5. On Selecting Policy Analysis Models by Forecast Accuracy

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1. Introduction

It is a pleasure to participate in a volume celebrating the contributions to economics of Michio Morishima, a colleague for many years. The eclectic nature of Michio's extensive publications makes it impossible to choose any topic on which he has never written, and our chapter is related to Morishima and Saito (1964), who developed a macro-econometric model of the US economy. While Morishima and Saito (1964) focus on econometric equations with a close eye on economic-policy analysis, we also consider the relationship between statistical forecasting devices and econometric models in the policy context. In particular, we investigate three aspects of this relationship. First, whether there are grounds for basing economic-policy analysis on the 'best' forecasting system. Secondly, whether forecast failure in an econometric model precludes its use for economic-policy analysis. Finally, whether in the presence of policy change, improved forecasts can be obtained by using 'scenario' changes, derived from the econometric model, to modify an initial statistical forecast. To resolve these issues, we analyze the problems arising when forecasting after a structural break (i.e., a change in the parameters of the econometric system), but before a regime shift (i.e., a change in the behaviour of non-modelled, often policy, variables), perhaps in response

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to the break (see Hendry and Mizon (1998), for discussion of this distinction). These three dichotomies, between econometric and statistical models, structural breaks and regime shifts, and pre and post forecasting events, are central to our results.

We envisage a statistical forecasting system as one having no economic-theory basis (the hallmark of an econometric model), so it will rarely have implications for economic-policy analysis – and may not even entail links between target variables and policy instruments. Consequently, being the 'best' available forecasting device is insufficient to ensure any value for policy analysis, and the main issue is the converse: does the existence of a dominating forecasting procedure invalidate the use of an econometric model for policy? Our answer is almost the opposite of the Lucas (1976) critique: since forecast failure often results from factors unrelated to the policy change in question, the econometric model may continue to characterize the response of the economy to the policy, despite its forecast inaccuracy. Indeed, when policy changes are implemented, forecasts from a statistical model may be improved by combining them with the predicted policy responses from an econometric model.

The rationale for our analysis is as follows. Using the taxonomy of forecast errors in Clements and Hendry (1996a), Hendry and Doornik (1997) establish that deterministic shifts are the primary source of systematic forecast failure in econometric models. Nevertheless, there exist devices that can robustify forecasting models against such breaks, provided they have occurred prior to forecasting (see e.g., Clements and Hendry (1996b), and Hendry and Clements (1999)). Such 'tricks' can help mitigate forecast failures, but the resulting models need not have useful policy implications. However, no methods are robust to unanticipated breaks that occur after forecasting, and Clements and Hendry (1999b) show that those same 'robustifying' devices do not offset post-forecasting breaks. Moreover, post-forecasting policy changes induce breaks in models that do not embody policy variables or links, so such models lose their robustness in that setting. Conversely, despite having experienced forecast failures from pre-forecasting breaks, econometric systems which do embody the relevant policy effects need not experience a post-forecasting structural break induced by the policy-regime shift.

Consequently, when both structural breaks and regime shifts occur, neither class of model alone is adequate: this suggests investigating whether, and if so how, they should be combined.

The structure of the analysis is as follows. In the next section, we discuss the relevant forecasting and economic-policy concepts and issues to motivate the paper. This is followed in §3 by an example of forecasting and policy in the presence of regime shifts. We analyze the impact of structural breaks and policy changes on forecasts in an open vector equilibrium-correction mechanism in §4, and present the case for combining the forecasts from robust statistical devices and policy-scenario changes in §5. Section 6 then provides an extensive empirical illustration using models of UK aggregate consumption. We present conclusions in §7.

2. Background

Much previous work on economic forecasting has considered the properties of forecasts when:

1. the data generation process (DGP) is known;
2. the DGP is constant; and
3. the econometric model coincides with the DGP.

These three assumptions are strong, and unlikely to be fulfilled in practice. As discussed in Hendry (1997) and Hendry and Doornik (1997), the failure of the first and third need not greatly affect the implications of forecasting theory. However, the failure of all three radically alters many aspects of that theory: for example, when the DGP is non-constant and the model is mis-specified, it cannot be proved that causal variables will dominate non-causal for forecasting. Moreover, while the three assumptions are sufficient to ensure that forecasts from the econometric model will be at least as good as those from purely statistical procedures, they are not necessary. For example, as discussed in Hendry (1979) and Miller (1978), stationarity ensures that, on average (i.e., excluding rare events), an incorrectly-specified model will forecast within its anticipated tolerances (providing these are correctly calculated). Moreover, even though such mis-specified models could be beaten by other methods based on correctly-specified equations, an encompassing model – however poor – will variance-dominate in-sample, and hence also when forecasting under unchanged conditions.

Since omniscience is not characteristic of economics, a better approach assumes that none of the three conditions applies. Clements and Hendry (1994) and Clements and Hendry (1998a) investigated a theory more relevant to practical economic forecasting in which:

a. the DGP is unknown;

b. the DGP is non-stationary (due to unit roots and structural breaks); and

c. the econometric model is mis-specified for the DGP.

These features seem descriptive of operational economic forecasting. Moreover, they provide a rationale for 'intercept corrections' to model-based forecasts (see Hendry and Clements (1994), and Clements and Hendry (1996b)), which is absent when 1.–3. hold. Further, differencing transformations, which arbitrarily impose unit roots and thereby eliminate cointegrating relations, also change permanent structural breaks in deterministic factors into 'blips' (see Clements and Hendry (1995)). Thus, despite being non-optimal under 1.–3., in practice such procedures can robustify forecasts against the form of structural break that Hendry and Doornik (1997) find to be the most pernicious source of forecast failure, namely shifts in deterministic factors.

A consequence of these results is that in a class of models for processes subject to structural breaks, the best available forecasting model need not be based on the 'causal determinants' of the actual economic process, and as the example in §3 shows, may be based on 'non-causal' variables. Thus, the best economic-policy analysis need not be based on the model that happens to forecast best, and the existence of a procedure that systematically produces better forecasts need not invalidate the policy use of another model.

The fact that a purely statistical device may provide the best available forecasts induces an apparent paradox. In a world characterized by a.–c. above, forecasts based on the currently-best econometric model may be beaten by statistical devices when forecasting after a structural break. Assume for the moment that the statistical forecasting model does not depend on any policy variables, and hence has neither policy implications, nor produces any revisions to forecasts following policy changes. These 'best' forecasts for some future period are presented to the finance minister of a given country, who thereupon decides that a major policy initiative is essential, and implements it. That the statistical forecasts are not then revised would justifiably be greeted with incredulity. More pertinently, providing the policy model did not fall foul of the critique in Granger and Deutsch (1992), so that changes to policy variables did indeed alter target variables, then a better forecast seems likely by adding the policy change effects predicted by the econometric model to the previous forecasts. But this contradicts any claim to the effect that the statistical devices produced the best forecasts in a world of structural change.

The resolution, of course, depends on distinguishing between unknown breaks – where (e.g.) differencing may deliver the best achievable forecast – and known changes, the consequences of which are partly measurable. The conclusion is that a combination of robustified statistical forecasts with the scenario changes from econometric systems subject to policy interventions may provide improved forecasts. This is the subject of §5.

An independent issue is that there is no unique measure of forecast accuracy, since predictability depends on intertemporal transformations (see e.g., Hendry (1997)). Measures such as mean square forecast errors (MSFEs) are often used for forecast comparisons across alternative models or methods (see e.g., Wallis and Whitley (1991)), but as shown in Clements and Hendry (1993), these lack invariance to non-singular, scale-preserving, linear transforms across isomorphic members of a model class for multi-step forecasts in systems of equations. Even the (invariant) generalized forecast-error second moment criterion (GFESM) which they propose is not thereby unique – a monetary measure is quite conceivable (see West (1993)). Our present concern does not depend on such a difficulty, and we assume that the agent desiring the forecast has a well-specified loss measure by which to judge forecast accuracy, and there is a unique optimum for the specified criterion. However, we recognize the additional practical difficulty of determining how to evaluate the outcomes of the forecasts or the policies.

The sources of forecast errors can be categorized into six classes as discussed in Clements and Hendry (1994), for example:

(*i*) slope change;

(*ii*) intercept change;

(*iii*) model mis-specification;

(*iv*) parameter estimation;

(*v*) initial forecast conditions;

(*vi*) error accumulation.

The first two are distinguished here because their consequences seem very different in practice: zero-mean changes are not easily detected, whereas shifts in equilibrium means can induce dramatic forecast failure. Such shifts need not, although they could, alter the partial derivatives of target variables with respect to instruments, in which case, the reasons for predictive failure need not impugn a policy model. Assuming they do not, e.g., because the regime shift is not due to causes that affect policy connections, then a better forecast can be derived by using the scenario change to modify the forecast obtained from a robustified method.

Alternatively, the policy model will be invalid when:

a] it embodies the wrong causal attributions;

b] its target-instrument links are not autonomous;

c] its parameters are not invariant to the policy change under analysis.

These are distinct from the causes of forecast failure, though they could be a subset of the factors in any given situation. We now consider a case where poor forecasts need not invalidate policy advice.

3. Forecasting and policy analysis across regime shifts

Hendry (1997) illustrates the potential role for statistical forecasting methods when an economy is subject to structural breaks, and the econometric model is mis-specified for the data generation process. He considers an economy where gross national product (GNP, denoted by y) is 'caused' solely by the exchange rate over a sample prior to forecasting, then the DGP changes to one in which y is only caused by the interest rate, but this switch is not known by the forecaster. The DGP is non-dynamic, and in particular, the lagged value of y does not affect its behaviour (i.e., y_{t-1} is non-causal). Nevertheless, when forecasting after the regime change, on the criterion of forecast unbiasedness, a forecasting procedure that ignores the information on both causal variables, and only uses y_{t-1}, namely predicting a constant change in y by $\mathsf{E}[y_t|y_{t-1}] = y_{t-1}$ can outperform (in terms of bias) compared to forecasts from models which included the correct causal variable. Here, neither the statistical model, nor the econometric model based on past causal links, is useful for policy.

Since policy analysis conducted on an incorrect model is not useful, we now consider what can be concluded in general settings. The paradigmatic example we have in mind is an econometric model of (say) the tax and benefits system which accurately portrays the relevant links, and yields a good approximation to the changes in revenues and expenditures resulting from changes in the basic rates. However, it would not necessarily provide good time-series forecasts in an economy subject to structural breaks that affected macroeconomic variables such as total consumers' expenditure and inflation.

The policy implications of any given model in use may or may not change with a particular regime shift. For the setting above, if the exchange rate (e_t) did not alter when the interest rate (r_t) was changed in the first regime, so r_t had no direct or indirect effect on y in that regime, then the policy implications of the first-regime model would be useless in the second regime. That seems unlikely here, though such may well occur in practice. If e_t is in fact altered by changes in r_t, so will y_t in both regimes. Policy analysis involves estimation of the target-instrument responses, which in this case means $\partial y_{t+h}/\partial r_t$ when y_t is the target variable and r_t the policy instrument. For the statistical model $\Delta y_t = \varsigma_t$, this response is zero at all forecast horizons h, and so despite its robust forecasting abilities, such a model is uninformative for policy analysis. The first-regime econometric model, on the other hand, does provide an estimate of $\partial y_{t+h}/\partial r_t$ via (e.g.):

$$\widehat{\frac{\partial y_{t+h}}{\partial r_t}} = \sum_{i=0}^{h} \frac{\partial y_{t+h}}{\partial e_{t+i}} \frac{\partial e_{t+i}}{\partial r_t}. \tag{5.1}$$

In regime-2, the actual policy response is $\partial y_{t+h}/\partial r_t$, so the regime-1 econometric model policy responses in (5.1) will be valuable when they have the same sign, and do not over-estimate the response by more than double, whereas the statistical model is always uninformative in that it gives a zero policy response.

The next section formalizes results for forecasting in the face of both structural breaks and regime shifts, when the DGP is a cointegrated system dependent on policy-determined variables. In §5, we explore the possibility that some combination of statistical forecasts and estimated policy responses could dominate either alone.

4. Structural breaks and regime shifts in policy models

Previous studies of the impacts on forecasting of structural breaks have looked at closed models (e.g., Clements and Hendry (1999b), and Hendry and Clements (1999)). We now generalize these results to open models to investigate the effects of regime shifts in non-modelled variables which are often policy instruments. We focus on deterministic shifts following Hendry and Doornik (1997), although other parametric changes could be envisaged. To establish the appropriate conditions, we first ascertain the impacts of structural breaks and regime shifts in two models. These are a second-differenced predictor (denoted DDV) and a vector equilibrium-correction mechanism (VEqCM). Clements and Hendry (1999b) show that these predictors have the same forecast biases for breaks that occur after forecasts are announced, but that the DDV is robust to deterministic breaks that have occurred before forecasting: this section draws on their approach, extending it to open models and to forecasts of growth rates (rather than levels). Thus, we consider forecasting after a structural break (due to a change in the parameters of the econometric system), but before a regime shift (here, a change in the policy rule). Since the VEqCM has some response to policy, but the DDV does not, such comparisons yield insights into the effects of using robustified forecasting methods, then exploiting policy-change information via an econometric system.

We envisage a policy rule as comprising drawings of the k policy variables $\mathbf{z}_t$ from a distribution centered on $\boldsymbol{\rho}$, perhaps dependent on recent past information in the economy, which we write as:

$$\mathbf{z}_t = \boldsymbol{\rho} + \mathbf{g}\left(\mathcal{I}_{t-1}\right) \tag{5.2}$$

where $\mathsf{E}[\mathbf{g}(\mathcal{I}_{t-1})] = \mathbf{0}$. The policy variables $\mathbf{z}_t$ are under the control of a policy agency, which, within regime, makes a drawing from (5.2), but when introducing a regime shift, changes $\boldsymbol{\rho}$ to $\boldsymbol{\rho}^*$. The in-sample DGP consists of the marginal process for the $\mathsf{I}(0)$ policy variables $\mathbf{z}_t$, and an open VEqCM, conditional on $\mathbf{z}_t$, representing the behaviour of the n private-sector $\mathsf{I}(1)$ variables $\mathbf{x}_t$:

$$\Delta\mathbf{x}_t = \boldsymbol{\tau} + \boldsymbol{\alpha}\boldsymbol{\beta}'\mathbf{x}_{t-1} + \Gamma\mathbf{z}_t + \boldsymbol{\epsilon}_t \text{ where } \boldsymbol{\epsilon}_t \sim \mathsf{IN}_n\left[\mathbf{0}, \Sigma\right] \tag{5.3}$$

where $\boldsymbol{\alpha}$ and $\boldsymbol{\beta}$ are $n \times r$ of rank r. To ensure that $\mathbf{x}_t$ is $\mathsf{I}(1)$, and not $\mathsf{I}(2)$, rank$(\boldsymbol{\alpha}'_{\perp}\boldsymbol{\beta}_{\perp}) = n-r$, where $\boldsymbol{\alpha}_{\perp}$ and $\boldsymbol{\beta}_{\perp}$ are $n\times(n-r)$ matrices such that $\boldsymbol{\alpha}'_{\perp}\boldsymbol{\alpha} = \mathbf{0}$, $\boldsymbol{\beta}'_{\perp}\boldsymbol{\beta} = \mathbf{0}$ with $(\boldsymbol{\alpha} : \boldsymbol{\alpha}_{\perp})$ and $(\boldsymbol{\beta} : \boldsymbol{\beta}_{\perp})$ being rank-n matrices. Also, $\mathsf{IN}_n[\mathbf{0}, \Sigma]$ denotes independent drawings from an n-dimensional normal distribution with mean zero and variance Σ. For $t < T$, the $\mathsf{I}(0)$ variables are stationary, so let:

$$\mathsf{E}\left[\Delta\mathbf{x}_t\right] = \boldsymbol{\gamma}; \;\; \mathsf{E}\left[\boldsymbol{\beta}'\mathbf{x}_t\right] = \boldsymbol{\mu}; \;\text{ and }\; \mathsf{E}\left[\mathbf{z}_t\right] = \boldsymbol{\rho}. \tag{5.4}$$

Taking expectations in (5.2) and (5.3):

$$\mathsf{E}\left[\Delta\mathbf{x}_t\right] = \boldsymbol{\gamma} = \boldsymbol{\tau} + \boldsymbol{\alpha}\mathsf{E}\left[\boldsymbol{\beta}'\mathbf{x}_t\right] + \Gamma\mathsf{E}\left[\mathbf{z}_t\right] = \boldsymbol{\tau} + \boldsymbol{\alpha}\boldsymbol{\mu} + \Gamma\boldsymbol{\rho},$$

using (5.4), so that:

$$\boldsymbol{\tau} = \boldsymbol{\gamma} - \boldsymbol{\alpha}\boldsymbol{\mu} - \Gamma\boldsymbol{\rho}, \tag{5.5}$$

where $\mathsf{E}\left[\Delta\boldsymbol{\beta}'\mathbf{x}_t\right] = \boldsymbol{\beta}'\boldsymbol{\gamma} = \mathbf{0}$. From (5.3) and (5.5), therefore:

$$\Delta\mathbf{x}_t = \boldsymbol{\gamma} + \boldsymbol{\alpha}\left(\boldsymbol{\beta}'\mathbf{x}_{t-1} - \boldsymbol{\mu}\right) + \Gamma\mathbf{z}_t^{+} + \boldsymbol{\epsilon}_t, \tag{5.6}$$

where $\mathbf{z}_t^+ = \mathbf{z}_t - \boldsymbol{\rho}$.

The system in (5.6) can be re-written as two distinct blocks, respectively obtained on pre-multiplying by $\boldsymbol{\beta}'$ and $\boldsymbol{\alpha}'_\perp$:

$$\begin{aligned} (\boldsymbol{\beta}'\mathbf{x}_t - \boldsymbol{\mu}) &= \Lambda\left(\boldsymbol{\beta}'\mathbf{x}_{t-1} - \boldsymbol{\mu}\right) + \boldsymbol{\beta}'\Gamma\mathbf{z}_t^+ + \boldsymbol{\beta}'\boldsymbol{\epsilon}_t \qquad (5.7) \\ \boldsymbol{\alpha}'_\perp\Delta\mathbf{x}_t &= \boldsymbol{\alpha}'_\perp\boldsymbol{\gamma} + \boldsymbol{\alpha}'_\perp\Gamma\mathbf{z}_t^+ + \boldsymbol{\alpha}'_\perp\boldsymbol{\epsilon}_t, \end{aligned}$$

where $\Lambda = \mathbf{I}_r + \boldsymbol{\beta}'\boldsymbol{\alpha}$. For explicit results under regime shifts, we assume that $\mathbf{z}_t^+$ does not permanently alter the growth rate of the system, so that $\boldsymbol{\alpha}'_\perp\Gamma = \mathbf{0}$, or $\Gamma = \boldsymbol{\alpha}\Psi$. We also assume that the parameters of (5.2) can change freely from those of (5.3), although in principle, the analysis could be generalized to allow for dependencies (e.g., through $\mathbf{g}(\cdot)$ depending on the disequilibria from the equilibrium corrections in (5.3)), or for $\mathsf{I}(1)$ policy variables that entered the cointegration vectors.

The dependence assumptions made about deterministic terms are fundamental to the outcome of the following analysis. For example, if $\boldsymbol{\gamma}$, $\boldsymbol{\mu}$, and $\boldsymbol{\rho}$ were unconnected, (5.6) has 'policy ineffectiveness', in that only deviations of $\mathbf{z}_t$ from $\boldsymbol{\rho}$ have an impact, and changes in $\boldsymbol{\rho}$ have no effect when implemented by keeping $\mathbf{z}_t^+$ fixed. If so, only impulse responses would be of interest. However, we consider that shifts in $\boldsymbol{\rho}$ are likely to have an impact on $\mathbf{x}$ in practice, and hence assume $\boldsymbol{\tau}$, $\boldsymbol{\alpha}$, $\boldsymbol{\beta}$, Ψ, and $\boldsymbol{\rho}$ may change freely, with $\boldsymbol{\mu}$ altering in response to shifts in $\boldsymbol{\rho}$. Since there is no impact of changes in $\boldsymbol{\rho}$ on $\boldsymbol{\gamma}$, $\partial\boldsymbol{\tau}/\partial\boldsymbol{\rho}' = \mathbf{0}$ entails $\boldsymbol{\mu} = \boldsymbol{\psi} - \Psi\boldsymbol{\rho}$, which is an assumption of contemporaneous mean co-breaking (see Hendry (1995b) and Hendry and Mizon (1998)), leading to $\boldsymbol{\tau} = \boldsymbol{\gamma} - \boldsymbol{\alpha}\boldsymbol{\psi}$ in (5.5). Thus, the final formulation in-sample (i.e., before breaks occur) is:

$$\Delta\mathbf{x}_t = \boldsymbol{\gamma} + \boldsymbol{\alpha}\left(\boldsymbol{\beta}'\mathbf{x}_{t-1} - \boldsymbol{\psi} + \Psi\boldsymbol{\rho}\right) + \boldsymbol{\alpha}\Psi\mathbf{z}_t^+ + \boldsymbol{\epsilon}_t. \qquad (5.8)$$

In the face of either regime shifts or structural breaks that directly alter deterministic terms:

$$\begin{aligned} \mathsf{E}[\Delta \mathbf{x}_t] &= \boldsymbol{\gamma}_t & (5.9) \\ \mathsf{E}[\boldsymbol{\beta}'\mathbf{x}_t] &= \boldsymbol{\psi}_t - \Psi_t \boldsymbol{\rho}_t & (5.10) \\ \mathsf{E}[\mathbf{z}_t] &= \boldsymbol{\rho}_t. & (5.11) \end{aligned}$$

The assumption of a non-constant mean vector in (5.11) is essential to consider policy regime shifts, the non-constant means in (5.9) and (5.10) are required if structural breaks occur, (dependence on t), and co-breaking in (5.10) is needed if mean shifts in policy are to be effective (dependence on $\boldsymbol{\rho}$). To the extent that $\Psi_t \neq \Psi$, policy will not have its anticipated consequences.

We first investigate a single structural break at time T which shifts the DGP parameters from $\boldsymbol{\gamma}$ to $\boldsymbol{\gamma}^*$, $\boldsymbol{\psi}$ to $\boldsymbol{\psi}^*$, and Ψ to Ψ^*, but leaves $\boldsymbol{\alpha}$ and $\boldsymbol{\beta}$ unchanged, such that, just prior to forecasting, the DGP becomes:

$$\Delta \mathbf{x}_T = \boldsymbol{\gamma}^* + \boldsymbol{\alpha}\left(\boldsymbol{\beta}'\mathbf{x}_{T-1} - \boldsymbol{\psi}^* + \Psi^*\boldsymbol{\rho}\right) + \boldsymbol{\alpha}\Psi^*\mathbf{z}_T^+ + \boldsymbol{\epsilon}_T \quad (5.12)$$

so:

$$\boldsymbol{\tau}^* = \boldsymbol{\gamma}^* - \boldsymbol{\alpha}\boldsymbol{\psi}^*,$$

but the forecaster is unaware that the parameters of the DGP have changed. The changes in $\boldsymbol{\gamma}$ and $\boldsymbol{\psi}$ induce forecast failure in the VEqCM, whereas the change in Ψ reduces the predictability of policy. When the $\mathbf{z}_{T+j}$ are the realized values of the policy vectors for $j = 1, 2$, the data outcomes are:

$$\Delta \mathbf{x}_{T+j} = \boldsymbol{\gamma}^* + \boldsymbol{\alpha}\left(\boldsymbol{\beta}'\mathbf{x}_{T+j-1} - \boldsymbol{\psi}^* + \Psi^*\boldsymbol{\rho}\right) + \boldsymbol{\alpha}\Psi^*\mathbf{z}_{T+j}^+ + \boldsymbol{\epsilon}_{T+j}. \quad (5.13)$$

Ignoring estimator variances, and assuming accurate data, we consider two forecasting rules for period $T+1$. One investigator uses the in-sample DGP with a provisional setting for the deviation of the policy variable $\mathbf{z}_{T+1}$ from its mean of $\boldsymbol{\rho}$ to obtain the provisional 1-step forecast (called procedure (a)):

$$\widehat{\Delta \mathbf{x}}_{T+1|T} = \boldsymbol{\gamma} + \boldsymbol{\alpha} \left(\boldsymbol{\beta}' \mathbf{x}_T - \boldsymbol{\psi} + \Psi \boldsymbol{\rho} \right) + \boldsymbol{\alpha} \Psi \mathbf{z}^{+}_{T+1}, \tag{5.14}$$

whereas the DDV (procedure (b)) is given by the simple rule:

$$\widetilde{\Delta^2 \mathbf{x}}_{T+1|T} = \mathbf{0},$$

which exploits the fact that few economic variables accelerate indefinitely, so that:

$$\widetilde{\Delta \mathbf{x}}_{T+1|T} = \Delta \mathbf{x}_T.$$

The analysis is then extended to the 2-step case, namely forecasting $T+2$ from T. Section 4.1 discusses the setting of breaks where $\boldsymbol{\rho}^* = \boldsymbol{\rho}$; policy revisions and their effects on constant parameters when $\boldsymbol{\rho}^* \neq \boldsymbol{\rho}$ are discussed in §4.2, whereas §4.3 allows for both shifts. Although we focus on forecast biases, the variances of the alternative forecasting devices are noted as these become increasingly important as the horizon increases (see e.g., Clements and Hendry (1999b)).

Since deterministic shifts induce non-stationary behaviour in all the data moments even after reduction to an I(0) representation, all the moments need to be derived recursively through time, and cannot be replaced by their asymptotic equivalents. For example, while $\mathsf{E}[\boldsymbol{\beta}'\mathbf{x}_{T+j}] = \boldsymbol{\mu}^* = \boldsymbol{\psi}^* - \Psi^* \boldsymbol{\rho}$ for $j \gg 0$, even though $\boldsymbol{\mu}$ has shifted fully to $\boldsymbol{\mu}^*$ at time T, from (5.7), $\mathsf{E}[\boldsymbol{\beta}'\mathbf{x}_T] = \boldsymbol{\mu}^* - \Lambda \left(\boldsymbol{\mu}^* - \boldsymbol{\mu} \right)$. These unconditional moments are summarized in (5.15) when there is a structural break, but no regime shift, for $j = 0$, 1, 2.

$$\begin{aligned} \mathsf{E}\left[\boldsymbol{\beta}'\mathbf{x}_{T-1}\right] &= \boldsymbol{\mu}; \;\; \mathsf{E}\left[\Delta \mathbf{x}_{T-1}\right] = \boldsymbol{\gamma}; \\ \mathsf{E}\left[\boldsymbol{\beta}'\mathbf{x}_{T+j}\right] &= \boldsymbol{\mu}^* - \Lambda \left(\boldsymbol{\mu}^* - \mathsf{E}\left[\boldsymbol{\beta}'\mathbf{x}_{T+j-1}\right] \right) = \boldsymbol{\mu}^* - \Lambda^{j+1} \boldsymbol{\delta}_{\mu}; \\ \mathsf{E}\left[\Delta \mathbf{x}_{T+j}\right] &= \boldsymbol{\gamma}^* + \boldsymbol{\alpha} \left(\mathsf{E}\left[\boldsymbol{\beta}'\mathbf{x}_{T+j-1}\right] - \boldsymbol{\mu}^* \right) = \boldsymbol{\gamma}^* - \boldsymbol{\alpha} \Lambda^{j} \boldsymbol{\delta}_{\mu}, \end{aligned} \tag{5.15}$$

where $\boldsymbol{\delta}_{\mu} = \boldsymbol{\mu}^* - \boldsymbol{\mu}$. As $\boldsymbol{\mu} = \boldsymbol{\psi} - \Psi \boldsymbol{\rho}$ and $\boldsymbol{\mu}^* = \boldsymbol{\psi}^* - \Psi^* \boldsymbol{\rho}$, then $\boldsymbol{\delta}_{\mu} = \boldsymbol{\delta}_{\psi} - \boldsymbol{\delta}_{\Psi} \boldsymbol{\rho}$, where $\boldsymbol{\delta}_{\psi} = \boldsymbol{\psi}^* - \boldsymbol{\psi}$ and $\boldsymbol{\delta}_{\Psi} = \Psi^* - \Psi$; also $\boldsymbol{\delta}_{\gamma} = \boldsymbol{\gamma}^* - \boldsymbol{\gamma}$.

4.1. No policy revision ($\boldsymbol{\rho}^* = \boldsymbol{\rho}$)

We first consider the forecast errors that result when the investigator is the policy maker, and sets $\mathbf{z}_{T+j}$ as a deviation from $\boldsymbol{\rho}$; §4.2 considers what happens when $\boldsymbol{\rho}$ is changed to $\boldsymbol{\rho}^*$, where such a response could be in the light of the forecasts from either procedures (a) or (b). Since $\boldsymbol{\rho}$ is unchanged, we replace $\boldsymbol{\psi} - \Psi\boldsymbol{\rho}$ and $\boldsymbol{\psi}^* - \Psi^*\boldsymbol{\rho}$ by $\boldsymbol{\mu}$ and $\boldsymbol{\mu}^*$.

4.1.1 One-period ahead forecast errors. The respective forecasting errors of procedures (a) and (b), conditional on known $\mathbf{z}_{T+i}$, are:

$$\widehat{\boldsymbol{\epsilon}}_{T+1|T} = \boldsymbol{\delta}_\gamma - \boldsymbol{\alpha}\left(\boldsymbol{\delta}_\mu - \boldsymbol{\delta}_\Psi \mathbf{z}^+_{T+1}\right) + \boldsymbol{\epsilon}_{T+1} \tag{5.16}$$

where $\widehat{\boldsymbol{\epsilon}}_{T+1|T} = \mathbf{x}_{T+1} - \widehat{\mathbf{x}}_{T+1|T} = \Delta\mathbf{x}_{T+1} - \widehat{\Delta\mathbf{x}}_{T+1|T}$; and:

$$\widetilde{\boldsymbol{\epsilon}}_{T+1|T} = \Delta\mathbf{x}_{T+1} - \widetilde{\Delta\mathbf{x}}_{T+1|T} = \boldsymbol{\alpha}\boldsymbol{\beta}'\Delta\mathbf{x}_T + \boldsymbol{\alpha}\Psi^*\Delta\mathbf{z}_{T+1} + \Delta\boldsymbol{\epsilon}_{T+1}. \tag{5.17}$$

Note that although the 1-step ahead forecast errors are the same for levels and differences, this is not so for multi-step ahead forecasts. Since $\mathsf{E}\left[\boldsymbol{\beta}'\mathbf{x}_{T-1}\right] = \boldsymbol{\mu}$ from (5.12), then:

$$\mathsf{E}\left[\Delta\mathbf{x}_T \mid \mathbf{z}_{T+1}, \mathbf{z}_T\right] = \boldsymbol{\gamma}^* - \boldsymbol{\alpha}\left(\boldsymbol{\delta}_\mu - \Psi^*\mathbf{z}^+_T\right),$$

and maintaining $\boldsymbol{\beta}'\boldsymbol{\gamma}^* = \mathbf{0}$ (so the cointegration vectors do not trend):

$$\boldsymbol{\beta}'\mathsf{E}\left[\Delta\mathbf{x}_T \mid \mathbf{z}_{T+1}, \mathbf{z}_T\right] = -\boldsymbol{\beta}'\boldsymbol{\alpha}\left(\boldsymbol{\delta}_\mu - \Psi^*\mathbf{z}^+_T\right).$$

Thus, the two forecast errors have conditional means:

$$\mathsf{E}\left[\widehat{\boldsymbol{\epsilon}}_{T+1|T} \mid \mathbf{z}_{T+1}, \mathbf{z}_T\right] = \boldsymbol{\delta}_\gamma - \boldsymbol{\alpha}\left(\boldsymbol{\delta}_\mu - \boldsymbol{\delta}_\Psi \mathbf{z}^+_{T+1}\right), \tag{5.18}$$

and:

$$\mathsf{E}\left[\widetilde{\boldsymbol{\epsilon}}_{T+1|T} \mid \mathbf{z}_{T+1}, \mathbf{z}_T\right] = -\boldsymbol{\alpha}\left(\boldsymbol{\beta}'\boldsymbol{\alpha}\right)\left(\boldsymbol{\delta}_\mu - \Psi^*\mathbf{z}^+_T\right) + \boldsymbol{\alpha}\Psi^*\Delta\mathbf{z}_{T+1}. \tag{5.19}$$

When $\Psi^* = \Psi$ and $\boldsymbol{\tau}^* = \boldsymbol{\tau}$, the DDV does worse on conditional bias if $\Delta\mathbf{z}_{T+1} \neq \mathbf{0}$. However, if the VEqCM parameters change, and the policy vector does not change, so $\mathsf{E}[\Delta\mathbf{z}_{T+1}] = \mathbf{0}$, then the DDV does better on average, noting that $\mathsf{E}[\mathbf{z}_{T+i}^{+}] = \mathbf{0}$.

Treating the $\mathbf{z}_{T+i}$ as fixed, the respective variance matrices are:

$$\mathsf{V}\left[\widehat{\boldsymbol{\epsilon}}_{T+1|T} \mid \mathbf{z}_{T+1}, \mathbf{z}_T\right] = \Sigma, \tag{5.20}$$

and, for $\Phi = \mathbf{I}_n - \boldsymbol{\alpha}\boldsymbol{\beta}'$:

$$\mathsf{V}\left[\widetilde{\boldsymbol{\epsilon}}_{T+1|T} \mid \mathbf{z}_{T+1}, \mathbf{z}_T\right] = \Sigma + \boldsymbol{\alpha}\left(\boldsymbol{\beta}'\boldsymbol{\alpha}\right)\mathsf{V}\left[\boldsymbol{\beta}'\mathbf{x}_{T-1}\right]\left(\boldsymbol{\alpha}'\boldsymbol{\beta}\right)\boldsymbol{\alpha}' + \Phi\Sigma\Phi'. \tag{5.21}$$

Thus, the DDV always loses on variance when $\boldsymbol{\alpha} \neq \mathbf{0}$. However, if $\boldsymbol{\alpha}$ is small, in the sense that the feedbacks are slow (as is often found in practice), then $\mathsf{V}\left[\widetilde{\boldsymbol{\epsilon}}_{T+1|T}\right] \simeq 2\Sigma$, to be compared with the bias gains of (5.19) over (5.18). For large breaks, such as oil crises shifting mean inflation, the DDV could have a much smaller mean square forecast error: from the results in Clements and Hendry (1993), this would apply to all linear transforms of the data.

4.1.2 Two-periods ahead forecast errors. The last comment is not applicable to multi-step forecasts: here we focus on measuring the forecast errors in the metric of the changes, denoted by $\widehat{\boldsymbol{\epsilon}}_{\Delta,T+2|T} = \Delta\mathbf{x}_{T+2} - \widehat{\Delta\mathbf{x}}_{T+2|T}$ and $\widetilde{\boldsymbol{\epsilon}}_{\Delta,T+2|T} = \Delta\mathbf{x}_{T+2} - \widetilde{\Delta\mathbf{x}}_{T+2|T}$. Setting the provisional policy vector at the value $\mathbf{z}_{T+2}$ and using (5.5), the 2-step ahead VEqCM forecast is:

$$\begin{aligned}
\widehat{\Delta\mathbf{x}}_{T+2|T} &= \boldsymbol{\tau} + \boldsymbol{\alpha}\boldsymbol{\beta}'\widehat{\mathbf{x}}_{T+1|T} + \boldsymbol{\alpha}\Psi\mathbf{z}_{T+2} \\
&= \boldsymbol{\gamma} + \boldsymbol{\alpha}\Lambda\left(\boldsymbol{\beta}'\mathbf{x}_T - \boldsymbol{\mu}\right) + \boldsymbol{\alpha}\Psi\mathbf{z}_{T+2}^{+} + \boldsymbol{\alpha}\boldsymbol{\beta}'\boldsymbol{\alpha}\Psi\mathbf{z}_{T+1}^{+}.
\end{aligned}$$

For the DDV:

$$\widetilde{\Delta\mathbf{x}}_{T+2|T} = \widetilde{\Delta\mathbf{x}}_{T+1|T} = \Delta\mathbf{x}_T.$$

Since:

$$
\begin{aligned}
\Delta \mathbf{x}_{T+2} &= \boldsymbol{\gamma}^* + \boldsymbol{\alpha}\left(\boldsymbol{\beta}'\mathbf{x}_{T+1} - \boldsymbol{\mu}^*\right) + \boldsymbol{\alpha}\Psi^*\mathbf{z}_{T+2}^{+} + \boldsymbol{\epsilon}_{T+2} \\
&= \boldsymbol{\gamma}^* + \boldsymbol{\alpha}\Lambda\left(\boldsymbol{\beta}'\mathbf{x}_{T} - \boldsymbol{\mu}^*\right) + \boldsymbol{\alpha}\Psi^*\mathbf{z}_{T+2}^{+} + \boldsymbol{\alpha}\boldsymbol{\beta}'\boldsymbol{\alpha}\Psi^*\mathbf{z}_{T+1}^{+} + \boldsymbol{\epsilon}_{T+2} + \boldsymbol{\alpha}\boldsymbol{\beta}'\boldsymbol{\epsilon}_{T+1},
\end{aligned}
$$

as:

$$
\left(\boldsymbol{\beta}'\mathbf{x}_{T+1} - \boldsymbol{\mu}^*\right) = \Lambda\left(\boldsymbol{\beta}'\mathbf{x}_{T} - \boldsymbol{\mu}^*\right) + \boldsymbol{\beta}'\boldsymbol{\alpha}\Psi^*\mathbf{z}_{T+1}^{+} + \boldsymbol{\beta}'\boldsymbol{\epsilon}_{T+1},
$$

then their respective forecasting errors conditional on $\mathbf{z}_{T+i}$ are:

$$
\begin{aligned}
\widehat{\boldsymbol{\epsilon}}_{\Delta,T+2|T} &= \boldsymbol{\delta}_{\gamma} - \boldsymbol{\alpha}\Lambda\boldsymbol{\delta}_{\mu} \\
&\quad + \boldsymbol{\alpha}\left(\boldsymbol{\delta}_{\Psi}\mathbf{z}_{T+2}^{+} + \boldsymbol{\beta}'\boldsymbol{\alpha}\boldsymbol{\delta}_{\Psi}\mathbf{z}_{T+1}^{+}\right) \\
&\quad + \boldsymbol{\epsilon}_{T+2} + \boldsymbol{\alpha}\boldsymbol{\beta}'\boldsymbol{\epsilon}_{T+1} \qquad (5.22)
\end{aligned}
$$

where $\boldsymbol{\delta}_{\mu} = \boldsymbol{\delta}_{\psi} - \boldsymbol{\delta}_{\Psi}\boldsymbol{\rho}$, and:

$$
\begin{aligned}
\widetilde{\boldsymbol{\epsilon}}_{\Delta,T+2|T} &= -\boldsymbol{\alpha}\left(\mathbf{I}_r + \Lambda\right)\left(\boldsymbol{\beta}'\boldsymbol{\alpha}\right)\boldsymbol{\delta}_{\mu} \\
&\quad + \boldsymbol{\alpha}\left(\mathbf{I}_r + \Lambda\right)\left(\boldsymbol{\beta}'\boldsymbol{\alpha}\right)\left(\boldsymbol{\beta}'\mathbf{x}_{T-1} - \boldsymbol{\mu}\right) \\
&\quad + \boldsymbol{\alpha}\Psi^*\mathbf{z}_{T+2}^{+} + \boldsymbol{\alpha}\boldsymbol{\beta}'\boldsymbol{\alpha}\Psi^*\mathbf{z}_{T+1}^{+} - \mathbf{D}\boldsymbol{\alpha}\Psi^*\mathbf{z}_{T}^{+} \\
&\quad + \boldsymbol{\epsilon}_{T+2} + \boldsymbol{\alpha}\boldsymbol{\beta}'\boldsymbol{\epsilon}_{T+1} - \mathbf{D}\boldsymbol{\epsilon}_{T}, \qquad (5.23)
\end{aligned}
$$

where $\mathbf{D} = \mathbf{I}_n - \boldsymbol{\alpha}\boldsymbol{\beta}' - \left(\boldsymbol{\alpha}\boldsymbol{\beta}'\right)^2$, as $2\mathbf{I}_r + \left(\boldsymbol{\beta}'\boldsymbol{\alpha}\right) = \mathbf{I}_r + \Lambda$, and:

$$
\boldsymbol{\beta}'\Delta\mathbf{x}_{T+1} = \left(\boldsymbol{\beta}'\boldsymbol{\alpha}\right)\left(\boldsymbol{\beta}'\mathbf{x}_{T} - \boldsymbol{\mu}^*\right) + \boldsymbol{\beta}'\boldsymbol{\alpha}\Psi^*\mathbf{z}_{T+1}^{+} + \boldsymbol{\beta}'\boldsymbol{\epsilon}_{T+1}.
$$

Since:

$$
\mathsf{E}\left[\boldsymbol{\beta}'\mathbf{x}_{T+1} - \boldsymbol{\mu}^*\right] = \Lambda\mathsf{E}\left[\boldsymbol{\beta}'\mathbf{x}_{T} - \boldsymbol{\mu}^*\right] = \Lambda^2\boldsymbol{\delta}_{\mu},
$$

these have expected values:

$$
\mathsf{E}\left[\widehat{\boldsymbol{\epsilon}}_{\Delta,T+2|T} \mid \mathbf{z}_{T+2}, \mathbf{z}_{T+1}\right] = \boldsymbol{\delta}_{\gamma} - \boldsymbol{\alpha}\Lambda\boldsymbol{\delta}_{\mu} + \boldsymbol{\alpha}\left(\boldsymbol{\delta}_{\Psi}\mathbf{z}_{T+2}^{+} + \boldsymbol{\beta}'\boldsymbol{\alpha}\boldsymbol{\delta}_{\Psi}\mathbf{z}_{T+1}^{+}\right),
$$

and:

$$\mathsf{E}\left[\widetilde{\boldsymbol{\epsilon}}_{\Delta,T+2|T} \mid \mathbf{z}_{T+2},\mathbf{z}_{T+1}\right] = -\boldsymbol{\alpha}\left(\mathbf{I}_r+\Lambda\right)\left(\boldsymbol{\beta}'\boldsymbol{\alpha}\right)\boldsymbol{\delta}_{\mu}+\boldsymbol{\alpha}\Psi^{*}\mathbf{z}^{+}_{T+2} +\boldsymbol{\alpha}\boldsymbol{\beta}'\boldsymbol{\alpha}\Psi^{*}\mathbf{z}^{+}_{T+1}-\mathbf{D}\boldsymbol{\alpha}\Psi^{*}\mathbf{z}^{+}_{T}.$$

Unconditionally, as $\mathsf{E}[\mathbf{z}^{+}_{T+i}]=\mathbf{0}$, then:

$$\mathsf{E}\left[\widehat{\boldsymbol{\epsilon}}_{\Delta,T+2|T}\right]=\boldsymbol{\delta}_{\gamma}-\boldsymbol{\alpha}\Lambda\boldsymbol{\delta}_{\mu},$$

and for $\mathbf{B}=\left(\mathbf{I}_r+\Lambda\right)\left(\boldsymbol{\beta}'\boldsymbol{\alpha}\right)$:

$$\mathsf{E}\left[\widetilde{\boldsymbol{\epsilon}}_{\Delta,T+2|T}\right]=-\boldsymbol{\alpha}\mathbf{B}\boldsymbol{\delta}_{\mu}.$$

Conversely, if no parameters change, then $\mathsf{E}[\widehat{\boldsymbol{\epsilon}}_{\Delta,T+2|T}]=\mathbf{0}=\mathsf{E}[\widetilde{\boldsymbol{\epsilon}}_{\Delta,T+2|T}]$. Finally:

$$\mathsf{E}\left[\widehat{\boldsymbol{\epsilon}}_{\Delta,T+2|T}\right]-\mathsf{E}\left[\widetilde{\boldsymbol{\epsilon}}_{\Delta,T+2|T}\right]=\boldsymbol{\delta}_{\gamma}-\boldsymbol{\alpha}\mathbf{D}\left(\boldsymbol{\delta}_{\psi}-\boldsymbol{\delta}_{\Psi}\boldsymbol{\rho}\right),$$

which could take either sign.

Again treating the $\mathbf{z}_{T+i}$ as fixed, their variance matrices are:

$$\mathsf{V}\left[\widehat{\boldsymbol{\epsilon}}_{\Delta,T+2|T} \mid \mathbf{z}_{T+2},\mathbf{z}_{T+1}\right]=\Sigma+\boldsymbol{\alpha}\boldsymbol{\beta}'\Sigma\boldsymbol{\beta}\boldsymbol{\alpha}' \tag{5.24}$$

and:

$$\mathsf{V}\left[\widetilde{\boldsymbol{\epsilon}}_{\Delta,T+2|T} \mid \mathbf{z}_{T+2},\mathbf{z}_{T+1}\right]=\boldsymbol{\alpha}\mathbf{B}\mathsf{V}\left[\boldsymbol{\beta}'\mathbf{x}_{T-1}\right]\mathbf{B}'\boldsymbol{\alpha}'+\Sigma+\boldsymbol{\alpha}\boldsymbol{\beta}'\Sigma\boldsymbol{\beta}\boldsymbol{\alpha}'+\mathbf{D}\Sigma\mathbf{D}'. \tag{5.25}$$

Thus, (5.25) always exceeds (5.24). Nevertheless, there are values for the structural breaks such that the MSFE of (b) is less than that of (a), and we consider such cases here – otherwise, the open VEqCM is best on both bias and variance criteria, so the issue of pooling forecasts does not arise.

4.2. Policy-regime shift ($\boldsymbol{\rho}^* \neq \boldsymbol{\rho}$)

We now allow for a shift in policy regime from ρ to ρ^*, which only affects data from time $T+1$ onwards, all other parameters remaining constant. Thus:

$$\Delta \mathbf{x}_T = \boldsymbol{\gamma} + \boldsymbol{\alpha} \left(\boldsymbol{\beta}' \mathbf{x}_{T-1} - \boldsymbol{\psi} + \Psi \boldsymbol{\rho} \right) + \boldsymbol{\alpha} \Psi \mathbf{z}_T^+ + \boldsymbol{\epsilon}_T$$

whereas

$$\Delta \mathbf{x}_{T+j}^a = \boldsymbol{\gamma} + \boldsymbol{\alpha} \left(\boldsymbol{\beta}' \mathbf{x}_{T+j-1}^a - \boldsymbol{\psi} + \Psi \boldsymbol{\rho}^* \right) + \boldsymbol{\alpha} \Psi \mathbf{z}_{T+j}^* + \boldsymbol{\epsilon}_{T+j} \tag{5.26}$$

where $\mathbf{x}_{T+j}^a$ denotes the after-policy-change data. The key feature of such a policy shift is that it comes after the DDV forecasts are made, so does not alter its forecasts, whereas the VEqCM includes the policy variables, and hence produces different forecasts. We denote after-policy forecasts by $\widehat{\Delta \mathbf{x}}_{T+i|T}^a$, and let $\mathbf{z}_{T+i}^a = \boldsymbol{\rho}^* + \mathbf{z}_{T+i} - \boldsymbol{\rho}$, so that $\mathbf{z}_{T+i}^* = \mathbf{z}_{T+i}^a - \boldsymbol{\rho}^* = \mathbf{z}_{T+i}^+$ to focus the whole change in the values of the policy variables on the regime shift for ease of comparability across cases. The unconditional moments for $j = 1, 2$ are summarized in (5.27) when there is a regime shift, but no structural break, using $\boldsymbol{\delta}_\rho = \boldsymbol{\rho}^* - \boldsymbol{\rho}$.

$$\begin{aligned} \mathsf{E}\left[\boldsymbol{\beta}' \mathbf{x}_{T+j}^a \right] &= \boldsymbol{\psi} - \Psi \boldsymbol{\rho}^* + \Lambda^j \Psi \boldsymbol{\delta}_\rho; \\ \mathsf{E}\left[\Delta \mathbf{x}_{T+j}^a \right] &= \boldsymbol{\gamma} + \boldsymbol{\alpha} \Lambda^{j-1} \Psi \boldsymbol{\delta}_\rho. \end{aligned} \tag{5.27}$$

4.2.1 One-period ahead forecast errors. Now:

$$\Delta \mathbf{x}_{T+1}^a = \boldsymbol{\gamma} + \boldsymbol{\alpha} \left(\boldsymbol{\beta}' \mathbf{x}_T - \boldsymbol{\psi} + \Psi \boldsymbol{\rho}^* \right) + \boldsymbol{\alpha} \Psi \mathbf{z}_{T+1}^* + \boldsymbol{\epsilon}_{T+1}$$

and as the policy maker knows the regime shift has occurred:

$$\widehat{\Delta \mathbf{x}}_{T+1|T}^a = \boldsymbol{\gamma} + \boldsymbol{\alpha} \left(\boldsymbol{\beta}' \mathbf{x}_T - \boldsymbol{\psi} + \Psi \boldsymbol{\rho}^* \right) + \boldsymbol{\alpha} \Psi \mathbf{z}_{T+1}^*$$

so both the data and the VEqCM forecasts are shifted by the 'policy-scenario' difference, $\boldsymbol{\alpha} \Psi \boldsymbol{\delta}_\rho$. The corresponding forecast errors, given (5.26), are $\widehat{\boldsymbol{\epsilon}}_{T+1|T}^a =$

$\Delta \mathbf{x}^a_{T+1} - \widehat{\Delta \mathbf{x}}^a_{T+1|T}$, so:

$$\widehat{\boldsymbol{\epsilon}}^a_{T+1|T} = \boldsymbol{\epsilon}_{T+1}. \tag{5.28}$$

Equation (5.28) has zero conditional and unconditional expectations. The DDV remains:

$$\widetilde{\Delta \mathbf{x}}^a_{T+1|T} = \Delta \mathbf{x}_T,$$

with forecast errors $\widetilde{\boldsymbol{\epsilon}}^a_{T+1|T} = \Delta \mathbf{x}^a_{T+1} - \widetilde{\Delta \mathbf{x}}^a_{T+1|T}$, so:

$$\widetilde{\boldsymbol{\epsilon}}^a_{T+1|T} = \boldsymbol{\alpha}\Psi\boldsymbol{\delta}_\rho + \boldsymbol{\alpha}\boldsymbol{\beta}'\Delta \mathbf{x}_T + \boldsymbol{\alpha}\Psi\left(\mathbf{z}^*_{T+1} - \mathbf{z}^+_T\right) + \Delta\boldsymbol{\epsilon}_{T+1},$$

which on average equals:

$$\mathsf{E}\left[\widetilde{\boldsymbol{\epsilon}}^a_{T+1|T} \mid \mathbf{z}_{T+1}, \mathbf{z}_T\right] = \boldsymbol{\alpha}\Psi\boldsymbol{\delta}_\rho + \boldsymbol{\alpha}\Psi\Delta\mathbf{z}_{T+1} + \boldsymbol{\alpha}\boldsymbol{\beta}'\boldsymbol{\alpha}\Psi\mathbf{z}^+_T. \tag{5.29}$$

Compared to (5.19) under no structural break, the errors in (5.29) are increased by $\boldsymbol{\alpha}\Psi\boldsymbol{\delta}_\rho$, which is also the unconditional bias, and the additional bias relative to the VEqCM when only a regime shift occurred. The variances remain as in §4.1.1. Thus, for a pure regime shift, the VEqCM is unequivocally better.

4.2.2 Two-periods ahead forecast errors. Now:

$$\begin{aligned}
\Delta \mathbf{x}^a_{T+2} &= \boldsymbol{\gamma} + \boldsymbol{\alpha}\left(\boldsymbol{\beta}'\mathbf{x}^a_{T+1} - \boldsymbol{\psi} + \Psi\boldsymbol{\rho}^*\right) + \boldsymbol{\alpha}\Psi\mathbf{z}^*_{T+2} + \boldsymbol{\epsilon}_{T+2} \\
&= \boldsymbol{\gamma} + \boldsymbol{\alpha}\Lambda\left(\boldsymbol{\beta}'\mathbf{x}_T - \boldsymbol{\psi} + \Psi\boldsymbol{\rho}^*\right) + \boldsymbol{\alpha}\Psi\mathbf{z}^*_{T+2} + \boldsymbol{\alpha}\boldsymbol{\beta}'\boldsymbol{\alpha}\Psi\mathbf{z}^*_{T+1} + \boldsymbol{\epsilon}_{T+2} + \boldsymbol{\alpha}\boldsymbol{\beta}'\boldsymbol{\epsilon}_{T+1},
\end{aligned}$$

as:

$$\left(\boldsymbol{\beta}'\mathbf{x}^a_{T+1} - \boldsymbol{\psi} + \Psi\boldsymbol{\rho}^*\right) = \Lambda\left(\boldsymbol{\beta}'\mathbf{x}_T - \boldsymbol{\psi} + \Psi\boldsymbol{\rho}^*\right) + \boldsymbol{\beta}'\boldsymbol{\alpha}\Psi\mathbf{z}^*_{T+1} + \boldsymbol{\beta}'\boldsymbol{\epsilon}_{T+1}.$$

Their 2-step ahead forecasting rules conditional on $\mathbf{z}^p_{T+2}$, $\mathbf{z}^p_{T+1}$ are respectively:

$$\begin{aligned}
\widehat{\Delta \mathbf{x}}^a_{T+2|T} &= \boldsymbol{\gamma} + \boldsymbol{\alpha}\left(\boldsymbol{\beta}'\widehat{\mathbf{x}}^a_{T+1|T} - \boldsymbol{\psi} + \Psi\boldsymbol{\rho}^*\right) + \boldsymbol{\alpha}\Psi\mathbf{z}^*_{T+2} \\
&= \boldsymbol{\gamma} + \boldsymbol{\alpha}\Lambda\left(\boldsymbol{\beta}'\mathbf{x}_T - \boldsymbol{\psi} + \Psi\boldsymbol{\rho}^*\right) + \boldsymbol{\alpha}\Psi\mathbf{z}^*_{T+2} + \boldsymbol{\alpha}\boldsymbol{\beta}'\boldsymbol{\alpha}\Psi\mathbf{z}^*_{T+1},
\end{aligned}$$

whereas the DDV still uses $\widetilde{\Delta \mathbf{x}}^{a}_{T+2|T} = \Delta \mathbf{x}_{T}$. The forecast errors are denoted by $\widehat{\boldsymbol{\epsilon}}^{a}_{\Delta,T+2|T} = \Delta \mathbf{x}^{a}_{T+2} - \widehat{\Delta \mathbf{x}}^{a}_{T+2|T}$ and $\widetilde{\boldsymbol{\epsilon}}^{a}_{\Delta,T+2|T} = \Delta \mathbf{x}_{T+2} - \widetilde{\Delta \mathbf{x}}^{a}_{T+2|T}$, so that conditional on $\mathbf{z}_{T+i}$:

$$\widehat{\boldsymbol{\epsilon}}^{a}_{\Delta,T+2|T} = \boldsymbol{\epsilon}_{T+2} + \boldsymbol{\alpha}\boldsymbol{\beta}'\boldsymbol{\epsilon}_{T+1},$$

and:

$$\begin{aligned}\widetilde{\boldsymbol{\epsilon}}^{a}_{\Delta,T+2|T} &= \boldsymbol{\alpha}\Lambda\left(\boldsymbol{\beta}'\mathbf{x}_{T} - \boldsymbol{\psi} + \Psi\boldsymbol{\rho}^{*}\right) - \boldsymbol{\alpha}\left(\boldsymbol{\beta}'\mathbf{x}_{T-1} - \boldsymbol{\psi} + \Psi\boldsymbol{\rho}\right) \\ &\quad + \boldsymbol{\alpha}\Psi\mathbf{z}^{*}_{T+2} + \boldsymbol{\alpha}\boldsymbol{\beta}'\boldsymbol{\alpha}\Psi\mathbf{z}^{*}_{T+1} - \boldsymbol{\alpha}\Psi\mathbf{z}^{+}_{T} \\ &\quad + \boldsymbol{\epsilon}_{T+2} + \boldsymbol{\alpha}\boldsymbol{\beta}'\boldsymbol{\epsilon}_{T+1} - \boldsymbol{\epsilon}_{T}.\end{aligned}$$

These have expected values:

$$\mathsf{E}\left[\widehat{\boldsymbol{\epsilon}}^{a}_{\Delta,T+2|T} \mid \mathbf{z}_{T+2}, \mathbf{z}_{T+1}\right] = \mathbf{0},$$

and:

$$\mathsf{E}\left[\widetilde{\boldsymbol{\epsilon}}^{a}_{\Delta,T+2|T} \mid \mathbf{z}_{T+2}, \mathbf{z}_{T+1}\right] = \boldsymbol{\alpha}\Lambda\Psi\boldsymbol{\delta}_{\rho} + \boldsymbol{\alpha}\Psi\mathbf{z}^{*}_{T+2} + \boldsymbol{\alpha}\boldsymbol{\beta}'\boldsymbol{\alpha}\Psi\mathbf{z}^{*}_{T+1} - \mathbf{D}\boldsymbol{\alpha}\Psi\mathbf{z}^{+}_{T}.$$

As $\mathsf{E}[\mathbf{z}^{*}_{T+i}] = \mathbf{0}$, then:

$$\mathsf{E}\left[\widetilde{\boldsymbol{\epsilon}}^{a}_{\Delta,T+2|T}\right] = \boldsymbol{\alpha}\Lambda\Psi\boldsymbol{\delta}_{\rho}.$$

Again, the regime shift acts as a post-forecasting break, and hence uniformly worsens the bias of the DDV relative to the VEqCM, exacerbating its variance loss.

4.3. Structural break and regime shift

We now allow for both the shift in policy regime from $\boldsymbol{\rho}$ to $\boldsymbol{\rho}^{*}$, affecting data from time $T+1$ onwards, and the previous deterministic shift at time T. Now, the regime shift might be in response to the forecasts from either the VEqCM or the DDV. Thus, $\mathbf{x}^{p}_{T+j}$ denotes the post-break and policy-change data, for $j = 1$, 2:

$$\Delta \mathbf{x}^p_{T+j} = \boldsymbol{\gamma}^* + \boldsymbol{\alpha}\left(\boldsymbol{\beta}'\mathbf{x}^p_{T+j-1} - \boldsymbol{\psi}^* + \Psi^*\boldsymbol{\rho}^*\right) + \boldsymbol{\alpha}\Psi^*\mathbf{z}^*_{T+j} + \boldsymbol{\epsilon}_{T+j}. \tag{5.30}$$

We denote post-policy forecasts by $\widehat{\Delta\mathbf{x}}^p_{T+i|T}$, and let $\mathbf{z}^p_{T+i} = \boldsymbol{\rho}^* + \mathbf{z}_{T+i} - \boldsymbol{\rho}$, so that $\mathbf{z}^*_{T+i} = \mathbf{z}^+_{T+i}$ as before.

4.3.1 One-period ahead. Now:

$$\Delta \mathbf{x}^p_{T+1} = \boldsymbol{\gamma}^* + \boldsymbol{\alpha}\left(\boldsymbol{\beta}'\mathbf{x}_T - \boldsymbol{\psi}^* + \Psi^*\boldsymbol{\rho}^*\right) + \boldsymbol{\alpha}\Psi^*\mathbf{z}^*_{T+1} + \boldsymbol{\epsilon}_{T+1},$$

whereas:

$$\widehat{\Delta\mathbf{x}}^p_{T+1|T} = \boldsymbol{\gamma} + \boldsymbol{\alpha}\left(\boldsymbol{\beta}'\mathbf{x}_T - \boldsymbol{\psi} + \Psi\boldsymbol{\rho}^*\right) + \boldsymbol{\alpha}\Psi\mathbf{z}^*_{T+1}. \tag{5.31}$$

The corresponding forecast error, given (5.30), is ($\widehat{\boldsymbol{\epsilon}}^p_{T+1|T} = \Delta\mathbf{x}^p_{T+1} - \widehat{\Delta\mathbf{x}}^p_{T+1|T}$):

$$\widehat{\boldsymbol{\epsilon}}^p_{T+1|T} = \boldsymbol{\delta}_\gamma - \boldsymbol{\alpha}\boldsymbol{\delta}_\psi + \boldsymbol{\alpha}\boldsymbol{\delta}_\Psi\left(\boldsymbol{\rho}^* + \mathbf{z}^*_{T+1}\right) + \boldsymbol{\epsilon}_{T+1}. \tag{5.32}$$

Equation (5.32) has the same conditional and unconditional expectation as (5.18) only when $\Psi^* = \Psi$, since:

$$\mathsf{E}\left[\widehat{\boldsymbol{\epsilon}}^p_{T+1|T} \mid \mathbf{z}_{T+1}, \mathbf{z}_T\right] = \boldsymbol{\delta}_\gamma - \boldsymbol{\alpha}\boldsymbol{\delta}_\psi + \boldsymbol{\alpha}\boldsymbol{\delta}_\Psi\left(\boldsymbol{\rho}^* + \mathbf{z}^*_{T+1}\right). \tag{5.33}$$

The DDV forecast remains:

$$\widetilde{\Delta\mathbf{x}}^p_{T+1|T} = \Delta\mathbf{x}_T,$$

with forecast error $\widetilde{\boldsymbol{\epsilon}}^p_{T+1|T} = \Delta\mathbf{x}^p_{T+1} - \widetilde{\Delta\mathbf{x}}^p_{T+1|T}$:

$$\widetilde{\boldsymbol{\epsilon}}^p_{T+1|T} = \boldsymbol{\alpha}\Psi^*\boldsymbol{\delta}_\rho + \boldsymbol{\alpha}\boldsymbol{\beta}'\Delta\mathbf{x}_T + \boldsymbol{\alpha}\Psi^*\left(\mathbf{z}^*_{T+1} - \mathbf{z}^+_T\right) + \Delta\boldsymbol{\epsilon}_{T+1},$$

which on average equals:

$$\begin{aligned}\mathsf{E}\left[\widetilde{\boldsymbol{\epsilon}}^p_{T+1|T} \mid \mathbf{z}_{T+1}, \mathbf{z}_T\right] &= \boldsymbol{\alpha}\Psi^*\boldsymbol{\delta}_\rho - \boldsymbol{\alpha}\left(\boldsymbol{\beta}'\boldsymbol{\alpha}\right)\left(\boldsymbol{\delta}_\psi - \boldsymbol{\delta}_\Psi\boldsymbol{\rho}\right) \\ &\quad + \boldsymbol{\alpha}\Psi^*\Delta\mathbf{z}^*_{T+1} - \left(\mathbf{I}_n - \boldsymbol{\alpha}\boldsymbol{\beta}'\right)\boldsymbol{\alpha}\Psi^*\mathbf{z}^+_T. \end{aligned} \tag{5.34}$$

Compared to (5.19), the errors in (5.34) are 'increased' by $\boldsymbol{\alpha}\Psi^{*}\boldsymbol{\delta}_{\rho}$ (there could be offsets between changes in parameters). When the only structural break is a change in Ψ to Ψ^{*} in response to the policy shift, (as in, say, the Lucas (1976), critique), then:

$$\mathsf{E}\left[\widehat{\boldsymbol{\epsilon}}^{p}_{T+1|T}\right]-\mathsf{E}\left[\widetilde{\boldsymbol{\epsilon}}^{p}_{T+1|T}\right]=\boldsymbol{\alpha}\left(\boldsymbol{\delta}_{\Psi}\boldsymbol{\rho}^{*}-\Psi^{*}\boldsymbol{\delta}_{\rho}-\boldsymbol{\beta}'\boldsymbol{\alpha}\boldsymbol{\delta}_{\Psi}\boldsymbol{\rho}\right),$$

which could take either sign for any element. Thus, despite agents possibly responding to a regime shift by a structural break, the VEqCM forecasts of the policy effect could be of value relative to the 'time-series' forecasts.

4.3.2 Two-periods ahead. Since:

$$\begin{aligned}\Delta\mathbf{x}^{p}_{T+2} &= \boldsymbol{\gamma}^{*}+\boldsymbol{\alpha}\left(\boldsymbol{\beta}'\mathbf{x}^{p}_{T+1}-\boldsymbol{\psi}^{*}+\Psi^{*}\boldsymbol{\rho}^{*}\right)+\boldsymbol{\alpha}\Psi^{*}\mathbf{z}^{*}_{T+2}+\boldsymbol{\epsilon}_{T+2}\\ &= \boldsymbol{\gamma}^{*}+\boldsymbol{\alpha}\Lambda\left(\boldsymbol{\beta}'\mathbf{x}_{T}-\boldsymbol{\psi}^{*}+\Psi^{*}\boldsymbol{\rho}^{*}\right)+\boldsymbol{\alpha}\Psi^{*}\mathbf{z}^{*}_{T+2}+\boldsymbol{\alpha}\boldsymbol{\beta}'\boldsymbol{\alpha}\Psi^{*}\mathbf{z}^{*}_{T+1}\\ &\quad +\boldsymbol{\epsilon}_{T+2}+\boldsymbol{\alpha}\boldsymbol{\beta}'\boldsymbol{\epsilon}_{T+1},\end{aligned}$$

the respective forecasting errors conditional on $\mathbf{z}^{p}_{T+2}$, $\mathbf{z}^{p}_{T+1}$ are:

$$\begin{aligned}\widehat{\Delta\mathbf{x}}^{p}_{T+2|T} &= \boldsymbol{\gamma}+\boldsymbol{\alpha}\left(\boldsymbol{\beta}'\widehat{\mathbf{x}}^{p}_{T+1|T}-\boldsymbol{\psi}+\Psi\boldsymbol{\rho}^{*}\right)+\boldsymbol{\alpha}\Psi\mathbf{z}^{*}_{T+2}\\ &= \boldsymbol{\gamma}+\boldsymbol{\alpha}\Lambda\left(\boldsymbol{\beta}'\mathbf{x}_{T}-\boldsymbol{\psi}+\Psi\boldsymbol{\rho}^{*}\right)+\boldsymbol{\alpha}\Psi\mathbf{z}^{*}_{T+2}+\boldsymbol{\alpha}\boldsymbol{\beta}'\boldsymbol{\alpha}\Psi\mathbf{z}^{*}_{T+1},\end{aligned}$$

for the VEqCM, as:

$$\widehat{\mathbf{x}}^{p}_{T+1|T}=\mathbf{x}_{T}+\boldsymbol{\gamma}+\boldsymbol{\alpha}\left(\boldsymbol{\beta}'\mathbf{x}_{T}-\boldsymbol{\psi}+\Psi\boldsymbol{\rho}^{*}\right)+\boldsymbol{\alpha}\Psi\mathbf{z}^{*}_{T+1},$$

so:

$$\left(\boldsymbol{\beta}'\widehat{\mathbf{x}}^{p}_{T+1|T}-\boldsymbol{\psi}+\Psi\boldsymbol{\rho}^{*}\right)=\Lambda\left(\boldsymbol{\beta}'\mathbf{x}_{T}-\boldsymbol{\psi}+\Psi\boldsymbol{\rho}^{*}\right)+\boldsymbol{\beta}'\boldsymbol{\alpha}\Psi\mathbf{z}^{*}_{T+1};$$

and the DDV remains $\widetilde{\Delta\mathbf{x}}^{p}_{T+2|T}=\Delta\mathbf{x}_{T}$. The forecast errors are denoted by $\widehat{\boldsymbol{\epsilon}}^{p}_{\Delta,T+2|T}=\Delta\mathbf{x}^{p}_{T+2}-\widehat{\Delta\mathbf{x}}^{p}_{T+2|T}$ and $\widetilde{\boldsymbol{\epsilon}}^{p}_{\Delta,T+2|T}=\Delta\mathbf{x}_{T+2}-\widetilde{\Delta\mathbf{x}}^{p}_{T+2|T}$, so that conditional on $\mathbf{z}_{T+i}$:

$$\begin{aligned}\widehat{\boldsymbol{\epsilon}}^{p}_{\Delta,T+2|T} &= \boldsymbol{\delta}_{\gamma} - \boldsymbol{\alpha}\Lambda\left(\boldsymbol{\delta}_{\psi} - \boldsymbol{\delta}_{\Psi}\boldsymbol{\rho}^{*}\right)\\ &\quad +\boldsymbol{\alpha}\boldsymbol{\delta}_{\Psi}\mathbf{z}^{*}_{T+2} + \boldsymbol{\alpha}\boldsymbol{\beta}'\boldsymbol{\alpha}\boldsymbol{\delta}_{\Psi}\mathbf{z}^{*}_{T+1}\\ &\quad +\boldsymbol{\epsilon}_{T+2} + \boldsymbol{\alpha}\boldsymbol{\beta}'\boldsymbol{\epsilon}_{T+1},\end{aligned}$$

and:

$$\begin{aligned}\widetilde{\boldsymbol{\epsilon}}^{p}_{\Delta,T+2|T} &= \boldsymbol{\alpha}\Lambda\left(\boldsymbol{\beta}'\mathbf{x}_{T} - \boldsymbol{\psi}^{*} + \Psi^{*}\boldsymbol{\rho}^{*}\right) - \boldsymbol{\alpha}\left(\boldsymbol{\beta}'\mathbf{x}_{T-1} - \boldsymbol{\psi}^{*} + \Psi^{*}\boldsymbol{\rho}\right)\\ &\quad +\boldsymbol{\alpha}\Psi^{*}\mathbf{z}^{*}_{T+2} + \boldsymbol{\alpha}\boldsymbol{\beta}'\boldsymbol{\alpha}\Psi^{*}\mathbf{z}^{*}_{T+1} - \boldsymbol{\alpha}\Psi^{*}\mathbf{z}^{+}_{T}\\ &\quad +\boldsymbol{\epsilon}_{T+2} + \boldsymbol{\alpha}\boldsymbol{\beta}'\boldsymbol{\epsilon}_{T+1} - \boldsymbol{\epsilon}_{T}.\end{aligned}$$

These have expected values:

$$\mathsf{E}\left[\widehat{\boldsymbol{\epsilon}}^{p}_{\Delta,T+2|T} \mid \mathbf{z}_{T+2}, \mathbf{z}_{T+1}\right] = \boldsymbol{\delta}_{\gamma} - \boldsymbol{\alpha}\Lambda\left(\boldsymbol{\delta}_{\psi} - \boldsymbol{\delta}_{\Psi}\boldsymbol{\rho}^{*}\right) + \boldsymbol{\alpha}\left(\boldsymbol{\delta}_{\Psi}\mathbf{z}^{*}_{T+2} + \boldsymbol{\beta}'\boldsymbol{\alpha}\boldsymbol{\delta}_{\Psi}\mathbf{z}^{*}_{T+1}\right),$$

and:

$$\begin{aligned}\mathsf{E}\left[\widetilde{\boldsymbol{\epsilon}}^{p}_{\Delta,T+2|T} \mid \mathbf{z}_{T+2}, \mathbf{z}_{T+1}\right] &= \boldsymbol{\alpha}\left(\mathbf{I}_{r} - \Lambda^{2}\right)\left(\boldsymbol{\delta}_{\psi} - \boldsymbol{\delta}_{\Psi}\boldsymbol{\rho}\right) + \boldsymbol{\alpha}\Lambda\Psi^{*}\boldsymbol{\delta}_{\rho}\\ &\quad +\boldsymbol{\alpha}\Psi^{*}\mathbf{z}^{*}_{T+2} + \boldsymbol{\alpha}\boldsymbol{\beta}'\boldsymbol{\alpha}\Psi^{*}\mathbf{z}^{*}_{T+1} - \mathbf{D}\boldsymbol{\alpha}\Psi^{*}\mathbf{z}^{+}_{T}.\end{aligned}$$

As $\mathsf{E}[\mathbf{z}^{*}_{T+i}] = \mathbf{0}$, then:

$$\mathsf{E}\left[\widehat{\boldsymbol{\epsilon}}^{p}_{\Delta,T+2|T}\right] = \boldsymbol{\delta}_{\gamma} - \boldsymbol{\alpha}\Lambda\left(\boldsymbol{\delta}_{\psi} - \boldsymbol{\delta}_{\Psi}\boldsymbol{\rho}^{*}\right), \tag{5.35}$$

and:

$$\mathsf{E}\left[\widetilde{\boldsymbol{\epsilon}}^{p}_{\Delta,T+2|T}\right] = \boldsymbol{\alpha}\left(\mathbf{I}_{r} - \Lambda^{2}\right)\left(\boldsymbol{\delta}_{\psi} - \boldsymbol{\delta}_{\Psi}\boldsymbol{\rho}\right) + \boldsymbol{\alpha}\Lambda\Psi^{*}\boldsymbol{\delta}_{\rho}. \tag{5.36}$$

4.4. Overview

We summarize the unconditional forecast-error biases in table 5.1. The coefficients represent the impacts of the different breaks on the two forecasting pro-

Table 5.1: Unconditional bias effects of structural breaks and regime shifts

	$\boldsymbol{\delta}_\gamma$	$\boldsymbol{\delta}_\psi$	$\boldsymbol{\delta}_\Psi\boldsymbol{\rho}^*$	$\boldsymbol{\delta}_\rho$	$\boldsymbol{\delta}_\Psi\boldsymbol{\delta}_\rho$
$\mathsf{E}\left[\widehat{\boldsymbol{\epsilon}}^p_{T+1\vert T}\right]$	$\mathbf{I}_n$	$-\boldsymbol{\alpha}$	$\boldsymbol{\alpha}$	$\mathbf{0}$	$\mathbf{0}$
$\mathsf{E}\left[\widetilde{\boldsymbol{\epsilon}}^p_{T+1\vert T}\right]$	$\mathbf{0}$	$\boldsymbol{\alpha}(\mathbf{I}_r-\Lambda)$	$-\boldsymbol{\alpha}(\mathbf{I}_r-\Lambda)$	$\boldsymbol{\alpha}\Psi^*$	$-\boldsymbol{\alpha}(\mathbf{I}_r-\Lambda)$
$\mathsf{E}\left[\widehat{\boldsymbol{\epsilon}}^p_{\Delta,T+2\vert T}\right]$	$\mathbf{I}_n$	$-\boldsymbol{\alpha}\Lambda$	$\boldsymbol{\alpha}\Lambda$	$\mathbf{0}$	$\mathbf{0}$
$\mathsf{E}\left[\widetilde{\boldsymbol{\epsilon}}^p_{\Delta,T+2\vert T}\right]$	$\mathbf{0}$	$\boldsymbol{\alpha}(\mathbf{I}_r-\Lambda^2)$	$-\boldsymbol{\alpha}(\mathbf{I}_r-\Lambda^2)$	$\boldsymbol{\alpha}\Lambda\Psi^*$	$-\boldsymbol{\alpha}(\mathbf{I}_r-\Lambda^2)$

cedures, one and two steps ahead, for growth rates. To clarify the patterns for longer horizons, the regime shift has been partitioned into the effect through any changes in the policy-reaction coefficient $\boldsymbol{\delta}_\Psi$, scaled by the second-regime policy mean, the policy mean change $\boldsymbol{\delta}_\rho$, and the interaction term $\boldsymbol{\delta}_\Psi\boldsymbol{\delta}_\rho$ (note that $-\boldsymbol{\alpha}(\mathbf{I}_r-\Lambda) = -\boldsymbol{\alpha}(\boldsymbol{\beta}'\boldsymbol{\alpha})$). Since the roots of Λ are inside the unit circle, $\Lambda^j \to \mathbf{0}$ as $j \to \infty$, so in the limit, for $\boldsymbol{\delta}_\psi$ and $\boldsymbol{\delta}_\Psi\boldsymbol{\rho}^*$, the DDV biases converge to the same magnitude, but the opposite sign, as the VEqCM at one-step, whereas the VEqCM biases converge to zero. Thus, only short-horizon benefits result from using the DDV as the baseline for such breaks. Conversely, the VEqCM is systematically wrong for changes in the growth rate $\boldsymbol{\gamma}$. Finally, the DDV biases from a regime shift converge to zero when $\boldsymbol{\delta}_\Psi = \mathbf{0}$. The table emphasises the different susceptibilities of the two approaches to the different shifts, thereby indicating possibilities for using each to 'correct' the other.

5. Policy-change corrections to robust forecasts

Any need to combine two disparate models on the same information set is evidence that both are incomplete: see Clements and Hendry (1998a). The encompassing principle argues for finding the congruent representation which can explain the failures of both models, but in the short-run that may prove infeasible. When the two models are differently susceptible to the causes of predictive failure, certain combinations could be beneficial: however, the relevant combination must reflect the motivation for pooling (namely, the impacts of breaks), rather than the usual grounds as discussed in (say) Bates and Granger (1969).

5.1. Pooling policy changes and DDV forecasts

The case of interest is when the robust forecast is made from the DDV, and that prompts a policy response to change the provisional setting $\mathbf{z}^p_{T+h}$ to the actual outcome $\mathbf{z}_{T+h}$. However, by construction, the DDV forecast is unaltered, so its forecast error changes one-for-one with the policy change. Since a deterministic shift happened one period earlier, a major change in $\Delta\mathbf{x}_{T+1}$ would just have occurred, inducing a correspondingly changed value for $\Delta\mathbf{x}_{T+2}$, and leading to forecast failure in the VEqCM. Conversely, forecasts from the open VEqCM are revised unconditionally by the difference between (5.14) and (5.31):

$$\mathsf{E}\left[\widehat{\Delta\mathbf{x}}^p_{T+1|T} - \widehat{\Delta\mathbf{x}}_{T+1|T}\right] = \boldsymbol{\alpha}\Psi\boldsymbol{\delta}_\rho.$$

Under the assumptions used here, the change in the realization over what it would have been provisionally, namely the difference between (5.13) and (5.30), is:

$$\mathsf{E}\left[\Delta\mathbf{x}^p_{T+1} - \Delta\mathbf{x}_{T+1}\right] = \boldsymbol{\alpha}\Psi^*\boldsymbol{\delta}_\rho. \tag{5.37}$$

If the policy-reaction matrix remained constant ($\Psi^* = \Psi$), the econometric model would correctly predict the impact of the regime shift, despite the deterministic structural break. Otherwise, the policy-reaction mistake is:

$$\boldsymbol{\alpha}\boldsymbol{\delta}_\Psi\boldsymbol{\delta}_\rho.$$

The DDV forecast error due to the policy change is equal to (5.37). Consequently, a combined forecast of the form:

$$\overline{\Delta\mathbf{x}}_{T+1|T} = \widetilde{\Delta\mathbf{x}}^p_{T+1|T} + \widehat{\Delta\mathbf{x}}^p_{T+1|T} - \widehat{\Delta\mathbf{x}}_{T+1|T} \tag{5.38}$$

implies an unconditional forecast-error bias from (5.34) of ($\overline{\boldsymbol{\epsilon}}_{T+1|T} = \Delta\mathbf{x}^p_{T+1} -$

$\overline{\Delta \mathbf{x}}_{T+1|T}$):

$$\mathsf{E}\left[\overline{\boldsymbol{\epsilon}}_{T+1|T}\right]=\mathsf{E}\left[\Delta \mathbf{x}^{p}_{T+1}-\Delta \mathbf{x}_{T}\right]-\boldsymbol{\alpha}\Psi\boldsymbol{\delta}_{\rho}=\boldsymbol{\alpha}\left[\boldsymbol{\delta}_{\Psi}\boldsymbol{\delta}_{\rho}+\left(\mathbf{I}_{r}-\Lambda\right)\left(\boldsymbol{\delta}_{\psi}-\boldsymbol{\delta}_{\Psi}\boldsymbol{\rho}\right)\right],$$

which avoids much of the structural break, yet captures some, and possibly all, of the policy effect. This exploits the fact that the DDV is robust to the past change in the intercept, whereas the VEqCM takes account of the current change in policy. Further, as the modification from the VEqCM is deterministic, the combined procedure has the same variance as the DDV forecast would have had in the absence of policy change ($\boldsymbol{\rho}^{*}=\boldsymbol{\rho}$), so after the change, for $\Psi^{*}=\Psi$, (5.38) dominates both the DDV and the VEqCM forecasts in mean, but loses to the latter in variance.

Similarly, at two-periods ahead, let:

$$\overline{\Delta \mathbf{x}}_{T+2|T}=\widetilde{\Delta \mathbf{x}}^{p}_{T+2|T}+\widehat{\Delta \mathbf{x}}^{p}_{T+2|T}-\widehat{\Delta \mathbf{x}}_{T+2|T},$$

then, as:

$$\mathsf{E}\left[\widehat{\Delta \mathbf{x}}^{p}_{T+2|T}-\widehat{\Delta \mathbf{x}}_{T+2|T}\right]=\boldsymbol{\alpha}\Lambda\Psi\boldsymbol{\delta}_{\rho}$$

for $\overline{\boldsymbol{\epsilon}}_{\Delta,T+2|T}=\Delta \mathbf{x}^{p}_{T+2}-\overline{\Delta \mathbf{x}}_{T+2|T}$:

$$\mathsf{E}\left[\overline{\boldsymbol{\epsilon}}_{\Delta,T+2|T}\right]=\boldsymbol{\alpha}\left[\Lambda\boldsymbol{\delta}_{\Psi}\boldsymbol{\delta}_{\rho}+\left(\mathbf{I}_{r}-\Lambda^{2}\right)\left(\boldsymbol{\delta}_{\psi}-\boldsymbol{\delta}_{\Psi}\boldsymbol{\rho}\right)\right].$$

When $\Psi^{*}=\Psi$, so the policy response does not change:

$$\mathsf{E}\left[\overline{\boldsymbol{\epsilon}}_{\Delta,T+2|T}\right]=\boldsymbol{\alpha}\left(\mathbf{I}_{r}-\Lambda^{2}\right)\boldsymbol{\delta}_{\psi}=-\boldsymbol{\alpha}\boldsymbol{\beta}'\boldsymbol{\alpha}\left(\mathbf{I}_{r}+\Lambda\right)\boldsymbol{\delta}_{\psi}, \qquad (5.39)$$

which compares favourably with (5.36), and will be smaller than (5.35) when the roots $\boldsymbol{\beta}'\boldsymbol{\alpha}$ are small. As before, there is no variance impact from the scenario-change correction. We now illustrate the empirical relevance of such combinations.

6. Empirical illustration: DHSY revisited

Davidson, Hendry, Srba, and Yeo (1978) developed an equilibrium-correction model of constant-price UK consumers' expenditure on non-durables and services (c) as a function of real personal disposable income (i) and annual inflation ($\Delta_4 p_t$), where lower case letters denote logs and $\Delta_4 = (1 - L^4)$ when L is the lag operator. The sample period was 1959(2)–1976(2) after initial values for lags, less 8 observations for forecasts. On estimating a variant of their model, our results are (all computations and graphics were produced by GiveWin and PcFiml: see Doornik and Hendry (1996) and Doornik and Hendry (1997)):

$$\begin{aligned}\Delta_4 c_t &= \underset{(0.04)}{0.25}\ \Delta_4 i_t + \underset{(0.04)}{0.24}\ \Delta_4 i_{t-1} - \underset{(0.11)}{0.42}\ \Delta_4 p_t + \underset{(0.11)}{0.35}\ \Delta_4 p_{t-1} \qquad (5.40)\\ &\quad - \underset{(0.015)}{0.086}\ (c-i)_{t-4} + \underset{(0.22)}{0.66}\ \Delta_4 d_t,\\ \mathsf{R}^2 &= 0.958 \ \ \widehat{\sigma} = 0.0061 \ \ SC = -9.92\\ \mathsf{F}_{\mathsf{ar}}(5,54) &= 0.25 \ \ \mathsf{F}_{\mathsf{arch}}(4,51) = 1.62 \ \ \mathsf{F}_{\mathsf{het}}(12,46) = 0.81 \ \ \mathsf{F}_{\mathsf{res}}(1,58) = 0.10\\ \chi^2(2) &= 0.39 \ \ \mathsf{F}_{\mathsf{Ch}}(8,59) = 0.79 \ \ V = 0.092 \ \ Jt = 1.31\end{aligned}$$

In (5.40), d_t is a dummy variable equal to zero except in 1968(1)–(2) when it takes the values $+0.01$, -0.01, so its coefficient is interpretable as a percentage effect: $\widehat{\sigma}$ denotes the residual standard deviation, expressed as a percentage of the level of the associated variable, and SC is the Schwarz criterion (see e.g., Hendry (1995a)). The residual diagnostic tests are of the form $\mathsf{F}_j(k, T-l)$, which denotes an F-test against the alternative hypothesis j for: 5^{th}-order serial correlation (F_{ar}; see Godfrey (1978)), 4^{th}-order autoregressive conditional heteroscedasticity ($\mathsf{F}_{\mathsf{arch}}$; see Engle (1982)), heteroscedasticity ($\mathsf{F}_{\mathsf{het}}$; see White (1980)), the RESET test ($\mathsf{F}_{\mathsf{res}}$; see Ramsey (1969)), a parameter constancy test over 1974(3)–1976(2) (F_{Ch}; see Chow (1960)), a chi-square test for normality ($\chi^2_{\mathsf{nd}}(2)$; see Doornik and Hansen (1994)), and the variance-change and joint-parameter constancy tests from Hansen (1992) (denoted Jt and V): * and ** denote significance at the 5% and 1% levels respectively. In (5.40), we have left $\Delta_4 p_t$ to enter freely, rather than as part of a cointegrating relation - which the results in Davidson, Hendry, Srba, and Yeo (1978) could be interpreted as supporting - as we wish to consider models that exclude inflation. The cointegration relation $c_t - i_t$ is the log of the average propensity to consume, and so is denoted apc_t in the sequel. For later analyses of the performance of (5.40), see *inter alia* Hendry and von Ungern-Sternberg (1981), Davis (1982), Birchenhall, Bladen-Hovell, Chui, Osborn, and Smith (1989), Carruth and Henley (1990), Hendry, Muellbauer, and Murphy (1990), Muellbauer and Murphy (1989), Harvey and Scott (1994), Hendry (1994) and Muellbauer (1994).

Here, we embed their model, denoted by the acronym DHSY, in a 3-equation VAR for c_t, i_t and $\Delta_4 p_t$ and replicate the main features of their results. Next, we drop the inflation variable from the system, and develop a model for $(c_t,\ i_t)$ which reproduces the consumption-income relation, but fails on forecast tests (and did so at the time). Thus, inflation, responding to the impact of the first 'Oil crisis', induced a shift in the equilibrium mean of apc_t, and our test period – commencing in 1974(3) – is after that shift. We also develop a 'time-series' model for c_t which does not fail on forecasting, but which would not respond to policy changes that affected income, such as altered income-tax rates. Then we generate new data from the DHSY system treating it as the DGP, but with income *growth* increased by 2.5% throughout the forecast period. Since the in-sample data are unaltered, the time-series model produces identical forecasts of the changed data, but the policy-modified econometric model delivers altered forecasts. Finally, we use the difference between these two 'runs' of the econometric model (a measure of the policy effect) to intercept-correct the time-series forecast, to reflect both income-tax changes and the hidden effects of the omitted variable which induced the structural break.

6.1. A three-equation VAR

The variables $(c_t,\ i_t,\ \Delta_4 p_t)$ were treated as $\mathsf{I}(1)$ and analyzed over the whole sample using a VAR with 5 lags, including a constant, linear deterministic trend, and $\Delta_4 d_t$. Table 5.2 shows the individual equation and system goodness-of-fit and evaluation statistics. Vector tests are shown as $\mathsf{F}_{\mathsf{j}}^{\mathsf{v}}(k, T-l)$, and their outcomes are consistent with a congruent system.

Table 5.2: System goodness of fit and evaluation

statistic	c	i	$\Delta_4 p$	VAR
$\widehat{\sigma}$	0.88%	2.06%	0.82%	
$\mathsf{F}_{\mathsf{ar}}(5,46)$	0.88	2.06	0.19	
$\mathsf{F}_{\mathsf{arch}}(4,43)$	0.81	0.25	3.55^{*}	
$\mathsf{F}_{\mathsf{het}}(32,18)$	0.36	0.44	1.63	
$\chi^2_{\mathsf{nd}}(2)$	2.65	3.02	2.90	
$\mathsf{F}^{\mathsf{v}}_{\mathsf{ar}}(45,101)$				1.08
$\mathsf{F}^{\mathsf{v}}_{\mathsf{het}}(192,85)$				0.54
$\chi^{2\,\mathsf{v}}_{\mathsf{nd}}(6)$				10.6

Table 5.3: System residual cross correlations

	c	i
i	0.71	–
$\Delta_4 p$	−0.47	−0.31

Table 5.3 records the inter-correlation structure of the residuals, which reveals important features to model in all the equations, but we will focus on those between c and $(i,\, \Delta_4 p)$. The eigenvalues of the long-run matrix are -0.71, and $-0.23 \pm 0.08\iota$, using ι to denote $\sqrt{-1}$ to avoid confusion with income, i, so the rank is non-zero, and is unlikely to be three given the data. The system dynamics are represented in table 5.4 by the eigenvalues of the companion matrix (denoted λ), where we also record the modulus $(|\lambda|)$. These eigenvalues are difficult to interpret, comprising the four roots of unity, a further unit root, and four large

Table 5.4: System dynamics

λ	-1	ι	$-\iota$	$0.95 \pm 0.32\iota$	$0.89 \pm 0.15\iota$	$0.13 \pm 0.84\iota$
$\lvert\lambda\rvert$	1	1	1	1.00, 1.00	0.90, 0.90	0.84, 0.84

λ	$-0.51 \pm 0.55\iota$	-0.48	-0.68	$0.51 \pm 0.26\iota$
$\lvert\lambda\rvert$	0.75, 0.75	0.48	0.68	0.57, 0.57

complex roots, with the remainder neither zero nor unity. To understand their composition, consider the simplest version of the VAR written as:

$$\begin{aligned}\Delta_4 c_t &= -0.1\,(c-i)_{t-4} + v_{1,t} \\ \Delta_4 i_t &= v_{2,t} \\ \Delta_4 p_t &= \Delta_4 p_{t-1} + v_{3,t}.\end{aligned}$$

This system has 15 eigenvalues: six are zero, with $\pm 1.0\iota$, ± 1.0, $\pm 0.974\iota$, ± 0.974, and 1, thereby inducing the four roots of unity, the extra unit root, and the four large roots, with the zeroes replaced by non-zero values in table 5.4, corresponding to the additional short-run dynamics. Thus, despite the five unit roots, the data are $\mathsf{I}(1)$.

For c_t, lags 3–5 were significant (on $\mathsf{F}(3,49)$, at 5% or less), for i_t, lags 1 and 5, and for $\Delta_4 p_t$, only lag 1: the trend was insignificant. As fig. 5.1 shows, the first two equations are constant, with their 1-step residuals having constant 95% confidence bands, but the equation for inflation is not constant: as a consequence, the system break-point Chow (1960) tests lie above the 1% critical values for a short period.

6.2. Cointegration

The fitted and actual values of this system in levels have correlations of 0.998, 0.992 and 0.991, so we turn to reductions to $\mathsf{I}(0)$. The cointegration analysis restricted the trend to the cointegration space, and yielded table 5.5 (see Doornik and Hendry (1997), Banerjee, Dolado, Galbraith, and Hendry (1993), Johansen (1995), and Doornik, Hendry, and Nielsen (1998)). For each value of the rank r of the long-run matrix in the Johansen (1988) procedure, table 5.5 reports the log-likelihood values (ℓ), eigenvalues (μ) and associated maximum eigenvalue (Max) and trace (Tr) statistics together with the estimated cointegrating vectors ($\widehat{\boldsymbol{\beta}}$) and feedback coefficients ($\widehat{\boldsymbol{\alpha}}$). Although the null of no cointegration is not rejected at conventional $\mathsf{I}(1)$ critical values (even ignoring the degrees-of-freedom corrections to the Tr statistic suggested by Reimers (1992)), given that (5.40) has a feedback coefficient with a t-value of 7 when the constant is excluded, the first cointegrating vector may be a consumers' expenditure relation. Consistent with

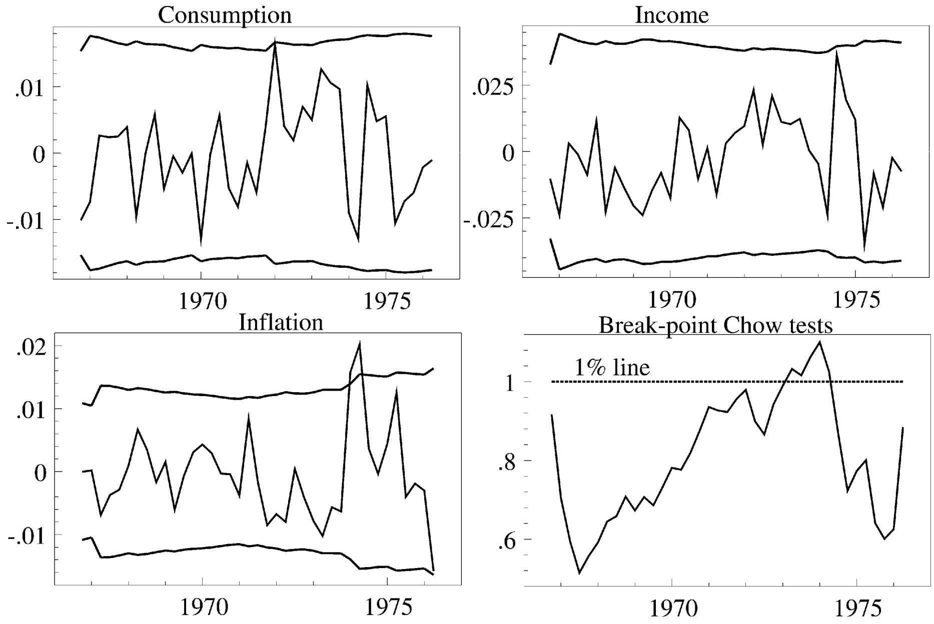

Figure 5.1: System recursive graphical statistics

this, when the trend is excluded from the cointegration relation, the coefficients for the income and inflation 'elasticities' become 0.93 and -0.69 respectively ($\chi^2(1) = 0.30$). The sizes of the other feedback coefficients might suggest a violation of long-run weak exogeneity, but enforcing that together with a unit income elasticity yielded $\chi^2(4) = 6.7$. The results are shown in table 5.6. These results are close to the long-run relation in (5.40) so their long-run analysis was not much distorted by being single equation.

Table 5.5: Cointegration analysis

r	1	2	3
ℓ	980	986	990
μ	0.21	0.17	0.11
Max	15.9	12.7	7.8
Tr	36.4	20.4	7.8

$\widehat{\boldsymbol{\alpha}}$	1	2	3
c	−0.18	0.31	−0.04
i	0.63	0.32	−0.12
$\Delta_4 p$	−0.26	−0.38	−0.01

$\widehat{\boldsymbol{\beta}}'$	c	i	$\Delta_4 p$	t
1	1	−0.68	0.45	−0.0014
2	−2.28	1	−0.44	0.0063
3	−1.93	2.40	1	−0.0075

Table 5.6: Restricted cointegration analysis

	$\widehat{\boldsymbol{\alpha}}$	SE
c	−0.10	(0.036)
i	0	(−)
$\Delta_4 p$	0	(−)

	c	i	$\Delta_4 p$	t
$\widehat{\boldsymbol{\beta}}'$	1	−1	1.18	0
SE	−	−	(0.21)	(−)

Table 5.7: DHSY vector model goodness of fit and diagnostics

statistic	value
$\mathsf{F}^{\mathsf{v}}_{\mathsf{ar}}(36,124)$	0.79
$\mathsf{F}^{\mathsf{v}}_{\mathsf{het}}(204,103)$	0.93
$\chi^{2\,\mathsf{v}}_{\mathsf{nd}}(6)$	11.2
$\mathsf{F}^{\mathsf{v}}_{\mathsf{Ch}}(24,56)$	1.20

6.3. A simultaneous-equations model

A model of the system was developed by sequential simplification in fourth differences for c and i, and first differences for $\Delta_4 p$, incorporating the DHSY feedback term. This yielded (5.41).

DHSY vector-model FIML estimates

$$\begin{aligned}
\Delta_4 c_t &= \underset{(0.08)}{0.27}\,\Delta_4 i_t + \underset{(0.07)}{0.21}\,\Delta_4 i_{t-1} - \underset{(0.19)}{0.22}\,\Delta_4 p_t + \underset{(0.19)}{0.14}\,\Delta_4 p_{t-1} \\
&\quad - \underset{(0.017)}{0.081}\,apc_{t-4} + \underset{(0.24)}{0.67}\,\Delta_4 d_t \\
\Delta_4 i_t &= \underset{(0.29)}{0.69}\,\Delta_4 c_{t-1} + \underset{(0.14)}{0.53}\,\Delta_4 i_{t-1} - \underset{(0.12)}{0.55}\,\Delta_4 i_{t-4} \\
&\quad + \underset{(0.12)}{0.35}\,\Delta_4 i_{t-5} + \underset{(0.006)}{0.004} \qquad (5.41) \\
\Delta_1\Delta_4 p_t &= \underset{(0.08)}{0.14}\,\Delta_1\Delta_3 p_{t-1} + \underset{(0.07)}{0.28}\,\Delta_4 c_{t-2} + \underset{(0.10)}{0.28}\,\Delta_4 c_{t-4} \\
&\quad - \underset{(0.05)}{0.17}\,\Delta_4 i_{t-5} - \underset{(0.002)}{0.007} \\
apc_t &\equiv apc_{t-4} + \Delta_4 c_t - \Delta_4 i_t
\end{aligned}$$

The likelihood-ratio test of the over-identifying restrictions on the $\mathsf{I}(0)$ VAR yielded $\chi^2(38) = 44.7$, and table 5.7 reports the model goodness-of-fit and diagnostic statistics. On the vector diagnostic tests, the model is congruent with the sample evidence, and remains constant over the forecast period. The residual

Table 5.8: DHSY vector model residual correlations

$$\begin{bmatrix} & c & i \\ i & 0.01 & - \\ \Delta_4 p & -0.27 & -0.50 \end{bmatrix}$$

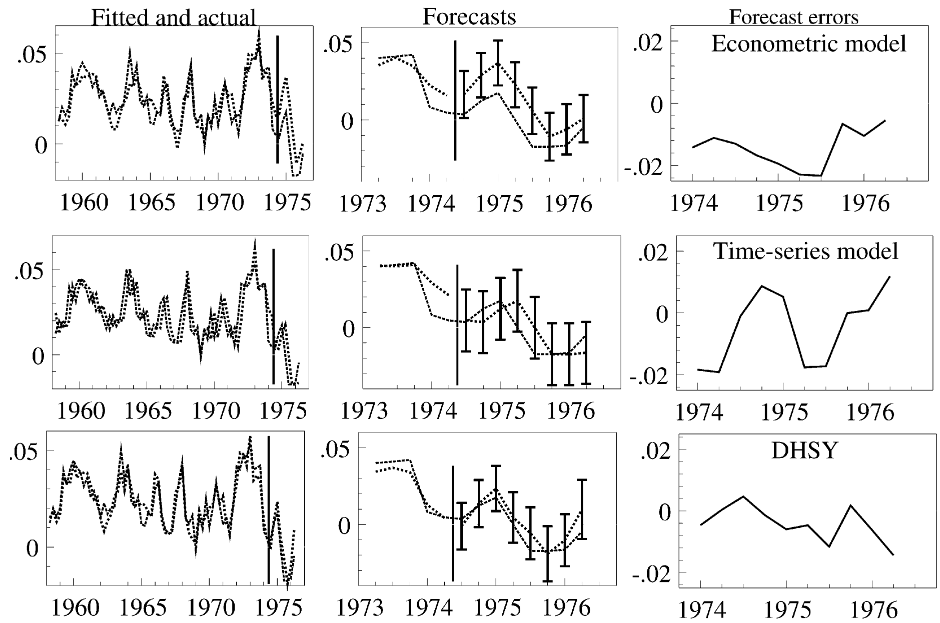

Figure 5.2: Fitted and actual values, 1-step forecasts, and forecast errors from all models

$\widehat{\sigma}$s are 0.64%, 2.00%, and 0.74%: and their cross correlations for the consumption equation are dramatically smaller than those in the VAR, consistent with finding closely similar estimates in OLS and FIML. For the rest of the paper, we treat (5.41) as the DGP against which to compare the remaining findings. Figure 5.2 reports the comparative forecast accuracy of (5.41), a mis-specified version which omits inflation, and a 'time-series' model, described in the next two sections.

6.4. A mis-specified econometric model

The natural mis-specification to consider is one where the econometrician omits inflation from the analysis of c and i, as DHSY did initially, since this induces a

Table 5.9: Mis-specified model statistics

statistic	value
$\mathsf{F}^{\mathsf{v}}_{\mathsf{ar}}(16,94)$	0.89
$\mathsf{F}^{\mathsf{v}}_{\mathsf{het}}(72,87)$	1.03
$\chi^{2\,\mathsf{v}}_{\mathsf{nd}}(4)$	3.57
$\mathsf{F}^{\mathsf{v}}_{\mathsf{Ch}}(16,56)$	2.95**

shift in the equilibrium mean of apc_t, causing the model to suffer forecast failure. Accordingly, we now develop such a bivariate system. As the form of analysis is close to that of the previous model, we only record the final model in (5.42) and its statistics.

Mis-specified model FIML estimates

$$
\begin{aligned}
\Delta_4 c_t &= \underset{(0.07)}{0.35}\,\Delta_4 i_t + \underset{(0.06)}{0.15}\,\Delta_4 i_{t-1} - \underset{(0.010)}{0.049}\,apc_{t-4} + \underset{(0.27)}{0.60}\,\Delta_4 d_t \\
\Delta_4 i_t &= \underset{(0.33)}{0.93}\,\Delta_4 c_{t-1} - \underset{(0.30)}{0.35}\,\Delta_4 c_{t-1} + \underset{(0.16)}{0.45}\,\Delta_4 i_{t-1} \\
&\quad - \underset{(0.14)}{0.47}\,\Delta_4 i_{t-4} + \underset{(0.13)}{0.37}\,\Delta_4 i_{t-5} + \underset{(0.007)}{0.007} \\
apc_t &\equiv apc_{t-4} + \Delta_4 c_t - \Delta_4 i_t
\end{aligned}
\qquad (5.42)
$$

The residual $\widehat{\sigma}$s are 0.73% and 1.94%, the residual cross-correlation is -0.11 and the likelihood-ratio test of the over-identifying restrictions is $\chi^2(16) = 24.6$. Table 5.9 reports the model goodness-of-fit and diagnostic statistics. The forecast test strongly rejects the null of parameter constancy, whereas the in-sample tests easily accept. Despite the forecast failure, the consumption-income nexus is well modelled, and remains close to that in the postulated (DHSY) DGP.

6.5. A 'time-series' model

The time-series analyst is assumed to have differenced the data twice, to remove both seasonal and $\mathsf{I}(1)$ unit roots, and so investigates $\Delta_1\Delta_4 c_t$ and $\Delta_1\Delta_4 i_t$: see

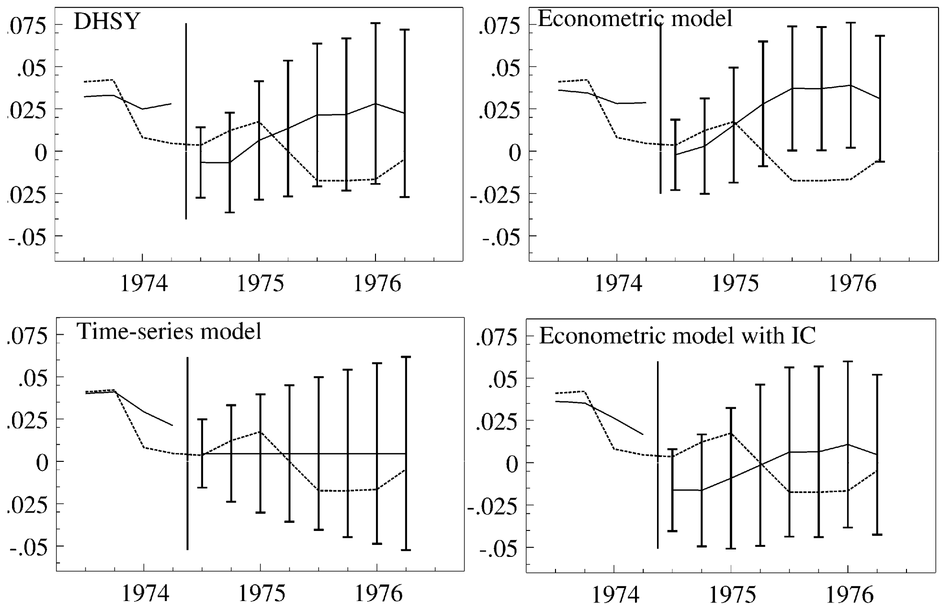

Figure 5.3: 8-step ahead forecasts of $\Delta_4 c_t$ from all models

Prothero and Wallis (1976). However, (5.43) treats these variables as unrelated, mimicking a univariate analysis of the former to illustrate the policy analysis aspects.

$$
\begin{array}{rcll}
\multicolumn{4}{l}{\text{`Time-series' model FIML estimates}} \\
\Delta_1\Delta_4 c_t & = & \underset{(0.32)}{1.30}\ \Delta_4 d_t & \widehat{\sigma} = 1.01\% \\
\Delta_1\Delta_4 i_t & = & \underset{(-)}{0.00} & \widehat{\sigma} = 2.48\%
\end{array}
\tag{5.43}
$$

In sample, the residual standard deviations greatly exceed those of the mis-specifed econometric model, and many of the diagnostic statistics reject the adequacy of this model, with some of the mis-specifications due to the moving-average error induced by over-differencing, and others perhaps to the resulting incorrectly-estimated standard errors: see table 5.10. However, modelling the moving-average error would lose some of the robustness to deterministic shifts (see Clements and Hendry (1999a): also, Clements and Hendry (1997) investi-

Table 5.10: 'Time-series' model statistics

statistic	value
$\mathsf{F}^{\mathsf{v}}_{\mathsf{ar}}(16, 104)$	4.97**
$\mathsf{F}^{\mathsf{v}}_{\mathsf{het}}(18, 147)$	2.03*
$\chi^{2\,\mathsf{v}}_{\mathsf{nd}}(4)$	6.78
$\mathsf{F}^{\mathsf{v}}_{\mathsf{Ch}}(16, 60)$	1.06

gate the impact of seasonal shifts on forecasts). Despite the considerable non-congruency on these in-sample tests, the forecast test does not reject over the same period that the 'econometric model' failed. This reflects better forecasting and not just incorrectly-wider confidence bands, as fig. 5.2 shows, where the first row records the outcomes in the space of $\Delta_4 c_t$ for the 'econometric model', with the 'time-series model' in the second row, and the DHSY model in the third row.

Nevertheless, (5.43) is constructed to deliver the same forecasts before and after the income-tax change, although (5.42) would reflect the consequences thereof if the policy effect were included (this is to mimic the role of an 'exogenous' policy variable). Thus, it may be possible to beat the 'time-series' forecasts in such a setting, as §6.7 considers.

6.6. Dynamic forecasts

The above analysis recorded the sequence of 1-step forecasts, so we now evaluate the forecast performances on 8-step forecasts, including an attempt to intercept correct (IC) the 'econometric' model using the residual at the forecast origin to set it back on track (see Clements and Hendry (1996b)). The outcomes for $\Delta_4 c_t$ are shown in fig. 5.3, and for levels in fig. 5.4. The IC improves the mis-specified econometric model, but would have worsened DHSY if also used there.

These 8-step results are after the break induced by the oil crisis, so we also record 12-step forecasts as a set of pre-break forecasts. As anticipated from the theory in Clements and Hendry (1999a), all the pre-break outcomes are poor, and relatively similar – (5.43) is indeed not robust to unanticipated breaks, although it remains the least affected.

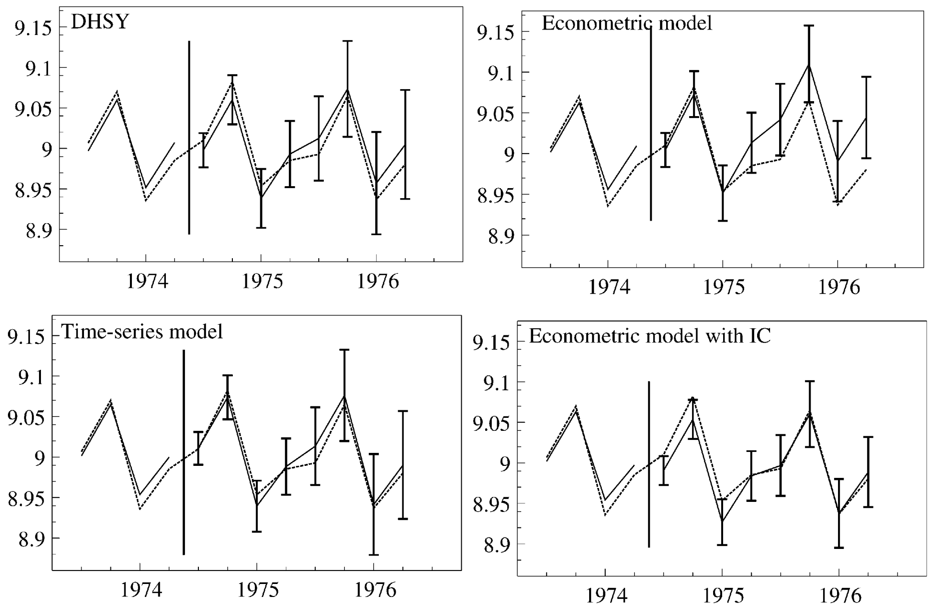

Figure 5.4: 8-step ahead forecasts for c_t from all models

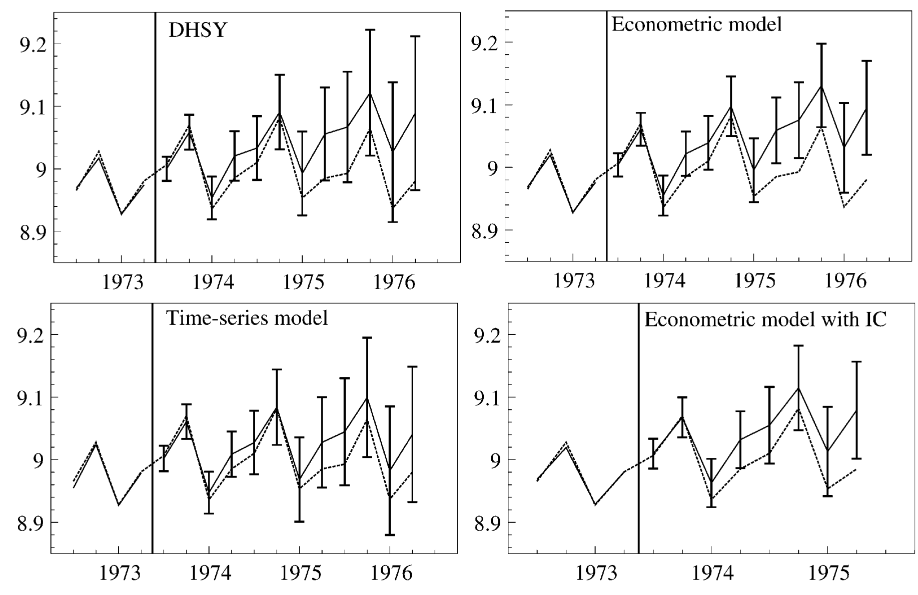

Figure 5.5: 12-step ahead forecasts for c_t from all models

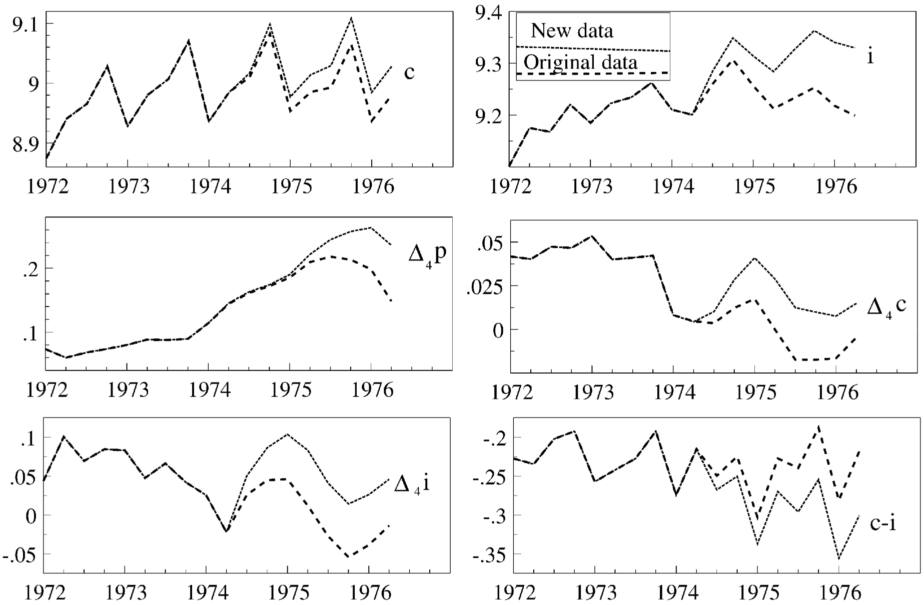

Figure 5.6: Original and post-policy data for c, y, $\Delta_4 p$, $c-y$, $\Delta_4 c$ and $\Delta_4 y$

6.7. Post-policy forecasts

The final stage was to construct the data that would have resulted after a substantial policy change aimed at preventing the large fall in income and expenditure that actually occurred. To simulate a large income-tax reduction, income was increased by 2.5%, over what it would otherwise have been, by adding 0.025 to i using an indicator variable for the remainder of the forecast period. The data on $\Delta_4 p$, i and c were sequentially generated, observation by observation, using the coefficients in (5.41) and adding on its residuals. Thus, had the policy indicator been zero, the original data would have been reproduced precisely by this process. Figure 5.6 compares the original and post-policy data, showing that the policy successfully raised expenditure, but also induced some additional inflation from the cross-equation feedbacks.

Next, each of the four models was used to forecast this altered future data. We already know that the time-series forecasts are unaltered, so those errors change to the extent the data are shifted. The DHSY and mis-specified econometric models include the policy dummy with an imposed coefficient of unity. The IC

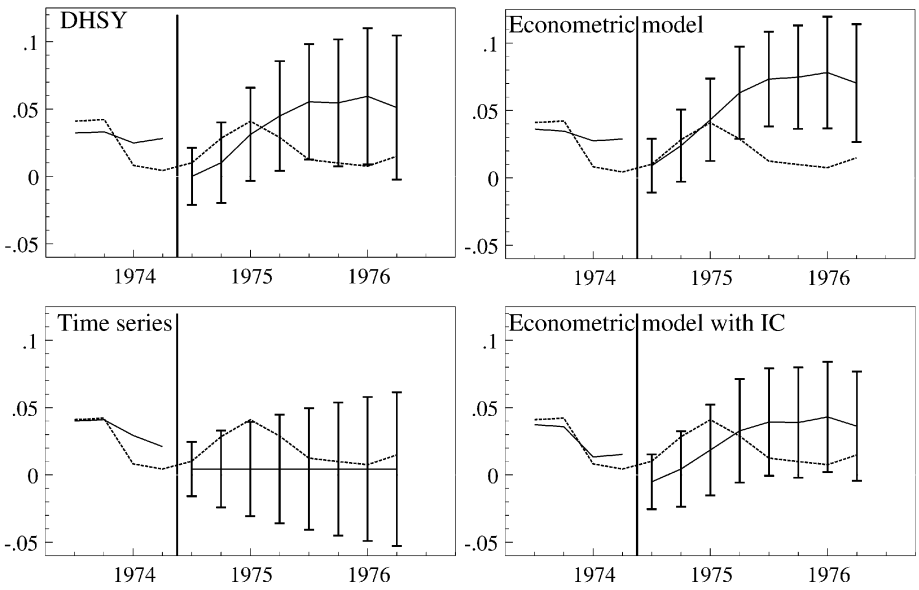

Figure 5.7: 8-step ahead forecasts of $\Delta_4 c_t$ from all models on the post-policy data

model needed two non-zero periods before the forecast to avoid perfect collinearity with the policy indicator, but otherwise was unaltered. Figure 5.7 records the four sets of 8-step forecasts. The time-series forecasts remain useful, especially compared to those from the econometric model, and the great improvement from intercept-correcting the latter is obvious.

Finally, we now 'scenario correct' using the theory developed above, by computing the differences between the trajectories of the econometric models with and without the policy indicator, and add that to the time-series forecasts to have a 'doubly-robust' forecast as in (5.38). Figure 5.8 reports the comparative policy responses of the DHSY and econometric models. As can be seen, the policy responses generated by the econometric model are close to those from the DHSY DGP, despite the former omitting inflation. From this perspective, the econometric model remains valuable for policy, and would have correctly predicted the impact of the regime shift. In turn, this suggests the scenario changes could be a useful basis for 'correcting' the time-series forecasts, since the data have been shifted by the amounts computed by the DHSY DGP.

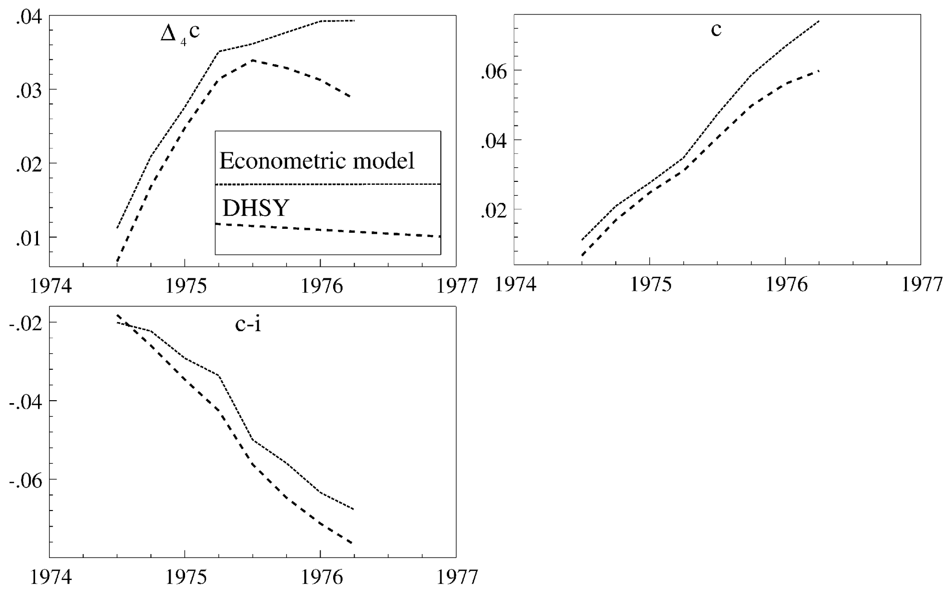

Figure 5.8: Comparative policy responses of DHSY and econometric model

Figure 5.9 reports the forecast errors from all the methods for comparison.

There is a clear benefit from the new form of intercept correction relative to the econometric models' forecasts, and the accuracy is close to that achieved by the extended DHSY system. However, the time-series forecasts remain accurate even if not corrected. Table 5.11 records the percentage forecast errors and their standard deviations (TS, Ect, and DHSY, respectively denote (5.43), (5.42) and (5.41); IC is the intercept-corrected econometric model; and TS(+Ect) is the scenario-corrected TS forecasts based on adding the differences between the Ect based on the new and the original data, and TS(+IC) is the scenario-corrected TS using the difference between IC based on the new and original data.[2]

Thus, for the new data TS is the most accurate, with IC close, and on mean forecast error, the latter does best. Both TS(+Ect) and TS(+IC) perform reasonably, but do not dominate because the policy shift happens to induce an insignificant positive bias in TS (see fig. 5.9c). It is interesting how poorly the

[2] The two sets of DHSY outcomes differ only because the policy indicator needed a small non-zero value in-sample to allow estimation.

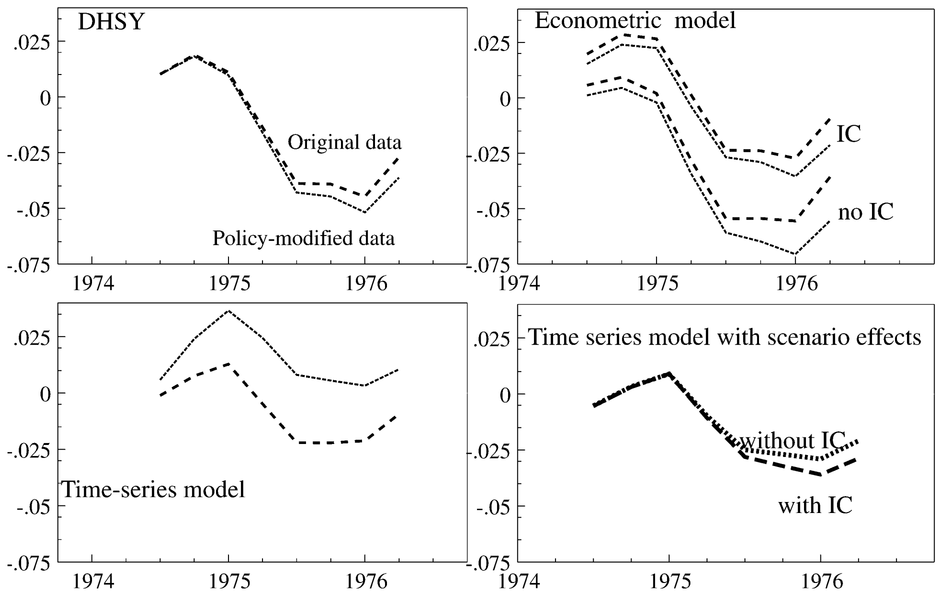

Figure 5.9: 8-step ahead forecast errors from all models on the post-policy data for $\Delta_4 c_t$

			Original Data			
	Model	DHSY	Ect	TS	IC	
	Mean	-1.54	-2.64	-0.75	-0.10	
	SD	2.58	2.84	1.36	2.35	
			New Data			
Model	DHSY	Ect	TS	IC	TS(+Ect)	TS(+IC)
Mean	-1.92	-3.53	1.48	-0.68	-1.61	-1.31
SD	2.85	3.20	1.21	2.46	1.45	1.73

Table 5.11: 8-period ahead forecast-error means and standard deviations

Original Data						
Model	DHSY	Ect	TS	IC		
Mean	0.67	−0.28	0.36	1.91		
SD	1.40	1.71	0.80	1.24		
New Data						
Model	DHSY	Ect	TS	IC	TS(+Ect)	TS(+IC)
Mean	0.55	−0.77	2.27	1.45	−0.10	−0.07
SD	1.49	1.80	1.27	1.28	0.87	0.84

Table 5.12: 4-period ahead forecast-error means and standard deviations

DHSY forecasting model performs, given it is the DGP.

Differences are more dramatic when the first four-period ahead forecasts are considered as in table 5.12.

Now, for the new data TS(+IC) is a clear winner, closely followed by TS(+Ect), with TS on the original data next best. Thus, the scenario corrections can be useful in modifying a statistical device for forecasting.

7. Conclusion

The main conclusions relate to the three issues posed in the introduction. The dominance in forecasting of an econometric policy model by a purely statistical device is not sufficient to sustain the use of the latter for policy: a statistical forecasting procedure which embodies no links between target variables and policy instruments has no implications for economic policy analysis, so outperforming on forecasting is clearly insufficient to justify policy analysis. Further, since the sources of forecast failure may be unrelated to the policy issue under analysis, forecast dominance does not by itself demonstrate the invalidity of the econometric model for the policy: the empirical example illustrated this proposition. However, combining robustified forecasts with policy-scenario changes may dominate either alone in a world subject to regime shifts: forecasting procedures designed to be robust to deterministic shifts that have occurred prior to forecasting could be improved by 'intercept correcting' them using the policy-change effects entailed by the econometric model. For short-horizon forecasts, the UK consumers' expenditure model illustrated this result. These findings exploited the different forecast biases of the various models to breaks pre and post forecasting, discussed in Hendry and Clements (1999) and Clements and Hendry (1999b), but ignored the variance consequences.

The present paper is more in the form of an existence theorem for the combination of robust forecasts and policy-change corrections than a practical manifesto, in that we have not yet developed criteria for when the proposal will outperform. The usual 'combination of forecasts' approach (see e.g., Bates and Granger (1969), Diebold (1989), and Coulson and Robins (1993)) does not seem appropriate, since in-sample correlations between forecast errors are unlikely to be a useful guide when deterministic shifts occur. Moreover, the corrections proposed above involve the difference between two dynamic forecasts of the econometric model, and not simply its second set of forecasts. A first step would be to determine when a deterministic shift occurred just before the forecast origin, and we are currently developing directed tests for such an event. That would enhance the decision to adopt a robust device. A second step would involve checking if the policy predictions from the econometric system remained reliable in the face of the shift, which is bound to involve judgement, perhaps supported by the results of tests of parameter invariance to previous shifts (see e.g., Favero and Hendry (1992), Engle and Hendry (1993), and Ericsson and Irons (1995)), and of the policy relevance of the model (see Granger and Deutsch (1992)). We have assumed that the in-sample econometric model coincides with the DGP, and allowing for model mis-specification and estimation must weaken the results. An alternative we are also investigating is using the time-series forecasts to 'intercept correct' the post-policy forecasts of the econometric model: as analyzed in Clements and Hendry (1998b), this may provide a useful route to avoiding forecast failure when structural breaks and regime shifts occur.

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