Does Representation Induce Polarization? Preferences Over Representatives*

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Abstract

Few elected representatives can unilaterally implement their platforms: rather, they choose between options generated by other actors and/or external events. When this is the case, voters' preferences over candidates' platforms will almost always be asymmetric even if their preferences over policy outcomes are symmetric. When uncertainty about the alternatives to be chosen from is single-peaked and symmetric, then voters comparing two equidistant candidates will *always* have a preference for the more extreme candidate. When the status quo is known *a priori*, then voters will have a preference for the extreme candidate when the degree of divergence between the candidates is low or when the voter is sufficiently distant from both the center of the distribution of bills and the status quo policy. On the other hand, when the voter is located sufficiently close to the center of the bill distribution, the voter may prefer a moderate candidate when comparing two candidates that are sufficiently distant from the voter's ideal point.

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Spatial theories of voting assume that voters reduce a candidates' platform to an ideological position.[1] A voter's perception of a candidate's platform represents how the voter believes that the candidate will, if elected, affect policy outcomes.[2] Most theories of electoral competition assume that voters presume that the platform of the winning candidate will be the policy that is implemented after the election. This is a convenient and productive simplification but, of course, it *is* a simplification: few, if any, political offices in a democracy allow the officeholder to unilaterally impose his or her will by fiat. Rather, the official must work through an institutionalized process in order to have some effect on public policy.[3]

In this article, we focus on a near-ubiquitous characteristic of policymaking processes: the menu of choices from which the representative may choose is at least partially determined by actors and/or events beyond the representative's control. From legislators to executives to judges, most officials with decision-making authority spend most of their time making decisions about issues and between choices that were chosen by someone else. For example, in legislatures, the bills that will be voted on are at least partially beyond the control of most legislators. At the end of the day, such a legislator can implement his or her "platform" only through voting on bills that are not necessarily representative of the policies that he or she would implement if given unilateral authority. Both internal procedures and external events, such as disasters, force legislators to vote on options other than their most-preferred policies.[4]

[1]For discussions of spatial preferences, see Grofman (2004), Dewan and Shepsle (2011), Hinich and Munger (1992, 1996), Eguia (2013). In terms of electoral competition, this basic framework is sometimes extended to include a candidate-specific "valence" dimension (*e.g.*, Groseclose (2001), Schofield (2004), Ashworth and Bueno de Mesquita (2009), Carter and Patty (2015)).

[2]Exceptions to this include XXXXXX. The question of the degree to which a candidate can choose (or, "commit to") a given platform has been considered in depth by many scholars (*e.g.*, Osborne and Slivinsky (1996), Besley and Coate (1997), and Dhillon and Lockwood (2002)). For our purposes, it is irrelevant where platforms "come from." Rather, we are interested in how voters should evaluate and compare various platforms when the agenda is at least partially beyond the control of the candidate.

[3]The institutionally imposed divergence between goals and actions is treated very generally in Penn, Patty and Gailmard (2011) and Gailmard, Patty and Penn (2008).

[4]We mention and set to the side for future work the fact that electoral incentives within a legislature could have similar effects, to the degree that some individuals seek to stake out positions on issues through dilatory tactics or other forms of obstruction (Patty (2016)).

We explore the implications of this reality in this article. The key finding is that incorporating this exogeneity into a voter's strategic calculation about which candidates to vote for induces asymmetric preferences on the part of voters. Specifically, the voter's expectations about what decisions (*e.g.*, votes) a member will confront affect how the voter views ideological positions distinct from his or her own. We show that, in a variety of agenda-setting environments, these expectations induce a strict preference for more extreme candidates, where "extremity" is relative to the agenda: if the voter tends to be (say) to the right of the alternatives brought up, then a more extreme candidate is one whose platform is even more likely to be to the right of the alternatives brought up on the agenda.

We demonstrate that this tendency can be reversed under some circumstances. One such circumstance is when the legislative agenda is expected to be extreme relative to the status quo. In this case, voters will prefer moderate candidates: candidates whose platforms are more likely to "fall between" the status quo and the alternatives brought up on the agenda.

Our goal in this article is not to offer a theory of electoral competition: we are not attempting to explain how candidates choose the platforms they offer to voters. Rather, we are interested solely in voters' induced preferences over candidates' platforms when those candidates will be voting (or more generally, choosing between options) on the voters' behalf. With that introduction in hand, we now present our theory.

1 The Model

We denote the set of potential policies by $X \subseteq \mathbf{R}$ and denote the set of voters by $N = \{1, \ldots, n\}$.[5] A voter i is characterized by an *ideal point*, $v_i \in \mathbf{R}$, and his or her preferences are represented by a policy payoff function $u(x, v_i)$, where $u(x, v_i)$ is a quasi-concave (and hence, "single-peaked") function of x. For expository purposes (to highlight the asymmetric preferences our model induces) it is useful to assume that u is a strictly decreasing function of the distance between x and v_i, and thus that u is symmetric about the voter's ideal point.[6]

[5]We discuss generalizing the model to multidimensional policy spaces in Section 3.

[6]Our general argument that induced preferences over candidates differ from preferences over policies does not require this assumption, and our equations that calculate these induced preferences do not require

Assumption 1 *For any ideal point v and pair of policies x and y,*

$$u(x, v) > u(y, v) \Leftrightarrow |x - v| < |y - v|.$$

With Assumption 1 in hand, Figure 1 displays the voter's optimal vote choice for all pairs of bills and status quos. This figure depicts every vote a candidate could take, with each vote represented by the pair (q, b), with q on the $x-$ axis denoting a preexisting "status quo" and b on the $y-$axis denoting a "bill" that has been proposed to replace the status quo.

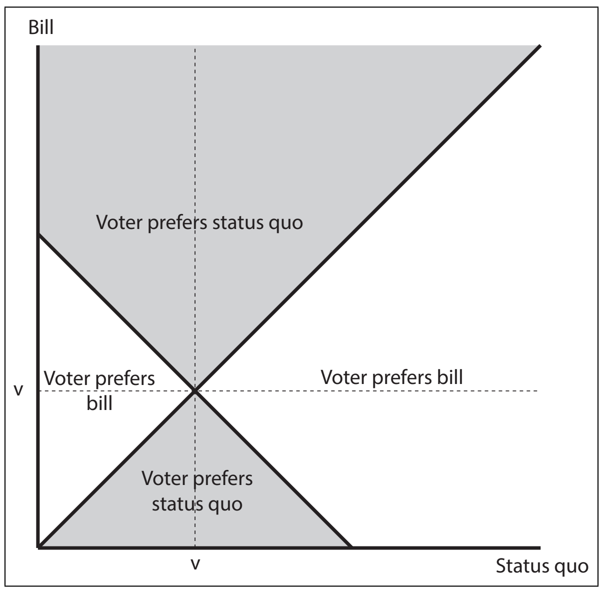

Figure 1: The Voter's Preferred Voting Behavior

Clearly when $b = q$ the voter is indifferent between the bill and status quo, because they are the same. The voter is also indifferent between b and q when $b = 2v - q$, because along this line $-|v - b| = -|v - (2v - q)| = -|v - q|$. These two lines along which the voter is indifferent between b and q are pictured in Figure 1; they intersect at the point (v, v). Above and below this point of intersection, in the shaded regions, the voter strictly prefers the status quo to the bill: $-|v - q| < -|v - b|$. To the left and right of the point of intersection the voter strictly prefers the bill to the status quo: $-|v - b| < -|v - q|$. This figure will

symmetry.

be referred to later, in order to characterize the regions along which the voter agrees or disagrees with a candidate's vote choice.

Candidates' Platforms. Any candidate, c, is characterized by his or her platform, $p_c \in X$. If elected, candidate c's platform, p_c, will determine the candidate's subsequent voting behavior as follows. For any pair consisting of a bill and status quo, (q, b), a candidate with realized platform p will vote for the bill, b, if and only if $u(b, p_c) > u(q, p_c)$. That is, any candidate c would vote as the voter would vote if the voter had ideal point equal to p_c. For any realized platform p_c, this voting behavior is represented formally by the following function:

$$V(b, q, p_c) = \begin{cases} b & \text{if } u(b, p_c) > u(q, p_c), \\ q & \text{otherwise.} \end{cases}$$

The Agenda. To capture the idea that the voter is choosing a platform to represent his or her interests in the face of an exogenous agenda, we represent the "agenda" as a probability measure, α, over X^2. In Section 2, we consider in detail two specific representations of the agenda. Prior to that, we summarize our theory of voter choice and establish a few general results.

Choosing Between Platforms. Viewed at its most general, our theory of voter choice is that the voter votes for (or selects) the candidate whose platform maximizes the following expected payoff function:

$$EU(p, v) = \int_{X^2} u(V(b, q, p), v)\mathrm{d}\alpha. \tag{1}$$

We can describe this function in some detail without specifying the agenda, α. To do so, we first introduce the notion of "disagreement sets." The disagreement set between a voter and any given candidate/platform identifies the set of bill/status quo pairs on which the candidate's vote would differ from how the voter would vote.

Disagreement Sets. For any voter i with ideal point $v_i \in X$ and for any platform $p \in X$, let

$$D(p, v_i) \equiv \{(q, b) : V(b, q, p) \neq V(b, q, v_i)\}$$

denote voter i's *disagreement set* with respect to the platform p. This region is illustrated in Figure 2. In this figure a platform p_L is depicted that is to the left of the voter's ideal point v. The regions of disagreement between voter and candidate are generated by superimposing the voter's preferred voting behavior (shown in Figure 1) on the candidate's voting behavior (also Figure 1, but with (v, v) replaced by (p_L, p_L)). For (q, b) combinations in the darker region, the voter prefers b while the candidate prefers q. For (q, b) combinations in the lighter region, the voter prefers q while the candidate prefers b. For the rest of the possible votes that could occur, the voter and candidate agree.

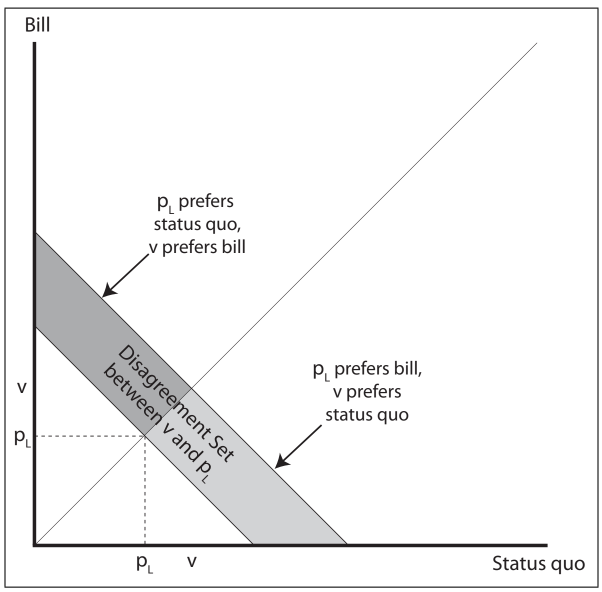

Figure 2: Voter-Candidate Disagreement Set

Our first result is that the voter's expected payoff is maximized by a representative whose platform is equal to the voter's ideal point.

Theorem 1 *For any ideal point $v_i \in X$ and any agenda α, the function $EU(p, v_i)$ is maximized at $p = v_i$.*

Proof: Fix an agenda α and an ideal point v_i. Then consider any platform $p > v_i$ (the case of $p < v_i$ is symmetric). Letting $\Delta U(v,p) \equiv EU(v,v_i) - EU(p,v_i)$, we obtain

$$\begin{aligned}\Delta U(v,p) &= \int_X \int_X u(V(b,q,v),v_i) f(b) f(q) \mathrm{d}\alpha - \int_X \int_X u(V(b,q,p),v_i) f(b) f(q) \mathrm{d}\alpha, \\ &= \int_{D(p,v_i)} \left[u(V(b,q,v),v_i) - u(V(b,q,p),v_i)\right] f(b) f(q) \mathrm{d}\alpha, \\ &\geq 0.\end{aligned}$$

Furthermore, the inequality is strict if $D(p,v_i)$ has positive measure under α. Thus, any platform other than the voter's ideal point yields an expected payoff that is no greater than the voter's expected payoff from a platform equal to his or her ideal point, as was to be shown. ∎

The next theorem extends Theorem 1 by establishing that the expected payoff function is single-plateaued.

Theorem 2 *For any ideal point $v_i \in X$ and any agenda α, the function $EU(p,v_i)$ is single plateaued: if $\underline{p} \leq p \leq v_i$ then $EU(\underline{p},v_i) \leq EU(p,v_i)$ and, if $\overline{p} \geq p \geq v_i$ then $EU(\overline{p},v_i) \leq EU(p,v_i)$.*

Proof: **Incomplete.** Let $\underline{p} < p < v_i$. We will show that every time candidates at p and $\underline{p}$ vote differently, voter i prefers the vote of the candidate at p. Suppose that (b,q) is a (bill, status quo) pair for which a candidate at p and $\underline{p}$ vote differently, with $b \neq q$.

First, if a candidate at p votes for b while $\underline{p}$ votes for q then it must be the case that $b > q$. This is because $|p-b| \leq |p-q|$ and $|\underline{p}-b| \geq |\underline{p}-q|$ can be rewritten $b^2 - 2bp + p^2 \leq q^2 - 2qp + p^2$ and $b^2 - 2b\underline{p} + \underline{p}^2 \geq q^2 - 2q\underline{p} + \underline{p}^2$. This reduces to $2b(p-\underline{p}) \geq 2q(p-(\underline{p}))$, or $b > q$. Second, if $b,q > p$ or if $b,q < \underline{p}$, then p and $\underline{p}$ will vote the same way. Therefore if they vote differently then either (a) $b \geq p \geq q \geq \underline{p}$, or (b) $p \geq b \geq q \geq \underline{p}$, or (c) $p \geq b \geq \underline{p} \geq q$. Our results are proved for the cases of (b) and (c) by the assumption that $u(x,v_i)$ is single peaked, as $v_i > b \geq q$ in these cases. For case (a) we know that $b - p < p - q$. ∎

Theorems 1 and 2 jointly establish a weak version of the "ally principle" (Bendor and Meirowitz (2004)) in this setting. Theorem 1 establishes that each voter should, if he or she can, appoint his or her ideological clone to vote on his or her behalf. Theorem 2 goes a step farther and implies that, when choosing among candidates whose platforms are all on the same side of the voter's ideal point (*i.e.* $p_c \leq v_I$ for all $c \in C$ or $p_c \geq v_i$ for all $c \in C$), then the voter should appoint the candidate whose platform is closest to his or her ideal point. These establish a "weak version" of the ally principle because they do not imply that "all else equal, a rational boss should choose her closest ally as an agent."[7] Specifically, the theorems do not address situations in which at least one candidate is offering a platform strictly larger, and another candidate is offering a platform that is strictly lower, than the voter's ideal point.

2 Specific Families of Agendas

Known Status Quo Model. The first type of agenda we consider is referred to as the *known status quo model*. In this model we suppose that the vote is between a fixed and known status quo, $q = 0$, and a bill, b, drawn according to a continuously differentiable cumulative distribution function $F : \mathbf{R} \to [0,1]$, with probability density function $f : \mathbf{R} \to \mathbf{R}$. In this case, for any set $Y = Y_b \times Y_q \subseteq \mathbf{R}^2$ where Y_b and Y_q are both Borel sets, the agenda α is defined as follows:

$$\alpha(Y) = \begin{cases} F(Y_b) & \text{if } 0 \in Y_q, \\ 0 & \text{otherwise.} \end{cases}$$

Symmetric Uncertainty Model. The second type of agenda we consider is referred to as the *symmetric uncertainty model*. In this model, we assume that the bill and status quo policies are each independently and identically distributed according to a probability density function, f, that is

- single-peaked around mode μ, and
- symmetric around μ.

[7] Bendor and Meirowitz (2004), p. 300.

In this setting, for any set $Y = Y_b \times Y_q \subseteq \mathbf{R}^2$ where Y_b and Y_q are both Borel sets, the agenda α is defined as follows:

$$\alpha(Y) = F(Y_b)F(Y_q).$$

The two models of the legislative agenda generate related, but distinct, results. The known status quo model matches the standard spatial bargaining framework utilized by political economy scholars for the past 40 years.[8] In addition to comparability with existing scholarship, this model is the most parsimonious way to consider "asymmetric" agendas. However, while the fixed status quo model reflects the standard political economy approach to the study of legislatures, it is not so clear that it is the best representation of many other political economy settings. For example, when choosing a delegate (such as a judge or bureaucrat) to adjudicate disputes that arise exogenously, it is probably better to incorporate uncertainty about both options that the appointed representative will be choosing between.[9] Furthermore, because we are assuming that the voter is purely instrumental (in the sense that he or she does not care about the labels of the alternatives), there is no gain from considering asymmetric uncertainty about (*i.e.*, different probability distributions governing) the two options.[10] Both models assume that the two alternatives are independently distributed. This assumption is not innocuous. However, space constraints prevent us from exploring its relaxation in this article.[11]

2.1 Analysis: The Known Status Quo Model

We begin by considering a setting with a known status quo policy, which we normalize to zero. Given a distribution of bills, F, the expected payoff for a voter with ideal point v

[8]For example, Romer and Rosenthal (1978), Cox and McCubbins (1993), and Krehbiel (1998).

[9]Clearly, our framework is still very restrictive in the sense that we are constraining the representative to a binary choice, but we leave this extension for future work. However, it is worth noting that allowing the representative to choose from a set of more than two randomly drawn options will reduce the asymmetry in the voters' induced preferences.

[10]Asymmetric uncertainty would matter if, ceteris paribus, the voter preferred the alternative labeled the "status quo" to that labeled the "bill" (or vice-versa).

[11]We explore the implications of an agenda in which the bill is dependent upon the realized status quo (as would be predicted in the Romer and Rosenthal (1978) "setter" model) in Patty and Penn (2017).

from a candidate with a platform equal to p is

$$EU(p,v) = \int_X u(V(b,0,p),v) f(b) \mathrm{d}b. \tag{2}$$

This is simply the voter's expected utility for a vote between status quo $q = 0$ and a bill distributed according to F, taken by a candidate with platform p. Our key results for this setting presume that the distribution of bills is single-peaked, which we define formally as follows.

Definition 1 *For any number μ, the probability density function f* is single peaked about *μ if f satisfies the following for all $b, c \in \mathbf{R}$:*

$$b < c < \mu \Rightarrow f(b) < f(c), \textit{ and}$$
$$\mu < b < c \Rightarrow f(b) > f(c).$$

Two points are worth noting: first, when f is single peaked about μ, then μ is the mode of the distribution and, second, single-peakedness of f about μ does not necessarily imply symmetry of f. Indeed, somewhat interestingly, symmetry of f does not provide much additional purchase in this model.

Extremism and Moderation. In our model, voter preferences for a candidate are determined solely by the candidate's platform and the legislative agenda. Consequently, our notions of extremism and moderation are indirect in that we define them solely in terms of platforms and agendas. In other words, we consider candidate extremism relative to the legislative agenda α, conceiving of the agenda as a proxy for political climate. The "middle" of the agenda represents something akin to moderation.

In keeping with this focus on the agenda as a indicator of extremism, we take a weak view of the concept of moderation. If, for example, the voter lies in the "center" of the agenda distribution, then it is unclear whether a vote to the left or right of the voter constitutes moderation. However, if the voter is to one side of this distribution–say, the right of both the status quo and the mode of the bill distribution–then a vote for a farther right candidate represents an extremist vote, and a vote for a farther left candidate represents a vote for moderation. This is defined formally below.

Definition 2 *In the known status quo model, a candidate A with platform p_A is* ***moderate*** *relative to a candidate B with platform p_B if $|p_A - q| \leq |p_B - q|$ and $|p_A - \mu| \leq |p_B - \mu|$, with at least one of the two inequalities strict. When this is the case, we refer to candidate B as* ***extreme*** *relative to candidate A. Otherwise, we cannot rank the candidates in terms of moderation.*

In words, a candidate (A) is more moderate than another (B) if A is both closer to the status quo *and* the mode of the bill distribution. We assume, without loss of generality, that $v > 0$. In order to consider voter tastes for extremism versus moderation, we also assume that $\mu < v$, so that both the mode of the bill distribution and the status quo lie to the left of the voter. This assumption implies that for any two candidates with platforms that are symmetric about v, the leftmost candidate is necessarily more moderate than the rightmost.

We consider preferences for more or less extreme candidates by considering the voter's preferences between pairs of distinct platforms that are equidistant from the voter's ideal point. We refer to the distance between the platforms and the voter's ideal point as the *degree of policy divergence*, δ. The platform on the right, $p_R = v + \delta$ is referred to as the "extreme" platform. The platform on the left, $p_L = v - \delta$, is referred to as the "moderate" platform. This is pictured in Figure 3.

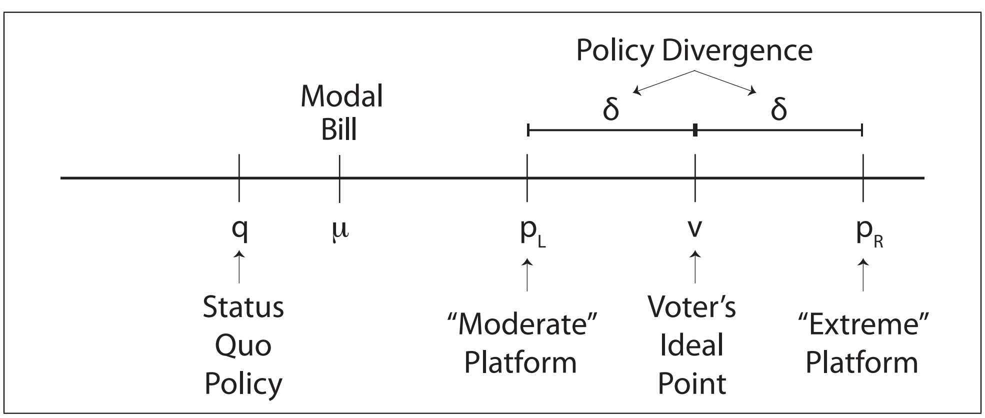

Figure 3: Policy Divergence, Extreme and Moderate Platforms

Induced Preferences Over Candidates. If we suppose that the chosen representative is decisive—that is, the alternative he or she chooses is actually implemented as policy—then our theory of voter preferences over candidates is simply the voter's expected instrumental policy payoffs induced by the candidates' platforms. However, if the voter's chosen representative influences policy only indirectly (for example, as a member voting within a larger legislature), then these preferences neglect the details of that institution's decision-making processes.

In such situations, one can interpret our theory in either of two ways, depending on one's tastes. The simplest interpretation is that these preferences reflect the "expressive" motivations of the voter.[12] Within an "expressive" interpretation, our theory interprets the voter's motivations as being simply to support candidates whose voting behavior would maximize the voter's payoffs if those votes were converted into actual policy, regardless of whether or how those votes actually influence the implemented policy. Accordingly, a second, *instrumental*, interpretation of our theory of voter preferences is available by supposing that there is a some positive probability that the elected representative's vote choice will be decisive. Such an interpretation can be derived from a larger model of (for example) probabilistic voting within a legislature.[13] A technical issue with such an interpretation regards whether this probability of the elected legislator being decisive is invariant to the elected legislator's platform. Unfortunately, a detailed consideration of this interesting issue is beyond the scope of this article. However, it is clear that if one does not assume that there is some positive probability of one's representative's vote being decisive, then an instrumental voter would be indifferent between all platforms, which is at odds with both intuition and the lengthy empirical literature on voter behavior.[14]

[12]For example, Buchanan (1954), Tullock (1971), Brennan and Buchanan (1984), Brennan and Lomasky (1993), Brennan and Hamlin (1998, 2000), Schuessler (2000), Hillman (2010), and Hamlin and Jennings (2011).

[13]There are multiple possible sources for such probabilistic voting, including uncertainty about the platforms of the legislators elected from other districts and/or the preferences of the voters electing the legislators from other districts.

[14]Among *many* others, see Tomz and Van Houweling (2008), Jessee (2012), and Montagnes and Rogowski (2015).

Asymmetric Preferences. The focus of this article is on asymmetries in voter preferences over representatives' platforms. Specifically, for any given divergence from the voter's ideal point, $\delta > 0$, when does the voter strictly prefer the extremist candidate, with platform $p_R = v + \delta$, to the moderate candidate, with platform $p_L = v - \delta$ (or vice versa)? We denote the voter's net expected payoff from the extremist candidate, $p_R = v + \delta$, relative to that from the moderate candidate, $p_L = v - \delta$, by:

$$\Delta(\delta, v) \equiv EU(p_R, v) - EU(p_L, v).$$

To begin, note that there is a single interval of bills on which the two candidates will vote differently from each other. When $b < 2p_L$ then both the left and right candidate (and the voter) prefer the status quo $q = 0$ to b. Similarly, when $b > 2p_R$ then both candidates and the voter prefer the status quo to the bill. We can therefore restrict attention to the interval $[2p_L, 2p_R]$; for any bill drawn from this interval the preferences of the two candidates diverge. We call this interval the *disagreement region*. Figures 4, 5, and 6 show this region for the following three cases: $p_L \in [\frac{v}{2}, v]$, $p_L \in [0, \frac{v}{2}]$, and $p_L < 0$.

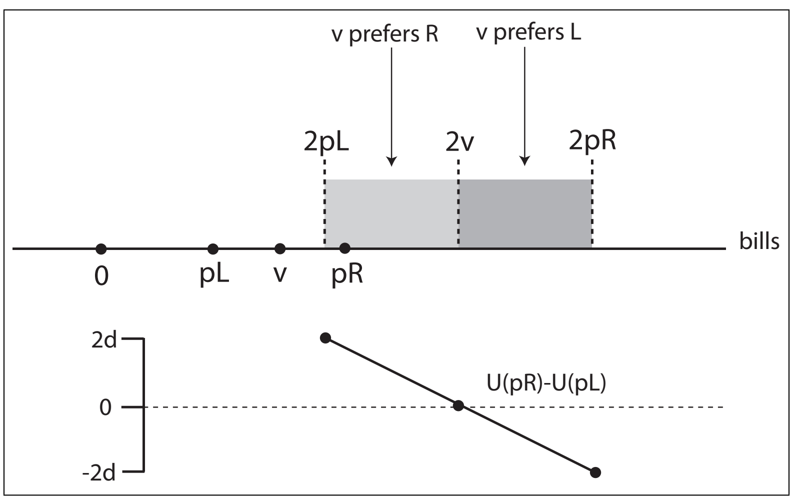

Figure 4: Disagreement region when $p_L \in [\frac{v}{2}, v]$

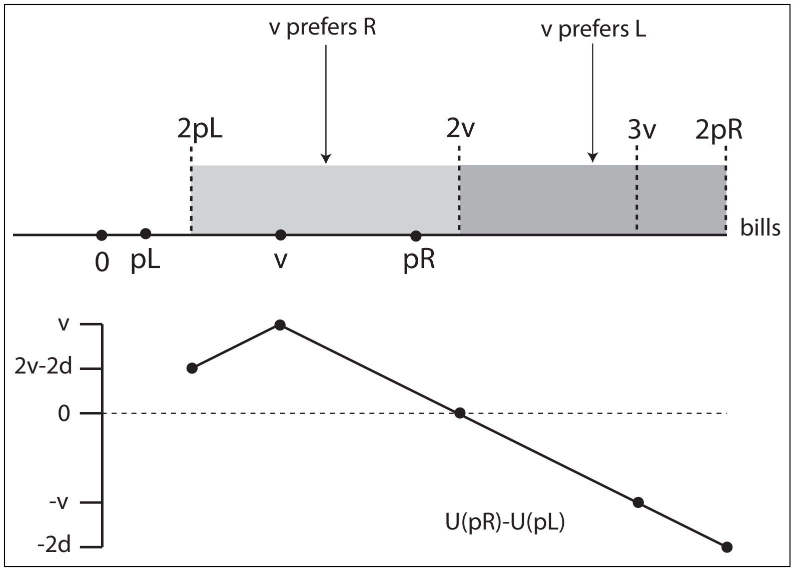

Figure 5: Disagreement region when $p_L \in [0, \frac{v}{2}]$

When $p_L \geq 0$ (Figures 4 and 5) the left candidate will always vote for status quo $q = 0$ and the right candidate will always vote for bill b on the entire disagreement region. The voter prefers the bill on $[2p_L, 2v]$ and prefers the status quo on $[2v, 2p_R]$. What distinguishes these figures is the voter's expected utility calculation for the right candidate over the left. When the bill distribution extends below v, as in Figure 5, the difference between a vote for p_R over p_L begins to decrease. This difference is always positive on the interval $b \in [2p_L, 2v]$, but the magnitude of the difference gets smaller for smaller bills; when $b = 0$ the voter is indifferent between the two candidates (because the bill *equals* the status quo, so the candidates cannot be distinguished by their votes).

When $p_L < 0$ candidate behavior and voter preferences change slightly; in this case the left candidate will vote for status quo $q = 0$ and the right candidate will vote for bill b when $b \in [0, 2p_R]$. When $b \in [2p_L, 0]$ then the left candidate will vote for the bill and the right candidate will vote for the status quo. On this disagreement region the voter prefers the bill on $[0, 2v]$ and prefers the status quo on $[2p_L, 0] \cup [2v, 2p_R]$. In this case, voter preference for the right candidate over the left gets smaller for smaller bills until $b = 0$; then

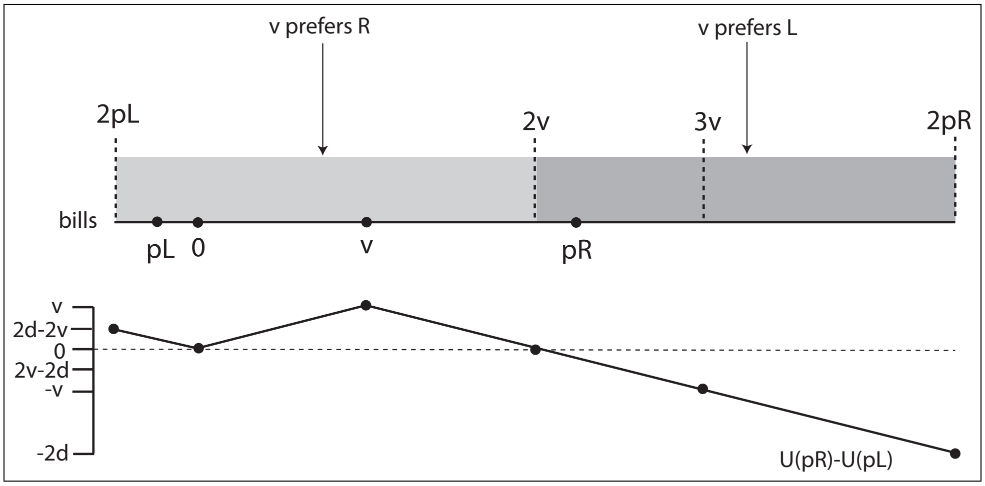

Figure 6: Disagreement region when $p_L \leq 0$

this magnitude starts to rise as bills move left, past zero.

To evaluate voter taste for extremism over moderation, first note that when $v \geq p_L > \frac{v}{2}$ the disagreement region region lies to the right of v; in this case, $b \geq v$ for all b in the disagreement region. Thus,

$$\Delta(\delta, v) = \int_{2p_L}^{2p_R} \left(-(b-v)+v\right) f(b)db,$$

or

$$\Delta(\delta, v) = \int_{2p_L-q}^{2p_R-q} (2v-b)f(b)db. \quad (3)$$

If $0 \leq p_L \leq \frac{v}{2}$ then the voter's net expected payoff from the extremist candidate changes to:

$$\Delta(\delta, v) = \int_{2p_L}^{v} bf(b)db + \int_{v}^{2p_R} (2v-b)f(b)db. \quad (4)$$

Finally, if $p_L \leq 0$ it changes to:

$$\Delta(\delta, v) = \int_{2p_L}^{0} -bf(b)db + \int_{2p_L}^{v} bf(b)db + \int_{v}^{2p_R} (2v - b)f(b)db. \tag{5}$$

When $p_L \in [\frac{v}{2}, v]$ (Figure 4) the voter always has a weak taste for extremism; he always prefers R to L. Intuitively, this is because the mode of the bill distribution is to the left of v, and so more likelihood is placed on bills arising from the left side of the disagreement region, where the voter prefers p_R. Additionally, $\Delta(\delta, v)$ is symmetric on the region overall, and so the increased likelihood of a bill being drawn from $[2p_L, 2v]$ leads to a preference for the right candidate over the left.

Proposition 1 *When $\delta \leq \frac{v}{2}$ (i.e. for small enough policy divergence) the voter always prefers the extremist candidate to the moderate candidate.*

Proof: The voter prefers the right candidate over the left if and only if

$$-(E(b|b \in [2p_L, 2p_R] - v) \geq -v,$$

which we can rewrite as

$$2v \geq E(b|b \in [2p_L, 2p_R]).$$

By the definition of single-peakedness of f and the assumption that $\mu < v \leq 2p_L$, we have that $E[b|b \in [2p_L, 2p_R]] \leq \frac{2p_L + 2p_R}{2} = 2v$. Thus, the voter prefers the right candidate over the left for small enough policy divergence between the two candidates (i.e. for $\delta \leq \frac{v}{2}$). □

Proposition 1 can be used to derive sufficient conditions for extremism when $\delta \geq \frac{v}{2}$.

Corollary 1 *When $\delta \in [\frac{v}{2}, v]$ a sufficient condition for the voter to prefer the right candidate to the left is*

$$2v \geq E(b|b \in [3v, 2p_R]) - E(b|b \in [2p_L, v]) \left(\frac{F(v) - F(2p_L)}{F(2p_R) - F(3v)} \right).$$

Proof: If $\delta \in [\frac{v}{2}, v]$ the voter's net expected payoff from R over L is given by Equation 4, which defines $\Delta(\delta, v) = \int_{2p_L}^{v} bf(b)db + \int_{v}^{2p_R}(2v - b)f(b)db$. We can rewrite

$$\Delta(\delta, v) = E(b|b \in [2p_L, v])(F(v) - F(2p_L)) + (2v - E(b|b \in [v, 3v]))(F(3v) - F(v)) \\ +(2v - E(b|b \in [3v, 2p_R]))(F(2p_R) - F(3v)).$$

By the same logic as in Proposition 1, we know that $(2v - E(b|b \in [v, 3v]))(F(3v) - F(v)) \geq 0$, as $E(b|b \in [v, 3v]) \geq 2v$. The corollary follows immediately. □

Finally, for the case of $\delta \geq v$ we get the next corollary. The proof is similar to the proof of Corollary 1, but utilizes Equation 5 instead of Equation 4. We omit the proof.

Corollary 2 *When $\delta \geq v$ a sufficient condition for the voter to prefer the right candidate to the left is*

$$2v \geq E(b|b \in [3v, 2p_R]) - E(|b||b \in [2p_L, v])\left(\frac{F(v) - F(2p_L)}{F(2p_R) - F(3v)}\right).$$

Uniformly distributed bills. The uniform distribution on bills represents a conservative test for extremist tastes in our model; this is because we assume that the expected value of a bill is to the left of the voter, and because the voter prefers the moderate candidate to the extreme candidate for bills that fall to the right of $2v$. With a large enough support, the uniform distribution doesn't penalize extreme-right bills in terms of the likelihood they will arise. Thus, a uniform distribution of bills whose support contains the entire disagreement region represents a "best case" scenario for moderation, and we can show that in this case the voter always has a weak taste for moderation. In this best case scenario, when $\delta \leq \frac{v}{2}$ then $\Delta(\delta, v)$ equals zero; the voter is indifferent between the moderate and extreme candidates. The condition described in Corollary 1—the sufficient condition for extremism—is also a necessary condition for extremism in this scenario. This condition reduces to $2v \geq 2\delta + v$, which never holds when $\delta > \frac{v}{2}$. Similarly, the sufficient condition outlined in Corollary

2 is also a necessary condition when F is uniform and its support contains the entire disagreement region. In this case the condition requires $2v \geq \frac{4\delta^2}{2\delta - v}$, which never holds when $\delta > v$.

The best case scenario for moderation described above requires that the distribution of bills spans an interval with length at least $2v + 4\delta$, since the distribution of bills spans the entire disagreement region and $b < v$. The geometry of the disagreement region gives us a similar *best case scenario* for extremism; if the distribution of bills does not extend past the point $3v$ then the voter will always have a weak preference for extremism, with this preference being strict so long as the probability that a bill on the interval $[2p_L, v)$ has a strictly positive likelihood of being drawn (and, implicitly, so long as $2p_L < v$). The following proposition formalizes this result, and provides a necessary and sufficient condition for extremist preferences when F is uniform.

Proposition 2 *Let bills be drawn from a uniform distribution with support $[v - \kappa - \gamma, v - \kappa + \gamma]$. Thus, the distribution of bills is centered at $v - \kappa$ and the variance of the distribution is $\frac{1}{3}\gamma^2$. When when $v > \frac{\gamma - \kappa}{2}$, the voter has a weak preference for extremism. When*

$$2\delta + \gamma - \kappa > v > \gamma - \kappa \tag{6}$$

this preference for extremism is strict.

Proof: The result follows immediately from the definition of the disagreement region. Over the region $[v, 3v]$, the net utility for R over L (or $\Delta(\delta, v)$) cancels to zero. Thus, if a voter has a preference for the moderate candidate over the extreme, it is because the bills drawn from $[3v, 2p_R]$ (a region on which the voter prefers the moderate candidate to the extreme) outweigh the bills drawn from $[2p_L, v]$ (a region on which the voter prefers the extreme candidate). However, if $3v > v - \kappa + \gamma$, no bill on $[3v, 2p_R]$ can arise with positive probability. Thus, the voter has a weak preference for the extreme candidate in this case. □

Equation 6 leads to the following conclusions for the case where bills are drawn from a uniform distribution. First, if v is sufficiently extreme relative to the median of the bill distribution (in particular, if $v > \frac{\gamma-\kappa}{2}$,[15] or phrased differently, if $\mu + \gamma < 3v$) then the voter can never strictly prefer a moderate candidate to an extreme candidate. Moreover, in this case, increasing δ (the spread between the two candidates) will eventually lead the voter to strictly prefer the extreme candidate to the moderate candidate.

Second, when $\gamma < \kappa$ the voter always has a weak taste for extremism. This is because $v > 0$ by assumption, and so if $\gamma - \kappa < 0$ then $v > \frac{\gamma-\kappa}{2}$. This implies that when the support of the bill distribution $f(b)$ is sufficiently small, the voter will weakly prefer the extremist candidate.

Finally, unlike previous results focusing on voter preference for extremism when the difference between the candidates is small (e.g. Proposition 1), large δ will always correspond to the voter preferring the extreme candidate when the voter himself is extreme relative to the bill mode (greater κ) and when the variance of the bill distribution is small (smaller γ). This is because the disagreement region is centered on $2v$ regardless of the size of δ. If the distribution of bills never crosses $3v$ then an increase in δ simply expands the region on which the voter disagrees with the moderate candidate.

2.2 Analysis: The Symmetric Uncertainty Model

We now turn to the symmetric uncertainty model. In this setting, given a probability density function, f, the expected payoff for a voter with ideal point v from a candidate with a platform equal to p is given by the following.

$$EU(p, v) = \int_X \int_X u(V(b, q, p), v) f(b) f(q) \mathrm{d}b \mathrm{d}q. \tag{7}$$

We first provide some intuition behind our results by considering a baseline case, the "Symmetric Uniform Model," in which the bill and status quo are each independently distributed according to the Uniform$[0, 1]$ distribution.[16] In this setting, if $v_i < 1/2$ and $\delta > 0$,

[15] In the general symmetric uncertainty model, the mode and the median are identical by assumption.

[16] Formally, letting λ denote Lebesgue measure on $\mathbf{R}$, then for every Lebesgue measurable subset $Y \subseteq X$,

$$\mathcal{B}(Y) = \mathcal{Q}(Y) = \lambda(Y \cap [0, 1]).$$

then the voter always strictly prefers candidate L to candidate R. Figure 7 illustrates this. Specifically, it highlights a voter's payoffs from two example platforms, p_L and p_R, equidistant from the voter's ideal point. Because the ideal point, v_i, is less than $^1/_2$ (specifically, $v = 0.35$), the platform of candidate R, p_R, represents the "moderate" platform and that of candidate L, p_L, represents the "extreme" platform. The voter receives a higher expected payoff from the extreme platform than he or she would receive from the moderate platform.

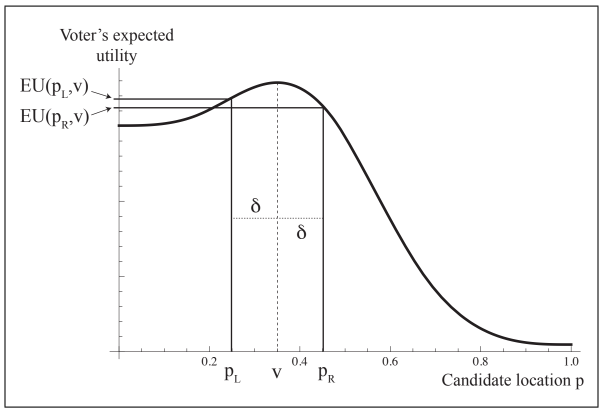

Figure 7: Expected Payoffs In the Symmetric Uniform Model

The preference for extreme platforms illustrated in Figure 7 is a general conclusion in the Symmetric Uniform Model, as we state formally in the next proposition.

Proposition 3 *Suppose that α is the Symmetric Uniform Model. Then, for any ideal point v_i and any $\delta > 0$, the voter receives a weakly higher expected payoff from the extreme platform than from the moderate one:*

$$v_i \leq {}^1/_2 \quad \Leftrightarrow \quad EU(v_i - \delta, v_i) \geq EU(v_i + \delta, v_i),$$
$$v_i \geq {}^1/_2 \quad \Leftrightarrow \quad EU(v_i + \delta, v_i) \geq EU(v_i - \delta, v_i).$$

Furthermore, the inequality is strict if and only if $v_i \neq {}^1/_2$.

The logic behind this result is illustrated in Figure 8. The two dark triangles represents (b, q) pairs in which the moderate platform will vote against the voter's interests and for which there is no analogous pair in which the extremist would vote against the voter's interests. That is, for every point in the voter's disagreement region with respect to the extremist candidate's platform, a unique point with exactly the same disutility for the voter exists in his or her disagreement region with the moderate candidate's platform, but the converse does not hold whenever $v \neq 1/2$ Because every (b, q) pair is equally likely, this establishes the conclusion of Proposition 3.

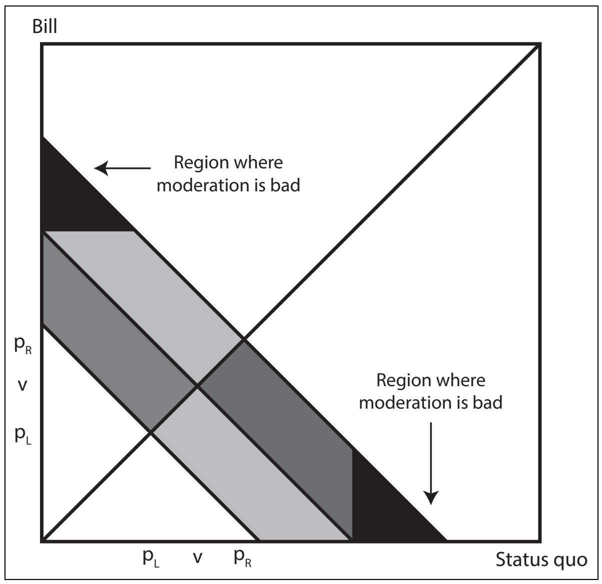

Figure 8: Why Extremists Are Preferred in Symmetric Uniform Model

As displayed in Figure 8, the logic of the Symmetric Uniform Model extends to all single-peaked, symmetric distributions.

Theorem 3 *In the Symmetric Uncertainty Model, for any probability density function f that is single-peaked and symmetric about μ, it follows that for all $\delta > 0$ and any $v \geq \mu$,*

$$EU(v + \delta) \geq EU(v - \delta).$$

Furthermore, the inequality is strict if and only if $v \neq \mu$.

Sketch.: Fix a value $\mu \in \mathbf{R}$ and a probability density function, f, that is single-peaked and symmetric around mode μ. Consider any line segment like the dotted 45° line pictured in Figure 9. All points long such a line represent equal "disutility" to the voter in terms of the incorrect votes (falling in the light (in this case, moderate candidate) and dark (in this case, extreme candidate) disagreement regions.

The dotted circles represent the iso-density curves for α. The key to the result is that the line integral over the light gray region along any such 45° line will be strictly larger in absolute magnitude than for the corresponding dark gray region. This implies that the moderate candidate yields a lower expected payoff to the voter than the extreme candidate. Note that both single-peakedness *and* symmetry of f are key to this result. ∎

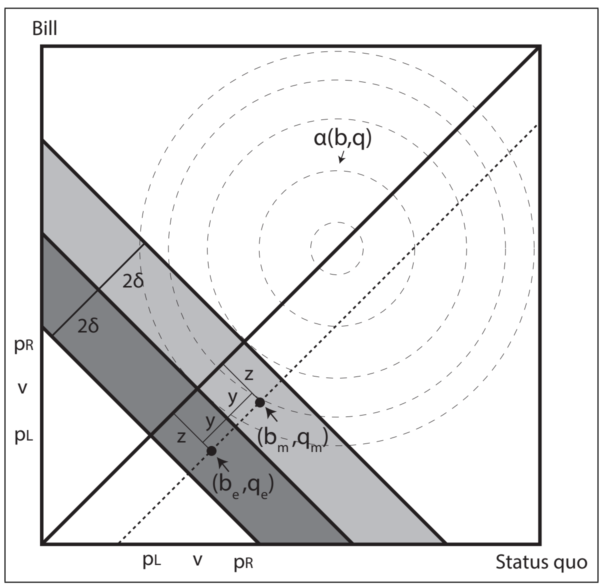

Figure 9: Why Extremists Are Preferred in Symmetric Uncertainty Model

3 Extensions and Robustness

Multidimensional Policy Space. It is straightforward to generalize our theory to allow for multiple dimensions. For reasons of space, we briefly consider only the case of the naive agenda model. Let $X = \mathbf{R}^M$ denote the policy space, for $M \geq 2$. Each voter i's ideal point is then a point $v_i \in X$ and the naive agenda process is represented by the uniform distribution on $[0,1]^{2M}$, where the first M components represent the bill, b, and the second M components represent the status quo, q. Extending the theory to multiple dimensions allows for a richer set of possible collective choice mechanisms. At one extreme of such mechanisms is a "closed rule" variant in which a representative must vote for either b or q and, at the other extreme is a "dimension-by-dimension" rule in which the representative casts M votes, one for each of the dimensions of the comparison.

The Voter's Payoff Function. As presented in this article, the voter's payoff function is based on the votes cast by the representative, rather than outcomes actually implemented. At first, this seems at odds with consequentialism and therefore incompatible with many standard conceptions of "rational" decision-making. This divergence is only in appearance, however, because the voter is unable to affect anything except choose which platform he or she sends to the legislature to cast votes on his or her behalf. Introducing a more explicitly consequentialist account would lead to considering only those cases in which the vote cast by the representative is decisive (or "pivotal").

Independence of the Legislative Agenda. We have assumed that the legislative agenda is entirely independent of the voter's choice of representative. This is obviously a strong assumption. However, it is straightforward to see that the qualitative characteristics of our conclusions will remain true if this independence is relaxed somewhat. For example, suppose that the representative will set policy equal to his or her platform with probability $\phi \in (0,1)$ and vote on an exogenous pair of policies as assumed in the article with probability $1 - \phi$. In such a model, it is clear that the voter's expected payoff function will converge to his or her policy payoff function, u, as $\phi \to 1$.

4 Conclusion

We have presented a theory of voting for representatives. The key finding is that, when candidates' platforms represent how they will vote over an exogenous agenda, the voter's preferences over these platforms will not generally be the same as his or her preferences over those platforms if they represented the policy that would be implemented by the candidate. Once recognized, this finding is intuitive but, to our knowledge, relatively unaccounted for in theoretical and empirical investigations of voting. We find that, when the uncertainty about the alternatives to be chosen from is single-peaked and strongly symmetric (in the sense that the two alternatives are iid and the marginal distribution of either alternative is single-peaked and symmetric about its mode), then voters will *always* have a preference for a more extreme candidate over a more moderate one, holding the degree of divergence from the voter's ideal platform constant.

When the status quo is known *a priori*, then almost all voters will always have a preference for extreme candidates when the degree of divergence between the candidates is low or when the voter is sufficiently distant from both the center of the distribution of bills and the status quo policy. Otherwise, the voter in some cases may have a preference for the moderate candidate. Specifically and intuitively, when the voter is located sufficiently close to the center of the bill distribution in the sense of there being a sufficiently high probability of bills being proposed that fall on the opposite side of the voter's ideal point from the status quo, the voter may prefer a moderate candidate when comparing two candidates that are sufficiently distant from the voter's ideal point.

This somewhat convoluted conclusion is interesting when considered in substantive terms. In particular, the voters who, on the margin, might prefer moderate candidates, are those who are likely to be "in the middle" of, and face non-trivial trade-offs regarding, the comparisons that will confront their representative. In ongoing work, we are exploring how various models of strategic (and non-strategic) candidacy and partisan motivations interact with the voter incentives we have identified in this article.

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