THE PROPERTY-RIGHTS THEORY OF THE FIRM WITH ENDOGENOUS TIMING OF ASSET PURCHASE*

by
David de Meza
London School of Economics and University of Exeter
and
Ben Lockwood
University of Warwick and CEPR

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The Suntory Centre
Suntory and Toyota International Centres
for Economics and Related Disciplines
London School of Economics and Political Science
Houghton Street
London WC2A 2AE
Tel.: 0171-405 7686

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Abstract

The standard property-rights theory of the firm assumes that prior to investing in human capital, team members meet and negotiate asset ownership. This paper endogenizes the event sequence in a matching model of market equilibrium. Equilibria exist in which, for strategic and efficiency reasons, agents invest in human capital and buy assets prior to matching and simple ownership arrangements are chosen. As in the original work, ownership of physical assets affects the incentive to invest. However, in this setting ownership creates rent shifting, search and asset transfer advantages, so new results emerge. It is no longer necessarily true that key agents own. As for the form of integration, there may be multiple Pareto-rankable equilibria.

Keywords: Property-rights; incomplete contracts; matching; asset ownership.

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Contact address: Professor David de Meza, Department of Economics, University of Exeter, Streatham Court Annex, Streatham Drive, Exeter, EX4 4QR.
Email: D.E.de-Meza@exeter.ac.uk

1. Introduction

According to the property rights approach to the theory of the firm, in a world of incomplete contracting, the ownership of assets matters for incentives. By influencing bargaining power over decisions not governed by contract, ownership affects the division of relationship surplus and so each agent's return to creating that surplus[1]. As developed in the seminal papers of Grossman and Hart (1986) and Hart and Moore (1990) - henceforth GHM - the assumed timing is that agents first negotiate ownership, after which unverifiable investment decisions are taken. The incentive effects of asset ownership then imply that there will be a determinate efficient asset ownership structure (i.e. one that maximizes the joint surplus of the agents, net of investment costs). As wealth constraints are assumed away, the theory predicts that parties adopt at the outset this efficient ownership structure.

In many cases this timing does not appear to describe how events unfold. For example, the proprietor of a new business typically purchases premises and plant and expends considerable planning effort before hiring employees. Employees too may have undertaken prior investments in skills which, because of non-contractibility and search frictions, will eventually be vulnerable to hold-up problems.

Rather than predetermining the timing of events (matching, asset purchase, investment), it seems to us preferable to allow the timing of decisions to be endogenous. This paper builds a matching model in which agents can decide whether to purchase assets and invest either before or after they are matched with others. We show that there exist equilibria in which the event sequence differs from that in GHM and study the effect this has on the pattern of asset ownership.

To illustrate further our motivation, consider the case of human capital. According to GHM, property rights theory concerns non-human assets. As summarized in Hart (1998, p. 332) "The reason is that non-human assets are alienable while human assets are inalienable, i.e. in the absence of slavery they cannot be bought or sold. To put it another way, residual control rights over human assets resides with the human beings concerned: they cannot (easily) be transferred to someone else. Thus it is changes in the allocation of residual control rights over non-human assets that are relevant for a theory of incentives (or for a theory of the firm...)"

The extent to which *ex post* inalienability excludes all human assets from the analysis is open to question. Suppose in some two-person productive relationship, success requires that at least one of the parties has an MBA. Achievement of the qualification is verifiable, so there is no problem in the parties negotiating *ex ante* over which of them should take the course and the payments to be made in the event of successful completion. The possessor of the degree enjoys enhanced power in subsequent bargaining over the returns to unverifiable investments. So to all intents and purposes, 'ownership' of the MBA could be allocated in much the same

[1]There are two routes by which asset ownership influences *ex post* bargaining power (de Meza and Lockwood, (1998), Chiu(1998)). First, ownership may increase payoff flows during bargaining (the inside option). Second, if the relationship breaks up, the available opportunities (outside options) may be enhanced by ownership. As the division of the relationship surplus depends on ownership, so does the incentive to undertake relationship specific investments.

way as a physical asset and will involve similar incentive effects. It is not so much inalienability that puts this kind of asset outside the GHM framework but the ordering of decisions. Firms do sometimes sponsor employees to obtain an MBA, but it is usually an agent's unilateral choice to take the course.

There are a number of reasons why this is sensible. One is that it may be anticipated that the MBA will be useful in a series of relationships each of limited duration. Here though the emphasis is on coordination problems and strategic considerations. If the course is only commenced after meeting a suitable partner, team production for both of them is postponed for the duration of study. Matching with an already qualified partner is much more attractive. So it is easy to see the possibility of an equilibrium in which agents seek jobs only when qualified. Moreover, since an MBA confers an *ex post* bargaining advantage there is a strategic incentive to capture this gain or neutralise a potential partner's power by owning prior to matching.

In outline, our model is as follows[2]. Agents live over $T \geq 2$ periods. At any date, an agent can choose to buy an asset (i.e. they have a choice between becoming firm owners or workers). The asset is the physical capital required to run a small business (such as a shop, a client list, etc.). Agents may also invest in a skill at any date. Agents with assets can produce on their own(home production), but are more productive in pairs (team production). There is a matching process that agents can enter at the end of the first time period. We assume that this matching process is efficient in the sense that there is no other way to match agents so that at least some are better off. Once matched, agents then bargain over the revenue from production, and finally production and consumption take place.

The main focus of the paper is the extent to which the GHM results carry over to this setting[3]. It is useful to begin by emphasizing two key differences between this set-up and GHM. When we refer to GHM it is helpful to think of the two-agent versions of the theory, with either one or two assets, as in Grossman-Hart(1986), rather than the more general treatment in Hart-Moore(1990). We also emphasize the implicit assumption of GHM that when agents meet, they then negotiate over asset purchase, and the gainer from the purchase compensates the loser through a side-payment, so that the ownership decision is efficient.

In GHM, asset ownership and investments are determined *after* the two agents are matched whereas in our set up, there may also be an *ex ante* equilibrium where these decisions are made *before* agents are matched.

In this equilibrium, there are two interesting incentives for asset purchase which are absent from GHM and which can be explained as follows. First, in both GHM and our setting, asset purchase raises the value of the outside option of an agent, and thus his share of the surplus *ex post*, though the bargaining process. We refer

[2]Our model builds on work by Rubinstein and Wolinsky (1985), Gale (1987) and Wolinsky (1987) but allows asset ownership and investment to be choice variables. Ramey and Watson (1996) also look at ownership issues in a matching model. They model a continuing relationship in which investment is unproductive in any other match and in stand-alone production. This assumption rules out the mechanism which in GHM's analysis determines the pattern of ownership.

[3]Although not the main focus of our analysis, it is clear that unlike the GHM case, our equilibrium is usually not socially efficient (Proposition 10).

to this as the *rent-shifting* incentive for asset purchase. However, in GHM, rent-shifting ultimately has no effect on asset purchase decisions, as the rent shifted to the purchaser *ex post* is completely offset by an adjustment of side-payments *ex ante*. In our set-up, this is of course not possible, as *ex ante*, the two agents are not yet paired, so, rent-shifting provides an additional incentive for asset purchase. Note that the rent-shifting incentive is related to the revenue from home production; the higher this is, the stronger the incentive for rent-shifting.

The second incentive for of asset purchase not in GHM is that ownership affects an agent's probability of making a productive match and thereby realizing the gains from team production. We call this the *search* incentive.

Bearing in mind these two additional incentives, we can say the following. First, the fundamental GHM observation, that ownership matters for investment, carries over to our setting. Generally, agents who buy assets invest more than those who do not. However, the difference in investment levels may occur even when in the GHM setting, ownership has no effect on investment (Proposition 1).

A second point concerns ownership patterns when agents are heterogenous. A general observation here is that in GHM, the optimal ownership structure depends only on the *marginal* productivities of the agents. A specific case is if there are two types of agent, where only one type has a positive marginal product in *both* home and team production (a key agent). According to GHM, it is unambiguous that only such key agents should (and do) own the asset.

By contrast, in our setting, the *average* productivities of agents also help determine ownership through rent-shifting and search incentives, and this may lead to conflict with GHM results. For example, suppose that key agents have a low average revenue in home production, and non-key agents have a high average revenue in home production. Then, non-key agents have a stronger rent-shifting incentive to own, and this may dominate the efficiency loss that is incurred if they own, causing them to own in equilibrium (Proposition 3). The above argument also suggests that if average and marginal products move together, then the GHM result that key agents should own carries over in our model, and this is indeed the case (Proposition 5).

Our basic model assumes that only one asset is required for production, and additional assets are superfluous i.e. assets are *independent* in Hart's terminology. Therefore, an interesting extension is to suppose that two assets are essential for production (*complementary* assets). GHM argue that complementary assets will be owned jointly (one agent owns both assets, and the other none) in order to capture incentives to invest. In our model equilibria where the assets are owned jointly (some agents own two assets, others none), and separately (all agents own one asset) may co-exist, and whenever they do co-exist, the separate ownership equilibrium Pareto-dominates the joint ownership one[4].

The model developed in this paper also allows us to address Maskin and Tirole's (1998) critique of property rights theory. They argue that even though it may

[4]Hart(1996) refers to joint and separate ownership as integration and non-integration respectively. This latter terminology is more appropriate to his model, which is one of possible vertical integration between an upstream and downstream firm.

be impossible to write complete contracts (i.e. contracts directly on investment levels), the contracts that can be written on assets are often sufficient to achieve first-best levels of investment In many settings, including the main ones studied in the property rights theory of the firm, the contracts that are required are not particularly complex. For example, all that may be needed is that the initial contract specifies that after investment one party is given the option to buy (or depending on the bargaining structure, sell) the productive assets at a predetermined price. Such results are in a sense confirmation that property rights matter, albeit that the optimal allocation is more complex than has so far been examined in the literature.[5] The problem is that the optimal form of ownership that is identified does not seem to correspond with what appears to be observed in practice. As Maskin and Tirole(1998) say of their work: "*Our aim is ...to offer the cautionary tale that straight ownership is not explained by the current incomplete contracting methodology. ......We conclude that the pervasiveness of straight ownership contracts more likely has to do with either robustness or bounded rationality considerations.*"

This paper offers a different reason why simple ownership rights matter and cannot be supplanted by option contracts or anything more elaborate. That is, we have a model where in equilibrium, the identity of eventual trading partners is *unknown* when investments are made, even though agents can choose to invest after matching and use Maskin-Tirole contracts. We show that even in this equilibrium, where simple ownership contracts cannot be improved upon, many of the GHM results extend.

The rest of the paper is arranged as follows. Section 2 develops the basic model where only one asset is required for production. Section 3 establishes some general properties of equilibrium. Section 4 deals with the case of heterogenous agents, and Section 5 extends the model to the case of complementary assets. Sections 6 and 7 deal with two extensions, namely (non-trivial) endogenous timing and relationship-specific investments. Some simple welfare properties of equilibrium are noted in Section 8 and Section 9 concludes.

2. The Model

2.1. Preliminaries

The economy evolves over two time periods, and is populated by a large number of agents. We suppose that agents may differ in their productivity. Specifically, agents are of types $i = A, B$, and the proportion of type i in the total population is θ_i. We will give more detail on how types differ in productivity below. Agents do not discount future payoffs.

Agents can engage in productive activity, either alone or in partnership with other agents, if they have invested in human capital and purchased a physical asset.

[5] With double-sided moral hazard a first-best solution requires that a third agent be introduced, which may be problematical (see Hart and Moore (1998)). In the absence of such an agent, the Maskin-Tirole scheme may still outperform simple ownership structures but is equivalent in its consequences to random ownershp.

Agents may invest and buy assets either *ex ante* (before matching with a partner) or *ex post* (after matching with a partner).

Agents are born without assets, but there is a capital market where agents can purchase an asset[6] at price p in either period. We assume that there is a time lag of one period in producing the asset. Assets have a scrap value of zero. Investment at level e costs e, and the level of investment is bounded above i.e. $0 \leq e \leq \bar{e}$. Investment also takes one period before it is productive.

If a type i agent has purchased one asset and invested at level e, she can produce individually revenue per period of $h_i(e)$; we refer to this activity as *home production* and $h_i(.)$ as the home production function. Two agents of types i, j with investment levels e, e' and at least one asset can produce revenue per period of

$$g_i(e) + g_j(e')$$

We refer to this joint production activity as *team production*, and $g_i(.)$ as the team production function. Additional assets in excess of one do not add to home or team revenue.

Our assumptions on individual and team production functions are as follows. We assume that g_i, h_i are concave and increasing in their arguments and that $\lim_{e\to 0} g_i'(e) = \lim_{e\to 0} h_i'(e) = +\infty$. Moreover, we assume that teams are more productive than individuals, both on average and at the margin;

$$g_i(e) > h_i(e),\ g_i'(e) > h_i'(e) \tag{1}$$

2.2. Timing

All agents are born at the beginning of period 0 with no training and no asset. Within period 0, the order of events is as follows.

1. At the beginning of the period, agents decide whether to buy an asset or not. We allow randomization, so an agent of type i chooses $\rho_i \in [0, 1]$, where ρ_i is the probability that an asset is purchased.

2. Conditional on the asset purchase decision, an agent of type i chooses investment (i.e. a type i agent with $k = 0, 1$ assets invests $e_k^i \in [0, \bar{e}]$).

3. At the end of period 0, agents enter a matching process, described in more detail below. In outline, every agent can specify whether they wish to meet an asset owner[7] (an owner in what follows), or a non-owner (worker), and a "co-ordinator" attempts to reconcile these requests efficiently.

4. Once having been matched, a pair of agents can decide whether to accept or reject the match. If both accept, they bargain over the revenue from team production. If one or more rejects the match, each engages in home production (if he can).

An agent may arrive at the beginning of period 1 either matched or not, and having invested and made an asset purchase or not. If she has not invested and

[6] As noted above, the team and home production technologies do not require more than one asset, so in equilibrium, any agent will own either no or one asset.

[7] That is, someone who has ordered an asset, and will have it delivered at the beginning of period 1. So, we assume that asset orders are observable and verifiable.

purchased an asset, she may choose to order an asset and invest at the beginning of period 1 also. However, due to the one-period delay in investment and asset purchase, no production will be possible if either investment and asset purchase is done *ex post* i.e. after matching. Consequently, without loss of generality, we can assume that all agents make their investment and asset purchase decisions before matching (*ex ante*) at the beginning of period 0. So, all that happens in period 1 is production and consumption[8].

We now describe the model backwards, starting with the bargaining stage.

2.3. Bargaining

Once two agents are matched, they bargain over the division of revenue from production, knowing that there is no opportunity to find a replacement partner. The bargaining protocol is that the proposer is decided by the flip of a fair coin, and if the responder rejects the first offer, the game ends and each agent engages in home production (if feasible).

So, the expected payoff to two agents of type i and type j who are paired together is given by the usual "split-the-difference" formula where each bargainer receives his disagreement payoff plus half the surplus from the match. Agent i's disagreement payoff is of course, the revenue from home production which is $mh_i(e^i_m)$ where m is the number of assets owned by i. So denote by $u_{i,j}(m,n)$ the payoff of a type i agent with m assets matched with a type j agent with n assets. Then the "split-the-difference" formula is may be written

$$u_{i,j}(m,n) = \begin{cases} mh_i(e^i_m) + 0.5(g_i(e^i_m) + g_j(e^j_n) - mh_i(e^i_m) - nh_j(e^j_n)) & \text{if } m+n>0 \\ 0 & \text{otherwise} \end{cases} \tag{2}$$

In what follows, we refer to home production as the *outside option* and $mh_i(e^i_m)$ as the *outside option payoff* for a type i. This is correct terminology in that home production is an alternative activity that can only be engaged in if team production does not take place. Note however, that due to our simple bargaining protocol, the outside option does not affect payoffs in (2) according to the outside option principle (Osborne and Rubinstein(1990), Muthoo(1998)); rather, the outside option payoff appears as a disagreement payoff, or "inside option" payoff, as in GHM. If the bargaining were repeated, rather than an ultimatum game, then the outside option principle would apply. The formulation adopted here allows us to focus on how timing issues affect the GHM results.[9]

Note finally for future reference that an owner of either type both loses and gains from being matched with an owner relative to a worker i.e.

$$u_{i,j}(1,1) - u_{i,j}(1,0) = [g_j(e^j_1) - g_j(e^j_0)] - h_j(e^j_1) \tag{3}$$

[8] In Section 6, we extend the number of periods to $T+1$, $T>1$. Then, the choice between ex ante and ex post asset purchase and investment becomes a non-trivial one.

[9] Introducing alternating-offers bargaining and interpreting home production as an outside option merges the present model with de Meza and Lockwood (1998). Rent transfer remains as an additional motive for ownership but now owners tend to invest less than workers and other things equal, owners are those with the least impact impact on team production.

The loss $-h_j(e_1^j)$ is due to *rent-shifting* i.e. the asset increases the disagreement payoff of the type j, and enables him to capture more of the surplus. The gain is increased team productivity (from increased investment), assuming[10] that $e_1^j \geq e_0^j$. Note also that a worker *always* prefers to match with an owner i.e. $u_{i,j}(0,1) > u_{i,j}(0,0) = 0$.

2.4. Matching

We model matching in a very simple but quite general way[11]. There is a market "co-ordinator" who brings together owners and workers. For simplicity, we assume that the market co-ordinator knows the structure of the model, and therefore the matching preferences of the agents. In particular, it follows from the above discussion that;

1. Any worker is indifferent between a WW match and no match (as two matched workers never produce), and strictly prefers an OW match to either.

2. Any owner strictly prefers either a OW match or a OO match to no match.

We will also *assume:*

3. Any owner strictly prefers a OW to a OO match.

This is a natural assumption, in the sense that it *always* holds when agents' investments are fixed (recall (3)). From (3), a more general sufficient condition for 3. is the following assumption;

A1. $g_j(0) - g_j(\overline{e}) < h_j(0)$

i.e. the team revenue increment induced by ownership is always less that the home revenue of an owner, whatever the investments.

The market coordinator chooses an Pareto-efficient matching rule. The definition is the usual one: a rule is said to be *efficient* if there is no other rule that makes all agents at least as well off and no single agent strictly better off[12]. In fact, it is clear from 1.-3. above that any efficient rule is of the following form. First, if there are more owners than workers, the coordinator matches all workers with a owner, and matches the remaining owners with each other. Second, if there are more workers than owners, the coordinator matches all owners with a worker. The efficient rule is not unique as it does not matter for efficiency whether WW matches are formed or not. Without loss of generality, we assume that the coordinator never forms WW matches.

Let $\rho = \sum_{i=A,B} \rho_i \theta_i$ be the fraction in the aggregate of agents who buy assets. Then given this matching rule, the probabilities of an OW match for owners and

[10] It is shown below that $e_1^j \geq e_0^j$ in equilibrium.

[11] This is in contrast to much of the literature on labour market matching (e.g. Pissarides(1990)), which assumes "two-urn matching" i.e. that the matching process matches owners with workers only, and vice versa. There, the implicit assumption in that literature is that production is only possible with an owner and a worker.

[12] It is now easy to see that two-urn matching (where only OW matches are formed) is not always efficient. For if there are more owners than workers, two-urn matching leaves some owners unmatched, whereas it is clear from 2. above that the remaining owners would prefer to be matched with each other.

workers respectively are:

$$\pi_1(\rho) = \min\{\frac{1-\rho}{\rho}, 1\}, \ \pi_0(\rho) = \min\{\frac{\rho}{1-\rho}, 1\} \tag{4}$$

Note that we indicate owners and workers by subscripts "1" and "0", denoting how many assets they own. Finally, when $\rho > 0.5$, the probability of an OO match is $1 - \pi_1(\rho)$.

Note that once matched, every agent will agree to produce with her match, as the alternative is to (at best) to engage in home production, which from (1) above, always yields each agent less than team production, given how the surplus is divided at the bargaining stage.

2.5. Asset Purchase and Investment

First, define θ_0^i, θ_1^i, to be the conditional probabilities that a worker or owner selected at random is type i. Note that from Bayes' rule, these are

$$\theta_0^i = \frac{(1-\rho_i)\theta_i}{1-\rho}, \ \theta_1^i = \frac{\rho_i\theta_i}{\rho}, \ i = A, B \tag{5}$$

Also, define

$$\overline{g}_0 = \sum_{j=A,B} \theta_0^j g_j(e_0^j), \ \overline{g}_1 = \sum_{j=A,B} \theta_1^j g_j(e_1^j), \ \overline{h}_1 = \sum_{j=A,B} \theta_1^j h_j(e_1^j) \tag{6}$$

So, $\overline{g}_0$ (resp. $\overline{g}_1$) is the average revenue generated by workers (resp. owners) in team production, and $\overline{h}_1$ is the average revenue generated by owners in home production.

From (6),(2), if an agent of type i decides to own, her expected payoff, net of the cost of investment (but not the price of the asset) is

$$\begin{aligned} v_i(1) &= \pi_1 \sum_{j=A,B} \theta_0^j u_{i,j}(1,0) + (1-\pi_1) \sum_{j=a,b} \theta_1^j u_{i,j}(1,1) - e_1^i \\ &= 0.5\left(g_i(e_1^i)) + h_i(e_1^i)\right) + 0.5\pi_1\overline{g}_0 + 0.5(1-\pi_1)(\overline{g}_1 - \overline{h}_1) - e_1^i \end{aligned} \tag{7}$$

The explanation of (7) is as follows. With probability π_1, the owner is matched with a worker, and with complementary probability, the owner is matched with an owner. So, from (7), the first equilibrium condition is that an owner chooses investment e_1^i to maximise $v_i(1)$ i.e. that

$$0.5g_i'(e_1^i) + 0.5h_i'(e_1^i) = 1 \tag{8}$$

In the same way, from (6),(2), if an agent of type i decides to not to own, her expected payoff, net of the cost of investment (but not including the price of the asset) is

$$\begin{aligned} v_i(0) &= \pi_0 \sum_{j=A,B} \theta_1^j u_i(0,j) - e_0^i \\ &= 0.5\pi_0\left[g_i(e_0^i)) + \overline{g}_1 - \overline{h}_1\right] - e_0^i \end{aligned} \tag{9}$$

Again, the explanation of (9) is as follows. With probability π_0, the worker is matched with an owner, and with complementary probability, the worker is unmatched, and gets zero payoff. So, from (9), the second equilibrium condition is that an owner chooses investment e_0^i to maximise $v_i(0)$ i.e. that

$$0.5\pi_0 g_i(e_1^j)) = 1 \tag{10}$$

Now, let $\Delta_i = v_i(1) - v_i(0)$ be the gross gain to a type i agent from buying an asset. The asset ownership equilibrium conditions is that an agent buys an asset only if $\Delta_i \geq p$, and will randomize iff $\Delta_i = p$ i.e.

$$\Delta_i \begin{array}{c} > \\ = \\ < \end{array} p \Longrightarrow \rho_i = \left\{ \begin{array}{l} 1 \\ 0 \leq x \leq 1 \\ 0 \end{array} \right. \tag{11}$$

2.6. Equilibrium

We can now formally define an equilibrium. An *equilibrium with ex ante investment and asset purchase* is a vector $(\hat{e}_1^i, \hat{e}_0^i, \hat{\rho}^i)_{i=A,B}$ such that (8),(10),(11) are all satisfied. Note that we qualify the name of the equilibrium in this way because we allow agents to choose these variables *ex ante* or *ex post*. At the moment, for simplicity, the model is currently "rigged" so that *ex post* investment and asset purchase is always inferior to *ex ante*. By introducing additional periods of production into the model, we can make this "timing" decision non-trivial. This issue is addressed in Section 6, where we show that under some conditions, an *ex ante* equilibrium still prevails.

Some general properties of equilibrium, including existence, are discussed in the next section. However, as our focus in this paper is on the determinants of equilibrium asset purchase, it is helpful to conclude by studying the crucial asset ownership condition (11) in more detail. To bring out the issues clearly, we assume $g_i' = h_i' = 0$ in which case all agents choose the minimum investment $e = 0$, and we suppose that there is only one type of agent, so $g_i = g$, $h_i = h$. Then, simplification of (7), (9) yields

$$\Delta_i = (1 - \pi_0)g + 0.5(\pi_1 + \pi_0)h$$

The first term, $(1 - \pi_0)g$, measures the *search incentive* for asset ownership. If an agent owns an asset, he engages in team production with probability one, whereas if an agent does not own an asset, he engages in team production only with probability π_0, the probability he is matched with an owner.

The second term $0.5(\pi_1 + \pi_0)h$ is the *rent-shifting incentive* for asset ownership. With probability π_1, an owner is matched with a worker, in which case he captures $0.5h$ units of revenue ("rent") by having a disagreement payoff of h rather than zero. Conversely, with probability π_1, a worker is matched with an owner, in which case he *loses* $0.5h$ units of rent by having a disagreement payoff of zero rather than h.

Neither of these asset ownership incentives are present[13] in GHM, and they

[13] In GHM, rent shifting is present, but it has no implications for ownership, as ownership is contractible. To put it another way, the mechanism outlined above will simply raise the price a non-owner is willing to pay for the asset.

both play a crucial role below in generating results about asset ownership that differ significantly from GHM.

3. Some General Properties of Equilibrium

Before looking at asset ownership in detail, we begin with some useful results[14] which hold quite generally:

Proposition 1. *An equilibrium with ex ante asset purchase and investment always exists. In any equilibrium, owners of type i always invest at least as much as workers ($\hat{e}_1^i \geq \hat{e}_0^i$), and strictly more unless $\hat{p} > 0.5$ and $h_i'(e) = 0$. Worker investment $\hat{e}_0^i$ is increasing in the proportion of agents who own.*

The results on investment follow directly from inspection of (8) and (10). Both the result and the mechanism at work are the same as in GHM. Possession of the asset allows the owner to produce even if agents cannot agree on team production, and so (as long as $h_i' > 0$), the owner has an additional incentive to invest.

Note, however, a new feature of our model relative to GHM. Suppose that $h_i' = 0$. Then, in the GHM setting, asset ownership has no effect on investment. Here, by contrast, owners will still invest more than workers if $\rho < 0.5$. This is a consequence of *search effect on investment*, namely that from (10) that workers will invest more when owners are abundant (ρ is high).

Two more observations are worth making at this point. First, the result on owners investing more does, however, rely on the feature our the model that owners have a higher probability of team production than workers. This is ultimately due to the fact that the matching process is one-shot. In an earlier version of this paper(de Meza and Lockwood(1998)), we allow matching to be repeated over time in an infinite horizon setting, in a model where there is only one agent type. Then, it may be optimal for two matched owners to reject their match, and wait for a match with a worker (recall from Section 2.4 that owners may prefer to match with workers). In this case, the owner enters into team production with probability only π_1, and so (ignoring the complications that arise in a dynamic model), the first-order condition determining $\hat{e}_1$ is

$$(1 - 0.5\pi_1)h'(\hat{e}_1) + 0.5\pi_1 g'(\hat{e}_1) = 1$$

Then if $h' = 0$, and $\hat{\rho} > 0.5$, $\pi_1 < \pi_0$, and so owners will invest less than workers.

The second observation is that without some restrictions on the parameters (notably the price of the asset), we cannot rule out an equilibrium where no-one buys and asset or everyone does. Such equilibria are of little interest to us as the results of the property rights literature mainly concern allocation of assets *between* agents. We say that an equilibrium is *mixed* if there are always owners and workers in equilibrium ($\hat{\rho}_i = 1 \Longrightarrow \hat{\rho}_j < 1$ and $\hat{\rho}_i = 0 \Longrightarrow \hat{\rho}_j > 0$), and non-mixed otherwise.

[14] All Propositions are proved in the Appendix, where proof is required.

In general, conditions for an equilibrium to be mixed are cumbersome. However, if we assume only one type of agent, it is easy to establish such conditions. Such conditions will also be useful for results in later sections. Define the prices

$$\begin{aligned} \overline{p} &= max_e\{0.5g(e)) + 0.5h(e) - e\} + 0.5g(0) & (12) \\ \underline{p} &= max_e\{0.5g(e)) + 0.5h(e) - e\} - max_e\{0.5g(e) - e\} & (13) \end{aligned}$$

and note that $\overline{p} > \underline{p} > 0$ from the assumptions made on the team production function (namely $g(e) \geq 0$, and g strictly concave with $lim_{e\to 0}g'(e) = \infty$). We then have the following result;

Proposition 2. *If there is only one type of agent, for any for any asset price $\hat{p} \in (\underline{p}, \overline{p})$ there exists a mixed equilibrium where a fraction $\hat{\rho} \in (0,1)$ of agents buy assets.*

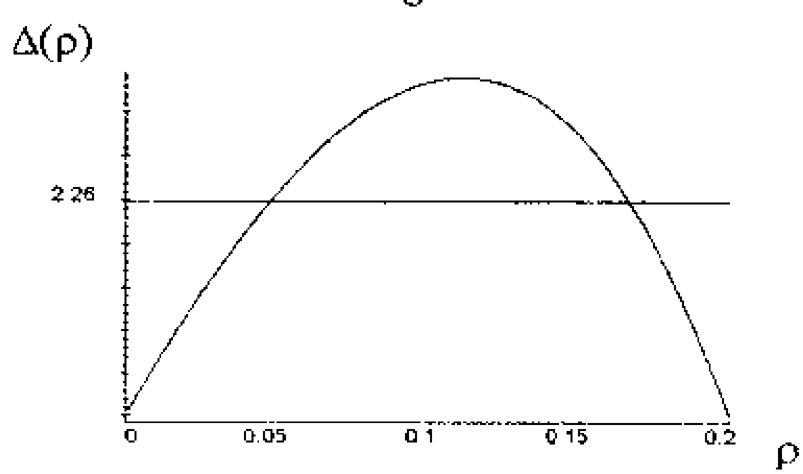
Proposition 2 asserts that as long as the asset price is not too high or too low, agents will diversify i.e. some will own assets, and some not. The upper and lower bounds on prices are the gains to asset purchase when nobody (everybody) buys the asset. As $\overline{p} > \underline{p}$, it is more profitable to buy the asset when nobody has it than when everybody has it. However, as the following example shows, it is *not* the case that the gain from asset purchase is monotonically decreasing in ρ, the proportion of agents that buy the asset, and this means that there may be multiple equilibria at some asset prices.

Example 1
Let $g(e) = 2e^{0.5}, h = 2\eta e^{0.5}$, $\eta < 1$. Then it is straightforward to calculate that $\hat{e}_1 = (0.5(1+\eta))^2$, $\hat{e}_0 = (0.5\pi_0)^2$. In the proof of Proposition 1, a general formula (A.1) for the gain to asset purchase is obtained. In the example, this reduces to

$$\begin{aligned} \Delta(\rho) &= 0.25(1+\eta)^2 - 0.25(\pi_0)^2 \\ &\quad +0.25\pi_1\pi_0 + 0.25(1 - \pi_1 - \pi_0)(1-\eta^2) \end{aligned}$$

Recalling that π_1, π_0 depend on ρ as in (4), we can plot $\Delta(\rho)$ as a function of ρ. For $0 \leq \rho \leq 0.2$, it turns out that Δ is non-monotonic. So, there exist prices for which there are multiple equilibria, as shown. □

Figure 1



Multiple Mixed Equilibria

Note that the "stable" equilibria are $\rho = 0$ and the largest solution of $\Delta(\rho) = p$. This latter equilibrium Pareto-dominates the others since agents are identical *ex-ante* and worker welfare is increasing in ρ.

Multiple equilibria are not unusual in search models, but they normally arise when increasing returns to the matching process are assumed. Here, there are constant returns to matching; if the number of agents in the model is doubled, it makes no difference to the match probabilities, and so this finding is of some interest.

As the diagram makes clear, multiple equilibria arise because (starting at a low value of ρ), the gain to asset ownership is increasing in ρ over a range. As the proportion of owners rises, workers enjoy a direct benefit through a higher matching probability implying that Δ should fall with ρ. However, there is an offsetting effect: a higher matching probability induces workers to invest more, and consequently, the productivity of workers rises. As the extra revenue is shared equally but workers bear the effort cost, this second effect may dominate and owners benefit more than workers from a higher ρ.

4. Heterogenous Agents

As observed in the introduction, the GHM theory makes both general and specific predictions about who should own which assets. The most general observation here is that in GHM, the optimal ownership structure depends only on the *marginal* productivities of the agents[15].

More specific predictions about ownership are easy to make in some special cases. The one we focus on here is where there are two types of agent (consistently with the model laid out in Section 2), and where only one type has a positive marginal product of investment in team *and* home production. Call such an agent[16] a *key* agent. According to GHM, it is unambiguous that only such key agents should (and do) own the asset[17]. To see this, note that the non-key agent *either* has zero marginal product of investment in team production, in which case it may be possible to induce him to invest by giving him the asset, but this does not enhance team revenue; *or* has zero marginal product of investment in home production, in which

[15] A second quite general kind of result is about *indeterminacy* of ownership. Specifically, that GHM have *no* predictions about asset ownership in the cases (i) where asset ownership does not impact on the marginal productivity of investment in the outside option for any agent (in our model, $h_i' = 0$, $i = A, B$), or (ii) where asset ownership does not impact on the marginal productivity of investment in team production n for any agent (in our model, $g_i' = 0$, $i = A, B$).

[16] A key agent may also be one who responds particularly strongly to marginal incentives because her marginal investment cost is relatively flat. We do not focus on this case here, as we assume that all agents have the same investment cost of unity. It can be demonstrated though that if marginal investment cost is smoothly increasing for one agent, and totally inelastic for the other, it will be the agent with *no* discretion over investment that owns if its investment is higher than the elastic agent's. The GHM model implies the opposite allocation.

[17] Hart(1995) says "a party is more likely to own an asset if he or she has an important investment decision (where the investment decision might represent figuring out how to make the asset more productive or looking after the asset."

case may be desirable to induce him to invest more by giving him the asset but this is impossible.

In this section, we show that the *average* productivities of agents also help determine ownership through rent-shifting and search incentives, and this may lead to conflict with GHM results. Specifically, we show that key agents may not own, when non-key agents have sufficiently stronger rent-shifting and/or search incentives than do key agents.

It is convenient (and involves little loss of generality) to model agent heterogeneity as follows. The two types of agents, $i = A, B$ have home and team production functions;

$$h_i(e) = h_i + \eta_i \kappa(e),\ g_i(e) = g_i + \gamma_i \kappa(e) \tag{14}$$

where $\kappa(.)$ is some strictly increasing, strictly concave function with $\kappa(0) = 0$. With this formulation, we can vary the average and marginal products of agents separately. We can also define key agents very easily: a type A agent is key in the above sense if

$$\gamma_A, \eta_A > 0,\ \gamma_B = 0 \text{ or } \eta_B = 0 \tag{15}$$

We assume in what follows, without loss of generality, that the A-types are key agents. So, in the GHM setting, in an AB match, the A would always own the asset. However, in our model, there is also a strategic payoff to ownership, namely the *rent shifting* alluded to above. Specifically, asset ownership increases the value of the outside option, which in turn increases the share of the surplus that the owner can get *ex post* through bargaining. So, the strength of the rent-shifting incentive is measured by $h_i(e)$, or for η_i small, by h_i. It is easy to show that if a type B's rent-shifting incentive is high enough relative to a type A's, then Bs, and not As, will own assets in equilibrium. Specifically:

Proposition 3. *Suppose that $h_B > h_A$ i.e. type Bs have a larger rent shifting incentive than type As. Then, it is possible to find $(\gamma_i, \eta_i)_{i=A,B}$ and an open interval $(\underline{p}, \overline{p})$ such that (i) type As are key agents i.e. (15) holds, and (ii) for any $p \in (\underline{p}, \overline{p})$, there is a unique equilibrium in which only type Bs own assets, and only type As do not $(\rho_B = 1, \rho_A = 0)$.*

This Proposition[18] asserts that the rent-shifting incentive for ownership can dominate the GHM incentive, so that equilibrium ownership may be inefficient.

We now close down the rent-transfer incentive for asset ownership, by setting $h_B = h_A = 0$. In this case,does our model yield the same predictions about ownership as GHM? In fact, the answer is no, as there is a second new incentive for asset ownership in our setting, the search incentive referred to in the Introduction.

Specifically, suppose that owners are scarce in the sense that here is a probability that a worker will not match with an owner and therefore cannot produce ($\pi_0 < 1$). Then by buying an asset, a type i agent can boost her payoff by half the expected

[18] Note also that this Proposition holds when (i) is replaced with $\gamma_i = \eta_i = 0$, $i = A, B$, so that we have the result that equilibrium asset ownership may be determinate even when it is indeterminate in GHM -recall footnote 16.

gain from asset purchase i.e. $0.5(1-\pi_0)g_i$. So, if $g_B > g_A$, the gain to type B agents from buying an asset is greater than that of a type A. This effect may dominate the incentive that key agents have to buy assets, as the following result shows. Define $\pi_0 = \theta_B/(1-\theta_B) < 1$. Then, we have the following result.

Proposition 4. *Suppose that type Bs are somewhat more productive than type As when no investment is made ($g_A/\pi_0 > g_B > g_A$), and that they are also more scarce (i.e. $\implies \theta_B < 0.5$). Then, it is possible to find $(\gamma_i, \eta_i)_{i=A,B}$ and an open interval $(\underline{p}, \overline{p})$ such that (i) type As are key agents i.e. (15) holds, and (ii) for any $p \in (\underline{p}, \overline{p})$, there is a unique equilibrium in which only type Bs own assets, and only type As do not ($\rho_B = 1, \rho_A = 0$).*

The requirement that type Bs be scarce, plus the upper bound g_A/π_0 on g_B rules out an equilibrium where owners are in a majority, in which case the gain to ownership is the same for both types.

So, we have seen that either the rent-shifting or search incentive to own assets may dominate the efficiency gain from having key agents own. However, it is clear from the above discussion that this arises only when one type (type A) has (weakly) higher *marginal* productivity in both team and home production but strictly lower *average* productivity in either team or home production.

If we impose the condition that agents that have the higher marginal productivity of investment in both team and individual production also have the higher average productivity i.e. that average and marginal productivity move together, then this is sufficient for key agents to own, even in our setting.

Proposition 5. *Assume that $g_A \geq g_B, h_A \geq h_B$, $\gamma_A \geq \gamma_B, \eta_A \geq \eta_B$ with at least one strict inequality. Then in any equilibrium, some type A agents must own assets, and if any type B agents own an asset, all type A agents must own also.*

It is immediate that under the conditions stated, key agents must own. For if As are key, one of $\gamma_A \geq \gamma_B, \eta_A \geq \eta_B$ must hold strictly.

5. Complementary Assets

One of the main contributions of GHM was to provide a set of specific predictions about when asset ownership would be dispersed across agents, or concentrated in a few (or one) hands. Specifically, the setting of Grossman-Hart(1986), and Hart(1995) is one where there are two agents, each of whom initially owns an asset. There are then three ownership structures; *non-integration*, where the two agents continue to own their assets, and *type i integration*, where agent $i = 1,2$ owns both assets.

In Hart(1995), the concepts of independent and complementary assets are introduced. A pair of assets is said to be *independent* if acquisition of a second asset does not increase the marginal return to investment to either agent, in the event that the agents decide not to produce together. First, a pair of assets is said to be *complementary* if acquisition of only one asset by an agent does not increase the marginal return to investment to that agent, in the event that the agents decide not to produce together. Key results in the GHM setting are: (i) if assets are independent, then integration is *never* an equilibrium outcome; (ii) if assets are complementary, then integration is *always* an equilibrium outcome.

The assumptions we have made so far correspond to the case of independent assets, as the payoff to home production, $h_i(e)$, is not affected by the acquisition of a second asset. It is also clear that our analysis so far confirms the GHM result (i) above: in our setting, when assets are independent, no agent will buy more than one asset in equilibrium.

In this section, we show that by contrast, result (ii) for complementary assets does *not* extend to our setting. We adapt our model to allow for complementary assets as follows. Our definition of complementarity is that two assets are complements if neither individual nor team production is possible unless two assets are available. This is a very strong form of complementarity which certainly implies complementarity in the GHM sense.

At the asset purchase stage, every agent randomizes over the choice of zero, one or two assets. The investment decision is as before. At the matching stage, agents with zero assets strictly prefer to be matched with two-asset agents (relative to being unmatched), agents with one asset strictly prefer to be matched with one- or two-asset agents(relative to being unmatched), and, assuming A1 holds, agents with two assets strictly prefer to be matched with agents with less than two assets than with another two-asset agent. So, the match coordinator then chooses the efficient rule given these preferences.

Throughout, we assume for clarity that all agents are of one type (i.e. $g_i(e) = g(e), h_i(e) = h(e)$), and revenue from home production is independent of e i.e. $h(e) = h(0) = h$, although these assumptions could easily be relaxed at the cost of some notational burden[19].

Even though our definition of complementarity is stronger than GHM's, we can construct an equilibrium where, even though assets are complementary in the GHM sense, no agent buys two assets.

Proposition 6. *Assume that assets are complements. Then, if $h \leq p \leq g(\hat{e}) - \hat{e}$ where $0.5g'(\hat{e}) = 1$, there exists an equilibrium where every agent buys exactly one asset (a non-integration equilibrium). In this equilibrium, every agent invests $\hat{e}$.*

The intuition is that if all other agents are following the equilibrium strategy, any agent is equally likely to find a match whether he has one or two assets (given the above matching preferences, all two-asset agents are paired with one-asset agents, and the remaining one-asset agents are paired with each other). With no search advantage to owning a second asset, only the rent-shifting advantage remains. The latter is measured by h, and consequently, if h is small relative to the price of the asset p (specifically, $h \leq p$), this gain does not compensate for the cost of purchasing an additional asset.

[19] If $h' > 0$, Proposition 6 generalises to the case where the lower bound on p is $max_e\{0.5(g(e) + h(e)) - e\} - max_e\{0.5g(e) - e\}$.

This result can be contrasted with GHM, where (as remarked above) if assets are complementary, one agent in a team will always own both assets. The reason for the difference is that in GHM, asset ownership is contractible, so the agent that buys both assets is always compensated by his partner. In our setting, by contrast, lack of coordination means that if an agent acquires a second asset, there is no compensating cancelled purchase by the other partner. So, acquisition of a second asset only pays if there is some other advantage, i.e. rent shifting.

We have established that for a range of asset prices, there is an equilibrium where assets are owned separately, even though they are strictly complementary. We might ask whether this is the only equilibrium. The answer is no, as the following result indicates.

Proposition 7. *If $\dot{e} < g(0) - h(0)$, then whenever there is an equilibrium in which strictly complementary assets are owned separately, there will also be an equilibrium in which a proportion $\hat{\rho} \in (0,1)$ own both assets, and a proportion $1 - \hat{\rho}$ own none (an integration equilibrium). However, all agents in this equilibrium are strictly worse off than the non-integration equilibrium.*

This second result indicates the possibility of integration a la GHM as an equilibrium outcome. However, differs from GHM because this equilibrium is less efficient than the equilibrium with one-asset ownership. The intuition for this is as follows. First, the agent who buys two assets in the integration equilibrium is *ceteris paribus*, worse off by the price of an asset than all agents in the non-integration equilibrium (who buy one asset each). On the other hand, due to rent-shifting, the owner in the integration equilibrium is better off by at most $0.5h$ due to rent-shifting. But if these equilibria coexist, $p \geq h > 0.5h$. So, overall, the owner is worse off.

6. Endogenous Timing

We now extend the model to allow for non-trivial timing of asset purchase as follows. First, we add additional periods of production to the model; we assume $T+1$ periods, with $T > 1$, where matching takes place at the end of period 0, as before (so far we have assumed $T = 1$). This gives agents the option of delaying either asset purchase or investment to after matching (*ex post*) and still producing for $T - 1$ periods, as compared with T periods if they purchase and invest *ex ante*. So, the decision to invest and/or purchase assets *ex ante* or *ex post* is now non-trivial.

To simplify the analysis, we suppose that investment is fixed at some level. This can be justified in the model e.g. by assuming that the cost of investment is zero, so all agents invest at the maximum level possible i.e. $e = \overline{e}$. We will also assume that agents are homogenous. Then every agent generates revenue $g = g(\overline{e})$ in team production, and $h = h(\overline{e})$ in home production.

As investment is fixed, every agent has only one timing decision, whether to buy an asset *ex ante* or *ex post* i.e. before or after matching. But to buy an asset *ex ante* is to be an owner. If an agent does not buy an asset *ex ante* (and is a worker), there are just two possibilities. One is that he is matched with an owner. In this case, he has an incentive to buy an asset *ex post* simply to shift rent to himself; by buying an asset *ex post*, he can generate a disagreement payoff of $(T-1)h$ (home production in $T-1$ periods) and thus capture $0.5(T-1)h$ of the rent. So, this will not be optimal if $p > 0.5(T-1)h$, which we assume in what follows[20].

The second possibility is that he is matched with a worker, in which case two workers will jointly purchase an asset ex post and produce iff the revenue from $T-1$ periods of production exceeds the purchase price $(T-1)2g \geq p$.

We call an equilibrium a *pure ex ante equilibrium* if all assets are purchased *ex ante*. If some assets are purchased *ex post* and some *ex ante*, we call such an equilibrium a *partial ex ante equilibrium*. If all assets are purchased ex post, we call such an equilibrium a *pure ex post equilibrium*. It is part of the definition of this last equilibrium that $(T-1)2g > p$ i.e. positive net revenue is generated in equilibrium.

Before discussing the existence of these equilibria, it is important to discuss how the bargaining and matching stages are modified in the T-period version of this model.

First, the "split-the-difference" formula remains valid, except that if two matched agents have at least one asset between them, they enjoy T periods of production, and if two agents with no assets are matched, they will produce in the last $T-1$ periods as long as $(T-1)2g > p$. So, the formula changes as follows. If two agents with m and n assets are matched at the end of period 0, payoffs, not including the cost of asset purchase, are

$$u(m,n) = \begin{cases} T(mh + 0.5(2g - mh - nh)) & \text{if } m+n > 0 \\ 0.5(T-1)g & \text{if } m,n = 0 \text{ and } (T-1)2g > p \end{cases} \tag{16}$$

and $u(m,n) = 0$ otherwise. We have also simplified (2) using our assumptions of identical agents and constant investment.

As for matching, worker and owner rankings of matches remain the same as before[21], except that now workers *strictly* prefer WW matches to no matches when $(T-1)2g > p$. So, we assume that when $(T-1)2g > p$ the coordinator forms WW matches when there are more workers than owners, and otherwise the matching rule is as before.

We can now establish the following results about equilibrium.

Proposition 8. *There exists a pure ex ante equilibrium iff $p \leq Th$, or $Tg + 0.5Th > p \geq 2(T-1)g$. There exists a partial ex ante equilibrium iff $Th < p \leq \min\{2g+Th, 2(T-1)g\} = \tilde{p}$. A pure ex post equilibrium exists iff $(T-1)2g > p \geq 2g + Th$. Consequently, pure and partial ex ante equilibria exist for all T, but pure ex post equilibria exist only for $T \geq 3$.*

The result is illustrated in Figure 2 below.

[20] This is a peculiar incentive for asset ownership as it leads to wasteful duplication of assets.

[21] Note that workers always prefer OW matches to WW matches if $Tg - 0.5Th > (T-1)g - 0.5p$. But this inequality implies $p > 0.5(T-1)h$, which holds by assumption.

Figure 2 in here

Some intuition for these results is as follows. When $p \leq Th$, the asset price is so low that more than half the agents buy an asset, ensuring that a worker is always matched with an owner and so workers never need to buy assets *ex post*. When p moves into the region $(Th, \tilde{p})$, less than half the agents buy an asset, implying that some workers are matched with workers; as $p < 2(T-1)g$, any such worker-worker pair will buy an asset *ex post*. As p rises into the region $[\tilde{p}, (T-1)2g]$, then - assuming this region non-empty - the price is too high for any agent to buy *ex ante*, knowing that he can match with another workers, and buy an asset *ex post*.

Finally, as p reaches the region $(2(T-1)g, Tg+0.5Th)$, the option of not buying and matching with another worker now yields only zero. So, even though the asset is now more expensive, agents are indifferent between buying or not. In this case, less than half the agents buy an asset, implying that some workers are still matched with workers; but now as $p > 2(T-1)g$, no such worker-worker pair will buy an asset *ex post*[22].

The intuition for non-existence of *ex post* equilibrium when $T = 2$ is simple: in such an equilibrium, the equilibrium payoff $g - 0.5p$, must be greater than the payoff from deviating by buying an asset and matching with a worker with probability one, namely $2g + h - p$. So, to ensure non-deviation, we require a very high price $p \geq 2(g+h) > 2g$; this price is now so high that worker-worker matches cannot generate strictly positive revenue.

So, in the simplest case, we are able to show that even with truly endogenous timing, an *ex ante* equilibrium will still exist for some parameter values. This will also be true (by continuity) if the marginal return to investment in home and team production is positive but small, even if agents use Maskin-Tirole contracts to motivate investment post matching. So, as claimed in the introduction, we can find equilibria with simple ownership contracts, even though Maskin-Tirole contracts are available o agents. The reason why agents do not make use of these contracts is that they have to wait to be matched before Maskin-Tirole contracts become feasible, and the resulting lost production outweighs any efficiency gain from using such contracts.

7. Relationship-Specific Investment

By definition, investment in our model is not relationship-specific, as investment precedes matching. In the GHM model investment in *partially* relationship-specific; all their results depend on investment being productive even if the relationship breaks up, but marginal returns are higher still when the team stays together[23]. Continuity arguments imply that if we made investments slightly relationship-specific (e.g. by making them slightly more productive if they were made ex-post), our main results would carry over.

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[22] When $p \geq Tg + 0.5Th$, nobody buys an asset, so we have an non-interior equilibrium.

[23] Felli and Harris (1996) make a similar assumption but in the context of a rather different model in which investment is not a choice variable.

Nevertheless, it is of some interest to consider the other polar case, where investments which are entirely relationship specific i.e. must be made subsequent to matching (investment is now interpreted as the intensity with which unverifiable effort is applied to team activities).

The simplest way to capture relationship-specific investment in our model is to retain the assumption of a one period delivery lag on asset purchase but to suppose that investment is now only possible post matching and is then instantaneously effective. So, timing is now as follows. As before, assets are ordered at the beginning of the first period, and then agents match. Once matched, the two agents may then choose to transfer the ownership[24] of the asset from the initial owner to the other. Next, following any transfer of ownership, the non-contractible investments are chosen. Finally, the division of revenue is negotiated prior to production taking place.

We will assume that there are no transaction costs[25] associated with the transfer of the title to the asset between matched agents.

It is important to specify the way in which the initial owner is compensated for the loss of the asset, should asset transfer take place. Consistently with our assumptions about bargaining over revenue from team production, we assume that the two agents bargain over the gains from asset transfer as follows: each agent is selected at with equal probability to make a take-it-or-leave-it offer to the other. If the offer is rejected, no ownership transfer occurs and the game moves on to the investment stage. Consequently, asset transfer only takes place if it is efficient and the efficiency gains are shared equally between the two agents, so that final asset ownership will always be as predicted by GHM.

What is therefore of interest is whether the *initial* (unilateral) purchase decisions will ever be such as to require asset transfer. If this does occur then there is a sense in which ownership does not follow the GHM pattern. Our finding is that equilibria of this sort do arise.

To see how this works, note that if an agent for whom it is *not* efficient to own buys an asset *ex ante*, he can then appropriate half the net efficiency gain from asset transfer; we call this the *asset transfer incentive* for asset purchase. This creates a further reason why initial ownership is not as predicted by GHM.

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[24] For simplicity, we assume that only simple ownership contracts are possible (i.e. either one agent owns the asset, or the other does). This rules out more sophisticated contracts, such as random ownership, or Maskin-Tirole(1998) contracts, which would induce more efficient ex post investments, but allowing such contracts would not change the substance of our argument, while adding to analytical complexity.

[25] The other polar case is where the legal cost in transferring asset ownership is so high that it exceeds the efficiency gains of transfer (but not so high as to preclude initial purchase of the asset). Then of course no transfer occurs. Now the model is very similar to that already analysed. Consider the case in which workers are in the majority. In moving from ex-ante to ex-post investment, there would be no change in the level chosen by owners, but matched workers invest more since in the ex-post case it is sure that there will be a payoff. As extra revenue is equally shared but the workers bear all the investment cost, the returns to ownership are boosted by more than those to being a worker. So, other things equal, the proportion of the population choosing to own goes up. Which type of agent owns is not affected. For example, if agents are identical except that $h_A > h_B$, then in any equilibrium rent shifting ensures that As will always own.

As an illustration of how this incentive comes into play, consider the case where g_i, h_i are parameterized as in (14), and suppose that the output of type A agents in either team or home production is unaffected by investment, whereas that of type Bs does respond to investment, so Bs are unambiguously key agents. We now show that there is a range of asset prices where there is an equilibrium where only As own, even if As and Bs have the same rent-shifting incentive.

Proposition 9. *Assume that g_i, h_i are as in (14) and investments are relationship-specific. If $\eta_A = \gamma_A = 0, \eta_B, \gamma_B > 0$ and $\theta_A = 0.5$, in equilibrium all As own and no Bs do iff*

$$\left[g_B(e_1^B) - g_B(e_0^B)\right] + \left[h_A(0) - h_B(e_1^B)\right] > 0 \tag{17}$$

where e_0^B, e_1^B solve $0.5g'_B(e_1^B) + 0.5h'_B(e_1^B) = 1,\ 0.5g'_B(e_0^B) = 1$.

Note that condition (17) indicates that As now have two strategic incentives to purchase the asset. The first, which we have already encountered, is the rent-shifting incentive; $h_A(0) - h_B(e_1^B)$ measures the additional incentive, relative to Bs, that As have to shift rent. The second incentive which is specific to the case of *ex post* investment is the asset transfer incentive referred to above, and $g_B(e_1^B) - g_B(e_0^B)$ measures the size of this incentive for As (this incentive is by definition zero for Bs).

8. Welfare

Although the question is not directly addressed by GHM, it is evident that in their set up the equilibrium is constrained efficient. Here, government intervention can be strictly Pareto-improving, due to the presence of matching externalities. This is now demonstrated in the simplest context of identical agents.

First consider an equilibrium in which workers are in the majority so some are unemployed. Now introduce a subsidy to asset purchase paid for by a poll tax. More agents opt for ownership, the process continuing till unemployment falls by so much that expected payoffs in the two activities are restored to equality. In the new equilibrium owners are better off by the difference between the subsidy to the asset and the poll tax, which must be positive if any workers remain. Since workers must be as well off as owners, everyone gains.

A similar analysis applies when the price of the asset is so low that in equilibrium owners are in the majority. Consider a tax on asset purchase with the proceeds distributed as a poll subsidy. There will be fewer owners in the new equilibrium, but if the tax is not too great, the chance of an owner matching with a worker will remain sufficiently high for them still to be in the majority. Hence, workers benefit by the amount of the subsidy and owners, who are equally well off in equilibrium, must also gain.

Proposition 10. *When agents are identical ex ante and in equilibrium workers are in the majority (minority) a small subsidy (tax) on asset purchase paid for by means of a poll tax (subsidy) yields a strict Pareto gain.*

This result clearly indicates that equilibrium is inefficient. In the case where owners are in the minority, the source of the inefficiency is simple: when an additional owner buys an asset, he convey an external benefit (namely higher probability of matching) on the worker. In the case where owners are in the majority, when an additional agent buys an asset, the purchase imposes an external cost on other owners, namely a slightly lower probability of matching with a worker, rather than another owner (recall from Section 2.3 above, owners always strictly prefer to match with workers).

9. Conclusions

Much acquisition of physical and human capital takes place prior to the formation of a productive relationship. Employees typically join going concerns having already undertaken considerable investment in skill creation. To the extent this is the case, the GHM analysis is not directly applicable for it relies on what is in effect a cooperative assignment of ownership to create appropriate investment incentives. Nevertheless, the anticipation of noncooperative ownership decisions does influence investment incentives.This paper shows how filling in background assumptions concerning the market in teammates allows the GHM model to be extended to cover such cases. It remains true that ownership matters, but specific results are modified. Since ownership is not contractible, there are rent-shifting and search as well as incentive effects of asset purchase. There remains a tendency, but not a necessity, for the agents with the greatest influence on team performance to own.

Identifying the strategic and efficiency factors implicit in the GHM set up inducing agents to purchase assets and invest when they are still strangers also provides a possible reason why the conditional ownership schemes of Maskin and Tirole are not normally observed, so providing a justification for studying the incentive effects of simple property rights allocations.

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Appendix

Proof of Proposition 1

The result on investment levels is obvious from inspection of (8),(10) and the properties of g and h. To prove existence, note first, $\hat{e}_1^i$ is independent of $\hat{\rho}^i$, and $\hat{e}_0^i$ is a continuous function of ρ. So, we may write the gain to a type i as a continuous function of (ρ_A, ρ_B) i.e. $\Delta_i(\rho_A, \rho_B)$. Also, (11) is a u.h.c. convex-valued correspondence ϕ from $\Re$ to subsets of $[0,1]$. Define the composition $\tau_i = \phi \circ \Delta_i$ and $\tau = (\tau_A, \tau_B)$. Then τ is an u.h.c. convex-valued correspondence from $[0,1]^2$ to $[0,1]^2$ and so has a fixed point by Kakutani's theorem.

Proof of Proposition 2

From (7), (9) we may write

$$\begin{aligned} v(1) &= 0.5\,(g(\hat{e}_1)) + h(\hat{e}_1)) - e_1 + 0.5\pi_1 g(\hat{e}_0) + 0.5(1-\pi_1)(g(\hat{e}_1) - h(\hat{e}_1)) \\ v(0) &= 0.5\pi_0 g(\hat{e}_0) - \hat{e}_0 + 0.5\pi_0(g(\hat{e}_1) - h(\hat{e}_1)) \end{aligned}$$

So, using the first-order conditions for $\hat{e}_0, \hat{e}_1$, the gain to ownership may be written

$$\begin{aligned} \Delta(\rho) &= v(1) - v(0) \\ &= max_{e_1}\{0.5g(e_1)) + 0.5h(e_1) - e_1\} - max_{e_0}\{0.5\pi_0 g(e_0) - e_0\} + \\ &\quad 0.5\pi_1 g(\hat{e}_0) + 0.5(1-\pi_1-\pi_0)(g(\hat{e}_1) - h(\hat{e}_1)) \end{aligned} \tag{A.1}$$

Also, $\Delta(\rho)$ is continuous in ρ as π_0, π_1 are continuous in ρ. So, to prove the result, it is sufficient to show that $\Delta(0) > \Delta(1) > 0$. For then, at any $p \in (\Delta(1), \Delta(0))$, there will be at least one solution to $\Delta(\rho) = p$.

From (4), we have

$$\begin{aligned} \rho &= 0 \Longrightarrow \pi_1 = 1, \pi_0 = 0 \\ \rho &= 1 \Longrightarrow \pi_1 = 0, \pi_0 = 1 \end{aligned} \tag{A.2}$$

Consequently, $\rho = 0$ implies $e_0 = 0$, and so from (A.1),(A.2), we have

$$\Delta(0) = max_e\{0.5g(e)) + 0.5h(e) - e\} + 0.5g(0) \tag{A.3}$$
$$\Delta(1) = max_e\{0.5g(e)) + 0.5h(e) - e\} - max_e\{0.5g(e) - e\} \tag{A.4}$$

Now, by assumption of $\lim_{e\to 0} g'(e) = +\infty$, $\lim_{e\to 0} h'(e) = +\infty$, and the concavity of g, h, the right-hand side of (A.4) must be strictly positive i.e. $\Delta(1) > 0$. Moreover, we have

$$\Delta(0) - \Delta(1) = 0.5g(0) + max_e\{0.5g(e) - e\} \tag{A.5}$$

Now it is clear that both terms on the right-hand side of (A.5) is positive, as $g > h$, $g' > h'$, and $g \geq 0$. So, we have established that$\Delta(0) > \Delta(1) > 0$. Setting $\overline{p} = \Delta(0)$, $\underline{p} = \Delta(1)$ completes the proof. □

Proof of Proposition 3

(i) *Existence.* We will assume that $g_A = g_B = g$, and $\eta_i = \gamma_i = 0$, and construct an equilibrium where Bs strictly prefer to buy assets and As strictly prefer not to. As preferences are strict, such an equilibrium will continue to prevail if the parameters are perturbed slightly so that $(\eta_i, \gamma_i)_{i=A,B}$ become non-zero while satisfying (15).

First, as $\eta_i = \gamma_i = 0$, then all agents invest zero, and so $h_i(e) = h_i$, $g_i(e) = g$, $i = A, B$. Then, from (7) and (9), we have

$$\begin{aligned} v_i(1) &= 0.5\,(g + h_i) + 0.5\pi_1 g + 0.5(1 - \pi_1)(g - \overline{h}_1) \\ v_i(0) &= 0.5\,(2g - \overline{h}_1) \end{aligned}$$

and consequently,

$$\Delta_i(\overline{h}_1) = 0.5h_i + (\pi_0 + \pi_1 - 1)\overline{h}_1 + (1 - \pi_0)g \tag{A.6}$$

So, from (A.6),

$$\Delta_B(\overline{h}_1) - \Delta_A(\overline{h}_1) = 0.5(h_B - h_A) > 0$$

so there will always be a range of prices for which hypothesized equilibrium exists, where only Bs own, and only As do not. Specifically, note that in the equilibrium, $\overline{h}_1 = h_B$. So if we take

$$\begin{aligned} \underline{p} &= \Delta_A(h_B) = 0.5h_A + (\pi_0 + \pi_1 - 1)h_B + (1 - \pi_0)g \\ \overline{p} &= \Delta_B(h_B) = 0.5h_B + (\pi_0 + \pi_1 - 1)h_B + (1 - \pi_0)g \end{aligned}$$

then for any $p \in (\underline{p}, \overline{p})$ there is an equilibrium where only Bs own, and only As do not.

(ii) *Uniqueness.* From (A.6), we see that $\Delta_B(\overline{h}_1) > \Delta_A(\overline{h}_1)$ whatever the pattern of ownership. So, there are only two other possible equilibria for some $p \in (\underline{p}, \overline{p})$: (a) $\Delta_B(\overline{h}_1) = p \geq \Delta_A(\overline{h}_1)$ and type Bs randomize i.e. choose ownership with probability $0 < \rho_B < 1$; (b) $\Delta_B(\overline{h}_1) > p = \Delta_A(\overline{h}_1)$ and type As randomize i.e. choose ownership with probability $0 < \rho_A < 1$. Suppose first that a type (a) equilibrium exists for $p \in (\underline{p}, \overline{p})$; then $\overline{h}_1 = h_B$, and so $p = \Delta_B(h_B) = \overline{p}$, which contradicts $p \in (\underline{p}, \overline{p})$.

Suppose next that a type (b) equilibrium exists. Then $\overline{h}_1 < h_B$, and using the fact that Δ_i is increasing in $\overline{h}_1$ (as $\pi_0 + \pi_1 > 1$), we have

$$p = \Delta_A(\overline{h}_1) < \Delta_A(h_B) = \underline{p}$$

again contradicting $p \in (\underline{p}, \overline{p})$. □

Proof of Proposition 4

(i) *Existence.* We will assume that $h_A = h_B = 0$, and $\eta_i = \gamma_i = 0$, and construct an equilibrium where Bs strictly prefer to buy assets and As strictly prefer not to. As preferences are strict, such an equilibrium will continue to prevail if the parameters are perturbed slightly so that $(\eta_i, \gamma_i)_{i=A,B}$ become non-zero while satisfying (15).

First note that for *any* $\pi_0 < 1$, from (7), (9), we have

$$\Delta_A(z) = 0.5(1 - \pi_0)g_A + 0.5z \tag{A.7}$$

$$\Delta_B(z) = 0.5(1 - \pi_0)g_B + 0.5z \tag{A.8}$$

where $z = \overline{g}_0 - \pi_0\overline{g}_1$.

Now, note that as $\theta_B < 0.5$, if only As own, $\hat{\pi}_0 = \theta_B/(1-\theta_B) < 1$. So, from (A.7),(A.8);

$$\Delta_B(z) - \Delta_A(z) = 0.5(1-\hat{\pi}_0)(g_B - g_A) > 0 \tag{A.9}$$

So there always exists a range of prices for which hypothesized equilibrium exists. Specifically, note that if only Bs own,

$$z = 0.5(g_A - \hat{\pi}_0 g_B) = \hat{z}$$

So if we take

$$\begin{aligned} \underline{p} &= \Delta_A(\hat{z}) = 0.5(1-\hat{\pi}_0)g_A + 0.5(g_B - \hat{\pi}_0 g_A) & \text{(A.10)} \\ \overline{p} &= \Delta_B(\hat{z}) = 0.5(1-\hat{\pi}_0)g_B + 0.5(g_B - \hat{\pi}_0 g_A) & \text{(A.11)} \end{aligned}$$

then for any $p \in (\underline{p}, \overline{p})$ there is an equilibrium where only Bs own, and only As do not, as required.

(ii) *Uniqueness.* First, assume that $\pi_0 < 1$. Then, from (A.9), $\Delta_A(z) > \Delta_B(z)$, whatever the pattern of ownership. So, the only other possible equilibria are either (a) $\Delta_B(z) = p > \Delta_A(z)$ and type Bs randomize i.e. choose ownership with probability $0 < \rho_B < 1$; (b) $\Delta_B(z) > p = \Delta_A(z)$ and type As randomize i.e. choose ownership with probability $0 < \rho_A < 1$. But then, it can be shown that

$$\begin{aligned} z^a &= 0.5(\overline{g}_0^a - \pi_0^a g_B) > 0.5(g_A - \hat{\pi}_0 g_B) \text{ (case (a))} & \text{(A.12)} \\ z^b &= 0.5(g_A - \pi_0^b \overline{g}_1^b) < 0.5(g_A - \hat{\pi}_0 g_B) \text{ (case (b))} & \text{(A.13)} \end{aligned}$$

Inequalities (A.12),(A.13) are implied by the following facts. First, in case (a), some type Bs do not buy, so $\pi_0^a < \hat{\pi}_0$ and $\overline{g}_0^a > g_A$, which implies (A.12) directly. Second, in case (b), some type As do buy, so $\pi_0^b > \hat{\pi}_0$. Also, from (4),(5), we have

$$\hat{\pi}_0 \overline{g}_1^b = (\theta_B g_B + \rho_A \theta_A g_A)/(1 - \rho_A\theta_A - \theta_B) > \theta_B g_B/(1-\theta_B) = \hat{\pi}_0 g_B$$

But then from (A.12),(A.13), for equilibrium in case (a) or (b), we require, noting also $\pi_0^b > \hat{\pi}_0$, that

$$\begin{aligned} p &= \Delta_B = 0.5(1-\pi_0^a)g_B + 0.5z^a > \overline{p} \text{ (case (a))} \\ p &= \Delta_A = 0.5(1-\pi_0^b)g_A + 0.5z^b < \underline{p} \text{ (case (b))} \end{aligned}$$

both of which are inconsistent with $p \in (\underline{p}, \overline{p})$.

Now consider possible equilibria with $\pi_0 = 1$. In this case from (7) and (9), and using the simplifying assumptions made,

$$\begin{aligned} v_i(0) &= 0.5(g_i + \overline{g}_1) & \text{(A.14)} \\ v_i(1) &= 0.5\pi_1(g_i + \overline{g}_0) + 0.5(1-\pi_1)(g_i + \overline{g}_1) & \text{(A.15)} \end{aligned}$$

So, from (A.14),(A.15), both types have the same gain from ownership i.e.

$$\Delta_i = v_i(1) - v_i(0) = 0.5\pi_1(\overline{g}_0 - \overline{g}_1)$$

Now, since this gain is bounded above by $0.5(g_B - g_A)$, as long as $p > 0.5(g_B - g_A)$ an equilibrium with asset purchase cannot exist when $\pi_0 = 1$.

So, to ensure that the equilibrium is unique we require the lower bound (A.10) on price to be modified to

$$\underline{p}' = max\{0.5(1-\pi_0)g_A + 0.5(g_A - \pi_0 g_B),\ 0.5(g_B - g_A)\}$$

Now note that $\overline{p} > \underline{p}'$ if $0.5(g_B - g_A) < \overline{p}$, which is equivalent to $g_B < g_A/\pi_0$. As this holds by assumption, we have found the required open interval of prices for which a unique equilibrium exists, namely $(\underline{p}', \overline{p})$. □

Proof of Proposition 5

To compare ownership incentives in an equilibrium, we take as fixed ρ and therefore π_0, π_1, and also the average values $\overline{g}_0, \overline{g}_1, \overline{h}_0$. Then, from (8), (10), we have

$$\Delta_i = \max_{e_i}\{0.5g_i(e_i) + 0.5h_i(e_i) - e_i\} - \max_{e_i}\{0.5\pi_0 g_i(e_i) - e_i\} + \sigma$$

where σ depends only on $\overline{g}_0, \overline{g}_1, \overline{h}_0$. Now, from (14), this simplifies to

$$\begin{aligned}\Delta_i &= 0.5(1-\pi_0)g_i + 0.5h_i + \max_{e_i}\{(0.5\gamma_i + 0.5\eta_i)\kappa(e) - e_i\} \\ &\quad - \max_{e_i}\{0.5\pi_0\gamma_i\kappa(e_i) - e_i\} + \sigma\end{aligned} \tag{A.16}$$

If $h_A \geq h_B$, $g_A \geq g_B$, $\gamma_A \geq \gamma_B$, and $\eta_A \geq \eta_B$, it is immediate from (A.16) that $\Delta_A > \Delta_B$, and so the result follows. □

Proof of Proposition 6

Postulate an equilibrium where all agents buy one asset. Then, all agents prefer to match with an owner than not match, and so given the behaviour of the matching coordinator described in Section 2.4, all agents are matched with probability one. So, the utility from a match between agents with investment levels e, e' is simply

$$v(e, e') = 0.5[g(e) + g(e')] - e \tag{A.17}$$

Let $\hat{e}$ be equilibrium investment. So, $\hat{e}$ must maximise $v(e, e') - e$, and consequently from (A.17) it solves

$$0.5g'(\hat{e}) = 1 \tag{A.18}$$

To establish that this is an equilibrium, we must show that no agent wishes to deviate by buying either no asset or two assets.

If an agent has no asset, no match he can enter into can be productive, and consequently his payoff is zero. So, it is sufficient that the purchase of one asset gives a non-negative payoff in equilibrium, i.e.

$$v(\hat{e}, \hat{e}) - p \geq 0 \tag{A.19}$$

Now consider a deviation by an agent who buys two assets and acquires investment e'. He is still matched with probability one (see the discussion in the text following Proposition 5). However, he can now engage in home production, and can get a maximum of

$$v'(e', \hat{e}) = 0.5[g(\hat{e}) + g(\hat{e}) + h] - e$$

So, for this deviation to be unprofitable, we also need $v'(e',\hat{e})-v(\hat{e},\hat{e}) \leq p$, or $h \leq p$. So, overall, we need

$$g(\hat{e}) - \hat{e} \geq p \geq h \qquad \text{(A.20)}$$

which is the condition in the Proposition. □

Proof of Proposition 7.

(i) First we derive condition under which an interior integration equilibrium exists, where $\bar{p}$ of the agents buy two assets, and the remainder, none. Provisionally suppose the two assets were indivisible. Then there would exist an equilibrium of the form already described in Proposition 2 in which some but not all agents own. Now allow the two assets to be separated and consider whether, starting from the initial equilibrium, there is an incentive for an individual agent to deviate by owning a single asset. Since two assets are required for production and every match is with a partner owning two or no assets the deviant has the same gross payoff as a non-owner but must pay for the asset.

This implies that an integration equilibrium exists when $p \in (\underline{p}, \overline{p})$ where formulae for the bounds (12),(13) are modified to take account of the fact that two assets are purchased and $h' = 0$ i.e.

$$2\overline{p} = 0.5h + max_e\{0.5g(e)) - e\} + 0.5g(0) \qquad \text{(A.21)}$$

$$2\underline{p} = 0.5h \qquad \text{(A.22)}$$

So, using the definition of $\hat{e}$ in the proof of Proposition 5 above, an integration equilibrium exists when

$$0.25h < p < 0.25h + 0.5(0.5g(\hat{e}) - \hat{e}) + 0.25g(0) = p' \qquad \text{(A.23)}$$

So, comparing (A.20), $(A.23)$, we see that $p' < g(\hat{e}) - \hat{e}$ using the properties of g, h, so the two equilibria coexist iff $p' > h$, or

$$3h < g(\hat{e}) - 2\hat{e} + g(0) \qquad \text{(A.24)}$$

From assumption A1, we have $h > g(\hat{e}) - g(0)$, so it is sufficient that $h < g(0) - \hat{e}$ for two equilibria to coexist.

(ii) It is sufficient to compare the payoff of an agent in the non-integration equilibrium with the payoff of an agent who buys two assets in the integration equilibrium, as in the integration equilibrium the payoffs of asset-owners and non-owners are equal (cf. Proposition 2). Denote by w_{NI}, w_I payoffs net of the asset price for asset owners in the non-integration and integration equilibria respectively.

Now, from the proofs of Propositions 2 and 7, we may write

$$\begin{aligned}
w_{NI} &= max_e\{0.5(g(e) - e\} + 0.5g(\hat{e}) - p \\
&\geq max_e\{0.5g(e) - e\} + 0.5g(\hat{e}) + h - 2p \\
&> max_e\{0.5g(e) - e\} + 0.5[\pi_1 g(\hat{e}_0) + (1 - \pi_1)g(\hat{e})] + 0.5\pi_1 h - 2p \\
&= w_I
\end{aligned}$$

where we have used the following facts: (i) $\hat{e}$ solves (A.18), and $\hat{e}_0$ solves (12), and consequently $\hat{e} > \hat{e}_0$; (ii) $h \leq p$, by the existence condition for the non-integration equilibrium. So, $w_{NI} > w_I$, as claimed. □

Proof of Proposition 8.
(i) A pure *ex ante* equilibrium exists if either $p > 2(T-1)g$, or if $\rho \geq 0.5$ so workers match with owners with probability one. We investigate the second possibility first. A necessary and sufficient condition for $\rho \geq 0.5$, is that either

$$\left(\frac{1-\rho}{\rho}\right)(Tg + 0.5Th - p) + \left(\frac{2\rho - 1}{\rho}\right)(Tg - p) = Tg - 0.5Th \tag{A.25}$$

or

$$Tg - p \geq Tg - 0.5Th \tag{A.26}$$

Equation (A.25) is the condition for an agent be indifferent about owning and not when $1 > \rho \geq 0.5$.

The left-hand side of (A.25) is the expected payoff to owning: with probability $(1-\rho)/\rho$, the owner will be matched with a worker, and get $Tg + 0.5Th - p$ from (16), and with complementary probability, the owner will be matched with another owner, and get only $Tg - p$ from (16). The right-hand side is the payoff to not owning: the worker is matched with an owner with probability one, and gets $Tg - 0.5Th$.

Now solving (A.25), we see

$$\rho = \frac{0.5Th}{p}$$

As $0.5 \leq \rho \leq 1$, this implies $0.5Th \leq p \leq Th$.

The other possibility, which arises when (A.26) holds, is that all agents buy an asset, which requires $p \leq 0.5Th$. So, for all $p \leq Th$ there is a pure *ex ante* equilibrium.

(ii) Now consider the existence of a pure *ex ante* equilibrium where $0 < \rho \leq 0.5$. As owners are scarce, WW matches may form so we need $p > 2(T-1)g$ to ensure that no assets are purchased *ex post*. Given this, a necessary and sufficient condition for $0 < \rho \leq 0.5$, with no assets bought *ex post*, is that an agent is indifferent between owning and not i.e.

$$Tg + 0.5Th - p = \frac{\rho}{1-\rho}(Tg - 0.5Th) \tag{A.27}$$

The owner is matched with a worker with probability one, and gets $Tg + 0.5Th - p$. With probability $\rho/(1-\rho)$, the worker will be matched with an owner, and get $Tg - 0.5Th$ from (16). With complementary probability, the worker will be matched with another worker, and gets zero. Solving, we get

$$\rho = \frac{Tg + 0.5Th - p}{2Tg - p} \tag{A.28}$$

As $0 < \rho \leq 0.5$, from (A.28), we need

$$Tg + 0.5Th > p \geq Th$$

But as $2(T-1)g > Th$, we conclude that if $Tg + 0.5Th > p \geq 2(T-1)g$, a pure *ex ante* equilibrium also exists.

(*iii*) We now establish when a partial *ex ante* equilibrium exists. Necessary and sufficient conditions for this is that $0 < \rho < 0.5$, $p < 2(T-1)g$, so that some WW matches form and buy assets *ex post*. Now, the indifference condition (A.27) is modified to

$$Tg + 0.5Th - p = \frac{\rho}{1-\rho}(Tg - 0.5Th) + \frac{1-2\rho}{1-\rho}((T-1)g - 0.5p) \tag{A.29}$$

The interpretation is as before, except that with probability $(1-2\rho)/(1-\rho)$ the worker will be matched with another worker, and get $(T-1)g - 0.5p$ from (16). Now solving (A.29), we see

$$\rho = \frac{g + 0.5Th - 0.5p}{2g} \tag{A.30}$$

As $0 < \rho < 0.5$, (A.30) implies $Th < p < 2g+Th$. We also require that $2(T-1)g > p$. So, if $Th < p < \min\{2g + Th, 2(T-1)g\}$, there exists an equilibrium where some assets are bought *ex post*.

(iv) We conclude by studying *ex post* equilibrium. In this equilibrium, all agents are workers; all are paired and buy assets *ex post* iff $(T-1)2g \geq p$. For this to be an equilibrium, it must pay no worker to deviate by buying an asset and getting matched with probability one i.e.

$$(T-1)g - 0.5p \geq Tg + 0.5Th - p$$

So, an *ex post* equilibrium requires that

$$(T-1)2g \geq p \geq 2g + Th$$

as claimed. For this to be possible, we need $2(T-2)g \geq Th$, so if $h > 0$, $T \geq 3$ for an *ex post* equilibrium to exist. □

Proof of Proposition 9.

We solve the model backwards, beginning with the second stage where the asset transfer may have taken place. The first possibility is that As own when investments are made. In this case, Bs invest at level e_0^B, which solves

$$0.5g'_B(e_0^B) = 1$$

Note that whether they own or not, As do not invest, as $\eta_A = \gamma_A = 0$. Then, payoffs to As and Bs are:

$$\begin{aligned} u_A^* &= 0.5(g_A(0)) + g_B(e_0^B) + h_A(0)) - p \\ u_B^* &= 0.5(g_A(0) + g_B(e_0^B) - h_A(0)) - e_0^B \end{aligned}$$

The other possibility is that Bs own. In this case, Bs invest at level e_1^B, which solves

$$0.5g'_B(e_1^B) + 0.5h'_B(e_1^B) = 1$$

Then, payoffs to As and Bs are:

$$\begin{aligned}\hat{u}_A &= 0.5(g_A(0)) + g_B(e_1^B) - h_B(e_1^B)) - p + q \\ \hat{u}_B &= 0.5(g_A(0) + g_B(e_1^B) - h_B(e_1^B)) - e_1^B - q\end{aligned}$$

where q is the expected price at which A agrees to sell the asset to B.

We are hypothesizing an equilibrium where As own initially. So, note that the gain to transfer of ownership to Bs is

$$\hat{u}_A + \hat{u}_B - (u_A^* + u_B^*) = g_B(e_1^B) - g_B(e_0^B) - (e_1^B - e_0^B) = \Delta > 0$$

So, reallocation will always take place in equilibrium.

By the bargaining protocol, the expected price q splits the gain 50:50, so we may write

$$\begin{aligned}\hat{u}_A &= 0.5(g_A(0) + g_B(e_0^B) + h_A(0)) + 0.5\Delta - p && \text{(A.31)} \\ \hat{u}_B &= 0.5(g_A(0) + g_B(e_0^B) - h_A(0)) + 0.5\Delta - e_0^B && \text{(A.32)}\end{aligned}$$

(ii) We now move to the stage of initial asset purchase. Consider a deviation by a type A i.e. a decision not to purchase an asset. An assetless type A will be matched with another type A, so his deviation payoff is

$$u'_A = 0.5(2g_A(0) - h_A(0)) \tag{A.33}$$

So, the condition that A prefers not to deviate is $\hat{u}_A \geq u'_A$, or using (A.31),(A.33), after some simplification,

$$g_B(e_1^B) - g_A(0) + 2h_A(0) - (e_1^B - e_0^B) > 2p \tag{A.34}$$

Consider next a deviation by a type B i.e. a decision to purchase an asset. An asset-owning type B will be matched with another type B, and no asset reallocation will take place (recall only simple ownership contracts are assumed feasible) so his deviation payoff is

$$u'_B = 0.5(g_B(e_1^B) + g_B(e_0^B) + h_B(e_1^B)) - e_1^B - p \tag{A.35}$$

So, the condition that B prefers not to deviate is $\hat{u}_B \geq u'_B$, or using (A.32),(A.35), after some simplification,

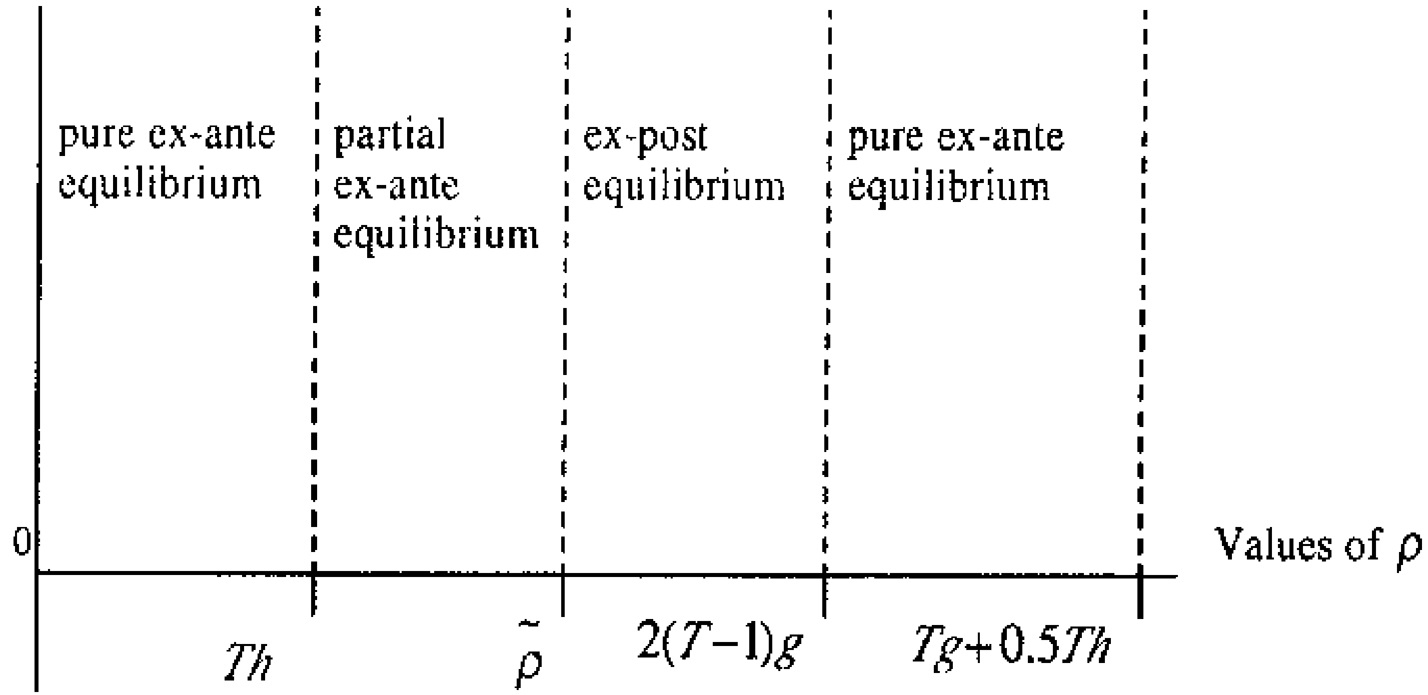
$$2p > -e_1^B + e_0^B + h_B(e_1^B) + h_B(0) + g_B(e_0^B) - g_A(0) \tag{A.36}$$

So, by inspection of (A.34), (A.36), we can find an non-empty interval of prices where neither As nor Bs wish to deviate iff

$$g_B(e_1^B) - g_B(e_0^B) + (h_A(0) - h_B(e_1^B)) > 0$$

which is the condition in the Proposition. □

Figure 2


pure ex-ante equilibrium
partial ex-ante equilibrium
ex-post equilibrium
pure ex-ante equilibrium
0
Values of ρ
Th
$\tilde{\rho}$
$2(T-1)g$
$Tg+0.5Th$