Market Provision of Public Goods: The Case of Broadcasting*

Abstract
This paper presents a theory of the market provision of broadcasting and uses it to address the nature of market failure in the industry. Advertising levels may be too low or too high, depending on the relative sizes of the nuisance cost to viewers and the expected benefits to advertisers from contacting viewers. Market provision may allocate too few or too many resources to programming and these resources may be used to produce programs of the wrong type. Monopoly may produce a higher level of social surplus than competition and the ability to price programming may reduce social surplus.

JEL Classification: D43, L13, L82
Keywords: public goods, broadcasting, advertising, market failure.

Simon P. Anderson  
Department of Economics  
University of Virginia  
Charlottesville VA 22901  
sa9w@virginia.edu

Stephen Coate  
Department of Economics  
Cornell University  
Ithaca NY 14853  
sc163@cornell.edu

*We thank Preston McAfee and Sharon Tennyson for helpful comments. We also thank Dan Bernhardt, Tim Besley, John Conley, Simon Cooper, Maxim Engers, Antonio Rangel, Joel Waldfogel, and numerous seminar and conference participants for useful discussions, and Sadayuki Ono for research assistance.
1 Introduction

Individuals in western countries spend a remarkable portion of their lives watching television and listening to radio. In the United States, the average adult spends around four hours a day watching television and three hours a day listening to the radio.\(^1\) Television and radio are also key ways that producers advertise their products. In the United States, television advertising accounted for 23.4\% of total advertising expenditures in 1999 and radio accounted for 8\%.\(^2\) All of this makes television and radio broadcasting of central economic importance.

In the United States, the bulk of radio and television broadcasts have always been provided by private commercial broadcasters. By contrast, in Europe and Japan, broadcasting has historically been provided by public broadcasters, financed through a mixture of television license fees, appropriations from general taxation, and advertising. Since the 1980s, however, commercial broadcasting has dramatically expanded in these countries. The market now plays a significant role in providing broadcasting in almost all western countries.

Despite this, little is known about the welfare economics of market-provided broadcasting. Will market provision lead to excessive advertising levels? Will it allocate too few or too many resources to programming and will these resources be used to produce programs of the appropriate quality and diversity? How will the ownership structure of broadcasting stations impact market outcomes?

Such questions arise continually in debates about the appropriate regulation of the broadcasting industry. Excessive advertising is an issue in the United States where total non-program minutes now exceed 20 minutes per hour on some network television programs and 30 minutes per hour

\(^1\) The Radio Advertising Bureau reports that in 1998 the average weekday time spent listening by adults is 3 hrs and 17 minutes; weekend time spent listening is 5 hrs and 30 mins (http://www.rab.com/station/mgb99/fac5.html). The Television Advertising Bureau reports that in 1999 the average adult man spent 4 hours and 2 minutes watching television per day, while the average adult woman spent 4 hours and 40 minutes (http://www.tvb.org/tvfacts).

\(^2\) Total advertising expenditures were $215 billion. Other important categories were newspapers (21.7\%); magazines (5.3\%); direct mail (19.2\%) and yellow pages (5.9\%) (http://www.tvb.org/tvfacts).
on certain radio programs. In Europe, advertising ceilings are imposed on broadcasters and it is natural to wonder if the United States should follow suit. Concerns about the programming provided by commercial radio led the Federal Communications Commission to announce that it was setting up hundreds of free “low-power” radio stations for non-profit groups across the United States (Leonhardt (2000)). More generally, such concerns are key to the debate about the role for public broadcasting in modern broadcasting systems (see, for example, the Davies Report (1999)). The effect of ownership structure is currently an issue in the United States radio industry which, following the Telecommunications Act of 1996, has seen growing concentration. Concern has been expressed that this will lead to higher prices for advertisers and less programming (see, for example, Ekelund, Ford, and Jackson (1999)).

Our purpose in this paper is to present a theory of the market provision of broadcasting and use it to explore the nature of market failure in the industry. The theory is distinctive in yielding predictions on both the programming and advertising produced by a market system. It therefore permits an analysis of how well commercial broadcasting fulfills its dual role of providing programming to viewers/listeners and permitting producers to contact potential customers.

In the next section, we explain how our analysis relates to previous work on broadcasting and the theory of public goods more generally. Sections 3 and 4 introduce the model and explore how market provision of broadcasts differs from optimal provision. Section 5 extends the model

---

3 Non-program minutes include commercials, station and networks promos, and public service announcements. The 1999 Television Commercial Monitoring Report indicates that non-program minutes on prime time network shows in November 1999 ranged from 12.54 minutes per hour to 21.07 minutes. Commercial minutes ranged from 9.31 minutes to 15.07 minutes. Good information on non-program minutes on radio is more difficult to find. However, in an article about a new technology that allows radio stations to wedge in more commercial minutes by truncating sounds and pauses in talk programs, Kuczynski (2000) reports that commercial minutes exceed 30 minutes per hour on some programs.

4 The ceilings chosen vary from country to country. In the United Kingdom the limit for private television channels is 7 minutes per hour on average. In France, it is 6 minutes and, in Germany, 9 minutes (Motta and Polo (1997)). In the United States, the National Association of Broadcasters, through its industry code, once set an upper limit on the number of commercial minutes per hour and this was implicitly endorsed by the Federal Communications Commission. In 1981, this practice was declared to violate the antitrust laws and no such agreement exists today (Owen and Wildman (1992)). In 1990, Congress enacted the Children’s Programming Act which limits advertising on children’s programming to 12 minutes per hour on weekdays and 10 minutes per hour on weekends.
to illustrate some further problems with market provision. Section 6 analyzes whether market provision produces better outcomes under monopoly or competition and how the possibility of pricing programming impacts market performance. Section 7 concludes with a summary of the main lessons of the analysis.

2 Relationship to the literature

Previous normative work on the market provision of broadcasting (see Owen and Wildman (1992) or Brown and Cave (1992) for reviews) has focused on the type of programming that would be produced and the viewer/listener benefits it generates. The literature concludes that the market may provide programming inoptimally: popular program types will be excessively duplicated (Steiner (1952)) and speciality types of programming will tend not to be provided (Spence and Owen (1977)). To illustrate, consider a radio market in which 3/4 of the listening audience like country music and 1/4 like talk, and suppose that the social optimum calls for one station serving each audience type. Then, the literature suggests that the market equilibrium might well involve two stations playing country music. Duplication arises because attracting half of the country listening audience is more profitable than getting all the talk audience. The lack of a talk station arises because capturing 1/4 of the audience does not generate enough advertising revenues to cover operating costs. This is so despite the fact that aggregate benefits to talk listeners exceed operating costs.

While these conclusions are intuitively appealing, the literature’s treatment of advertising is unsatisfactory. First, advertising levels and prices are assumed fixed. Thus, each program is assumed to carry an exogenously fixed number of advertisements and the revenue raised from

---

5 The fact that broadcasting is used by both viewers and advertisers and that the latter also create surplus has been largely ignored. One exception is Berry and Waldfogel’s (1999) study of the U.S. radio broadcasting industry, which addresses empirically the question of whether free entry leads to a socially excessive number of radio stations. Their study is distinctive in clearly distinguishing between the social benefits of additional radio stations stemming from delivering more listeners to advertisers and more programming to listeners.
each advertisement equals the number of viewers times an exogenously fixed per viewer price (Steiner (1952), Beebe (1977), Spence and Owen (1977) and Doyle (1998)).

Second, the social benefits and costs created by advertisers’ consumption of broadcasts are not considered. These features preclude analysis of the basic issue of whether market-provided broadcasts will carry too few or too many advertisements. More fundamentally, since advertising revenues determine the profitability of broadcasts, one cannot understand the nature of the programming the market will provide without understanding the source of advertising revenues. Since these revenues depend on both the prices and levels of advertising, the literature offers an incomplete explanation of advertising revenues and hence its conclusions concerning programming choices are suspect.

The theory developed in this paper provides a detailed treatment of advertising, while preserving the same basic approach to thinking about the market developed in the literature. To enable a proper welfare analysis, the model incorporates the social benefits and costs of advertising. The benefits are that advertising allows producers to inform consumers about new products, facilitating the consummation of mutually beneficial trades. The costs stem from its nuisance value. In addition, the model assumes that broadcasters choose advertising levels taking account of their

---

6 There are a number of exceptions. Assuming that a broadcaster’s audience size is reduced by both higher subscription prices and higher advertising levels, Wildman and Owen (1985) compare profit maximizing choices under pure price competition and pure advertising competition and conclude that viewer surplus would be the same in either case. However, theirs is not an equilibrium analysis. Making a similar assumption that viewers are turned off by higher levels of advertisements, Wright (1994) and Vaglio (1995) develop equilibrium models of competition in an advertiser supported system. However, their models are both too ad hoc and too intractable to yield insight into the normative issues. Masson, Mudambi, and Reynolds (1990) develop an equilibrium model of competition by advertiser supported broadcasters in their analysis of the impact of concentration on advertising prices but their model permits neither an analysis of the provision of programming nor a welfare analysis.

7 While this is the most obvious role of advertising, not all advertising on radio and television is of this form. Advertisements for new cars, movies and toys fit this model, but advertisements for established soft drinks and beers do not. The advertising literature identifies a number of other possible functions of advertising. An alternative informational perspective is that advertising acts as a signal. Consumers are aware of the prices and availability of goods, but cannot observe product quality (Nelson (1974), Kihlstrom and Riordan (1984), Milgrom and Roberts (1986)). Advertising signals high quality, because only a firm that would get repeat business could afford the upfront outlay represented by advertising. Chwe (1997) argues that advertising might solve coordination problems. If consumers benefit from consuming the same type of good, advertising can serve as a coordinating device for their consumption decisions. Critics of advertising often argue that its purpose is to persuade rather than to inform. Under this persuasive view, advertisements can directly alter consumers’ tastes for a product. Even if post-advertising tastes are taken as the correct ones, market determined levels of advertising can be excessive (Dixit and Norman (1978)). Clearly, each of these perspectives implies a different view of the social benefits of advertising. Thus, the results obtained in this paper will be sensitive to the assumed function of advertising.
effect on the number of viewers and on advertising prices. In this way, advertising revenues and hence program profitability are determined endogenously.

Since the first version of this paper was completed, a spate of papers on broadcasting have appeared. For our purposes, particularly noteworthy is Hansen and Kyhl’s (2001) welfare comparison of pay per view broadcasting with pure advertiser-supported provision of a single event (like a boxing match). Their analysis takes into account both advertiser surplus and the nuisance cost of advertisements to viewers and endogenizes advertising levels. Our analysis of pricing in Section 6.2 generalizes their welfare comparison beyond the case of a single monopoly-provided program. Also related are Gabszewicz, Laussel and Sonnac (2001) and Dukes and Gal-Or (2001) who develop spatial models of broadcasting competition in which two broadcasters compete in both programming and advertising levels. Gabszewicz, Laussel and Sonnac use their model to argue that advertising ceilings will lead stations to choose more similar programming. Dukes and Gal-Or provide a more detailed treatment of the product market in which advertisers compete and argue that product market competition can provide stations with an incentive to choose less differentiated programming. While both of these papers develop models that endogenize programming and advertising levels, neither is focused on the welfare issues motivating this paper.

The paper also contributes to the theory of public goods (see Cornes and Sandler (1996) for a comprehensive review). It shows that radio and television broadcasts can be thought of as public goods that are “consumed” by two types of agents. The first are viewers/listeners who receive a direct benefit from the broadcast. The second are advertisers who by advertising on the broadcast receive an indirect benefit from contacting potential customers. The nuisance value of advertisements means that advertisers’ “consumption” of a broadcast imposes an externality on viewers. However, advertisers can be excluded and, by charging advertisers for accessing their

---

8 A selection of these papers were presented at a recent conference and can be found at http://www.core.ucl.ac.be/media/default.html.
broadcasts, broadcasting firms can earn revenues, enabling market provision.

The special features of broadcasts make them a distinct type of public good the market provision of which raises interesting theoretical issues. In particular, it is not clear a priori in what ways market provision will diverge from optimal provision. Since advertisers’ consumption of a broadcast imposes an externality on viewers, optimal provision requires that advertisers face a Pigouvian corrective tax for accessing programming. The price advertisers must pay to broadcasters to advertise on their programs may be thought of as playing this role. Accordingly, the basic structure of market provided broadcasting - free provision to viewers/listeners financed by charges to advertisers - appears similar to that of an optimal structure. The issues are how well equilibrium prices of advertising internalize the externality and whether advertising revenues generate appropriate incentives for the provision of broadcasts.

3 The model

We are interested in modeling a basic broadcasting system in which programs are broadcast over the air and viewers/listeners can costlessly access such programming. Thus, we will be assuming that viewers/listeners have the hardware (i.e., televisions and radios) allowing them to receive broadcast signals. Moreover, we assume that broadcasters cannot exclude consumers by requiring special decoders, etc.\(^9\)

There are two channels each of which can carry one program. There are two types of program, indexed by \(t \in \{1, 2\}\). Examples of program types are “top 40” and “country” for radio, and “news” and “sitcom” for television. For concreteness, we focus on the television case and henceforth refer to consumers as viewers. Programs can carry advertisements. Each advertise-

\(^9\) This is still a reasonable model of radio broadcasting in the United States. It is also a reasonable model for television in countries, like the United Kingdom, in which most viewers still pick up television signals via a rooftop antenna. In the United States, however, the majority of households receive television via cable. The cable company picks up signals and retransmits to households via cable. This yields superior picture quality and permits reception of more channels. The cable company charges a monthly fee and can exclude consumers from viewing certain channels, which permits the use of subscription prices. Our basic model applies in the cable case when all consumers are hooked up and subscription prices are not used. We introduce subscription prices in section 6.2.
ment takes a fixed amount of time and thus advertisements reduce the substantive content of a program. Each program type is equally costly to produce and producing advertisements costs the same as producing regular programming. The cost of producing either type of program with an advertisements is denoted $K$.

There are $2N$ potential viewers, each of whom watches at most one program. Viewers are distinguished by the type of program they prefer and the degree to which their less preferred program can substitute for their preferred program. Formally, each viewer is characterized by a pair $(t, \lambda)$ where $t$ represents the viewer’s preferred type of program and $\lambda$ denotes the fraction of the gross viewing benefits he gets from his less preferred type of program. A type $(t, \lambda)$ viewer obtains a net viewing benefit $\beta - \gamma a$ from watching a type $t$ program with advertisements and a benefit $\lambda \beta - \gamma a$ from watching the other type of program with advertisements. The parameter $\gamma$ represents the nuisance cost of advertisements and is the same for all viewers.

There are $N$ viewers of type $t$ and, for each group, the parameter $\lambda$ is distributed uniformly on the interval $[-\varepsilon, 1]$, where $\varepsilon \geq 0$. Not watching any program yields a zero benefit so that if $\lambda < 0$, a viewer finds it costly to watch his less-preferred program. This might be the case, for example, for teenagers watching a public affairs program. The larger is $\varepsilon$, the more viewers in this category.

Advertisements are placed by producers of new goods and inform viewers of the nature and prices of these goods. Having watched an advertisement for a particular new good, a viewer knows his willingness to pay for it and will purchase it if this is no less than its advertised price. There are $m$ producers of new goods, each of which produces at most one good. New goods can be produced at a constant cost per unit, which with no loss of generality we set equal to zero. Each new good is characterized by some type $\sigma \in [0, \overline{\sigma}]$ where $\overline{\sigma} < 1$. New goods with higher types are more likely to be attractive to consumers. Specifically, if a new good is of type $\sigma$, then a viewer will have willingness to pay $\omega > 0$ with probability $\sigma$ and willingness to pay 0 with probability $1 - \sigma$. The fraction of producers with new goods of type less than or equal to $\sigma$ is (approximately)
The assumption that all consumers have a willingness to pay of $\omega$ or 0 means that each new producer will advertise a price of $\omega$. A lower price does not improve the probability of a sale. Thus, a new producer with a good of type $\sigma$ is willing to pay $\sigma \omega$ to contact a viewer. Accordingly, if $p$ denotes the per-viewer price of an advertisement (i.e., if the advertisement is seen by $n$ viewers it costs $np$), then the number of firms wishing to advertise is (approximately) $a(p) = m \cdot [1 - p/\omega \sigma]$.\(^{10}\)

This represents the demand curve for advertising. The corresponding inverse demand curve is $p(a) = \omega \sigma \cdot [1 - a/m]$.

The fact that each new producer sets a price of $\omega$ implies that consumers receive no expected benefits from their purchases of new products: the new producers extract all the surplus from the transaction. This implies that viewers get no informational benefit from watching a program with advertisements. Viewers therefore allocate themselves across their various options in such a way as to maximize their net viewing benefits, which simplifies the analysis.\(^{11}\) We assume that a consumer who is indifferent between watching any two programs is equally likely to watch either.

This completes the description of the model. In the analysis to follow, we maintain the following assumption concerning the values of the parameters:

**Assumption 1**  
(i) $\gamma \in (0, 2\beta/m)$ and (ii) $\omega \sigma \in (0, 4\beta/m)$.

The role of these restrictions will become clear later in the paper.

\(^{10}\) This approximation is designed to circumvent the analytical difficulties created by a step demand function. The larger is $m$, the better the approximation.

\(^{11}\) The model can be extended to incorporate informational benefits by assuming that each consumer’s valuation of a new producer of type $\sigma$’s product is the realization of a random variable $\nu$ which is uniformly distributed on $[\omega, \sigma \omega]$ with probability $\sigma$ and is 0 with probability $1 - \sigma$. Under the assumption that $\omega > \frac{\sigma \omega}{2}$, the type $\sigma$ new producer’s optimal price is $\omega$ and, hence, if a consumer watches an advertisement placed by a type $\sigma$ new producer, he obtains an informational benefit $\sigma \omega - \omega^2$. The introduction of informational benefits in this way does not significantly change any of the conclusions of the paper. The details are available in the appendix of the version of the paper available at the web site http://www.people.virginia.edu/~sa9w/.
4 Optimal vs market provision

4.1 Optimal provision

To understand optimal provision, it is helpful to think of the two types of program as discrete public goods each of which costs $K$ to provide and each of which may be consumed by two types of agents - viewers and advertisers. By an advertiser “consuming” a program, we simply mean that its advertisement is placed on that program. The optimality problem is to decide which of these public goods to provide and who should consume them. We first analyze the desirability of providing one program rather than none, and then consider adding the second program.

Given that the number of viewers of each type is the same, if one program is provided, its type is immaterial. For concreteness, we consider a type 1 program. Following the Samuelson rule for the optimal provision of a discrete public good, provision of the program will be desirable if the sum of benefits it generates exceed its cost. Typically, the aggregate benefit associated with a public good is just the sum of all consumers’ willingnesses to pay. However, in the case of broadcasts, account must be taken of externalities between the two types of consumers.

More specifically, suppose that the program has $a \leq \beta / \gamma$ advertisements and hence is “consumed” by $a$ new producers. Then, all type 1 viewers will watch the program and obtain a benefit $\beta - \gamma a$. Type 2 viewers for whom $\lambda \geq \frac{\gamma a}{\beta}$ will watch and obtain a benefit $\lambda \beta - \gamma a$. Clearly, the $a$ advertisements should be allocated to those new producers who value them the most, so the aggregate benefits generated by the program are

$$B_1(a) = N[\beta - \gamma a + \int_{\frac{\gamma a}{\beta}}^{1} (\lambda \beta - \gamma a) \frac{d \lambda}{1 + \varepsilon}] + N(1 + \frac{1 - \gamma a / \beta}{1 + \varepsilon}) \int_{0}^{a} p(\alpha) d\alpha.$$  

The first term represents viewer benefits, while the second measures the benefits to advertisers.

The level of advertising that maximizes these benefits, denoted $a_1^*$, satisfies the first order

---

12 A level of advertisements in excess of $\beta / \gamma$ yields no viewers and hence no benefits to either viewers or advertisers.
condition\textsuperscript{13}:

\[ p(a_1^0) \leq \gamma + \frac{\gamma \int_0^{\alpha_1} p(\alpha) d\alpha}{\beta(2 + \varepsilon) - \gamma a_1^0} \text{ with equality if } a_1^0 > 0. \]

Essentially, this says that the per viewer marginal social benefit of advertising must equal its per viewer marginal social cost. The per viewer marginal social benefit is the marginal advertiser’s willingness to pay per viewer, \( p(a) \). The per viewer marginal social cost is the consequent decrease in each viewer’s utility, \( \gamma \), plus a term that reflects the losses to existing advertisers resulting from their advertisements being seen by a smaller audience. Increased advertising drives type 2 viewers from the market meaning that they are not exposed to the existing advertisements and the net social benefits that they engender. Letting \( B_1^a = B_1(a_1^a) \) denote maximal aggregate benefits, providing the program is desirable if the operating cost \( K \) is less than \( B_1^a \).

The determination of the optimal advertising level with one program is illustrated in Figure 1. The horizontal axis measures the level of advertising, while the vertical axis measures dollars per viewer. The downward sloping line is the inverse demand curve \( p(a) \), measuring the willingness to pay of the marginal advertiser to contact a viewer. The horizontal line is the nuisance cost \( \gamma \) and the upward sloping line is the graph of the function \( \gamma + \frac{\gamma \int_0^{\alpha} p(\alpha) d\alpha}{\beta(2 + \varepsilon) - \gamma a_1^0} \), which represents the per viewer marginal social cost of advertising. The optimal advertising level is determined by the intersection of the inverse demand curve with the marginal cost curve.

It is natural to interpret the price \( p(a_1^0) \) as a Pigouvian corrective tax. Each new producer’s consumption of the program imposes an externality both on viewers through the nuisance cost and on other advertisers through the loss of audience. Advertisers’ consumption of the program should thus be taxed and the optimal tax is \( p(a_1^0) \).

\textsuperscript{13} It will be shown below that the constraint that \( a_1^0 \) be no greater than \( \beta/\gamma \) is not binding.

\textsuperscript{14} It is also the case that each viewer who chooses to view the program confers an external benefit on the advertisers since he/she might purchase one of their goods. It would therefore be desirable to subsidize viewers to watch. However, we do not consider such subsidies since they would seem difficult to implement. Even if it were possible to monitor use of a radio or television, the difficulty would be making sure that a viewer/listener was actually watching/listening. That said, commercial radio stations sometimes give out prizes to listeners by inviting them to call in if they have the appropriate value of some random characteristic (like a telephone number) and this
Adding a type 2 program will be desirable if the increase in aggregate benefits it generates exceeds its cost $K$. When both programs are provided, advertising levels on the two programs should be the same. Divergent advertising levels cause some viewers to watch a less preferred program and, because all viewers are of equal value to advertisers, this situation is dominated by one in which net aggregate advertising benefits are the same but levels are equalized. If the common level of advertisements is $a \leq \beta/\gamma$, all viewers will watch their preferred programs and obtain a benefit $\beta - \gamma a$. Since the $a$ advertisements are allocated to those new producers who value them the most, the aggregate benefits from providing both programs are

$$B_2(a) = 2N(\beta - \gamma a) + 2N \int_0^a p(\alpha) d\alpha.$$  

The two terms represent viewer and advertiser benefits, respectively.

The benefit maximizing advertising level, denoted $a^o_2$, satisfies the first order condition:

$$p(a^o_2) \leq \gamma \text{ with equality if } a^o_2 > 0.$$  

The per viewer marginal social cost is now simply the nuisance cost, since a marginal increase in advertising causes no viewers to switch off (each viewer is watching his favorite program and hence there are no marginal viewers). The optimal advertising level is illustrated in Figure 1.16

Letting $B^o_2 = B_2(a^o_2)$ denote maximal aggregate benefits, the gain in benefits from the second program is $\Delta B^o = B^o_2 - B^o_1$. Accordingly, if $K$ is less than $\Delta B^o$ provision of both programs is desirable. Notice that the incremental benefit from broadcasting the second program, $\Delta B^o$, is strictly less than the direct benefits that the second program generates, $B^o_2/2$. Because the two public goods are substitutes, some of the benefit of the second one comes at the expense

---

15 Notice that the same new producers advertise on both programs. This is because the two programs are watched by different viewers and, since marginal production costs are constant, contacting one set of consumers does not alter the willingness to pay to contact another set.

16 Using the fact that $p(a) = \omega \sigma \cdot [1 - a/m]$, we see that $a^o_2$ equals $m[1 - \gamma/\omega \sigma]$ if $\gamma \leq \omega \sigma$ and 0 if $\gamma > \omega \sigma$. Note that Assumption 1(ii) guarantees that $a^o_2 < \beta/\gamma$ for all $\gamma$. Since $a^o_1$ is clearly less than $a^o_2$ (see Figure 1), Assumption 1(ii) also implies that $a^o_1 < \beta/\gamma$.  

11
of reducing the benefits generated by the first one. This feature of broadcasts is important to understanding some of the results below.

4.2 Market provision

We suppose that the two channels are controlled by firms that make programming and advertising choices to maximize their profits. In standard fashion, we model the interaction between the firms as a two stage game. In Stage 1, each firm chooses whether to operate its channel and what type of program to put on. In Stage 2, given the types of program offered on each channel, each firm chooses a level of advertising.

The (subgame perfect Nash) equilibrium of the game may be solved for in the usual way via backward induction. We first solve for advertising levels and revenues in Stage 2, taking the firms’ Stage 1 choices as given. Suppose that only one firm decides to provide a program. If the firm runs \( a \leq \beta/\gamma \) advertisements, its program will be watched by \( N(1 + \frac{1-\gamma a/\beta}{1+\epsilon}) \) viewers. To sell \( a \) advertisements it must set a per-viewer price \( p(a) \) meaning that its revenues will be

\[
\pi_1(a) = N(1 + \frac{1-\gamma a/\beta}{1+\epsilon})R(a),
\]

where \( R(a) = p(a)a \) denotes revenue per viewer. The revenue maximizing level of advertisements is given by \( a_1^* \) where

\[
R'(a_1^*) = \frac{\gamma R(a_1^*)}{\beta(2+\epsilon) - \gamma a_1^*}.
\]

At advertising level \( a_1^* \), the increase in per viewer revenue from an additional advertisement equals the decrease in revenue from lost viewers measured in per viewer terms.\(^{17}\) This advertising level is illustrated in Figure 2. The downward sloping line is marginal revenue per viewer, \( R'(a) \), and the lower of the two hump-shaped curves represents the per viewer marginal cost stemming from lost viewers. The revenue maximizing advertising level is determined by the intersection of the two curves.

---

\(^{17}\) Since \( R'(m/2) = 0 \), we know that \( a_1^* < m/2 \) and hence \( a_1^* < \beta/\gamma \) by Assumption 1(ii).
If both firms provide programs, they will provide different programs. For if they choose the same type of program, competition for viewers will drive advertising levels and revenues to zero. Call the two firms A and B and suppose that firm A chooses a type 1 program and firm B a type 2. If firm $J \in \{A, B\}$ sets an advertising level $a_J$ it must charge a per viewer price $p(a_J)$. This price is independent of the advertising level of the other channel. The assumption that each viewer watches only one program, means that each channel has a monopoly in delivering its viewers to advertisers. Furthermore, the assumption that each new producer has a constant marginal production cost means that its demand for advertising on one channel is independent of whether it has advertised on the other. $^{18}$

This said, the number of viewers that each firm gets will depend on the advertising level of its competitor. If firm A has the lower advertising level, its program is watched by all the type 1 viewers and those type 2 viewers for whom $\lambda\beta - \gamma a_A > \beta - \gamma a_B$. If it has the higher advertising level, then its program is watched by all the type 1 viewers for whom $\beta - \gamma a_A > \lambda\beta - \gamma a_B$. In either case, viewers not watching firm A’s program watch firm B’s and the two firms’ revenues are given by

$$\pi^A_2(a_A, a_B) = N[1 + \frac{\gamma(a_B - a_A)}{\beta(1 + \varepsilon)}]R(a_A),$$

and

$$\pi^B_2(a_A, a_B) = N[1 + \frac{\gamma(a_A - a_B)}{\beta(1 + \varepsilon)}]R(a_B).$$

The equilibrium advertising levels $(a^*_A, a^*_B)$ are such that each firm is maximizing its revenue given its rival’s choice. Each firm must balance the negative effect of higher advertising levels on viewers with the positive effect on revenue per viewer. Using the first order conditions for each

$^{18}$ This implication of our micro-founded model of advertising demand should be contrasted with the assumptions made concerning inverse demand functions for advertising elsewhere in the literature. Berry and Waldfogel (1999) assume that the per viewer price received by each radio station is the same and depends only on the total share of the population who are listening; i.e., $p = f(\sum n_J/n)$ where $n$ is the total population and $n_J$ is the number listening to station $J$. Masson, Mudambi, and Reynolds (1990) assume the per viewer price received by each broadcaster is the same and depends on the total number of “viewer-minutes”; i.e., $p = f(\sum n_Ja_J)$. In light of our results, it would be worth understanding exactly what assumptions on the primitives would generate these formulations.
firm’s optimization, it is straightforward to show that the equilibrium advertising levels \( (a^*_A, a^*_B) \) are such that \( a^*_A = a^*_B = a^*_2 \), where \( a^*_2 \) satisfies:

\[
R'(a^*_2) = \frac{\gamma}{\beta(1 + \varepsilon)} R(a^*_2).
\]

The left hand side measures the increase in per viewer revenue a firm would earn from an additional advertisement in equilibrium and the right hand side measures the loss in revenue resulting from lost viewers in per viewer terms. Figure 2 illustrates the determination of this advertising level. The higher of the two hump shaped curves is the graph of lost revenues per viewer, \( \frac{\gamma}{\beta(1 + \varepsilon)} R(a^*_2) \).

Turning to Stage 1, let \( \pi^*_1 = \pi_1(a^*_1) \) denote the firm’s revenues in the one firm case and \( \pi^*_2 = \pi_2(a^*_2, a^*_2) \) each firm’s revenues in the two firm case. Since \( a^*_1 \) exceeds \( a^*_2 \), revenues in the one firm case exceeds those with two firms. It follows that neither firm will provide a program if \( K \) exceeds \( \pi^*_1 \); one firm will provide a program if \( K \) lies between \( \pi^*_1 \) and \( \pi^*_2 \); and both firms will provide programs if \( \pi^*_2 \) exceeds \( K \).

### 4.3 Optimal and market provision compared

Conditional on the market providing one or both of the programs, will they have too few or too many advertisements? When only one firm provides a program, the advertising level \( (a^*_1) \) may be bigger or smaller than the optimal level \( (a^*_o) \) depending on the nuisance cost. It is clear from Figures 1 and 2 that as \( \gamma \) tends to 0, \( a^*_o \) tends to \( m \) while \( a^*_1 \) approaches \( m/2 \). At the other extreme, if \( \gamma \) exceeds \( \omega \), then \( a^*_o = 0 \) and \( a^*_1 \) is positive. Similar remarks apply when both firms provide programs. In either case, there exists a critical nuisance cost such that the market under-provides advertisements when \( \gamma \) is less than this value and over-provides them otherwise.\(^{19}\)

**Proposition 1** Suppose that the market provides \( i \in \{1, 2\} \) types of programs. Then, there exists a critical nuisance cost \( \gamma_i \in (0, \omega) \) such that the market provided advertising level is lower (higher) than the optimal level as \( \gamma \) is smaller (larger) than \( \gamma_i \). Moreover, \( \gamma_1 \) is less than \( \gamma_2 \).

\(^{19}\) The proofs of this and the subsequent propositions can be found in the Appendix.
Another way of phrasing this conclusion is that the market price of advertising will be higher than the Pigouvian corrective tax for low values of the nuisance cost and lower for high values. Thus, while it is possible for the market price of advertising to be “just right”, there are no economic forces ensuring the equivalence of the two prices. While the Pigouvian corrective tax reflects the negative externality that advertisers impose on each other and on viewers, the market price of advertising reflects the dictates of revenue maximization. Revenue maximization only accounts for viewers’ disutility of advertising to the extent that it induces viewers to switch off or over to another channel.

The most striking thing about the proposition is the possibility that market provided programs may have too few advertisements. While the governments of many countries set ceilings on advertising levels on commercial television and radio, we are not aware of any governments subsidizing advertising levels. Under-advertising arises in our model because each broadcaster has a monopoly in delivering its audience to advertisers. This means that broadcasters hold down advertisements in order to keep up the prices that they receive. While this effect might be mitigated if there were alternative ways that advertisers could reach viewers, the possibility of under-provision will still arise. For if the nuisance cost imposed by advertisers is small, then efficiency demands that programs be consumed by many advertisers. But the only way this can happen is if the market price is sufficiently low and this will make profitable provision impossible.

Turning to programming, the question is will the market provide too few or too many types of program. Recall that both types of programs should be provided if \( K < \Delta B^o \), while only

---

20 That said, as noted in the introduction, concern about increasing concentration in the United States radio industry is partly motivated by fears about high advertising prices and hence (presumably) low advertising levels.

21 If broadcasters could perfectly price discriminate across advertisers, then they would not need to hold down advertising levels to drive up prices and the market outcome is always excessive advertising (see also Hansen and Kyhl (2000)).

22 The analysis here compares the number of program types provided by the market with the optimal number. A slightly different problem, in the spirit of Mankiw and Whinston (1986), would be to compare the number of program types provided by the market with the number in an optimal “second-best” system which treated as a constraint the fact that with \( i \in \{1,2\} \) types of programs, the advertising levels would be \( a_i^* \). Our choice is motivated by the desire to understand if market provision can actually achieve the first best.
one should be provided if $\Delta B^o < K < B_1^o$. The market provides both types if $K < \pi_2^*$, one if $\pi_2^* < K < \pi_1^*$, and none if $K > \pi_1^*$. In the one channel case, the revenues the firm earns, $\pi_1^*$, are less than the gross advertisers’ benefits from the program. Accordingly, they must be less than the optimized sum of viewer and advertiser benefits, implying that $\pi_1^* < B_1^o$. Thus, if $\pi_1^* < K < B_1^o$, the market under-provides programs.

It is quite possible that $\pi_1^*$ is less than the optimized gain in aggregate benefits from adding a second program, $\Delta B^o$. Then, there will be a range of operating costs for which both programs should be provided, while the market provides none! This case arises when the benefits to viewers from watching their preferred programs are large (large $\beta$ and $\varepsilon$), while the expected benefits from new producers contacting consumers ($m\omega \sigma$) are small. Since broadcasting firms only capture a share of advertiser benefits and these are small relative to viewer benefits, advertising revenues are considerably less than the aggregate benefits of programming. This produces the type of market failure previously identified in the broadcasting literature (see, for example, Spence and Owen (1977)).

With two channels, the revenue each firm earns, $\pi_2^*$, is again less than the gross advertisers’ benefits from the program it provides, implying that each firm’s revenue is less than $B_2^o/2$. However, as noted earlier, $\Delta B^o$ is less than $B_2^o/2$ because some of the direct benefits of the second program come at the expense of the first. Moreover, $\pi_2^*$ includes revenues that are obtained from “stealing” the advertising revenues of the first program. Accordingly, it is unclear whether $\pi_2^*$ exceeds or is smaller than $\Delta B^o$. In the latter case, the market always under-provides programs. In the former, there exists a range of operating costs for which the optimal number of programs is one, while the market provides two. Programs are then over-provided by the market. The following proposition provides some sufficient conditions for under- and over-provision.

**Proposition 2** (i) If $m\omega \sigma < \frac{2\beta(1+2\varepsilon)}{1+\varepsilon}$, the market does not overprovide programs, and underprovides them for some values of $K$. (ii) If $\varepsilon = 0$ and $m\omega \sigma > 2\beta$, there exist values of $K$ such that
the market overprovides programs for \( \gamma \) sufficiently small.

To understand the result, note that as \( \gamma \) gets small, the equilibrium advertising level \( a_2^* \) converges to \( \frac{m}{\pi} \), implying that equilibrium revenues \( \pi_2^* \) converge to \( N \frac{m-\bar{\pi}}{\pi} \). This represents an upper bound, since equilibrium revenues are decreasing in \( \gamma \). Part (i) of the proposition now follows from the fact (established in the Appendix) that \( \Delta B^o \geq N \frac{\beta(1+2\varepsilon)}{2(1+\varepsilon)} \). Part (ii) follows from the fact that when \( \varepsilon = 0 \), \( \Delta B^o \) converges to \( N \frac{\beta}{2} \) as \( \gamma \) gets small. Thus, if the stated inequality holds, \( \pi_2^* \) exceeds \( \Delta B^o \) for \( \gamma \) sufficiently small. To see why \( \Delta B^o \) converges to \( N \frac{\beta}{2} \), note that as \( \gamma \) gets small, the optimal advertising levels with one and two programs converge to \( m \). Moreover, all viewers watch even if only one program is provided. Thus, the only gain from providing the second program is the increase in viewing benefits enjoyed by the \( N \) viewers who now see their preferred program and this is given by \( N \frac{\beta}{2} \).

Since the literature on market provision of public goods emphasizes the problem of under-provision, the possibility of over-provision of broadcasting is noteworthy.\(^{23}\) The key feature which permits over-provision is that the social benefit of an additional program is less than the direct benefits that program generates. This is because programs are substitutes for viewers. Although the entering firm’s revenues are less than the direct benefits it generates, they may exceed the social benefits since part of its revenues are offset by the reduction in revenues of the incumbent firm. This is a familiar problem with private decision making when entry is costly (Mankiw and Whinston (1986)).

The previous two propositions establish that there is no guarantee that market outcomes are optimal. Nonetheless, since both over- and under-provision of advertising and programs is possible, the market may produce something close to the optimum for a range of parameter values\(,^{24}\)

\(^{23}\) The possibility of overprovision is also stressed by Berry and Waldfogel (1999). They structurally estimate a model of radio broadcasting based on the work of Mankiw and Whinston (1986). This model implies that the equilibrium number of stations will always exceed the number that maximizes total non-viewer surplus (broadcasting stations plus advertisers) and they quantify the extent of this overprovision. While they are unable to observe viewer surplus, they are able to compute the values of programming that would make the equilibrium optimal.
Accordingly, the model does not suggest that the market necessarily provides broadcasting inefficiently.

5 Duplication and other problems

This section extends the basic model to illustrate some distortions in the market allocation of resources across types of programs. We start with the duplication of popular program types, which the existing literature has seen as a major problem with market provision. Our basic model is not an appropriate framework for studying duplication because it assumes that both types of programs are equally popular. However, extending the model to allow one program type to be more popular does not generate duplication. If both firms choose the more popular program type, competition for viewers will drive advertising levels and revenues down to zero. Thus, firms will choose not to duplicate even when doing so would increase viewers.

The fierce advertising competition driving this conclusion reflects the strong assumption (also made in the existing literature) that two programs of the same type are perfect substitutes for viewers. In reality, there is considerable variation within a type of program: talk programs can discuss current affairs or offer personal advice; country programs can play classics or current hits; etc. Such variation means that programs of the same broad type are not perfect substitutes and hence broadcasters can and do offer programs of the same type. However, the welfare consequences of duplication are then less clear because there is a viewer benefit to having multiple differentiated programs of the same type. Thus, whether the market produces too much duplication is unclear.

This question can be addressed with a minor extension of the model. Suppose there are two

\[ \Delta B^o > K \]

To see this, suppose that \( \Delta B^o \) exceeds \( K \) so that the optimum involves providing both programs. Suppose further that the Pigouvian corrective tax, \( \gamma \), is sufficiently high that the revenues it would generate are sufficient to finance the provision of both programs; i.e., \( N \gamma a_2^o > K \). Then, if \( \gamma \) is close to \( \gamma_2 \), the critical nuisance cost defined in Proposition 1, the market will provide two channels showing different types of programs with an advertising level close to \( a_2^o \). By continuity, \( a_2^o \) is close to \( a_2^o \) which means that \( p(a_2^o) \) is close to \( \gamma \). This, in turn, implies that \( \pi_2^o > K \) which ensures that the market will operate both channels.

\[ 25 \]

In our basic model, equilibrium involves the firms choosing different programs even under the literature’s assumption that advertising levels are fixed.
varieties of each program type, denoted $i$ and $j$. Each viewer is now characterized by a triple $(t, k, \tau)$ where $t$ denotes his preferred type of program, $k \in \{i, j\}$ his preferred variety and $\tau$ the fraction of the gross viewing benefits he gets from his less preferred variety. Thus, a type $(t, k, \tau)$ viewer gets gross viewing benefits $\beta$ from watching a type $t$ program of his preferred variety $k$ and benefits $\tau \beta$ from his less preferred variety. We assume that viewers receive no benefits from watching either variety of their less preferred type of program, which allows us to make use of our earlier results in characterizing market equilibrium.\textsuperscript{26}

Suppose there are $N_t$ viewers preferring type $t$ programs and that type 1 programs are more popular (i.e., $N_1 > N_2$). Viewers of each type are evenly split in terms of their preferred variety and the parameter $\tau$ is uniformly distributed on the interval $[\tau, 1]$. The lower bound $\tau$ is a measure of how close substitutes the two varieties are. As $\tau$ increases the two varieties become closer substitutes. As $\tau \to 1$ the situation approaches our basic model with $\varepsilon \to \infty$ (i.e., there are effectively only two types of program - type 1 and type 2 - and viewers will only watch programs of their own type).

When both firms provide programs, there are two possible market outcomes: duplication in which the two firms broadcast type 1 programs of different varieties and diversity in which the two firms broadcast different types of program. Following our analysis of two firm competition in the previous section, the equilibrium advertising level under duplication, denoted $a^*_d$, satisfies

$$R'(a^*_d) = \frac{\gamma}{\beta(1 - \tau)} R(a^*_d),$$

and equilibrium revenue for each firm is $\frac{N_t}{2} R(a^*_d)$. Under diversity, the assumption that viewers will not watch either variety of their less preferred type of program implies that each firm has a monopoly in delivering its viewers. Thus, our earlier analysis of the one firm case applies and the

\textsuperscript{26} Without this assumption, the model becomes significantly more complicated and analyzing it would require a separate paper.
equilibrium advertising level under diversity, denoted \( a_v^* \), satisfies\(^{27}\)

\[
R'(a_v^*) = \frac{\gamma}{\beta(2 - \tau)} - \gamma a_v^* R(a_v^*).
\]

The firm serving type \( t \) viewers will obtain revenues

\[
\pi^v_t = \frac{N_t}{2} \left[ 1 + \frac{1 - \gamma a_v^*/\beta}{1 - \tau} \right] R(a_v^*).
\]

The market outcome will be duplication if \( \frac{N_t}{2} R(a_d^*) > \pi^v_2 \) and diversity otherwise.

Since a higher value of \( \tau \) means that the two varieties are closer substitutes, intuition suggests that the market outcome will be diversity for \( \tau \) sufficiently large and duplication for \( \tau \) sufficiently small. This is confirmed in the following proposition.

**Proposition 3** When both firms provide programs, there exists a critical level of \( \tau \), denoted \( \tau^* \), such that the market outcome will be duplication for \( \tau < \tau^* < 1 \) and diversity for \( \tau > \tau^* \).

If \( \tau \) is small, providing both varieties of a type 1 program generates significant viewing benefits for type 1 viewers. Since these viewers are more numerous than type 2 viewers, optimal provision may involve duplication in these circumstances. The key question is whether the market generates duplication in circumstances when optimal provision involves diversity. Our next proposition provides sufficient conditions for this to occur.

**Proposition 4** Suppose that \( K \) is such that both optimal and market provision involve both channels operating. Then, if \( N_2 \in (\frac{N_1}{3}, \frac{N_1}{2}) \) and \( \tau > 0 \), market provision involves duplication and optimal provision involves diversity for sufficiently small \( \gamma \).

To prove this result, note that equilibrium advertising levels under both duplication and diversity converge to \( \frac{m}{2} \) as \( \gamma \) becomes small. Moreover, since \( \tau > 0 \), under diversity, all type \( t \) citizens would watch the type \( t \) channel. Thus, the market outcome will be duplication if \( \frac{N_t}{2} > N_2 \). With

\(^{27}\) This equation holds for \( \tau \) less than \( \hat{\tau} \) where \( R'(\beta \hat{\tau}^2/\gamma) = \gamma R(\beta \hat{\tau}^2/\gamma)/2\beta(1 - \hat{\tau}) \). If \( \hat{\tau} \) lies between \( \hat{\tau} \) and \( \gamma m/2\beta \) the solution to the firm’s problem is to set \( a_v^* = \beta \hat{\tau}/\gamma \), while if \( \hat{\tau} \) exceeds \( \gamma m/2\beta \) then the firm should set \( a_v^* = m/2 \). In these latter two ranges, the solution involves all type \( t \) viewers watching the type \( t \) program. The existence of these ranges requires that \( \tau \) be significantly bigger than 0.
optimal provision, advertising levels under duplication and diversity converge to \( m \) as \( \gamma \) becomes small and, under diversity, all type \( t \) citizens would watch the type \( t \) channel. Moving from duplication to diversity would create new viewing benefits of \( \frac{N_t}{2}\beta[1 + \frac{1 + \tau}{2}] \) for type 2 viewers and lead to a loss of viewing benefits of \( \frac{N_t}{2}\beta\frac{1 - \tau}{2} \) for type 1 viewers. The gain is bigger than the loss if \( N_2 > \frac{N_1}{3} \). Since the total viewing audience is greater under diversity (\( N_1 + N_2 \) vs \( N_1 \)), advertisers must also be better off and hence diversity dominates duplication from a welfare standpoint.

While this proposition restores the conclusion that the market can produce socially inefficient duplication, it does not imply that this is the only possible type of distortion. In principle, there might be conditions under which the market outcome is diversity when optimal provision is duplication. The fiercer competition in advertising levels under duplication may encourage firms to provide diversity before it is socially optimal. While we have not been able to come up with a counter-example in our simulations, we have been unable to prove that this cannot happen.\(^{28}\)

Duplication is by no means the only reason that the market may misallocate resources across types of programming. As has been pointed out by numerous critics, market provided programming will be biased towards those programs favored by consumers who have a high value to advertisers. This can be illustrated in our model by assuming that the number of type 1 viewers is larger than the number of type 2 viewers but the latter are more likely to have a positive willingness to pay for new products. In these circumstances, there are conditions under which the market provides one program of type 2 when the optimum calls for one program of type 1.\(^{29}\)

A further problem arises when the type of programming a viewer is exposed can change his willingness to pay for products.\(^{30}\) As Sunstein (1999) notes, “advertisers want programming

---

\(^{28}\) We have been able to prove that there are conditions under which market provision involves diversity when aggregate surplus would be higher with duplication under the “second best” constraint that advertising levels are \( a^*_d \) under duplication and \( a^*_v \) under diversity.

\(^{29}\) Our discussion paper (Anderson and Coate (2000)) provides the details.

\(^{30}\) The link between programming and the demand for products is a key issue in the regulation of children’s television. The concern is that programs themselves can become quasi-commercials. For example, to sell a new action toy, an effective strategy would be to advertise on a cartoon program about the character’s adventures. This
that will put viewers in a receptive mood, and hence not be too ‘depressing’”. This type of bias can be illustrated in our model by assuming that the number of type 1 viewers is larger than the number of type 2 viewers but that watching the latter type of program raises the probability that viewers have a positive willingness to pay for new products. In these circumstances, there are again conditions under which the market provides one program of type 2 when the optimum calls for one program of type 1.

6 Further issues concerning market provision

In this section, we extend the analysis to address two important questions concerning market provision. The first is whether market provision produces better outcomes under monopoly or competition. This has been a key question in the literature (see Steiner (1952), Beebe (1977) and Spence and Owen (1977)) and remains a policy relevant issue today, given the current spate of mergers in the broadcasting industry. The second issue is how the possibility of pricing programming impacts market performance. This has long been of interest to public good theorists (see Samuelson (1958), (1964) and Minasian (1964)). The issue was the central concern of Spence and Owen (1977) and continues to attract attention in the broadcasting literature (Doyle (1998), Hansen and Kyhl (2000) and Holden (1993)). It is of policy interest since, in the television industry, it is becoming increasingly possible to exclude viewers and monitor their viewing choices.

This permits the pricing of individual programs as well as access to particular channels.\footnote{In Europe, direct broadcast satellite channels like Canal Plus are partially financed by subscription pricing. In the United States, premium cable channels such as HBO and Showtime are often priced individually. Other cable channels, such as ESPN and CNN, are “bundled” and sold as a package. In this case, both cable companies and the cable networks are involved in pricing decisions. In our model, bundling does not make sense because viewers watch at most one program. Obviously, it would be interesting to extend the analysis to incorporate bundling.}
6.1 Is monopoly better than competition?

To analyze the issue, suppose that the two channels are owned by a single firm, rather than two separate firms. If the monopoly chooses to operate both channels, its revenue maximizing level of advertisements is \( a = m/2 \) and its revenues will be \( 2NR(\frac{m}{2}) \). If it operates only one channel, its revenue maximizing advertising level will be \( a_1^* \) and its revenues \( \pi_1^* \). Letting \( \Delta \pi \) be the incremental profit from offering the second program, the monopoly will provide both programs if \( K \) is less than \( \Delta \pi \), and one program if \( K \) is between \( \Delta \pi \) and \( \pi_1^* \).

The following proposition describes the impact of monopoly on advertising levels.

**Proposition 5** Suppose that both channels would be operated under competition. Then, monopoly produces higher advertising levels than competition.

To understand this result, recall that advertising levels are \( a_2^* \) on each channel under competition. If the monopoly operates both channels, it chooses an advertising level \( m/2 \), which is larger than the competitive level \( a_2^* \). If the monopoly operates only one channel, it chooses \( a_1^* \) advertisements, which exceeds \( a_2^* \). In both cases per viewer advertising prices are lower under monopoly, suggesting that concerns about increasing concentration raising prices to advertisers are misguided.\(^{32}\) The logic is exactly that of Masson, Mudambi, and Reynolds (1990). Under competition firms compete by reducing advertising levels to render their programs more attractive. A monopoly, by contrast, is only worried about viewers turning off completely and so advertises more.\(^ {33}\) This greater quantity of advertisements sells at a lower per viewer price.

\(^{32}\) Notice, however, that advertiser surplus will not necessarily be higher if the monopoly shuts down one channel, because fewer viewers will be exposed to advertisements. In this case, the total price of an advertisement may be higher under monopoly because each advertisement reaches more viewers.

\(^{33}\) This finding is consonant with the explanation offered by some observers of the United States radio industry that increased concentration of ownership explains increased advertising levels. For example, Duncan’s American Radio analysts J.T. Anderton and Thom Moon argue that “As bottom-line pressures increase from publicly-traded owners, the number of commercials on the air has risen. The biggest change when a new owner takes over seems to be the addition of one new stopset per hour. The rationalization offered by most owners is that they limited unit loads because they needed to compete effectively with a direct format competitor: “Fewer commercials gives the listener more reasons to stay with me.” Now the reasoning is, “We own the other station they’re most likely to change to, so we have them either way. Why limit spot loads?”"
The impact of monopoly on programming is more difficult to discern. On the one hand, the monopoly internalizes the business stealing externality (i.e., it takes into account the fact that introducing additional programming means that existing programs will earn less revenue), which favors the provision of less programming. On the other, the monopoly puts on more advertisements implying that each program earns more revenue than under competition. This second effect, which suggests that monopoly will provide more programming, has been ignored by the literature because of its assumption of fixed advertising levels. We can show that the first effect dominates the second when $\varepsilon = 0$, which yields the following proposition.34

**Proposition 6** If $\varepsilon = 0$, there exist values of $K$ such that monopoly provides fewer programs than competition.

What can be said about the welfare comparison of monopoly and competition? In contrast to standard markets, there is no presumption that monopoly in broadcasting produces worse outcomes than competition. If monopoly leads to the same amount of programming, then the welfare comparison simply depends on relative advertising levels. If advertising levels are too high with competition, then they are even higher with monopoly, so that monopoly must reduce welfare. If they are too low, then monopoly can raise welfare. If monopoly changes the amount of programming, then the welfare analysis needs to take account of both changes in advertising levels and programming. Even if one knows the direction of the changes in programming, welfare comparisons are complicated by the fact that both advertising and programming could be either over- or under-provided under competition.35

---

34 For $\varepsilon > 0$, the proposition continues to hold for sufficiently small $\gamma$. This is because with two channels, advertising levels under competition converge to the monopoly level as $\gamma$ approaches zero and the second effect disappears. However, we have been unable to establish that the result holds for all $\gamma$.

35 If monopoly leads to the loss of a program, society saves $K$ but aggregate benefits are reduced by an amount $\Delta B^* = B_2(a_2^*) - B_1(a_1^*)$. The latter can be decomposed into a change in viewer benefits and a change in advertiser benefits (gross of payments to broadcasters). Viewer benefits must decrease, but the effect on advertiser benefits is ambiguous. The per viewer price of advertisements is lower, bringing in a greater range of products advertised and an associated increase in advertiser benefits on that account, but each previously advertised product now reaches a smaller potential market due to viewers who switch off. Conditions under which $\Delta B^*$ exceeds or is smaller than $K$ can be derived by carefully considering the determinants of these benefit changes. The interested reader can
This analysis of the relative merits of monopoly and competition should be contrasted with
the classic discussion in Steiner (1952). In our model, the fact that the monopoly internalizes the
business stealing externality is a force working in the direction of less programming. By contrast,
in Steiner’s analysis it is a force working in favor of monopoly producing more variety. Steiner
argued that, while competition would duplicate popular program types, a monopoly would have no
incentive to duplicate programs because it would simply be stealing viewers from its own stations.
It would, however, have an incentive to also provide less popular programming to the extent that
this attracted more viewers. While this argument does not emerge from our basic model, it does
in the extension considered in section 5. Under the assumptions of Proposition 4, the monopoly
outcome would be diversity for precisely the reasons Steiner suggests.

6.2 Does pricing help?

Characterizing market outcomes when firms can both run advertisements and charge viewers
subscription prices is conceptually straightforward but somewhat involved, so we relegate the
details to the Appendix. The main point to note is that the equilibrium advertising level is the
same whether the market provides one or two programs and equals one half of the two program
optimal level. To understand this, note that the number of viewers a firm gets is determined by its
“full price”; i.e., the nuisance cost plus subscription price. Hence, for any given full price, the firm
will choose the advertising level and subscription price that maximize revenue per viewer. More
precisely, if the firm is charging a full price \( r \) its advertising level \( a \) and subscription price \( s \) must
maximize \( R(a) + s \) subject to the constraint that \( \gamma a + s = r \). To see the implications of this, observe
that if the subscription price were reduced by \( \gamma \) and one more advertisement were transmitted,
the full price would stay constant and revenue per viewer would be raised by \( R'(a) - \gamma \). Thus,
consult Anderson and Coate (2000) for the details.

36 As Beebe (1977) pointed out, if there were a “lowest common denominator” program that all viewers would
watch, then a monopoly would have no incentive to provide anything else even if viewers had strong and idiosyncratic
preferences for other types of programs.
the equilibrium advertising level must satisfy the first order condition \( R'(a) \leq \gamma \) with equality if \( a > 0 \). Since the linearity of the demand function implies that \( R'(a) = p(2a) \), this means that the equilibrium advertising level is \( a_2^0/2 \).

It follows from this result and the fact (established in the Appendix) that \( a_2^0/2 < a_2^* < a_1^* \), that pricing reduces advertising levels. Intuitively, pricing allows broadcasters to respond to viewers’ dislike of commercials by reducing advertisements and raising subscription prices. Not surprisingly, it can be shown that equilibrium profits are higher with pricing. Pricing allows broadcasters to extract direct payments for their programming and hence makes programs more profitable. This means that the market provides at least as many programs with pricing (see also Spence and Owen (1977) and Doyle (1998)). Thus, we have the following proposition.

**Proposition 7** With pricing, the market provides lower advertising levels than without. Moreover, with pricing the market provides at least as many types of programs as without and strictly more for some values of \( K \).

Will the market generate a higher level of welfare with pricing?\textsuperscript{37} There are many circumstances in which it will. For example, when \( \gamma \geq \omega \sigma \) and \( K < \frac{N \beta}{2} \), optimal provision involves two programs each of which have no advertising. Without pricing, the market cannot achieve this. With pricing, however, market provision is fully optimal. Viewers are charged a subscription price \( \beta \) and exposed to no advertisements (since \( a_2^0/2 = 0 \)). Each firm earns revenues \( N \beta \), more than sufficient to cover operating costs.

However, there are also circumstances under which pricing reduces welfare. If pricing leads to no changes in the number of programs provided, it must reduce surplus if advertising levels are already underprovided without pricing. Pricing may also reduce welfare when it increases programs. Suppose that the market provides one program without pricing and two with. If nuisance costs are close to 0, the equilibrium advertising level with and without pricing is almost

\textsuperscript{37} A more systematic analysis of how pricing impacts welfare can be found in Anderson and Coate (2000).
the same. Thus, pricing holds constant the advertising level but generates a new program. The extra advertising benefits this produces is that associated with the extra $N_{1+\varepsilon}$ viewers, and is (approximately) $N_{1+\varepsilon} - \frac{3\omega m}{8}$. The extra viewing benefits are $N_{1+\varepsilon}1+\varepsilon\frac{1}{2(1+\varepsilon)}$. Thus, if $K$ is greater than the sum of these two terms, aggregate surplus is lower with pricing.\(^{38}\)

Irrespective of its impact on aggregate surplus, pricing has some interesting distributional consequences. Consider again the case in which $\gamma \geq \omega\sigma$ and $K < N_{\frac{\beta}{2}}$ and suppose that the market would provide at least one program without pricing. Then, since viewers would pay a price $\beta$ and there would be no advertising, introducing pricing would eliminate both viewer and advertiser benefits. All the surplus from the market would be extracted by the two broadcasting firms!

7 Conclusion

This paper has presented a theory of the market provision of broadcasting and used it to address the nature of market failure in the industry. Equilibrium advertising levels can be greater or less than socially optimal levels, depending on the relative sizes of the nuisance cost to viewers and the expected benefits to producers from contacting viewers. This reflects a trade off between two factors. On the one hand, broadcasters do not fully internalize the costs of advertisements to viewers - they only care to the extent that they induce viewers to switch off or switch channels. On the other, broadcasters hold down advertising levels in order to drive up advertising prices. Broadcasters have market power because only they can deliver their viewers to advertisers.

It is perhaps surprising that the analysis does not provide a clear cut case for regulatory limits on advertising levels. Of course, the possibility that advertising levels could actually be too low reflects our assumption that advertising is informative. Moreover, even assuming that advertising is informative, different conclusions might be reached under alternative assumptions.

\(^{38}\) It is straightforward to show that there exist values of $K$ satisfying this inequality that are consistent with the assumption that one program is provided without pricing and two with.
about the product markets in which advertisers operate. In particular, in our model advertisers face the correct incentives to advertise because their payoff from selling their products perfectly reflects the surplus that they generate. This is because advertisers are monopolies and extract all the gains from trade.\textsuperscript{39} In a world of differentiated products, advertisers’ payoffs from selling their goods may overstate the surplus they generate since their sales come at the expense of their competitors (Grossman and Shapiro (1984)). This would decrease the likelihood of there being too few commercials. Even when the market provides excessive advertising, however, ceilings may be undesirable because of their impact on programming.\textsuperscript{40}

Market provision can allocate too few or too many resources to programming. A broadcaster’s choice of whether to provide programming does not account for the extra viewer and advertiser surpluses generated, nor for the loss of advertising revenue inflicted on competitors. Underprovision will arise when the benefits of programming to viewers are high relative to the benefits advertisers get from contacting viewers. This may explain the prevalence of public broadcasting in the early stages of a country’s development when advertising benefits are likely to be low. Overprovision can arise when program benefits are low relative to advertiser benefits and nuisance costs are low.

The market may also misallocate resources across types of programs. In particular, it may provide multiple varieties of a popular program type, when society would be better served by having programs of different types. The problem, once again, is that stations do not take account of the lost advertising revenues to competitors when they choose their format. In addition, market provision will be biased towards demand-inducing programming and programming preferred by those viewers more likely to buy advertisers’ products.

\textsuperscript{39} If they did not capture all the surplus from trade, they would underadvertise as noted by Shapiro (1980). This case arises when there are informational benefits in the set up discussed in footnote 11.

\textsuperscript{40} Advertising ceilings reduce advertising revenues and hence may negatively impact the number of programs provided. They may also reduce program quality as argued by Wright (1994) and may reduce program differentiation as argued by Gabszewicz, Laussel, and Sonnac (2001).
With respect to the debate concerning the role of public or not-for-profit broadcasting, the results make clear that there are circumstances under which socially valuable programming will not be provided by the market. However, the possibility of the market overproviding programming means that arguments for public broadcasting should not be made on a priori grounds (as in, for example, the Davies Report (1999)). Any assessment of the case for public broadcasting should also include a consideration of how programming and funding decisions are made in the public sector, an interesting subject for further study.\footnote{For an entertaining discussion of this issue see Coase (1966).}

There should be no presumption that increased concentration in the broadcasting industry is necessarily detrimental to social welfare. Such concentration may be expected to raise advertising levels, but this is not necessarily undesirable. The impact on the amount of resources allocated to programming is (in general) ambiguous, but increased concentration may yield a broader variety of programming. Welfare analysis is complicated by the fact that even if one knows the changes in programming concentration brings, one needs to know whether advertising and programming were over- or under-provided in the status quo.

Finally, the ability to price programming does not necessarily solve the problems associated with market provision. With such pricing, broadcasters can internalize the costs of advertisements to viewers by substituting prices for advertising at the margin. In addition, pricing enables more revenue to be extracted from the market by raising money directly from viewers. However, lower advertising levels and more programming are not necessarily socially desirable. Pricing may also have significant distributional consequences, redistributing surplus away from viewers and advertisers towards broadcasters.

The theory presented here could be extended in many ways. To make it more amenable for empirical investigation, it would be useful to allow for more channels and program types. It would then be interesting to allow for programs of different quality and study how equilibrium quality
is impacted by the number of channels. This would permit analysis of the familiar argument that channel proliferation leads to lower program quality.

A key extension is to relax the assumption that viewers watch only one program. One might assume that individuals watch say, two programs per day and that channels offered a program in each time slot. In addition to deciding what type of programs to produce, broadcasters must also decide how to schedule them against the line-up of their competitors.\footnote{See Cancian, Bills and Bergstrom (1995) for a discussion of some technical difficulties that may arise in modelling program scheduling.} The possibility of viewers switching stations over the course of a viewing period would have interesting implications for the advertising market as it means that broadcasters no longer necessarily have a monopoly in delivering their viewers. Moreover, with subscription pricing for each channel, broadcasters would strive to bundle their programs in a way to attract the most viewers.

In this context, one could also study the market for programming in more detail. Here we have assumed that each station can obtain any of the available types of programming for some uniform cost. In reality, the producers of programming seem to find it desirable to restrict access to their programming, by selling stations the exclusive rights to broadcast it. Such restrictions may have significant negative implications for welfare in a world of subscription pricing as discussed in Armstrong (1999).
References


Chwe, Michael, [1997], “Believe the Hype: Solving Coordination Problems with Television Advertising,” mimeo, University of Chicago.


Gabszewicz, J, Laussel, Didier and Nathalie Sonnac, [2001], “TV Broadcasting Competition and Advertising,” mimeo, CORE.


Sunstein, Cass, [1999], “Private Broadcasters and the Public Interest: Notes Toward a ‘Third Way’,” mimeo, University of Chicago.


Wright, Donald, [1994], “Television Advertising Regulation and Program Quality,” The Economic Record, 70 (211), 361-367.
8 Appendix

Proof of Proposition 1: We begin with the case in which the market provides one type of program. For clarity, write \( a_1^* \) as \( a_1^*(\gamma) \) and similarly for \( a_i^*, i = 1, 2 \). We already know that \( a_1^*(\gamma) > a_1^*(\gamma) \) for \( \gamma \) sufficiently small and that \( a_1^*(\gamma) < a_1^*(\gamma) \) for all \( \gamma \geq \omega \sigma \). Thus, by continuity, there exists \( \gamma_1 \in (0, \omega \sigma) \) such that \( a_1^*(\gamma_1) = a_1^*(\gamma_1) \). We need to show that this is unique.

We know that if \( a_1^*(\gamma) > 0 \) then
\[
(\beta(2 + \varepsilon) - \gamma a_1^*)(p(a_1^*) - \gamma) = \gamma \int_{0}^{a_1^*} p(\alpha)d\alpha,
\]
or, equivalently, using the linear demand specification,
\[
(\beta(2 + \varepsilon) - \gamma a_1^*)[\omega \sigma(1 - \frac{a_1^*}{m}) - \gamma] = \gamma \omega \sigma a_1^*(1 - \frac{a_1^*}{2m}).
\]
In addition, from the first order conditions that characterize \( a_1^*(\gamma) \), we have that \( (\beta(2 + \varepsilon) - \gamma a_1^*)R'(a_1^*) = \gamma R(a_1^*) \) or, equivalently,
\[
(\beta(2 + \varepsilon) - \gamma a_1^*)(1 - \frac{2a_1^*}{m}) = \gamma (1 - \frac{a_1^*}{m})a_1^*.
\]
Thus, if \( a_1^*(\gamma) = a_1^*(\gamma) = a \), we have that
\[
\frac{\gamma \omega \sigma a(1 - \frac{a}{2m})}{[\omega \sigma(1 - \frac{a}{m}) - \gamma]} = \frac{\gamma (1 - \frac{a}{m})a}{(1 - \frac{2a}{m})},
\]
which implies, after some simplification, that
\[
\frac{a}{m} = \frac{1}{1 + \frac{\omega \sigma}{2\gamma}}.
\]
We can now establish uniqueness. Suppose, to the contrary, that there exists \( \gamma \) and \( \gamma' \) such that \( \gamma < \gamma' \) with the property that \( a_1^*(\gamma) = a_1^*(\gamma) = a \) and \( a_1^*(\gamma') = a_1^*(\gamma') = a' \). Then, we know that \( a > a' \) and hence the above equation implies that
\[
\frac{1}{1 + \frac{\omega \sigma}{2\gamma}} > \frac{1}{1 + \frac{\omega \sigma}{2\gamma'}}.
\]
But this is inconsistent with the hypothesis that $\gamma < \gamma'$.

Turning to the case in which the market provides both programs, again since we already know that there exists $\gamma_2 \in (0, \omega \sigma)$ such that $a^o_2(\gamma_2) = a^*_2(\gamma_2)$, the task is to show that $\gamma_2$ is unique. We know that if $a^o_2(\gamma) > 0$, then $p(a^o_2) = \gamma$. In addition, from our characterization of $a^*_2(\gamma)$ we have that

\[
\frac{\beta(1+\varepsilon)R'(a^*_2)}{R(a^*_2)} = R(a^*_2) = p(a^*_2),
\]

or, equivalently,

\[
\frac{(1 - \frac{2a}{m})}{a(1 - \frac{2a}{m})} = \frac{\omega \sigma (1 - \frac{a}{m})}{\beta(1 + \varepsilon)}.
\]

We will show that this equation has a unique solution for $a$ in the relevant range which, since both $a^o_2(\gamma)$ and $a^*_2(\gamma)$ are decreasing functions, will imply that the solution $\gamma_2$ to the equation $a^o_2(\gamma) = a^*_2(\gamma)$ is unique. Letting $\varsigma = a/m$ and $\Upsilon = \frac{\omega \sigma m}{\beta(1+\varepsilon)}$, we may rewrite the equation as

\[
\frac{1 - 2\varsigma}{1 - \varsigma} = \Upsilon \varsigma (1 - \varsigma).
\]

Since we know that $a^*_2(\gamma) \leq m/2$, the relevant range is $\varsigma \in (0, 1/2)$. The left-hand side is decreasing in $\varsigma$ while the right hand side is increasing in $\varsigma$ over the relevant range, so the solution is unique.

For the final part of the proposition (that $\gamma_2$ is less than $\gamma_1$), we simply need to show that $a^o_2(\gamma_1) > a^*_2(\gamma_1)$. Using Figures 1 and 2, we have that $a^o_2(\gamma_1) > a^o_1(\gamma_1) = a^*_1(\gamma_1) > a^*_2(\gamma_1)$.

**Proof of Proposition 2:** To complete the argument in the text, we need to show that $\Delta B^o \geq N \beta \frac{1+2\varepsilon}{2(1+\varepsilon)}$. It is clear that

\[
\Delta B^o \geq B_2(a^o_1) - B_1(a^o_1),
\]

so that it suffices to show that

\[
B_2(a^o_1) - B_1(a^o_1) \geq N \beta \frac{1+2\varepsilon}{2(1+\varepsilon)}.
\]

Consider, then, the effect of adding an additional program holding constant the advertising level at $a^o_1$. Suppose that the existing program is a type 1 program and hence that the additional program
is a type 2 program. Type 1 viewers experience no change in their welfare. A type (2, λ) viewer enjoys a welfare increase of \((1 - \lambda)\beta\) if \(\lambda \geq \gamma a_0^\lambda / \beta\) and \(\beta - \gamma a_0^\lambda\) otherwise (these gains corresponding to those who were and who were not watching before). Advertisers get an additional \(N \frac{\gamma a_0^\lambda}{1 + \varepsilon}\) viewers and experience a gain of \(N \left[\frac{\gamma a_0^\lambda}{1 + \varepsilon} + \varepsilon\right] R_{\alpha_1} 0 p\left(a_0^\lambda\right)\). Aggregating these gains up, we obtain

\[
B_2(a_0^\lambda) - B_1(a_0^\lambda) = N \int_0^{a_0^\lambda} (1 - \lambda) \beta \frac{d\lambda}{1 + \varepsilon} + \left[\frac{\gamma a_0^\lambda + \varepsilon}{1 + \varepsilon}\right] \left[(\beta - \gamma a_0^\lambda) + \int_0^{a_0^\lambda} p(a) da\right].
\]

Since \(p(a_0^\lambda) \geq \gamma\), the right hand side is at least as large as

\[
N \beta \left[\int_0^{1} (1 - \lambda) \frac{d\lambda}{1 + \varepsilon} + \left[\frac{\gamma a_0^\lambda + \varepsilon}{1 + \varepsilon}\right]\right].
\]

To complete the proof, observe that

\[
N \beta \left[\int_0^{1} (1 - \lambda) \frac{d\lambda}{1 + \varepsilon} + \left[\frac{\gamma a_0^\lambda + \varepsilon}{1 + \varepsilon}\right]\right] \geq N \beta \left[\int_0^{1} (1 - \lambda) \frac{d\lambda}{1 + \varepsilon} + \left[\frac{\varepsilon}{1 + \varepsilon}\right]\right] = N \beta \frac{1 + 2\varepsilon}{2(1 + \varepsilon)}.
\]

**Proof of Proposition 3:** First recall that for \(\tau = 1\), duplication yields zero profits because equilibrium advertising levels are zero. However, diversity yields strictly positive profits for both firms, so the outcome is diversity. At the opposite extreme, for \(\tau \to -\infty\) viewers in each type will only watch if their preferred variety is aired, and firms can set advertising levels at the revenue maximizing level, \(m_2\), without losing their natural constituencies. This means that the firm in the smaller type market (2) earns a profit \(N_2 R(m_2)\) under diversity, but it earns \(N_2 R\left(m_2\right)\) under duplication. Hence the equilibrium outcome is duplication under the maintained assumption that \(N_1 > N_2\).

It remains to show that there is a single value of \(\tau\), denoted \(\tau^*\), that separates the \(\tau\) values for which the different outcomes prevail. Both duplication and diversity profits are continuous functions of \(\tau\). Equilibrium profits under diversity, \(\pi^\nu\), are weakly increasing in \(\tau\). For values of \(\tau\) less than \(\hat{\tau}\) where \(R'(\beta\hat{\tau}/\gamma) = \gamma R(\beta\hat{\tau}/\gamma)/2\beta(1 - \hat{\tau})\) not all viewers watch, and applying the
envelope theorem to the profit expression in the text \( \pi^*_i = \frac{N_i}{\tau}[1 + \frac{1 - \gamma}{1 - \frac{\gamma}{\beta}}]R(a^*_i) \) shows that profits are strictly increasing in \( \tau \). For values of \( \tau \) between \( \frac{\gamma}{\beta} \) and \( \frac{m}{2} \), all viewers watch and \( a^*_\nu = \frac{\beta \tau}{\gamma} < \frac{m}{2} \). Profits are \( \pi^*_i = N_i R(a^*_i) \) and are strictly increasing in \( \tau \). Lastly, for values of \( \tau \) higher than \( \frac{\gamma}{\beta} \), all viewers watch and \( a^*_\nu = \frac{m}{2} \) and profits are \( \pi^*_i = N_i R(\frac{m}{2}) \), independent of \( \tau \). Hence the varieties being closer substitutes (higher \( \tau \)) helps the firm under diversity because it more readily captures those viewers that prefer the other variety.

By contrast, duplication profits strictly fall with \( \tau \) since less heterogeneity means greater competition. The result can be seen from rewriting the equilibrium advertising condition as

\[
\frac{R'(a^*_i)}{R(a^*_i)} = \frac{\gamma}{\beta(1 - \frac{\gamma}{2})},
\]

from which we see that higher \( \tau \) elicits lower advertising levels (the left hand side of the above equation being strictly decreasing in \( a^*_i \)) and hence lower profit \( \left( \frac{N_i}{\tau} R(a^*_i) \right) \) because \( a^*_d < \frac{m}{2} \).

Because duplication profits are higher for low enough \( \tau \) and end up below diversity profits for high enough \( \tau < 1 \), while diversity profits are non-decreasing, there is a unique \( \tau^* < 1 \) for which the two profits are equal. ■

**Proof of Proposition 6:** We need to show that if \( \varepsilon = 0 \), \( \Delta \pi < \pi_2^* \) for all \( \gamma \in (0, 2\beta/m) \). By definition \( \Delta \pi = 2NR(\frac{m}{2}) - \pi_1^* \), so that \( \Delta \pi < \pi_2^* \) if and only if \( \pi_2^* + \pi_1^* > N \frac{m \sqrt{\gamma}}{2} \). Since \( \pi_2^* \) and \( \pi_1^* \) are decreasing in \( \gamma \), this inequality will hold for all \( \gamma \in (0, 2\beta/m) \) if it holds at \( \gamma = 2\beta/m \). We can rewrite the inequality as

\[
a_2^*[1 - \frac{a_2^*}{m}] + a_1^*[1 - \frac{a_1^*}{m}][1 + \frac{1 - \gamma a_1^*/\beta}{1 + \varepsilon}] > m/2.
\]

Writing \( a_i^* \) as \( a_i^*(\gamma) \), we can also show that

\[
a_1^*(\frac{2\beta}{m}) = m[\frac{2}{3} + \frac{\varepsilon}{6} - \frac{\sqrt{\varepsilon^2 + 4 + 2\varepsilon}}{6}]
\]

and

\[
a_2^*(\frac{2\beta}{m}) = m[1 + \frac{\varepsilon}{2} - \frac{\sqrt{\varepsilon^2 + 2 + 2\varepsilon}}{2}].
\]
Substituting in the values of $a_1^*(\frac{2\beta}{m})$ and $a_2^*(\frac{2\beta}{m})$, we see that the inequality will hold for all $\gamma \in (0, 2\beta/m)$, if $\Psi(\varepsilon) > 1/2$, where

$$
\Psi(\varepsilon) = \left[ 1 + \frac{\varepsilon}{2} - \frac{\varepsilon^2 + 2 + 2\varepsilon}{2} \cdot \frac{\varepsilon}{2} \right] \cdot \left[ 1 + \frac{\varepsilon}{2} - \frac{\varepsilon^2 + 2 + 2\varepsilon}{2} \cdot \frac{\varepsilon}{2} \right] + \left[ \frac{2}{3} + \frac{\varepsilon}{6} - \frac{\varepsilon^2 + 4 + 2\varepsilon}{6} \cdot \frac{\varepsilon}{6} + 1 - \frac{\varepsilon}{6} \cdot \left( \frac{2}{3} + \frac{\varepsilon^2 + 4 + 2\varepsilon}{3(1+\varepsilon)} \right) \right].
$$

But we have that

$$
\Psi(0) = \left[ 1 - \frac{\sqrt{2}}{2} \right] \cdot \left( \frac{\sqrt{2}}{2} \right) + \left[ \frac{2}{3} - \frac{\sqrt{4}}{6} \cdot \frac{\sqrt{4}}{6} + 1 \cdot \frac{2}{3} + \frac{\sqrt{4}}{3} \right] = 0.5034 > 1/2.
$$

Characterization of Market Outcomes with Excludable Viewers: We amend the game considered in section 4.2 by supposing that in Stage 2 each firm chooses an advertising level and a subscription price. Again, we solve for equilibria via backward induction. To this end, consider the second stage and suppose that in Stage 1, only one firm provides a program. Let $r = \gamma a + s$ denote the full price charged to viewers by the firm. If $r \leq \beta$, then its program will be watched by $N(1 + \frac{1-\gamma}{1+\gamma})$ viewers. For any given full price $r$, the firm will choose the advertising level and subscription price that maximizes revenue per viewer. These are given by

$$
(\hat{a}(r), \hat{s}(r)) = \arg \max_{(a, s)} \{ R(a) + s : \gamma a + s = r, \ a \geq 0, \ s \geq 0 \}.
$$

After observing that $R^*(\frac{a_2^*}{2}) \leq \gamma$ with equality if $\frac{a_2^*}{2} > 0$ it is easy to show that

$$
\hat{a}(r) = \begin{cases} 
\frac{r}{\gamma} & \text{if } r \leq \gamma a_2^*/2 \\
\frac{a_2^*}{2} & \text{if } r > \gamma a_2^*/2
\end{cases},
$$

and

$$
\hat{s}(r) = \begin{cases} 
0 & \text{if } r \leq \gamma a_2^*/2 \\
\frac{r}{\gamma} & \text{if } r > \gamma a_2^*/2
\end{cases}.
$$

It follows that if $P(r)$ denotes maximal revenue per viewer, then

$$
P(r) = \begin{cases} 
R(r/\gamma) & \text{if } r \leq \gamma a_2^*/2 \\
R(\frac{a_2^*}{2}) + r - \gamma a_2^*/2 & \text{if } r > \gamma a_2^*/2
\end{cases}.
$$
Note that $P(r)$ is continuous and differentiable in $r \in (0, \beta)$. It is also concave.

With this notation, the firm’s revenues can be written as

$$\pi_{1s}(r) = N(1 + \frac{1 - r/\beta}{1 + \varepsilon})P(r).$$

The profit maximizing full price is then $r_1^*$, where $r_1^*$ satisfies the equation

$$P'(r_1^*) \geq \frac{P(r_1^*)}{(2 + \varepsilon)\beta - r_1^*} \quad \text{(with equality if } r_1^* < \beta).$$

Assuming $r_1^*$ is greater than $\gamma a_2^2/2$, then $P'(r_1^*) = 1$ and the above equation implies that

$$r_1^* = \min\{\beta, \beta(1 + \varepsilon/2) - \frac{R(a_2^2) - \gamma a_2^2/2}{2}\}.$$

This is consistent with the assumption that $r_1^*$ is greater than $\gamma a_2^2/2$ if $\beta > R(a_2^2)/2 + \gamma a_2^2/4$. This follows from Assumption 1 (ii). Thus, we may conclude that the firm sets an advertising level $a_2^2/2$ and a subscription price $s_1^* = r_1^* - \gamma a_2^2/2 > 0$. Its maximized revenues can be written as

$$\pi_{1s}^* = N(1 + \frac{1 - (s_1^* + \gamma a_2^2/2)/\beta}{1 + \varepsilon})[s_1^* + R(a_2^2/2)].$$

Now suppose that in Stage 1 both firms provide programs, with firm $A$ showing a program of type 1. Let $r_J = \gamma a_J + s_J$ denote the full price charged to viewers by firm $J$. If $r_A \leq r_B \leq \beta$, then firm $A$’s program is watched by all the type 1 viewers and those type 2 viewers for whom $\lambda \beta - r_A > \beta - r_B$. If $\beta \geq r_A \geq r_B$, firm $A$’s program is watched by all the type 1 viewers for whom $\beta - r_A > \lambda \beta - r_B$. In either case, consumers not watching channel $A$ watch channel $B$. If either firm charges a full price in excess of $\beta$, it will attract no viewers. Using our earlier notation, the two firms’ revenues can be written as

$$\pi_{2s}^A(r_A, r_B) = N[1 + \frac{r_B - r_A}{\beta(1 + \varepsilon)}]P(r_A)$$

and

$$\pi_{2s}^B(r_A, r_B) = N[1 + \frac{r_A - r_B}{\beta(1 + \varepsilon)}]P(r_B).$$

39
The equilibrium full price levels \((r^*_{A}, r^*_{B})\) satisfy the first order condition

\[
N[1 + \frac{r^*_B - r^*_A}{\beta(1 + \varepsilon)}]P'(r^*_A) \geq N \frac{1}{\beta(1 + \varepsilon)}P(r^*_A) \quad (\text{with equality if } r^*_A < \beta)
\]

and similarly for firm \(B\) (transposing \(A\) and \(B\) subscripts). The two first order conditions imply that \(r^*_B = r^*_A = r^*_2\), where the common full price \(r^*_2\) is uniquely defined by the equation

\[
P'(r^*_2) \geq \frac{1}{\beta(1 + \varepsilon)}P(r^*_2) \quad (\text{with equality if } r^*_2 < \beta).
\]

If \(r^*_2\) is greater than \(\gamma a^*_2/2\), \(P'(r^*_2) = 1\) and the above equation implies that

\[
r^*_2 = \min\{\beta, \beta(1 + \varepsilon) - [R(\frac{a^*_2}{2}) - \gamma a^*_2/2]\}.
\]

For this to be consistent with the supposition that \(r^*_2\) is greater than \(\gamma a^*_2/2\), we require that \(\beta(1 + \varepsilon) > R(\frac{a^*_2}{2})\). Since \(R(a) \leq R(m/2) = \omega m/4\), this inequality follows from Assumption \(1\) (ii). Since the function \(P(r)\) is concave, then so are the firms’ revenue functions, and so \(r^*_2\) is indeed the equilibrium full price. We may conclude that the firms choose a common advertising level \(a^*_2/2\) and subscription price \(s^*_2 = r^*_2 - \gamma a^*_2/2\) and that each firm earns revenues

\[
\pi^*_2s = N(s^*_2 + R(a^*_2/2)).
\]

Turning to Stage 1, neither firm will provide a program if \(K > \pi^*_1s\) and only one firm will provide a program if \(\pi^*_1s > K > \pi^*_2s\). If \(\pi^*_2s > K\), both firms will provide programs.

In summary, then, neither firm will find it worthwhile to provide a broadcast if \(K\) exceeds \(\pi^*_1s\). If \(K\) lies between \(\pi^*_1s\) and \(\pi^*_2s\), one firm will provide a program and it will carry \(a^*_2/2\) advertisements and have a subscription price \(s^*_1 = r^*_1 - \gamma a^*_2/2\). If \(K\) is less than \(\pi^*_2s\), the two firms will offer different types of programs and each will carry \(a^*_2/2\) advertisements and have a subscription price \(s^*_2 = r^*_2 - \gamma a^*_2/2\).

**Proof of Proposition 7:** For the first part, we need to show that \(a^*_2/2 < a^*_2\). The result follows immediately if \(\gamma \geq \omega \sigma\) (since \(a^*_2/2 = 0 < a^*_2\)), so consider \(\gamma < \omega \sigma\). In this case, \(R'(\frac{a^*_2}{2}) = p(a^*_2) = \gamma\).
But,

\[ R'(a^*_2) = \frac{\gamma}{\beta(1 + \varepsilon)} R(a^*_2) \leq \frac{\gamma}{\beta(1 + \varepsilon)} R\left(\frac{m}{2}\right) = \frac{m\omega\sigma}{4\beta(1 + \varepsilon)}. \]

Assumption 1(ii) therefore implies that

\[ R'(a^*_2) \leq R\left(\frac{a^*_2}{2}\right) \quad \text{with strict inequality if } \gamma > 0. \]

This implies the result.

For the second part, we need only show that equilibrium revenues are higher with pricing in both the one and two firm cases; i.e., \( \pi_1^* < \pi_{1s}^* \) and \( \pi_2^* < \pi_{2s}^* \). In the one firm case, this is obvious. With pricing, the firm could always choose to set prices equal to zero and to raise revenue solely through advertising. But, as shown above, the (uniquely) optimal strategy is to reduce advertisements and charge viewers a price \( s > 0 \). By revealed preference, revenues must be higher with pricing.

In the two firm case, the result is not immediate because the price and advertising levels are determined strategically and firms compete on two fronts rather than one, which might a priori increase competition so much as to reduce equilibrium revenues. We could rule out this possibility if we knew that the full price is higher with pricing. To see this, suppose that \( r^*_2 > \gamma a^*_2 \) and that \( \pi^*_2 \geq \pi^*_{2s} \). Note that, by symmetry, each firm attracts \( N \) viewers with or without pricing. With pricing, each firm has the option of setting the advertising level \( a^*_2 \) and a subscription price of \( 0 \). Since \( r^*_2 > \gamma a^*_2 \) by hypothesis, this would result in strictly more than \( N \) viewers and revenues strictly higher than \( \pi^*_2 \). This contradicts the fact that each firm choosing \((s^*_2, a^*_2)\) is an equilibrium.

We now establish that it is indeed the case that \( r^*_2 > \gamma a^*_2 \). It is clear that the result holds if \( r^*_2 = \beta \). As shown in section 4, Assumption 1 (ii) implies that \( \gamma a^*_2 < \beta \). Thus, it remains to consider the case in which

\[ r^*_2 = \beta(1 + \varepsilon) - \left[ R\left(\frac{a^*_2}{2}\right) - \gamma a^*_2 \right] < \beta \]

and hence that \( \gamma < \omega\sigma \) (otherwise we have \( a^*_2 = 0 \) and a contradiction). Figure 3 depicts the
determination of the equilibrium full prices in the two regimes in this case. The equilibrium full price with non-excludability, $\gamma a^*_2$, is determined by the intersection of the downward sloping line $R'(\xi)/\gamma$ and the hump shaped curve $R(\xi)/\beta(1+\varepsilon)$. With excludability, the equilibrium full price is determined by $1 = P(r^*_2)/\beta(1+\varepsilon)$, which in the graph is the intersection of the horizontal line emanating from the point $(0,1)$ and the upward sloping curve $$\frac{R(a^*_2)-\gamma a^*_2+r}{\beta(1+\varepsilon)}.$$ The result will hold if the slope $\frac{1}{\beta(1+\varepsilon)}$ is less than the absolute value of the slope of $R'(\xi)/\gamma$ so that $R'(\xi)/\gamma$ crosses $R(\xi)/\beta(1+\varepsilon)$ (which here is sloping up since $R(a^*_2) > R(a^*_0)$) before $\frac{R(a^*_2)-\gamma a^*_2+r}{\beta(1+\varepsilon)}$ crosses the horizontal line emanating from $(0,1)$.

We know that $R'(a) = \omega \sigma [1 - \frac{2a}{m}]$, so that $\frac{dR'(\xi)/\gamma}{dr} = -\frac{2\omega \sigma}{m\gamma}$. The required condition is therefore $\beta(1+\varepsilon) > \frac{m\gamma^2}{2\omega \sigma}$. But, since $\gamma < \omega \sigma$ and, by Assumption 1 (i), $\gamma < 2\beta/m$, we have

$$\frac{m\gamma^2}{2\omega \sigma} < \frac{m\gamma}{2} < \beta \leq \beta(1+\varepsilon),$$

as desired. \(\blacksquare\)