

Antitrust Leniency with Multi-Product Colluders*

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Abstract

We use a global games approach to model alternative implementations of an antitrust leniency program as applied to multi-product colluders. We derive several policy design lessons; e.g., we show that linking leniency across products may increase the likelihood of conviction in the first product investigated but reduce it in subsequent products. Thus, firms may have an incentive to form sacrificial cartels and apply for leniency in less valuable products to reduce convictions in more valuable products. This undesirable effect can be mitigated by cartel profiling, but cartel profiling also reduces the probability of conviction in the first product investigated.

Keywords: Penalty Plus, Cooperation Policy, Cartels, Collusion, Multimarket Contact

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1 Introduction

In recent years, antitrust leniency programs in the United States, European Union, Australia, and elsewhere have played an important role in allowing competition authorities to successfully prosecute major price fixing conspiracies.¹ A review of the European Commission (EC) decisions in cartel cases for 2001–2012 shows that a firm received a 100% reduction in the fine through the leniency program in 55 (54%) of the 101 products in which firms were prosecuted.²

Table 1: EC cartel cases 2001–2012 with a firm receiving a 100% fine reduction based on the leniency program

Cases in which a firm received a 100% fine reduction		
Airfreight	Elevators and escalators	Methylglucamine
Aluminium Fluoride	Exotic fruit (bananas)	Monochloroacetic Acid
Animal Feed Phosphates	Fine art Auction Houses	Mountings for windows and window-doors
Bananas	Fittings	Needles
Bathroom fittings & fixtures	Food flavour enhancers	Netherlands beer market
Bitumen Nederland	Freight forwarding	Organic peroxide
Bitumen Spain	Gas insulated switchgear	Power transformers
Calcium carbide	Hard haberdashery: fasteners	Prestressing steel
Candle waxes	Heat stabilisers	Refrigeration compressors
Carbonless paper	Hydrogen peroxide	Rubber chemicals
Chloroprene rubber	Industrial bags	Sodium Chlorate
Choline chloride	LCD	Sorbates
Consumer Detergents	Luxembourg brewing industry	Specialty graphite
Copper plumbing tubes	Marine hoses	Synthetic rubber (BR/ESBR)
CRT glass bulbs	Methacrylates	Vitamins
DRAM	Methionine	Water management products
Electrical and mechanical carbon and graphite products		

Antitrust leniency programs can take different forms and have evolved over time.³ One of the key changes to the U.S. antitrust leniency program in 1993 was to allow firms

¹“The Antitrust Division’s Leniency Program is its most important investigative tool for detecting cartel activity. Corporations and individuals who report their cartel activity and cooperate in the Division’s investigation of the cartel reported can avoid criminal conviction, fines, and prison sentences if they meet the requirements of the program.” (United States Department of Justice website, <http://www.justice.gov/atr/public/criminal/leniency.html>, accessed October 22, 2012). As Chairman of the Australian Competition and Consumer Commission (ACCC), Graeme Samuel stated that ACCC’s Immunity Policy for Cartel Conduct was “absolutely vital” in the Australian government’s efforts to crack cartels and credited it with exposing potential cases at the rate of about one a month (Beaton-Wells and Fisse, 2011, p.379). See also Beaton-Wells (2008a, 2008b) and Wils (2007).

²Some EC decisions apply to more than one product. For example, the EC decision in Vitamins covers multiple vitamin products, with a separate application of the leniency program for each product. The EC’s leniency program also offers smaller fine reductions for cooperators other than the first to apply for leniency. In 87 (86%) of the products, a firm received some reduction in the fine. In the United States, an official at the DoJ has stated that, in addition to the initial leniency applicant, as many as four firms may receive a “substantial assistance” discount on their fine of as much as 25–30%. (Statements of Lisa Phelan, head of the National Criminal Enforcement Section, at the 61st ABA Antitrust Law Spring Meeting, April 10–12, 2013, as reported by MLex, “Up to Four Companies Can Be ‘Second-In’ To Get Antitrust Cooperation Discount, Official Says,” April 10, 2013.) In Australia, only one firm can obtain a discount under the Immunity Policy for Cartel Conduct, but others may obtain a discount under the Cooperation Policy.

³For a description of the evolution of U.S. and EC leniency programs, see Wils (2008a, Chapter 5).

to apply for leniency even after the DoJ had received information about illegal antitrust activity (so-called Type B leniency).⁴ Changes to the EU antitrust leniency program in 2002 also allowed for leniency after an investigation had been opened.⁵ U.S. programs directed at multi-product colluders include Amnesty Plus, introduced in 1999, under which a firm being prosecuted for collusion that has not received leniency can qualify for reduced fines if it applies for leniency in a separate product in which it is also engaged in collusion,⁶ and Penalty Plus, under which the failure to report collusion in separate products can put firms at risk for increased penalties should they later be prosecuted for collusion in those products.⁷ In addition, there have been changes related to the treatment of ringleaders, the scope for individual leniency, and the use of “carve outs” that can exclude certain individuals from being covered under corporate leniency.⁸

Just as policies related to antitrust leniency have evolved, undoubtedly so have cartel strategies for dealing with leniency. This raises questions about cartel strategies to undermine or even benefit from leniency policies. As stated by Wils (2008a, p.137):

[S]uccessful cartels tend to be sophisticated organisations, capable of learning. It is thus safe to assume that cartel participants will try to adapt their organisation to leniency policies, not only so as to minimise the destabilising effect, but also, where possible, to exploit leniency policies to facilitate the creation and maintenance of cartels. This raises the question whether there could be features of leniency programmes that risk being exploited to perverse effects.

⁴“A company will qualify for leniency even after the Division has received information about the illegal antitrust activity, whether this is before or after an investigation is formally opened, if the following [seven] conditions are met:” (Hammond and Barnett, 2008, p.5) According to Motta and Polo (2003, p.349), “The key mechanism of leniency programs is the rule that allows firms to receive fine reductions even after an investigation is opened.”

⁵See Spagnolo (2008, Section 7.2.2) and Stephan (2009, p.554 and Table 4). In Australia, leniency applications are permitted until the ACCC has received written legal advice that it has sufficient evidence to commence proceedings in the case.

⁶See Lefouili and Roux (2012) for a discussion and theoretical model of Amnesty Plus. See also Wils (2008a, Chapter 5.4.4).

⁷See Masoudi (2007, p.8).

⁸The DoJ may “carve out” individuals from the protection of corporate leniency, including, historically, “culpable employees, employees who refuse to cooperate with the Division’s investigation, and employees against whom the Division is still developing evidence.” (Scott D. Hammond, Deputy Assistant Attorney General for Criminal Enforcement, Antitrust Division, U.S. Department of Justice, “Charting New Waters in International Cartel Prosecutions,” March 2, 2006, available at <http://www.usdoj.gov/atr/public/speeches/214861.htm>). However, recently, the DoJ has indicated that it “will no longer carve out employees for reasons unrelated to culpability,” which presumably includes a refusal to cooperate (Bill Baer, Assistant Attorney General Press Release, Antitrust Division, U.S. Department of Justice, “Changes to Antitrust Division’s Carve-Out Practice Regarding Corporate Plea Agreements,” April 12, 2013, available at <http://www.justice.gov/opa/pr/2013/April/13-at-422.html>).

In this paper we focus on the effect of leniency policies on multi-product colluders.⁹ The list of firms engaged in collusion in more than one product is long. Table 2, which is based on EC cartel cases, lists multi-product colluders that have received a 100% fine reduction through the leniency program in at least one of the products in which they were prosecuted. Table 3 shows firms colluding in three or more products that did not receive a complete fine reduction in any of the products where they were prosecuted.¹⁰

Table 2: Multi-product colluders that received a complete fine reduction in at least one product in EC cartel cases 2001–2012

Firm	Number of products	No fine reduction	Incomplete fine reduction	Complete fine reduction
Akzo Nobel	9	2	4	3
Takeda	6	4	1	1
Aventis	5		2	3
William Prym	5	1	3	1
Bayer	4		2	2
KONE	4	1	1	2
Otis	4		3	1
Degussa	3		1	2
Merck	3	1	1	1
Samsung	3		1	2
Shell	3	2		1
ABB Ltd	2	1		1
Boliden	2		1	1
BP	2			2
Chemtura	2			2
Chiquita	2			2
DHL and Exel	2			2
GrafTech International	2			2
Kemira Oyj	2	1		1
Mueller	2			2
Siemens	2	1		1

Table 3: Multi-product colluders that colluded in three or more products and did not receive a complete fine reduction in any product in EC cartel cases 2001–2012

Firm	Number of products	No fine reduction	Incomplete fine reduction	Complete fine reduction
Roche	13	4	9	
BASF	11	2	9	
Arkema	6	3	3	
Coats	6	3	3	
Elf Aquitaine	4	1	3	
Schindler	4	3	1	
SGL	4		4	
ThyssenKrupp	4	2	2	
AC Treuhand	3	3		
Barbour Threads	3	1	2	
Hitachi	3	3		
Schenker	3	1	2	
Toshiba	3	3		
UPS	3	3		
YKK	3	1	2	

We construct a model that allows us to examine the effects on multi-product colluders of different implementations of an antitrust leniency program by a competition authority.

⁹In what follows, we sometimes refer to “cartels” in more than one product, but these may be appropriately viewed as components of a single overarching conspiracy.

¹⁰There were an additional 38 multi-product colluders that were colluding in only two products and that did not receive a complete fine reduction in either product.

We developed our model using information obtained in detailed interviews with defense attorneys experienced in taking firms through the leniency process at the DoJ. Based on these interviews, corporate leniency applications occur under three general sets of circumstances: applications under Type A leniency, which means the DoJ has not yet opened an investigation; applications under Type B leniency, which means the DoJ has already opened an investigation; and follow-up leniency applications, where a firm being prosecuted for collusion in one product applies for leniency in a separate product. The division between Type A and Type B leniency is approximately 80-90% Type B and 10-20% Type A.¹¹ In the model, we focus on type B leniency and follow-up leniency applications.¹²

An application for type B leniency would unfold as follows: Potential collusion in a product comes to the attention of the DoJ, perhaps because buyers of the product or their trade association have approached the DoJ with economic circumstantial evidence suggestive of collusion.¹³ The DoJ opens an investigation. When the colluding firms become aware of the investigation, they retain outside legal counsel. It is natural to expect firms to become aware of an investigation at approximately the same time because public information would be available to all and subpoenas would typically be served

¹¹In the United States, the DoJ maintains the confidentiality of leniency applicants, although in some cases the identity of a leniency applicant is available through other sources. In Europe, EC decisions in cartel cases identify leniency applicants. A review of these cases shows that the percentage of cases in which a firm applies for leniency prior to the start of an investigation by the EC is greater than the 10-20% indicated for the United States. However, in many of these cases, it may be that the firm was applying for leniency in Europe as a response to an investigation in the United States. According to Bloom (2007), roughly half of the leniency applications received by the EC follow leniency applications in the United States: “One important factor that is likely to lead to an overestimate of the success of the EC leniency program is where applications to the Commission either followed on from those to the US Department of Justice (DOJ) or were simultaneous. The prime aim of any applicant is normally to avoid US criminal sanctions. But once a US investigation is stimulated by an amnesty application, other authorities will start investigations as they become aware at some stage of the US one. Hence applications need to be made simultaneously to other authorities or as soon as possible after one to the DOJ. It is the US powers rather than the EC (or other jurisdiction) powers which drive these applications. However, if the applicants could not secure leniency in the EC as well as the US it is highly likely that a significant proportion of them would not apply for US amnesty as they would not be able to avoid heavy EC fines. In approaching half of the EC cases from 2000 there was a prior or simultaneous application for amnesty under the US program.” (Bloom, 2007, pp.8–9).

¹²An application for type A leniency would unfold as in the case of type B leniency, except that events are typically triggered when the involvement of the firm in potentially illegal activity comes to the attention of an employee, who would typically report the concerns to the firm’s general counsel, who decides whether to bring in outside counsel to investigate.

¹³As stated in Wils (2008a, pp.127–128), aside from obtaining information from colluding firms themselves, competition authorities could “monitor markets, observing publicly available information and data, and possibly use economic analysis of these data to try to detect and prove violations” or “customers or competitors harmed by antitrust violations may bring complaints to the authorities, and third parties may otherwise volunteer to provide information.” See Kovacic, Marshall, Marx, and White (2011) on the use of economic evidence to prove the existence of a cartel.

on the same day. Outside counsel contacts the DoJ to find out whether leniency is still available. If it is, counsel starts an internal investigation at the firm to assess whether the firm has been engaged in illegal activity, in particular whether there is sufficient evidence to enable the firm to admit definitively to a violation of the antitrust laws. Counsel reports the results of the investigation to the firm's board of directors. The board will weigh the tradeoffs between applying for leniency and not. Because this scenario plays out in all of the colluding firms at roughly the same time, firms must be concerned that co-conspirators will beat them in the race to be first to apply for leniency.

In all cases, firms being prosecuted for collusion are asked if there are any other products in which they are colluding. At that point, the board of directors must make decisions related to that. If the firm denies colluding in other products, and if the DoJ later incurs the expense to investigate and prosecute the firm's activities in another product, the firm would not necessarily have the option of applying for leniency in that product, and individuals might be vulnerable to prosecution for obstruction of justice and/or perjury.

Our model focuses on leniency applications that are triggered either by the initiation of a DoJ investigation (type B leniency) or by the prosecution of a firm for collusion in a separate product. Whether the cartel is successfully prosecuted depends on a number of factors, including (i) whether the potential existence of the cartel comes to the attention of the competition authority, (ii) the strength of the evidence uncovered by the competition authority's investigation, and (iii) whether cartel members apply for leniency. If more than one cartel member applies for leniency, then only one, chosen at random, is designated as receiving leniency. If a cartel is successfully prosecuted, cartel members not covered by the leniency policy are fined.

We show that leniency programs enhance the detection of cartels but that the incentives for leniency application and hence the probability of successful prosecutions are affected by linkages across markets in the antitrust leniency program. Specifically, a penalty-plus antitrust leniency program that asks firms convicted of collusion to attest to whether or not they are colluding in any other product markets can increase leniency applications in the first product investigated but reduce the probability of prosecution in the other products. Such a linkage in the leniency program can create incentives for firms to form sacrificial cartels and apply for leniency in small products where penalties would be limited in order to reduce the probability of conviction in larger, more valuable products. We show that these undesirable effects can be mitigated by cartel profiling, that is by increasing the probability of investigation for other products produced by firms found to be engaged in collusion. At the same time, by reducing the incentives of firms to apply for leniency in the first product under investigation for collusion, cartel profiling reduces the probability of conviction in the first product investigated. We also show that

the effectiveness of leniency programs for detecting cartels is improved if there is a greater likelihood that firms' internal investigations into possible antitrust offenses will be successful, which suggests there is value in policies that enhance cooperation by employees and facilitate the discovery of incriminating evidence. We consider implications for the allocation of antitrust enforcement resources and show that resources directed at investigations and prosecutions are strategic complements for generating convictions and that resources must be devoted to both investigation and prosecution in order for a leniency program to be effective in terms of improving detection and deterrence of cartels.

In Section 2, we discuss related literature. In Section 3, we present the model and provide a benchmark result for the case without a leniency program. In Section 4 we identify the continuation equilibrium in the second market under investigation, while in Section 5 we derive the full equilibrium for both a standard leniency and a penalty-plus leniency program. Section 6 contains the main policy insights of the paper. Section 7 concludes and argues that our model and its insights have wider applicability, as they apply to any situation where a group of agents in a coalition (e.g., a criminal organization or gang) face an external threat to the stability of their relationship (e.g., by law enforcement).

2 Literature

There is a substantial economics literature on antitrust leniency.¹⁴ The theoretical literature, including Chen and Rey (2013), Choi and Gerlach (2012b), Lefouili and Roux (2012), Harrington (2008), Chen and Harrington (2007), Aubert, Rey, and Kovacic (2006), Buccirosi and Spagnolo (2005, 2006), Spagnolo (2004), Motta and Polo (2003), and Spagnolo (2000), has focused on repeated games models and on the self-enforcement of a cartel structure. The collusive behavior is supported as an equilibrium in a supergame without need for communication and without interfirm transactions.¹⁵ In the context of these models, one can analyze how the range of discount factors or the range of collusive payoffs under which collusion can be supported is affected by various implementations of leniency programs. These papers provide important insights related to the optimal design of leniency programs. In general, they suggest that the introduction of a leniency program makes it more difficult for firms to support collusion, although they recognize that to the extent that leniency programs reduce expected fines, they may reduce deterrence.

A different approach is taken by Harrington (2011), who considers the case of a cartel that has ended, so deviations from the collusive agreement are no longer an issue, but where the threat remains that firms might disclose the cartel to authorities and apply for

¹⁴For surveys, see Rey (2003) and Spagnolo (2008). See also Wils (2008a, Chapter 5).

¹⁵See Green, Marshall, and Marx (2013) for a discussion of the role of communication in supporting collusion.

leniency. Harrington (2011) assumes, as we do, that the firms face uncertainty over the probability that the cartel will be discovered and prosecuted in the absence of a leniency applicant, but his model differs in many ways from ours. Other approaches are taken by Brisset and Thomas (2004), who provide an auction-based model, and Motchenkova (2004), who considers an optimal stopping model. Angelucci and Han (2012) consider the interaction of leniency with the within-firm principal-agent problem. For empirical analysis of leniency, see Sokol (2012), Zhou (2012), Miller (2009), and Stephan (2009), and for experimental results, see Bigoni et al. (2012a,b) and Hinlopen and Soetevent (2008).

The literature has also addressed the potential for the strategic use of leniency by cartels. The potential benefits to a cartel from explicitly including leniency applications in their collusive strategy in order to obtain the benefits of reduced fines are considered by Chen and Rey (2013), Chen and Harrington (2007), Spagnolo (2004), and Motta and Polo (2003). This literature suggests that generous leniency programs may be exploited by cartels.

Our approach differs in two fundamental ways from the existing literature. First, we are interested in understanding the incentive to be the first firm to apply for leniency after an investigation has been started by the competition authority. To do so, we abstract from dynamic self-enforcing constraint and use instead a modeling approach based on global games to solve the coordination game induced by a leniency program.¹⁶ Second, our focus is on multi-product colluders.¹⁷

Coordination games commonly result in multiple equilibria. For example, if a firm expects its co-conspirator to apply for leniency, then the firm expects to be prosecuted, so it would typically have an incentive also to apply, hoping to be first in the door and avoid paying a fine. But if a firm expects that its co-conspirators will not apply for leniency, then it may be a best response also not to apply if that allows collusive profits to continue. The theory of global games has shown that often the existence of multiple equilibria relies on common knowledge of payoffs, but that if players have private information, the equilibrium is unique (see Carlsson and van Damme, 1993a, 1993b; and Morris and

¹⁶Often colluding firms are able to set up the necessary structures to control secret deviations, such as the pricing allocation, and enforcement structures outlined by Stigler (1964). For further discussion of collusive structures, see Marshall and Marx (2012, Chapter 6). For example, the colluding firms may have taken steps to limit their ability to achieve large one-time payoffs through deviations, for example by putting in place contractual restrictions governing their relationship with the input market that prevent them from being able to expand their output to take advantage of any opportunities they might have for secret price cutting. These provisions, together with effective monitoring structures ensure that any feasible deviation by a cartel member would be small and detected immediately and so could be quickly rectified through interfirm transfers.

¹⁷See Choi and Gerlach (2012a) on the effects on multi-product cartels of demand linkages among products.

Shin, 2002).¹⁸ The theory of global games presents a natural way to look at the issue of leniency, where each player has two main strategies and where it is natural to view the probability of conviction as not being common knowledge, but known with error by the firms in a cartel. Although the coordination game aspect of leniency applications typically generates multiple equilibria and is a key issue in studying the effects of leniency programs, the global games approach allows us to identify a unique Bayesian equilibrium that survives iterated elimination of dominated strategies.

3 Model

We consider two symmetric firms that have chosen to form an illegal cartel in each of two markets.¹⁹ Consistent with the U.S. experience, we focus on leniency applications that happen after the cartel is under investigation by the competition authority (type B leniency). If a firm comes under investigation by the competition authority in one of the two markets, the firm’s board of directors brings in outside counsel to do an internal investigation. Such an investigation leaves open the possibility that insufficient evidence is uncovered to support a leniency application even though a cartel was, in fact, active. This is especially true because the ability of a firm’s outside antitrust counsel to uncover evidence of collusion depends on the cooperation of managers with knowledge of the conspiracy, whose interest may be to avoid detection.²⁰ Furthermore, as part of their service firms that offer cartel management services provide counseling on avoiding detection and the maintenance of incriminating documents at a site out of the reach of key antitrust authorities, making detection more difficult.²¹

¹⁸Applications of global games results related to financial market issues include Morris and Shin (1998, 1999), Danielson et al. (2001), Heinemann and Illing (2002), Hellwig (2002), Metz (2002), and Prati and Sbracia (2002). Other contributions to the global games literature include Levin (2001), who examines a global game with overlapping generations, and Chamley (1999) and Angeletos, Hellwig, and Pavan (2007), who consider models of regime change. Frankel, Morris, and Pauzner (2003) prove an equilibrium uniqueness result for a family of supermodular, global games, with an arbitrary number of players and actions. See also extensions by Giannitsarou and Toxvaerd (2012) and Ordoñez (2012). Heinemann, Nagel, and Ockenfels (2004) study global games in an experimental context and obtain results consistent with the global game solution.

¹⁹We discuss later how our results could be extended to more general settings where firms are not symmetric, there are more than two firms, and the cartels in the two markets are composed by different firms; see footnotes 26 and 29.

²⁰By cooperating, a manager promotes the prosecution of the cartel, which would potentially leave the manager labeled as someone who has engaged in illegal price fixing, fired from his or her current position, and have severe future career consequences. Furthermore, if a manager cooperates, the firm may not get leniency, or if it does, that manager may be “carved out” by the antitrust authority from the corporate leniency agreement and so face criminal prosecution. (See footnote 8.)

²¹For example, colluding firms might expend resources to engage a third party facilitator for the cartel that could manage incriminating evidence. The EC Decision in *Organic Peroxides*, states that the cartel

If the internal investigation does not uncover evidence that would allow a leniency application, which happens with probability $1 - \rho$, then there is no option of applying for leniency. If the internal investigation at firm i related to product j does uncover evidence, which happens with probability ρ , then the investigation also provides outside counsel with a signal θ_{ij} as to the probability τ_j that the cartel would be prosecuted in the absence of any leniency applicant. Outside counsel then advises the board of directors on next steps and the board of directors makes the choice between applying for leniency or not. At the time of this choice, the board does not know whether the internal investigation at the other firm has uncovered evidence sufficient to allow a leniency application by that firm, or if it has, what choice was made by the other firm.

We assume the firms are symmetric and that π_j is each firm’s payoff in product $j \in \{1, 2\}$ when it does not apply for leniency and is not prosecuted. A firm’s payoff when it is successfully prosecuted and fined (with no leniency granted) is $-f\pi_j$. We let $-\ell\pi_j$ be the payoff when granted leniency in product j , where $\ell < f$, so that the payoff is higher than when prosecuted without applying for leniency.²²

Table 4: Payoffs in the model for product j

Outcome	Payoffs
Not caught	π_j
Caught and granted leniency	$-\ell\pi_j$
Caught and pay fines	$-f\pi_j$

The timeline is as follows:

maintained certain documents at the premises of the consulting firm AC Treuhand in Switzerland: “[AC Treuhand] produced, distributed and recollected the so called ‘pink’ and ‘red’ papers with the agreed market shares which were, because of their colour, easily distinguishable from other meeting documents and were not allowed to be taken outside the AC Treuhand premises.” (EC Decision in *Organic Peroxides* at par. 92(b)) In addition, AC Treuhand “reimbursed the travel expenses of the participants, in order to avoid traces of these meetings in the companies’ accounts” (par. 92(d)) and “instructed all participants on the legal dangers of parts of these meetings and on what measures to take to avoid detection of these arrangements’ bearing on Europe.” (par. 92(j)) One would expect this type of strategy to reduce the ability of cartel firms to be able to produce sufficient evidence to qualify for leniency. In *Organic Peroxides*, there were leniency applications: “[Peroxid Chemie] and Laporte [later Degussa] provided in their submission the original of the initial main agreement of 1971, which they obtained from AC Treuhand while preparing the leniency application. It was printed on pink paper, as were other confidential cartel documents which were not allowed to be taken out of the premises of AC Treuhand.” (EC Decision in *Organic Peroxides* at par.83) (http://ec.europa.eu/competition/antitrust/cases/dec_docs/37857/37857_100_1.pdf)

²²In the United States, firms receiving leniency may still be subject to penalties from civil litigation; however, exposure to those penalties is reduced for successful leniency applicants. “Under the Antitrust Criminal Penalty Enhancement and Reform Act of 2004, Pub. L. No. 108-237, Title 2, §§ 211-214, 118 Stat. 661, 666-668, a leniency applicant may qualify for detrebling of damages if the applicant cooperates with plaintiffs in their civil actions while the applicant’s former co-conspirators will remain liable for treble damages on a joint and several basis.” (Hammond and Barnett, 2008, p.18.)

1. In the first round, which focuses on product 1, the following leniency game is played for product 1:

- (a) In the first stage, both firms observe signal $s_1 \in \{0, 1\}$, where $\Pr(s_1 = 1) = h \in (0, 1)$. The realization $s_1 = 1$ denotes that the competition authority has received some evidence about illegal antitrust activity in product 1 and has started an investigation, while $s_1 = 0$ means that this has not happened.
- (b) In the second stage, nothing happens if $s_1 = 0$, but if $s_1 = 1$, each firm brings in outside counsel to do an internal investigation. The internal investigation uncovers evidence sufficient to support a leniency application with probability $\rho \in (0, 1)$, in which case the outside counsel observes a conditionally independent random variable θ_{i1} uniformly distributed in the interval $[\tau_1 - \epsilon, \tau_1 + \epsilon]$, where $\epsilon > 0$, centered on the realized value of the random variable τ_1 , defined below in (d). We will think of ϵ as “small”, so that τ is “almost” perfectly observed by each firm and focus on the limit as $\epsilon \downarrow 0$.²³
- (c) In the third stage, nothing happens if $s_1 = 0$ or if $s_1 = 1$ and the internal investigation did not uncover evidence sufficient to support a leniency application. But if $s_1 = 1$ and the internal investigation did uncover such evidence, then the outside counsel advises the board of directors by reporting the observed value θ_{i1} and the board decides whether to apply for leniency or not. If only one firm applies for leniency, it receives leniency. If both firms apply for leniency, one (and only one) is randomly designated as receiving leniency.
- (d) In the fourth stage, the competition authority concludes its investigation after observing an additional signal $v_1 \in \{0, 1\}$ indicating the strength of the case; $v_1 = 1$ signifies that the authority has enough evidence to convict the firms, while $v_1 = 0$ denotes insufficient evidence and the need to drop the case. We assume that $v_1 = 1$ if there is at least one leniency applicant. If there is no leniency applicant, $\Pr(v_1 = 1 \mid s_1 = 0) = 0$ and $\Pr(v_1 = 1 \mid s_1 = 1) = \tau_1$. From the point of view of the firms, τ_1 is a random variable with positive density $g(\tau_1)$ and distribution $G(\tau_1)$ with support on the interval $(0, 1)$; let $\tau^E = \int_0^1 \tau g(\tau) d\tau$ be the expected value of τ_1 .

²³As described in Carlsson and van Damme (1993), the global game result that iterated dominance forces each player to select the risk-dominant equilibrium of the game corresponding to his observation provided that ϵ is sufficiently small relies only on the posterior beliefs being approximately symmetric (the likelihood that i assigns to j observing θ_{j1} given θ_{i1} is approximately equal to the likelihood that j assigns to i observing θ_{i1} given θ_{j1}). Symmetry holds exactly if the prior is uniform but holds approximately for general priors if the observation errors are small.

2. In the second round, which focuses on product 2, we need to distinguish between penalty-plus and standard leniency. Under *standard leniency*, except for the probability that the competition authority starts an investigation, the same game as in the first round is played for product 2. We let the probability that the competition authority starts an investigation to be $h_C \geq h$ if in the first round firms were convicted, i.e., if $v_1 = 1$ and h otherwise. This reflects “cartel profiling”; that is, the conviction in product 1 may cause the competition authority to be more attentive to the potential for collusion in other products produced by the same firms, increasing the probability of an investigation in product 2.²⁴

Under *penalty-plus leniency*, if $v_1 = 0$, then the game played for product 2 is the same as the game for product 1; but if $v_1 = 1$, then a penalty-plus game is played for product 2, in which firms prosecuted in product 1 are asked about potential collusion in product 2 and must decide whether to apply for leniency without having observed the signal s_2 (i.e., without knowing whether the competition authority has received evidence about illegal antitrust activity). Firms that deny any involvement in a collusive agreement in product 2 are not allowed to apply for leniency at a later stage; e.g., after the competition authority has started an investigation.²⁵ Formally, the penalty-plus game has four stages:

- (a) In the first stage, each firm brings in outside counsel to do an internal investigation. The internal investigation uncovers evidence sufficient to support a leniency application with probability $\rho > 0$, in which case the outside counsel observes a conditionally independent random variable θ_{i2} centered on the realized value of the random variable τ_2 , which, for simplicity, has the same density and support as τ_1 .
- (b) In the second stage, if the internal investigation uncovered evidence sufficient to support a leniency application, then the board decides whether to apply for leniency or not.
- (c) In the third stage, if no firm has applied for leniency, then the competition authority receives evidence about illegal antitrust activity, $s_2 = 1$, with probability h_C .

²⁴ “The [Antitrust] Division [of the DoJ] will target its proactive efforts in industries where we suspect cartel activity in adjacent markets or which involve one or more common players from other cartels.” (Hammond, 2004, p.15)

²⁵ Because, as we shall prove in Proposition 2, firms never apply for leniency in the second product under penalty-plus leniency, if we allowed firms to apply for leniency after an investigation has started, then penalty-plus leniency would be equivalent to standard leniency.

- (d) In the fourth stage, the competition authority concludes its investigation after observing the additional signal $v_2 \in \{0, 1\}$. As in the first round, $\tau_2 = \Pr(v_2 = 1 \mid s_2 = 1)$.

In the benchmark case without a leniency program in place, the cartel is convicted in the first product with probability $\Psi_1^N = h\tau^E$ and in the second product with probability $\Psi_2^N = h\tau^E h_C \tau^E + (1 - h\tau^E) h\tau^E$. A cartel firm's expected payoff in product i is $V_i^N \pi_i$, where $V_i^N = (1 - \Psi_i^N) - \Psi_i^N f$.

We will use the following three assumptions to reduce the number of cases we need to analyze and to focus on the most interesting setting.

Assumption A1:

$$\rho < \min \left\{ \frac{2(1+\ell)}{2+f+\ell}, \frac{2(1+\ell)}{2+f+\ell} + 2h\rho \left(\frac{2(1+f)\tau^E}{2+f+\ell} - 1 \right) \frac{\pi_2}{\pi_1} \right\}.$$

Assumption A2:

$$h_C < \frac{1+\ell}{1+f} - \frac{\rho(f-\ell)}{2(2-\rho)(1+f)}.$$

Assumption A3:

$$E[\tau \mid \tau > t] \leq \frac{1+t}{2}.$$

Assumption A1 puts an upper bound on the value that ρ can take. Note that it implies that

$$\rho < \frac{2(1+\ell)}{2+f+\ell}. \tag{1}$$

Indeed, most of our results only require that condition (1) holds. The full force of Assumption A1 is only needed in Proposition 4 (and Lemma 3), to guarantee that under a penalty-plus program for some parameter values it is optimal for firms not to apply for leniency in the first product.

Assumption A2 puts an upper bound on the probability h_C , and consequently h , that the competition authority acquires evidence of collusion on its own. It will be used in Proposition 2 to show that firms never apply for leniency when the penalty-plus leniency game is played in product 2 after a conviction in product 1.

Assumption A3 is a restriction on the right tail of the distribution of τ ; note that it is satisfied by the uniform and other common distributions. Assumption A3 is used to prove Lemma 2, parts (ii) and (iii), and Lemma 3.

4 Second product equilibrium

Using backward induction, we begin by considering the second product coming to the attention of the competition authority. We need to distinguish the standard leniency setting from the case of penalty-plus leniency after a first period conviction. The difference between the two is that after a conviction in the first-market, under penalty plus a firm must decide whether to apply for leniency before the competition authority starts an investigation.

In both cases, firms must decide whether to apply for leniency after having conducted an internal investigation. If a firm does not uncover evidence, then it has no choice to make; it cannot apply for leniency. After uncovering evidence, a firm faces a strategic game (the *basic leniency game*). The firm (the row player) must decide whether to apply for leniency (L) or not (N) and its payoff depends on whether the other firm (the column player) applies for leniency in case it has uncovered evidence. The payoff of the row player is given by adding the baseline payoff $-f\pi_2$ to the entries in the table below.²⁶

	L	N	
L	$(1 - \frac{\rho}{2}) \pi_2 (f - \ell)$	$\pi_2 (f - \ell)$	(2)
N	$(1 - \rho) (1 - \beta\theta_{i2}) \pi_2 (1 + f)$	$(1 - \beta\theta_{i2}) \pi_2 (1 + f)$	

The perceived probability of successful prosecution in case of no leniency application is $\beta\theta_{i2}$, where $\beta = 1$ in the case of standard leniency or penalty-plus without a first market conviction (because in those cases the firms only decide whether to apply for leniency after having observed that the competition authority has started an investigation), and where $\beta = h_C$ in the case of penalty-plus after a conviction in the first market (because in this case firms must decide whether to apply for leniency before the competition authority starts an investigation, which will happen with probability h_C).

We can think of $-f\pi_2$, the firm's payoff when prosecuted and fined, as the baseline payoff of the row player. If the row player applies for leniency, then it receives leniency and a payoff of $\pi_2 (f - \ell)$ above the baseline if the other firm does not apply after uncovering evidence (upper right cell). It receives leniency and a payoff of $\pi_2 (f - \ell)$ above the baseline with probability $1 - \frac{\rho}{2}$ if the other firm does apply after uncovering evidence (upper left cell). This is because the only event in which the applying firm does not receive leniency is when the other firm uncovers evidence (which occurs with probability ρ), applies, and is selected to receive leniency by the random draw (which occurs with probability $\frac{1}{2}$).

²⁶The symmetry of firms does not play any role in the game; we could replace π_2 with a different payoff π_{2i} for each firm i without affecting the analysis.

When the row player does not apply for leniency, it is not prosecuted and receives a payoff of $\pi_2(1+f)$ above the baseline $-f\pi_2$ with probability $1-\beta\theta_{i2}$ if the other firm does not apply after uncovering evidence (lower right cell) and with probability $(1-\rho)(1-\beta\theta_{i2})$ if the other firm applies for leniency after uncovering evidence (lower left cell).

Based on the basic leniency game, we can distinguish between the following four cases:

1. Applying for leniency is a strictly dominant strategy and (L, L) is the unique Nash equilibrium. This holds if and only if

$$\beta\theta_{i2} > \frac{1+\ell}{1+f}. \quad (3)$$

2. There are two pure strategy Nash equilibria (L, L) and (N, N) , and equilibrium (L, L) is risk dominant. Because the basic leniency game is symmetric, (L, L) is risk dominant if L is the best reply to the opponent's strategy of randomizing with equal probability between L and N . This holds if and only if

$$\frac{1+\ell}{1+f} - \frac{\rho(f-\ell)}{2(2-\rho)(1+f)} < \beta\theta_{i2} < \frac{1+\ell}{1+f}. \quad (4)$$

3. There are two pure strategy Nash equilibria (L, L) and (N, N) , and (N, N) is risk dominant. This holds if and only if

$$\frac{1+\ell}{1+f} - \frac{\rho(f-\ell)}{2(1-\rho)(1+f)} < \beta\theta_{i2} < \frac{1+\ell}{1+f} - \frac{\rho(f-\ell)}{2(2-\rho)(1+f)}. \quad (5)$$

4. No leniency is a dominant strategy and (N, N) is the unique Nash equilibrium. This holds if and only if

$$\beta\theta_{i2} < \frac{1+\ell}{1+f} - \frac{\rho(f-\ell)}{2(1-\rho)(1+f)}. \quad (6)$$

We see from condition (3) that if the punishment for being convicted without a leniency application is sufficiently severe, i.e., f is sufficiently large, then applying for leniency is a dominant strategy, as long as being convicted is possible, i.e., $\theta_{i2} > 0$. Large values of f may be appropriate if, for example, colluding firms can be held jointly and severally liable or damages tripled or to account for prison sentences for managers.

We see from condition (6) that, without enough probability $\beta\theta_{i2}$ that the competition authority prosecutes the cartel in the absence of a leniency application, it is a dominant strategy for firms not to apply for leniency. This provides a modeling foundation for the view that for leniency to work it is important that, in the words of Wils (2008a, p.130),

“the companies and individuals concerned perceive a risk that the competition authorities will detect and establish the antitrust violation without recourse to leniency.”

When there is a threat of prosecution in the absence of a leniency application, the threat that a co-conspirator may apply increases leniency applications. If it were known that the rival could not apply for leniency, perhaps because it would be viewed as a ringleader or coercing others to join and so not eligible for leniency, then the firm applies for leniency if and only if $\beta\theta_{i2} > \frac{1+\ell}{1+f}$. However, as shown below, with the threat that a co-conspirator may apply, a firm applies for leniency for the larger range of values $\beta\theta_{i2} > \frac{1+\ell}{1+f} - \frac{\rho(f-\ell)}{2(2-\rho)(1+f)}$. Thus, a “race to the courthouse” can amplify the incentive for a colluding firm to apply for leniency.²⁷

4.1 Standard leniency in the second product

Because we are interested in the case of a small error in the observation by firm i of the probability of successful prosecution, θ_{i2} is approximately equal to τ_2 . Assumption A1 guarantees that in the case of standard leniency, or penalty-plus with no prior conviction, when $\beta = 1$, the parameter configuration does not rule out any of the four equilibrium cases. For high values of τ_2 the leniency program is certainly effective and for low values of τ_2 the leniency program is ineffective; more formally, (3) holds for θ_{i2} sufficiently close to one and (6) holds for θ_{i2} sufficiently close to zero because by Assumption A1, inequality (1) holds and, as a result, the right side of (6) is positive.

We can think of the signal θ_{i2} received by firm i as i 's type. The strategy of firm i can then be represented as the probability $\alpha_{i2}(\theta_{i2})$ with which the firm chooses pure strategy L after observing signal θ_{i2} . Define the cut-off value for the probability of prosecution below which (N, N) is risk dominant and above which (L, L) is risk-dominant in the basic leniency game by:

$$\tau_2^* \equiv 1 - \frac{(4 - \rho)(f - \ell)}{2(2 - \rho)(1 + f)} > 0. \quad (7)$$

We are now in a position to prove the following result, which exploits the fact that τ_2 is a random variable that is imperfectly observed by the firms. The proof is contained in Appendix A, as are all other proofs.

Proposition 1 *Under Assumption A1, with standard leniency or penalty-plus with no prior conviction, in the basic leniency game for product 2, for ϵ sufficiently small, the subgame taking place after a signal $s_2 = 1$ has a unique Bayesian equilibrium that survives the iterated elimination of strictly dominated strategies. In such an equilibrium, when*

²⁷For discussion of the desirability of this effect, see Hammond (2004, p.10) and Wils (2008a, p.130).

firm i uncovers evidence, it applies for leniency (i.e., $\alpha_{i2}(\theta_{i2}) = 1$) if it receives a signal $\theta_{i2} > \tau_2^*$, and does not apply (i.e., $\alpha_{i2}(\theta_{i2}) = 0$) if it receives a signal $\theta_{i2} < \tau_2^*$.

As Proposition 1 shows, depending on the signals firms receive, firms for which leniency is feasible may choose to apply for leniency or may not. Henceforth, when computing payoffs and probabilities of successful prosecution, we take the limit as $\epsilon \downarrow 0$, with the implication that the firms coordinate on either both applying for leniency when that is feasible or both not applying for leniency.

The ex ante probability that the cartel will be convicted in the basic leniency game is:

$$\Psi_{2i}^S = h_2 \left[1 - (1 - \rho)^2(1 - \tau^E) - \rho(2 - \rho) \int_0^{\tau_2^*} (1 - \tau) g(\tau) d\tau \right], \quad (8)$$

where $h_2 = h$ and $\Psi_{2i}^S = \Psi_{2B}^S$ if there was no conviction in product 1, while $h_2 = h_C$ and $\Psi_{2i}^S = \Psi_{2C}^S$ otherwise. To understand this expression, note that, conditional on the competition authority acquiring evidence, which occurs with probability h_2 , the cartel is not convicted if neither firm finds evidence to apply for leniency and then the competition authority is unable to convict, which occurs with probability $(1 - \rho)^2(1 - \tau^E)$ – the second term in the square brackets – or if at least one firm finds evidence (probability $\rho(2 - \rho)$) but τ is less than τ_2^* and the authority is unable to convict – the last term in the square brackets.

The expected payoff of a cartel firm from a basic leniency game in product 2 is $V_{2i}^S \pi_2$, where

$$V_{2i}^S = 1 - \Psi_{2i}^S(1 + f) + h_2 \rho \left(1 - \frac{\rho}{2} \right) (f - \ell) (1 - G(\tau_2^*)), \quad (9)$$

where V_{2i}^S is denoted as V_{2B}^S or V_{2C}^S depending on whether $h_2 = h$ or $h_2 = h_C$. Note that a firm gets π_2 with probability $1 - \Psi_{2i}^S$ and a baseline payoff of $-f\pi_2$ with probability Ψ_{2i}^S ; in addition, it gets $(f - \ell)\pi_2$ if it is the only firm to apply for leniency (probability $h_2 [\rho(1 - \rho) + \frac{1}{2}\rho^2] \Pr(\tau > \tau_2^*)$), which generates the last term in (9).

The next lemma follows from $h_C \geq h$.

Lemma 1 *Under Assumption A1, $\Psi_{2C}^S \geq \Psi_{2B}^S$ and $V_{2B}^S \geq V_{2C}^S$.*

4.2 Penalty-plus in the second product

If the penalty-plus leniency game is played in product 2 after firms have been convicted in product 1, then firms uncovering sufficient evidence from an internal investigation must decide whether to apply for leniency before the competition authority has collected any incriminating evidence; that is, before observing the signal s_2 . Because h_C is the

probability that $s_2 = 1$, this game corresponds to the case $\beta = h_C$, so that $\beta\theta_{i2}$ is approximately equal to $h_C\tau_2$.

Under Assumptions A1 and A2, for some parameter values N is a dominant strategy, while for others it is the risk dominant strategy; strategy L is never dominant or risk dominant. More precisely, (6) holds for θ_{i2} sufficiently close to zero, while by Assumption A2, (3) and (4) never hold.²⁸ We may now state the main result of this subsection.

Proposition 2 *Under Assumptions A1 and A2, for ϵ sufficiently small, with a penalty-plus program after a first-market conviction, the penalty-plus game for product 2 has a unique Bayesian equilibrium that survives the iterated elimination of strictly dominated strategies. In that equilibrium, no firm applies for leniency, $\alpha_{i2}(\theta_{i2}) = 0$, for all θ_{i2} .*

Since firms will not apply for leniency, it is straightforward to compute the probability that the cartel will be convicted in the penalty-plus game:

$$\Psi_{2C}^P = h_C\tau^E. \quad (10)$$

The expected payoff of a cartel firm from a penalty-plus leniency game in product 2 is $V_{2C}^P\pi_2$, where

$$V_{2C}^P = 1 - \Psi_{2C}^P(1 + f). \quad (11)$$

The next lemma shows that after a conviction in the first product, the probability of successful prosecution is greater and the expected payoff to the cartel is lower under standard leniency than in the penalty-plus regime; following a conviction, firms prefer penalty-plus to standard leniency. In addition, as long as the probability ratio h_C/h is below a threshold $r^* = \min\{r_A^*, r_B^*\} > 1$, the probability of prosecution is greater and the expected cartel payoff lower in the basic game with no previous conviction, than in the penalty-plus game; in this case, even without a prior conviction, the colluding firms prefer penalty-plus.

Lemma 2 *Under Assumptions A1 and A2: (i) There exists a threshold $r_A^* > 1$ such that the probability that the cartel will be convicted is lower in the penalty-plus game than in the basic game without a prior conviction, i.e., $\Psi_{2C}^P < \Psi_{2B}^S$, if and only if $h_C/h < r_A^*$; (ii) If in addition Assumption A3 holds, then there exists a threshold $r_B^* > 1$ such that the cartel's expected payoff is higher in the penalty-plus game than in the basic game without*

²⁸This conclusion remains true if the competition authority could increase the fine multiplier f by a small amount in case a firm is convicted of colluding in product 2 after having denied doing it. If f could be increased without bound, then a firm may find it optimal to apply for leniency and Proposition 2 would no longer hold. In practice, sentencing guidelines restrict the ability of competition authorities to impose fines above some upper bound.

a prior conviction, i.e., $V_{2C}^P > V_{2B}^S$, if and only if $h_C/h < r_B^*$; (iii) After a conviction in the first product, relative to standard leniency under penalty-plus the probability that the cartel will be convicted in product 2 is lower, $\Psi_{2C}^P < \Psi_{2C}^S$, and if, in addition, Assumption A3 holds, then the cartel's payoff is higher, $V_{2C}^P > V_{2C}^S$.

5 Multi-product equilibrium

In this section we study firms' decisions in the first product after their activity has come under investigation by the competition authority. The payoff of the row player in the first product game is given by adding the baseline payoff $-f\pi_1 + V_{2i}\pi_2$ to the entries in the table below; the second period payoff multiplier V_{2i} is equal to V_{2C}^S in the case of standard leniency and it is equal to V_{2C}^P in the case of penalty-plus leniency.²⁹

	L	N	
L	$(1 - \frac{\rho}{2}) \pi_1 (f - \ell)$	$\pi_1 (f - \ell)$	(12)
N	$(1 - \rho) (1 - \theta_{i1})$ $\cdot [\pi_1 (1 + f) + (V_{2B}^S - V_{2i}) \pi_2]$	$(1 - \theta_{i1})$ $\cdot [\pi_1 (1 + f) + (V_{2B}^S - V_{2i}) \pi_2]$	

The payoffs of the period 1 leniency game corresponds to payoff of the basic leniency game (2) with $\beta = 1$ once we add the baseline payoff $-f\pi_1 + V_{2i}\pi_2$ to the entries in all cells and we replace π_2 with π_1 in the first row, $\pi_2(1+f)$ with $\pi_1(1+f) + (V_{2B}^S - V_{2i})\pi_2$ in the second row, and θ_{i2} with θ_{i1} , which is approximately equal to τ_1 . If a firm applies for leniency in product 1, it guarantees itself a second product payoff of $\pi_2 V_{2i}$. If a conviction has not taken place in the first product, then the continuation payoff in the second product is $V_{2B}^S \pi_2$. Thus, we can think of $\pi_1(1+f) + (V_{2B}^S - V_{2i})\pi_2$ as the firm's net payoff if it does not apply for leniency and it is not prosecuted in product 1.

We now distinguish between the standard leniency and the penalty-plus regime.

5.1 Standard leniency in the multi-product game

With standard leniency, after a conviction in the first product, each firm has an expected payoff in the second product of $V_{2C}^S \pi_2$. Thus, in this case $V_{2i} = V_{2C}^S$ in table (12).

Following the same logic used to derive (3)–(6), we can show that under Assumption A1, the parameter configuration does not rule out any of the four equilibrium cases. This

²⁹It is not essential that the firms belonging to the cartel in the first product are involved in the same second-product cartel. The important ingredient of the model is that there is a second-product continuation payoff; that is, that the firms in the first-product cartel are also involved in cartels for other, possibly different, products with other, possibly different, firms.

is because if θ_{i1} is sufficiently close to one, then L is a dominant strategy, while if θ_{i1} is sufficiently close to zero, then N is a dominant strategy because $V_{2B}^S > V_{2C}^S$ by Lemma 1 and, by Assumption A1, inequality (1) holds. We can also define the cut-off value for the probability of prosecution below which (N, N) is risk dominant and above which (L, L) is risk-dominant:

$$\tau_{1B}^* \equiv 1 - \frac{(4 - \rho)(f - \ell)}{2(2 - \rho) \left[1 + f + (V_{2B}^S - V_{2C}^S) \frac{\pi_2}{\pi_1} \right]}, \quad (13)$$

where $\tau_{1B}^* > \tau_2^*$ because $V_{2B}^S > V_{2C}^S$.

Proposition 3 *Under Assumption A1, for ϵ sufficiently small, the model with a standard leniency program has a unique Bayesian equilibrium that survives the iterated elimination of strictly dominated strategies. When it uncovers evidence in the first product, firm i applies for leniency (i.e., $\alpha_{i1}(\theta_{i1}) = 1$) if it receives signal $\theta_{i1} > \tau_{1B}^*$, and does not apply (i.e., $\alpha_{i1}(\theta_{i1}) = 0$) if it receives signal $\theta_{i1} < \tau_{1B}^*$. In the second product firms conduct an investigation only if the competition authority has received some evidence of collusion (i.e., following $s_2 = 1$); when firm i uncovers evidence, it applies for leniency (i.e., $\alpha_{i2}(\theta_{i2}) = 1$) if it receives signal $\theta_{i2} > \tau_2^*$, and does not apply (i.e., $\alpha_{i2}(\theta_{i2}) = 0$) if it receives signal $\theta_{i2} < \tau_2^*$.*

Proposition 3 shows that, conditional on the competition authority uncovering some evidence of collusion (i.e., $s_1 = 1$ and $s_2 = 1$), the firms apply for leniency in product 1 with lower probability than in product 2. The probability that some firm applies for leniency in product 1 is $\rho(2 - \rho)(1 - G(\tau_{1B}^*))$, while the probability that some firm applies for leniency in product 2 is $\rho(2 - \rho)(1 - G(\tau_2^*))$, where the former is lower because $\tau_{1B}^* > \tau_2^*$.

We now define the ex ante probability that the cartel will be prosecuted and convicted in the first product:

$$\Psi_1^S = h \left[1 - (1 - \rho)^2(1 - \tau^E) - \rho(2 - \rho) \int_0^{\tau_{1B}^*} (1 - \tau) g(\tau) d\tau \right]. \quad (14)$$

In the multi-product game, the ex ante probability that the cartel will be prosecuted and convicted in the second product is

$$\Psi_2^S = \Psi_1^S \Psi_{2C}^S + (1 - \Psi_1^S) \Psi_{2B}^S. \quad (15)$$

The expected payoff from the first product in the multi-product game is $V_1^S \pi_1$, where

$$V_1^S = 1 - \Psi_1^S(1 + f) + \frac{1}{2} h \rho (2 - \rho) (f - \ell) (1 - G(\tau_{1B}^*)). \quad (16)$$

The total payoff in the multi-product game is $V^S \pi_1$, where

$$V^S = V_1^S + \Psi_1^S V_{2C}^S \frac{\pi_2}{\pi_1} + (1 - \Psi_1^S) V_{2B}^S \frac{\pi_2}{\pi_1}. \quad (17)$$

5.2 Penalty-plus in the multi-product game

In the case of penalty-plus, under Assumptions A1 and A2, after a conviction in the first product, each firm has an expected payoff in the second product of $V_{2C}^P \pi_2$. Thus, in this case $V_{2i} = V_{2C}^P$ in table (12).

If, in addition, Assumption A3 holds, then it is also true in this case that the parameter configuration does not rule out any of the four equilibrium cases. As before, if θ_{i1} is sufficiently close to one, then L is a dominant strategy, while if θ_{i1} is sufficiently close to zero, then Assumptions A1–A3 imply that N is a dominant strategy, as shown in Lemma 3 in the Appendix.

The cut-off value for the probability of prosecution below which (N, N) is risk dominant and above which (L, L) is risk-dominant is given by:

$$\tau_{1P}^* \equiv 1 - \frac{(4 - \rho)(f - \ell)}{2(2 - \rho) \left[1 + f + (V_{2B}^S - V_{2C}^P) \frac{\pi_2}{\pi_1} \right]}. \quad (18)$$

One can show that $\tau_{1P}^* < \tau_{1B}^*$ because $V_{2C}^P > V_{2C}^S$ by Lemma 2. Furthermore, one can show that $\tau_{1P}^* < \tau_2^*$ if and only if $V_{2C}^P > V_{2B}^S$, that is, if and only if h_C/h is below the threshold r_B^* defined in Lemma 2.

Proposition 4 *Under Assumptions A1–A3, for ϵ sufficiently small, the model with penalty-plus has a unique Bayesian equilibrium that survives the iterated elimination of strictly dominated strategies. When firm i uncovers evidence in the first product, it applies for leniency (i.e., $\alpha_{i1}(\theta_{i1}) = 1$) if it receives signal $\theta_{i1} > \tau_{1P}^*$, and does not apply (i.e., $\alpha_{i1}(\theta_{i1}) = 0$) if it receives signal $\theta_{i1} < \tau_{1P}^*$. If firms are not prosecuted in the first product (i.e., if $v_1 = 0$), then in the second product firms conduct an investigation only if the competition authority has received some evidence of collusion (i.e., following $s_2 = 1$); when firm i uncovers evidence, it applies for leniency (i.e., $\alpha_{i2}(\theta_{i2}) = 1$) if it receives signal $\theta_{i2} > \tau_2^*$, and does not apply (i.e., $\alpha_{i2}(\theta_{i2}) = 0$) if it receives signal $\theta_{i2} < \tau_2^*$. If firms are prosecuted in the first product (i.e., if $v_1 = 1$), then in the second product neither firm applies for leniency.*

Proposition 4 completes our analysis of the equilibrium of the game and shows that, conditional on the competition authority uncovering some evidence of collusion (i.e., $s_1 = 1$), the firms apply for leniency in product 1 with higher probability under penalty-plus

than standard leniency. With penalty-plus, the probability that some firm applies for leniency in product 1 is $\rho(2 - \rho)(1 - G(\tau_{1P}^*))$, while with standard leniency it is $\rho(2 - \rho)(1 - G(\tau_{1B}^*))$; the latter is lower because $\tau_{1B}^* > \tau_{1P}^*$.

In addition, even if firms are not convicted in product 1, if h_C/h is below the threshold r_B^* , then conditional on the competition authority uncovering some evidence of collusion (i.e., $s_1 = 1$ and $s_2 = 1$), the firms apply for leniency in product 1 with higher probability than in product 2 because $\tau_{1P}^* < \tau_2^*$.

We now define the ex ante probabilities Ψ_1^P and Ψ_2^P that the cartel will be prosecuted and convicted in the first and second product:

$$\Psi_1^P = h \left[1 - (1 - \rho)^2(1 - \tau^E) - \rho(2 - \rho) \int_0^{\tau_{1P}^*} (1 - \tau) g(\tau) d\tau \right], \quad (19)$$

$$\Psi_2^P = \Psi_1^P \Psi_{2C}^P + (1 - \Psi_1^P) \Psi_{2B}^S. \quad (20)$$

The expected payoff from the first product in the multi-product game is $V_1^P \pi_1$, where

$$V_1^P = 1 - \Psi_1^P(1 + f) + \frac{1}{2} h \rho (2 - \rho) (f - \ell) (1 - G(\tau_{1P}^*)). \quad (21)$$

The total payoff in the multi-product game is $V^P \pi_1$, where

$$V^P = V_1^P + \Psi_1^P V_{2C}^P \frac{\pi_2}{\pi_1} + (1 - \Psi_1^P) V_{2B}^S \frac{\pi_2}{\pi_1}. \quad (22)$$

6 Policy implications

In this section we describe several policy implications emerging from our model.

6.1 Leniency contributes to prosecution and preemption effects

As shown in Propositions 1–4, assuming the cartel firms receive accurate signals on the probability of prosecution in the absence of a leniency applicant, once a cartel comes under investigation, firms apply for leniency whenever the probability of prosecution without a leniency applicant, τ , is greater than a threshold (τ_{1B}^* , τ_{1P}^* , or τ_2^*), except in the penalty-plus environment after having been convicted in the first product, in which case the firms do not apply for leniency. The thresholds differ depending on the leniency environment and whether it is the first or second product. In product 1, the threshold is τ_{1B}^* for standard leniency and τ_{1P}^* for penalty-plus. In product 2, the threshold is τ_2^* for standard leniency or penalty-plus with no conviction in the first product, and essentially equal to 1 for penalty-plus with a conviction because the firms never apply for leniency in that case.

We can analyze the game in terms of the prosecution and preemption effects created by leniency (see Harrington, 2011). If τ is sufficiently large that L is the dominant strategy, then a firm will seek leniency even if it expects that the other firm will not. This is the prosecution effect. Firms have an incentive to apply for leniency in order to avoid the penalties associated with being prosecuted, which for high τ is relatively likely even in the absence of a leniency applicant. If τ is in the range where there are two Nash equilibria of the complete information game, but L is the risk dominant strategy, then a firm will seek leniency because it expects the other firm to apply for leniency. This is the preemption effect. A firm only prefers leniency as a means to preempt the leniency application of the other firm.

We can define the strength of the prosecution effect to be the probability that τ is in the region where L is a dominant strategy and the strength of the preemption effect to be the probability that τ is in the region where L is not dominant but is a risk dominant strategy. Using this definition, we can examine the effect of leniency on the prosecution and preemption effects.

Proposition 5 *Under Assumptions A1–A3, the strength of the prosecution and preemption effects increase as the payoff under leniency increases (ℓ decreases) (i) for the second product except in the penalty-plus environment after having been convicted in the first product, in which case there is no effect, and (ii) for the first product when $\frac{\pi_2}{\pi_1}$ is sufficiently small or h_C is sufficiently close to h .*

As shown in Proposition 5, in the environment with standard leniency, there is a double benefit on the probability of leniency in the first product from a more generous leniency program, which corresponds to a lower value of ℓ , because a decrease in ℓ results in an increase in both the prosecution and the preemption effects.³⁰

6.2 The effectiveness of internal investigations increases the preemption effect

An increase in the probability that an internal investigation uncovers evidence, ρ , means that a firm that has itself uncovered evidence sufficient to apply for leniency believes it is more likely that its co-conspirator will be in a similar position. This can increase the preemption effect.

Proposition 6 *Under Assumptions A1–A3, the strength of the preemption effect increases as the probability ρ that an internal investigation uncovers evidence increases (i)*

³⁰See Harrington (2011) on the “multiplier effect” of a more aggressive competition authority.

for the second product except in the penalty-plus environment after having been convicted in the first product, in which case there is no effect, and (ii) for the first product when $\frac{\pi_2}{\pi_1}$ is sufficiently small or h_C is sufficiently close to h .

If it is more likely that a co-conspirator has maintained incriminating evidence in house, then one would expect ρ to increase, and so as shown by Proposition 6 a firm has a greater incentive to apply for leniency. This suggests that leniency programs can be made more effective if the competition authority can take steps that enhance incentives for employees with knowledge of the conspiracy to cooperate and that facilitate the discovery of incriminating evidence, for example by limiting the ability of cartels to outsource the running of the cartel and control of incriminating evidence to third-party facilitators.³¹

6.3 Penalty-plus may reduce detection

Under a penalty-plus leniency program, firms have an additional incentive to apply for leniency in the first product. Conviction in the first product delivers the benefit to the cartel of committing the firms not to apply for leniency in the second product because that is the unique equilibrium of the penalty-plus game. As a result, penalty-plus leniency increases convictions in the first product, but it decreases convictions in the second product relative to standard leniency unless cartel profiling is so severe that after a conviction in the first product it is very likely that the competition authority will also start an investigation in the second product.

Proposition 7 *Under Assumptions A1–A3, relative to standard leniency, under a penalty-plus leniency program: (i) firms are ex ante more likely to be convicted in the first product, $\Psi_1^P > \Psi_1^S$, (ii) if $h_C/h < r_A^*$, where r_A^* is the threshold defined in Lemma 2, then firms are ex ante less likely to be convicted in the second product, $\Psi_2^P < \Psi_2^S$, and (iii) if $h_C/h < r_B^*$, where r_B^* is the threshold defined in Lemma 2 and the ratio π_2/π_1 is above a threshold $(\pi_2/\pi_1)^*$, then firms ex ante payoffs are higher under penalty-plus than under standard leniency, $V^P \pi_1 > V^S \pi_1$.*

Proposition 7 tells us that in the absence of extreme cartel profiling, penalty-plus leniency generates a trade-off. It increases the probability of a conviction in the first product and decreases it in the second product. In addition, if the second product is sufficiently more profitable than the first product, then firms prefer penalty-plus to standard leniency. This suggests that the competition authority has an incentive to attend to the more profitable product first, while, on the contrary, firms engaged in collusion in multiple

³¹See footnote 21 on the role AC Treuhand played in supporting cartels in organic peroxides and heat stabilisers.

products have an incentive to manipulate the order in which products are approached by the competition authority to the extent that is possible, potentially engaging in collusion in a minor product and revealing the existence of the cartel in order to decrease the probability of prosecution in the more valuable product.

Thus, penalty-plus can potentially cause more cartels to form than under standard leniency. In particular, minor products that were not worth cartelizing with standard leniency, perhaps because the incremental value from cartelization was insufficient given the costs of establishing the required collusive structures, may be worth cartelizing in an environment with penalty-plus leniency because the additional, sacrificial, cartels provide the potential benefit of insulating more valuable products from leniency applications. These undesirable effects can potentially be avoided by directing additional resources towards the investigation of potential collusion in other products produced by firms found to be engaged in collusion, with the effect of increasing h_C/h .

Other leniency policies also offer a type of commitment device similar to penalty-plus, which can be similarly abused by strategic multi-product cartels. Chen and Rey (2013) show that in their model prohibiting leniency for repeat offenders can reduce the effectiveness of leniency and increase the profitability of collusion. For example, Greece used to have the policy that firms with prior convictions for collusion could not apply for leniency.³² In this environment, firms have an incentive to collude and get convicted in a less valuable product to protect a more valuable one. Certain jurisdictions restrict the ability of firms identified as “ringleaders” or firms having “coerced” others into participation to apply for leniency.³³ This suggests the possibility that cartels may fabricate evidence that one, or perhaps all, of the cartel firms are ringleaders or coercers, in order to prevent leniency from being an option for those firms.

6.4 Profiling may reduce detection

We have assumed that the probability of investigation is h in the first product and is h_C in the second product following a conviction in the first product, with $h_C \geq h$. By allowing h_C to be greater than h , we allow the possibility that the competition authority responds to a conviction by increasing the intensity with which it pursues the other products of

³²See Wils (2008a, p.138, n.139).

³³The defense attorneys we interviewed stated that in the United States, when the 1993 leniency rules were put in place, there were few leniency applications in part because firms were afraid that they would be labeled as one of the ringleaders, and so they would disclose the cartel but not get leniency benefits. Under later revisions to the program, clarified through speeches by DoJ officials, in order to be denied leniency, a firm would have to be the one and only ringleader and possibly also have a role in coercing others to join. This has lessened firms’ fears that they might be denied leniency based on their status as a ringleader.

convicted colluders, which can be viewed as “profiling.”³⁴

Clearly, holding fixed firm behavior with respect to leniency in the first product, the greater is h_C , the greater is the probability of conviction in the second product. However, h_C also affects firms’ first-stage leniency choice. Indeed, by reducing the incentive that firms have to apply for leniency (i.e., decreasing the range of values of τ_1 for which the firms apply for leniency in the first product), profiling reduces the probability of a conviction in the first product. As long as h_C is sufficiently close to h , the reduced probability of conviction in the first product is not sufficient to lead to a reduced ex-ante probability of conviction in the second product.

Proposition 8 *Under Assumptions A1–A3, an increase in the probability h_C of an investigation in the second product after conviction in the first due to cartel profiling: (i) decreases the ex ante probability of conviction in the first product under both standard leniency and penalty-plus leniency, $\frac{\partial \Psi_1^S}{\partial h_C} < 0$ and $\frac{\partial \Psi_1^P}{\partial h_C} < 0$; (ii) increases the ex ante probability of conviction in the second product if h_C is sufficiently close to h under standard leniency, $\frac{\partial \Psi_2^S}{\partial h_C} > 0$, and if $h_C/h < r_A^*$, where r_A^* is the threshold defined in Lemma 2, under penalty-plus leniency, $\frac{\partial \Psi_2^P}{\partial h_C} > 0$.*

The competition authority faces a trade-off. Profiling makes a leniency application in the first product less appealing and hence reduces convictions in the first product, but it increases convictions in the second product. Thus, under penalty-plus, profiling may be a useful counter-measure against firms that form sacrificial cartels in less valuable products, as discussed in the previous subsection, in order to shelter more valuable products from leniency applications. In general, profiling makes it more appealing for a competition authority to start by investigating cartels in less valuable markets.

6.5 Antitrust resources for investigation and prosecution are complementary

Competition authorities may be able to choose whether to direct resources towards more preliminary investigations or towards more successful prosecutions without a leniency applicant. Focusing on the case of a single product (product 2), we consider a small increase in the probability of an investigation h_2 and a first order stochastic shift in the probability of prosecution in the absence of a leniency application τ_2 . Unsurprisingly, both increase the probability of conviction; more interestingly, we show that resources spent in the two activities are complementary.

³⁴The economics literature has analyzed profiling in law enforcement, primarily with regard to the issue of racial profiling (e.g., Knowles, Persico, and Todd (2001) and Bandyopadhyay and Chatterjee (2010)). Harcourt (2006) points out how profiling can have perverse effects on crime rates.

Proposition 9 *Under Assumption A1, with a leniency program in a single product, an increase in the probability of investigation (h_2) and a first order stochastic shift in the probability of prosecution in the absence of a leniency applicant (τ_2) both increase the ex ante probability of conviction Ψ_{2i}^S . Resources spent in investigation and prosecution are complementary, as a first order stochastic shift in the probability of prosecution τ_2 increases $\frac{\partial \Psi_{2i}^S}{\partial h_2}$, the marginal impact on the probability of conviction of an increase in the probability of investigation.*

If the competition authority eliminates resources directed at investigations, then no cartels are identified and no firms apply for leniency. If the competition authority eliminates resources directed at the prosecution of cartels under investigation but without a leniency applicant, then there is no threat to induce firms to apply for leniency and so no prosecutions. In order for a leniency program to be effective, the competition authority must maintain resources directed at both investigations and the prosecution of cartels where there is no leniency applicant.

The probability of investigations can potentially be increased through increased monitoring and reporting requirements that allow the competition authority to more easily identify anomalies. The probability of successful prosecution in the absence of a *corporate* leniency applicant can potentially be increased by encouraging whistleblowers (see Aubert, Rey, and Kovacic, 2006) or allowing *individual* leniency applicants, although one would need to consider whether the evidence provided by a whistleblower or individual applicant would be as extensive or as valuable in terms of facilitating prosecution as that of a corporate applicant.³⁵

7 Conclusion

The U.S. antitrust leniency program has been in place in roughly its current form since 1993. The past twenty years have given colluding firms an opportunity to adjust their behavior to account for the presence of the leniency program. We should expect colluding firms to optimize given the existence of leniency. Our results point to the possibility that colluding firms might turn to their advantage an enforcement approach that links the availability of leniency across products for firms engaged in collusion in multiple products. Firms might create sacrificial cartels in minor products in order to protect cartels in more valuable products from the threat that a cartel member might apply for leniency.

³⁵In our model, we focus on responses by firms to an investigation (type B leniency). Based on interviews with defense attorneys, in the United States, individual leniency does not come up very often.

The results and insights we derived in this paper apply more generally to any situation where a group of agents in a coalition face an external threat to the stability of their relationship. Take for example crime gangs and criminal organizations.

In the United States, during a criminal investigation, which may involve several potential crimes, an individual may refuse to cooperate by appealing to the Fifth Amendment privilege against self-incrimination. To combat organized crime, government attorneys have several tools at their disposal that resemble corporate leniency. First, they may enter into a non-prosecution agreement in exchange for an individual's cooperation. Second, they may agree to reduce the charges against the individual.³⁶ Third, they may seek "use immunity," which requires the individual to testify or provide information, but promises not to use that against the individual.³⁷

It is well known that crime gangs typically require members to pass some initiation procedure that involves committing a crime. Like applying for leniency in a sacrificial cartel, such an initiation procedure raises the cost of defecting and is a form of commitment to be loyal to the gang in the future.

A number of policy implications follow from the results of this paper. We focus on antitrust leniency, but they could be stated to apply more widely. Competition authorities should (1) use leniency programs to enhance the detection of cartels; (2) take steps to improve the likelihood that internal investigations into possible antitrust offenses will be successful, including steps that enhance cooperation by employees and facilitate the discovery of incriminating evidence; (3) avoid policies that offer avenues for firms to commit themselves not to apply for leniency and, in general, use care when linking leniency procedures for firms participating in cartels in multiple products; (4) consider directing additional resources towards the investigation of potential collusion in other products produced by firms found to be engaged in collusion; and (5) maintain resources to investigate and uncover cartels as well as resources to prosecute cartels even in the absence of a leniency applicant. The overarching lesson is to consider how clever cartels will respond to the programs put in place.

³⁶This involves filing a motion according to Sentencing Guideline 5K1.1 or Rule 35 of the Federal Rules of Criminal Procedure.

³⁷This involves a court order under 18 U.S.C. §§ 6001-6003.

A Proofs

Proof of Proposition 1. Assume that ε is sufficiently close to zero so that $[\theta_{12} - 2\varepsilon, \theta_{12} + 2\varepsilon] \in (0, 1)$; after observing θ_{12} the density of firm 1's posterior about τ is

$$g(\tau | \theta_{12}) = \begin{cases} \frac{g(\tau)}{G(\theta_{12} + \varepsilon) - G(\theta_{12} - \varepsilon)}, & \text{if } \tau \in [\theta_{12} - \varepsilon, \theta_{12} + \varepsilon] \\ 0, & \text{otherwise.} \end{cases} \quad (23)$$

The conditional density $h(\theta_{22} | \theta_{12})$ of the other firm's observation θ_{22} is

$$h(\theta_{22} | \theta_{12}) = \int_{-\varepsilon}^{+\varepsilon} g(\theta_{22} - x | \theta_{12}) \frac{1}{2\varepsilon} dx$$

with support $\theta_{22} \in [\theta_{12} - 2\varepsilon, \theta_{12} + 2\varepsilon]$.

Because $\lim_{\varepsilon \rightarrow 0} E[\tau | \theta_{i2}] = \theta_{i2}$, it follows that for ε sufficiently small, if $\theta_{i2} > \frac{1+\ell}{1+f}$, then i 's conditionally expected payoff from L is greater than from N regardless of the rival's choice, so L is conditionally (strictly) dominant for i when firm i observes $\theta_{i2} > \frac{1+\ell}{1+f}$.

Letting H denote the cdf of the density h , if firm 2 plays L for $\theta_{22} > \frac{1+\ell}{1+f}$, then firm 1 observing $\theta_{12} = \frac{1+\ell}{1+f}$ must assign at least probability

$$1 - H(\theta_{12} | \theta_{12}) = 1 - \int_{\theta_{12} - 2\varepsilon}^{\theta_{12}} \left[\int_{-\varepsilon}^{+\varepsilon} g(\theta_{22} - x | \theta_{12}) \frac{1}{2\varepsilon} dx \right] d\theta_{22}$$

to firm 2's choosing L ; but this equals $\frac{1}{2}$ as ε converges to zero because

$$\begin{aligned} \lim_{\varepsilon \rightarrow 0} \int_{\theta_{12} - 2\varepsilon}^{\theta_{12}} \left[\frac{\int_{-\varepsilon}^{+\varepsilon} g(\theta_{22} - x | \theta_{12}) dx}{2\varepsilon} \right] d\theta_{22} &= \lim_{\varepsilon \rightarrow 0} \int_{\theta_{12} - 2\varepsilon}^{\theta_{12}} \frac{g(\theta_{22} - \varepsilon | \theta_{12}) + g(\theta_{22} + \varepsilon | \theta_{12})}{2} d\theta_{22} \\ &= \lim_{\varepsilon \rightarrow 0} \int_{\theta_{12} - 2\varepsilon}^{\theta_{12}} g(\theta_{22} + \varepsilon | \theta_{12}) \frac{1}{2} d\theta_{22} \\ &= \lim_{\varepsilon \rightarrow 0} \frac{G(\theta_{12} + \varepsilon) - G(\theta_{12} - \varepsilon)}{G(\theta_{12} + \varepsilon) - G(\theta_{12} - \varepsilon)} \frac{1}{2} = \frac{1}{2}, \end{aligned}$$

where the first equality is obtained by applying l'Hopital's Rule to the expression in square brackets, the second equality follows from $g(\tau | \theta_{12}) = 0$ for $\tau < \theta_{12} - \varepsilon$, and the third follows from the definition of $g(\tau | \theta_{12})$ in (23) for $\tau \in [\theta_{12} - \varepsilon, \theta_{12} + \varepsilon]$.

Let $\alpha \geq \frac{1}{2}$ be the probability that firm 1 assigns to firm 2's choosing L . Firm 1's expected payoff from L is

$$\alpha \left(1 - \frac{\rho}{2}\right) (f - \ell) \pi_2 + (1 - \alpha) (f - \ell) \pi_2 = \left(1 - \frac{\alpha\rho}{2}\right) (f - \ell) \pi_2,$$

and firm 1's conditionally expected payoff from N is

$$\alpha(1 - \rho)(1 - \theta_{12})(1 + f)\pi_2 + (1 - \alpha)(1 - \theta_{12})(1 + f)\pi_2 = (1 - \alpha\rho)(f - \ell)\pi_2,$$

where the equality uses $\theta_{12} = \frac{1+\ell}{1+f}$, which is less than the expected payoff from L . Thus, N can be excluded by iterated dominance for $\theta_{i2} = \frac{1+\ell}{1+f}$.

Let θ_{i2}^* be the largest observation for which L cannot be established by iterated dominance, i.e., θ_{i2}^* is the lower bound on the iterated dominance region. By symmetry, $\theta_{12}^* = \theta_{22}^* \equiv \theta_2^*$. Let α be the probability which firm 1 assigns to firm 2 choosing L . Iterated dominance requires firm 2 to play L for any $\theta_{22} > \theta_2^*$, so if firm 1 observes θ_2^* , it will be $\alpha \geq \frac{1}{2}$. By the definition of θ_2^* , it must be that firm 1's conditionally expected payoff from N is greater than or equal to its expected payoff from L , i.e.,

$$\alpha \left(1 - \frac{\rho}{2}\right) (f - \ell) \pi_2 + (1 - \alpha) (f - \ell) \pi_2 \leq \alpha(1 - \rho)(1 - \theta_2^*)(1 + f)\pi_2 + (1 - \alpha)(1 - \theta_2^*)(1 + f)\pi_2,$$

which we can rewrite as

$$\begin{aligned} \left(1 - \frac{\alpha\rho}{2}\right) (f - \ell) &\leq (1 - \alpha\rho)(1 - \theta_2^*)(1 + f), \quad \text{or} \\ \theta_2^* &\leq 1 - \frac{(2 - \alpha\rho)(f - \ell)}{(2 - 2\alpha\rho)(1 + f)} \leq 1 - \frac{(4 - \rho)(f - \ell)}{2(2 - \rho)(1 + f)} = \tau_2^*, \end{aligned} \quad (24)$$

where the second inequality follows from $\alpha \geq \frac{1}{2}$.

Similarly, as long as ε is sufficiently small, if $\theta_{i2} < 1 - \frac{(2-\rho)(f-\ell)}{2(1-\rho)(1+f)}$, then i 's conditionally expected payoff from N is greater than from L regardless of the rival's choice, so N is conditionally (strictly) dominant for i when firm i observes $\theta_{i2} < 1 - \frac{(2-\rho)(f-\ell)}{2(1-\rho)(1+f)}$.

If firm 2 plays N for $\theta_{22} < 1 - \frac{(2-\rho)(f-\ell)}{2(1-\rho)(1+f)}$, then firm 1 observing $\theta_{12} = 1 - \frac{(2-\rho)(f-\ell)}{2(1-\rho)(1+f)}$ must assign at least probability $\frac{1}{2}$ to firm 2's choosing N . Let $\alpha \leq \frac{1}{2}$ be the probability that firm 1 assigns to firm 2 choosing L . Firm 1's expected payoff from L is once again $(1 - \frac{\alpha\rho}{2})(f - \ell)\pi_2$, and firm 1's conditionally expected payoff from N is

$$\alpha(1 - \rho)(1 - \theta_{12})(1 + f)\pi_2 + (1 - \alpha)(1 - \theta_{12})(1 + f)\pi_2 = (1 - \alpha\rho) \frac{(2 - \rho)(f - \ell)}{2(1 - \rho)} \pi_2,$$

where the equality uses $\theta_{12} = 1 - \frac{(2-\rho)(f-\ell)}{2(1-\rho)(1+f)}$, which is greater than the expected payoff from L . Thus, L can be excluded by iterated dominance for $\theta_{i2} = 1 - \frac{(2-\rho)(f-\ell)}{2(1-\rho)(1+f)}$.

Let θ_{i2}^{**} be the smallest observation for which N cannot be established by iterated dominance, i.e., θ_{i2}^{**} is the upper bound on the iterated dominance region. By symmetry, $\theta_{12}^{**} = \theta_{22}^{**} \equiv \theta_2^{**}$. Let α be the probability which firm 1 assigns to firm 2 choosing L . Iterated dominance requires firm 2 to play N for any $\theta_{22} < \theta_2^{**}$, so if firm 1 observes θ_2^{**} ,

it will be $\alpha \leq \frac{1}{2}$. By the definition of θ_2^{**} , it must be that firm 1's conditionally expected payoff from L is greater than or equal to its expected payoff from N , i.e.,

$$\begin{aligned} & \alpha \left(1 - \frac{\rho}{2}\right) (f - \ell) \pi_2 + (1 - \alpha) (f - \ell) \pi_2 \\ & \geq \alpha(1 - \rho) (1 - \theta_2^{**}) (1 + f) \pi_2 + (1 - \alpha) (1 - \theta_2^{**}) (1 + f) \pi_2, \end{aligned}$$

which we can rewrite as

$$\begin{aligned} \left(1 - \frac{\alpha\rho}{2}\right) (f - \ell) & \geq (1 - \alpha\rho) (1 - \theta_2^{**}) (1 + f), \quad \text{or} \\ \theta_2^{**} & \geq 1 - \frac{(2 - \alpha\rho) (f - \ell)}{2(1 - \alpha\rho) (1 + f)} \geq 1 - \frac{(4 - \rho) (f - \ell)}{2(2 - \rho) (1 + f)} = \tau_2^*, \end{aligned} \quad (25)$$

where the second inequality follows from $\alpha \leq \frac{1}{2}$.

Since $\theta_2^{**} \leq \theta_2^*$ and $\theta_2^{**} \geq \tau_2^* \geq \theta_2^*$, it must be $\theta_2^{**} = \tau_2^* = \theta_2^*$ and the result follows. ■

Proof of Lemma 1. The proof that $\Psi_{2C}^S \geq \Psi_{2B}^S$ follows from (8) and $h_C \geq h$. Given that $h_C \geq h$, to show that $V_{2B}^S \geq V_{2C}^S$, it is sufficient to show that $\frac{\partial V_2^S}{\partial h_2} \leq 0$. Using (9) and (8),

$$\begin{aligned} \frac{\partial V_2^S}{\partial h_2} & = - \left(1 - (1 - \rho)^2 (1 - \tau^E) - \rho(2 - \rho) \int_0^{\tau_2^*} (1 - \tau) g(\tau) d\tau\right) (1 + f) \\ & \quad + \rho \left(1 - \frac{\rho}{2}\right) (f - \ell) (1 - G(\tau_2^*)) \\ & = - \left(\tau^E + \rho(2 - \rho) \int_{\tau_2^*}^1 (1 - \tau) g(\tau) d\tau\right) (1 + f) + \rho \left(1 - \frac{\rho}{2}\right) (f - \ell) (1 - G(\tau_2^*)) \\ & = - \left(\tau^E + \rho(2 - \rho) (1 - G(\tau_2^*)) - \rho(2 - \rho) \int_{\tau_2^*}^1 \tau g(\tau) d\tau\right) (1 + f) \\ & \quad + \rho \left(1 - \frac{\rho}{2}\right) (f - \ell) (1 - G(\tau_2^*)) \\ & = -\tau^E (1 + f) + \rho(2 - \rho) (1 + f) \int_{\tau_2^*}^1 \tau g(\tau) d\tau - (2 + \ell + f) \frac{1}{2} \rho (2 - \rho) (1 - G(\tau_2^*)). \end{aligned}$$

Because $\rho \leq 1$ and $\rho(2 - \rho)$ is maximized at $\rho = 1$, it follows that

$$\begin{aligned} \frac{\partial V_2^S}{\partial h_2} & \leq -\tau^E (1 + f) + (1 + f) \int_{\tau_2^*}^1 \tau g(\tau) d\tau - (2 + \ell + f) \frac{1}{2} \rho (2 - \rho) (1 - G(\tau_2^*)) \\ & = -(1 + f) \int_0^{\tau_2^*} \tau g(\tau) d\tau - (2 + \ell + f) \frac{1}{2} \rho (2 - \rho) (1 - G(\tau_2^*)) \\ & < 0. \quad \blacksquare \end{aligned}$$

Proof of Proposition 2. The proof of Proposition 2 parallels the proof of Proposition 1

starting from the paragraph after equation (24) and is omitted.³⁸

Proof of Lemma 2. (i) Using (8) and (10) we have:

$$\begin{aligned}\Psi_{2B}^S - \Psi_{2C}^P &= \frac{h}{h_C} (\Psi_{2C}^S - \Psi_{2C}^P) + (h - h_C) \tau^E \\ &= h\rho(2 - \rho) \int_{\tau_2^*}^1 (1 - \tau) g(\tau) d\tau + (h - h_C) \tau^E.\end{aligned}$$

The first term is positive, while the second is negative and of larger size if and only if $\frac{h_C}{h}$ is above a threshold.

(ii) Using (9) and (11) we have:

$$\begin{aligned}V_{2B}^S - V_{2C}^P &= \frac{h}{h_C} (V_{2C}^S - V_{2C}^P) + (h_C - h) \tau^E (1 + f) \\ &= h \left[\frac{\rho(2-\rho)}{2} [2(1+f) E[\tau | \tau > \tau_2^*] - 2 - \ell - f] (1 - G(\tau_2^*)) \right. \\ &\quad \left. + \left(\frac{h_C}{h} - 1\right) \tau^E (1 + f) \right].\end{aligned}\quad (26)$$

Using Assumption A3, the first term inside the square brackets is negative, while the second is positive and of larger size if and only if $\frac{h_C}{h}$ is above a threshold.

(iii) Using the definitions of Ψ_{2C}^S and Ψ_{2C}^P in (8) and (10):

$$\begin{aligned}\Psi_{2C}^S - \Psi_{2C}^P &= h_C \left[1 - (1 - \rho)^2 (1 - \tau^E) - \rho(2 - \rho) \int_0^{\tau_2^*} (1 - \tau) g(\tau) d\tau \right] - h_C \tau^E \\ &= h_C \rho(2 - \rho) \int_{\tau_2^*}^1 (1 - \tau) g(\tau) d\tau > 0.\end{aligned}\quad (27)$$

Using (9) and (11), we have

$$\begin{aligned}V_{2C}^S - V_{2C}^P &= 1 - \Psi_{2C}^S (1 + f) + h_C \rho \left(1 - \frac{\rho}{2}\right) (f - \ell) (1 - G(\tau_2^*)) - (1 - \Psi_{2C}^P (1 + f)) \\ &= h_C \rho \left(1 - \frac{\rho}{2}\right) (f - \ell) (1 - G(\tau_2^*)) - (1 + f) (\Psi_{2C}^S - \Psi_{2C}^P) \\ &= h_C \rho \left(1 - \frac{\rho}{2}\right) (f - \ell) (1 - G(\tau_2^*)) - (1 + f) h_C \rho (2 - \rho) \int_{\tau_2^*}^1 (1 - \tau) g(\tau) d\tau \\ &= h_C \rho (2 - \rho) (1 - G(\tau_2^*)) (1 + f) \frac{1}{2} \left(\frac{f - \ell}{1 + f} - 2 \frac{\int_{\tau_2^*}^1 (1 - \tau) g(\tau) d\tau}{(1 - G(\tau_2^*))} \right) \\ &= h_C \rho (2 - \rho) (1 - G(\tau_2^*)) (1 + f) \frac{1}{2} \left(\frac{f - \ell}{1 + f} - 2 E[\tau | \tau > \tau_2^*] \right) \\ &< h_C \rho (2 - \rho) (1 - G(\tau_2^*)) (1 + f) \frac{1}{2} \left(\frac{f - \ell}{1 + f} - 1 + \tau_2^* \right) \\ &= h_C \rho (2 - \rho) (1 - G(\tau_2^*)) (1 + f) \frac{1}{2} \left(\tau_2^* - \frac{1 + \ell}{1 + f} \right) < 0,\end{aligned}\quad (28)$$

where the first equality uses (9) and (11), the third equality uses (27), the fifth equality

³⁸More precisely, N is conditionally (strictly) dominant for i when firm i observes θ_{i2} sufficiently close to zero and the upper bound on the iterated dominance region of N can be shown to be $\theta_{i2}^{**} > 1$.

uses the definition of $E[\tau \mid \tau > \tau_2^*]$, the first inequality uses Assumption A3, and the final inequality uses $\tau_2^* < \frac{1+\ell}{1+f}$. ■

Proofs of Propositions 3 and 4. The proofs of Propositions 3 and 4 parallel the proof of Proposition 1 and are omitted.

Lemma 3 *Under Assumptions A1–A3, if θ_{i1} is sufficiently close to zero, then N is a dominant strategy in the first product game under penalty-plus leniency.*

Proof. Using Assumptions A1 and A2, for the first product game under penalty-plus leniency $V_2 = V_{2C}^P$ in Table (12). Strategy N is dominant when θ_{i1} is sufficiently close to zero, if and only if

$$(1 - \rho) [\pi_1 (1 + f) + (V_{2B}^S - V_{2C}^P) \pi_2] > \left(1 - \frac{\rho}{2}\right) \pi_1 (f - \ell). \quad (29)$$

It is necessary because it is the condition for N to be a strict best reply to L , and it is sufficient because (29) implies that $\pi_1 (1 + f) + (V_{2B}^S - V_{2C}^P) \pi_2 > \pi_1 (f - \ell)$, so that N is also a strict best reply to N . We can write (29) equivalently as

$$\begin{aligned} 1 + f + (V_{2B}^S - V_{2C}^P) \frac{\pi_2}{\pi_1} &> \frac{(2 - \rho)(f - \ell)}{2(1 - \rho)}, \text{ or} \\ (V_{2B}^S - V_{2C}^P) \frac{\pi_2}{\pi_1} &> \frac{(2 - \rho)(f - \ell) - 2(1 - \rho)(1 + f)}{2(1 - \rho)} \\ &= \frac{\rho(2 + f + \ell) - 2(1 + \ell)}{2(1 - \rho)} \end{aligned}$$

Using (26) we can rewrite the above inequality as:

$$\begin{aligned} \left(h \frac{\rho(2 - \rho)}{2} (2(1 + f) E[\tau \mid \tau > \tau_2^*] - 2 - \ell - f) (1 - G(\tau_2^*)) + [(h_C - h) \tau^E (1 + f)] \right) \frac{\pi_2}{\pi_1} \\ > \frac{\rho(2 + f + \ell) - 2(1 + \ell)}{2(1 - \rho)} \end{aligned}$$

Dropping the term in square brackets, we have the following sufficient condition:

$$\begin{aligned} h\rho(2 - \rho)(1 + f) \int_{\tau_2^*}^1 \left(\tau - \frac{2 + \ell + f}{2(1 + f)} \right) g(\tau) d\tau \frac{\pi_2}{\pi_1} &> \frac{\rho(2 + f + \ell) - 2(1 + \ell)}{2(1 - \rho)}, \text{ or} \\ \frac{2(1 - \rho)}{2 + f + \ell} h\rho(2 - \rho)(1 + f) \int_{\tau_2^*}^1 \left(\tau - \frac{2 + \ell + f}{2(1 + f)} \right) g(\tau) d\tau \frac{\pi_2}{\pi_1} &> \rho - \frac{2(1 + \ell)}{2 + f + \ell}, \text{ or} \end{aligned}$$

$$\rho < \frac{2(1 + \ell)}{2 + f + \ell} + \frac{2(1 - \rho)}{2 + f + \ell} h\rho(2 - \rho)(1 + f) \int_{\tau_2^*}^1 \left(\tau - \frac{2 + \ell + f}{2(1 + f)} \right) g(\tau) d\tau \frac{\pi_2}{\pi_1},$$

which, given $\tau_2^* = \frac{2+\ell+f}{2(1+f)} - \frac{(f-\ell)}{(1+f)(2-\rho)}$ and $\int_0^{\tau_2^*} \left(\tau - \frac{2+\ell+f}{2(1+f)} \right) g(\tau) d\tau < 0$, certainly holds if

$$\begin{aligned} \rho &< \frac{2(1+\ell)}{2+f+\ell} + \frac{2(1-\rho)}{2+f+\ell} h\rho(2-\rho)(1+f) \int_0^1 \left(\tau - \frac{2+\ell+f}{2(1+f)} \right) g(\tau) d\tau \frac{\pi_2}{\pi_1}, \\ &= \frac{2(1+\ell)}{2+f+\ell} + \frac{2(1-\rho)}{2+f+\ell} h\rho(2-\rho)(1+f) \left(\tau^E - \frac{2+\ell+f}{2(1+f)} \right) \frac{\pi_2}{\pi_1}. \end{aligned}$$

Assumption A3 states that $E[\tau | \tau > t] \leq \frac{1+t}{2}$, which implies that $\tau^E - \frac{2+\ell+f}{2(1+f)} \leq \frac{1}{2} - \frac{2+\ell+f}{2(1+f)} = -\frac{1+\ell}{2(1+f)} < 0$. Hence the above inequality certainly holds if

$$\begin{aligned} \rho &< \frac{2(1+\ell)}{2+f+\ell} + \frac{2}{2+f+\ell} h\rho 2(1+f) \left(\tau^E - \frac{2+\ell+f}{2(1+f)} \right) \frac{\pi_2}{\pi_1}, \text{ or} \\ \rho &< \frac{2(1+\ell)}{2+f+\ell} + 2h\rho \left(\frac{2(1+f)\tau^E}{2+f+\ell} - 1 \right) \frac{\pi_2}{\pi_1}, \end{aligned}$$

which holds by Assumption A1. ■

Proof of Proposition 5. The strengths of prosecution and preemption effects are as given in Table 5.

Table 5: Strength of prosecution and preemption effects by environment and product

	Prosecution Effect (L dominant)	Preemption Effect (L risk dominant)	Relevant Prop.
Product 1			
Standard leniency	$\int_{\frac{1+\ell+(V_{2B}^S-V_{2C}^S)\frac{\pi_2}{\pi_1}}{1+f+(V_{2B}^S-V_{2C}^S)\frac{\pi_2}{\pi_1}}}^1 g(\tau) d\tau$	$\int_{\tau_{1B}^*}^{\frac{1+\ell+(V_{2B}^S-V_{2C}^S)\frac{\pi_2}{\pi_1}}{1+f+(V_{2B}^S-V_{2C}^S)\frac{\pi_2}{\pi_1}}} g(\tau) d\tau$	Prop. 3
Penalty-plus	$\int_{\frac{1+\ell+(V_{2B}^S-V_{2C}^P)\frac{\pi_2}{\pi_1}}{1+f+(V_{2B}^S-V_{2C}^P)\frac{\pi_2}{\pi_1}}}^1 g(\tau) d\tau$	$\int_{\tau_{1P}^*}^{\frac{1+\ell+(V_{2B}^S-V_{2C}^P)\frac{\pi_2}{\pi_1}}{1+f+(V_{2B}^S-V_{2C}^P)\frac{\pi_2}{\pi_1}}} g(\tau) d\tau$	Prop. 4
Product 2			
Standard leniency & penalty-plus w/o conviction	$\int_{\frac{1+\ell}{1+f}}^1 g(\tau) d\tau$	$\int_{\tau_2^*}^{\frac{1+\ell}{1+f}} g(\tau) d\tau$	Prop. 1
Penalty-plus with conviction	0	0	Prop. 2

We show that the prosecution and the preemption effects are decreasing in ℓ . To do so, we show that the lower bounds of integration in the prosecution effect column of Table 1 (the same as the upper bounds of integration in the preemption effect column)

are increasing in ℓ approximately linearly, that the lower bounds of integration in the preemption effect column of Table 1 are increasing in ℓ approximately linearly, and that the lower bounds of integration in the preemption effect columns are increasing at a faster rate than the upper bounds. For product 2, the result follows from Table 1 and the fact that τ_2^* and $\frac{1+\ell}{1+f}$ are both linear and increasing in ℓ , with $\frac{\partial \tau_2^*}{\partial \ell} > \frac{\partial(\frac{1+\ell}{1+f})}{\partial \ell}$. For product 1 with standard leniency, using (13),

$$\frac{\partial \tau_{1B}^*}{\partial \ell} = \frac{(4-\rho)2(2-\rho) \left[1+f + (V_{2B}^S - V_{2C}^S) \frac{\pi_2}{\pi_1} \right] + (4-\rho)(f-\ell) \left(\frac{\partial V_{2B}^S}{\partial \ell} - \frac{\partial V_{2C}^S}{\partial \ell} \right) \frac{\pi_2}{\pi_1}}{\left(2(2-\rho) \left[1+f + (V_{2B}^S - V_{2C}^S) \frac{\pi_2}{\pi_1} \right] \right)^2},$$

which is positive for $\frac{\pi_2}{\pi_1}$ sufficiently small or for h_C sufficiently close to h , in which case $V_{2B}^S - V_{2C}^S$ and $\frac{\partial V_{2B}^S}{\partial \ell} - \frac{\partial V_{2C}^S}{\partial \ell}$ are close to zero. The analysis is similar for product 1 with penalty-plus, using (18). The bounds of integration in Table 1, $\frac{1+\ell+(V_{2B}^S-V_{2C}^S)\frac{\pi_2}{\pi_1}}{1+f+(V_{2B}^S-V_{2C}^S)\frac{\pi_2}{\pi_1}}$ and $\frac{1+\ell+(V_{2B}^S-V_{2C}^P)\frac{\pi_2}{\pi_1}}{1+f+(V_{2B}^S-V_{2C}^P)\frac{\pi_2}{\pi_1}}$ are also increasing in ℓ for $\frac{\pi_2}{\pi_1}$ sufficiently small or for h_C sufficiently close to h . Furthermore, for $\frac{\pi_2}{\pi_1}$ sufficiently small or for h_C sufficiently close to h ,

$$\frac{\partial \tau_{1B}^*}{\partial \ell} \approx \frac{(4-\rho)2(2-\rho)(1+f)}{(2(2-\rho)(1+f))^2} = \frac{(4-\rho)}{2(2-\rho)(1+f)} > \frac{1}{1+f} \approx \frac{\partial \left(\frac{1+\ell+(V_{2B}^S-V_{2C}^S)\frac{\pi_2}{\pi_1}}{1+f+(V_{2B}^S-V_{2C}^S)\frac{\pi_2}{\pi_1}} \right)}{\partial \ell},$$

and similarly for the comparison of $\frac{\partial \tau_{1P}^*}{\partial \ell}$ and $\frac{\partial \left(\frac{1+\ell+(V_{2B}^S-V_{2C}^P)\frac{\pi_2}{\pi_1}}{1+f+(V_{2B}^S-V_{2C}^P)\frac{\pi_2}{\pi_1}} \right)}{\partial \ell}$. ■

Proof of Proposition 6. Using Table 5, it is sufficient to show that τ_2^* , τ_{1B}^* , and τ_{1P}^* are decreasing in ρ , which is straightforward to show for $\frac{\pi_2}{\pi_1}$ sufficiently small or for h_C sufficiently close to h . ■

Proof of Proposition 7. By (14) and (19), under Assumptions A1–A3,

$$\Psi_1^P - \Psi_1^S = h\rho(2-\rho) \int_{\tau_{1P}^*}^{\tau_{1B}^*} (1-\tau)g(\tau)d\tau > 0. \quad (30)$$

By (15) and (20),

$$\begin{aligned} \Psi_2^P - \Psi_2^S &= \Psi_1^P \Psi_{2C}^P + (1 - \Psi_1^P) \Psi_{2B}^S - \Psi_1^S \Psi_{2C}^S - (1 - \Psi_1^S) \Psi_{2B}^S \\ &< (\Psi_1^P - \Psi_1^S) \Psi_{2C}^P - (\Psi_1^P - \Psi_1^S) \Psi_{2B}^S \\ &= (\Psi_1^P - \Psi_1^S) (\Psi_{2C}^P - \Psi_{2B}^S) < 0, \end{aligned}$$

where the first inequality follows from $\Psi_{2C}^P < \Psi_{2C}^S$ by Lemma 2(iii) and the second

inequality follows from (30) and Lemma 2(i) as long as $h_C/h < r_A^*$.

Finally, by (17) and (22),

$$\begin{aligned} V^P \pi_1 - V^S \pi_1 &= (V_1^P - V_1^S) \pi_1 + (\Psi_1^P V_{2C}^P - \Psi_1^S V_{2C}^S) \pi_2 + (\Psi_1^S - \Psi_1^P) V_{2B}^S \pi_2 \\ &> (V_1^P - V_1^S) \pi_1 + (\Psi_1^P - \Psi_1^S) (V_{2C}^P - V_{2B}^S) \pi_2 > 0, \end{aligned}$$

where the first inequality follows from $V_{2C}^P > V_{2C}^S$ by Lemma 2(iii) and the second inequality follows, as long as π_2/π_1 is above a threshold, because $(\Psi_1^P - \Psi_1^S) (V_{2C}^P - V_{2B}^S) > 0$ by (30) and Lemma 2(ii), which says that $V_{2C}^P > V_{2B}^S$ if $h_C/h < r_B^*$. ■

Proof of Proposition 8. By (8) and (9), $\frac{\partial \Psi_{2B}^S}{\partial h_C} = \frac{\partial V_{2B}^S}{\partial h_C} = 0$, while $\frac{\partial \Psi_{2C}^S}{\partial h_C} = \frac{\Psi_{2C}^S}{h_C} > 0$ and $\frac{\partial V_{2C}^S}{\partial h_C} = -\frac{\Psi_{2C}^S}{h_C} (1+f) < 0$. Hence it follows from (13) and (14) that

$$\frac{\partial \tau_{1B}^*}{\partial h_C} = -\frac{(4-\rho)(f-\ell)2(2-\rho)\frac{\pi_2}{\pi_1}}{\left(2(2-\rho)\left[1+f+(V_{2B}^S-V_{2C}^S)\frac{\pi_2}{\pi_1}\right]\right)^2} \frac{\partial V_{2C}^S}{\partial h_C} > 0$$

and

$$\frac{\partial \Psi_1^S}{\partial h_C} = -h\rho(2-\rho)(1-\tau_{1B}^*)g(\tau_{1B}^*)\frac{\partial \tau_{1B}^*}{\partial h_C} < 0.$$

By (15)

$$\begin{aligned} \frac{\partial \Psi_2^S}{\partial h_C} &= \frac{\partial \Psi_1^S}{\partial h_C} (\Psi_{2C}^S - \Psi_{2B}^S) + \Psi_1^S \frac{\partial \Psi_{2C}^S}{\partial h_C} \\ &= \frac{\partial \Psi_1^S}{\partial h_C} \left(\Psi_{2C}^S - \frac{h\Psi_{2C}^S}{h_C} \right) + \Psi_1^S \frac{\Psi_{2C}^S}{h_C} \\ &= \frac{\Psi_{2C}^S}{h_C} \left(\Psi_1^S + (h_C - h) \frac{\partial \Psi_1^S}{\partial h_C} \right), \end{aligned}$$

which is positive for h_C sufficiently close to h .

By (10) and (11), $\frac{\partial \Psi_{2C}^P}{\partial h_C} = \tau^E > 0$ and $\frac{\partial V_{2C}^P}{\partial h_C} = -\tau^E(1+f) < 0$. Hence it follows from (18) and (19) that

$$\frac{\partial \tau_{1P}^*}{\partial h_C} = -\frac{(4-\rho)(f-\ell)2(2-\rho)\frac{\pi_2}{\pi_1}}{\left(2(2-\rho)\left[1+f+(V_{2B}^S-V_{2C}^P)\frac{\pi_2}{\pi_1}\right]\right)^2} \frac{\partial V_{2C}^P}{\partial h_C} > 0$$

and

$$\frac{\partial \Psi_1^P}{\partial h_C} = -h\rho(2-\rho)(1-\tau_{1P}^*)g(\tau_{1P}^*)\frac{\partial \tau_{1P}^*}{\partial h_C} < 0. \quad (31)$$

By (20),

$$\frac{\partial \Psi_2^P}{\partial h_C} = \frac{\partial \Psi_1^P}{\partial h_C} (\Psi_{2C}^P - \Psi_{2B}^S) + \Psi_1^P \frac{\partial \Psi_{2C}^P}{\partial h_C},$$

where the second term is positive and the first term is also positive by (31) and Lemma 2(i) as long as h_C/h is less than the threshold r_A^* . ■

Proof of Proposition 9. First, note that

$$\frac{\partial \Psi_{2i}^S}{\partial h_2} = \frac{\Psi_{2i}^S}{h_2} > 0. \quad (32)$$

Second, by (8) we have

$$\begin{aligned} \Psi_{2i}^S &= h_2 \left[1 - (1 - \rho)^2 \int_{\tau_2^*}^1 (1 - \tau) g(\tau) d\tau - \int_0^{\tau_2^*} (1 - \tau) g(\tau) d\tau \right] \\ &= h_2 \left[1 - (1 - \rho)^2 \left(\int_{\tau_2^*}^1 G(\tau) d\tau - (1 - \tau_2^*) G(\tau_2^*) \right) - \left((1 - \tau_2^*) G(\tau_2^*) + \int_0^{\tau_2^*} G(\tau) d\tau \right) \right] \\ &= h_2 \left[1 - (1 - \rho)^2 \int_{\tau_2^*}^1 G(\tau) d\tau - \rho(2 - \rho)(1 - \tau_2^*) G(\tau_2^*) - \int_0^{\tau_2^*} G(\tau) d\tau \right], \end{aligned}$$

where the second equality follows from integration by parts. It is immediate that a first order stochastic shift in the distribution of τ , by reducing $G(\tau)$, increases Ψ_{2i}^S . By (32), it is also immediate that a first order stochastic shift in the distribution of τ increases $\frac{\partial \Psi_{2i}^S}{\partial h_2}$, and hence resources for investigation and prosecution are strategic complements. ■

References

- Angeletos, George-Marios, Christian Hellwig, and Alessandro Pavan (2007), “Dynamic Global Games of Regime Change: Learning, Multiplicity, and Timing of Attacks,” *Econometrica* 75, 711–756.
- Angelucci, Charles and Martijn A. Han (2012), “Private and Public Control of Management,” Working Paper, Toulouse School of Economics.
- Aubert, Cécile, Patrick Rey, and William E. Kovacic (2006), “The Impact of Leniency and Whistle-blowing Programs on Cartels,” *International Journal of Industrial Organization* 24(6), 1241–1266.
- Bandyopadhyay, Siddhartha and Kalyan Chatterjee (2010) “Crime Reporting: Profiling and Neighbourhood Observation,” *The B.E. Journal of Theoretical Economics* 10(1) (Advances), Article 7, available at: <http://www.bepress.com/bejte/vol10/iss1/art7>.
- Beaton-Wells, Caron (2008a), “Forks in the Road: Challenges Facing the ACCC’s Immunity Policy for Cartel Conduct: Part 1,” *Competition & Consumer Law Journal* 16, 71–113.
- Beaton-Wells, Caron (2008b), “Forks in the Road: Challenges Facing the ACCC’s Immunity Policy for Cartel Conduct: Part 2,” *Competition & Consumer Law Journal* 16, 246–285.
- Beaton-Wells, Caron and Brent Fisse (2011), *Australian Cartel Regulation: Law, Policy and Practice in an International Context*, Cambridge, UK: Cambridge University Press.
- Bigoni, Maria, Sven-Olof Fridolisson, Chloé Le Coq and Giancarlo Spagnolo (2012a), “Trust, Leniency and Deterrence: Evidence from an Antitrust Experiment,” Working Paper, University of Bologna.
- Bigoni, Maria, Sven-Olof Fridolisson, Chloé Le Coq and Giancarlo Spagnolo (2012b), “Fines, Leniency, and Rewards in Antitrust,” *RAND Journal of Economics* 43(2), 368–390.
- Bloom, Margaret (2007), “Despite Its Great Success, the EC Leniency Program Faces Great Challenges,” in *European Competition Law Annual 2006: Enforcement of Prohibition of Cartels*, ed. by Claus-Dieter Ehlermann and Isabela Atanasiu, Portland, OR: Hart Publishing.
- Brisset, Karine and Lionel Thomas (2004), “Leniency Programs: A New Tool in Competition Policy to Deter Cartel Activity in Procurement Auctions,” *European Journal of Law and Economics* 17, 5–19.
- Buccirossi, Paolo and Giancarlo Spagnolo (2005), “Optimal Fines in the Era of Whistleblowers: Should Price Fixers Still Go to Prison?,” Lear Research Paper No 05-01.
- Buccirossi, Paolo and Giancarlo Spagnolo (2006), “Leniency Policies and Illegal Transactions,” *Journal of Public Economics* 90, 1281–1297.

- Carlsson, Hans and Eric van Damme (1993a), “Global Games and Equilibrium Selection,” *Econometrica* 61(5), 989–1018.
- Carlsson, Hans and Eric van Damme (1993b), “Equilibrium Selection in Stag Hunt Games,” in *Frontiers of Game Theory*, ed. by Kenneth Binmore, Alan Kirman, and Piero Tani, Cambridge, MA: MIT Press, 237–253.
- Chamley, Christophe (1999), “Coordinating Regime Switches,” *Quarterly Journal of Economics* 114, 869–905.
- Chen, Joe, and Joseph E. Harrington, Jr. (2007), “The Impact of the Corporate Leniency Program on Cartel Formation and the Cartel Price Path,” in Vivek Ghosal and Johan Stennek, eds., *The Political Economy of Antitrust*, New York: Elsevier, 59–80.
- Chen, Zhijun and Patrick Rey (2013), “On the Design of Leniency Programs,” *Journal of Law and Economics* 56, 917–957.
- Choi, Jay Pil and Heiko Gerlach (2012a), “Global Cartels, Leniency Programs and International Antitrust Cooperation,” *International Journal of Industrial Organization* 30(6), 528–540.
- Choi, Jay Pil and Heiko Gerlach (2012b), “Multi-Market Collusion with Demand Linkages and Antitrust Enforcement,” Working Paper, University of New South Wales.
- Danielson, Jon, Paul Embrechts, Charles Goodhart, Felix Muennich, and Hyun Song Shin (2001), “An Academic Response to Basel II,” Financial Markets Group Special Paper No. 130, London School of Economics.
- Frankel, David M., Stephen Morris, and Ady Pauzner (2003), “Equilibrium Selection in Global Games With Strategic Complementarities,” *Journal of Economic Theory* 108, 1–44.
- Giannitsarou, Chryssi and Flavio Toxvaerd (2012), “Recursive Global Games,” Working Paper, University of Cambridge.
- Green, Edward J., Robert C. Marshall, and Leslie M. Marx (2013), “Tacit Collusion in Oligopoly,” Working Paper, Penn State University.
- Hammond, Scott D. (2004), “Cornerstones of an Effective Leniency Program,” U.S. Department of Justice, <http://www.justice.gov/atr/public/speeches/206611.pdf>.
- Hammond, Scott D. and Belinda A. Barnett (2008), “Frequently Asked Questions Regarding the Antitrust Division’s Leniency Program and Model Leniency Letters,” U.S. Department of Justice, <http://www.justice.gov/atr/public/criminal/239583.pdf>.
- Harcourt, Bernard E. (2006): *Against Prediction: Sentencing, Policing and Punishing in an Actuarial Age*, Chicago: Chicago University Press.
- Harrington, Joseph E., Jr. (2008), “Optimal Corporate Leniency Programs,” *Journal of Industrial Economics* 56, 215–246.
- Harrington, Joseph E., Jr. (2011), “Corporate Leniency Programs when Firms have Private Information: The Push of Prosecution and the Pull of Pre-emption,” forthcoming in *Journal of Industrial Economics*.

- Heinemann, Frank and Gerhard Illing (2002), "Speculative Attacks: Unique Sunspot Equilibrium and Transparency," *Journal of International Economics* 58, 429–450.
- Heinemann, Frank, Rosemarie Nagel, and Peter Ockenfels (2004), "The Theory of Global Games on Test: Experimental Analysis of Coordination Games with Public and Private Information," *Econometrica* 72(5), 1583–1599.
- Hellwig, Christian (2002), "Public Information, Private Information, and the Multiplicity of Equilibria in Coordination Games," *Journal of Economic Theory* 107, 191–222.
- Hinlopen, Jeroen and Adriaan R. Soetevent (2008), "Laboratory Evidence on the Effectiveness of Corporate Leniency Programs," *RAND Journal of Economics* 39, 607–616.
- Knowles, John, Nicola Persico, and Petra Todd (2001), "Racial Bias in Motor-Vehicle Searches: Theory and Evidence," *Journal of Political Economy* 109, 203–29.
- Kovacic, William E., Robert C. Marshall, Leslie M. Marx, and Halbert L. White (2011), "Plus Factors and Agreement in Antitrust Law," *Michigan Law Review* 110(3), 393–436.
- Lefouili, Yassine and Catherine Roux (2012), "Leniency Programs for Multimarket Firms: The Effect of Amnesty Plus on Cartel Formation," *International Journal of Industrial Organization* 30, 624–640.
- Levin, Jonathan (2001), "A Note on Global Games With Overlapping Generations," Mimeo, Stanford University.
- Marshall, Robert C. and Leslie M. Marx (2012), *The Economics of Collusion: Cartels and Bidding Rings*, Cambridge, MA: MIT Press.
- Masoudi, Gerald F. (2007), "Cartel Enforcement in the United States (and Beyond)," Presentation at the Cartel Conference Budapest, Hungary, available at <http://www.justice.gov/atr/public/speeches/221868.pdf> (accessed November 16, 2012).
- Metz, Christina E. (2002), "Private and Public Information in Self-Fulfilling Currency Crises," *Journal of Economics* 76, 65–85.
- Miller, Nathan H. (2009), "Strategic Leniency and Cartel Enforcement," *American Economic Review* 99(3), 750–768.
- Morris, Stephen and Hyun Song Shin (1998), "Unique Equilibrium in a Model of Self-Fulfilling Currency Attacks," *American Economic Review* 88(3), 587–597.
- Morris, Stephen and Hyun Song Shin (1999), "A Theory on the Onset of Currency Attacks," in *The Asian Financial Crisis: Causes, Contagion and Consequences*, ed. by P.-R. Agenor, M. Miller, D. Vines, and A. Weber, Cambridge, U.K.: Cambridge University Press, 230–255.
- Morris, Stephen and Hyun Song Shin (2002), "Global Games: Theory and Applications," in *Advances in Economics and Econometrics, The Eighth World Congress*, ed. by M. Dewatripont, L. Hansen, and S. Turnovsky, Cambridge, U.K.: Cambridge University Press, 56–114.

- Motta, Massimo and Michele Polo (2003), “Leniency Programs and Cartel Prosecution,” *Journal of Industrial Organization* 21, 347–379.
- Motchenkova, Evgenia (2004), “Effects of Leniency Programs on Cartel Stability,” Center for Economic Research Discussion Paper No. 2004-98, Tilburg University.
- Ordoñez, Guillermo (2012), “Fragility of Reputation and Clustering of Risk Taking,” Working Paper, University of Pennsylvania.
- Prati, Alessandro, and Massimo Sbracia (2002), “Currency Crises and Uncertainty about Fundamentals,” IMF Working Paper WP 02/03.
- Rey, Patrick (2003), “Towards a Theory of competition Policy,” in M. Dewatripont, L.P. Hansen, and S.J. Turnovsky, eds., *Advances in Economics and Econometrics: Theory and Applications*, Cambridge, UK: Cambridge University Press.
- Sokol, Daniel D. (2012), “Cartels, Corporate Compliance and What Practitioners Really Think About Enforcement,” *Antitrust Law Journal* 78, 201–240.
- Spagnolo, Giancarlo (2000), “Self-Defeating Antitrust Laws: How Leniency Programs Solve Bertrand’s Paradox and Enforce Collusion in Auctions,” FEEM Working Paper 52.00, Fondazione Eni Enrico Mattei.
- Spagnolo, Giancarlo (2004), “Divide et Impera: Optimal Leniency Programs,” CEPR Discussion Paper No. 4840.
- Spagnolo, Giancarlo (2008), “Leniency and Whistleblowers in Antitrust,” Ch. 7 of P. Buccirossi (Ed.) *Handbook of Antitrust Economics*, Cambridge, MA: MIT Press.
- Stephan, Andreas (2009), “An Empirical Assessment of the European Leniency Notice,” *Journal of Competition Law & Economics* 5, 537–561.
- Stigler, George J. (1964), “A Theory of Oligopoly,” *Journal of Political Economy* 72(1), 44–61.
- Wils, Wouter P. J. (2008a), *Efficiency and Justice in European Antitrust Enforcement*, Portland, OR: Hart Publishing.
- Wils, Wouter P. J. (2008b), “The Use of Settlements in Public Antitrust Enforcement: Objectives and Principles,” *World Competition* 31(3), 335–352.
- Wils, Wouter P. J. (2007), “Leniency in Antitrust Enforcement: Theory and Practice,” *World Competition* 30(1), 25–64.
- Zhou, Jun (2012), “Evaluating Leniency with Missing Information on Undetected Cartels: Exploring Time-Varying Policy Impacts on Cartel Duration,” Working Paper, Bonn University.