q)$. The value of a type-z buyer matched with a seller offering q satisfies:

$$rV_z(q) = \gamma(zq - p) + \alpha_B(\theta)\Big(z\int_0^{\bar{q}} x dF(x) + \int_q^{\bar{q}} (V_z(x) - V_z(q)) dF(x) - p\Big) + \delta(\bar{V}_z - V_z(q)). \tag{2}$$

The Proposition characterizes a buyer's decision to participate and his reservation qualities.

Proposition 2 *Given $F(\cdot)$ and θ:*

1. *The value of participating in the market for a type-z buyer satisfies:*

$$r\bar{V}_z = \alpha_B(\theta)\Big(z\int_0^{\bar{q}} x dF(x) + z\int_{\frac{p}{z}}^{\bar{q}} \frac{\gamma(1 - F(x))}{r + \delta + \alpha_B(\theta)(1 - F(x))} dx - p\Big). \tag{3}$$

2. *The optimal reservation quality is*

$$R_z = \frac{p}{z}. \tag{4}$$

3. *There exists $\hat{z}(F,\theta)$ such that a type-z buyer participates in the market if and only if $z \geq \hat{z}(F,\theta)$.*

Proof. See Appendix A. ■

Notice that the reservation quality is decreasing in buyers' marginal utility. For a given q, a buyer's utility from consuming is increasing in z and, thus, so is his willingness to match with the sellers who offers q. Furthermore, R_z does not depend on the distribution of offered qualities, $F(\cdot)$, because the arrival rate of new sellers is the same when matched and unmatched.[14]

We now aggregate buyers' decisions, thereby obtaining the measure of participating buyers and the buyer-seller ratio θ. Conditional on the quality distribution $F(\cdot)$ and the number of participating sellers S, we have the following characterization of the market:

Proposition 3 *Given $F(\cdot)$ and S:*

1. *If $p \geq \bar{z}\int_0^{\bar{q}} xdF(x)$, then there is no buyer entry: $B = 0$.*

2. *If $p < \bar{z}\int_0^{\bar{q}} xdF(x)$, then there is a marginal buyer type $z^* \leq \bar{z}$ such that a buyer participates in the market if and only if $z \geq z^*$.*

3. *The marginal buyer type is given by the solution to:*

$$\alpha_B\left(\frac{\bar{B}(1-\bar{M}(z^*))}{S}\right)\left(z^*\int_0^{\bar{q}} xdF(x) + z^*\int_{p/z^*}^{\bar{q}} \frac{\gamma(1-F(x))}{r+\delta+\alpha_B\left(\frac{\bar{B}(1-\bar{M}(z^*))}{S}\right)(1-F(x))}dx - p\right) = K_B. \tag{5}$$

Proof. See Appendix A. ■

The Proposition's results are intuitive. If the average quality in the market is low enough that the most eager buyer receives no surplus from a purchase, then no buyers enter the market.[15] Otherwise, there is a unique marginal type such that buyers enter only if their marginal utility of consuming drugs is higher.

[14] See Galenianos *et al.* (2012) for a different modeling assumption where R_z does depend on the full distribution.

[15] Though intuitive, this is not immediate because the option value of climbing the quality ladder needs to be taken into account.

We now describe the distribution of buyer types and reservation qualities. Let $z(R)$ denote the buyer type whose reservation quality equals R. Rearranging equation (4) we have:

$$z(R) = \frac{p}{R}.$$

Moreover, note that $R_{z(R)} = R$ and $z \leq z(R) \Leftrightarrow R_z \geq R$. Given z^*, the equilibrium distribution of reservation qualities mirrors the distribution of marginal utilities. The corollary summarizes the results.

Corollary 4 *The marginal buyer type z^* completely characterizes buyers' behavior.*

1. *The measure of buyers in the market is:*

$$B = \bar{B}(1 - \bar{M}(z^*)).$$

2. *The distribution of reservation qualities in the market retains the log-concavity of $\bar{M}(\cdot)$ and satisfies:*

$$H(R) = \begin{cases} 0 & \text{if } R \leq \underline{R}, \\ \frac{1-\bar{M}\left(\frac{p}{R}\right)}{1-\bar{M}(z^*)} & \text{if } R \in [\underline{R}, \bar{R}], \\ 1 & \text{if } R \geq \bar{R}, \end{cases}$$

where $\underline{R} = R_{\bar{z}} = \frac{p}{\bar{z}}$ and $\bar{R} = R_{z^*} = \frac{p}{z^*}$.

4.3 The Sellers

This Section derives sellers' optimal decisions, taking as given the marginal buyer type z^*, which determines the measure B of buyers who participate and the (log-concave) distribution of reservation qualities $H(\cdot)$.

An individual seller's transactions come from two sources: new buyers, who purchase from that seller for the first time, and repeat buyers, who purchased from that seller in the past and decided to match with him. The flow of transactions is $t(q) = t_N + t_R(q)$, where t_N represents sales to new buyers, and $t_R(q)$ represents sales to the seller's regular buyers. Steady-state profits, therefore, equal:

$$\pi_c(q) = \Big(t_N + t_R(q)\Big)\Big(p - cq\Big).$$

The flow of new transactions equals the rate at which new buyers contact an individual seller, which does not depend on the quality offered:

$$t_N = \alpha_S(\theta) = \theta\alpha_B(\theta).$$

The flow of repeat transactions to a seller who offers quality q depends on the number of regular buyers, denoted by $l(q)$, and the rate γ at which these buyers contact their regular seller:

$$t_R(q) = \gamma l(q).$$

Unlike new transactions, the flow of repeat transactions depends on the seller's offered quality. A seller who offers quality q gains regular customers when he meets with unmatched buyers whose reservation is below q and with matched buyers whose regular seller offers less than q. Similarly, he loses his regular customers when they meet with a seller offering quality greater than q and when the match exogenously dissolves, which happens at rate δ.

We characterize sellers' actions in three steps. First, we derive some necessary conditions for the distribution of offered qualities. Second we derive an individual seller's profits. Third we characterize the full distribution of offered qualities and sellers' entry decisions.

In equilibrium, the quality distribution satisfies the following features:

Lemma 5 *In equilibrium, the quality distribution $F(\cdot)$ is continuous on $[0, \bar{q}]$ and has support on a subset of $\{0\} \cup [\underline{q}, \bar{q}]$ for some $\underline{q} \in [\underline{R}, \bar{R}]$.*

Proof. See Appendix A. ■

The following Proposition characterizes the steady-state profits of a type-c seller, taking the action of buyers and other sellers as given.

Proposition 6 *Given $H(\cdot)$, θ and $F(\cdot)$, the steady-state profits of a type-c seller who offers quality q are:*

$$\pi_c(q) = \begin{cases} \alpha_B(\theta)\theta\left(1 + \frac{\gamma\delta H(q)}{\left(\delta+\alpha_B(\theta)(1-F(q))\right)^2}\right)(p - cq) & \text{if } q \geq \underline{R}, \\ \alpha_B(\theta)\theta p & \text{if } q < \underline{R}. \end{cases}$$

Proof. See Appendix A. ■

We characterize the optimal decisions of sellers, taking as given the measure of buyers B and buyers' reservation distribution $H(\cdot)$.

Proposition 7 *Given B and $H(\cdot)$,*

1. *Positive quality is always offered in the market. There is a unique marginal seller type $\bar{c}$ such that sellers with $c \leq \bar{c}$ enter the market and offer quality $q^*(c)$, which is strictly decreasing in c.*

2. *The measure $S_0 > 0$ of sellers who offer zero quality is uniquely determined. $S_0 > 0$ if and only if the measure of potential sellers is low enough.*

Proof. See Appendix A. ■

The subsequent quantitative analysis will focus on the case where cheating occurs in equilibrium ($S_0 > 0$) as this is the empirically relevant case.

Proposition 7 illustrates the importance of imperfect observability of quality and moral hazard for market outcomes: sellers with high cost of providing drugs participate in the market but specialize in cheating their buyers—i.e., offering zero quality—and still retain a positive flow of sales. In a market with perfect observability of quality, such sellers would not be in the market.

Corollary 8 *Seller behavior is summarized as follows:*

1. *The measure of sellers in the market is $S = \bar{S}\ \bar{D}(\bar{c}) + S_0$.*

2. *The type distribution of sellers in the market who offer positive quality is $D(c) = \frac{\bar{D}(c)}{\bar{D}(\bar{c})}$ for $c \leq \bar{c}$.*

3. *The quality distribution is $F(q) = 1 - (1 - F(0))D\Big(q^{*-1}(q)\Big)$ for $q > 0$ and $F(0) = \frac{S_0}{S}$.*

4.4 Equilibrium

The equilibrium is a fixed point on the marginal buyer type. Given z^*, the measure of participating buyers B and the distribution of their reservations $H(\cdot)$ are uniquely determined (Corollary 4). This, in turn, determines the marginal seller type $\bar{c}$, the measure of sellers who enter the market S, (Proposition 7) and the quality distribution $F(\cdot)$ (Corollary 8). Finally, $F(\cdot)$ and S determine the marginal buyer type (Proposition 3, equation (5)). The marginal type is defined on a closed and bounded set $[0, \bar{z}]$, and Proposition 9 follows.

Proposition 9 *An equilibrium exists.*

5 Quantitative Analysis

The model does not admit an analytic solution for all endogenous outcomes. Hence, we choose the parameters that best match moments of the data with the corresponding moments computed from the model's numerical solution. We then study the quantitative implications of the model evaluated at the estimated parameters.

5.1 Parametric Assumptions

We estimate the model using the data described in Section 3, assuming that they are generated from the model's steady state. We further assume that the empirical quality distribution obtained from STRIDE corresponds to the distribution $F(\cdot)$ that first-time buyers face. We set the unit of time to be one month, as this is the period over which we observe consumption frequencies in ADAM.

Unfortunately, the data lack some detailed information to identify all parameters. Therefore, we fix some values. Specifically, the discount rate r is traditionally difficult to identify, and we set it to $r = .01$. Moreover, since we use the normalized variable Pure Grams per $100, we set the price equal to $p = \$100$. Furthermore, we set sellers' monthly opportunity cost K_S to be $\$1,500$, which is broadly in line with the bottom of the distribution of drug dealers' earnings reported by Levitt and Venkatesh (2000).

We further make parametric assumptions about the distributions of buyers' and sellers' heterogeneity. Specifically, we assume that the distribution $\bar{M}(\cdot)$ of buyers' taste for crack cocaine z is a mixture distribution: a fraction λ has taste $z = 0$—i.e., they will never enter the market—and a fraction $1 - \lambda$ has taste z that follows a lognormal distribution with unknown parameters μ_z and σ_z; hence, the maximum number of active buyers is $(1 - \lambda)\bar{B}$. Since the reservation quality of a buyer with type $z > 0$ is $R_z = \frac{p}{z}$, it follows that the distribution $H(\cdot)$ of reservation qualities is also lognormal with parameters $\mu_R = \log p - \mu_z$ and σ_z.

Moreover, we assume that the distribution of the inverse of sellers' costs $1/c$ is a Pareto distribution with lower bound $\frac{1}{c_M}$ and shape parameter $\xi \geq 1$. This implies that the distribution of costs c is:

$$\bar{D}(c) = \left(\frac{c}{c_M}\right)^{\xi}, \ c \in [0, c_M] \tag{6}$$

and, thus, the truncated distribution of costs of active sellers equals

$$D(c) = \left(\frac{c}{\bar{c}}\right)^{\xi}, \ c \in [0, \bar{c}]$$

where $\bar{c}$ is the cost of the marginal active seller that offers the lowest quality level $\underline{q}$. The shape parameter ξ captures the dispersion of costs. If $\xi = 1$, the cost distribution is uniform. As ξ increases, the relative number of high-cost sellers increases, and the cost distribution is more concentrated at these higher cost levels. As ξ goes to infinity, the cost distribution becomes degenerate at the upper bound.[16]

We further assume that drug qualities q are measured with error. More specifically, we assume that the reported qualities $\hat{q}$ and the "true" qualities q are related as:

$$\hat{q} = q\epsilon,$$

where ϵ is a measurement error. We assume that ϵ follows a lognormal distribution and restrict its mean to equal 1—i.e., measurement is unbiased—which implies that the parameters μ_ϵ and σ_ϵ of the lognormal distribution satisfy $\mu_\epsilon = -.5\sigma_\epsilon^2$.

The assumption of measurement error on wages is quite common in the literature that structurally estimates search models of the labor market. In our application, it is plausible as well, and it could also account for some unobserved seller behavior that the model does not consider (i.e., price discrimination), thereby allowing us to fit the quality distribution better. For example, as Lemma 5 highlights, the model implies a gap in the quality distribution between the complete rip-offs $q^* = 0$ and the minimum positive quality $\underline{q}$. Figure 1 shows that the empirical distribution displays this qualitative feature, and the measurement ϵ allows it to more precisely match its magnitude. Similarly, Figure 1 shows that the quality distribution displays a long right tail, and the measurement ϵ allows the model to capture some of these high-quality transactions.

Finally, we explicitly model the selection into the ADAM sample. Specifically, we assume that a buyer of type z is in ADAM if $\log(z) + \eta \geq 0$, where η is a random variable which is independent of z and is distributed according to a normal distribution with mean μ_η and standard deviation σ_η. Hence, this selection equation highlights that buyers with higher preferences for drugs (and, thus, greater drug consumption) are more likely to be in the ADAM dataset. Appendix B reports the details of the derivation of the density of drug users' preferences z in ADAM; we will use this density to compute simulated moments that we match to their empirical counterparts.

[16]We could non-parametrically estimate the distribution $D(c)$ from the empirical distribution of q for $q > 0$, as $D(c) = \frac{1-F(q)}{1-F(0)}$. However, we specify a parametric distribution for $D(c)$ to perform the counterfactual out-of-sample analyses of Section 5.4.

5.2 Estimation and Identification

We estimate the vector of parameters $\psi = \left\{\alpha, \gamma, \delta, K_B, \mu_R, \sigma_R, \bar{c}, \xi, \sigma_\epsilon, \mu_\eta, \sigma_\eta, \lambda\right\}$ using a minimum-distance estimator that matches key moments of the data with the corresponding moments of the model. More precisely, for any value of these parameters, we solve the model of Section 4 to find its equilibrium: the mass B of active buyers and their distribution of reservation qualities $H(\cdot)$, and the mass S of active sellers and their distribution $F(\cdot)$ of offered qualities. We then calculate two sets of moments, one that we match to a set of moments computed from the STRIDE dataset, and one that we match to a set of moments computed from the ADAM dataset.

The first set $\mathbf{m}_1(\psi)$ is composed of these moments of the offered quality distribution $F(\cdot)$:

1. The fraction of rip-offs $q = \hat{q} = 0$.[17]
2. The mean of quality for $\hat{q} > 0$.
3. The standard deviation of quality for $\hat{q} > 0$.
4. The median of quality for $\hat{q} > 0$.
5. The skewness of quality for $\hat{q} > 0$.
6. The kurtosis of quality for $\hat{q} > 0$.

Moreover, at each value of the parameters, we simulate buyer-seller meetings and consumption patterns (i.e., the α_B, δ and γ shocks), using the distributions of preferences z and buyer-seller matches that take into account the selection into ADAM (see Appendix B). We then compute the second set $\mathbf{m}_2(\psi)$ composed of these moments:

1. The fraction of individuals who purchased crack cocaine in the last 30 days.
2. The fraction of users who made their last purchase of crack cocaine from their regular dealer, among those who purchased crack cocaine in the last 30 days (in the simulation, a purchase from a regular dealer is defined as a purchase from the same seller as the previous purchase).
3. The average number of purchases of those who purchased crack cocaine in the last 30 days and made their last purchase from their regular dealer.

[17] Note that $\hat{q} = 0$ if and only if $q = 0$.

4. The average number of purchases of those who purchased crack cocaine in the last 30 days and did not make their last purchase from their regular dealer.

5. The standard deviation of the number of purchases of those who purchased crack cocaine in the last 30 days and make their last purchase from their regular dealer.

6. The standard deviation of the number of purchases of those who purchased crack cocaine in the last 30 days and did not made their last purchase from their regular dealer.

We further add to this set of moments two aggregate statistics that identify the parameters that determine the selection into the ADAM sample and the fraction λ of the population who has no taste for crack cocaine.

7. The fraction of individuals arrested. While the ADAM data do not report this number directly, the Federal Bureau of Investigation reports that there were 8,608,479 persons arrested in 2002, and the U.S. Census reports that the U.S population over 15 years of age in July 2002 was equal to 228 million.[18] These figures imply an arrest rate of approximately 3.7 percent, and we use this figure in the estimation.

8. The fraction of the population that consumes crack cocaine. Our data do not report this fraction directly, but the 2002 National Survey on Drug Use and Health (NSDUH) reports that the fraction of non-institutionalized individuals aged 12 and older who self-report to have consumed crack cocaine in the past year equals 0.7 percent. Because the NSDUH does not include arrestees, who are more likely to consume drugs, and includes young individuals aged between 12 and 15, whom we do not include among our potential buyers, we set this fraction to be equal to one percent in the estimation.[19]

The minimum-distance estimator chooses the parameter vector ψ that minimizes the criterion function

$$(\mathbf{m}(\psi) - m_S)' \, \Omega \, (\mathbf{m}(\psi) - m_S),$$

[18]Sources: `http://www.fbi.gov/about-us/cjis/ucr/crime-in-the-u.s/2011/crime-in-the-u.s.-2011/tables/table-32` and `https://www.census.gov/popest/data/historical/2000s/vintage_2002/index.html`.

[19]The confidence interval of the estimate of λ does not depend on the sampling variability of the empirical fraction of the population that consumes crack cocaine (which we cannot compute), but exclusively on the sampling variability of the moments that identify μ_z, σ_z and z^*.

where $\mathbf{m}(\psi) = \begin{bmatrix} \mathbf{m}_1(\psi) \\ \mathbf{m}_2(\psi) \end{bmatrix}$ is the vector of stacked moments computed from the model evaluated at ψ, and m_S is the vector of corresponding sample moments. Ω is a symmetric, positive-definite matrix. In practice, we use a diagonal matrix whose elements are those on the main diagonal of the inverse of the matrix $E(m_S' m_S)$.

The identification of the model is similar to that of structural search models of the labor market which follow the framework of Burdett and Mortensen (1998); see, for example, Bontemps *et al.* (1999). Specifically, although the model is highly nonlinear, so that (almost) all parameters affect all outcomes, the identification of some parameters relies on some key moments in the data.

The moments of the quality distribution identify the parameters of the distribution $D(\cdot)$ of sellers' heterogeneity and of the distribution of the measurement error, and they contribute to the identification of the parameters of the distribution $M(\cdot)$ of buyers' heterogeneity.[20] More precisely, as in structural search models of the labor market, the data sometimes display events that should not occur according to the model, and these "zero-probability events" identify the variance of the measurement error. In search models of the labor markets, these events are job-to-job transitions that feature a wage decrease; instead, our model implies a gap in the quality distribution between $q = 0$ and $\underline{q}$ that is larger than that observed in the data—i.e., no seller offers a small positive quality, as this quality is strictly more expensive than zero quality and does not induce buyers to purchase again from the seller offering it—and this gap identifies the variance of the measurement error.

The moments of buyers' consumptions identify the meeting rates α_B and γ and the destruction rate δ, and they contribute to the identification of the parameters of the distribution $M(\cdot)$ of buyers' heterogeneity. The fraction of individuals who purchased drugs in the last 30 days and the arrest rate identify the parameters of the distribution of the unobservable η that contributes to the selection into the ADAM sample.[21] From the distribution of buyers' heterogeneity, we then recover buyers' cost K_B.

[20] Recall that, in the absence of measurement error on q, the empirical distribution of q for $q > 0$ non-parametrically identifies the distribution $D(c)$.

[21] Since we use the aggregate arrest rate, obtained for an external source, this moment has no sampling variability. Hence, the confidence interval of the estimate of σ_η does not depend on the sampling variability of the arrest rate, but exclusively on the sampling variability of the moments that identify the distribution of the taste z.

Table 2: Parameter Estimates

α_B	1.211 $[1.204, 1.236]$	λ	0.989 $[0.983, 0.989]$
γ	19.165 $[19.165, 24.124]$	μ_z	3.711 $[3.710, 3.721]$
δ	0.723 $[0.707, 0.740]$	σ_z	0.158 $[0.116, 0.168]$
K_B	110.079 $[65.750, 149.386]$	c_M	29.832 $[29.766, 33.784]$
σ_ϵ	0.414 $[0.381, 0.436]$	ξ	12.098 $[11.724, 18.319]$
σ_η	17.090 $[12.812, 24.173]$	μ_η	6.855 $[3.272, 13.263]$

Notes—This table reports the estimates of the parameters. 95-percent confidence intervals in brackets are obtained by bootstrapping the data using 100 replications.

5.2.1 Estimates

Table 2 reports estimates of the parameters, along with 95-percent confidence intervals obtained by bootstrapping the data using 100 replications.

The magnitude of the parameter α_B indicates that a buyer meets a new seller, on average, approximately every $\frac{30}{\alpha_B} \approx 25$ days. The parameter γindicates that a matched buyer purchases, on average, approximately 19 times every month. However, the buyer-seller exogenously breaks, on average, every $\frac{30}{\delta} = 40$ days. Buyers' monthly cost K_B is quite low, approximately \$110.

The estimates of the parameters of the distribution of buyers' heterogeneity imply that almost all individuals have taste z equal to zero, and, thus, the market for crack cocaine is limited in size, presumably because crack cocaine is one of the most addictive and dangerous drugs. Of those individuals with positive preferences, buyers with taste $z \geq z^* = 34.39$ are active in the market, and among those active, the average taste is approximately equal to 43 and the standard deviation is equal to 5.72. The parameters $\bar{c}$ and ξ of sellers' cost distribution imply that the range of sellers' cost is $[0, 29.83]$, but their average cost is 27.55, as $\xi = 12.09$ implies that almost all sellers have costs quite close to the upper bound $\bar{c}$. Moreover, the comparison between buyers' average valuation and sellers' average cost implies that the surplus of each pure gram traded equals approximately 15 dollars.

Finally, we estimate that σ_ϵ equals 0.41, which means that the variance of the measurement error equals 0.18. The estimates imply that the variance of the "true" quality q equals 1.28, indicating that the model without any error captures quite well the dispersion of drug quality observed in the data, and the measurement error improves the fit, in particular by "filling the gap" between $q = 0$ and $\underline{q}$, which the estimated model sets at 2.54.[22]

5.2.2 Model Fit

Before considering some broader implications of our results, we examine the fit of the estimated model. Table 3 presents a comparison between the empirical moments and the moments calculated from the model at preliminary parameters. Overall, the model matches the moments of the quality distribution quite well. The largest discrepancy is in the fraction of purchases from regular dealers, which we observe in the data less frequently than the model predicts. Nonetheless, the model captures both the fraction of rip-offs and the higher-order moments of the quality distribution well. The model matches the moments of the distribution of buyers' consumptions slightly less precisely than those of the quality distribution, but, overall, it captures the difference in consumption rates between matched and unmatched buyers well. Moreover, the model matches perfectly the fraction of individuals purchasing drugs in the ADAM sample, the fraction of individuals consuming crack cocaine in the population as well as the arrest rate, indicating that our empirical model captures very well the over-representation of drug users into the ADAM sample.

To further appreciate how the model compares to the quality data in, perhaps, a more-intuitive way, Figure 2 displays the histogram of the quality distribution obtained from a model simulation using the estimated parameters reported in Table 2. The comparison with the empirical distribution of Figure 1 corroborates that the model does well at matching the qualitative and quantitative features of the distribution of drug quality.

5.3 Model Implications

The estimated parameters reported in Table 2 imply that 15.6 percent of all sellers rip their buyers off by choosing $q = 0$. For sellers with costs $0 \leq c \leq \bar{c} = 29.83$, $q^* (c)$ is the solution to the differential equation (11) in Appendix A: sellers' quality choices are strictly decreasing in their costs, as Lemma 7 says. The estimated parameters imply that sellers' positive qualities lie in the interval $[2.54, 3.67]$. Hence, the model implies a non-trivial dispersion of drug qualities, and the measurement error helps to match the large dispersion.

[22] Of course, if we estimate the model *without* measurement error, the value of $\underline{q}$ is lower—i.e., it equals approximately 1.7.

Table 3: Model Fit

	Data	Model
Fraction of Rip-offs (%)	15.411	15.599
Average Pure Grams per \$100, $\hat{q} > 0$	2.467	3.078
St. Dev. Pure Grams per \$100, $\hat{q} > 0$	1.592	1.359
Median Pure Grams per \$100, $\hat{q} > 0$	2.017	2.824
Skewness Pure Grams per \$100, $\hat{q} > 0$	1.513	1.416
Kurtosis Pure Grams per \$100, $\hat{q} > 0$	5.575	6.608
Fraction Obtained Drug in Last 30 Days (%)	16.900	16.900
Fraction Last Purchased from Regular Dealer (%)	52.481	83.972
Average Number of Purchases, Matched Buyer	16.331	15.051
Average Number of Purchases, Unmatched Buyer	11.548	9.169
St. Dev. Number of Purchases, Matched Buyer	11.124	6.967
St. Dev. Number of Purchases, Unmatched Buyer	10.419	8.194
Fraction Consuming Drug in the Population (%)	1.000	1.000
Arrest Rate (%)	3.776	3.776

Notes—This table reports the values of the empirical moments and of the simulated moments calculated at the estimated parameters reported in Table 2.

Sellers' quality choice $q^*(c)$ implies that sellers' markups $\frac{p-cq^*(c)}{p}$ are non-monotonic in c, with the lowest- and highest-cost sellers charging the highest ones (equal to 1, as either c or q equals 0), and the seller with cost $c = 27.04$ charging the lowest one; the average sellers' markup $\frac{\int_0^{\bar{c}}(p-cq^*(c))dD(c)}{p}$ equals 15.4 percent. On average, sellers make approximately 175 transactions $t(q)$ per month, and the distribution of transactions $t(q)$ has a large range—the lowest-quality (i.e., highest-cost) sellers make approximately 60 monthly deals and the highest-quality sellers make approximately 415 monthly deals—and is skewed towards sellers with fewer transactions. Sellers' profits have a large range and are highly skewed, as well: the lowest-quality seller is earning $\$1,500$ per month; the highest-quality seller is earning approximately $\$19,000$ per month; and the average seller is earning approximately $K_S = \$2,750$. The shape of the distribution of profits matches the evidence reported by Levitt and Venkatesh (2000) reasonably well.

The distribution $G(q)$ of qualities from which matched buyers consume differs substantially from the distribution $F(q)$ of qualities from which unmatched buyers consume. The distribution $F(q)$ displays the key features of the distribution of qualities characterized in

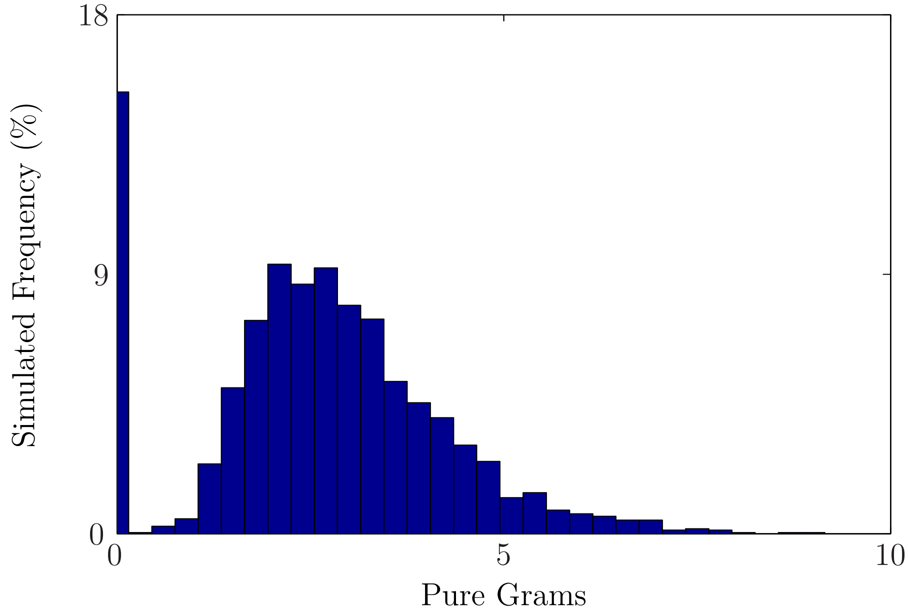

FIG. 2: Histogram of pure grams per \$100, simulated data.

Lemma 5, most notably the mass point at zero quality. Of course, no matched buyer purchases zero quality from his regular dealer. Moreover, as buyers move up over time in the offered quality distribution by switching to sellers that offer higher-quality drugs, they are more likely to be matched to higher-quality sellers. Hence, the cumulative $G(q)$ first-order stochastically dominates the cumulative $F(q)$. Matched buyers consume drugs that have an average quality of $\int_{\underline{q}}^{\bar{q}} qg(q)\,dq = 3.21$, whereas unmatched buyers consume drugs that have an average quality of $\int_0^{\bar{q}} qf(q)\,dq = 2.60$, indicating that buyers' switching behavior and buyer-seller relationships have a substantial effect on the qualities that regular buyers consume relative to the distribution of qualities that first-time buyers consume.

5.4 Counterfactual Analyses

In this Section, we use our model to quantitatively analyze two key features of illegal markets: 1) the effect of moral hazard on market outcomes; and 2) the effect of changing penalties on market outcomes.

Both analyses are out-of-sample and, thus, require that we specify the measures $\overline{B}$ and $\overline{S}$ of potential buyers and potential sellers, along with the functional form of the matching function $m(B, S)$ that determines the aggregate number of new meetings between B active buyers and S active sellers.

We set the measure $\overline{B}$ of potential buyers to 228 million, which, as we mentioned before when reporting the fraction of individuals arrested, is the U.S population over 15 years of age reported in July 2002 (the midpoint of our sample period) by the U.S. Census. We set the measure $\overline{S}$ of potential sellers to five percent of potential buyers $\overline{B}$. Our theoretical model imposes an upper bound on this measure in order to match the fraction of rip-offs observed in the data (see Proposition 7), and our choice satisfies this restriction.[23] Following Corollary 8, the measure $\overline{S}$ allows us to recover the upper bound c_M of the aggregate distribution of costs $\bar{D}(c)$.

We further assume a Cobb-Douglas functional form:

$$m(B,S) = \omega B^{1/2} S^{1/2},$$

where ω is the efficiency of the matching function. With our estimated parameters, we can calculate ω as

$$\omega = \frac{m(B,S)}{B^{1/2}S^{1/2}} = \frac{\alpha_B(\theta)B}{B^{1/2}S^{1/2}},$$

where we estimated α_B using the ADAM data, we obtained B from the fraction of the population consuming crack cocaine, and we inferred S from sellers' free-entry condition.

(In the following counterfactual analyses, we report quantitative results without measurement errors on drug quality.)

5.4.1 The Role of Sellers' Moral Hazard

In order to understand the quantitative role of moral hazard on market outcomes, we modify the baseline model by letting buyers observe quality before making a purchase, and compute the new equilibrium. This counterfactual highlights how the observability of q affects buyers' and sellers' incentives and, thus, the equilibrium distribution of quality q. Appendix C reports the full derivation of the equilibrium.

When quality is observable, a buyer who meets a new seller has two decisions to make, after observing q: whether to consume and whether to match. Regardless of whether the buyer is currently matched or not, he makes a purchase when the instantaneous payoff $zq - p$ is positive. The decision to match is similar to that of the baseline model: if matched, the buyer chooses between matching with the new seller or returning to his regular seller, after potentially taking advantage of the consumption opportunity; if unmatched, the buyer

[23] We have experimented with other measures of sellers that satisfy this theory-based restriction, and the results of the counterfactuals are quite similar.

chooses between matching with the new seller and remaining unmatched, as consuming and remaining unmatched is not optimal.

Hence, the value functions of a type-z buyer satisfy:

$$\begin{aligned} r\overline{V}_z &= \alpha_B(\theta)\int_0^{\overline{q}} \max\Big[zx - p + \max[V_z(x) - \overline{V}_z, 0], 0\Big] dF(x), \\ rV_z(q) &= \gamma(zq - p) + \alpha_B(\theta)\int_0^{\overline{q}} \max\Big[zx - p + \max[V_z(x) - V_z(q), 0], 0\Big] dF(x) + \delta(\overline{V}_z - V_z(q)). \end{aligned}$$

Simple calculations suggest that the reservation quality for consuming is the same as the reservation quality for matching (see Appendix C for details).

Quality observability affects sellers' payoffs because new buyers make a purchase only if the instantaneous payoff is positive. Therefore, the flow of transactions to new buyers for a seller offering quality q is equal to the meeting rate with new buyers $\alpha_S(\theta)$ times the probability that the buyer's reservation is below q, $H(q)$:

$$t_N(q) = \alpha_S(\theta)H(q).$$

The flow of sellers' transactions with regular buyers $t_R(q)$ is determined in an equivalent way to the baseline case. Steady-state profits are:

$$\pi_c(q) = (p - cq)(t_N(q) + t_R(q)),$$

Sellers' incentives differ from the baseline case and, thus, their choices do as well. Specifically, offering zero quality yields negative instantaneous payoff to all buyers and, thus, sellers no longer offer complete rip-offs. Therefore, the distribution of offered quality does not feature a mass point at zero. More generally, the flow of new customers depends on the quality offered, since buyers with low levels of z might choose not to purchase from low-q sellers.

We quantitatively assess the effect of eliminating moral hazard by introducing the estimated parameters in the observable quality model. We consider two separate cases: 1) a partial-equilibrium case in which the measures of participating buyers and sellers are unchanged relative to the baseline case, but the agents make optimal decisions in the new information environment; 2) a general-equilibrium case in which, in addition to the partial-equilibrium optimizations, buyers and sellers also make optimal entry decisions. We believe that the partial-equilibrium case is useful to focus exclusively on the effects of sellers' moral hazard due the imperfect observability of drugs' purity.

Table 4: Observable Quality

	Baseline	Observable q, Partial Eq.	Observable q, General Eq.
Fraction of Rip-offs (%)	15.599 [13.903; 18.735]	0.000 [0.000; 0.000]	0.000 [0.000; 0.000]
Average Pure Grams per \$100	2.613 [2.278; 2.635]	3.131 [2.755; 3.131]	3.136 [2.777; 3.136]
St. Dev. Pure Grams per \$100	1.135 [1.029; 1.167]	0.250 [0.171; 0.250]	0.243 [0.159; 0.243]
Active Buyers, in Millions	2.280 [2.269; 2.317]	2.280 [2.269; 2.317]	2.389 [2.321; 2.505]
Active Sellers, in Millions	0.184 [0.182; 0.190]	0.184 [0.182; 0.190]	0.148 [0.144; 0.155]
Fraction of Matched Buyers (%)	57.933 [56.722; 58.742]	62.612 [62.007; 63.296]	58.820 [58.024; 59.589]
Average Number of Purchases per Month	12.166 [12.122; 14.952]	13.071 [13.070; 16.140]	12.134 [12.134; 14.989]
Average Pure Grams Consumed per Month	38.531 [36.536; 43.906]	42.703 [38.938; 48.341]	39.500 [37.531; 45.048]

Notes—This table reports market outcomes in the counterfactual cases in which buyers can observe drugs' purity before purchasing.

Table 4 reports market outcomes for the counterfactuals of observable quality for the partial-equilibrium case and the general-equilibrium case. Overall, market outcomes differ substantially when buyers observe drug purity and when they do not. Moreover, the quantitative effects are very similar in the partial- and general-equilibrium cases.

Specifically, the partial-equilibrium case highlights that the average offered quality increases by approximately 20 percent and the standard deviation of quality decreases by approximately 75 percent relative to the baseline case. Moreover, zero-purity drugs disappear from the market when quality is observable, as we discussed above. Overall, this counterfactual indicates that unobservable quality, rather than search frictions, is the main determinant of the observed dispersion of quality. Finally, a larger fraction of buyers is matched to a regular seller, thereby increasing their number of purchases and consumption by approximately five and ten percent, respectively.

The general equilibrium case highlights additional effects relative to the partial-equilibrium case. First, buyers receive higher quality than in the baseline case, thereby increasing buyers' participation. Second, the intensified competition among sellers reduces their profits relative to the baseline case, thereby decreasing sellers' participation. As a result, the buyer-seller ratio increases relative to the baseline case, attenuating the partial-equilibrium effects. Overall, the increase in the number of buyers together with the increase in their drug consumption leads to an increase in the aggregate consumption of pure drugs by approximately 7.5 percent relative to the baseline case.

The results reported in Table 4 provide some insights on how outcomes would differ if drugs markets were legal. Buyers' imperfect information is one key way in which our model captures an illegal market, because, in legal markets for similar commodities (i.e., tea, coffee, cigarettes, alcohol), buyers are better (albeit perhaps not fully) informed about the quality of products that they are purchasing. While a full legalization counterfactual requires many additional assumptions (for example, on the destruction rate δ, the efficiency of the matching process and the entry costs K_B and K_S), our analysis illustrates that the average quality of drugs will increase if drugs markets were legal because of buyers' better information.

5.4.2 The Role of Penalties

The United States has witnessed a large increase of policies and penalties on buyers and sellers of drugs in the last 30 years. The principal aim of these changes was to disrupt the market for narcotics. Legal penalties on drug trade obviously affect sellers' costs K_S and buyers' costs K_B and, thus, in this Section, we use our model to understand how these costs K_S and K_B affect market outcomes.[24]

We estimated our model on data from 2001-2003, when penalties were high, and, thus, we now perform two counterfactuals with lower costs in order to understand their effects on market outcomes. More specifically, in the first one, we decrease sellers' cost K_S by 15 percent (i.e., from $\$1,500$ to $\$1,350$) relative to the baseline case of Section 4 with ex-ante unobservable quality; in the second one, we decrease buyers' cost K_B by 15 percent relative to the baseline case.

Table 5 reports the quantitative values of market outcomes for the two counterfactuals cases, displaying interesting results. The effect of lower penalties on sellers (i.e., a drop in K_S) on market participation is intuitive: the number of sellers is 16-percent higher. Thus, the rate at which buyers meet with new sellers, $\alpha_B(\theta)$ is higher, as well.

[24] Different policing interventions affect the market through different parameters. For example, prison sentencing guidelines more likely affect buyers' and sellers' costs, whereas police patrolling more likely affects the destruction rate δ and the efficiency of the matching function, and, thus, the meeting rate.

Table 5: The Effect of Penalties

	Baseline	Lower K_S	Lower K_B
Fraction of Rip-offs (%)	15.599	17.684	21.240
	[13.903; 18.735]	[9.479; 23.873]	[12.930; 30.383]
Average Pure Grams per \$100	2.613	2.523	2.460
	[2.278; 2.635]	[2.098; 2.713]	[1.954; 2.627]
St. Dev. Pure Grams per \$100	1.135	1.185	1.257
	[1.029; 1.167]	[0.801; 1.265]	[1.023; 1.370]
Active Buyers, in Millions	2.280	2.233	2.305
	[2.269; 2.317]	[2.170; 2.292]	[2.235; 2.374]
Active Sellers, in Millions	0.184	0.214	0.198
	[0.182; 0.190]	[0.191; 0.238]	[0.179; 0.224]
Fraction of Matched Buyers (%)	57.933	57.587	57.664
	[56.722; 58.742]	[56.548; 58.570]	[56.415; 58.620]
Average Number of Purchases per Month	12.166	12.150	12.180
	[12.122; 14.952]	[12.017; 15.157]	[12.180; 15.081]
Average Pure Grams Consumed per Month	38.531	37.976	38.255
	[36.536; 43.906]	[36.243; 42.741]	[35.984; 43.682]

Notes—This table reports market outcomes in the counterfactual cases in which buyers' cost K_B and sellers' cost K_S are 15-percent lower than in the baseline case, respectively.

The effect on the average offered quality, however, is quite striking: the average quality of a new purchase is lower, despite lower sellers' costs. The reason is that a larger value of $\alpha_B(\theta)$ means that the match between a buyer and a seller is, on average, shorter-lived. In turn, this reduction lowers sellers' value of forming long-term relationships with new buyers relative to the value of ripping them off and, thus, the proportion of rip-offs increases. Our quantitative exercise shows that, at the estimated parameter values, the proportion of sellers who specialize in rip-offs is approximately 13-percent higher (i.e., it increases from 15.59 to 17.68 percentage points), the average offered quality is 3.5-percent lower, and the standard deviation is 4.5-percent higher than in the baseline case.

This decrease in drug quality tends to decrease the number of active buyers in the market, whereas a higher meeting rate tends to increase it. As a result of these opposing forces, the equilibrium number of buyers decreases by a small amount, two percent. Similarly, the lower quality of drugs implies that the fraction of matched buyers decreases, as well as their average number of purchases and their average pure quantity of crack consumed. However, the magnitudes of all these changes are negligible—i.e., all these change by at most two percent relative to the baseline case.

Similarly, a 15-percent reduction in buyers' participation cost K_B leads to a small (i.e., one percent) increase in the number of buyers. A larger market size attracts a larger number of sellers (higher by 7.5 percent), indicating that the elasticity of supply is larger than the elasticity of demand. Therefore, the buyer-seller ratio is, again, lower than in the baseline case, thereby increasing buyers' meeting rate $\alpha_B(\theta)$. Hence, sellers have stronger incentives to make quick profits by selling $q = 0$, decreasing the offered qualities. The last column of Table 5 reports that these effects on the drug quality distribution are quantitatively sizable: the fraction of rip-offs increase by 36 percent, decreasing the average drug quality by 5.9 percent, and increasing the standard deviation of drug quality by 10 percent relative to the baseline case. However, because of the two contrasting effects—i.e., the increase in the meeting rate and the decrease in drug quality—the overall effects on the number of matched buyers, as well as on buyers' purchases and consumptions, are again negligible—i.e., all these change by at most one percent relative to the baseline case.

Overall, the results reported in Table 5 highlight that changes in penalties have larger effects on the distribution of drugs offered on the market than on buyers' purchase and consumption patterns, as buyers' switching between sellers allows them to adjust their consumption. Hence, these results suggest that it may be difficult to infer buyers' consumptions exclusively from samples of drug purchases.

Moreover, the results further highlight the role of buyers' incomplete information at the time of purchase, as sellers' moral hazard is fundamental to this, perhaps counterintuitive, relation between lower penalties—sellers' penalties, in particular—and *lower* quality. Specifically, when buyers have perfect information about quality in a frictional product market (e.g., the product-market version of Burdett and Mortensen, 1998), an increase in buyers' meeting rate $\alpha_B(\theta)$ unambiguously increases competition among sellers, thereby leading to an improvement in buyers' terms of trade—i.e. an increase in the average offered quality.

Finally, the results indicate that increasing penalties may help strengthen the long-term relationships between buyers and sellers that help overcome illegal markets' informational problems. Hence, our analysis suggests that increasing penalties may have contributed to the observed increased purity of retail drugs in the U.S. Similarly, the UN office for Drugs and Crime reports that price-adjusted purity of drugs is lower in Europe than in the United States,[25] whereas penalties for market participants—buyers, in particular—are lower in Europe than in the United States. Of course, there are potentially many other differences among

[25] See `http://www.unodc.org/unodc/secured/wdr/Cocaine_Heroin_Prices.pdf`.

U.S. markets over time, and between the U.S. market and the European one. Nonetheless, we find interesting that our model is consistent with these across-market differences.

6 Conclusions

This paper develops a framework to understand illicit drug markets. We focus on two key characteristics of illegal markets: 1) the inability to verify/contract the quality of the good; and 2) penalties on market participants. Buyers' inability to verify the quality of the good creates a trade-off for sellers. On the one hand, they can offer low-quality drugs, thus maximizing short-term profits. On the other hand, they can offer high-quality drugs, thus inducing buyers to purchase from them again. In equilibrium, a distribution of quality levels persists.

We estimate the model using data on the U.S. market for crack cocaine. The model fits the data well. Our counterfactual analysis implies that buyers' inability to verify the quality of the good and, thus, sellers' moral hazard reduce the average and increase the dispersion of drug purity, thereby reducing drug consumption. Moreover, the estimated model implies that increasing penalties may increase the purity and the affordability of the drugs traded because it increases sellers' relative profitability of targeting loyal buyers versus first-time buyers.

At the same time, there are several interesting extensions of our model and of our out-of-sample analyses. We believe that our cross-sectional data impose some important limitations on what our model can identify in the data, and richer panel-data on buyers and sellers would allow us to enrich our current framework. Specifically, additional forms of heterogeneity are difficult to identify with our data. Thus, our model focuses on a single dimension of heterogeneity across buyers—i.e., their taste for drugs. Similarly, we let sellers commit to a single level of quality: a model that allows sellers to discriminate across their buyers or to vary quality over time would be difficult to identify with the available data since we do not observe how sellers discriminate between first-time buyers and repeat buyers or whether they vary their strategy over time. Nonetheless, our theoretical framework delivers rich heterogeneity across sellers, and our empirical model successfully matches the large heterogeneity observed in the data.

References

Allen, J., Clark, R. and Houde, J.-F. (2014). *Search frictions and market power in negotiated price markets*. Tech. rep., National Bureau of Economic Research.

Arkes, J., Pacula, R. L., Paddock, S. M., Caulkins, J. P. and Reuter, P. (2008). *Why the DEA Stride data are still useful for understanding drug markets*. Tech. rep., National Bureau of Economic Research.

Becker, G. and Murphy, K. (1988). A theory of rational addiction. *The Journal of Political Economy*, **96** (4), 675–700.

Becker, G. S., Murphy, K. M. and Grossman, M. (2006). The market for illegal goods: the case of drugs. *Journal of Political Economy*, **114** (1), 38–60.

Board, S. and Meyer-ter Vehn, M. (2013). Reputation for quality. *Econometrica*, **81** (6), 2381–2462.

Bontemps, C., Robin, J.-M. and Van den Berg, G. (1999). An empirical equilibrium job search model with search on the job and heterogeneous workers and firms. *International Economic Review*, **40** (4), 1039–1074.

Boot, A. W. (2000). Relationship banking: What do we know? *Journal of Financial Intermediation*, **9** (1), 7 – 25.

Burdett, K. and Mortensen, D. (1998). Wage differentials, employer size, and unemployment. *International Economic Review*, **39** (2), 257–273.

Bushway, S. and Reuter, P. (2008). Economists' contribution to the study of crime and the criminal justice system. *Crime and Justice*, **37** (1), 389–451.

Caulkins, J. P. (2007). Price and purity analysis for illicit drug: data and conceptual issues. *Drug and Alcohol Dependence*, **90**, S61–S68.

Cebul, R. D., Rebitzer, J. B., Taylor, L. J. and Votruba, M. E. (2011). Unhealthy insurance markets: Search frictions and the cost and quality of health insurance. *The American Economic Review*, pp. 1842–1871.

Dave, D. (2008). Illicit drug use among arrestees, prices and policy. *Journal of Urban Economics*, **63** (2), 694–714.

Dranove, D. and Jin, G. (2010). Quality disclosure and certification: Theory and practice. *Journal of Economic Literature*, **48** (4), 935–963.

Eckstein, Z. and Van den Berg, G. J. (2007). Empirical labor search: A survey. *Journal of Econometrics*, **136** (2), 531–564.

Fryer, R. G., Heaton, P. S., Levitt, S. D. and Murphy, K. M. (2013). Measuring crack cocaine and its impact. *Economic Inquiry*, **51** (3), 1651–1681.

Gale, D. and Rosenthal, R. W. (1994). Price and quality cycles for experience goods. *The RAND Journal of Economics*, **25** (4), 590–607.

Galenianos, M., Pacula, R. L. and Persico, N. (2012). A search-theoretic model of the retail market for illicit drugs. *Review of Economic Studies*, **79**, 1239–1269.

Gavazza, A. (2013). *An Empirical Equilibrium Model of a Decentralized Asset Market*. Tech. rep., London School of Economics.

Hong, H. and Shum, M. (2006). Using price distributions to estimate search costs. *The RAND Journal of Economics*, **37** (2), 257–275.

Horowitz, J. L. (2001). Should the dea's stride data be used for economic analyses of markets for illegal drugs? *Journal of the American Statistical Association*, **96** (456), 1254–1271.

Hortaçsu, A. and Syverson, C. (2004). Search costs, product differentiation, and welfare effects of entry: a case study of sp 500 index funds. *Quarterly Journal of Economics*, **119** (4), 403–456.

Iizuka, T. (2007). Experts' agency problems: evidence from the prescription drug market in japan. *The RAND Journal of Economics*, **38** (3), 844–862.

— (2012). Physician agency and adoption of generic pharmaceuticals. *The American Economic Review*, **102** (6), 2826–2858.

Jacobi, L. and Sovinsky, M. (2013). *Marijuana on Main Street? Estimating Demand in Markets with Limited Access*. Working paper, Department of Economics, University of Zurich.

Kilmer, B., Everingham, S. S., Caulkins, J. P., Midgette, G., Pacula, R. L., Reuter, P. H., Burns, R. M., Han, B. and Lundberg, R. (2014). *What America's Users Spend on Illegal Drugs*. Research Reports RR-534-ONDCP, RAND Corporation.

Kuziemko, I. and Levitt, S. (2004). An empirical analysis of imprisoning drug offenders. *Journal of Public Economics*, **88** (9), 2043–2066.

Levitt, S. and Venkatesh, S. (2000). An economic analysis of a drug-selling gang's finances. *Quarterly Journal of Economics*, **115** (3), 755–789.

Reuter, P. and Caulkins, J. P. (2004). Illegal 'lemons': price dispersion in cocaine and heroin markets. *Bulletin on Narcotics*, **56** (1-2), 141–165.

Wildenbeest, M. R. (2011). An empirical model of search with vertically differentiated products. *The RAND Journal of Economics*, **42** (4), 729–757.

APPENDICES

A Proofs

Proof of Proposition 2. We determine the reservation quality of an unmatched buyer by equating the value of remaining unmatched with the value of becoming matched at R_z:

$$\bar{V}_z = V_z(R_z),$$

which implies equation (4). Integrating by parts, the value of being unmatched satisfies:

$$r\bar{V}_z = \alpha_B(\theta)\Big(z\int_0^{\bar{q}} qdF(q) + \int_{R_z}^{\bar{q}} V_z'(q)(1-F(x))dx - p\Big).$$

Differentiating equation (2) with respect to q, we obtain

$$V_z'(q) = \frac{\gamma z}{r+\delta+\alpha_B(\theta)(1-F(q))}.$$

Combining the previous two equations yields equation (3).

The value of participating in the market is negative for buyers who receive no utility from consuming and is strictly increasing in a buyer's marginal utility of consumption:

$$\begin{aligned} r\bar{V}_0 &= -\alpha_B(\theta)p < 0, \\ \frac{\partial r\bar{V}_z}{\partial z} &= \alpha_B(\theta)\Big(\int_0^{\bar{q}} xdF(x) + \int_{R_z}^{\bar{q}} \frac{\gamma(1-F(x))}{r+\delta+\alpha_B(\theta)(1-F(x))}dx + \frac{p}{z^2}\frac{\alpha_B(\theta)\gamma(1-F(R_z))}{r+\delta+\alpha_B(\theta)(1-F(R_z))}\Big) > 0. \end{aligned}$$

The value of participating can be made arbitrarily large by increasing z; therefore, a buyer participates in the market only if his type is high enough. ■

Proof of Proposition 3. Fix a type-z buyer and consider his value of participating in the market as a function of the buyer-seller ratio, θ. If θ is large, type-z buyer never meets with a seller ($\lim_{\theta\to\infty} \alpha_B(\theta) = 0$) and, thus, his value of participating is strictly below the entry cost:

$$\lim_{\theta\to\infty} r\bar{V}_z = 0 < K_B.$$

The value of participating in the market is strictly decreasing in θ:

$$\frac{\partial r\bar{V}_z}{\partial\theta} = \alpha_B'(\theta)\Big(z\int_0^{\bar{q}} xdF(x) - p\Big) + \frac{z\alpha_B'(\theta)(r+\delta)}{\alpha_B(\theta)^2}\int_{p/z}^{\bar{q}} \frac{\gamma(1-F(x))}{\big(\frac{r+\delta}{\alpha_B(\theta)} + 1 - F(x)\big)^2}dx < 0.$$

If θ is small, a buyer meets with a seller arbitrarily often ($\lim_{\theta\to 0} \alpha_B(\theta) = \infty$) and, thus, the value of participating satisfies:

$$\lim_{\theta\to 0} r\bar{V}_z = \lim_{\theta\to 0} \alpha_B(\theta)\Big(z\int_0^{\bar{q}} xdF(x) - p\Big),$$

and the buyer participates if:

$$\lim_{\theta\to 0} \alpha_B(\theta)\Big(z\int_0^{\bar{q}} xdF(x) - p\Big) > K_B.$$

Therefore, regardless of θ, a type-z buyer never participates in the market if $z\int_0^{\bar{q}} xdF(x) \leq p$. As a result, there is no buyer entry ($B = 0$) when $\bar{z}\int_0^{\bar{q}} xdF(x) \leq p$, which proves part 1 of the Proposition.

When $\bar{z}\int_0^{\bar{q}} xdF(x) > p$, a type-$z$ buyer participates if $z\int_0^{\bar{q}} xdF(x) > p$ *and* if θ is low enough. Conversely, given θ, there is a $z(\theta)$, with $z(\theta)\int_0^{\bar{q}} xdF(x) > p$, such that

$$r\bar{V}_{z(\theta)} = K_B,$$

so that a buyer participates if and only if $z \geq z(\theta)$ and the measure of buyers equals $B = \bar{B}(1 - \bar{M}(z(\theta)))$. Given S, this leads to

$$\theta(z(\theta)) \quad = \quad \frac{\bar{B}(1-\bar{M}(z(\theta)))}{S}.$$

We now show that, given S and $F(\cdot)$, there is a unique z^* such that $z^* = z(\theta(z^*))$. We show that as z^* increases, the participation value of the marginal buyer type increases after taking into account the effect on θ:

$$\frac{dr\bar{V}_{z^*}}{dz^*} = \frac{\partial r\bar{V}_{z^*}}{\partial z^*} + \frac{\partial r\bar{V}_{z^*}}{\partial\theta}\Big(-\bar{B}\bar{M}'(z^*)\Big) > 0.$$

Therefore, there is a unique z^* such that the unmatched value of the marginal buyer is exactly equal to K_B and it is defined by equation (5).

This completes the proof of parts 2 and 3. ■

Proof of Lemma 5. For $q \in [0, \underline{R})$ we have $t(q) = t_N$, which implies that $\pi_c(0) > \pi_c(q)$ for $q \in (0, \underline{R})$. Therefore, either $q = 0$ or $q \geq \underline{q}$ for some $\underline{q} \geq \underline{R}$ and F is constant (and, hence, continuous) on $[0, \underline{q}]$. If $\underline{q} > \bar{R}$, then $t(q) = t(\bar{R})$ for $q \in [\bar{R}, \underline{q}]$ which implies that $\pi_c(\bar{R}) > \pi_c(q)$ for $q \in (\bar{R}, \underline{q}]$. Therefore, $\underline{q} \leq \bar{R}$. Standard arguments (as in Burdett and Mortensen, 1998) prove continuity on $[\underline{q}, \bar{q}]$. ■

Proof of Proposition 6. We determine the number of unmatched buyers and their type distribution using the fact that, in steady state, the flow of buyers into and out of the matched state must be equal. Let $n(R)$ denote the number of buyers who are unmatched and whose type is less than R. The total number of unmatched buyers is $n(\bar{R}) \equiv \bar{n}$. An unmatched buyer of type R becomes matched after transacting with a seller who offers above-reservation quality, which occurs at rate $\alpha_B(\theta)(1 - F(R))$. A matched buyer exits the matched state when his match is exogenously destroyed, which occurs at rate δ. In steady state:

$$n'(R)\alpha_B(\theta)(1 - (F(R))) = \delta(BH'(R) - n'(R)) \Rightarrow n'(R) = \frac{\delta BH'(R)}{\delta + \alpha_B(\theta)(1 - F(R))}.$$

Hence, the mass $n(R)$ satisfies:

$$n(R) = \int_{\underline{R}}^{R} \frac{B\delta}{\delta + \alpha_B(\theta)(1 - F(x))} dH(x),$$

and, thus, the mass of matched buyers is:

$$B - \bar{n} = B\Big(1 - \int_{\underline{R}}^{\bar{R}} \frac{\delta}{\delta + \alpha_B(\theta)(1 - F(x))} dH(x)\Big) = \int_{\underline{R}}^{\bar{R}} \frac{B\alpha_B(\theta)(1 - F(x))}{\delta + \alpha_B(\theta)(1 - F(x))} dH(x).$$

Let $G(\cdot)$ denote the received quality distribution for matched buyers. The mass of matched buyers receiving quality up to q is given by $(B - \bar{n})G(q)$. An unmatched type-R buyer flows into this group if $R \leq q$, and he samples a seller who offers quality less than q, which occurs at rate $\alpha_B(\theta)(F(q) - F(R))$. A buyer flows out of this group if the match is exogenously destroyed or if he samples a new seller whose quality if greater than q, which occurs at rate $\delta + \alpha_B(\theta)(1 - F(q))$. Equating these flows yields:

$$\begin{aligned} \alpha_B(\theta) \int_{\underline{R}}^{q} (F(q) - F(x)) dn(x) &= (B - \bar{n})G(q)(\delta + \alpha_B(\theta)(1 - F(q))) \\ \Rightarrow (B - \bar{n})G(q) &= \frac{\alpha_B(\theta)B\delta \int_{\underline{R}}^{q} \frac{F(q) - F(x)}{\delta + \alpha_B(\theta)(1 - F(x))} dH(x)}{\delta + \alpha_B(\theta)(1 - F(q))}. \end{aligned}$$

Thus, $G'(q)$ satisfies (we assume, and later verify, that F is differentiable):

$$(B - \bar{n})G'(q) = \frac{\alpha_B(\theta)B\delta F'(q)H(q)}{\left(\delta + \alpha_B(\theta)(1 - F(q))\right)^2}.$$

The number of buyers who are matched with a seller offering q is $(B - \bar{n})G'(q)$, and the number of sellers offering quality q is $SF'(q)$. Hence, the number of matched buyers per seller at quality q is:

$$l(q) = \frac{(B - \bar{n})G'(q)}{SF'(q)},$$

which implies that the flow of transactions from regular buyers is:

$$t_R(q) = \frac{\gamma \alpha_B(\theta)\theta\delta H(q)}{\left(\delta + \alpha_B(\theta)(1 - F(q))\right)^2}. \tag{7}$$

Combining results completes the proof of Proposition 6. ■

Proof of Proposition 7. Suppose that all sellers who participate in the market offer $q = 0$. In equilibrium, their profits are equal to the entry cost K_S and they are given by $\pi_c(0) = \alpha_B(\theta)\theta p$. Consider a seller with cost c' who deviates offers $q' = \bar{R}$. His profits are

$$\pi_{c'}(\bar{R}) \quad = \quad \alpha_B(\theta)\theta\Big(1 + \frac{\gamma\delta}{\delta^2}\Big)(p - c'\bar{R}),$$

which is strictly greater than K_S for c' close enough to zero. Since the support of seller types is $(0, \infty)$, such a type exists and strictly prefers to participate in the market. Therefore, $F(0) = 1$ cannot occur in equilibrium.

Take as given the set of sellers who participate in the market and consider sellers 1 and 2 with $c_1 > c_2$ who choose quality q_1 and q_2. We show that $q_2 \geq \underline{R} \Rightarrow q_2 > q_1$ and $q_2 = 0 \Rightarrow q_1 = 0$. Suppose, by contradiction, that $\underline{R} \leq q_2 \leq q_1$. Since seller 1 chooses q_1 over q_2 and seller 2 chooses quality q_2 over q_1:

$$\begin{aligned}
(p - c_1 q_1)t(q_1) \quad &\geq \quad (p - c_1 q_2)t(q_2) \Rightarrow p(t(q_1) - t(q_2)) \geq c_1\big(t(q_1)q_1 - t(q_2)q_2\big), \\
(p - c_2 q_2)t(q_2) \quad &\geq \quad (p - c_2 q_1)t(q_1) \Rightarrow p(t(q_1) - t(q_2)) \leq c_2\big(t(q_1)q_1 - t(q_2)q_2\big),
\end{aligned}$$

which, combined with the assumption $c_1 > c_2$, yields a contradiction and therefore $q_2 > q_1$. Going through the same steps, it is immediate that $q_2 = 0 \Rightarrow q_1 = 0$.

Combining the two results, there is a marginal seller type $\bar{c} > 0$ such that sellers with $c > \bar{c}$ offer $q = 0$ and sellers with $c \leq \bar{c}$ offer strictly positive quality, which is strictly decreasing in their type. Furthermore, notice that profits are strictly decreasing in type for sellers with $c \in (0, \bar{c})$, since a lower-cost seller can always mimic a higher-cost seller's action and earn higher profits than him.

The free entry condition specifies that the profits of participating sellers are weakly greater than the entry cost. This implies that the profits of the marginal seller type are equal to K_S, all sellers with $c < \bar{c}$ participate in the market and the measure of sellers who offer positive quality is $S_+(\bar{c}) = \bar{S}\ \bar{D}(\bar{c})$. Let S_0 denote the measure of sellers who participate in the market and offer $q = 0$, where $S_0 \geq 0$. Their profits do not depend on their type and satisfy

$$p\alpha_S\Big(\frac{B}{S_+(\bar{c}) + S_0}\Big) \quad \leq \quad K_S, \tag{8}$$

which holds with equality if $S_0 > 0$.

We derive the marginal seller type and the measure of sellers offering zero quality in three steps. First, we characterize the optimal quality offered by the marginal seller conditional on his type and S_0. Second, given S_0, we show that there a unique marginal type $\bar{c}$ such that his profits are equal to K_S. Third, we characterize S_0.

The profits of the seller who offers the lowest positive quality (the marginal seller) satisfy:

$$\underline{\pi}_c(q) = \alpha_S(\theta)(p - cq)\Big(1 + \frac{\gamma\delta H(q)}{\left(\delta + \alpha_B(\theta)(1 - F(0))\right)^2}\Big), \tag{9}$$

where $F(0) = \frac{S_0}{S}$ and $S = S_+(c) + S_0$.

Denote the optimal choice for a type-c seller who offers the lowest positive quality by $\underline{q}(c)$ and note that, necessarily, $\underline{q}(c) \geq \underline{R}$. $\underline{q}(c)$ is determined by the root of:

$$\underline{\pi}'_c(q) = \alpha_S(\theta)\Big[-c\Big(1 + \frac{\gamma\delta H(q)}{\big(\delta + \alpha_B(\theta)(1 - F(0))\big)^2}\Big) + (p - cq)\frac{\gamma\delta H'(q)}{\big(\delta + \alpha_B(\theta)(1 - F(0))\big)^2}\Big], \tag{10}$$

where the the log-concavity of $H(\cdot)$ guarantees that the second-order condition is negative:

$$\underline{\pi}''_c(q) = \alpha_S(\theta)\frac{-2c\gamma\delta H'(q) + (p - cq)\gamma\delta H''(q)}{\big(\delta + \alpha_B(\theta)(1 - F(0))\big)^2} < 0.$$

We show that, given S_0, there is a unique marginal type $\bar{c}$ such that his profits are exactly equal to K_S when he offers $\underline{q}(\bar{c})$. Notice that $\underline{\pi}_c(\underline{q}(c))$ is greater than K_S when c is low, is lower than K_S when c is high and is strictly decreasing in c:

$$\begin{aligned}
\lim_{c\to 0} \underline{\pi}_c(\underline{q}(c)) &= \lim_{c\to 0} \alpha_S(\frac{B}{S_0})pt(\underline{q}(c)) > K_S, \\
\lim_{c\to\infty} \underline{\pi}_c(\underline{q}(c)) &= \alpha_S(\frac{B}{\bar{S}}) \lim_{c\to\infty} (p - c\underline{q}(c))t(\underline{q}(c)) < K_S, \\
\frac{d\underline{\pi}_c(\underline{q}(c))}{dc} &= \underline{\pi}'_c(\underline{q}(c))\frac{d\underline{q}(c)}{dc} + \frac{\partial\underline{\pi}_c(\underline{q}(c))}{\partial c} + \frac{\partial\underline{\pi}_c}{\partial\theta}\frac{d\theta}{d\bar{c}} < 0.
\end{aligned}$$

In the last equation, the first term is zero by the envelope condition, the second term is negative because higher costs reduce margins and the third term is negative because an increase in $\bar{c}$ increases the measure of participating sellers which reduces a seller's arrival rate of new buyers. Therefore, there is a unique $\bar{c}$ such that $\underline{\pi}(\underline{q}(\bar{c})) = K_S$, given S_0.

To determine S_0, note that an increase in $\bar{S}$ leads to an increase in $S_+(\bar{c})$. Suppose, by contradiction, that this is not true and that the marginal type drops enough to outweigh the increase in the measure of potential sellers. In that case, however, the (new) marginal type's profits will be strictly greater than K_S because he has lower costs and faces fewer competing sellers. As a result, given S_0 an increase in $\bar{S}$ leads to a drop in the buyer-seller ratio, θ.

Define $\hat{S}$ as the measure of potential sellers such that equation (8) holds with equality when $\bar{S} = \hat{S}$ and $S_0 = 0$. If $\bar{S} \geq \hat{S}$ then $S_0 = 0$, $S = S_+(\bar{c})$ and $\bar{c}$ is determined by equating (10) to zero and (9) to K_S. If $\bar{S} < \hat{S}$, then the measure of sellers S is determined by $\alpha_S(\frac{B}{S})p = K_S$, $\bar{c}$ is determined by equating (10) to zero and (9) to K_S, and $S_0 = S - S_+(\bar{c})$.

We now determine $q^*(c)$ for $c < \bar{c}$. We assume that an optimal schedule $q^*(c)$ exists, and we rewrite the profits of a type-c seller as if he decides which other type c' to imitate rather than which quality to offer—i.e., to imitate type c', who offers quality $q' = q^*(c')$. We have:

$$\pi_c(c') = \alpha_B(\theta)\theta(p - cq^*(c'))(1 + \frac{\gamma\delta H(q^*(c'))}{(\delta + \alpha_B(\theta)(1 - F(0))D(c'))^2}),$$

where $D(\cdot)$ is the distribution of sellers in the market who offer positive quality:

$$D(c) = \frac{\bar{D}(c)}{\bar{D}(\bar{c})}$$

The advantage of formulating the choice in terms of c' rather than q' is that the term in the denominator depends on the exogenous type distribution $D(\cdot)$ rather than on the endogenous quality distribution $F(\cdot)$. Differentiating profits with respect to c', we obtain:

$$\begin{aligned}
\pi_c'(c') &= \alpha_B(\theta)\theta c\Big(-q^{*\prime}(c')(1+\frac{\gamma\delta H(q^*(c'))}{(\delta+\alpha_B(\theta)(1-F(0))D(c'))^2}) \\
&+ (\frac{p}{c}-q^*(c'))\gamma\delta\frac{H'(q^*(c'))q^{*\prime}(c')(\delta+\alpha_B(\theta)(1-F(0))D(c'))-H(q^*(c'))2\alpha_B(\theta)(1-F(0))D'(c')}{(\delta+\alpha_B(\theta)(1-F(0))D(c'))^3}\Big).
\end{aligned}$$

By construction, profits are maximized when $c' = c$, and, thus, we can set the derivative to zero and rearrange to arrive at equation:

$$q^{*\prime}(c) = -\frac{2\gamma\delta(\frac{p}{c}-q^*(c))H(q^*(c))\alpha_B(\theta)(1-F(0))D'(c)}{(\delta+\alpha_B(\theta)(1-F(0))D(c))\left((\delta+\alpha_B(\theta)(1-F(0))D(c))^2+\gamma\delta H(q^*(c))-\gamma\delta(\frac{p}{c}-q^*(c))H'(q^*(c))\right)} \tag{11}$$

This differential equation and the initial condition $q^*(\bar{c}) = \underline{q}(\bar{c})$ determine $q^*(c)$ for $c < \bar{c}$. ■

B Selection into ADAM

In this Appendix, we derive the density of log preferences z in the ADAM dataset.

An individual with preferences z is arrested if $\log(z) + \eta \geq 0$, where η is normally distributed with mean μ_η and variance σ_η^2. A fraction λ of individuals has taste $z = \varepsilon \approx 0$ and, thus, their arrest rate equals

$$\Pr\left(\log(\varepsilon) + \eta \geq 0\right) = 1 - \Phi\left(\frac{-\log(\varepsilon) - \mu_\eta}{\sigma_\eta}\right) = \Phi\left(\frac{\log(\varepsilon) + \mu_\eta}{\sigma_\eta}\right),$$

where $\Phi(\cdot)$ denotes the cumulative distribution function of a standard normal. A fraction $1-\lambda$ has taste z that follows a lognormal distribution with parameters μ_z and σ_z and, thus, their arrest rate is

$$\Pr\left(\log(z) + \eta \geq 0\right) = 1 - \Phi\left(\frac{-\mu_z - \mu_\eta}{\sqrt{\sigma_z^2 + \sigma_\eta^2}}\right) = \Phi\left(\frac{\mu_z + \mu_\eta}{\sqrt{\sigma_z^2 + \sigma_\eta^2}}\right).$$

Thus, the aggregate arrest rate is

$$P(A) = \lambda\Phi\left(\frac{\log(\varepsilon) + \mu_\eta}{\sigma_\eta}\right) + (1-\lambda)\Phi\left(\frac{\mu_z + \mu_\eta}{\sqrt{\sigma_z^2 + \sigma_\eta^2}}\right).$$

Moreover, a buyer enters the market if $\log(z) \geq \log(z^*)$. Therefore, a buyer is both in ADAM and in the market if $\log(z) \geq \log(z^*)$ and $\log(z) + \eta \geq 0$:

$$P(M, A) = (1-\lambda)\int_{\log(z^*)}^{\infty} \Phi\left(\frac{\log(z) + \mu_\eta}{\sigma_\eta}\right)\frac{1}{\sigma_z}\phi\left(\frac{\log(z) - \mu_z}{\sigma_z}\right) d\log(z),$$

where $\phi(\cdot)$ denotes the probability density function of a standard normal.

Thus, the fraction of drug users in ADAM is:

$$P(M|A)=\frac{P(M,A)}{P(A)}=\frac{(1-\lambda)\int_{\log(z^*)}^{\infty}\Phi\left(\frac{\log(z)+\mu_\eta}{\sigma_\eta}\right)\frac{1}{\sigma_z}\phi\left(\frac{\log(z)-\mu_z}{\sigma_z}\right)d\log(z)}{\lambda\Phi\left(\frac{\log(\varepsilon)+\mu_\eta}{\sigma_\eta}\right)+(1-\lambda)\,\Phi\left(\frac{\mu_z+\mu_\eta}{\sqrt{\sigma_z^2+\sigma_\eta^2}}\right)},$$

and the density of the log of drug users' preferences z in ADAM satisfies:

$$\frac{\frac{1}{\sigma_z}\phi\left(\frac{\log(z)-\mu_z}{\sigma_z}\right)\Phi\left(\frac{\log(z)+\mu_\eta}{\sigma_\eta}\right)}{\int_{\log(z^*)}^{\infty}\Phi\left(\frac{\log(z)+\mu_\eta}{\sigma_\eta}\right)\frac{1}{\sigma_z}\phi\left(\frac{\log(z)-\mu_z}{\sigma_z}\right)d\log(z)}\text{ for } z\geq z^*, \tag{12}$$

and zero otherwise. In practice, the density (12) is the density of buyers' preferences $\frac{1}{\sigma_z}\phi\left(\frac{\log(z)-\mu_z}{\sigma_z}\right)$ weighted by the probability $\Phi\left(\frac{\log(z)+\mu_\eta}{\sigma_\eta}\right)$ of being in ADAM for those with $z>\varepsilon$.

C Observable Drug Purity

In this Appendix, we prove the existence of equilibrium and derive its characterization when buyers observe drug quality before making a purchase. The analysis mirrors that of the baseline model.

C.1 The Buyers

A type-z buyers takes as given sellers' behavior ($F(\cdot)$ and S) and makes two decisions when meeting a new seller: whether to consume (reservation $\hat{R}_z$) and, if so, whether to match (reservation R_z if unmatched and q if matched with a q-seller). Notice that a buyer will never match with a seller whose product he does not want to consume. The value functions of a type-z buyer satisfy:

$$\begin{aligned}
r\bar{V}_z &= \alpha_B(\theta)\int_0^{\bar{q}}\max\left[zx-p+\max[V_z(x)-\bar{V}_z,0],0\right]dF(x),\\
rV_z(q) &= \gamma(zq-p)+\alpha_B(\theta)\int_0^{\bar{q}}\max\left[zx-p+\max[V_z(x)-V_z(q),0],0\right]dF(x)+\delta(\bar{V}_z-V_z(q)).
\end{aligned}$$

Comparing the static costs and benefits of consumption and equating the value functions deliver the reservation quality for becoming matched:

$$R_z=\hat{R}_z=\frac{p}{z}.$$

Thus, we can rewrite the value functions as follows:

$$\begin{aligned}
r\bar{V}_z &= \alpha_B(\theta)\int_{\frac{p}{z}}^{\bar{q}}(zx-p+V_z(x)-\bar{V}_z)dF(x),\\
rV_z(q) &= \gamma(zq-p)+\alpha_B(\theta)\Big(\int_{\frac{p}{z}}^{\bar{q}}(zx-p)dF(x)+\int_q^{\bar{q}}(V_z(x)-V_z(q))dF(x)\Big)+\delta(\bar{V}_z-V_z(q)).
\end{aligned}$$

Using integration by parts, the value of being unmatched satisfies:

$$\begin{aligned} r\bar{V}_z &= \alpha_B(\theta)\Big((zq - p + V_z(q) - \bar{V}_z)F(q)|_{R_z}^{\bar{q}} - \int_{R_z}^{\bar{q}} (z + V_z'(x))F(x)dx\Big) \\ &= \alpha_B(\theta)\Big(z(\bar{q} - R_z) + V_z(\bar{q}) - V_z(R_z) - \int_{\frac{p}{z}}^{\bar{q}} (z + V_z'(x))F(x)dx\Big), \end{aligned}$$

where we used $R_z = \frac{p}{z}$ and $\bar{V}_z = V_z(R_z)$ in the last equality. Using the fundamental theorem of calculus, we obtain:

$$\begin{aligned} r\bar{V}_z &= \alpha_B(\theta)\Big(\int_{\frac{p}{z}}^{\bar{q}} (z + V_z'(x))dx - \int_{\frac{p}{z}}^{\bar{q}} (z + V_z'(x))F(x)dx\Big) \\ &= \alpha_B(\theta)\int_{\frac{p}{z}}^{\bar{q}} \big[z + V_z'(q)\big](1 - F(x))dx. \end{aligned}$$

Differentiating the value of being matched with respect to q and rearranging, we obtain:

$$V_z'(q) = \frac{\gamma z}{r + \delta + \alpha_B(\theta)(1 - F(q))}.$$

Hence, combining the previous two equations, we obtain:

$$r\bar{V}_z = z\alpha_B(\theta)\int_{\frac{p}{z}}^{\bar{q}} \Big(1 + \frac{\gamma}{r + \delta + \alpha_B(\theta)(1 - F(x))}\Big)(1 - F(x))dx.$$

We now determine whether a buyer of type z participates in the market, taking as given the actions of sellers $\{F(\cdot), S\}$ and other buyers (B). The actions of other agents are summarized by $\{F(\cdot), \theta\}$. Notice that:

$$\begin{aligned} r\bar{V}_0 &= -\alpha_B(\theta)p < 0, \\ \frac{\partial r\bar{V}_z}{\partial z} &= \alpha_B(\theta)\int_0^{\bar{q}} xdF(x) + \alpha_B(\theta)\int_{R_z}^{\bar{q}} \frac{\gamma(1 - F(x))}{r + \delta + \alpha_B(\theta)(1 - F(x))}dx + \\ &\quad \frac{p}{z^2}\frac{\alpha_B(\theta)\gamma(1 - F(R_z))}{r + \delta + \alpha_B(\theta)(1 - F(R_z))} > 0, \end{aligned}$$

which prove that there exists a $\tilde{z}(F, \theta)$ such that a buyer participates if and only if $z \geq \tilde{z}(F, \theta)$.

Aggregating the entry decision across buyers, we obtain:

Proposition 10 *Given $F(\cdot)$ and S:*

1. *If $\frac{p}{\bar{q}} \geq \bar{z}$, then there is no buyer entry: $B = 0$.*

2. *If $\frac{p}{\bar{q}} < \bar{z}$, then there is a unique buyer type $z^{**} \leq \bar{z}$ such that all buyers with $z > z^{**}$ participate in the market and all buyers with $z \leq z^{**}$ do not.*

3. *The marginal buyer type is given by the solution to:*

$$z^{**}\alpha_B(\theta)\int_{\frac{p}{z^{**}}}^{\bar{q}} \Big(1 + \frac{\gamma}{r + \delta + \alpha_B(\theta)(1 - F(q))}\Big)(1 - F(q))dq = K_B. \tag{13}$$

Proof. Buyers with $z \leq \frac{p}{\bar{q}}$ have no benefit from participating in the market and never enter. Consider a buyer with $z > \frac{p}{\bar{q}}$. Whether this buyer enters the market depends on how quickly he trades, which, in turn, depends on the buyer-seller ratio θ. We have:

$$\begin{aligned}
\lim_{\theta\to\infty} r\bar{V}_z &= 0 < K_B, \\
\lim_{\theta\to 0} r\bar{V}_z &= \lim_{\theta\to 0} \alpha_B(\theta)\int_{\frac{p}{z}}^{\bar{q}} (1-F(x))dx > K_B, \\
\frac{\partial r\bar{V}_z}{\partial\theta} &= z\int_{\frac{p}{z}}^{\bar{q}} \Big(\alpha_B'(\theta) + \frac{\alpha_B'(\theta)(r+\delta)}{\alpha_B(\theta)^2}\frac{\gamma}{\big(\frac{r+\delta}{\alpha_B(\theta)}+1-F(x)\big)^2}\Big)(1-F(x))dx < 0.
\end{aligned}$$

Therefore, for each buyer of type z with $z > \frac{p}{\bar{q}}$, there is a unique $\theta(z)$ such that he participates if $\theta \leq \theta(z)$ and stays out otherwise. Hence, the measure of buyers in the market is:

$$B = \bar{B}(1-\bar{M}(z(\theta))).$$

Given S, this leads to

$$\theta(z(\theta)) = \frac{\bar{B}(1-\bar{M}(z(\theta)))}{S}.$$

We now show that, given S and $F(\cdot)$, there is a unique z^{**} such that $z^{**} = z(\theta(z^{**}))$. We show that as z^{**} increases, the participation value of the marginal type increases after taking into account the effect on θ:

$$\frac{dr\bar{V}_{z^{**}}}{dz^{**}} = \frac{\partial r\bar{V}_{z^{**}}}{\partial z^{**}} + \frac{\partial r\bar{V}_{z^{**}}}{\partial\theta}\Big(-\bar{B}\bar{M}'(z^{**})\Big) > 0.$$

Therefore, there is a unique z^{**} such that the unmatched value of the marginal buyer equals K_B, as in equation (13). ■

As in the baseline model, the distribution of reservation qualities obtains:

$$H(R) = \begin{cases} 0 & \text{if } R \leq \underline{R}, \\ \frac{1-\bar{M}(\frac{p}{R})}{1-\overline{M}(z^{**})} & \text{if } R \in [\underline{R}, \bar{R}], \\ 1 & \text{if } R \geq \bar{R}, \end{cases}$$

where $\underline{R} = R_z = \frac{p}{z}$ and $\bar{R} = R_{z^{**}} = \frac{p}{z^{**}}$.

C.2 The Sellers

We derive sellers' actions, taking as given the measure of buyers who participate B and the distribution of reservation qualities $H(\cdot)$.

Free entry determines the measure S and type distribution $D(\cdot)$ of sellers in the market, subject to flow participation cost K_S. The problem of a seller of type c is to choose a level of quality $q^{**}(c)$ that maximizes his steady-state profits, which depend on the margin per transaction $(p - cq)$ and the steady-state flow of transactions $(t(q))$:

$$\pi_c(q) = (p - cq)t(q).$$

We first derive some necessary conditions on the distribution of offered qualities:

Lemma 11 *In equilibrium, the quality distribution F has support on a subset of $[\underline{q}, \bar{q}]$, where $\underline{q} \in [\underline{R}, \bar{R}]$, and is continuous on $[0, \bar{q}]$.*

Proof. For $q \in [0, \underline{R})$, we have $t(q) = 0$ and, therefore, $q \geq \underline{q}$ for some $\underline{q} \geq \underline{R}$. If $\underline{q} > \bar{R}$, then $t(q) = t(\bar{R})$ for $q \in [\bar{R}, \underline{q}]$, which implies that $\pi_c(\bar{R}) > \pi_c(q)$ for $q \in (\bar{R}, \underline{q}]$. Therefore, $\underline{q} \leq \bar{R}$. The previous point proves that F is constant (and, hence, continuous) on $[0, \underline{q}]$. Standard arguments (as in Burdett and Mortensen, 1998) prove continuity on $[\underline{q}, \bar{q}]$. ■

We take $H(\cdot)$, $F(\cdot)$ and θ as given and calculate the steady-state profits that a type-c seller would enjoy for any quality q. The main result is summarized in the next proposition.

Proposition 12 *The steady-state profits of a seller of type c who offers quality q are:*

$$\pi_c(q) \quad = \quad \alpha_B(\theta)\theta H(q)(p - cq)\Big(1 + \frac{\gamma\delta}{\big(\delta + \alpha_B(\theta)(1 - F(q))\big)^2}\Big), \qquad\qquad q \geq \underline{q}.$$

Proof. To determine profits, we need to first determine the flow of a seller's transactions as a function of the quality he offers. The rate at which an individual seller transacts with a new buyer equals the meeting rate times the probability that the seller's quality is above the buyer's reservation:

$$t_N(q) \quad = \quad \alpha_S(\theta)H(q) = \theta\alpha_B(\theta)H(q).$$

The flow of transactions from regular buyers is:

$$t_R(q) = \gamma l(q),$$

where $l(q)$ is the steady-steady number of regular buyers of a seller offering q. The number of regular buyers per seller offering q is:

$$l(q) = \frac{(B - \bar{n})G'(q)}{SF'(q)},$$

where $\bar{n}$ is the number of unmatched buyers; $(B - \bar{n})G'(q)$ is the number of buyers who are matched with a seller offering q; and $SF'(q)$ is the number of sellers offering quality q.

We now determine the number of unmatched buyers and their type distribution. In steady state, the flow of buyers from the unmatched to the matched state must equal the flow out of the matched state and into the unmatched state. Let $n(R)$ denote the number of buyers who are unmatched and whose type is less than R. The total number of unmatched buyers is, thus, given by $n(\bar{R}) \equiv \bar{n}$.

An unmatched buyer of type R becomes matched after transacting with a seller who offers above-reservation quality, which occurs at rate $\alpha_B(\theta)(1 - F(R))$. A matched buyer exits the matched state when his match is exogenously destroyed, which occurs at rate δ. As a result, in steady state, the following holds:

$$n'(R)\alpha_B(\theta)(1 - (F(R))) = \delta(BH'(R) - n'(R)) \Rightarrow n'(R) = \frac{\delta BH'(R)}{\delta + \alpha_B(\theta)(1 - F(R))},$$

which we can rewrite as:

$$n(R) = \int_{\underline{R}}^{R} \frac{B\delta}{\delta + \alpha_B(\theta)(1 - F(x))} dH(x).$$

Therefore, we have:

$$\begin{aligned} \bar{n} &= \int_{\underline{R}}^{\bar{R}} \frac{B\delta}{\delta + \alpha_B(\theta)(1-F(x))} dH(x), \\ B - \bar{n} &= B\Big(1 - \int_{\underline{R}}^{\bar{R}} \frac{\delta}{\delta + \alpha_B(\theta)(1-F(x))} dH(x)\Big) = \int_{\underline{R}}^{\bar{R}} \frac{B\alpha_B(\theta)(1-F(x))}{\delta + \alpha_B(\theta)(1-F(x))} dH(x). \end{aligned}$$

We now characterize $G(\cdot)$. The mass of matched buyers receiving quality up to q is $(B-\bar{n})G(q)$. An unmatched type-R buyer flows into this group if $R \leq q$ and he samples a seller who offers quality less than q, which occurs at rate $\alpha_B(\theta)(F(q) - F(R))$. A buyer flows out of this group if the match is exogenously destroyed or if he samples a new seller whose quality if greater than q, which occurs at rate $\delta + \alpha_B(\theta)(1 - F(q))$. Equating these flows yields:

$$\begin{aligned} \alpha_B(\theta) \int_{\underline{R}}^{q} (F(q) - F(R)) dn(R) &= (B - \bar{n})G(q)(\delta + \alpha_B(\theta)(1 - F(q))), \\ \Rightarrow (B - \bar{n})G(q) \frac{\alpha_B(\theta)B\delta \int_{\underline{R}}^{q} \frac{F(q)-F(x)}{\delta+\alpha_B(\theta)(1-F(x))} dH(x)}{\delta + \alpha_B(\theta)(1-F(q))}. \end{aligned}$$

Rearranging, we obtain:

$$(B - \bar{n})G'(q) = \frac{\alpha_B(\theta)B\delta F'(q)H(q)}{\left(\delta + \alpha_B(\theta)(1 - F(q))\right)^2},$$

which implies that the flow of transactions from regular buyers is:

$$t_R(q) = \frac{\gamma\alpha_B(\theta)\theta\delta H(q)}{\left(\delta + \alpha_B(\theta)(1 - F(q))\right)^2}.$$

Combining results completes the proof of the Proposition. ■

We characterize the distribution of offered qualities, $F(\cdot)$ and the number of sellers who enter the market, taking as given the number of buyers B and the distribution of their reservation values $H(\cdot)$.

Lemma 13 *Consider sellers 1 and 2 with costs c_1 and c_2 and denote their actions by q_1 and q_2 and their profits by π_1 and π_2. Then: $c_1 > c_2 \Rightarrow q_2 > q_1$ and $c_1 > c_2 \Rightarrow \pi_2 > \pi_1$.*

Proof. The proof for qualities is by contradiction. Suppose that $c_1 > c_2$ and $q_2 \leq q_1$. Recall that profits are given by $\pi_c(q) = (p - cq)t(q)$. Seller 1 chooses q_1 over q_2 and seller 2 chooses q_2 over q_1. Therefore:

$$\begin{aligned} (p - c_1q_1)t(q_1) \geq (p - c_1q_2)t(q_2) &\quad\Rightarrow\quad p(t(q_1) - t(q_2)) \geq c_1(t(q_1)q_1 - t(q_2)q_2) \\ (p - c_2q_2)t(q_2) \geq (p - c_2q_1)t(q_1) &\quad\Rightarrow\quad p(t(q_1) - t(q_2)) \leq c_2(t(q_1)q_1 - t(q_2)q_2). \end{aligned}$$

which yields the desired contradiction, after noting that $c_1 > c_2$.

Regarding profits, note that:

$$\pi_1 = (p - c_1q_1t(q_1)) < (p - c_2q_1)t(q_1) \leq (p - c_2q_2)t(q_2) = \pi_2$$

which completes the proof. ■

We characterize the marginal seller type $\bar{c}$ and the lowest quality that is offered, $\underline{q}$ (we know from the previous Lemma that $\underline{q}$ is offered by the $\bar{c}$-seller). Two conditions need to be satisfied: first, $\underline{q}$ must give higher profits to $\bar{c}$ than any other quality level; second, $\underline{q}$ must cover the seller's flow cost. The proposition summarizes the result.

Proposition 14 *Given B and $H(\cdot)$, there is a unique marginal seller type $\bar{c}$ such that sellers with $c \leq \bar{c}$ participate in the market and sellers with $c > \bar{c}$ do not. The marginal seller type is determined by the solution to:*

$$\alpha_B(\theta)\theta H(\underline{q}(\bar{c}))(p - c\underline{q}(\bar{c}))\Big(1 + \frac{\gamma\delta}{\left(\delta + \alpha_B(\theta)\right)^2}\Big) \quad = \quad K_S \tag{14}$$

where $\underline{q}(c)$ solves

$$cH(\underline{q}) = H'(\underline{q})(p - c\underline{q}) \tag{15}$$

Proof. It is immediate from Lemma 13 that a marginal type exists, he offers the lowest quality level and his profits are equal to the entry cost K_S.

The profits of a type-c seller who offers the lowest quality are given by:

$$\underline{\pi}_c(q) \quad = \quad \alpha_S(\theta)H(q)(p - cq)\Big(1 + \frac{\gamma\delta}{\left(\delta + \alpha_B(\theta)\right)^2}\Big).$$

The optimal choice of the lowest quality is given by the root of:

$$\underline{\pi}_c\prime(q) \quad = \quad \alpha_S(\theta)\Big(1 + \frac{\gamma\delta}{\left(\delta + \alpha_B(\theta)\right)^2}\Big)\Big(H'(q)(p - cq) - cH(q)\Big).$$

The second derivative of profits is always negative due to the log-concavity of $H(\cdot)$:

$$\underline{\pi}_c\prime\prime(q) \quad = \quad \alpha_S(\theta)\Big(1 + \frac{\gamma\delta}{\left(\delta + \alpha_B(\theta)\right)^2}\Big)\Big(H''(q)(p - cq) - 2cH(q) - cH'(q)\Big) < 0.$$

Therefore, the optimal quality choice for a type-c seller who offers the lowest quality $\underline{q}(c)$ is given by equation (15).

The marginal seller type determines the buyer-seller ratio according to $\theta = \frac{B}{S\ D(\bar{c})}$. The profits of the seller who offers $\underline{q}(c)$ are decreasing in his cost c:

$$\frac{d\underline{\pi}_c(\underline{q}(c))}{dc} \quad = \quad \frac{\underline{\pi}_c(\underline{q})}{d\underline{q}}\underline{q}'(c) + \frac{\partial\underline{\pi}_c(\underline{q}(c))}{\partial c} + \frac{\partial\underline{\pi}_c(\underline{q}(c))}{\partial\theta}\frac{d\theta}{dc} < 0$$

It is also immediate that:

$$\begin{aligned} \lim_{c\to 0}\underline{\pi}(\underline{q}(c)) \quad &> \quad K_S \\ \lim_{c\to \infty}\underline{\pi}(\underline{q}(c)) \quad &< \quad K_S \end{aligned}$$

Therefore, there is a unique marginal seller type $\bar{c}$ and is determined by equation (14). ■

Corollary 15 *The number of sellers who participate in the market is $S = \bar{S}\ \bar{D}(\bar{c})$. The distribution of their types is $D(c) = \frac{\bar{D}(c)}{\bar{D}(\bar{c})}$.*

We now determine sellers' optimal $q^{**}(c)$ for $c < \bar{c}$.

Proposition 16 *Given $H(\cdot)$ and B, the optimal quality choice for sellers of type c is given by the solution to the differential equation*

$$q^{**\prime}(c) = \frac{2H(q^{**}(c))(p - cq^{**}(c))\gamma\delta\alpha_B(\theta)D'(c)}{[H'(q^{**}(c))(p - cq^{**}(c)) - cH(q^{**}(c))]\Big(\delta + \alpha_B(\theta)D(c)\Big)[(\delta + \alpha_B(\theta)D(c))^2 + \gamma\delta]}, \quad (16)$$

where the initial condition $q^{**}(\bar{c}) = \underline{q}(\bar{c})$.

The distribution of qualities is:

$$F(q) = 1 - D(q^{**-1}(q)).$$

Proof. To characterize the optimal quality offer $q^{**}(c)$ for a type-c seller with $c < \bar{c}$, we rewrite his profits as if he decides which other type c' to imitate rather than which quality to offer. In other words, his profits from offering some quality q' are written in terms of imitating type c', who offers quality $q' = q^{**}(c')$. We have:

$$\pi_c(c') = \alpha_S(\theta)H(q^{**}(c'))(p - cq^{**}(c'))(1 + \frac{\gamma\delta}{(\delta + \alpha_B(\theta)D(c'))^2}).$$

The advantage of formulating the choice in terms of c' rather on than q' is that the term in the denominator depends on the exogenous type distribution $D(\cdot)$ rather than the endogenous quality distribution $F(\cdot)$. The quality distribution can be recovered once $q^{**}(c)$ is constructed.

We differentiate profits with respect to c':

$$\begin{aligned} \pi'_c(c') &= \alpha_S(\theta)\Big(H'(q^{**}(c'))q^{**\prime}(c')(p - cq^{**}(c'))(1 + \frac{\gamma\delta}{(\delta + \alpha_B(\theta)D(c'))^2}) \\ &\quad - \frac{cq^{**\prime}(c')H(q^{**}(c'))\gamma\delta}{(\delta + \alpha_B(\theta)D(c'))^2} - H(q^{**}(c'))(p - cq^{**}(c')\frac{\gamma\delta 2(\delta + \alpha_B(\theta)D(c'))D'(c')}{(\delta + \alpha_B(\theta)D(c'))^3}\Big) \end{aligned}$$

Equating the derivative to zero and setting $c' = c$ leads to equation (16) which, together with the initial condition $q^{**}(\bar{c}) = \underline{q}(\bar{c})$, defines $q^{**}(c)$ for $c \in (0, \bar{c})$. ■

C.3 Equilibrium

The proof of equilibrium existence is identical to the baseline model and is, therefore, omitted.