A New Method of Estimating Risk Aversion

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Abstract

This paper develops a method of estimating the coefficient of relative risk aversion ($\gamma$) from data on labor supply. I show that the curvature of the utility function can be inferred from labor supply wage and income elasticities for a given degree of complementarity between consumption and leisure. The degree of complementarity can in turn be inferred from data on consumption choices when employment is stochastic. Using a large set of existing estimates of wage and income elasticities, I find a mean estimate of $\gamma = 1$. I also give a calibration argument showing that a positive uncompensated wage elasticity, as found in most studies of labor supply, implies $\gamma < 1.25$. The estimate of $\gamma$ changes by at most 0.25 over the range of plausible values for the complementarity parameter.

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The curvature of utility over consumption, measured e.g. by the coefficient of relative risk aversion, is an important parameter for a broad set of economic problems. It is of obvious relevance in calibrating models of risk-taking behavior such as portfolio choice, insurance, and executive compensation. But the coefficient of relative risk aversion also enters into problems that require cardinal utility but do not explicitly involve uncertainty. It determines the deadweight cost of taxation and the benefits of redistribution, which are needed to identify optimal tax and welfare policies. In addition, if utility is time-separable, the curvature of utility is related to the path of consumption, savings, and labor supply over a lifetime.

Existing estimates of the coefficient of relative risk aversion ($\gamma$) are highly dispersed. Estimates of $\gamma$ from portfolio choice and equity premiums exceed 10.\textsuperscript{1} Estimates of $\gamma$ using data on insurance deductibles, premiums, and the fraction of insured assets range from 2 to 10.\textsuperscript{2} Finally, a more recent literature uses experimental data and yields estimates of $\gamma$ between 0 and 15.\textsuperscript{3} In the absence of a consensus, most economists rely on introspection to justify assumptions of $\gamma \in (1, 5)$ while others contend that higher values of $\gamma$ are appropriate.\textsuperscript{4}

\textsuperscript{1}These surprisingly high estimates of $\gamma$ are the “equity premium puzzle,” documented by Mehra and Prescott (1985) and summarized by Kocherlakota (1996).

\textsuperscript{2}Szpiro (1986) finds $\gamma = 2$ using aggregate time series on insured property and insurance premiums. Dreze (1987) uses the size of insurance deductibles and load factors and imputes $\gamma = 10$.

\textsuperscript{3}For example, Barsky et. al. (1997) use responses to hypothetical gambles over lifetime income on the Health and Retirement Survey to estimate $\gamma = 12$. Wolf and Pohlman (1983) cast doubt on this approach by studying the behavior of a bond trader. When inferred from respones to questions about hypothetical gambles his $\gamma = 1$; when inferred from actual bond trades, it is 4. Gertner (1993) and Metrick (1995) overcome this limitation by using actual behavior on game shows and find $\gamma = 4$ and $\gamma = 0$, respectively.

\textsuperscript{4}See e.g. Kandel and Stambaugh (1991), who argue that $\gamma = 10$ does not seem unreasonable when one introspects about small gambles.
This paper develops a method of estimating the coefficient of relative risk aversion from a combination of static labor supply choices and consumption choices when labor supply is stochastic. The basic logic of this approach is as follows. Consider an agent whose risk preferences are characterized by a von-Neumann Morgenstern (vN-M) utility function over outcomes. This agent uses the same utility when making decisions that do not involve risk, such as labor-leisure choices. Labor supply data cannot be used in isolation to identify cardinal properties of the utility function because data on certainty behavior only identify utility functions up to a monotonic transformation. However, for a fixed degree of complementarity between consumption and leisure ($u_{cl}$), there is only one vN-M utility (up to affine transformations) that can be consistent with a given set of labor-leisure choices. Hence, given a value of $u_{cl}$, labor supply data can be used to estimate risk aversion. It remains to estimate $u_{cl}$ from choices under uncertainty. This complementarity parameter can be inferred from data on consumption choices when employment is stochastic, e.g. when individuals anticipate being laid off with some probability.

The basic intuition for the connection between labor supply and risk aversion is that the compensated wage elasticity of labor supply is inversely related to the curvature of the utility function. Following a compensated wage increase, agents adjust their labor supply upward

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5In other words, identification of curvature requires a 1-1 map between observed choices and $\gamma$. In the general labor-leisure model, such a map does not exist because any monotonic transformation of utility generates the same labor supply choices.

6It is critical to have data on choices under uncertainty; barring additional assumptions, a cardinal value for $u_{cl}$ cannot be inferred from the usual certainty settings in which we typically think about estimating the degree of complementarity between consumption and leisure (e.g. timing-of-work, household production).
to the point where the marginal utility of an additional dollar is offset by the marginal disutility of the additional work necessary to earn that dollar. If utility is very curved (i.e. $\gamma$ is high) and the marginal disutility of labor is constant, this condition is met by a small increase in labor supply. Conversely, if utility is not very curved, the agent will increase labor supply significantly in response to a compensated wage increase. In the general setting where the marginal disutility of labor is not constant, $\gamma$ can be estimated using the income elasticity as well as the compensated wage elasticity.

There are several reasons to estimate risk aversion using labor supply data. First, the connection between risk aversion and labor supply can serve as a useful test of models of risk preference. If a risk preference model has a well-defined cardinal utility function and the curvature implied by labor supply behavior does not match the curvature implied by risky decisions, the assumptions of the risk preference model may be invalid.\(^7\) In this vein, the empirical results of this paper can be used to test whether the canonical expected utility model is consistent with both data on labor supply and choice under uncertainty.

Second, the growing “non-expected utility” literature argues that individuals do not take a linear expectation over utility outcomes when making decisions under uncertainty.\(^8\) Nonetheless, these models retain a utility over outcomes that is unique up to affine trans-

\(^7\) Chetty (2002) applies this method to test whether an expected utility model augmented with “consumption commitments” can explain observed risk preferences in the small and large.

formations. We may be interested in knowing the curvature of this utility for normative reasons (e.g. to calculate an optimal social insurance or tax policy). Since we will find that the estimates of $\gamma$ are not very sensitive to the $u_{cl}$ parameter, the labor-supply method effectively relies on the assumption that individuals maximize utility in a certainty setting. It can therefore be used to identify the underlying curvature of utility even if agents violate the axioms of expected utility when making choices under risk. Finally, from a practical standpoint, data on clean, large gambles are limited, especially for lower income individuals in the U.S; but labor supply data are available for many groups in different environments.

I implement the method empirically by using a large set of existing estimates of wage and income elasticities. I find an (unweighted) mean estimate of $\gamma \approx 1$. In addition, I give a calibration argument showing that if the uncompensated labor supply curve is upward sloping (as almost all studies of labor supply find), $\gamma < 1.25$. The estimate of $\gamma$ changes by at most 0.25 for plausible values of the degree of complementarity between consumption and leisure. Consequently, we are able to identify a relatively tight range for $\gamma$ despite the lack of consensus about the magnitudes of labor supply elasticities and the limited amount of evidence on the complementarity parameter.

The next section derives estimators for $\gamma$ in a standard labor supply model. To simplify the exposition, I begin with the case of additive utility ($u_{cl} = 0$) and then discuss the more

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9In the terminology of Kahneman et al (1997), the decision utility used to make labor supply decisions may be more directly related to experienced utility than the decision utility used to choose between gambles. Therefore, if one believes that a stable experienced utility over wealth $u(w)$ exists, the labor supply method may be the best way to identify the curvature of $u$.  

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general case of arbitrary $u_{cl}$. Section 2 uses existing wage and income elasticities to estimate risk aversion. Section 3 concludes.

1 Labor Supply and Risk Aversion

This section begins by giving intuition for the connection between risk aversion and labor supply with a graphical argument. I then derive an estimator for $\gamma$ in a model where (1) vN-M utility is additive in consumption and leisure, (2) agents have identical preferences, and (3) agents are able to make intensive labor supply choices. Each of these assumptions is subsequently relaxed.

(a) Intuition

Consider an agent who has a vN-M utility function $u(c, l)$ over consumption and labor. This agent’s expected utility from a gamble $(\bar{c}, \bar{l})$ is obtained by taking an expectation over the vN-M utility function:

$$U(\bar{c}, \bar{l}) = E u(\bar{c}, \bar{l})$$

The goal of this paper is to identify the curvature of the underlying cardinal utility over outcomes, $u(c, l)$.\(^{10}\)

To illustrate the connection between labor supply and risk aversion, I first derive an expression for compensated labor supply, $l^c(w)$, by solving an expenditure minimization

\(^{10}\)I focus on the curvature of utility over consumption $\gamma = \frac{\partial^2 u}{\partial c^2}$ here, but show below that deriving other curvature parameters of interest is straightforward once the value of $\gamma$ is known.
problem. Let $c$ denote consumption, $l$ labor supply, and $w$ the wage. The agent’s utility over consumption is $u(c)$; assume that $u_c > 0$ and $u_{cc} < 0$. I temporarily assume that the marginal disutility of labor, $\psi$, is constant. The EMP is

$$\min c + w(1 - l) \text{ s.t. } u(c) - \psi l \geq \overline{v}$$

At an interior optimum, $l^c(w)$ satisfies the first order condition

$$u_c(u^{-1}(\overline{v} + \psi l^c)) = \frac{\psi}{w}$$

This condition is intuitive: the individual chooses labor supply by equating the marginal utility of an extra dollar of consumption with the marginal disutility of working to earn that dollar. These choices are depicted by the intersections of the $u_c$ and $\frac{\psi}{w}$ curves in Figure 1.

Now consider the effect of an increase in $w$ from $w_0$ to $w_1$ on compensated labor supply. As shown in Figure 1, a change in $w$ shifts the flat marginal disutility of labor curve downward. If the utility function is highly curved (case A), the marginal utility of consumption ($u_c$) falls quickly as labor supply and income rise. Consequently, the increase in $w$ leads to a small increase in $l^c_A$. When the utility function is not very curved (case B), marginal utility declines slowly as a function of wealth and the same $\Delta w$ leads to a larger increase in $l^c_B$. Figure 1 therefore illustrates that the compensated elasticity of labor supply, $\varepsilon_{l,w}^c$ is inversely related to the curvature of utility over consumption.
Figure 1: Recovering $\gamma$ from Labor Supply

The intuition for this relationship is as follows. Following a compensated wage increase, agents increase their labor supply up to the point where the marginal utility of an additional dollar is offset by the marginal disutility of the additional work necessary to earn that dollar. If utility is very curved, this condition is met by a small increase in labor supply. If utility is not very curved, the agent needs to increase $l$ much more before his marginal utility of money falls sufficiently to equal the new $\frac{\psi}{w}$. 

The preceding argument relies on the assumption that disutility of labor, $\psi$, does not vary with $l$. When it does, the curvature of $\psi(l)$ is confounded with the curvature of $u$ and $\varepsilon_{l,w}^c$ is no longer sufficient to recover $\gamma$. In this case, the elasticity of labor supply with respect to unearned income, $\varepsilon_{l,y}$ is needed to separate the two curvature parameters. Abstractly, the
income and compensated wage elasticities are both functions of the two curvatures. One can therefore back out $\gamma$ and the curvature of $\psi$ by solving a system of two equations and two unknowns, while holding fixed the degree of complementarity between consumption and leisure.

The next section derives the relationship between $\gamma$ and labor supply elasticities formally.

(b) Base Case: Additive Utility

It is convenient to redefine $u(c, l)$ as the agent’s utility over both consumption and labor. Assume that $u_c > 0, u_l < 0, u_{cc} < 0, u_{ll} < 0$. For expositional clarity, I begin with the base case in which utility is additive in consumption and labor, i.e. $u_{cl} = 0$.\footnote{Note that this restriction is stronger than assuming that the utility function permits an additively separable representation. For example, Cobb-Douglas utility is “additively separable” but does \textit{not} satisfy the additivity restriction (however, the log of a Cobb-Douglas utility does).}

An agent with wage $w$ and unearned income $y$ chooses (Marshallian) labor supply $l$ by solving

$$\max_l u(y + wl, l)$$

At an interior optimum, $l$ satisfies the first order condition

$$wu_c(y + wl, l) = -u_l(y + wl, l)$$ (1)
Consider the effects of increasing \( w \) and \( y \) on \( l \):

\[
\frac{\partial l}{\partial y} = -\frac{wu_{cc}}{w^2 u_{cc} + u_l} \\
\frac{\partial l}{\partial w} = -\frac{u_c + wlu_{cc}}{w^2 u_{cc} + u_l}
\]

Using the Slutsky decomposition for compensated labor supply \( \frac{\partial l^c}{\partial w} \)

\[
\frac{\partial l^c}{\partial w} = \frac{\partial l}{\partial w} - l \frac{\partial l}{\partial y} \tag{2}
\]

it follows that

\[
\frac{\partial l / \partial y}{\partial l^c / \partial w} = \frac{wu_{cc}(y + wl, l)}{u_c(y + wl, l)} \tag{3}
\]

The definition of the coefficient of relative risk aversion at consumption \( c \) is

\[
\gamma(c) \equiv -\varepsilon_{u(c)} = \frac{\partial u(c)}{\partial c} \frac{c}{u(c)} = -\frac{u_{cc}(c)}{u_c(c)} c \tag{4}
\]

which implies, using (3), that

\[
\gamma(y + wl) = -\frac{y + wl}{w} \frac{\partial l / \partial y}{\partial l^c / \partial w} = -(1 + \frac{wl}{y}) \varepsilon_{l,y}(y, w, l) \tag{5}
\]

where \( \varepsilon_{l,y} \) denotes the income elasticity, \( \varepsilon_{l^c,w} \) the compensated wage elasticity, and \( \frac{wl}{y} \) the ratio of earned to unearned income. As expected, the coefficient of relative risk aversion is
inversely related to the compensated wage elasticity.

The reader may be puzzled that we can identify a unique value for $\gamma$ by observing only labor supply. Since non-linear monotonic transformations of $u(c, l)$ do not affect the choice of $l$, are there not infinitely many values of $\gamma$ that could be associated with observed labor supply behavior? While this is true in general, the key is to observe that any non-linear transformation of $u$ will change the value of $u_{cl}$. However, (5) was derived under the assumption that $u(c, l)$ is additive, i.e. $u_{cl} = 0$.

It should be emphasized that $\gamma$ is the curvature of utility over consumption. When labor supply is not fixed, $\gamma$ will not necessarily equal the curvature of utility over wealth. However, once we know $\gamma$, we have a complete map of the vN-M utility function, and can calculate any curvature of interest. For example, defining indirect utility over unearned income as

$$v(y) = u(y + wl(y)) - \psi(y)$$

it is shown in the appendix that the curvature of utility over unearned income is

$$-\frac{v_{yy}}{v_y} y = \gamma \frac{y + wl \varepsilon_{l,y}}{y + wl}$$

Finally, one may worry that adjustment costs which prevent agents from reoptimizing fully

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12 This is the parameter estimated by most studies of choice under uncertainty, insofar as labor supply is omitted from these analyses. More importantly, it is the relevant parameter for many models (e.g. portfolio choice and optimal social insurance).
in response to perturbations in \( w \) and \( y \) will affect the estimates of \( \gamma \). This is not true because the method is biased only by factors that affect the price and income elasticities differently. A status-quo bias that makes some individuals reluctant to change their labor supply or institutional constraints that make small adjustments in labor supply difficult do not affect the estimate.\(^{13}\)

(c) Complementarity Between Labor and Consumption

When \( u_{cl} \neq 0 \), (3) becomes

\[
\frac{\partial l}{\partial y} = \frac{\partial l}{\partial w} = w u_{cc}/u_c + u_{cl}/u_c
\]

which, after some rearrangement, implies that

\[
\gamma(y + wl) = (1 + \frac{wl}{y}) \frac{-\varepsilon_{l,y}}{\varepsilon_{l,w}} + (1 + \frac{y}{wl}) \varepsilon_{u_{cl}}
\]

where \( \varepsilon_{u_{cl}} \) denotes the elasticity of the marginal utility of consumption with respect to labor supply. Note that labor supply data is sufficient to identify \( \gamma \) given any value of \( \varepsilon_{u_{cl}} \) because no non-linear transformation of \( u \) will leave \( \varepsilon_{u_{cl}} \) unchanged.

It remains to estimate \( \varepsilon_{u_{cl}} \) from choices under uncertainty. One way to do so is by observing data on the consumption choices of individuals who face uncertainty in labor

\(^{13}\)To see this formally, note that an adjustment cost or status quo bias can be modeled as a cost \( k(l, l_0) \) of changing labor supply to \( l \) from \( l_0 \). Since we make no assumptions about the way in which labor supply \( l \) enters \( u(c, l) \), (5) still obtains. The reason is that curvature is identified from the ratio of income and price effects, and \( k > 0 \) attenuates both effects.
supply. For example, if an agent chooses to keep consumption fairly constant across states in which he is employed and unemployed, \( \varepsilon_{u,c,t} \) must be small.

To derive the relationship between consumption choices when employment is stochastic and \( \varepsilon_{u,c,t} \) formally, consider a world with two states. Agents supply \( l_1 \) units of labor in state 1 and \( l_2 \) units of labor in state 2. Suppose that the agent can trade consumption fairly between the two states by purchasing state-contingent commodities (e.g. using an insurance policy). He chooses consumption in the two states by maximizing expected utility

\[
\max_{c_1,c_2} p u(c_1, l_1) + (1 - p) u(c_2, l_2)
\]

s.t. \( pc_1 + (1 - p)c_2 = pw l_1 + (1 - p) w l_2 \equiv \overline{W} \)

The agent’s first-order condition for consumption is obtained by equating marginal utilities across the two states:

\[
u_c(c_1, l_1) = u_c(\overline{W} - c_1, l_2)\]

Now, suppose we observe data from the following experiment. Assume that the agent starts out supplying a constant \( l_1 = l_2 = \overline{l} \) units of labor in each state. Suppose there is a balanced-budget change in labor supply, increasing state 1 labor supply by \( \delta_1 \) units while decreasing state 2 labor supply by \( \delta_2 \) units to keep expected earnings fixed at \( \overline{W} \). We can think of this as a decision to increase work effort in state 1 to compensate for (partial) unemployment in state 2.
Differentiating the first order condition with respect to $l_1$ while holdings earnings fixed at $\bar{W}$ yields the following identity:

$$\varepsilon_{uc,l} = \gamma \varepsilon_{c_1,l_1}$$

Here, $\varepsilon_{c_1,l_1}$ denotes the elasticity of consumption with respect to labor supply in state 1. Plugging this expression into (7) and solving gives an estimator for risk aversion in terms of $\varepsilon_{c_1,l_1}$:

$$\gamma = (1 + \frac{wl}{y}) \frac{-\varepsilon_{l,y}}{\varepsilon_{c,w}/(1 - (1 + \frac{y}{wl})\varepsilon_{c_1,l_1})}$$

(8)

This formula reduces to (5) when utility is additive in labor and consumption ($\varepsilon_{c_1,l_1} = 0$). When consumption and labor are complements, $\varepsilon_{c_1,l_1} > 0$, and the actual $\gamma$ is higher relative to the estimate obtained when additive utility over $c$ and $l$ is assumed.\textsuperscript{14}

While one can introspect about the magnitude of $\varepsilon_{c_1,l_1}$, it is helpful to map existing estimates of the consumption drop during unemployment into values of $\varepsilon_{c_1,l_1}$. To do so, normalize the agent’s labor supply to 1 when working and 0 when unemployed. Since we are using a discrete change in labor supply to infer an elasticity, we must choose a functional form for consumption in terms of labor supply. For simplicity, I assume a linear form:

$$c = a + bl$$

\textsuperscript{14}The sign of $\varepsilon_{c_1,l_1}$ is theoretically ambiguous. If consumption requires time, as in Becker (1965), leisure and consumption are complements ($\varepsilon_{c_1,l_1} < 0$). On the other hand, work-related expenses can make labor and consumption complementary ($\varepsilon_{c_1,l_1} > 0$). I show below that regardless of the sign of $\varepsilon_{c_1,l_1}$, the estimates of $\gamma$ are not very sensitive to its magnitude.
In this case, $\varepsilon_{c_1,l_1}(l_1 = 1) = \frac{b}{a+b}$, which is precisely the percentage drop in consumption from the employed to unemployed state. Estimates of the consumption drop are small: for example, Gruber (1997) estimates $\frac{b}{a+b} = 0.068$ using data from the PSID.\footnote{Similarly, Browning and Crossley (2001) use data from the Canadian Out of Employment Panel to show that the consumption drop is not statistically distinguishable from zero for households that have positive liquid assets before their unemployment spell.} It should be noted that this estimate gives an upper bound for the true $\varepsilon_{c_1,l_1}$ if the agent is not able to smooth consumption across states to his desired level because of the inadequacy of available insurance policies. Hence, the empirical evidence suggests that the departure from additivity ($u_{cl} = 0$), if any, is not large.

(d) Unobservable Heterogeneity in Preferences

How should the estimators (5) and (8) be interpreted in a world with unobserved heterogeneity in preferences? To answer this question, suppose there are $N$ types of agents, who have utilities $u^1(c,l), u^2(c,l), \ldots, u^n(c,l)$. To simplify the exposition, assume that all the utilities are additive in $c$ and $l$. Let $\alpha_i$ denote the fraction of type $i$ agents.

Let us define a utility function $\bar{u}(c)$ for a representative agent by taking a weighted average of the individual utilities as follows:

$$\bar{u}(c,l) = \sum_{i=1}^J \omega_i u_i^i(c,l)$$

where the weights

$$\omega_i = \frac{\alpha_i / (w^2u_{cc} + u_{ll})}{\sum_{i=1}^J \alpha_i / (w^2u_{cc} + u_{ll})}$$
for $i = 1, \ldots, J$ sum to 1. Note that in general, the representative agent’s utility will differ depending on the values of $(y, w, l)$.

It is shown in the appendix that in a heterogeneous population, the estimate $\hat{\gamma}(y + wl)$ equals the coefficient of relative risk aversion ($\bar{\gamma}$) for the representative agent with utility $\bar{u}$:  

$$\bar{\gamma} = -(y + wl) \frac{\bar{u}_{cc}}{\bar{u}_c} = \hat{\gamma}$$

Hence, in a heterogeneous economy, the estimator $\hat{\gamma}(y + wl)$ can be interpreted as a weighted average of the risk aversions of agents in the marginal group at $(y, w, l)$. The marginal group consists of those who change their labor supply in response to perturbations in $w$ and $y$. The preferences of those who are constrained and cannot make such changes could be quite different.

(e) Extensive Labor Supply Decisions

Many individuals are unable to choose the number of hours they work. They face the narrower choice of either working for a fixed number of hours or not working at all. To model such extensive labor supply decisions, assume that the agent makes a binary decision to work and supply 1 unit of labor or not work. As above, let $y$ denote unearned income and $w$ the income earned by working. Returning temporarily to the world of additive utility

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16In general, $\bar{\gamma}$ is not equal to the weighted average of each type’s $\gamma$ because coefficients of relative risk aversion do not aggregate linearly.

17Note that we can observe many different marginal groups by estimating $\hat{\gamma}$ at various levels of $(y, w, l)$.

18The purpose of this section is to show that inferences about curvature can be made even when changes in labor supply are lumpy. The nature of the lumpiness itself (e.g. whether there are restrictions on hours worked in a week or weeks worked in a year) is not important.
over consumption and leisure, redefine \( u(c) \) as the utility from consumption. Let \( \psi \) denote disutility of supplying 1 unit of labor. The agent chooses labor supply by solving

\[
\max_{l \in \{0, 1\}} u(y + wl) - \psi l
\]

He works if his disutility of labor is less than the utility of an additional \( w \) units of consumption, i.e. if

\[
\psi < \hat{\psi}(y, w) \equiv u(y + w) - u(y)
\]

Let us model the heterogeneity of disutility of labor in the economy by a smooth density \( f(\psi) \). Then the fraction of workers who participate in the labor force is

\[
\theta(y, w) = \int_0^{\hat{\psi}(y, w)} f(\psi) d\psi
\]

It follows that

\[
- \frac{\partial \theta}{\partial y} = \frac{u_c(y) - u_c(y + w)}{u_c(y + w)}
\]

This expression shows that the percent change in marginal utility of wealth from \( y \) to \( y + w \) is equal to the ratio of the income and wage effects on labor supply. In the intensive labor supply world, we could compute \( \gamma(c) \) at any level \( c \) without making any functional form assumptions because we could observe how marginal utility changes for small changes in
income. In a world with extensive labor supply decisions, we observe only the change in marginal utility between \( y \) and \( y + w \). Consequently, we need to make a functional form assumption for \( u(c) \) to translate the change in marginal utilities into a coefficient of relative risk aversion. I assume CRRA utility:\(^{19}\)

\[
u(c) = \frac{c^{1-\gamma}}{1-\gamma}
\]

Under this assumption, (10) implies

\[
-\frac{\partial \theta / \partial y}{\partial \theta / \partial w} = \frac{y^{-\gamma} - (y + w)^{-\gamma}}{(y + w)^{-\gamma}}
\]

Solving for \( \gamma \) yields

\[
\gamma = \frac{\log[1 - \frac{\varepsilon \theta, w}{\varepsilon \theta, w, y}]}{\log[1 + \frac{w}{y}]} \tag{11}
\]

Finally, a model of unemployment analogous to that above can be used to derive an estimator for \( \gamma \) when utility is not additive:

\[
\gamma = \frac{\log[1 - \frac{\varepsilon \theta, y, w}{\varepsilon \theta, y, w, c}]}{\log[(1 - \frac{\Delta c}{c})(1 + \frac{w}{y})]} \tag{12}
\]

where \( \frac{\Delta c}{c} \) denotes the consumption drop associated with unemployment.

\(^{19}\)If \( \gamma(c) \) actually varies with \( c \), this method yields the best constant-\( \gamma \) fit of the data, which can be loosely interpreted as the average \( \gamma(c) \) in the region \( c \in [y, y + w] \).
2 Empirical Implementation

A large number of studies have estimated “static” wage and income elasticities using exogenous variation in unearned income and wages such as tax changes, cross-sectional differences, or lottery winnings.\(^\text{20}\) The traditional literature, summarized by Pencavel (1986) and Blundell and MaCurdy (1999), defines labor supply as hours worked or work participation. More recently, Feldstein (1995) emphasizes that hours worked is only one component of labor supply and that other margins such as effort or training might adjust as well. When a multi-dimensional definition of labor supply in incorporated into the model above, (5) still obtains except that the elasticity ratio \(\varepsilon_{l,y}/\varepsilon_{l,c,w}\) is replaced by \(\varepsilon_{Ll,y}/\varepsilon_{Ll,c,1-\tau}\), where \(Ll\) is labor income and \(1-\tau\) the net-of-tax rate. This result follows the lines of Feldstein (1999), who shows that the elasticity of taxable labor income with respect to the net-of-tax rate captures all margins on which taxable income can be adjusted. The reason is that the relative prices of each mechanism of adjustment remain fixed when tax rates change.

Table 1 presents a set of income and wage elasticities for three definitions of labor supply: (A) Hours worked, (B) Participation, and (C) Earned income.\(^\text{21}\) To get a sense of the plausible range for \(\gamma\) that is consistent with labor supply behavior, I include elasticity estimates

\(^{20}\)Of course, these estimates are obtained from a world in which consumers make dynamic choices. However, a wide variety of empirical strategies have been designed to identify precisely the relevant elasticities for a static model.

\(^{21}\)In part (C) of the table, it is important to distinguish between taxable labor income and total labor income. In the calculation of \(\gamma\), we are interested in changes in total earned income, irrespective of the form of compensation. The measure of income used in the table (AGI) may not capture forms of compensation that are not reported on tax returns such as office perks.
for a wide range of groups, such as prime age males, married women, retired individuals, and low income families. In addition, I consider studies with many different approaches to estimating the elasticities, such as welfare reforms, tax changes, cross-sectional variation, and lottery payouts. The inclusion of a diverse set of studies yields a substantial amount of variation in the elasticities, ranging from 0.035 to 1.0 for the compensated wage elasticity and -0.08 to -0.3 for the income elasticity. In general, elasticity estimates for groups who are not as attached to the labor force (married women and older individuals) tend to be higher than the elasticity estimates for groups with greater labor force attachment (prime age men). Despite this variation in the estimates, the implied range for $\gamma$ ends up being quite tight, largely because the ratios of the unearned income to wage elasticities are actually fairly stable across studies.

To estimate $\gamma$, I first consider the additive utility case and apply (5) and (11). Column (6) of Table 1 reports estimates of $\gamma$ at the average value of $\frac{y}{w}$ and $l$ in each study.$^{22}$ Rather than attempting to discern the coefficient of relative risk aversion at specific income levels, I group all the estimates together to identify a representative agent’s average $\gamma$ across a broad range of incomes. The (unweighted) mean $\gamma$ is 1.0 using hours, 0.29 using participation, and 1.19 using earned income elasticities. The overall mean estimate of $\gamma = 1.01$, which implies that a 10% increase in consumption reduces the marginal utility of consumption by

$^{22}$Following Hausman (1985), $y$ is defined as “virtual” unearned income to account for the progressivity of the U.S. tax system. Since the earned income estimates combine different studies, they are evaluated at $\frac{y}{wl} = \frac{1}{3}$, which reflects the median value of unearned to earned income in the US (see below).
Despite the lack of a consensus among labor economists about the magnitudes of income and wage elasticities, every study except one implies $\gamma < 1.5$. This similarity of the estimates of $\gamma$ despite the use of different methodologies, definitions of labor supply, and groups of the population is striking. The following calibration exercise sheds some light on the source of this consensus. Rewriting the Slutsky equation for compensated labor supply (2) in terms of elasticities yields

$$\varepsilon_{l,c,w} = \varepsilon_{l,w} - \frac{lw}{y} \varepsilon_{l,y}$$

Most studies of labor supply find that uncompensated labor supply curves are upward sloping ($\varepsilon_{l,w} > 0$). This places a lower bound on $\varepsilon_{l,c,w}$ of $-\frac{lw}{y} \varepsilon_{l,y}$ which implies using (5) that

$$\gamma < 1 + \frac{y}{wl}$$

The ratio of unearned income to earned income varies across the population, but in the aggregate it equals the ratio of capital income to labor income, which is $\frac{1}{2}$ in the U.S. This places an upper bound of $\gamma = 1.5$ for a representative agent whose utility is an income-weighted average of individual utilities. Since capital income is highly concentrated, if we are interested in the curvature of an hours-weighted average of utilities, the relevant value of $\frac{y}{wl}$ is much lower; a reasonable estimate is $\frac{1}{4}$.\footnote{Tabulations by the US Census Bureau (1999, Table E) adjusted for the progressivity of the income tax indicate that $\frac{y}{wl} \approx \frac{1}{4}$ for the median family in the U.S., which has an income of approximately $40,000.} In this case, $\varepsilon_{l,w} > 0$ implies $\gamma < 1.25$. 
In fact, to generate $\gamma > 2$ with an income elasticity of $-0.1$, we must have $\varepsilon_{l,w} < -0.15$. Almost every existing study of labor supply finds $\varepsilon_{l,w} > -0.15$.\textsuperscript{24}

Taking complementarity between labor and consumption into account does not change these results significantly. Column (7) of Table 1 reports estimates that account for the degree of complementarity implied by an unemployment consumption drop of $\Delta c = 10\%$. This adjustment increases the average estimate of $\gamma$ to 1.24. As noted above, if some of this drop in consumption reflects a lack of adequate mechanisms to smooth consumption across states rather than rational behavior, the observed $\Delta c$ overstates the degree of complementarity between consumption and labor. In this case, the estimates in column (7) give an upper bound for $\gamma$. In fact, generating a mean estimate of $\gamma > 2$ requires $\varepsilon_{c_l,l} = \Delta c > 0.25$, which seems implausibly large. Therefore, the band for $\gamma$ remains narrow even though our estimate of $\varepsilon_{u_c,l}$ may be imprecise.

\section{Conclusion}

This paper has shown that labor supply data can be used to infer risk aversion conditional on the degree of complementarity between consumption and leisure. The degree of complementarity can in turn be estimated from consumption choices when labor supply is stochastic.

Using a broad array of existing estimates of income and wage elasticities, I find an\footnote{Pencavel (1986), Blundell and MaCurdy (1999), and Gruber and Saez (2000) summarize more than sixty studies with an array of methodologies. All find uncompensated wage elasticities greater than $-0.15$.}
(unweighted) mean estimate of $\gamma \approx 1$ and show that the estimates of almost every study of labor supply imply $\gamma < 2$. The estimate of $\gamma$ changes by at most 0.25 over the range of plausible values for the complementarity parameter. Consequently, we are able to bound $\gamma$ quite tightly despite the lack of consensus about the magnitudes of labor supply elasticities and the limited amount of evidence on the complementarity parameter. The intuition underlying the low coefficient of relative risk aversion is clear when one thinks about the labor supply behavior we would observe if $\gamma$ were high. If $\gamma$ were high, the marginal utility of money would deteriorate quickly with income, and labor supply would diminish significantly as income rises. But existing empirical evidence strongly rejects this hypothesis.

Two caveats deserve mention: First, this estimate applies only to the range of incomes over which the relevant elasticities are estimated (roughly $10,000$-$200,000$). Second, the estimate only reflects the curvature of the utility function for agents who are able to adjust their labor supply in response to wage and income changes. Many people do not have such flexibility because of prior commitments, and it would be useful to identify the curvature of their utilities over wealth.\textsuperscript{25}

Is the estimate of $\gamma = 1$ implied by labor supply behavior consistent with empirical evidence on choice under uncertainty? Most economists find log utility reasonable based on introspection about risk preferences over large gambles. However, there is a considerable amount of evidence that individuals exhibit much higher degrees of risk aversion with respect

\textsuperscript{25}Chetty (2002) uses data on unemployment durations to show that the coefficient of relative risk aversion is an order of magnitude larger when individuals are constrained by prior commitments.
to gambles that involve moderate stakes, i.e. losses or gains between $1000 and $100,000.\textsuperscript{26} The analysis of this paper suggests that the canonical expected utility model cannot simultaneously explain observed labor supply behavior and high degrees of moderate-stake risk aversion.

\textsuperscript{26}Rabin (2000) gives a calibration argument showing that typical levels of moderate-stakes risk aversion can only be generated by highly curved utilities in the expected utility model.
References


Appendix

(a) Curvature of utility over unearned income
The Envelope theorem implies that

\[ v_y(y) = u_c(c(y), l(y)) \]

and it follows that

\[ \gamma^y = \frac{v_{yy}}{v_y} y = -\frac{u_{cc}}{u_c} \frac{\partial c}{\partial y} y \]

Recognizing that \( \frac{\partial c}{\partial y} = 1 + w \frac{\partial l}{\partial y} \), we obtain

\[ \gamma^y = \gamma \varepsilon_{c,y} \]

where \( \varepsilon_{c,y} \) denotes the income elasticity of consumption. Finally, observe that

\[ \varepsilon_{c,y} = \frac{y + wl \varepsilon_{l,y}}{y + wl} \]

where \( \varepsilon_{l,y} \) is the income elasticity of labor supply.

Note that \( \varepsilon_{c,y} < 1 \) implies \( \gamma^y < \gamma^c \). If utility over consumption is not very curved \( (\gamma = 1) \), utility over wealth must be even less curved.

(b) Heterogeneity in Preferences
Assume that the first \( J \) types supply exactly \( l \) units of labor when they have unearned income \( y \) and wage \( w \). The aggregate income and compensated wage effects at \( (y, w, l) \) are

\[
\begin{align*}
\frac{\partial l}{\partial y} &= \sum_{i=1}^{J} \alpha_i \frac{\partial l^i}{\partial y} = \sum_{i=1}^{J} \alpha_i \frac{wu_i^{c,i}}{w^2u_{cc}^i + u_{ll}^i} \\
\frac{\partial l^c}{\partial w} &= \sum_{i=1}^{J} \alpha_i \frac{\partial l^{c,i}}{\partial w} = \sum_{i=1}^{J} \alpha_i \frac{wu_i^{c,i}}{w^2u_{cc}^i + u_{ll}^i}
\end{align*}
\]

which implies that the estimator in (5) is

\[
\hat{\gamma}(y + wl) = -\frac{y + wl}{w} \frac{\partial l}{\partial y} = -\frac{y + wl}{w} \sum_{i=1}^{J} \alpha_i \frac{wu_i^{c,i}}{w^2u_{cc}^i + u_{ll}^i}
\]

It follows that

\[ \gamma^\Pi = -(y + wl) \frac{u_{cc}}{u_c} = -(y + wl) \frac{\sum_{i=1}^{J} \omega_i u_i^c}{\sum_{i=1}^{J} \omega_i u_i^c} = \hat{\gamma} \]
TABLE 1
Labor Supply Elasticities and Implied Coefficients of Relative Risk Aversion

<table>
<thead>
<tr>
<th>Study Sample Identification</th>
<th>Income Elasticity</th>
<th>Compensated Wage Elasticity&lt;sup&gt;a&lt;/sup&gt;</th>
<th>Additive</th>
<th>γ</th>
<th>γ&lt;sub&gt;Δc/c=0.1&lt;/sub&gt;</th>
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<td><strong>A. Hours</strong></td>
<td></td>
<td></td>
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<tr>
<td>Blundell and MaCurdy (1999)&lt;sup&gt;b&lt;/sup&gt;</td>
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<td>1.68</td>
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<td>-0.030</td>
<td>0.192</td>
<td>0.88</td>
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<td>Married Women, Inc &lt; 30K EITC Expansions</td>
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<td>Friedberg (2000)</td>
<td>Older Men (63-71) Soc. Sec. Earnings Test</td>
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<td><strong>B. Participation&lt;sup&gt;c&lt;/sup&gt;</strong></td>
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<td>0.44</td>
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<td>Average</td>
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<td><strong>C. Earned Income&lt;sup&gt;d&lt;/sup&gt;</strong></td>
<td></td>
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<tr>
<td>Imbens et al (2001)</td>
<td>Lottery Players in MA Lottery Winnings</td>
<td>-0.110</td>
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<td><strong>Overall Average</strong></td>
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</table>

<sup>a</sup>In part (C), this column gives the elasticity of earned income with respect to the net-of-tax rate.

<sup>b</sup>This row uses an average of the 20 elasticities reported in Blundell and MaCurdy (1999) and assumes y/wl=1/4.

<sup>c</sup>Participation elasticities assume CRRA utility.

<sup>d</sup>Since studies on earned income do not estimate income elasticities, I use the Imbens et. al. estimate in each case.