Innovation and competitive pressure

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Abstract

I analyze the effects of competition on process and product innovation and obtain robust results that hold for a variety of market structures, including markets with restricted or free entry and markets characterized by either price or quantity competition. It is found that increasing the number of firms tends to reduce R&D effort, whereas increasing the degree of product substitutability, with or without free entry, increases R&D effort—provided that the total market for varieties does not shrink. Increasing the total market size increases R&D effort and has ambiguous effects on the number of varieties while decreasing the cost of entry increases the number of entrants and varieties but reduces R&D effort per variety.

Keywords: R&D, cost reduction, X-inefficiency, market concentration, market size, substitutability, product introduction, entry, corporate governance, globalization

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1 Introduction

This paper provides general and robust results on the effect of indicators of competitive pressure on innovation, reconciles theory with available empirical results, and provides a framework to help guiding the empirical analysis with results that do not depend on the fine details of market structure. The central question to examine is whether competitive pressure fosters innovation.

Innovation is claimed to be the engine of growth (see e.g. Romer (1990), Aghion and Howitt (1992, 1998), Grossman and Helpman (1989, 1991,1994)) and therefore it is crucial to understand its determinants. Furthermore, questions arise about the impact of globalization and deregulation on the incentives to innovate. The impact of globalization comes typically with market enlargement; regulatory reform has introduced price caps, i.e., direct price pressure, and has lowered barriers to entry in different industries. What will be the impact of these developments on process and product innovation?

There is by now a large body of work, going back at least to Schumpeter and continuing with Arrow (1962) and many other scholars, with regard to the effect of competitive pressure on innovation effort. Schumpeter himself oscillated between thinking that monopoly profit or competitive pressure were the drivers of innovation although usually only the monopoly driver is emphasized in the interpretation of his work. One stumbling block in the analysis of competition and innovation is the use of particular functional forms. Another, the lack of agreement between theory and empirics.

Theoretical work, be it in industrial organization, agency theory or endogenous growth theory, has relied on particular functional specifications. Leading models of process innovations like Dasgupta and Stiglitz (1980) and Spence (1984) use constant elasticity functional forms; Bester and Petrakis (1993) and Qiu (1997) compare innovation incentives in Cournot and Bertrand markets with a linear-quadratic specification. Similarly, models of X-inefficiency in which there is an agency problem between owners and managers rely on very simple and parameterized specifications of market competition. This is the case, for instance, in the linear model of Martin (1993), the examples in Schmidt (1997), and the linear-quadratic model of Raith (2003). The constant elasticity specification, derived from Dixit and Stiglitz (1977), has become also a workhorse in endogenous growth models with product innovation providing an ever expanding variety of horizontally differentiated products in a growing market (Romer (1990) and Grossman and Helpman (1989)). Rivera-Batiz and Romer (1991) show that international economic integration by expanding the market size incentivates innovation. One may wonder whether the results obtained are robust to more general specifications.

Some theoretical results do not seem to agree with the empirical evidence. Indeed, quite a few theoretical models tend to conclude that competition reduces innovation effort despite the fact that available empirical evidence (Porter (1990), Geroski (1990, 1994), Baily and Gersbach (1995), Nickell (1996), Blundell, Griffith, and Van Reenen (1999), Symeonidis (2002a, b), and Galdón-Sánchez and Schmitz (2002)) is favorable to the positive effect of competition on innovation. For example, Dasgupta and Stiglitz (1980) and Spence (1984) find that increasing the number of firms, a typical measure of increased competitive pressure, reduces innovation effort.

The benchmark model for the analysis is a symmetric reduced–form non–tournament model, where the investment by a firm always yields some innovation results for the firm —in contrast to a patent race with its winner–take–all feature, where there are no spillovers and R&D investment has no strategic commitment value.

I will consider price and quantity competition as well as restricted and free entry. The models considered are the central workhorses in oligopoly theory: Bertrand (price) competition with product differentiation and Cournot (quantity) competition with homogenous products. All cases are empirically relevant although perhaps Bertrand competition with differentiated products and free entry is more salient. In this situation a firm decides whether to enter the market producing a new variety (product innovation), and paying a fixed cost of entry, and how much to invest in reducing variable costs of production (process innovation). The model displays thus the typical trade-off between fixed and variable costs of previous work in the literature. I perform also a robustness check when investments have strategic commitment value, when spillovers are present, and comment on how far the results extend to investment in quality.

It is worth noting that the trade-off between fixed and variable costs not only includes R&D and cost–reduction models but also agency models where owners incentivate managers to reduce costs in an asymmetric information context (X-inefficiency). That is, the firm (or the principal) incurs in a fixed cost in order to lower variable costs. In agency models the innovation incentive of owners typically translates monotonically, via the incentive scheme of the manager, into the managers' incentives. The owner must pay the manager his reservation utility, the cost of effort, and an information rent (owing to asymmetric information) in order to reduce costs. In this way, for example, more competition may induce a higher cost–reduction effort through an incentive scheme that is more sensitive to performance.[1]

The central scenario considered is plausible on empirical grounds. With regard to the non–tournament aspect, patents (inducing a patent race or tournament) do not seem to be the major source of returns to innovative activity (Schankerman (1991) and Cohen et al. (2000))[2] and, according to Cohen, "The empirical findings to date do not establish whether the net effect of appropriability on R&D incentives is positive or negative" (1995, p. 230). Furthermore, it is worth remarking that even though R&D investment typically precedes market interaction, this does not mean that it can be used strategically. That is, it does not follow that R&D investment, or contracts with managers that reward effort, are observable and that firms can commit to it. The evidence on the strategic commitment value of R&D is scant.[3] No claim is made about the realism of the symmetry assumption.[4]

I consider three (classical) different possible measures of enhanced competitive pressure

[1] Hubbard and Palia (1995) and Cuñat and Guadalupe (2002) provide evidence of how competition increases the performance-pay sensitivity, respectively, of CEOs in the U.S. banking industry after deregulation and of CEO, executives, and workers in a panel of U.K. firms after the pound's appreciation in 1996. Competition may also provide information (e.g., on the cost structure of firms) and enlarged opportunities for comparison, and therefore stronger incentives. The informational role of competition in enhancing efficiency has been highlighted in a series of models. I will not pursue this line of inquiry in this paper but see Hart (1983), Scharfstein (1988), Hermalin (1992, 1994), and Meyer and Vickers (1997).

[2] Recent empirical analysis does not seem to favor the patent race model (with its first-mover advantage). See Tellis and Golder (1996) and Lieberman and Montgomery (1998).

[3] At the same time, it is possible that strategic effects have been overplayed in the literature. For example: "Despite the considerable theoretical attention devoted to strategic interaction, we know surprisingly little about its empirical relevance" (Cohen (1995, p. 234; see also Griliches (1995)). Geroski (1991) also hints that strategic effects may be of second–order importance in determining innovation incentives.

[4] See Boone (2000) and Klette and Kortum (2004) for an analysis of innovation incentives with firm heterogeneity.

with free entry (endogenous market structure), as an increase

- in the degree of product substitutability; or
- in the total size of the market; or
- in the ease of entry (decrease in the entry cost).

With restricted entry (exogenous market structure), competitive pressure is measured as a variation[5]

- in the degree of product substitutability; or
- in the number of competitors.

In the scenario considered, individual firms' cost–reduction incentives depend on the output per firm because the value of a reduction in unit costs will increase with the output produced by the firm. Output per firm depends in turn on demand and price–pressure effects. For a given total market size, competition affects the effective market of a firm, its residual demand (a level or size effect), and the elasticity of the residual demand faced by the firm (an elasticity effect). For example, typically an increase in the number of competitors for a given total market size will decrease the residual demand for the firm and will increase the demand elasticity. The first effect will tend to decrease R&D effort because a unit cost reduction will benefit a diminished output, whereas the second will tend to increase R&D effort, because a unit reduction in costs will allow the firm to decrease price with a higher output impact.[6]

I obtain the following results in a market with restricted entry:

- Increasing the number of firms tends to reduce R&D effort. In Bertrand the result holds for all leading examples (including linear, constant elasticity, constant expenditure, and logit demand systems). In Cournot the result holds in the usual case of

[5] Sometimes a change from Cournot to Bertrand behavior is interpreted as an increase in competitive pressure. This may be so, since Bertrand equilibria tend to be more competitive than Cournot, but this interpretation need not make sense within a given industry. Indeed, the mode of competition is typically dictated by the structural conditions in the industry (see Vives (1999, Chap.7)).

[6] See Kamien and Schwartz (1970) and Willig (1987) for related analyses.

outputs being strategic substitutes. The residual demand (size) effect tends to dominate the price–pressure (elasticity) effect. The reason is that the demand effect is a direct one while the price–pressure effect is an induced one through the impact on the equilibrium price. However, it is still possible, and indeed likely, that increasing the number of firms increases R&D intensity (i.e. R&D expenditure over sales).

- Increasing the degree of product substitutability increases R&D effort provided the total market for varieties does not shrink. The reason is that the demand effect and the price–pressure effect both work, although perhaps weakly, in the same direction. This holds for leading examples such as linear (Shapley–Shubik specification), constant elasticity, and constant expenditure demand systems. With logit there is neither demand effect nor price–pressure effect.

The results in a market with restricted entry generalize those obtained by (among others) Dasgupta and Stiglitz (1980), Spence (1984), Tandon (1984), and Martin (1993) in the context of a Cournot model with constant elasticity or linear specifications, and by Raith (2003) with price competition and product differentiation à la Salop (1979).

The empirical literature is consistent with the findings. Cohen and Klepper (1996a, b) provide compelling evidence of the positive relationship between R&D expenditure and firm size (at the business unit level). Pagano and Schivardi (2003) find a positive relationship between average firm size and productivity growth. Bertrand and Kramarz (2002) and Ebell and Haefke (2003) provide evidence on the output expansion effect of competition.

In a market with free entry I find the following:

- Increasing the total market size increases per–firm output and R&D effort. However, the number of firms and varieties may increase or diminish. The results hold for either Bertrand or Cournot competition. Increasing the market size has a direct positive impact on R&D effort and output per firm, but at the same time it may increase the free–entry number of firms. However, the latter increases less than proportionately, owing to the reduction in margins, and the direct effect prevails. In fact, the free–entry number of firms may even decrease with market size. The reason is that the increase in market size may induce such an increase in expenditure on cost reduction that it may leave less room for entry. In a constant elasticity example with no entry cost, the free–entry number of firms is independent of market size. Nonetheless the case of a positive effect of market size on product innovation is more likely because of its direct profitability–enhancing impact.

- Decreasing the entry cost increases the number of firms (varieties) introduced but decreases output and R&D effort per firm. The first result is very intuitive and implies the second: once more firms have entered we know that there are less incentives to invest in cost reduction.

- Increasing the degree of product substitutability increases R&D effort (and per–firm output) provided the total market does not shrink. The number of varieties introduced may diminish (and it will do so if the market does not expand). The reason of the latter is that the increase in competitive pressure leaves less room for entrants. If this happens it should be clear that per–firm output and innovation effort should increase because of increased price pressure with less entry. If the market where to expand with the degree of product substitutability then the direct effect on per–firm output and innovation effort would overcome any possible adverse effect of a possible increase in the number of entrants.

Schmookler (1959, 1962) emphasized the role of demand and market size in the innovation incentive and stated that innovative activity is governed by the extent of the market. Process innovation is enhanced in larger markets but not necessarily product innovation. The result that larger markets may accommodate fewer firms (and varieties) is consistent with the parameterized examples in Sutton (1991). The intuition is that a larger market may make the R&D competition so fierce that fewer firms may be able to survive (and cover their fixed cost). Decreasing barriers to entry (by lowering the entry cost) will indeed induce more firms (and varieties) to enter but will diminish the incentive of each firm to produce and invest in cost reduction. The result that increasing product substitutability increases innovation effort but may decrease the number of varieties introduced is consistent with the findings in Boone (2000) for symmetric market structures. The results also generalize those obtained by Raith (2003).

The empirical literature tends to confirm the role of market size in explaining the incentives to innovate (see Scherer and Ross (1990) and Cohen (1995) for surveys as well as Symeonides (2002a, chap. 6)).[7] Acemoglu and Linn (2004) find a large effect of an increase in market size on the entry (quality improvement) of nongeneric drugs and new molecular entities in the pharmaceutical industry. The authors present also a theoretical model with a constant elasticity specification. Kremer (2002) also builds on the idea that market size drives pharmaceutical research. Syverson (2004a) finds in the ready-mixed concrete industry that higher (spatial) substitutability, created by transport costs, increases average productivity. The same author provides evidence that industries' median productivity levels are increasing in the degree of product substitutability of the industry products (Syverson (2004b)).

The results with free entry suggest also that market integration and opening of markets may yield unambiguous benefits in terms of innovation effort. Indeed, an increase in market size can result from international market integration or the dismantling of barriers to trade. We would thus have a connection between globalization, understood as the general lowering of transport costs and barriers to trade, and innovation effort.[8] Our results in particular are consistent with the findings in Baily and Gersbach (1995) that competition in the global marketplace is what gives companies a productivity advantage.

The plan of the paper is as follows. Section 2 considers the case of Bertrand markets with product differentiation. Section 3 deals with Cournot markets with homogeneous products. In both sections the cases of restricted and free entry are analyzed. Section 4 explores extensions of the results (an alternative measure of competitive pressure, quality innovation, and the effect of spillovers). Concluding remarks close the paper, and the Appendix collects several proofs and the details of the examples.

[7] Blundell, Griffith, and Van Reenen (1999) provide evidence on the positive impact of market share on innovation output for a panel of British manufacturing firms.

[8] See e.g. Krugman (1995).

2 Bertrand competition with product differentiation

In this section I consider Bertrand markets with differentiated products. I start with the case with restricted entry and perform a comparative statics exercise with the number of firms and with the degree of product substitutability (Section 2.1). A robustness exercise with respect to the strategic commitment effect of R&D is performed and I comment also on using a switch from Cournot to Bertrand competition to indicate a higher competitive pressure. Section 2.2 establishes connections with the empirical literature about the inverted U-shaped relationship found in some studies. I consider then the case of free entry and study the comparative statics properties with respect to the degree of product substitutability, size of the market and entry cost.

Consider a differentiated product market with n firms, where each firm produces a different variety and $F \geq 0$ is the sunk cost of entry. The demand system for the varieties is symmetric and is given by the smooth (whenever demand is positive) and exchangeable functions $x_i = SD_i(p)$, $p = (p_1, ..., p_n)$, $i = 1, ..., n$, where S denotes the size of the market (number of consumers, say).[9] Demand is downward sloping $\frac{\partial D_i}{\partial p_i} < 0$, products are gross substitutes $\frac{\partial D_i}{\partial p_j} > 0$, $j \neq i$, and the Jacobian of the demand system is negative definite.

Firm i can invest z_i to reduce its constant marginal cost of production c_i according to a smooth function $c_i = c(z_i)$ with $c(z) > 0$, $c'(z) < 0$, and $c''(z) > 0$ for all $z > 0$. The cost for firm i of producing output x_i is $c(z_i)\, x_i$.

The profits for firm i are therefore

$$\pi_i = (p_i - c(z_i))\, SD_i(p) - z_i - F.$$

2.1 Restricted entry: Price–pressure and demand effects

Let the number of firms n be fixed and $S = 1$. Consider the simultaneous–move game in which each firm chooses an investment–price pair. This can be interpreted also as an

[9] That is, interchanging the prices of rival goods does not affect the demand for any good (as a function of its own price) and any two goods that sell at the same price have the same demand. Formally, the demand system can be described by a unique demand function for any good depending on its own price and the prices of rivals, $D_i(p_i; p_{-i}) = D(p_i; p_{-i})$ for all i.

open–loop strategy in a two–stage investment–price game (or just a two–stage game where actions in the first stage are not observable).

Let $H(p;\alpha) \equiv D_i(p,...,p;\alpha)$ be the demand for a variety when all firms set the same price (the Chamberlininan DD function) where α is a parameter that affects demand. I will consider $\alpha = n$, the number of firms, and $\alpha = \sigma$, a measure of product substitutability (typically, the elasticity of substitution between any two products, either the Allen-Hicks or the direct elasticity of substitution). For convenience we will think of n as a continuous variable but all results hold with n discrete. It follows from our assumptions that $\frac{\partial H}{\partial p}(p;\alpha) \equiv \frac{\partial D_i}{\partial p_i}(p,...,p;\alpha) + \sum_{j \neq i} \frac{\partial D_i}{\partial p_j}(p,...,p;\alpha) < 0.$[10] Let $h(p;\alpha) \equiv \frac{\partial D_i}{\partial p_i}(p,...,p;\alpha)$ and note that $h(p;\alpha) < 0$. The parameter α will be suppressed to ease notation in functions when no confusion is possible. A very wide range of demand systems fulfill the assumptions.

Fix a symmetric profile of investment $z_i = z$ and consider an associated (interior) symmetric Bertrand equilibrium $p(z,\alpha)$. In equilibrium we have that

$$L \equiv \frac{p-c}{p} = \frac{1}{\eta},$$

where L is the Lerner index and

$$\eta \equiv -\frac{p}{H(p)}h(p)$$

is the (absolute value of the) elasticity of demand for an individual firm.

If $\alpha = n$ then typically $\frac{\partial \eta}{\partial n} > 0$, and increasing the number of firms increases the elasticity of demand and decreases prices. If $\alpha = \sigma$ then typically $\frac{\partial \eta}{\partial \sigma} > 0$, and increasing product substitutability decreases prices.[11] Table 1 provides properties of examples of several commonly used demand systems: linear (Shapley–Shubik (1969) and Bowley (1924)

[10] See Vives (1999, Sec. 6.3).

[11] Suppose that demands come from a representative consumer with (strictly quasiconcave) utility function $U(x_0,x)$, where x is the vector of differentiated commodities and x_0 is the numéraire (this a generalization of the quasilinear case, for which $W(x_0,x) = x_0 + U(x)$). For a symmetric allocation, denote by σ the (Allen–Hicks) elasticity of substitution between any pair of differentiated goods, by σ_0 the elasticity of substitution between the numéraire and a differentiated good, and by η^I the income elasticity of the demand for a differentiated good. Assuming that the latter is bounded, at a symmetric Bertrand equilibrium we have $\eta = \mu\sigma_0 + (1-\mu)\sigma(n-1)n^{-1} + (1-\mu)\eta^I n^{-1}$, where μ is the expenditure share of the numéraire good. It is clear that η increases with σ. Increasing n has a more complex effect in the formula, but typically it will (among other effects) increase η by weakly increasing σ. See Benassy (1989) and Vives (1999, Sec. 6.4).

specifications), location (Salop (1979)), constant elasticity, constant expenditure demand systems (with exponential and constant elasticity specifications) and logit.

From the structure of the profit function π_i it should be clear that the incentive to reduce cost is larger when the firm produces a larger output. The question therefore is how parameter changes affect output in equilibrium. Let $x(z;\alpha) \equiv H(p(z,\alpha);\alpha)$ be the equilibrium output per firm in the Bertrand equilibrium for a given z. The decomposition

$$\frac{\partial x}{\partial \alpha} = \frac{\partial H}{\partial p}\frac{\partial p}{\partial \alpha} + \frac{\partial H}{\partial \alpha}$$

is instructive. The term

$$\frac{\partial H}{\partial p}\frac{\partial p}{\partial \alpha}$$

is the price–pressure effect: increasing α decreases p (in the leading examples considered with either $\alpha = n$ or $\alpha = \sigma$), which in turn increases demand. The term

$$\frac{\partial H}{\partial \alpha}$$

is the demand effect: the direct impact of α on demand. We will see how, when $\alpha = n$, the price–pressure and the demand effects have different signs, provided that there is a limited market for the differentiated varieties ($\partial H/\partial n < 0$), and the latter typically dominates as we will see. The basic reason for the dominance is that the price–pressure effect is an indirect one while the demand effect is a direct one. On the other hand, if $\alpha = \sigma$ then typically both the price–pressure effect and the demand effect (weakly) work in favor of more output and R&D effort. Indeed, there is no presumption that increasing the elasticity of substitution will decrease the symmetric demand for varieties.

A symmetric and interior equilibrium will be *regular* if (at the equilibrium) $B \equiv \left((p-c)\frac{\partial h}{\partial p} + h + \frac{\partial H}{\partial p}\right) c'' H + (c')^2 h \frac{\partial H}{\partial p} < 0$. Proposition 1 provides the comparative statics analysis of innovation effort with respect to the number of firms ($\alpha = n$) and product substitutability ($\alpha = \sigma$).

Proposition 1 *Let the demand system fulfill* $\frac{\partial D_i}{\partial p_i} < 0$ *and* $\frac{\partial D_i}{\partial p_j} > 0$ *for* $j \neq i$ *with negative definite Jacobian, and let* $c' < 0$ *and* $c'' > 0$*. Consider a symmetric regular interior equilibrium* (p^*, z^*)*. Then the following statements hold.*

(i)

$$sign\left\{\frac{dz^*}{d\alpha}\right\} = sign\left\{\frac{\partial x\left(z;\alpha\right)}{\partial\alpha}\right\} = sign\left\{\frac{\partial H}{\partial p}\frac{\partial p(z,\alpha)}{\partial\alpha} + \frac{\partial H}{\partial\alpha}\right\},$$

where $(p(z,\alpha), x\left(z;\alpha\right))$ *is the symmetric Bertrand equilibrium for given* α *and* z*.*

(ii) When changing the number of firms n *for linear, constant elasticity, logit, and constant expenditure demand systems, we have* $\frac{\partial H}{\partial n} < 0$ *and* $\frac{\partial\eta}{\partial n} > 0$*; the demand effect dominates the price–pressure effect, and* $\frac{\partial x(z;n)}{\partial n} < 0$*.*[12]

(iii) When varying product substitutability σ *in all cases considered,* $\frac{\partial\eta}{\partial\sigma} > 0$*. For linear (Shapley–Shubik specification), logit, and constant expenditure demand systems,* $\frac{\partial H}{\partial\sigma} = 0$*; for constant elasticity,* $\frac{\partial H}{\partial\sigma} > 0$*. For these cases, price–pressure and demand effects work (perhaps weakly) in the same direction and so* $\frac{\partial x(z;\sigma)}{\partial\sigma} > 0$*. The logit system (like classical location models) is a boundary case with neither price–pressure nor demand effects and so* $\frac{\partial x(z;\sigma)}{\partial\sigma} = 0$*. For the linear demand specification of Bowley,* $\frac{\partial H}{\partial\sigma} < 0$ *and* $\frac{\partial x(z;\sigma)}{\partial\sigma} < 0$*.*

Proof: Fix a symmetric profile of investment $z_i = z$ and consider an associated (interior) symmetric Bertrand equilibrium $p(z,\alpha)$. The first-order condition for a symmetric interior equilibrium is $(p-c)\frac{\partial D_i}{\partial p_i} + D_i = 0$, or

$$\phi\left(p;\alpha\right) \equiv \left(p-c\right)h\left(p;\alpha\right) + H\left(p;\alpha\right) = 0.$$

If $\frac{\partial\phi}{\partial p} = (p-c)\frac{\partial h}{\partial p} + h + \frac{\partial H}{\partial p} < 0$ it is immediate that

$$sign\left\{\frac{\partial p(z,\alpha)}{\partial\alpha}\right\} = sign\left\{(p-c)\frac{\partial h}{\partial\alpha} + \frac{\partial H}{\partial\alpha}\right\} = sign\left\{-\frac{\partial\eta}{\partial\alpha}\right\}.$$

A symmetric (interior) equilibrium of the full investment–price game[13] will satisfy the first–order condition for investment: $-xc'\left(z\right) - 1 = 0$ or

$$\Psi\left(z,\alpha\right) \equiv -H\left(p(z,\alpha);\alpha\right)c'\left(z\right) - 1 = 0.$$

A sufficient condition for $\partial\Psi/\partial z < 0$ is

$$B \equiv \left((p-c)\frac{\partial h}{\partial p} + h + \frac{\partial H}{\partial p}\right)c''H + \left(c'\right)^2 h\frac{\partial H}{\partial p} < 0.$$

[12] When $B < 0$ we have that $sign\,\frac{dz^*}{dn} = sign\left\{\frac{\partial H}{\partial p}\left(p-c\right)\frac{\partial h}{\partial n} - \frac{\partial H}{\partial n}\left(\left(p-c\right)\frac{\partial h}{\partial p} + h\right)\right\}$ and $sign\,\frac{dp^*}{dn} = sign\left\{\left((p-c)\frac{\partial h}{\partial n} + \frac{\partial H}{\partial n}\right)c''H + \left(c'\right)^2 h\frac{\partial H}{\partial n}\right\}$. A sufficient condition for $\frac{dz^*}{dn} < 0$ is that $\frac{\partial h}{\partial n} > 0$ and $\frac{\partial h}{\partial p} < 0$.

[13] If $\pi_i = \left(p_i - c\left(z_i\right)\right)D_i\left(p\right) - z_i$ is strictly concave in (p_i, z_i), then some mild boundary conditions ensure the existence of an interior equilibrium.

Note that $B < 0$ implies that $\frac{\partial \phi}{\partial p} < 0$.

We are now ready to assess the impact of the parameter α on the equilibrium z. From $\frac{dz}{d\alpha} = -\frac{\partial \Psi / \partial \alpha}{\partial \Psi / \partial z}$ we have that

$$sign \frac{dz}{d\alpha} = sign \frac{\partial \Psi}{\partial \alpha} = sign \left\{ -c' \frac{\partial x(z; \alpha)}{\partial \alpha} \right\},$$

where $x(z; \alpha) \equiv H(p(z, \alpha); \alpha)$ is the equilibrium output per firm in the Bertrand equilibrium for a given z and the result in (i) follows. Note that, indeed, we have that innovation effort and individual output move in the same direction: $sign \left\{ \frac{dx}{d\alpha} \right\} = sign \left\{ \frac{\partial x(z;\alpha)}{\partial \alpha} \right\}$ because $\frac{dx}{d\alpha} = \frac{\partial x(z;\alpha)}{\partial z} \frac{dz}{d\alpha} + \frac{\partial x(z;\alpha)}{\partial \alpha}$ and $sign \frac{\partial x(z;\alpha)}{\partial z} = sign \{-c'h\} > 0$.

The results in (ii) and (iii) follow from Table 1 and the characterization of the examples in the Appendix. ■

Remarks

- The parameter α could also be interpreted as "regulatory pressure". It is then akin to our product substitutability measure with $\frac{\partial \eta}{\partial \alpha} > 0$ and $\frac{\partial H}{\partial \alpha} = 0$. Increasing α would exert price pressure, increasing output and R&D effort. The same effect would be obtained with a binding price cap (or with an increase in a sales tax paid by the firms when the price is regulated).[14]
- We can extend the characterization in Proposition 1(i) removing regularity conditions using lattice-theoretic methods as long as we restrict attention to extremal equilibria (I do so explicitly for the Cournot case in Proposition 7 in the Appendix).

Table 1 provides the properties of the examples claimed in Proposition 1, as well as computed equilibrium solutions for $c(z) = \alpha z^{-\gamma}$ with $\alpha > 0$ and $\gamma > 0$ and for the demand systems of constant elasticity, constant expenditure (constant elasticity specification), and logit. (The Appendix provides details for each example.) For those computed examples we find also that R&D intensity (R&D expenditures over sales) $\frac{z^*}{p^* x^*}$ is increasing in n and

[14] With unregulated prices it is easy to see that an increase in a proportional sales tax paid by the firms would increase prices and reduce output and innovation effort.

σ. It is worth noting therefore that despite the fact that $z^*$is decreasing with n a usual measure of the firm's R&D intensity is in fact increasing in n in the examples.

2.1.1 Strategic commitment effects

It may be asked if the results are robust with respect to strategic effects. Toward this end I analyze the subgame-perfect equilibria (SPE) of the two-stage game in which firms first invest in cost–reducing R&D —the investments are observable— and then compete in prices. Denote by $p^*(z_i, z_{-i})$, $i = 1, ..., n$, a second–stage Bertrand equilibrium for a given investment profile z, and let

$$(p_i^*(z_i, z_{-i}) - c(z_i)) D_i(p^*(z_i, z_{-i})) - z_i$$

be the corresponding profit of firm i in the reduced–form game at the first stage.

It is not difficult to see that, at a symmetric interior SPE of the two-stage game $(z, p^*(z))$, we have $H(p)+(p - c(z)) h(p) = 0$ and $-c'(z) H - 1 + (p - c(z))(n-1) \frac{\partial D_i}{\partial p_j} \frac{\partial p_j^*}{\partial z_i} = 0$. The term

$$(p - c(z))(n-1) \frac{\partial D_i}{\partial p_j} \frac{\partial p_j^*}{\partial z_i}$$

is the strategic commitment effect and it does not appear in the characterization of the equilibrium in the simultaneous move game. With strategic complementarity in prices and the condition $\frac{\partial H}{\partial p} + h + (p - c) \frac{\partial h}{\partial p} < 0$, it follows that $\frac{\partial p_j^*}{\partial z_i} \leq 0$ and therefore $(p - c)(n-1) \frac{\partial D_i}{\partial p_j} \frac{\partial p_j^*}{\partial z_i} \leq 0$ because goods are gross substitutes $\frac{\partial D_i}{\partial p_j} \geq 0$. Increasing the innovation effort of firm i reduces the equilibrium prices of rivals, because firm i is more aggressive and best responses are upward sloping. In order to perform comparative statics with respect to n note that the SPE z will be characterized by

$$F(z; n) \equiv -c'(z) x(z; n) - 1 + (p^*(z, n) - c(z))(n-1) \frac{\partial D_i}{\partial p_j} \frac{\partial p_j^*}{\partial z_i} = 0$$

with $x(z; n) \equiv H(p^*(z, n); n)$ and, provided $\frac{\partial F}{\partial z} < 0$, we have that the strategic effect makes firms invest less –because this softens price competition– and $sign \frac{dz}{dn} = sign \frac{\partial F}{\partial n}$.

We know that $sign \frac{\partial}{\partial n}(-c'(z) H - 1) = sign \left(-c' \frac{\partial x(z;n)}{\partial n}\right) = sign \frac{\partial x(z;n)}{\partial n}$. This confirms the result in the simultaneous–move game, with $dz/dn < 0$ when $\partial x(z;n)/\partial n < 0$.

The derivative of the strategic effect has an ambiguous sign. The reason is that increasing the number of firms may induce the firms in the first stage to distort their investment more (because there will be more competition at the second stage) or to distort it less (because, with more firms, the possibilities of manipulating the second–stage price equilibrium diminish).

However, tedious algebra shows that, in the case of constant expenditure demand system (with constant elasticity specification for demand and constant elasticity innovation costs) as well as in the Shapley–Shubik linear demand system and the logit case (both for a general innovation cost function) the result of the simultaneous game holds and $dz/dn < 0$. In all these examples investments at the first stage are strategic substitutes. Furthermore, in these examples the same comparative statics with respect to σ hold: $dz/d\sigma > 0$ for the first and second cases and $dz/d\sigma = 0$ for the logit.[15] Using the Bowley linear demand system, Qiu (1997) finds that $sign\, \frac{dz}{d\sigma} = sign\, \frac{\partial x}{\partial \sigma} < 0$ in the strategic two-stage game. This is the same result as in the simultaneous game according to Proposition 1.

2.1.2 Bertrand and Cournot

We can think of still another way to change competitive pressure in the market: by switching from Bertrand to Cournot. It is well known that Bertrand equilibria tend to be more competitive than Cournot equilibria (see Vives (1985), Singh and Vives (1984), and Vives (1999, Chap. 6) for a precise statement of the needed conditions). Typically we would then have, at symmetric equilibria and for the same level of costs, that the Bertrand output will be larger than the Cournot output and hence the incentive for cost reduction is greater in the former. However, this conclusion need not be robust to strategic commitment effects. Indeed, in Cournot (with strategic substitutes competition) it pays a firm to overinvest in order to gain an advantage, whereas in Bertrand (with strategic complements) it pays to underinvest in order to gain an advantage (Fudenberg and Tirole (1984)). This means that Cournot competition may induce more cost–reduction effort owing to this strategic effect even though the output in Bertrand may be higher (see the linear-quadratic models of Bester and Petrakis (1993), Qiu (1997), and Symeonidis (2003); in the latter, R&D increases

[15] The (lengthy) computations of the examples are available upon request.

product quality in a quality-augmented version of the Bowley demand system). However, it should be noted that, in general, if we want to know how an increase of competitive pressure in a particular industry affects innovation effort, then a comparison between Bertrand and Cournot equilibria will not be appropriate. Indeed, institutional features of the market typically determine the mode of competition.[16]

2.2 About the inverted U-shaped relationship

The result obtained —that in markets with restricted entry, the innovation effort per firm decreases in the number of firms— should be contrasted with some results in the empirical literature where an inverted U-shaped relationship is found between market concentration and R&D effort or output (see e.g. Scherer and Ross (1990), Caves and Barton (1990), and Aghion et al. (2002)).[17] For highly concentrated markets, a decrease in concentration seems to benefit innovation, although the effect is reversed for lower concentration levels. Aghion et al. (2002) relate a measure of innovative output (the count of successful patent applications) to a measure of competition (the Lerner index[18]) as a proxy for competitive pressure given by σ in a market with a fixed number of firms. In their step-by-step innovation model there are two forces: competition may increase the incremental profit from innovating (i.e., escape the competition effect for firms that are neck-to-neck, that is, that they have similar cost levels) but also may reduce innovation incentives for laggards when it is intense enough (by reducing rents to innovation). When competition is low the first force dominates, yet when competition is intense the second does owing to a composition effect in the steady-state distribution of technology gaps.

These empirical results can be reconciled with the analysis in this paper provided that competition involves also a liquidation effect that induces cost–reduction effort.[19] Galdón-

[16] See Vives (1999, Chap.7).

[17] See also Ceccagnoli (2003) for a nonmonotonic effect in an increase in the number of non-innovating firms.

[18] In fact, they use average cost instead of marginal cost and hence their measure of competition (in terms of our model) is instead $\widehat{L} \equiv \frac{p-c-(z/x)}{p} = L - \frac{z}{px}$. Furthermore, in the following section we will see that in fact the Lerner index is a problematic measure of competition under free entry.

[19] We might also try to explain the inverted U-shaped relationship between an average Lerner index and average innovation output (or effort) in an industry with asymmetric firms and composition effects.

Sánchez and Schmitz (2002) provide evidence of the impact of an increased probability of closure of iron–ore mines on productivity gains. The escape–the–competition effect is akin to our elasticity effect when we measure the intensity of competition by the number of firms. In our general specification the elasticty effect is dominated by the direct demand effect, which is akin to the reducing rents effect. However, the first effect is made more dramatic whenever low profits may imply exit and bankruptcy costs (termination costs for the manager or owner of the firm). By reducing profits, competition may put in danger the survival or the company and/or its management and so induce more effort whenever there are liquidation costs (see e.g. Schmidt (1997)). This means, for example, that increasing the number of firms increases the probability of liquidation and thus tends to increase innovation effort. This effect is then dominated by the reduction in profit (or demand) effect when the number of firms grows large.

2.3 Free entry

In this section I analyze markets with free entry and perform a comparative statics analysis with the size of the market, the size of the entry cost $F \geq 0$, and the degree of product substitutability. Firms choose whether to enter or not at a first stage and then choose simultaneously investment and price.[20]

For any given n consider a regular (i.e. with $B < 0$) symmetric equilibrium at the second stage with associated profits per firm of π_n. At a free-entry equilibrium with n^e firms in the market, each firm makes nonnegative profits, $\pi_{n^e} \geq F$, and further entry would result in negative profits, $\pi_{n^e+1} < F$ (I assume that firms when indifferent enter). If π_n is strictly decreasing in n then there can be at most one free-entry equilibrium, and there will be one if π_n tends to zero as n grows.

[20] Alternative game forms involve firms choosing simultaneously whether to enter, their investment in cost reduction, and level of output (Dasgupta and Stiglitz (1980)); or entry and investment at a first stage followed by market competition (Boone (2000)); or a sequential three-stage entry–investment–market competition (Sutton (1991), Suzumura (1995)). See Mas-Colell, Whinston, and Green (1995, Sec.12E) for a careful discussion of the differences in the game forms when there is no investment in cost reduction. Novshek (1980) and Kihlstrom (1999) consider simultaneous entry and output or price decisions.

Strengthening the condition $B \equiv \left((p-c)\frac{\partial h}{\partial p}+h+\frac{\partial H}{\partial p}\right)c''H+(c')^2 h\frac{\partial H}{\partial p}<0$ to

$$\widehat{B} \equiv \left((p-c)\frac{\partial h}{\partial p}+h+\frac{\partial H}{\partial p}\right)c''H+(c')^2 h^2 \leq 0$$

(note that $|\partial H/\partial p| < |h|$), and assuming $\frac{\partial H}{\partial n} < 0$ and $\frac{\partial \eta}{\partial n} > 0$, yields that profits at the symmetric equilibrium, π_n, are strictly decreasing in n.[21]

I will finesse the game form positing a free-entry zero-profit condition. We will say that the free-entry equilibrium is regular if $d\pi_n/dn < 0$ for $n = n^e$. If n^e is such that $\pi_n = F$, then the free-entry number of firms is $[n^e]$.[22] Obviously, if we have a result, say, that $\frac{dn^e}{dS} > 0$, this means that $\frac{d[n^e]}{dS} \geq 0$.

2.3.1 Comparative statics with market size and entry cost

Let the demand function be parameterized by the number of varieties n, yielding $H(p) \equiv D_i(p, ..., p; n)$.

Proposition 2 *Consider a symmetric regular interior free–entry equilibrium* (p^e, z^e, n^e)*. Under the assumptions of Proposition 1, suppose that* $\frac{\partial H}{\partial n} < 0$ *and* $\frac{\partial \eta}{\partial n} > 0$*; then*

$$sign\left\{\frac{dz^e}{dS}\right\} = sign\left\{\frac{dx^e}{dS}\right\} = sign\left\{-\frac{dp^e}{dS}\right\} > 0$$

and

$$sign\,\frac{dn^e}{dS} = sign\left\{-\widehat{B}\right\}.$$

Furthermore,

$$\frac{dn^e}{dF} < 0,\ sign\left\{\frac{dz^e}{dF}\right\} = sign\left\{-\frac{dz_n}{dn}\right\},$$

and

$$sign\left\{\frac{dp^e}{dF}\right\} = sign\left\{-\frac{dp_n}{dn}\right\},$$

where (p_n, z_n) *is the equilibrium with exogenous* n *evaluated at* $n = n^e$*.*

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[21] This follows because

$$\frac{d\pi_n}{dn} = (p-c)\left(\frac{\partial H}{\partial n}+\frac{dp}{dn}\left(\frac{\partial H}{\partial p}-h\right)\right)$$

and $\frac{d\pi_n}{dn} < 0$ if and only if $-\frac{\partial H}{\partial n}\widehat{B}+\left(\frac{\partial H}{\partial p}-h\right)\left((p^*-c)\frac{\partial h}{\partial n}+\frac{\partial H}{\partial n}\right)c''H < 0$ (since $\frac{\partial H}{\partial p}-h > 0$, $c''H > 0$, and $sign\left\{(p-c)\frac{\partial h}{\partial n}+\frac{\partial H}{\partial n}\right\} = sign\left\{-\frac{\partial \eta}{\partial n}\right\}$). Alternatively, with $B < 0$, a sufficient condition for $d\pi_n/dn < 0$ is that $dp/dn < 0$.

[22] The brackets $[x]$ denote the largest integer less than x.

The symmetric regular interior free–entry equilibrium (p^e, z^e, n^e) will fulfill the FOCs for price and innovation effort as well as the zero profit condition $(p - c(z))SH(p; n) - z - F = 0$. The results follow by differentiating totally the equilibrium conditions under the assumptions (see the Appendix).

Increasing the size of the market reduces cost (process innovation) and may increase or decrease the number of varieties (product innovation). The first results follows from an output expansion effect in a larger market. Increasing market size increases the number of firms less than proportionately, if at all, and thus increases individual firm output and innovation effort. The potential downward pressure exerted on innovation effort by an increase in the number of firms is overwhelmed by the expanded market. However, increasing market size may decrease the number of firms and varieties. The reason is that increasing the market size may increase R&D rivalry so much, increasing R&D expenditure, as to leave less room for entrants. That is, profits are pushed down for a given number of firms because the direct profitability–enhancing effect of market expansion is overwhelmed the indirect effect of increased rivalry in R&D and prices. Obviously, when n^e increases with S, total R&D effort $n^e z^e$ increases with S.

Increasing the entry cost reduces the number of firms and products introduced (indeed, under our assumptions profits are decreasing in n), and it affects price and R&D effort depending on the impact of a decreased number of firms. Typically (see examples in Table 1) we have that decreasing n increases z and p, and increasing F will therefore decrease n and increase p and z. Increasing the entry cost then has the (perhaps paradoxical) effect of increasing innovation effort. The reason is that it decreases the number of entrants, and each entrant produces more and has more incentive to reduce costs. Decreasing entry barriers (F) will always increase the number of firms and varieties but it will also decrease individual cost reduction efforts. It still can be true that the total cost reduction effort $n^e z^e$ increases as F decreases. This is the case in the constant elasticity (provided $\beta \leq (1+\gamma)^{-1}$), constant expenditure–constant elasticity or logit examples.

All the demand systems considered (linear, constant elasticity, constant expenditure, and logit demand systems) fulfill $\frac{\partial H}{\partial n} < 0$ and $\frac{\partial \eta}{\partial n} > 0$ (see Table 1). Table 2 provides the endogenous market structure counterpart of Table 1 with computed examples. The

Appendix provides computational details of the results reported in Table 2. In all the cases considered in Table 2 when $c(z) = \alpha z^{-\gamma}$ with $\alpha >$ and $\gamma > 0$ we have that $sign\, \frac{dn^e}{dS} = sign\left\{-\widehat{B}\right\} \geq 0$ (with strict inequality for constant expenditure-CES, logit and CES with $F > 0$, and equality for CES and $F = 0$). In all those cases a larger market implies more variety.

The result that $sign\, \frac{dn^e}{dS} = sign\left\{-\widehat{B}\right\}$and the sufficient condition to obtain a unique free–entry equilibrium (profits decreasing in n), $\widehat{B} \leq 0$, suggest that we will find more often that increasing market size increases product variety than the opposite result. Indeed, when the direct profitability–enhancing effect of market expansion prevails we have that $\widehat{B} \leq 0$. This is so in the collected examples above but by no means a universal result. An example is provided by the constant expenditure-CES demand system with the innovation function $c(z) = 1/(A + z)$ and $F < A$. We have that $sign\, \frac{dn^e}{dS} = sign\,\{F - A\}$ and for $F < A$ there is less variety in a larger market.[23] It can be checked also that $\frac{dn^e z^e}{dS} > 0$ despite the fact that $\frac{dn^e}{dS} < 0$.

2.3.2 Comparative statics with product substitutability

Let the demand function be parameterized by α, yielding $H(p) \equiv D_i(p, ..., p; \alpha)$ with $\alpha = n$ or $\alpha = \sigma$.

Proposition 3 *Consider a symmetric regular interior free–entry equilibrium* (p^e, z^e, n^e). *Under the assumptions of Proposition 1, suppose that* $\frac{\partial H}{\partial n} < 0$, $\frac{\partial \eta}{\partial n} > 0$, $\frac{\partial H}{\partial \sigma} \geq 0$, *and* $\frac{\partial \eta}{\partial \sigma} > 0$, *then at the equilibrium*

$$sign\left\{\frac{dz^e}{d\sigma}\right\} = sign\left\{\frac{dx^e}{d\sigma}\right\} = sign\left\{-\frac{dp^e}{d\sigma}\right\} > 0$$

and $sign\left\{\frac{dn^e}{d\sigma}\right\}$ *is ambiguous but*

$$\frac{dn^e}{d\sigma} < 0 \text{ if } \frac{\partial H}{\partial \sigma} = 0 \text{ or if } \widehat{B} > 0.$$

The proof of the proposition follows along similar lines than that of Proposition 2 and can be found in the Appendix. The assumptions on demands are fulfilled for all the examples

[23] We have $sign\, \frac{dn^e}{dS} = sign\left\{-\widehat{B}\right\} = sign\,\{F - A\} < 0$. For a free-entry equilibrium to exist when $F < A$ we need that $\frac{2n^e - 1}{(n^e - 1)} < \sigma$ and this implies necessarily that $\widehat{B} > 0$.

(except the Bowley variation of linear demands). In the linear (Shapley–Shubik), constant expenditure, and logit demand systems, $\frac{\partial H}{\partial \sigma} = 0$ and therefore $\frac{dn^e}{d\sigma} < 0$. It should be clear why this is so. When changes in σ are demand–neutral, increasing σ decreases profits and so the zero–profit entry condition is restored, decreasing the number of entrants.[24] Increasing the degree of product substitutability increases output per firm and R&D effort, provided the total market does not shrink. The reason is that if $\frac{\partial H}{\partial \sigma} = 0$ increasing σ has no effect on demand but increases price pressure and decreases the number of entrants enticing a higher output and innovation effort per firm; if $\frac{\partial H}{\partial \sigma} > 0$ then increasing σ has the direct effect of expanding the market overcoming any possible adverse indirect effect on individual output and innovation effort if the number of entrants where to increase.

The parameter σ could also be interpreted as "regulatory pressure", with $\frac{\partial \eta}{\partial \sigma} > 0$ and $\frac{\partial H}{\partial \sigma} = 0$. An increase in regulatory pressure would then decrease price while increasing R&D effort and concentration. Again, this would be the effect of a binding price cap or the increase in a sales tax with regulated prices.

2.3.3 Market power, concentration, product substitutability, and innovation

In a free–entry equilibrium

$$L \equiv \frac{p-c}{p} = \frac{1+\frac{F}{z}}{1+\frac{1}{\gamma(z)}+\frac{F}{z}} = \frac{1}{\eta\left(p,n,\alpha\right)},$$

where $\gamma\left(z\right) \equiv -zc'\left(z\right)/c\left(z\right)$ and α may represent S or σ. From these expressions and our examples we can derive a series of observations, some of which run counter common intuition and even practice in empirical model–building.

- *The relationship between market power (Lerner index) and innovation effort is ambiguous*:
  $$sign\,\frac{\partial L}{\partial z} = sign\left\{-\frac{F}{z^2}\gamma^{-1}+\left(1+\frac{F}{z}\right)\frac{\gamma'}{\gamma^2}\right\}.$$
  If $\gamma' \leq 0$ and $F > 0$, then L is strictly decreasing in z. If $F = 0$ then $sign\,\frac{\partial L}{\partial z} = sign\left\{\gamma'\right\}$ and thus, if $\gamma' > 0$, L is strictly increasing in z. It follows that, if σ or S

[24] In the constant elasticity (CES) case we have that n^e is strictly decreasing in σ when $F = 0$ even tough $\partial H/\partial\sigma > 0$.

increase (and hence z also increases) then L decreases when $\gamma' \leq 0$ and $F > 0$ or if γ' < 0 and $F = 0$. This is the case in particular if γ is constant with $F > 0$.

- However, *R&D intensity including expenditure F on product introduction and market power move together*: $L = (z + F)/px$. If γ is increasing in z and $F = 0$, then increasing S increases z, L, and R&D intensity. If $F = 0$ and γ is constant, the degree of monopoly power L is determined by technological considerations (the elasticity of the innovation function) and R&D intensity is independent of market size S or product substitutability σ. This latter case would be consistent with the empirical evidence that indicates that R&D intensity is independent of firm size.[25]

- *The relationship between market power (Lerner index) and product substitutability is ambiguous.* Increases in product substitutability σ need not go together with decreases in the Lerner index L. This is so because increasing σ may bring an increase in concentration which more than compensates for the direct effect of the augmented substitutability. In particular, it could be that an increase in σ increases market power (L) and innovation effort z. This will happen, for example, with $\gamma' > 0$ [26] and $F = 0$, or F small as in the constant expenditure–constant elasticity case with the innovation function $c(z) = 1/(A+z)$ when $A > F$ (then $sign\, \frac{dL}{d\sigma} = sign\, \{A - F\} > 0$). In this latter case increasing σ ($\equiv 1+r$) decreases n^e so much that L diminishes despite the direct impact of the increase in σ. This situation would be at odds with work (e.g. Aghion et al. (2002)) in which the Lerner index, or an approximation to it, is taken as a proxy for competitive pressure measured by σ. However, we see from Table 2 that the Lerner index is decreasing in σ in the constant expenditure (constant elasticity)[27] and logit cases ($\sigma \equiv 1/\mu$)) with innovation function $c(z) = \alpha z^{-\gamma}$. Then the direct effect of the increase in σ overwhelms the indirect effect via the decrease in n^e.

- *The Lerner index and the level of concentration may move in opposite directions.* If $F > 0$ and γ is constant, then the Lerner index is strictly decreasing with σ. It follows

[25] The number of varieties introduced n^e increases with S if η is increasing in p. This is so because η is independent of S and strictly increasing in n, and also increasing S decreases p.

[26] Note that $sign\ \gamma' = sign \left\{ 1 + \gamma + \frac{c'' z}{c'} \right\}$. Then $-\frac{c' z}{c} + \frac{c'' z}{c'} < 0$ if and only if c is log-convex.

[27] The fact that L is decreasing in σ validates the conjecture of Aghion et al. (2002, p. 13, fn.9).

that increasing σ increases z, decreases L (and R&D intensity) and also decreases n^e if $\frac{\partial H}{\partial \sigma} = 0$. This does not happen in the constant expenditure–constant elasticity case or in the logit where $sign\left\{\frac{dn^e}{d\sigma}\right\} = sign\left\{\frac{dL^e}{d\sigma}\right\} < 0$ (see Table 2). The result that we may have a high level of (market power and) R&D intensity and a high number of varieties (low concentration) when substitutability (σ) is low is parallel to the result obtained by Sutton (1996) in a linear demand example with Cournot competition and quality-enhancing investments.

- *Market power (Lerner index) increases with higher entry barriers.* We see from Table 2 that the Lerner index is increasing in F in the constant elasticity, constant expenditure–constant elasticity and logit cases with constant elasticity innovation function as well as in the constant expenditure–constant elasticity case with the innovation function $c(z) = 1/(A + z)$.

Incentives in the Salop (1979) model (Raith (2003)) The incentive model by Raith (2003) provides a nice illustration of our results. The author considers the model of Salop (1979) with a mass of consumer S uniformly distributed on a circle of circumference 1 and with quadratic transportation costs having parameter t. Each of the n firms has a cost

$$c_i = \overline{c} - e_i - u_i,$$

where e_i is the unobservable effort exerted by the manager of the firm and u_i is normally distributed idiosyncratic noise with mean 0 and variance v. Owner i makes decisions and offers a linear contract to his manager, with compensation $w_i = s_i + b_i(\overline{c} - c_i)$, to reduce costs. After all managers have chosen their effort levels, costs are realized (and are private information to the firms), firms compete in prices, and a (Bayesian) Bertrand equilibrium obtains. Managers have constant absolute risk aversion ρ, quadratic cost of effort $\frac{k}{2}(e_i)^2$, and a reservation utility of 0. Given that the manager of i will choose $e_i = b_i/k$, firm i will set $s_i = -\frac{1}{2k}(1 - k\rho v)(b_i)^2$ so that the manager will obtain a zero expected utility. The expected compensation of the manager will be $\overline{w}_i = s_i + b_i e_i = \frac{1+k\rho v}{2k}(b_i)^2$; the expected cost, $\overline{c}_i = \overline{c} - b_i/k = \overline{c} - \sqrt{\frac{2\overline{w}_i}{k(1+k\rho v)}}$. In terms of our model, then,

$$c(z) = \overline{c} - \sqrt{\frac{2z}{k(1 + k\rho v)}}.$$

Under some parameter restrictions, and for a fixed number of firms n, Raith shows that there is a symmetric equilibrium for the overall game and that cost reduction effort is independent of $\sigma \equiv 1/t$. Furthermore, with free entry and with firms paying an entry cost F, cost–reduction effort is increasing in σ, S, and F. All these results follow from Propositions 1, 2, and 3 —noting that in the Salop model $\frac{\partial H}{\partial p} = \frac{\partial H}{\partial \sigma} = 0$.

In this section we have obtained robust results of the impact on process and product innovation of several standard indicators of increased competitive pressure in a world of price competition with product differentiation. In short, we have found that with restricted entry increasing the number of firms lowers incentives for process innovation while increasing product substitutability raises them; with free entry, increasing market size and/or product substitutability increases process innovation incentives and has an ambiguous impact on product introduction. Raising entry barriers decreases new product introduction but raises cost reduction efforts.

3 Cournot competition with homogenous product

In this section I check the robustness of the results obtained to the case of Cournot markets with homogenous products. I start with the case with restricted entry and perform a comparative statics exercise on the number of firms. I check also for the strategic commitment effect of R&D. I consider then the case of free entry and study the comparative statics properties with respect to the size of the market and entry cost.

Consider an n-firm Cournot market for a homogenous product with smooth inverse demand $P(\cdot)$, $P' < 0$. Parameterize the demand by the size of the market $S > 0$. Inverse demand is then given by $p = P(X/S)$. As before firm i can invest z_i to reduce its constant marginal cost of production c_i according to a smooth function $c_i = c(z_i)$ with $c(z) > 0$, $c'(z) < 0$, and $c''(z) > 0$ for all $z > 0$. A firm to enter the market has to incur a fixed cost $F \geq 0$. The profit to firm i is given by

$$\pi_i = P(X)\, x_i - c(z_i)\, x_i - z_i - F,$$

where x_i is the output of the firm and X is total output.

3.1 Restricted entry

Let $S = 1$ and consider a simultaneous–move game where firm i, for each i of a given number of firms n, chooses (z_i, x_i). Consider an (interior) symmetric equilibrium (z, x) of the game. We say that the equilibrium is regular if

$$D \equiv \left((n+1)\,P' + nP''x\right) c''x + \left(c'\right)^2 < 0.$$

With Cournot competition the price–pressure and the demand effects collapse into the output effect. It is to be expected again that the direct demand effect dominates and that output and innovation effort decrease with the number of firms. This is confirmed in the following proposition for the usual Cournot case of strategic substitutes competition (downward sloping best replies or $P' + P''x < 0$).

Proposition 4 *Let $P' < 0$ and let $c' < 0$, and $c'' > 0$. Consider a symmetric regular interior equilibrium (z^*, x^*). Then*

$$sign\left\{\frac{dz^*}{dn}\right\} = sign\left\{\frac{dx^*}{dn}\right\} = sign\left\{P'\left(nx^*\right) + P''\left(nx^*\right)x^*\right\}.$$

The proof is immediate by differentiating the first order conditions for equilibrium.[28] As before from the profit function we see that output and innovation effort have to move together since a larger output provides a larger benefit to reduce costs. How individual output in a Cournot equilibrium changes with the number of firms is dictated by the slope of best replies, which is determined by $sign\left\{P'\left(nx^*\right) + P''\left(nx^*\right)x^*\right\}$. The reason is that increasing n increases $(n-1)x_n^*$ and in a Cournot equilibrium of a symmetric market individual output equals the best response to the aggregate output of rivals. Therefore, individual output x_n^* increases or decreases according to the slope of the best reply to the aggregate output of rivals $(n-1)x_n^*$.[29] The normal case is that this slope is negative and best responses are decreasing (strategic substitutes case). Indeed, the conditions for upward sloping best replies (strategic complements) in Cournot oligopoly are quite stringent.

[28] Differentiating the FOCs yields $\frac{dx}{dn} = -\frac{x^2\left(P'+xP''\right)c''}{D} < 0$ so $\frac{dz}{dn} = \frac{xc'\left(P'+xP''\right)}{D} < 0$, and $sign\left\{\frac{dz^*}{dn}\right\} = sign\{\frac{dx^*}{dn}\} = sign\ \{P'\left(nx^*\right) + P''\left(nx^*\right)x^*\}$.)

[29] See p.106-107 in Vives (1999).

Letting the elasticity of the slope of inverse demand be

$$E \equiv -XP''(X)/P'$$

we have upward sloping best responses (with constant marginal costs) if $n+1 > E > n$. The first inequality yields uniqueness (and stability) of the symmetric Cournot equilibrium ($(n+1)P' + nP''x < 0$ is equivalent to $n+1 > E$); the second yields upward sloping best responses (see Seade (1980) and Vives (1999, Sec. 4.3.1)). If E is constant (encompassing the linear and constant elasticity cases)[30] upward sloping best responses will hold, if at all, for a single change in the number of firms n. If E is constant and we require $n+1 > E$ for all $n \geq 1$, then $2 > E$ and only $2 > E > 1$ is possible.[31]

Existence, uniqueness, and regularity conditions

- Sufficient conditions for $D < 0$ when $c'' > 0$ are that $P' + x_i P'' < 0$ and $(2P' + xP'')c''x_i + (c')^2 < 0$. These conditions imply that $\pi_i = P(X)x_i - c_i x_i - z_i$ is strictly concave in (x_i, z_i). Strict concavity plus a mild boundary condition implies the existence of an interior equilibrium.[32]

- If $D < 0$ at any candidate equilibrium then equilibrium is unique.

- Using lattice-theoretic methods is possible to extend Proposition 4 removing the regularity conditions, as long as we restrict attention to extremal equilibria. With downward sloping demand and a decreasing innovation function plus some mild boundary conditions interior extremal equilibria (x^*, z^*) exist and x^* and z^* are strictly decreasing (increasing) in n if Cournot best replies are strictly decreasing (increasing). The statement of the result and proof are in the Appendix (Proposition 7).

[30] Then demands are of the form $P(X) = a - bX^{1-E}$ if $E \neq 1$ or $P(X) = a - b \log X$ if $E = 1$, with $a \geq 0$ and $b > 0$ if $E \leq 1$ and $b < 0$ if $E > 1$, and they include linear ($E = 0$) and constant elasticity.

[31] See Amir (1996) and Vives (1999, Sec. 4.1)) for a discussion of why downward best replies are the normal case in Cournot.

[32] Profits π_i are strictly concave in (x_i, z_i) if $c'' > 0$, $2P' + x_i P'' < 0$, and $(2P' + xP'')c''x_i + (c')^2 < 0$. If $P' + x_i P'' < 0$ then a sufficient condition to have that $(2P' + xP'')c''x_i + (c')^2 < 0$ is that $c(\cdot)$ is sufficiently convex, that is, $-c''x/c' > c'/P' > 0$.

Examples The models of Dasgupta and Stiglitz (1980) and Tandon (1984) are particular cases of Proposition 4.

Constant elasticity (Dasgupta and Stiglitz (1980)). Let $P(X) = bX^{-\varepsilon}$ ($a = 0$, $E - 1 = \varepsilon > 0$) and let $c(z) = \alpha z^{-\gamma}$. The parameter α can be interpreted as the underlying scientific base in the industry, while the elasticity γ of $c(\cdot)$ would indicate innovation opportunities in the industry (with a higher γ increasing opportunities). The condition $n + 1 > E > n$ becomes in this case $n > \varepsilon > n - 1$. Assume that $\varepsilon(1 + \gamma)/\gamma \geq n > \varepsilon$ (this implies that $D < 0$); then there is a unique symmetric equilibrium with

$$z^* = \left[b\left(\gamma/n\right)^{\varepsilon}\alpha^{\varepsilon-1}\left(1 - \varepsilon/n\right)\right]^{1/(\varepsilon-\gamma(1-\varepsilon))}$$

and

$$x^* = (1/\gamma\alpha)\left[b\left(\gamma/n\right)^{\varepsilon}\alpha^{\varepsilon-1}\left(1 - \varepsilon/n\right)\right]^{(1+\gamma)/(\varepsilon-\gamma(1-\varepsilon))}.$$

If we require that $n > \varepsilon$ for all $n \geq 1$, then z^* and x^* increase with n only when going from monopoly to duopoly. Total output nx^* and industry R&D expenditure nz^* both increase with n. R&D intensity $\dfrac{z^*}{p^*x^*} = \gamma\left(1 - \dfrac{\varepsilon}{n}\right)\left(\dfrac{\alpha}{\beta}\right)^{\varepsilon-1}$ increases with n and with γ. It is immediate also that z^* and profit π^* increase (decrease) with α if $\varepsilon > 1$ $(\varepsilon < 1)$.

Linear demand (Tandon (1984)). Consider a market with linear demand $p = a - bX$ and $c(z) = a - \beta z^{\delta}$. We need $\delta < \frac{1}{2}$ to guarantee strict concavity of profits of firm i with respect to x_i and z_i (if $\delta < 1$ then $c(\cdot)$ is strictly convex). Then $z^* = \left(\frac{\delta\beta^2}{b(n+1)}\right)^{1/(1-2\delta)}$ and $x^* = \left(\frac{\beta}{(n+1)b}\right)\left(\frac{\delta\beta^2}{b(n+1)}\right)^{\delta/(1-2\delta)}$ are both decreasing in n for $\delta < \frac{1}{2}$, while R&D intensity z^*/p^*x^* may decrease or increase with n (it decreases for $\delta \in \left(\frac{1}{5}, \frac{1}{2}\right)$).

3.1.1 Strategic commitment effects

I analyze the subgame-perfect equilibria (SPE) of the two-stage game where firms first invest in cost reduction and then compete in quantities. Denote by $x^*(z_i, z_{-i})$, $i = 1, ..., n$, a second–stage Cournot equilibrium for a given investment profile and let

$$V(z_i, z_{-i}) \equiv P\left(X^*(z)\right)x_i^*(z) - c(z_i)\,x_i^*(z) - z_i$$

be the associated profit for firm i. The following proposition strengthens the requirements on demand to ensure that investments in the first stage are strategic substitutes ($\frac{\partial^2 V}{\partial z_i \partial z_j} < 0$,

$j \neq i$) and that increasing n reduces both output and innovation effort. When investments are strategic substitutes, increasing the number of firms will tend, but need not –see the example after the proposition, to decrease innovation effort of any firm because the aggregate investment of rivals increases. The following proposition states the result formally (with proof in the Appendix).

Proposition 5 *Consider a symmetric interior SPE of the two-stage game:* $(z^*, \{x^*(z_i, z_{-i})\}_{i=1}^n)$*. Suppose that* $P'' \leq 0$ *and that* $-P'$ *is log–concave (i.e.,* $P'P''' - (P'')^2 \leq 0$*). Then investments are strategic substitutes at the first stage, and we have* $\frac{dz^*}{dn} < 0$ *and* $\frac{dx^*}{dn} < 0$*.*

For the case of E constant we can provide an explicit expression for $sign\{dz^*/dn\}$. Let $E < 1 + n$, $n > 1$, and let $c(\cdot)$ be sufficiently convex ($-c''x/c' > c'/((1 + \min(n - E, 0))P')$ > 0), then[33]

$$sign\left\{\frac{dz^*}{dn}\right\} = sign\{E - 2(n - E)^2\}.$$

Therefore, $\frac{dz^*}{dn} < 0$ for $E \leq 0$ (or $P'' \leq 0$)[34] and $\frac{dz^*}{dn} > 0$ for $1 + n > E > n$ (strategic complementarity at the output stage). Note however that we could have $\frac{dz^*}{dn} > 0$ for E close to n and $0 < E < n$, i.e., with strategic substitutes at the output stage (as well as at the investment stage*???*). This is the case in the constant elasticity demand model considered by Spence (1984). Then $E = 1 - \varepsilon$ and, with an exponential innovation function (as in the following example), z^* increases from $n = 1$ to $n = 2$ for $\varepsilon = 1/2$; otherwise, z^* is decreasing with n. Note that for $\varepsilon < 1/2$ and $n = 1$, $E - 2(n - E)^2 > 0$ (whereas for $n \geq 2$ it is negative).

An agency model with linear demand (Martin (1993)) Here every firm has an owner and a manager and the manager's unobservable effort reduces cost. The constant marginal cost of firm i is given by

$$c(\theta_i) = m + \theta_i e^{-l_i}$$

[33] It can be checked after some tedious computations (see Suzumura (1995)) that with these assumptions $\frac{dz^*}{dn} = -\frac{\partial\varphi/\partial n}{\partial\varphi/\partial z}$ with $\partial\varphi/\partial z = -(1 + G(x, n))(c''x + (c')^2/(1 + n - E)P')) < 0$ and $\partial\varphi/\partial n = xc'(z)(n - 1)(2(n - E)^2 - E)/(1 + n - E)^2 n^2$.

[34] This actually follows from Proposition 2 because $-P'$ is log-concave if E is constant and $E \leq 0$ (i.e., $P'' \leq 0$).

for $m > 0$, θ_i a random variable (IID across firms) with compact positive support $\left[\underline{\theta}, \overline{\theta}\right]$, and l_i the labor input (effort) of the firm's manager. The manager observes θ_i and knows l_i but the owner does not. The latter sets up an incentive scheme with a cost target $c(\theta_i)$ and a payment schedule $\varphi(\theta_i)$. The interpretation is that, given a reported efficiency θ_i, the manager must achieve the cost target $c(\theta_i)$ in order to obtain the compensation $\varphi(\theta_i)$. The utility of the manager equals the compensation minus the disutility of effort λl_i, where $\lambda > 0$. It is easy to check that an incentive-feasible compensation schedule must satisfy $\varphi(\theta_i) = \lambda \log \frac{\overline{\theta}}{c(\theta_i) - m}$. Market competition is à la Cournot with linear demand, and in the first stage owners compete by setting cost targets. It is then immediate that the optimal cost target and the compensation are constant. We are thus in the frame of our model with an innovation function (or reduced–form cost function)

$$c(z) = m + \overline{\theta} \exp\{-z/\lambda\}, \ \lambda > 0.$$

Note that $c' < 0$ and $c'' > 0$. Given that demand is linear ($E = 0$) we have that $\frac{dz}{dn} < 0$ or that increasing the number of firms reduces cost-reduction effort and increases costs. Indeed, this is the result obtained by Martin (1993).

3.2 Free entry

We look for a free-entry equilibrium in which entering firms incur a fixed cost $F \geq 0$. Firms choose whether to enter or not at a first stage and then choose simultaneously investment and output.

For any given n consider a regular (i.e. with $D < 0$) symmetric equilibrium at the second stage with associated profits per firm of π_n. As before, we will say that the free–entry equilibrium is regular if $d\pi_n/dn < 0$ for $n = n^e$.[35]

Proposition 6 *Suppose that the assumptions of Proposition 4 hold and let* (x^e, z^e, n^e) *be*

[35] With $D < 0$ we have that

$$sign\{d\pi_n/dn\} = sign\{(2P' + xP'')c''x_i + (c')^2\},$$

and the second–order necessary condition yields $(2P' + xP'')c''x_i + (c')^2 \leq 0$. Profits are strictly decreasing in n if $\pi_i = P(X)x_i - c_i x_i - z_i$ is strictly concave in (x_i, z_i).

a symmetric regular interior free-entry equilibrium. Then

$$sign\left\{\frac{dz^e}{dS}\right\} = sign\left\{\frac{dx^e}{dS}\right\} > 0.$$

Furthermore,

$$sign\left\{\frac{dz^e}{dF}\right\} = sign\left\{\frac{dx^e}{dF}\right\} = sign\left\{-\left(P' + (x/S)\,P''\right)\right\} \;\; and \;\; \frac{dn^e}{dF} < 0.$$

At the equilibrium we will have a triple (x, z, n) fulfilling the FOCs for output and innovation effort as well as the zero profit condition $(P(xn/S) - c(z))x - z - F = 0$. The results follow by differentiating totally the equilibrium conditions under the assumptions (see the Appendix).

Some intuition for the market size S comparative statics result in the proposition can be gained as follows. Increasing S will have a positive direct impact on x and z and an indirect effect because of the changes in n. However, the indirect effect is always dominated because n increases (if at all) less than proportionately than S. The reason is that, with constant marginal costs, increasing the market size increases also the toughness of competition and puts pressure on margins, moderating the rate of entry.[36] In fact, n may even decline as a result of the intensity of the R&D competition. A condition for this not to be the case is strategic substitutability in outputs (i.e., $P' + (x/S)P'' < 0$) and $c(\cdot)$ sufficiently convex (i.e., $-c''x/Sc' > nc'/P' > 0$). Then $\frac{dn^e}{dS} > 0$.

The comparative statics results on F are very intuitive. Increasing the entry cost decreases the free–entry number of firms, and it increases (decreases) output and R&D effort whenever outputs are strategic substitutes (complements). In the usual strategic substitutes case decreasing entry barriers induces more entry and each firm is smaller and has less incentive to invest.

Remarks

- It is easy to check that in equilibrium

$$L \equiv \frac{p-c}{p} = \frac{1+\frac{F}{z}}{1+\frac{1}{\gamma(z)}+\frac{F}{z}} = \frac{\varepsilon(nx)}{n},$$

[36] For example, if $P(X/S) = (X/S)^{-1}$ then, letting $n(S)$ denote the free-entry number of firms for a given symmetric investment profile z and Cournot competition, we have $n(S)/S = (F/S)^{1/2}$.

where $\gamma(z) \equiv -zc'(z)/c(z)$ and $\varepsilon(X) \equiv -XP'(X)/P(X)$.[37] It is immediate also that $L = (z+F)/px$.

- If $F = 0$ then $L = z/px$ (R&D intensity) and

$$n^e = \varepsilon(n^e x^e)\frac{1+\gamma(z^e)}{\gamma(z^e)}.$$

  If γ is increasing in z, then increasing S increases z and R&D intensity. Note that, for a given inverse elasticity ε, increasing the technological opportunities γ will tend to increase concentration. This is consistent with the empirical findings that industries with more technological opportunities are more concentrated (see, e.g. Scherer and Ross (1990)).

- With constant elasticity innovation and demand functions and $F > 0$, we have that L decreases (strictly) with z and hence increasing S increases z, decreases L, and increases n. However, if $F = 0$, then $L = \gamma/(1+\gamma)$ and $n^e = \varepsilon(1+\gamma)/\gamma$ are independent of S.

Constant elasticity examples

- Let $p = (X/S)^{-\varepsilon}$ $(E - 1 = \varepsilon > 0)$, $c(z) = \alpha z^{-\gamma}$ and $F = 0$. Then, indeed, both

$$z^e = \left(S^\varepsilon \gamma^{2\varepsilon} \alpha^{\varepsilon-1} \varepsilon^{-\varepsilon} (1+\gamma)^{-(1+\varepsilon)}\right)^{1/(\varepsilon-\gamma(1-\varepsilon))}$$

  and

$$x^e = \frac{1}{\gamma\alpha}\left(S^\varepsilon \gamma^{2\varepsilon} \alpha^{\varepsilon-1} \varepsilon^{-\varepsilon} (1+\gamma)^{-(1+\varepsilon)}\right)^{(1+\gamma)/(\varepsilon-\gamma(1-\varepsilon))}$$

  increase with S and the free–entry number of firms is $[\varepsilon(1+\gamma)/\gamma]$ (Dasgupta and Stiglitz (1980)).

- Tandon (1984) considers a linear demand example $p = a - bX$ with $c(z) = a - \beta z^\delta$ and $F = 0$. Strict concavity of profits of firm i with respect to x_i and z_i requires $\delta < \frac{1}{2}$. Then $n^e = \frac{1-\delta}{\delta}$. Both $z^e = \left(\frac{\delta^2\beta^2}{b}\right)^{\frac{1}{1-2\delta}}$ and $x^e = \left(\frac{\delta\beta}{b}\right)\left(\frac{\delta^2\beta^2}{b}\right)^{\frac{\delta}{1-2\delta}}$, as well as R&D intensity $z^e/p^e x^e$, increase in S (decrease in b) since $\delta < \frac{1}{2}$, and $n^e z^e$ increases in δ.

[37] Note that $sign\ \varepsilon' = sign\ \{1 - \varepsilon - E\}$.

- Sutton (1991) considers a three-stage game featuring (i) an entry decision, (ii) investment in cost reduction, and (iii) quantity competition. Demand is given by $P(X/S) = (X/S)^{-1}$and the innovation curve by $c(z) = (z\gamma a^{-1} + 1)^{-1/\gamma}$, where $\gamma > max\{1, 2a/3F\}$ and F is the sunk cost of entry. Then, for S small, $z^e = 0$; for larger S, z^e is increasing in S while n^e decreases (increases) in S if $F < a/\gamma$ ($F > a/\gamma$). This model can also be given a quality investment or advertising interpretation. In this example, investment has a strategic commitment effect.

Product differentiation The results could be easily extended to product differentiation. In fact, Spence (1984) has shown how a certain class of cost–reduction Cournot models with homogenous product can be reinterpreted in a product differentiation environment. In the constant elasticity case, for example, it is possible to check that, under quantity competition, the same comparative statics with respect to S hold as in the Bertrand case. That is, $\frac{dn^e}{dS} > 0$ for $F > 0$ and $\frac{dn^e}{dS} = 0$ for $F = 0$.[38]

4 Extensions

4.1 An alternative measure of competitive pressure

Competitive pressure could be measured also by the extent that each firm internalizes the profits of other firms. This could arise, for example, when firms in the industry have cross-shareholdings. Suppose that firm i maximizes

$$\pi_i + \lambda \Sigma_{j \neq i} \pi_j,$$

where λ ranges from $\lambda = 0$ (no internalization as before) to $\lambda = 1$ (full internalization or collusion), and consider the simultaneous–move game. An increase in λ will then mean a decrease in competitive pressure. The parameter λ was called by Edgeworth the coefficient of "effective sympathy". It is possible to check (proofs available on request) that, under Cournot and under Bertrand competition, an increase in competitive pressure $1/\lambda$ will:

- increase output and innovation effort with restricted entry; and

[38] Inverse demand is given by $p_i = \frac{S^{1-\beta\theta}\alpha^{\theta}\beta\theta x_i^{\beta-1}}{\left(\Sigma_j x_j^{\beta}\right)^{1-\theta}}$ for $i = 1, ..., n$ (Koenker and Perry (1981)).

- increase innovation effort and decrease the number of entrants (and varieties) with free entry.

The intuition is straightforward. With restricted entry, if firms are more aggressive (a lower λ) then output per firm and the incentive to innovate will increase. With free entry, a firm (when deciding whether to enter) considers only its own profits but knows that, once in the market, competition will be softer if λ is higher. Tougher competition thus means that fewer firms will enter and that output per firm will be larger, inducing a larger innovation effort. The results with λ parallel those obtained in the Bertrand case with degree of substitutability σ whenever changes in σ are demand–neutral ($\frac{\partial H}{\partial \sigma} = 0$).

4.2 Investment in quality

The cost–reduction model can be interpreted as investment in quality (in terms of product enhancement) in the context of the Cournot model. The most straightforward case is when investment increases the intercept of the inverse demand function ($\phi(z)+P(X)$ with $\phi' > 0$). In the Cournot duopoly model of Vives (1990) a higher product substitutability increases investment in product enhancement that expands the market. Spence (1984) or Sutton (1991) present other cases where such extension is possible. Results by Sutton (1996) and Symeonidis (2000) are in line with those obtained in this paper. Sutton (1996) considers a linear demand example (a quality-augmented version of the Bowley demand system) with Cournot competition and quality-enhancing investments and finds, as in our model, that it is possible to have a high level of R&D intensity and a high number of varieties (low concentration) when substitutability (σ) is low. Symeonidis (2000) considers a (strategic) three-stage game of entry, investment in product quality, and quantity competition within a model in which horizontal and vertical product differentiation coexist. Demand functions are linear as in Sutton (1996) and the innovation function is of the power variety. The author finds that increasing the degree of horizontal product substitutability increases concentration and R&D effort and that increasing the market size increases R&D effort.[39]

[39] Interestingly, Berry and Waldfogel (2003) show that in the restaurant industry (where quality is produced mostly with *variable* costs) the range of qualities increases with market size, whereas in daily newspapers (where quality is produced mostly with *fixed* costs) the average quality increases with market size and there

It is worth noting that the same straightforward reinterpretation does not hold with price competition. In this case decreasing unit costs are not equivalent to an increase in the intercept of the demand function. However, in the Bertrand duopoly model of Vives (1990) it can be shown that a higher product substitutability increases investment in product enhancement that raises the willingness to pay of consumers.

4.3 Spillovers

When the effort of one firm affects (favorably) the cost reduction of other firms, we say that there are (positive) spillovers.[40] With high enough (positive) spillovers, the R&D cost reduction investments of rivals may be strategic complements in a two-stage game with investment at the first stage and Cournot competition in the second. This is what happens in the linear-quadratic specifications of d'Aspremont and Jacquemin (1988, 1990) and Ceccagnoli (2003).[41] In principle this suggests that, with high enough spillovers and with Cournot competition, it could be that increasing the number of firms increases individual firm innovation effort. However, it can be checked that this does not happen in the linear-quadratic specification where increasing the number of firms always lowers innovation effort.[42]

5 Concluding remarks

Does competitive pressure foster innovation?

The answer to the question is a qualified "yes" because it depends on what measure of competitive pressure we use and what type of innovation (process or product) we have in mind. The results and testable empirical implications of our analysis may be summarized as follows. (See Table 3.)

is no fragmentation as the market grows large.

[40] See Spence (1984), d'Aspremont and Jacquemin (1988, 1990) and Amir (2000).

[41] Ceccagnoli (2003) also shows that with fringe firms that do not invest in R&D and do not benefit from the spillover, strategic complementarity among the investing firms increases with the number of fringe firms.

[42] The setting is as follows: $P(X) = a - bX$ and $c(z) = \overline{c} - z_i - \beta \sum_{j \neq i} z_j$. If a firm invests $\gamma z_i^2/2$, then its marginal cost will be reduced by $z_i + \beta \sum_{j \neq i} z_j$, where $1 > \beta > 0$ is the spillover rate. When $\beta > 1/2$ investments at the first stage are strategic complements.

Table 3

	Restricted entry			Free entry	
Indicator	n	σ	S	σ	$1/F$
Process Innovation	$-$	$+$ (with no market contraction)	$+$	$+$ (with no market contraction)	$-$
Product Innovation			$\pm$	$-$ (with no market expansion)	$+$

- *In markets with restricted entry*: More competitive pressure in terms of a larger number of firms means less R&D effort per firm, whereas more competitive pressure in terms of a greater product substitutability (that does not shrink the total market for varieties) means more R&D effort per firm.

- *With free entry*: Increasing the market size or product substitutability (with no shrinking of the total market for varieties) increases innovation effort and per firm output. Increasing the market size may increase or decrease the number of varieties introduced (product innovation) although the former is more likely than the latter. Increasing product substitutability will decrease entry and product variety if the market does not expand. Lowering entry costs will increase the number of entrants and lower (individual) innovation effort.

Taking into account that with restricted entry increasing the number of firms tends to increase research intensity (effort over sales), and with free entry decreasing the cost of entry tends to increase total research effort, we could safely state that "competitive pressure fosters innovation".

These results are consistent with the available empirical evidence and help reconcile it with the theory when we take account of the different measures of competitive pressure used by different authors. It is reassuring that the results hold for both Bertrand and Cournot competition since the competition mode is not easy to distinguish empirically. Furthermore, the approach should help also to avoid pitfalls in empirical analysis by making clear what results are robust and which ones are not. For example, it cannot be taken for granted that a good proxy for the degree of product substitutability, as indicator of competitive pressure, is the Lerner index.

A potential application of these results is to regulated markets. For example, in banking both the deregulation process in Europe and the removal of restrictions on U.S. intrastate and interstate branching have been claimed (by Gual and Neven (1993) and Jayaratne and Strahan (1998), respectively) to deliver cost efficiencies. More in general, Alesina et al. (2004) show that in OECD countries deregulation has tended to increase investment. This is consistent with our analysis as long as deregulation is interpretable as increases in market size and/or product substitutability. If deregulation implies a reduction in entry barriers then it will increase product variety but decrease cost reduction effort by individual firms, although total investment may still increase. The effects of price caps on innovation are discussed in the pharmaceutical industry. A tighter price cap is akin to an increased product substitutability with neutral demand effects. The result then is more cost reduction effort but less product innovation.

Many extensions of the analysis could be envisioned. I have already commented on alternative ways of parameterizing competitive pressure, investments to enhance quality, and spillovers. An immediate extension would be to consider investment that affects the slope of (increasing) marginal costs. More substantial extensions would include asymmetric market structures and performing a welfare analysis with a view toward competition and industrial policy. Leahy and Neary (1997) have developed part of this program.

6 Appendix:

6.1 Proofs

Proof of Proposition 2: Consider the symmetric regular interior free-entry equilibrium (p^e, z^e, n^e). The equilibrium will be characterized by

$$\begin{aligned} (p - c(z))\,h(p;n) + H(p;n) &= 0 \\ -SH(p;n)c'(z) - 1 &= 0 \\ (p - c(z))SH(p;n) - z - F &= 0 \end{aligned}$$

It can be checked that the Jacobian of the system is negative definite under the assumptions ($B < 0$ and $\partial \pi_n / \partial n < 0$ for $n = n^e$). Differentiating totally the equilibrium conditions with respect to S and evaluating at the equilibrium, we find that

$$sign \frac{dp^e}{dS} = sign\left\{ H^2 S c''(p-c)\left[(p-c)\frac{\partial h}{\partial n} + \frac{\partial H}{\partial n}\right]\right\} < 0,$$

$$sign \frac{dz^e}{dS} = -H\left[(p-c)\frac{\partial h}{\partial n} + \frac{\partial H}{\partial n}\right] > 0,$$

and

$$sign \frac{dn^e}{dS} = sign\left\{H^2 S h^{-1} \widehat{B}\right\} = sign\left\{-\widehat{B}\right\}.$$

Differentiating totally the equilibrium conditions with respect to F and evaluating at the equilibrium, we find that

$$sign\,\frac{dn^e}{dF} = sign\left\{c'' H\left(h + \frac{\partial H}{\partial p} + (p-c)\frac{\partial h}{\partial p}\right) + (c')^2 h \frac{\partial H}{\partial p}\right\} = sign\{B\} < 0.$$

Furthermore,

$$sign\,\frac{dp^e}{dF} = -\,sign\left[(c')^2 h \frac{\partial H}{\partial n} + Hc''\left((p-c)\frac{\partial h}{\partial n} + \frac{\partial H}{\partial n}\right)\right] = -sign\,\frac{dp_n}{dn}$$

and

$$sign\,\frac{dz^e}{dF} = -\,sign\left[\frac{\partial H}{\partial p}\frac{\partial h}{\partial n}(p-c) - \frac{\partial H}{\partial n}\left(h + (p-c)\frac{\partial h}{\partial p}\right)\right] = -sign\,\frac{dz_n}{dn},$$

where (p_n, z_n) is the equilibrium with exogenous n evaluated at $n = n^e$. ■

Proof of Proposition 3: Similarly as in the proof of Proposition 6, differentiating totally the equilibrium conditions and evaluating at the equilibrium yields

$$sign\,\frac{dp^e}{d\sigma} = sign\left\{Hc''(p-c)\,\Omega\right\},$$

and

$$sign\,\frac{dz^e}{d\sigma} = sign\left\{Hc'\Omega\right\},$$

where

$$\Omega = \left[\frac{\partial H}{\partial \sigma}\left[(p-c)\frac{\partial h}{\partial n} + \frac{\partial H}{\partial n}\right] - \frac{\partial H}{\partial n}\left[(p-c)\frac{\partial h}{\partial \sigma} + \frac{\partial H}{\partial \sigma}\right]\right].$$

We obtain that $\frac{dp^e}{d\sigma} < 0$ and $\frac{dz^e}{d\sigma} > 0$ because $\Omega < 0$ ($\frac{\partial H}{\partial \sigma} \geq 0$, $sign\left\{-\left[(p-c)\frac{\partial h}{\partial n} + \frac{\partial H}{\partial n}\right]\right\} = sign\frac{\partial \eta}{\partial n} > 0$, $\frac{\partial H}{\partial n} < 0$, and $sign - \left[(p-c)\frac{\partial h}{\partial \sigma} + \frac{\partial H}{\partial \sigma}\right] = sign\,\frac{\partial \eta}{\partial \sigma} > 0$); and

$sign\,\frac{dn^e}{d\sigma} = sign\left\{-H\Gamma\right\}$, where

$$\begin{aligned}\Gamma \equiv\ & \tfrac{\partial H}{\partial \sigma}\left[c''(p-c)\left(h + \tfrac{\partial H}{\partial p} + (p-c)\tfrac{\partial h}{\partial p}\right) - (c')^2 h\right] \\ & - \left(H + (p-c)\tfrac{\partial H}{\partial p}\right)c''\left((p-c)\tfrac{\partial h}{\partial \sigma} + \tfrac{\partial H}{\partial \sigma}\right).\end{aligned}$$

In general $sign\,\dfrac{dn^e}{d\sigma}$ is ambiguous. If $\dfrac{\partial H}{\partial \sigma} = 0$ then $sign\,\dfrac{dn^e}{d\sigma} < 0$ because $\left(H + (p-c)\frac{\partial H}{\partial p}\right)c''\left((p-c)\frac{\partial h}{\partial \sigma} + \frac{\partial H}{\partial \sigma}\right)$ 0.

If $\widehat{B} > 0$ then $sign\,\dfrac{dn^e}{d\sigma} < 0$ because

$$\Gamma \equiv \frac{\partial H}{\partial \sigma}\frac{\widehat{B}}{-h} - \left(H + (p-c)\frac{\partial H}{\partial p}\right)c''\left((p-c)\frac{\partial h}{\partial \sigma} + \frac{\partial H}{\partial \sigma}\right).$$

■

Proposition 7 *Let $P' < 0$ and $c' < 0$, and let the following boundary conditions hold: There exist $\overline{c} > \underline{c} > 0$ and $\overline{X} > 0$ such that $\overline{c} > c(z) > \underline{c} > 0$, $c'(0+) = -\infty$, $c'(z) \to 0$ as $z \to \infty$, $P(xn) \leq \underline{c}$ if $xn \geq \overline{X}$, and $\lim_{x\to 0}\left\{P(xn) + xP'(xn)\right\} \geq \overline{c}$. Consider an extremal symmetric interior equilibrium (x^*, z^*). Then x^* and z^* are strictly decreasing (increasing) in n if Cournot best replies are strictly decreasing (increasing).*

Proof: Given a symmetric investment profile z and given that $P' < 0$, there exist extremal symmetric Cournot equilibria $\underline{x}(z)$ and $\overline{x}(z)$ that are increasing in z (Amir and

Lambson (2000), Vives (1999, pp. 106–107)). This means that there exist extremal symmetric equilibria in the game. Indeed, just consider $\underline{x}(\underline{z})$, where $\underline{z}$ is the smallest equilibrium associated to $\underline{x}(\cdot)$ and $\overline{z}$ is the greatest equilibrium associated to $\overline{x}(\cdot)$. At an extremal interior equilibrium (x^*, z^*), we have $P(xn) + xP^{'}(xn) - c(z) = 0$ and $-xc'(z) - 1 = 0$. Therefore, $\phi(z, n) \equiv -x(z, n)c'(z) - 1 = 0$, where $x(z, n)$ is an extremal Cournot equilibrium given z. We know that $\phi(\cdot, n)$ cannot jump down, since $x(z, n)$ is increasing in z; $\phi(0+, n) > 0$, since $c'(0+) = -\infty$; and $\phi(z, n) < 0$ for z large, since $c'(z) \to 0$ as $z \to \infty$. It follows that, for extremal z, $\phi(z, n)$ is decreasing in z (indeed, it could not otherwise be an extremal equilibrium) and therefore, if $\phi(z, n)$ is strictly increasing (decreasing) in n then so will z be. We have that $\phi(z, n)$ is strictly increasing (decreasing) in n if and only if $x(z, n)$ is strictly increasing (decreasing) in n. Given that $x(z, n)$ fulfills $\varphi(x, z, n) \equiv P(xn) + xP'(xn) - c(z) = 0$ and that, at extremal equilibria, φ is decreasing in x –because (a) $\varphi(x, z, n) < 0$ for x large (for $xn \geq \overline{X}$ we have $p \leq \underline{c}$) and (b) $\varphi(0+, z, n) > 0$ (since $\lim_{x\to 0}\left\{P(xn) + xP^{'}(xn)\right\} > \overline{c}$)– we conclude that $x(z, n)$ is strictly increasing (decreasing) in n if and only if $\varphi(x, z, n)$ is, and this happens if $P'(xn) + xP''(xn)$ is positive (negative). ■

Proof of Proposition 5: At the symmetric SPE we have that

$$\varphi(z) \equiv \frac{\partial V(z_i, z_{-i})}{\partial z_i} |_{z_i = z} = -xc'(z)\left[1 + (n-1)\frac{P' + xP''}{(n+1)P' + nxP''}\right] - 1$$
$$= -x(z, n)c'(z)(1 + G(x, n)) - 1 = 0,$$

where $G(x, n) = ((n-1)/n)(n-E)/(1+n-E)$.[43] Note that $E(X) \leq 0$ because $P'' \leq 0$ and therefore $E(X) < 1 + n$ (so that, for a given symmetric profile of investments, there is a unique and stable symmetric Cournot equilibrium). Hence, $\frac{dz^*}{dn} = -\frac{\partial\varphi/\partial n}{\partial\varphi/\partial z}$. We have $\frac{\partial\varphi}{\partial z} = \frac{\partial^2 V}{\partial z_i^2} + (n-1)\frac{\partial^2 V}{\partial z_i \partial z_j}$ evaluated at a symmetric solution.[44] Very tedious algebra shows

[43] With $n + 1 > E$, we have that $sign\, G = sign\{n - E\}$. That is, innovation effort is larger (smaller) in the two-stage (simultaneous) game depending on whether best responses in the Cournot game are downward (upward) sloping.

[44] Interestingly, Berry and Waldfogel (2003) show that in the restaurant industry (where quality is produced mostly with *variable* costs) the range of qualities increases with market size, whereas in daily newspapers (where quality is produced mostly with *fixed* costs) the average quality increases with market size and there

that $\frac{\partial^2 V}{\partial z_i \partial z_j} < 0$ when $P' < 0$, $P'' \leq 0$, and $P'P''' - (P'')^2 \leq 0$; therefore, investments are strategic substitutes at the first stage. Moreover, the second order necessary condition at the equilibrium is $\frac{\partial^2 V}{\partial z_i^2} \leq 0$ and so $\frac{\partial \varphi}{\partial z} < 0$. Under the assumptions it is possible to check also that $\partial\varphi/\partial n < 0$. ■

Proof of Proposition 6: We have (x, z, n) fulfilling:

$$\begin{aligned} P(xn/S) + (x/S)P'(xn/S) - c(z) &= 0 \\ -xc'(z) - 1 &= 0 \\ (P(xn/S) - c(z))x - z - F &= 0 \end{aligned}$$

Differentiating totally the equilibrium conditions and evaluating at the equilibrium, we find that

$$\frac{dx^e}{dS} = \frac{(xc'')(x/S^2)P'}{(2P' + (x/S)P'')c''(x/S) + (c')^2}$$

and

$$\frac{dz^e}{dS} = -\frac{c'(x/S^2)P'}{(2P' + (x/S)P'')c''(x/S) + (c')^2}.$$

We have that $sign\left\{\frac{dz^e}{dS}\right\} = sign\left\{\frac{dx^e}{dS}\right\} > 0$ because the denominator is negative (strict concavity of profits of firm i with respect to x_i and z_i implies $xc''(((n+1)P'/S)+((xn/S)(P''/S)))+(c')^2 < 0$ for any n, which in turn implies the result). Furthermore,

$$\frac{dn^e}{dS} = \frac{((n+1)P' + (x/S)nP'')c''(x/S^2) + (n/S)(c')^2}{(2P' + (x/S)P'')c''(x/S) + (c')^2}.$$

Sufficient conditions for $dn^e/dS > 0$ are that $P' + (x/S)P'' < 0$ and $-c''x/Sc' > nc'/P' > 0$. We obtain also

$$\begin{aligned} \frac{dx^e}{dF} &= \frac{c''\left(P' + \frac{x}{S}P''\right)}{-P'\left(\frac{xc''}{S}\left(2P' + \frac{xP''}{S}\right) + (c')^2\right)}, \\ \frac{dz^e}{dF} &= \frac{-c'\left(P' + \frac{x}{S}P''\right)}{-xP'\left(\frac{xc''}{S}\left(2P' + \frac{xP''}{S}\right) + (c')^2\right)}, \end{aligned}$$

and

$$\frac{dn^e}{dF} = \frac{\frac{xc''}{S}\left((n+1)P' + \frac{nxP''}{S}\right) + (c')^2}{\frac{x^2}{S}P'\left(\frac{xc''}{S}\left(2P' + \frac{xP''}{S}\right) + (c')^2\right)} < 0.$$

is no fragmentation as the market grows large.

As before, we have that $\frac{xc''}{S}\left(2P' + \frac{xP''}{S}\right) + (c')^2 < 0$, and the inequality follows because $D = \frac{xc''}{S}\left((n+1)P' + \frac{nxP''}{S}\right) + (c')^2 < 0$. ■

6.2 Examples

6.2.1 Exogenous market structure (restricted entry)

Denote by x and p the symmetric Bertrand equilibria for a given z, and let S parameterize total market size.

Linear demand (Shapley and Shubik (1969)).[45] Let $S = 1$ and $D_i(p) = \frac{S}{n}\left(\alpha - \beta\left[p_i + \gamma\left(p_i - \frac{1}{n}\sum_i p_i\right)\right]\right)$ for $i = 1, .., n$, where α, β, γ are positive constants. We have $H = (\alpha - \beta p)/n$. At a symmetric solution, the direct elasticity of substitution is given by $\sigma = (1+\gamma)(\alpha - nx)/nx$, and it increases with the substitutability parameter γ.[46] We have that $\frac{\partial H}{\partial n} < 0$ and that $\frac{\partial h}{\partial n} > 0$, $\frac{\partial \eta}{\partial n} > 0$, $\frac{\partial \eta}{\partial \gamma} > 0$, and $\frac{\partial H}{\partial \gamma} = 0$. For a given symmetric profile z, there is a unique and symmetric Bertrand equilibrium with price p and output per firm x. We have that $\frac{\partial p}{\partial n} < 0$, $\frac{\partial x}{\partial n} < 0$, $\frac{\partial x}{\partial \gamma} > 0$, and $\frac{\partial p}{\partial \gamma} < 0$. In summary, $\frac{\partial x}{\partial n} < 0$ and $\frac{\partial x}{\partial \sigma} > 0$.

Linear demand (Bowley (1924)). Let $D_i(p) = S\left(a_n - b_n p_i + c_n \sum_{j \neq i} p_j\right)$ for $i = 1..., n$, where $a_n = \alpha/(\beta + (n-1)\gamma)$,
$b_n = (\beta + (n-2)\gamma)/((\beta + (n-1)\gamma)(\beta - \gamma))$, and
$c_n = \gamma/((\beta + (n-1)\gamma)(\beta - \gamma))$ and where $\alpha > 0$ and $\beta > \gamma > 0$ are utility parameters.[47] At a symmetric solution, the direct elasticity of substitution $\sigma = p/(\beta - \gamma)x$ increases with γ. The Chamberlinian DD demand function is given by $H = (\alpha - p)/(\beta + (n-1)\gamma)$, where $\frac{\partial H}{\partial n} < 0$, $\frac{\partial h}{\partial n} < 0$, $\frac{\partial \eta}{\partial n} > 0$, $\frac{\partial \eta}{\partial \gamma} > 0$, and $\frac{\partial H}{\partial \gamma} < 0$. For a given symmetric profile z,

[45] This linear demand system can be derived from a quadratic utility function (with preferences linear in the numéraire) in which the number of firms n enters as a parameter. See Vives (1999, Chap. 6).

[46] For symmetric solutions (with demands arising from the maximization of a quasilinear utility function), the (direct) elasticity of substitution is given by $\sigma = (\varepsilon_{ij} + \varepsilon_i)^{-1}$, where ε_{ij} is the cross–elasticity of inverse demand, $\varepsilon_{ij} = \frac{q_j}{p_i}\frac{\partial P_i}{\partial q_j}$. Note also that $\varepsilon_{ij} \leq 0$ and $\varepsilon_i \geq 0$.

[47] This linear demand system can be derived also from a quadratic utility function (with preferences linear in the numéraire). See Vives (1999, Chap. 6).

there is a unique and symmetric Bertrand equilibrium with price p and output per firm x: $p = (a_n + b_n c(z)) / (2b_n - (n-1) c_n)$. We have that $\frac{\partial p}{\partial n} < 0$, $\frac{\partial x}{\partial n} < 0$, and $\frac{\partial p}{\partial \gamma} < 0$ but $\frac{\partial x}{\partial \gamma} < 0$. Hence, in this case, increasing competitive pressure by increasing the elasticity of substitution decreases output. With this particular demand system we have the unusual feature that $\frac{\partial H}{\partial \sigma} < 0$. In summary, $\frac{\partial x}{\partial n} < 0$ and $\frac{\partial x}{\partial \sigma} < 0$.

Location models (Salop (1979)). Although formally in models with localized competition the demand system is not exchangeable for $n > 2$, the analysis is easily adapted. A uniform mass of customers S is distributed within a circle in which n firms have located symmetrically and each produces at constant marginal cost c. Consumers have a linear transportation cost $t > 0$. Then the demand of firm i setting price p_i (with neighbors setting a price equal to p) is $\frac{S}{n} + \frac{p-p_i}{t}$ when there is direct competition among firms. We can take $\sigma \equiv 1/t$. Therefore $H = S/n$ and H is independent of p and σ. There is neither price–pressure effect nor a demand effect coming from σ. The unique Bertrand equilibrium is $p = c + t/n$ and $\eta = 1 + nc/t$, which for given c is increasing in n and in σ. If the transportation cost is quadratic with parameter t, then the Bertrand equilibrium is given by $p = c + t/n^2$.

Constant elasticity. Let

$D_i(p) = S(\beta\theta)^{\frac{1}{1-\beta\theta}} \frac{p_i^{\frac{1}{1-\beta}}}{\left(\sum_{j=1}^{n} p_j^{\frac{\beta}{\beta-1}}\right)^{\frac{1-\theta}{1-\beta\theta}}}$ for $i = 1..., n$, with $0 \le \beta < 1$ and also $0 \le \beta\theta < 1$.

The (direct) elasticity of substitution is $\sigma = 1/(1-\beta)$; for $\beta = 0$ goods are independent, and for $\beta = 1$ they are perfect substitutes. We have that $H = S(\beta\theta)^{1/(1-\beta\theta)} p^{-1/(1-\beta\theta)} n^{-(1-\theta)/(1-\beta\theta)}$ and that $\frac{\partial H}{\partial p} < 0$, $sign \frac{\partial H}{\partial n} = sign(\theta - 1)$, and $\frac{\partial H}{\partial \beta} > 0$. Restrict attention to the case $\theta - 1 < 0$ in order to ensure a limited market for the differentiated varieties: $\frac{\partial H}{\partial n} < 0$. We have that $\eta = \frac{1}{1-\beta}\left(1 - \frac{\beta}{n}\frac{1-\theta}{1-\beta\theta}\right)$, which is strictly increasing in n and β (and therefore the Lerner index L will be decreasing in n and β). For a given symmetric profile z, there is a unique and symmetric Bertrand equilibrium with price p and output per firm x (the price game is log-supermodular and there is a unique symmetric equilibrium, hence the symmetric equilibrium is the unique one). In equilibrium, $\frac{\partial h}{\partial n} > 0$, $\frac{\partial \eta}{\partial n} > 0$ (for $n > 1$), and $p = (n(1-\beta\theta) + \beta(\theta - 1)) c / (\beta n (1-\beta\theta) + \beta(\theta-1))$, and it is easily checked that $sign \frac{\partial p}{\partial n} = sign(\theta - 1) < 0$ and $\frac{\partial x}{\partial n} < 0$. Furthermore, $\frac{\partial p}{\partial \beta} < 0$ and $\frac{\partial x}{\partial \beta} > 0$ because

$\frac{\partial x}{\partial \beta} = \frac{\partial H}{\partial \beta} + \frac{\partial H}{\partial p}\frac{\partial p}{\partial \beta}$, $\frac{\partial H}{\partial \beta} > 0$, $\frac{\partial H}{\partial p} < 0$, and $\frac{\partial p}{\partial \beta} < 0$. In summary, $\frac{\partial x}{\partial n} < 0$ and $\frac{\partial x}{\partial \sigma} > 0$.

Assuming that $c(z) = \alpha z^{-\gamma}$ with $\alpha >$ and $\gamma > 0$, we can obtain a closed–form solution. It can be shown that, evaluating at a symmetric equilibrium, $B < 0$ if and only if $\beta\theta < \frac{1}{\gamma+1}$. Some computations then yield

$$z^* = \left(\alpha^{\beta\theta} (\gamma S)^{\beta\theta-1} n^{1-\theta} (\beta\theta)^{-1} \frac{n(1-\beta\theta) + \beta(\theta-1)}{\beta n(1-\beta\theta) + \beta(\theta-1)}\right)^{\frac{1}{\gamma\beta\theta+\beta\theta-1}},$$

and

$$p^* = \left[(S\alpha\gamma)^{\beta\theta-1} (\beta\theta)^{-1} n^{1-\theta} \left(\alpha \frac{n(1-\beta\theta) + \beta(\theta-1)}{\beta n(1-\beta\theta) + \beta(\theta-1)}\right)^{\frac{(\gamma+1)(1-\beta\theta)}{\gamma}}\right]^{\frac{\gamma}{1-\beta\theta-\beta\theta\gamma}}.$$

By Proposition 4 it follows that if $\beta\theta < \frac{1}{\gamma+1}$ then $\frac{dz}{dn} < 0$ and $\frac{dz}{d\beta} > 0$. Indeed, for $\beta\theta < \frac{1}{\gamma+1}$ we have $sign\,\frac{dz}{dn} = sign\,(\gamma\beta\theta + \beta\theta - 1) < 0$ and $\frac{dp}{dn} < 0$ because $sign\,\frac{dp}{dn} = -sign\frac{\gamma\beta\theta+\beta\theta-1}{\gamma\beta\theta+\beta\theta-1-\gamma} < 0$. Furthermore, $\pi_n^* = z^* \frac{1-\beta-\beta\gamma\left(1-\frac{1}{n}\frac{1-\theta}{1-\beta\theta}\right)}{\gamma\beta\left(1-\frac{1}{n}\frac{1-\theta}{1-\beta\theta}\right)} = z^*\left(\frac{1}{(\eta-1)\gamma} - 1\right)$ and $sign\ \widehat{B} = -sign\frac{1}{1-\beta}\left\{1 - \beta - \beta\gamma\left(1 - \frac{1}{n}\frac{1-\theta}{1-\beta\theta}\right)\right\}$. As a result, $\pi_n^* > 0$ if and only if $\widehat{B} < 0$. This means that π_n^* is strictly decreasing in n whenever positive. Note also that $\beta \leq \frac{1}{\gamma+1}$ guarantees that $\widehat{B} < 0$ for all n.

Constant expenditure model. Let $D_i(p) = \frac{S}{p_i}\frac{g(p_i)}{\sum_{j=1}^{n} g(p_j)}$, $i = 1, ..., n$, with $g > 0$, $g' < 0$, and $S > 0$. We have that $H = S/np$ and therefore $\frac{\partial H}{\partial n} < 0$ and $\frac{\partial H}{\partial \sigma} = 0$. We have also that $\frac{d\eta}{dn} > 0$ because $\frac{\partial H}{\partial n} + (p-c)\frac{\partial h}{\partial n} = -\frac{S}{pn^2}\left[\frac{c}{p} - \frac{g'(p)}{g(p)}\right] < 0$.

Let $g(p) \equiv e^{-\beta p}$ with $\beta > 0$. Observe that goods are independent for $\beta = 0$ yet are perfect substitutes for $\beta \to \infty$. Let $S = 1$. For a given symmetric profile z, there is a unique and symmetric Bertrand equilibrium with price p and output per firm x (the price game is log-supermodular and symmetric and there is a unique symmetric equilibrium, so the symmetric equilibrium is the unique one). We have $p = \left(c + \left(c^2 + (4cn/(\beta(n-1)))\right)^{1/2}\right)/2$, $x = S/np$, $\frac{\partial p}{\partial n} < 0$, $\frac{\partial x}{\partial n} < 0$, $\frac{\partial p}{\partial \beta} < 0$, $\frac{\partial H}{\partial \beta} = 0$, and $\frac{\partial x}{\partial \beta} > 0$.

Another example is of the constant elasticity variety: $g(p) \equiv p^{-r}$ where $r > 0$ (see Anderson, de Palma and Thisse (1992, Chap. 7)).[48] Goods are perfect substitutes when $r \to$

[48] This is also the specification in Aghion et al. (2002).

∞ but are independent when $r \to 0$. We can take $\sigma = 1+r$. (The demand system may arise from $W(x_0, x) = \left(\sum_i x_i^{\frac{r}{1+r}}\right)^{\frac{1+r}{r}} x_0^{\alpha}$ for $\alpha > 0$, yielding $S = I/(1+\alpha)$, where I is the income of the representative consumer.) We have that $h = -S\frac{n(r+1)-r}{(np)^2}$, $\frac{\partial h}{\partial p} = -S\frac{2(n(r+1)-r)}{n^2p^3} < 0$, and $\eta = \frac{n(r+1)-r}{n}$ (which increases with n and r). For a given symmetric profile z, there is a unique and symmetric Bertrand equilibrium with price p and output per firm x (the price game is log-supermodular and symmetric and there is a unique symmetric equilibrium, hence the symmetric equilibrium is the unique one). We have that $p = c\frac{n(r+1)-r}{r(n-1)}$, $\frac{\partial p}{\partial n} = -\frac{c}{r(n-1)^2} < 0$, $\frac{\partial x}{\partial n} < 0$, $\frac{\partial p}{\partial r} = -\frac{cn}{r^2(n-1)} < 0$, and $\frac{\partial x}{\partial r} > 0$ because $\frac{\partial H}{\partial p} < 0$ and $\frac{\partial H}{\partial r} = 0$.

Assuming that $c(z) = \alpha z^{-\gamma}$ with $\alpha >$ and $\gamma > 0$, we can obtain a closed–form solution. It can be shown that $B < 0$ if and only if $\frac{\gamma+1}{\gamma} > \frac{r(n-1)}{r(n-1)+4n}$. This is always true. The equilibrium solution is $z^* = \frac{S\gamma r(n-1)}{n(n(r+1)-r)}$ and $p^* = \alpha\left[\frac{S\gamma}{n}\left(\frac{r(n-1)}{n(r+1)-r}\right)^{\frac{\gamma+1}{\gamma}}\right]^{-\gamma}$. Indeed, we have that $sign\,\frac{dz}{dn} =$
$sign\left\{\frac{d}{dn}\left(\frac{r(n-1)}{n(r+1)-r}\right) < 0\right\}$ and $sign\,\frac{dp}{dn} =$
$-sign\left[-\frac{1}{n^2}\left(\frac{r(n-1)}{n(r+1)-r}\right)^{1/\gamma}\frac{r}{n(r+1)-r}\left(1-\frac{\gamma+1}{\gamma}\frac{n}{n(r+1)-r}\right)\right]$. We have also that $\frac{dx}{dn} < 0$.

Profits are given by $\pi_n = S\left[\frac{n-\gamma r(n-1)}{n(r(n-1)+n)}\right]$; they are strictly decreasing in n, and $\pi_n > 0$ if and only if $n > \gamma r(n-1)$. This holds for all n if $\gamma r < 1$. Positive profits imply that $\widehat{B} < 0$ $\left(\widehat{B} < 0 \text{ if and only } \frac{\gamma+1}{\gamma}\frac{n}{r(n-1)+n} > \frac{r(n-1)}{r(n-1)+4n}\right)$ and $\frac{dp}{dn} < 0$.

We have that $L = \frac{n}{n(r+1)-r}$, which is decreasing in n and r. The R&D expenditure/sales ratio $\frac{z^*n}{p^*x^*n} = \frac{z^*n}{S} = \frac{\gamma r(n-1)}{n(r+1)-r}$ is increasing in r and n.

Logit. Let $D_i(p) = \frac{e^{-p_i/\mu}}{\sum_j e^{-p_j/\mu}}S$, $i = 1, ..., n$, $\mu > 0$. We have that goods are perfect substitutes for $\mu = 0$ and are independent for $\mu = \infty$, and the elasticity of substitution is pn/μ. Furthermore, $H(p) = S/n$ and $h(p) = -(S/n)(1-1/n)/\mu$, $\frac{\partial H}{\partial n} < 0$, and $\frac{\partial h}{\partial n} > 0$. We have that $\eta = \frac{p(n-1)}{\mu n}$, which is increasing in n and $\sigma \equiv 1/\mu$. For a given symmetric profile z, there is a unique and symmetric Bertrand equilibrium with price p and output per firm x (the price game is log-supermodular and symmetric and there is a unique symmetric equilibrium, so the symmetric equilibrium is the unique one). We have that $p = c + n\mu/(n-1)$, $\frac{\partial p}{\partial n} < 0$, and $\frac{\partial x}{\partial n} < 0$. There is no price–pressure effect because $\frac{\partial H}{\partial p} = 0$, but there is a demand effect $\frac{\partial H_n}{\partial n} < 0$. Furthermore, $\frac{\partial H}{\partial \sigma} = 0$ and therefore there is no demand effect. Neither there is a price–pressure effect, (because $\frac{\partial H}{\partial p} = 0$) and hence, despite that $\frac{\partial p^*}{\partial \sigma} < 0$, we have that

$\frac{\partial x^*}{\partial \sigma} = 0$. (In this case $B < 0$ always because $\frac{\partial H}{\partial p} = 0$.)

As before, assuming that $c(z) = \alpha z^{-\gamma}$ with $\alpha >$ and $\gamma > 0$ yields a closed–form solution: $p = \frac{n\mu}{n-1} + \alpha \left[\frac{S\alpha\gamma}{n}\right]^{-\frac{\gamma}{\gamma+1}}$ and $z = \left[\frac{S\alpha\gamma}{n}\right]^{\frac{1}{\gamma+1}}$. We have that $L = \left[1 + \frac{n-1}{\mu\gamma S}\left(\frac{S\alpha\gamma}{n}\right)^{\frac{1}{\gamma+1}}\right]^{-1}$, which is decreasing in n and $\sigma \equiv 1/\mu$ and that $\frac{z^*}{p^* x^*} = \left[\frac{1}{S\gamma} + \frac{\mu}{n-1}\left(\frac{n}{S\alpha\gamma}\right)^{\frac{1}{\gamma+1}}\right]^{-1}$, which is increasing in n and σ.

6.2.2 Endogenous market structure (free entry)

Constant elasticity. It can be shown that, evaluating at a symmetric equilibrium, $B < 0$ if and only if $\beta\theta < \frac{1}{\gamma+1}$ and $sign\,\widehat{B} =$
$-sign\left\{\frac{1}{1-\beta}1 - \beta - \beta\gamma\left(1 - \frac{1}{n}\frac{1-\theta}{1-\beta\theta}\right)\right\}$. We have, that for given n,

$$z_n = \left(\alpha^{\beta\theta}(\gamma S)^{\beta\theta-1} n^{1-\theta}\frac{n(1-\beta\theta)+\beta(\theta-1)}{\beta n(1-\beta\theta)+\beta(\theta-1)}\right)^{\frac{1}{\gamma\beta\theta+\beta\theta-1}}$$

and

$$p_n = \left[(S\alpha\gamma)^{\beta\theta-1}(\beta\theta)^{-1} n^{1-\theta}\left(\frac{\alpha(n(1-\beta\theta))+\beta(\theta-1)}{\beta[n(1-\beta\theta)+(\theta-1)]}\right)^{\frac{(\gamma+1)(1-\beta\theta)}{\gamma}}\right]^{\frac{\gamma}{1-\beta\theta-\beta\theta\gamma}}.$$

The free–entry number of firms is $[n^e]$, where n^e is the solution to

$$\pi_n = z_n \frac{1-\beta-\beta\gamma\left(1-\frac{1}{n}\frac{1-\theta}{1-\beta\theta}\right)}{\gamma\beta\left(1-\frac{1}{n}\frac{1-\theta}{1-\beta\theta}\right)} - F = 0$$

given that variable profits (whenever positive) are strictly decreasing with n. It is straightforward to check that profits are strictly increasing in S because $\partial z/\partial S > 0$. It follows then that $\frac{dn^e}{dS} > 0$.

The following expression implicitly defines n^e:

$$\left((\gamma S\alpha)^{\beta\theta-1} n^{1-\theta}(\beta\theta)^{-1}\alpha\frac{n(1-\beta\theta)+\beta(\theta-1)}{\beta n(1-\beta\theta)+\beta(\theta-1)}\right)^{\frac{1}{\gamma\beta\theta+\beta\theta-1}} = \frac{F\gamma\beta(n(1-\beta\theta)-(1-\theta))}{(1-\beta-\beta\gamma)n(1-\beta\theta)+(1-\theta)\beta\gamma}.$$

In equilibrium it should hold that $z^e = \frac{F\gamma\beta([n^e](1-\beta\theta)-(1-\theta))}{(1-\beta-\beta\gamma)[n^e](1-\beta\theta)+(1-\theta)\beta\gamma}$ or $z^e = \frac{F\gamma(\eta-1)}{1-\gamma(\eta-1)}$, where $\eta = \frac{1}{1-\beta}\left(1-\frac{\beta}{n}\frac{1-\theta}{1-\beta\theta}\right)$. From this expression, knowing that $\frac{dn^e}{dS} > 0$ it follows that

$\frac{dz^e}{dS} > 0$. (This holds even if $[n^e]$ stays constant for increasing S; in this case, the direct impact of S increases z.)

With constant elasticity demand and γ constant, the Lerner index is decreasing in z. Therefore, increasing S increases z, decreases L, and increases η. The result is that n must increase.

We know also that increasing F increases z (because $sign\,\frac{dz^e}{dF} = -sign\,\frac{dz_n}{dn} > 0$) and increases p (because $sign\,\frac{dp^e}{dF} = sign\,\frac{dp_n}{dn} > 0$). It can be checked that $\frac{dz^e n^e}{dF} < 0$ if $\beta \leq \frac{1}{\gamma+1}$. If $F = 0$ and $\beta \leq \frac{1}{\gamma+1}$, then profits are strictly positive for all n and $n^e = \infty$.

If $F = 0$, $\beta > \frac{1}{\gamma+1}$, and $\beta\theta < \frac{1}{\gamma+1}$, then we still know that profits (whenever positive) are strictly decreasing with n . Then the free–entry number of firms is $\left[\frac{\beta\gamma(1-\theta)}{(1-\beta\theta)(\beta\gamma+\beta-1)}\right]$ because, at this n, adding one more firm would result in negative profits. In this case the free–entry number of firms is independent of S, and $n^e \geq 1$ as long as $\beta > \frac{1}{\gamma+1}$; as before, under our assumptions $\left(\beta\theta < \frac{1}{\gamma+1}\right)$, $\frac{dz}{dS} > 0$ and $\frac{dp}{dS} < 0$. (Note that for $n = \frac{\beta\gamma(1-\theta)}{(1-\beta\theta(\beta\gamma+\beta-1))}$ we have $\widehat{B} = 0$.) Furthermore, $\frac{dn^e}{d\beta} < 0$ (using the assumption $\beta\theta < \frac{1}{\gamma+1}$).

<u>Constant expenditure</u> (and constant elasticity). Given n, $z_n = \frac{S\gamma r(n-1)}{n(n(r+1)-r)}$ and profits are given by $\pi_n = S\left[\frac{n-\gamma r(n-1)}{n(r(n-1)+n)}\right]$. They are strictly decreasing in n, and $\pi_n > 0$ if and only if $n > \gamma r\,(n-1)$. This holds for all n if $\gamma r < 1$. Positive profits imply that $\widehat{B} < 0$ ($\widehat{B} < 0$ if and only if $\frac{\gamma+1}{\gamma}\frac{n}{r(n-1)+n} > \frac{r(n-1)}{r(n-1)+4n}$).

Using the zero profit–entry condition we obtain

$$n^e = \frac{(F - S\gamma)\,r + S + \sqrt{(Fr + S - S\gamma r)^2 + 4\gamma r SF\,(r+1)}}{2F\,(r+1)},$$

which is strictly increasing in S provided that $\gamma r < 1$. Furthermore, as expected, $\frac{dz^e}{dS} > 0$, $\frac{dz^e}{dr} > 0$, and $sign\,\frac{dn}{dr} = sign\,\frac{d\pi}{dr} < 0$ (recall that $\partial H/\partial r = 0$). We know also that increasing F increases z (because $sign\,\frac{dz^e}{dF} = -sign\,\frac{dz_n}{dn} > 0$) and increases p (because $sign\,\frac{dp^e}{dF} = -sign\,\frac{dp_n}{dn} > 0$). It is immediate then that $\frac{dn^e z^e}{dF} < 0$. The Lerner index is given by $L = \frac{n}{r(n-1)+n}$, and it can be checked that (a)$\frac{dL}{dr} < 0$ whenever $n > r\gamma\,(n-1)$ and (b) L is increasing in F because n is decreasing in F.

<u>Logit</u>. Given n, we have $p_n = c\,(z) + \frac{\mu n}{n-1}$ and $z_n = \left[\frac{S\alpha\gamma}{n}\right]^{\frac{1}{\gamma+1}}$. Profits (gross of fixed costs) are given by $\pi_n = \frac{S\mu}{n-1} - \left[\frac{S\alpha\gamma}{n}\right]^{\frac{1}{\gamma+1}}$. For profits to be decreasing in n we need

$\frac{S\mu(\gamma+1)}{n-1} - \frac{n-1}{n}\left[\frac{S\alpha\gamma}{n}\right]^{\frac{1}{\gamma+1}} > 0$ (which is equivalent to $\widehat{B} < 0$ and is implied by positive profits $\pi_n > 0$).[49] We conclude that if π_n is positive then it is strictly decreasing in n. The free–entry number of firms is implicitly defined by $[n^e]$ where n^e solves $\frac{S\mu}{n-1} - \left[\frac{S\alpha\gamma}{n}\right]^{\frac{1}{\gamma+1}} = F$. Consistent with our other results, we have $\frac{dn^e}{dS} > 0$ and $\frac{dz^e}{dS} > 0$; $\frac{dx}{d\mu} < 0$ or $\frac{dx}{d\sigma} > 0$; $\frac{dz}{d\mu} < 0$ or $\frac{dz}{d\sigma} > 0$; and $\frac{dn^e}{d\mu} > 0$ or $\frac{dn^e}{d\sigma} < 0$. Increasing F increases z^e (because $sign\,\frac{dz^e}{dF} = -sign\,\frac{dz_n}{dn} > 0$), decreases $n^e z^e$ (because $n^e z^e = [S\alpha\gamma]^{\frac{1}{\gamma+1}} n^{\frac{\gamma}{\gamma+1}}$) and the impact on p is ambiguous (because $sign\,\frac{dp^e}{dF} = -sign\,\frac{dp_n}{dn}$). We have that $L = \left[1 + \left(\frac{\mu}{n-1}\right)^{-1}\left(\frac{S\alpha\gamma}{n}\right)^{\frac{1}{\gamma+1}}\frac{1}{S\gamma}\right]^{-1}$ and $dL/d\sigma < 0$ (taking into account the impact of σ on L), and $dL/dF > 0$ because L decreases in n and n decreases with F.

Constant elasticity demand with innovation function $c(z) = \frac{1}{A+z}$. Let $D_i(p) = S\frac{p_i^{-\sigma}}{\Sigma_j p_i^{-\sigma}}$ and $A > 0$. It can be shown that at a symmetric equilibrium $B < 0$ always and $sign\widehat{B} = sign\left[(\sigma-1)(n-1) - n\right]$. We have that for given n, $z_n = \frac{S(\sigma-1)(n-1)}{n[n\sigma-(\sigma-1)]} - A + F$ and $p_n = \frac{n[n\sigma-(\sigma-1)]^2}{S[(\sigma-1)(n-1)]^2}$. We have also that $sign\frac{dz}{dn} = sign\left[-n\sigma(n-2) - \sigma + 1\right] < 0$ and $sign\frac{dp}{dn} = sign\left[n\sigma(n-3) + (\sigma-1)(n+1)\right]$ is positive for $n \geq 3$ and ambiguous for $n \in (1,3)$. The free entry number of firms is n^e, where n^e solves $\frac{S[n-(\sigma-1)(n-1)]}{n[n\sigma-(\sigma-1)]} + A - F = 0$ given that profits are decreasing in n. (Whenever $n > 2$, $\frac{\partial\pi}{\partial n} < 0 \Rightarrow \sigma < \frac{2n-1}{n-2}$.) It is straightforward that $sign\frac{\partial\pi}{\partial n} = -sign\,(A-F)$. It follows that $sign\frac{dn^e}{dS} = -sign\,(A-F)$. As expected $\frac{dn^e}{d\sigma} < 0$. Also, increasing F increases z ($sign\frac{dz}{dF} = -sign\frac{dz}{dn} > 0$) and has an ambiguous effect on p ($sign\frac{dp}{dF} = -sign\frac{dp}{dn}$). The Lerner index is $L = \frac{n}{n\sigma-(\sigma-1)}$ and it can be checked that $sign\frac{dL}{dF} = -sign\frac{dn}{dF} > 0$. We have also that $sign\frac{dL}{d\sigma} = sign\,(A-F)$.

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[49] It can be checked that $B^* < 0$ at $n = n^*$: observe that $B^* < 0$ if and only if $\gamma + 1 + \frac{1-n}{\mu n}\gamma\alpha z^{-\gamma} > 0$, and this holds at equilibrium if and only if $\frac{S\mu}{n-1} - F = \left[\frac{S\alpha\gamma}{n}\right]^{\frac{1}{\gamma+1}} < \frac{\mu S(\gamma+1)}{n-1}$.

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Table 1: Exogenous market structure

Demand system	Linear (Shapley-Shubik)	Linear (Bowley)	Location	CES	Constant Exp. (Exponential)	Constant Exp. (CES)	Logit
$D_i(p) =$	$S(\frac{\alpha-\beta p_i}{n} - \frac{\beta\gamma p_i}{n} + \frac{\beta\gamma\Sigma_j p_j}{n^2})$	$S(a_n - b_n p_i + c_i\Sigma_{j\neq i}p_j)$	$\frac{St+n(p-p_i)}{nt}$	$\frac{S(\beta\theta)^{\frac{1}{1-\beta\theta}} p_i^{\frac{1}{1-\beta}}}{[\Sigma_j p_j^{\frac{\beta}{\beta-1}}]^{\frac{1-\theta}{1-\beta\theta}}}$	$\frac{S\exp -\beta p_i}{p_i\Sigma_j \exp -\beta p_j}$	$\frac{Sp_i^{-r}}{p_i\Sigma_j p_j^{-r}}$	$\frac{S\exp\left[-\frac{p_i}{\mu}\right]}{\Sigma_i \exp\left[-\frac{p_i}{\mu}\right]}$
$\sigma \equiv$	γ	γ	$\frac{1}{t}$	β	β	$1+r$	$\frac{1}{\mu}$
$H =$	$\frac{S(\alpha-\beta p)}{n}$	$\frac{S(\alpha-p)}{\beta+(n-1)\gamma}$	$\frac{S}{n}$	$\frac{S(\beta\theta)^{\frac{1}{1-\beta\theta}} p^{\frac{1}{\beta\theta-1}}}{n^{\frac{1-\theta}{1-\beta\theta}}}$	$\frac{S}{np}$	$\frac{S}{np}$	$\frac{S}{n}$
$\eta =$	$\frac{(n+n\gamma-\gamma)p}{(\alpha-\beta p)n}$	$\frac{[\beta+(n-1)\gamma]p}{(\beta-\gamma)(\alpha-p)}$	$\frac{np}{t}$	$\frac{1-\frac{\beta}{n}\frac{1-\theta}{1-\beta\theta}}{1-\beta}$	$\frac{n+\beta(n-1)p}{n}$	$\frac{n(r+1)-r}{n}$	$\frac{p(n-1)}{\mu n}$
$sign\ \partial H/\partial n$	$-$	$-$	$-$	$-$	$-$	$-$	$-$
$sign\ \partial \eta/\partial n$	$+$	$+$	$+$	$+$	$+$	$+$	$+$
$sign\ \partial H/\partial \sigma$	0	$-$	0	$+$	0	0	0
$sign\ \partial \eta/\partial \sigma$	$+$	$+$	$+$	$+$	$+$	$+$	$+$
$sign\ \partial x/\partial n$	$-$	$-$	$-$	$-$	$-$	$-$	$-$
$sign\ \partial x/\partial \sigma$	$+$	$-$	0	$+$	$+$	$+$	0
Innovation Function $c(z) = \alpha z^{-\gamma}$							
$z^* =$				$\left[\frac{(S\alpha\gamma)^{\beta\theta} n\eta}{n^\theta S\beta\theta\gamma(\eta-1)}\right]^{\frac{1}{\beta\theta(\gamma+1)-1}}$		$\frac{S\gamma r(n-1)}{n(n(r+1)-r)}$	$\left[\frac{S\alpha\gamma}{n}\right]^{\frac{1}{\gamma+1}}$
$p^* =$				$\frac{\eta}{\eta-1}\alpha(z^*)^{-\gamma}$		$\frac{\alpha(z^*)^{-\gamma}[n(r+1)-r]}{r(n-1)}$	$\frac{n\mu}{n-1} + \alpha[z^*]^\gamma$
$\pi^* =$				$z^*\left[\frac{1-\gamma(\eta-1)}{\gamma(\eta-1)}\right]$		$\frac{S[n-\gamma r(n-1)]}{n(n(r+1)-r)}$	$\frac{S\mu}{n-1} - z^*$
$\frac{z^*}{p^*x^*} =$				$\frac{\gamma(\eta-1)}{\eta}$		$\frac{\gamma r(n-1)}{n(r+1)-r}$	$\frac{1}{\frac{1}{S\gamma}+\frac{\mu}{(n-1)z^*}}$

Table 2: Endogenous Market Structure with innovation function $c(z) = \alpha z^{-\gamma}$ | with innovation function $c(z) = \frac{1}{A+z}$

Demand system	CES $F=0$	CES $F>0$	Constant Expenditure (CES)	Logit	Constant Expenditure (CES) $A>F$	Constant Expenditure (CES) $A<F$
$D_i(p) =$	$\frac{S(\beta\theta)^{\frac{1}{1-\beta\theta}} p_i^{\frac{1}{1-\beta}}}{\left(\Sigma_j p_j^{\frac{\beta}{\beta-1}}\right)^{\frac{1-\theta}{1-\beta\theta}}}$		$\frac{S p_i^{-r-1}}{\Sigma_j p_j^{-r}}$	$\frac{S\exp\left(-\frac{p_i}{\mu}\right)}{\Sigma_j \exp\left(-\frac{p_j}{\mu}\right)}$	$\frac{S p_i^{-\sigma}}{\Sigma_j p_j^{1-\sigma}}$	
n^e	$= \frac{\beta\gamma(1-\theta)}{(1-\beta\theta)(\beta\gamma+\beta-1)}$	$\left[\frac{(S\alpha\gamma)^{\beta\theta}(n^e)^{1-\theta}\eta^e}{S\beta\theta\gamma(\eta^e-1)}\right]^{\frac{1}{\gamma\beta\theta+\beta\theta-1}} = \\ = \frac{F\gamma(\eta^e-1)}{1-\gamma(\eta^e-1)}$	$= \frac{(F-S\gamma)r+S}{2F(r+1)} + \frac{[(Fr+S-\gamma rS)^2+4\gamma rFS(r+1)]^{1/2}}{2F(r+1)}$	$\frac{S\mu}{n^e-1} - \left[\frac{S\alpha\gamma}{n^e}\right]^{\frac{1}{\gamma+1}} = F$	$\frac{S[n-(\sigma-1)(n-1)]}{n[n\sigma-(\sigma-1)]} + A = F$	
$z^e =$	$\left[\frac{(S\alpha\gamma)^{\beta\theta}(n^e)^{1-\theta}}{S\beta\theta\gamma}\frac{\eta^e}{\eta^e-1}\right]^{\frac{1}{\gamma\beta\theta+\beta\theta-1}}$		$\frac{S\gamma r(n^e-1)}{n^e(n^e(r+1)-r)}$	$\left[\frac{S\alpha\gamma}{n^e}\right]^{\frac{1}{\gamma+1}}$	$\frac{S(\sigma-1)(n^e-1)}{n^e[n^e\sigma-(\sigma-1)]} - A$	
$p^e =$	$\frac{\eta^e}{\eta^e-1}\alpha(z^e)^{-\gamma}$		$\alpha\left[\frac{S\gamma}{n^e}\left(\frac{r(n^e-1)}{n^e(r+1)-r}\right)^{\frac{\gamma+1}{\gamma}}\right]^{-\gamma}$	$\frac{n^e\mu}{n^e-1} + \alpha\left[\frac{S\alpha\gamma}{n^e}\right]^{-\frac{\gamma}{\gamma+1}}$	$\frac{Sn^e[n^e\sigma-(\sigma-1)]^2}{[(\sigma-1)(n^e-1)]^2}$	
$L^e\left(=\frac{1}{\eta^e}\right) =$	$\frac{\beta n^e(1-\beta\theta)(1-\beta)}{n^e(1-\beta\theta)+\beta(\theta-1)}$		$\frac{n^e}{n^e(r+1)-r}$	$\frac{1}{1+\frac{n^e-1}{S\mu\gamma}\left(\frac{S\alpha\gamma}{n^e}\right)^{\frac{1}{\gamma+1}}}$	$\frac{n^e}{n^e\sigma-(\sigma-1)}$	
$sign\ \partial n^e/\partial S$	0	+	+	+	−	+
$sign\ \partial n^e/\partial \sigma$			−	−	−	
$sign\ \partial L^e/\partial \sigma$			−	−	+	−
$sign\ \partial L^e/\partial F$	+		+	+	+	
$sign\ \partial z^e/\partial F$	+		+	+	+	
$sign\ \partial p^e/\partial F$	+		+	ambiguous	+ for $n \geq 3$ ambiguous otherwise	