Politics and Administration*

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Abstract

This paper develops a theory of the effectiveness of government programs. In the model, a bureaucrat chooses a mechanism for assigning a good (such as a license, or a benefit payment) to a client of uncertain qualifications. The mechanism uses a means test to verify the client's eligibility. A politician who values the payoffs of a subset of client types exercises oversight by designating the resources available for means testing and the population of clients that can be served. The model makes predictions about common administrative pathologies, including inefficient and politicized distribution of resources, program errors, and backlogs. When the politician favors marginally qualified clients, programs will tend to be universal, with low per capita spending and high error rates. When the politician favors highly qualified clients, programs have smaller client populations, higher per capita spending, and lower error rates. Notably, the bureaucrat can only use discriminatory testing in the latter case, and programs with larger budgets do not spend more per client.

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1 Introduction

What determines the quality of government administration? In both developing and advanced countries, the ability of the bureaucracy to deliver on stated policy goals is considered a key component of the overall quality of governance. Accordingly, there are today many efforts to measure both national and cross-national government performance. In the United States, several states and every recent presidential administration have implemented performance measurement initiatives.\(^1\) Internationally, interest groups and non-government organizations have compiled numerous well-known measures of government performance.\(^2\)

In addition to the data collection efforts, a variety of theoretical perspectives are relevant to this question. Perhaps the predominant theme throughout this work is the political principal’s trade-off between ideological outcomes and delegation that would exploit some (possibly endogenous) capability possessed by the agency. This capability might be policy expertise (e.g., Epstein and O’Halloran 1994, Huber and Shipan 2002), the ability to achieve policy outcomes with precision (Huber and McCarty 2004), or valence (Ting 2011, Hirsch and Shotts 2012). Other models disaggregate the bureaucracy somewhat by focusing on the incentives and abilities of government personnel, particularly in the presence of civil service rules (Horn 1995, Rauch 1995, Gailmard and Patty 2007).

This paper takes a different approach and develops a simple theory of public administration “on the ground.” Instead of an ideological policy space, the model focuses directly on outcomes such as the misallocation of resources, Type I and Type II errors, politicized bureaucratic decision-making, backlogs, inflexible service, and resource constraints. These are common results in almost any setting where bureaucrats must make costly judgments that affect the payoffs of citizens or clients. Prominent examples include public assistance grants by a welfare agency, indictments by a prosecutor’s office, residency applications by an immigration agency, tax-exempt status by a tax agency, or research grants from a scientific funding agency.

The set of common administrative pathologies suggests four important features for a model. First, there is adverse selection: bureaucratic allocation problems arise because it

\(^{1}\)The 1993 Government Performance and Results Act requires agencies to state objectives, develop performance metrics, and report performance in a standardized fashion. This law was significantly amended in 2010. The Bush administration developed the Performance Assessment Rating Tool in 2002 (since discontinued) to analyze the execution of individual programs.

is not clear which clients are most deserving of some benefit. Second, bureaucrats have expertise, and thereby have some ability to discern the appropriateness of an allocation. Third, bureaucratic resources for using expertise are endogenous. This is the main source of a political principal’s control over the bureaucrat. Fourth, the principal has political preferences that do not reflect social welfare maximization. A natural approach for this setting is to consider the bureaucrat as a mechanism designer, with a principal who can set a limited number of parameters of the mechanism to suit his interests.

The theory shares a number of features with and is inspired in part by Banerjee’s (1997) seminal article on government corruption. In it, a bureaucrat designs a mechanism or screen to allocate scarce slots to a population of clients with private information about their valuation of a slot. The mechanism includes the bureaucrat’s price to each type, as well as “red tape” that is costly to both the bureaucrat and the client. A politician oversees the bureaucrat by punishing her when a mechanism is found to be improper, for example due to excessively high prices (i.e., bribes). By contrast, my model does not incorporate corruption or any explicit notion of red tape, and is therefore perhaps most applicable to societies where efficiency rather than corruption is the main concern about bureaucratic performance.\(^3\)

More specifically, in the model a large number of clients apply in sequence to a bureaucrat to receive a good. Each client has a private type, which determines both her valuation of the good and the effect of a means test.\(^4\) The test embodies the legal and procedural requirements for receiving the good, and thus the client receives the good if and only if she passes. For some types, a higher level of testing is more likely to produce a passing result. The remaining types are are less likely to pass as testing increases, and also find testing to be costly. I refer to these sets of types as having *increasing* and *decreasing valuations*, respectively. This dichotomy roughly represents a distinction between types that are highly and marginally qualified for the good in question. A bureaucrat administers the allocation of the good by committing to a screen that assigns announced client types to an examination level, subject to a budget constraint. The bureaucrat is motivated by career concerns and therefore wishes to make “correct” allocations. This entails passing types with increasing valuations and failing types with decreasing valuations, weighted by the distribution of types in the population and idiosyncratic preferences.

A political principal oversees this procedure. His role is to choose the bureaucrat’s total

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\(^3\)In many settings, of course, the practice of charging different prices to different clients is illegal.

testing budget and client population before she designs the screening mechanism. The bu-
reaucrat cannot serve any clients beyond the number designated by the principal. Thus, for
example, the principal could grant a high budget but mandate low enrollment. This would
force the bureaucrat to spend a high average per capita amount on testing, but would also
create shortages due to a large population of unserved clients. The budget and population
are crude instruments because the principal cannot steer resources toward particular types.
However, these parameters do reduce the bureaucrat’s discretion to a choice over the distri-
bution of testing levels across client types. The principal has distributive motivations and
wishes to maximize the payoff of one type in society while minimizing overall testing costs.

The model makes several predictions about how the politician and bureaucrat generate
trade-offs between breadth and quality of service. A first intuition is that the bureaucrat’s
ability to discriminate across types will be highly limited. This follows from a crucial dif-
fERENCE between the model here and more standard screening models. In the latter, the
screen designer can impose side payments on certain types in order to give incentives for
agents to truthfully reveal their information. In the public sector context, these payments
can be interpreted either as corruption or bureaucratic “red tape.” Here, the bureaucrat
would at a first best solution like to discriminate across types by focusing testing on types
whose probability of passage are most sensitive to testing, as well as those for which she
has the highest idiosyncratic payoff weights. However, she cannot impose transfers, and so
truthful revelation requires that similar types receive similar testing levels. In particular, all
types with increasing valuations must be tested at one level, while all types with decreasing
valuations must be tested at a level that is no higher. Any other configuration would give
an incentive for some client type to misrepresent her type.

The principal faces three possibilities in this environment. First, if his preferred client
type has decreasing valuations, then the simple solution is to starve the bureaucrat and
mandate broad service: members of that client type collectively benefit from wide eligibil-
ity, maximized acceptance probabilities, and minimized testing costs. This system creates
widespread bureaucratic errors. Second, if his preferred client type has increasing valuations,
then the bureaucrat may be forced to test all types identically, because she is unable to test
types with decreasing valuations at a higher level than those with increasing valuations.
This generates a high per capita cost that reduces the program’s client population. Third,
if the principal’s preferred client type has increasing valuations and the bureaucrat can dis-
criminate by testing types with increasing valuations at a higher level, then the principal
sets a larger budget in order to reach a larger population. Discrimination results in lower
per capita spending. This in turn causes the bureaucrat to commit more Type I errors, as marginally qualified types (i.e., those with decreasing valuations) become more likely to receive the good.

Taken together, the results suggest two broad patterns in program implementation that may be amenable to empirical investigation. First, distributive preferences matter. A principal who is a proponent of a marginally qualified type or group of citizens will tend toward broad but administratively shallow programs with high error rates. A principal who instead is a proponent of a highly qualified type will favor smaller but more capable programs. Second, within (and only within) the latter category, bureaucratic preferences and the examination technology are important. A bureaucrat who is more inclined toward rewarding highly qualified types (i.e., those with increasing valuations) will induce a “bigger” program than one who is more inclined toward punishing marginally qualified types. In this environment, two somewhat counter-intuitive implications emerge: testing discrimination is associated with higher error rates, and higher budgets are associated with higher populations but lower per capita spending.

The paper joins a number of literatures related to the administration of government policy. As noted, there are models of government corruption and red tape that use a similar mechanism design or screening technology (e.g., Laffont and N’Guessan 1999, Guriev 2004, Banerjee, Hanna, and Mullainathan 2012). Baron (2000) develops a screening model of legislative committees, and Gailmard (2009) examines the bureaucratic oversight in a mechanism design setting. Other models that consider errors by a bureaucrat who assesses client “applications,” but in a game theoretic setting, include Prendergast (2003) and Carpenter and Ting (2007).

More generally, theoretical and empirical studies of government quality have focused heavily on corruption (Shleifer and Vishny 1993, Rose-Ackerman 1999, Svensson 2005, Bandiera, Prat, and Valletti 2009). However, a number of prominent empirical papers have used broader notions of administrative quality as either a dependent or independent variable (e.g., Knack and Keefer 1995, La Porta et al. 1999, Rauch and Evans 2000). In the American context, numerous studies have examined the links between administrative quality and political control of the bureaucracy (e.g., Moe 1989, Derthick 1990, Lewis 2008, Moynihan, Herd, and Harvey 2014).

The paper proceeds as follows. The next section describes the model. Section 3 derives the results and discusses some implications. Section 4 develops extensions of the model that explore minimum or maximum testing standards and goods that are costly to the principal.
Section 5 concludes.

2 Model

The model is a simple mechanism by which bureaucrats allocate a good to clients, under the supervision of a political principal. There are three types of players; a politician or principal, a bureaucrat, and clients or citizens.

There is large population of $N$ potential clients. Each has a private type drawn i.i.d. from the set $\Theta$, where $|\Theta|$ is finite and $|\Theta| \geq 2$. Elements of $\Theta$ satisfy $\theta_1 > 1$ and $\theta_i < \theta_{i+1}$ for all $i$. The type is the valuation that the client places on a good whose allocation is controlled by a bureaucrat. Let $\pi_i$ denote the probability that a client is of type $\theta_i$.

The bureaucrat uses a means test to determine whether to allocate the good to the client. For a given type, the testing level $t \geq 1$ can be interpreted as the per capita effort expended on the test. Testing generates a binary result corresponding to “fail” and “pass,” where $\phi(t; \theta_i)$ is the probability of passage. The good is allocated if and only if the client passes. The probability of passing is either increasing and concave or decreasing and convex, as follows:

$$
\phi(t; \theta_i) = \begin{cases} 
1 - \frac{1}{c(\theta_i) t^\alpha} & \text{if } \theta_i \in \Theta^h \\
\frac{1}{c(\theta_i) t^\alpha} & \text{if } \theta_i \in \Theta^l
\end{cases}
$$  

(1)

The sets $\Theta^h$ and $\Theta^l$ partition $\Theta$. In the interesting case of the model both subsets are non-empty, so that different types “disagree” on whether they benefit from more testing. The parameter $\alpha > 0$ is a measure of bureaucratic expertise. Higher values of $\alpha$ will generate larger increases in the probability of acceptance (respectively, rejection) for types in $\Theta^h$ (respectively, $\Theta^l$) for low values of $t$. The parameter $c(\theta_i) \in [1, \theta_i)$ can the considered a technological parameter, with higher values increasing the “default” probability of acceptance (respectively, rejection) at $t = 1$ for types in $\Theta^h$ (respectively, $\Theta^l$). For example, at $t = 1$ types in $\Theta^h$ fail with certainty and types in $\Theta^l$ pass with certainty when $c(\theta_i) = 1$, and all types pass with probability $1/2$ if $c(\theta_i) = 2$. This functional form eliminates many (but not all) corner solutions.

The bureaucrat maximizes her weighted ability to deliver the proper benefit to each type. Let the weight for type $\theta_i$ be denoted $w_i$, where for example $w_1 < 0$ and $w_2 > 0$ implies that the bureaucrat would like the benefit to go only to type $\theta_2$. Assume further that $w_i < 0$ if and only if $\theta_i \in \Theta^l$. This assumption represents the bureau’s authority and expertise in designing its screening technology: for “qualified” and “marginal” types (i.e., $\Theta^h$ and $\Theta^l$)
it can fashion a testing methodology that make the desired outcome more and less likely, respectively. Note that \( \phi(\cdot) \) then gives identical marginal returns to investigation for types \( \theta_i \) and \( \theta_j \) when \( w_i = -w_j \); thus, \( |w_i| \) serves as a measure for the extent to which the bureaucrat is interested in investigating a type-\( \theta_i \) client.

The bureaucrat chooses a direct mechanism or screen \((t(\theta_i))\) that treats clients at level \( t(\theta_i) \) for a report of type \( \theta_i \). Clients “arrive” at the bureaucrat in i.i.d. fashion, and so the probability of a type-\( \theta_i \) client is always simply \( \pi_i \). Given truthful reporting, the bureaucrat’s objective is then:

\[
u_b(t(\theta_1), \ldots, t(\theta_{|\Theta|})) = \sum_i \pi_i w_i \phi(t(\theta_i); \theta_i)\tag{2}\]

The politician specifies the pair \((p,T)\) for the bureaucrat prior to her mechanism choice. The parameter \( p \) \((0 \leq p \leq N)\) is the size of the population that the bureaucrat is mandated to serve through means tests and allocations.\(^5\) The parameter \( T \geq p \) is the bureaucrat’s budget, which constrains the ex ante number of clients the bureaucrat can test, as follows:

\[
p \sum_i \pi_i t(\theta_i) \leq T.\tag{3}\]

Thus, the bureaucrat’s expected service cost is linear in \( p \), and each client “costs” \( t(\theta_i) \). Since the bureaucrat’s objective (2) is increasing in all \( t(\theta_i) \), this constraint must bind.

Clients care about receiving the allocation as well as testing levels. In particular, testing at a level \( t \) imposes a cost \( k(\theta_i)t \). I assume that \( k(\theta_i) = 0 \) for \( \theta_i \in \Theta^\ell \), while \( k(\theta_i) > 0 \) for \( \theta_i \in \Theta^h \). While it is certainly conceivable that testing should be costly for all types, this assumption captures the notion that testing will be less costly for types that the bureaucrat wants to pass, perhaps to the point that she will endeavor to make testing costs negligible.\(^6\)

A type-\( \theta_i \) client who announces type-\( \theta_j \) therefore receives:

\[
u_c(\theta_j; \theta_i) = \phi(t(\theta_j); \theta_i) \theta_i - k(\theta_i)t(\theta_j).\tag{4}\]

Finally, the principal wishes to maximize the net surplus of citizens of some type \( \theta_j \), but faces a cost of providing resources to the bureaucrat. Given truth-telling under the

\(^5\)The population choice might represent an explicit limit on the bureaucrat’s services, or it may represent the selection of clients based on some observable characteristic that is independent of the type distribution, such as geography.

\(^6\)As an example, a bureaucrat might choose to qualify applicants for a program simply by using available administrative data, rather than requiring applicants to produce evidence of eligibility. Under the Affordable Care Act, enrollment in state Medicaid programs can (at the state’s discretion) be handled largely through pre-existing data from other public assistance programs. See http://medicaid.gov/AffordableCareAct/Medicaid-Moving-Forward-2014/Targeted-Enrollment-Strategies/targeted-enrollment-strategies.html.
bureaucrat’s screen, the principal’s objective can be written:

\[ u_p(p, T; \theta_j) = \pi_j p [\phi(t(\theta_j); \theta_j) \theta_j - k(\theta_j) t(\theta_j)] - \frac{1}{2} T^2. \]  

(5)

3 Results

As is standard, the incentive compatibility (IC) constraints require that each client type \( \theta_i \) prefer reporting \( \theta_i \) to any \( \theta_j \neq \theta_i \):

\[ \phi(t(\theta_i); \theta_i) \theta_i - k(\theta_i) t(\theta_i) \geq \phi(t(\theta_j); \theta_i) \theta_i - k(\theta_i) t(\theta_j). \]

Additionally each type \( \theta_i \) has an individual rationality (IR) constraint:

\[ \phi(t(\theta_i); \theta_i) \theta_i - k(\theta_j) t(\theta_i) \geq 0. \]

The specific interpretation of the IR constraints is that while all potential clients under the population cap are entitled to be considered for the good, any client can choose to fail the exam (e.g., by not showing up) and not receive the good.

Observe that a type-\( \theta_i \) client’s report only matters insofar as it affects the bureaucrat’s inspection level. Thus it is helpful to write the client’s expected utility as a function of inspection level \( t(\theta_i) \):

\[ \hat{u}_c(\theta_i, t(\theta_i)) = \begin{cases} 
(1 - \frac{1}{c(\theta_i) t(\theta_i)^\alpha}) \theta_i & \text{if } \theta_i \in \Theta^h \\
\frac{1}{c(\theta_i) t(\theta_i)^\alpha} \theta_i - t(\theta_i) & \text{if } \theta_i \in \Theta^l
\end{cases} \]  

(6)

The valuation for type \( \theta_i \) is obviously decreasing over \( t(\theta_i) \) if and only if \( \theta_i \in \Theta^l \). Thus, types in \( \Theta^h \) always benefit from greater scrutiny, while types in \( \Theta^l \) are hurt by it.

Finally, because the bureaucrat must respect the client’s IR constraint, denote by \( t_{iR} \) the value of \( t(\theta_i) \) at which \( \hat{u}_c(\theta_i, t(\theta_i)) = 0 \) for \( \theta_i \in \Theta^l \). By assumption, \( t_{iR} \) exists and is unique, and \( t_{iR} > 1 \). Clients with higher valuation will have higher values of \( t_{iR} \), and hence also will make IR easier for the bureaucrat to satisfy.

3.1 First Best

As is standard, I first derive the bureaucrat’s solution under the assumption that client types are known. For notational simplicity, I hereafter abuse notation slightly and let \( t_i = t(\theta_i) \). The bureaucrat then maximizes her objective (2) subject to her budget constraint (3), taking as given her budget \( T \) and population mandate \( p \). Performing the straightforward constrained optimization problem yields the following relationship between testing levels.
Lemma 1 First Best. Under the first best, at an interior solution the testing levels for types \( \theta_j \) and \( \theta_k \) satisfy:

\[
    t^f_j = \left( \frac{c(\theta_i)|w_j|}{c(\theta_j)|w_i|} \right)^{\frac{1}{\alpha}} t^f_k. \tag{7}
\]

Proof. All proofs are in the Appendix.

At an interior solution, the testing level for each type is a fixed proportion of every other inspection level, independent of \( T \) and \( p \). This implies that there is a unique set of testing levels \( \{t^f_i\} \) that satisfy the budget constraint with equality. Corner solutions for inspection levels are also possible. Each \( t^f_i \) is constrained to be at least 1, and to satisfy the IR constraint, \( t^f_i < t^IR_i \) must also hold for types in \( \Theta^l \).

The comparative statics on the bureaucrat’s relative inspection level for a given type \( \theta_j \) are mostly intuitive. It is increasing in the bureaucrat’s absolute payoff weight \( |w_j| \) on that type, and also in the extent to which inspections can change the probability of passage, as measured by \( c(\theta_j) \). (Recall that these parameters jointly capture the bureaucrat’s incentive to test type \( \theta_j \).) Because there are no fixed costs to inspecting any given type, inspection levels are independent of the distribution of types. Finally, the effect of expertise (\( \alpha \)) is ambiguous: the testing level is increasing in \( \alpha \) if \( |w_j| \) is low, and decreasing otherwise.

For the principal’s maximization problem, the effects of the budget and population follow directly from Lemma 1 and the fact that the budget constraint binds. Since the ratio between testing levels is independent of \( T \), a change in budget from \( T \) to \( kT \) will, at an interior solution, simply change all inspection levels from \( t^f_i \) to \( kt^f_i \). The effect of \( p \) is simply the inverse of the effect of \( T \). From the principal’s objective (5), it is straightforward to see that a principal who wishes to benefit type \( \theta_j \) will therefore have an incentive to select a large budget and client population if \( \theta_j \in \Theta^h \), \( \pi_j \) is high, and if \( |w_j| \) is large and \( c(\theta_j) \) is small.

3.2 Decreasing and Increasing Valuations

To develop some intuition, I begin by examining two extreme cases, where all client types are either in \( \Theta^l \) or \( \Theta^h \). In the former case (i.e., decreasing valuations), each client type’s expected utility \( \hat{u}_c(\theta_i, t_i) \) is decreasing in the bureaucrat’s inspection level. In the latter case (i.e., increasing valuations), \( \hat{u}_c(\theta_i, t_i) \) is increasing in \( t_i \) for all types. These cases might correspond, for example, to the administration of construction permits by an anti-development (respec-
tively, pro-development) municipal buildings department. While such cases are probably uncommon, the results will be useful for developing the main results in the next subsection.

The first result is that a feasible client screen must have a uniform inspection level $t^*$ for all types. Under decreasing valuations, the common inspection level must also be sufficiently low. This follows from straightforward manipulation of the IC and IR constraints.

**Lemma 2** Uniform Testing Under Decreasing and Increasing Valuations. Under decreasing valuations or increasing valuations, $t^*_i = t^*$ for all $\theta_i$. Under decreasing valuations, $t^* \leq \min_i \{t_i^{IR}\}$. 

This result is a consequence of the assumption that the bureaucrat has only one dimension – the level of means testing – to control each type’s payoff. When all types in the population have the same preferences over whether to have more or less testing, all clients would choose an identical testing level – the highest available under increasing valuations, or the lowest available under decreasing valuations. Thus IC can be preserved only by implementing a uniform test. This contrasts with a standard screening framework, where the uninformed player typically has the ability impose different side-payments on different types. If the bureaucrat could impose side payments, then she would be able to test some types at a higher level in exchange for a side payment that precludes other types from claiming that testing level.

Solving for $t^*$ is straightforward. Since there is a common testing level $t^*$ and the budget constraint binds, there is a unique testing level that is feasible and satisfies IC. Substituting into (3), this testing level must satisfy:

$$t^* = \frac{T}{p}.$$ 

This value is the solution to the bureaucrat’s problem under increasing valuations, and if it satisfies IR it is also the solution under decreasing valuations if $t^* < \min_i \{t_i^{IR}\}$ for all $i$. Otherwise, the corner solution is $t^* = \min_i \{t_i^{IR}\}$.

Compared to the first-best solution, the optimal testing level under decreasing or increasing valuations is not weighted by the importance (to the bureaucrat) of each type. The allocative inefficiency is therefore especially large when the type distribution or the distribution of weights $u_i$ is highly skewed toward some types.

The principal then chooses a budget and client service level to maximize the surplus of
her preferred group, $j$. Substituting $t^*$ into her objective yields:

$$u^*_p(p, T, \theta_j) = \begin{cases} 
\pi_j p \left(1 - \frac{p^*}{c(\theta_j)^\alpha}\right) \theta_j - \frac{1}{2} T^2 & \text{if } \theta_j \in \Theta^h \\
\pi_j p \left[\frac{p^*}{c(\theta_j)^\alpha} \theta_j - \frac{T}{p}\right] - \frac{1}{2} T^2 & \text{if } \theta_j \in \Theta^l
\end{cases}$$

Manipulation of this expression produces the first main result.

**Proposition 1** Budgets and Client Service Under Decreasing and Increasing Valuations. Under decreasing valuations, $T^* = N$, $p^* = N$, and $t^* = 1$. Under increasing valuations, an interior solution must satisfy $T^* = \frac{\alpha \pi_j \theta_j c(\theta_j) \theta_j}{(\alpha + 1)^{\frac{\alpha}{\alpha + 1}}}$, $p^* = \frac{\alpha \pi_j \theta_j c(\theta_j) \theta_j}{(\alpha + 1)^{\frac{\alpha}{\alpha + 2}}}$, and $t^* = \left(\frac{\alpha + 1}{c(\theta_j)}\right)^{1/\alpha}$. 

Under decreasing valuations, the principal’s problem is simple: members of her preferred group benefit from broad implementation (i.e., high $p$), and low testing (i.e., low $t$). These goals are simultaneously achieved by minimizing the budget and mandating universality. Under increasing valuations, these goals are in conflict: higher testing levels help clients, especially when $c(\theta_j)$ is low, but are also costly and reduce the population that can be served.

The comparative statics for $T^*$ and $p^*$ under increasing valuations are identical. Both are increasing in $\theta_j$, as this mechanically increases the surplus enjoyed by the politician’s favored clients. They are also increasing in the default probability of acceptance (through the parameter $c(\theta_i)$), as this reduces the losses from clients who are inspected but do not receive the allocation, and the proportion of $\pi_j$ of favored types in the population. The role of $\alpha$ is ambiguous: $T^*$ and $p^*$ are decreasing in $\alpha$ for low values of $\alpha$, but possibly increasing for higher values. Finally, the testing level $t^*$ is decreasing in $c(\theta_i)$, which dampens the potential impact of testing, but the effect of $\alpha$ is again ambiguous.

### 3.3 Mixed Valuations

Now consider the case where both $\Theta^h$ and $\Theta^l$ are non-empty. This case is more plausible than those in the previous section, since the bureaucrat desires opposite outcomes for different types of clients.

The first result shows that the logic of Lemma 2 still applies: the set of types with increasing and decreasing valuations must still be tested at the same level. Somewhat more interestingly, these levels might be different, with the high types receiving the higher level of testing.

**Lemma 3** Uniform Testing Under Mixed Valuations. Under mixed valuations, $t^*_i = t^{l*} \leq \min \{t^{lR} \}$ for all $\theta_i \in \Theta^l$, $t^*_i = t^{h*}$ for all $\theta_i \in \Theta^h$, and $t^* \leq t^{h*}$. 

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Lemma 3 implies that there can be at most two levels of testing across the entire client population. A second testing level can exist because more testing attracts types in $\Theta^h$ but deters those in $\Theta^l$. Thus types that the bureaucrat would like to pass can be given higher levels of testing. Unfortunately it is not necessarily the case that the bureaucrat would want to do so; under the first best she may actually wish to test types in $\Theta^l$ more heavily, but this outcome would violate IC.

To characterize testing levels, it is useful to begin by deriving a version of the first best that recognizes the constraints of Lemma 3. In other words, all types in $\Theta^l$ are tested at one level and all types in $\Theta^h$ are tested at another. If the derived $\Theta^h$ testing level is greater than the $\Theta^l$ testing level, then IC is satisfied. If to the contrary the bureaucrat would prefer to test $\Theta^l$ types at a higher level, then it can be shown that IC is satisfied only by the “uniform” standard $T/p$ used for decreasing or increasing valuations. In both cases, the screening mechanism is implementable if the testing levels are feasible (i.e., at least 1) and satisfy IR.

The next result gives a simple condition under which each kind of screening mechanism is implementable, as well as the resulting testing levels.

**Proposition 2** Testing Under Mixed Valuations. Let $I^l = \{i|\theta_i \in \Theta^l\}$ and $I^h = \{i|\theta_i \in \Theta^h\}$. Under mixed valuations, at an interior solution:

(i) If

\[
\frac{\sum_{i \in I^h} \pi_i}{\sum_{i \in I^l} \pi_i} < \frac{\sum_{i \in I^h} \frac{\pi_i w_i}{c(\theta_i)}}{\sum_{i \in I^l} \frac{\pi_i w_i}{c(\theta_i)}},
\]

then $t^l^* < t^h^*$, where:

\[
t^l^* = \frac{T}{p \left(1 - \left(1 - \frac{\left(\sum_{i \in I^h} \pi_i \right) \left(\sum_{i \in I^l} \frac{\pi_i w_i}{c(\theta_i)}\right)}{\left(\sum_{i \in I^l} \pi_i \right) \left(\sum_{i \in I^h} \frac{\pi_i w_i}{c(\theta_i)}\right)}\right) \left(\sum_{i \in I^h} \pi_i\right)\right)} \geq 1
\]

\[
t^h^* = \frac{T}{p \left(1 - \left(1 - \frac{\left(\sum_{i \in I^h} \pi_i \right) \left(\sum_{i \in I^l} \frac{\pi_i w_i}{c(\theta_i)}\right)}{\left(\sum_{i \in I^l} \pi_i \right) \left(\sum_{i \in I^h} \frac{\pi_i w_i}{c(\theta_i)}\right)}\right) \left(\sum_{i \in I^h} \pi_i\right)\right)} \geq 1.
\]

(ii) If (9) does not hold, then $t^l^* = t^h^* = T/p$.

Proposition 2 has two main implications. First, the bureaucrat is able to perform some simple discrimination across types under one important condition. This condition, given by expression (9), is satisfied when her payoff weights ($w_i$) on types in $\Theta^h$ and the potential
gain from testing for those types (i.e., low $c(\theta_i)$) are sufficiently high. Loosely speaking, the condition can be interpreted as higher agreement on type treatment between the principal and the bureaucrat. Second, when this discrimination is possible, $t^{hs} > T/p$ and $t^{ls} < T/p$; that is, the bureaucrat over-tests types in $\Theta^h$ relative to the uniform benchmark, and under-tests types in $\Theta^l$.

For any given $p$ and $T$, the screening mechanism is most inefficient for the bureaucrat when she wishes to test types in $\Theta^l$ more heavily. That is, when (9) does not hold, the bureaucrat is both more interested in rejecting “marginal” types than accepting “qualified” ones, and faces relatively high marginal returns to testing for those types. The resulting constraint on equal treatment implies that, relative to her optimum, the bureaucrat will make more allocation errors on types in $\Theta^l$ and less on types in $\Theta^h$.

The principal then chooses a budget and client population to maximize the surplus of her preferred type, $\theta_j$. The key observation is that in two cases, her problem is essentially identical to her problem in the simple decreasing or increasing valuations world. First, when $\theta_j \in \Theta^l$, she again maximizes her clients’ probability of acceptance by minimizing the bureaucrat’s resources and mandating universal implementation. Second, when $\theta_j \in \Theta^h$ and (9) does not hold, testing must be uniform and her objective is given by (8). Proposition 1 describes these cases.

In the final case, $\theta_j \in \Theta^h$ and some testing discrimination is possible. Notationally, define the following term, which is simply the ratio between $t^{hs}$ under mixed valuations and $t^*$ under increasing valuations:

$$m = 1 - \left( 1 - \frac{\left( \sum_{i \in \mathcal{I}^h} \pi_i \left( \sum_{i \in \mathcal{I}^l} \frac{\pi_i | w_i |}{c(\theta_i)} \right) \right)^{1/1+r}}{\left( \sum_{i \in \mathcal{I}^l} \pi_i \left( \sum_{i \in \mathcal{I}^h} \frac{\pi_i | w_i |}{c(\theta_i)} \right) \right)} \right) \sum_{i \in \mathcal{I}^l} \pi_i$$

Note that condition (9) implies that $m < 1$. This allows the principal’s objective to be written to reflect type $\theta_j$’s higher level of testing as follows.

$$u^*_p(p, T; \theta_j) = \pi_j p \left( 1 - \frac{p^{\alpha} m^{\alpha}}{c(\theta_j) T^{\alpha}} \right) \theta_j - \frac{1}{2} T^2$$ (10)

Combined with the preceding observations, maximizing (10) produces the following result on budgets and client service under mixed valuations.

**Proposition 3** Budgets and Client Service Under Mixed Valuations. If $\theta_j \in \Theta^l$, then
If $\theta_j \in \Theta^h$, then at an interior solution:

\[
T^* = \begin{cases} 
\frac{\alpha \pi_j \theta_j c(\theta_j)_{1/\alpha}}{m(\alpha+1)^{(\alpha+1)/\alpha}} & \text{if (9) holds} \\
\frac{\alpha \pi_j \theta_j c(\theta_j)_{1/\alpha}}{(\alpha+1)^{(\alpha+2)/\alpha}} & \text{if (9) does not hold,}
\end{cases}
\]

\[
p^* = \begin{cases} 
\frac{\alpha \pi_j \theta_j c(\theta_j)_{2/\alpha}}{m^2(\alpha+1)^{(\alpha+2)/\alpha}} & \text{if (9) holds} \\
\frac{\alpha \pi_j \theta_j c(\theta_j)_{2/\alpha}}{(\alpha+1)^{(\alpha+2)/\alpha}} & \text{if (9) does not hold,}
\end{cases}
\]

\[
t^{h*} = \left(\frac{\alpha + 1}{c(\theta_j)}\right)^{1/\alpha},
\]

and $t^{l*}$ is given by Proposition 2. ■

Proposition 3 identifies three program implementation styles. As noted, two cases are identical to the decreasing and increasing valuations cases characterized in Proposition 1. In the third, the principal’s favored type $\theta_j$ has increasing valuations and condition (9) holds. Here the politician induces the testing level from the basic increasing valuations case for types in $\Theta^h$, but also gives a larger budget to service a larger population, while testing types in $\Theta^l$ at a lower level.

3.4 Implications

Proposition 3 implies some basic comparative statics relationships on key measures of program performance. The outputs are as follows:

- **Budget** received by the agent, measured by $T$.
- **Client population**, measured by $p$, where $p < N$ implies a shortage or backlog.
- **Per capita budget**, measured by $T/p$.
- **Type I error avoidance rate**, measured by $1 - \phi(t_i; \theta_i)$ for $\theta_i \in \Theta^l$.
- **Type II error avoidance rate**, measured by $\phi(t_i; \theta_i)$ for $\theta_i \in \Theta^h$.
- **Flexibility**, measured by whether $t^{l*} < t^{h*}$.

Note that Type I errors are defined as approvals of the good to types in $\Theta^l$ (e.g., approving a bad drug), while Type II errors are denials of the good to types in $\Theta^h$ (e.g., rejecting a good drug). The errors therefore reflect the bureaucrat’s idiosyncratic preferences over approving
different types. Both types of errors are conditional upon participation in the bureaucrat’s mechanism, and therefore do not count Type II errors arising from failure to serve eligible clients.

There are two sets of results, summarized by Comments 1 and 2. The first comment describes performance parameters in the case where the principal favors a marginal type (i.e., $\theta^j \in \Theta^l$). It also compares the equilibrium parameters to the case where the principal favors a qualified type. The result is a simple consequence of the fact that when her favored type is marginal, the principal mandates universality along with minimal resources. Since the bureaucrat’s solution is at a corner, the comparison with the case where $\theta^j \in \Theta^l$ is straightforward. The result is stated without proof.

**Comment 1** Favored Marginal Types. Let $\theta_j \in \Theta^l$ be the principal’s favored type. (i) All performance measures are independent of all $|w_i|$ and $c(\theta_i)$.

(ii) The client population is higher, and the per capita budget, Type I and II error avoidance rates, and flexibility are (weakly) lower than in the case where $\theta^j \in \Theta^l$. ■

The next comment describes the case of a principal who favors a qualified group. In this case, changes in the testing technology and the bureaucrat’s idiosyncratic weights affect all of the observable performance measures. Note that in most cases, the result accounts for both whether (9) holds, as well as comparative statics conditional upon whether it holds.

**Comment 2** Favored Qualified Types. Let $\theta_j \in \Theta^h$ be the principal’s favored type. (i) The budget and client population are weakly increasing in $|w_i|$ for $\theta_i \in \Theta^h$ and $c(\theta_i)$ for $\theta_i \in \Theta^l$, and weakly decreasing in $|w_i|$ for $\theta_i \in \Theta^l$ and $c(\theta_i)$ for $\theta_i \in \Theta^h$ and $\theta_i \neq \theta_j$.

(ii) The per capita budget is weakly decreasing in $|w_i|$ for $\theta_i \in \Theta^h$ and $c(\theta_i)$ for $\theta_i \in \Theta^l$, and weakly increasing in $|w_i|$ for $\theta_i \in \Theta^l$ and $c(\theta_i)$ for $\theta_i \in \Theta^h$ and $\theta_i \neq \theta_j$.

(iii) Flexibility occurs when $|w_i|/c(\theta_i)$ for $\theta_i \in \Theta^h$ is sufficiently high or $|w_i|/c(\theta_i)$ for $\theta_i \in \Theta^l$ is sufficiently low.

(iv) The Type I error avoidance rate is lower for any combination of $|w_i|$ and $c(\theta_i)$ excluding $c(\theta_j)$ satisfying (9) than for any such combination that does not. The Type II error avoidance rate is decreasing in $c(\theta_j)$ and constant in all $|w_i|$ and other $c(\theta_i)$. ■

Comment 2 has two notable implications. The first lies in the contrast between parts (i) and (ii): overall spending and per capita spending move in opposite directions. In equilibrium, larger budgets are spent on larger client populations and less expert investigations.
The second follows from the contrast between parts (iii) and (iv): since bureaucratic flexibility results from satisfying condition (9), it is associated with lower testing levels and in turn a higher frequency of Type I errors. In summary, conditions that induce higher budgets will reduce per capita spending and increase client population, flexibility, and Type I errors.

The parametric conditions that generate these outcomes are loosely related to the extent to which the bureaucrat “agrees” with the principal on the distribution of effort across types. The bureaucrat’s motivation to test is generally increasing in both how much she cares about delivering desired outcomes to particular types ($|w_i|$), and the available “headroom” for testing to do so (i.e., low $c(\theta_i)$). Condition (9) is not satisfied when the bureaucrat is more interested in denying marginal types than approving qualified ones (i.e., $|w_i|/c(\theta_i)$ is relatively high for types in $\Theta^l$). As Comment 2 establishes, this generates a smaller total budget. By contrast, increasing the bureaucrat’s motivation for any type in $\Theta^h$ helps to satisfy and (9) and allows the bureaucrat to secure a larger budget, as long as the principal is merely interested in helping one type in $\Theta^h$. This is a consequence of the bureaucrat’s inability to discriminate across types with increasing valuations.

4 Extensions

4.1 Testing Standards

Suppose that, in addition to $T$ and $p$, the bureaucrat were bound by a minimum or maximum testing level for all clients. In some environments, such as education, legislatures impose testing requirements that bureaucrats use in order to advance students. In others, such as law enforcement, courts can impose “due process” requirements on politicians and bureaucrats. The obvious trade-off is that a principal could conceivably improve the payoff of his favored group through higher mandated testing levels, but such testing levels would require higher expenditures.

Formally, suppose that instead of allowing any testing level satisfying $t(\theta_i) \geq 1$, testing levels for all types are constrained to satisfy $t(\theta_i) \in [t_{min}, t_{max}]$. For simplicity, assume that at $t_{min}$, all IR constraints are satisfied. When it is binding, the effect of such a restriction is easily calculated in the cases where the bureaucrat does not discriminate across types. For example, if the principal’s favored type is some $\theta_j \in \Theta^l$, then by the same argument as in the previous section he simply minimizes testing and chooses a budget to match: $T^* = Nt_{min}$.

\footnote{The comparative statics on the Type I error avoidance rate are ambiguous, as it may be increasing or decreasing when (9) holds.}
\( p^* = N, \) and \( t^* = t_{\text{min}}. \) In these cases, it is clear that the principal cannot benefit from a testing standard. Since the bureaucrat’s budget constraint is binding, the uniform testing level will be \( T/p \) and any testing standard can only constrain the principal.

More generally, the following comment shows that the principal cannot benefit from a testing restriction. The intuition is that uniform testing restrictions are too crude of a means of inducing higher testing for a favored client type. A testing ceiling of \( t_{\text{max}} \) simply constrains the extent to which a politician can help a qualified type. And for any given budget, a binding testing floor of \( t_{\text{min}} \) also necessarily implies (through the budget constraint) a lower level of testing for qualified types.

**Comment 3** Testing Restrictions. *The principal cannot benefit from a testing standard.*

What might explain the presence of testing standards? One possibility is that they can be useful when the principal is unable to specify other parameters of the bureaucrat’s behavior. For example, if the principal cannot designate \( p \), then a mandated maximum testing level could establish a floor on client population. Relatedly, a second possibility is that testing standards can be imposed by other principals. A second principal who cared about not approving too many types in \( \Theta^l \) could use a minimum testing standard to force more scrutiny upon types that the principal and bureaucrat would otherwise ignore. This second principal might be a court, or another legislator whose support is necessary for enacting the program in question.

### 4.2 Costly Goods

One important assumption of the preceding analysis is that the bureaucratically allocated good is costless to the principal. In many instances, this is not the case: social welfare benefits are obviously quite costly, and building permits create opportunity costs. Costly goods create an important trade-off, as they reduce the principal’s incentive to test types in \( \Theta^h \), but increase his incentive to test types in \( \Theta^l \).

To see the effects of costly goods, suppose that each allocated good has a unit cost \( \gamma < 1. \) I focus on the case where \( \theta_j \in \Theta^h \) and (9) does not hold, so the bureaucrat cannot discriminate across types. The principal’s objective can then be written:

\[
 u^*_p(p, T; \theta_j) = \pi_j p \left( 1 - \frac{p^\alpha}{c(\theta_j) T^\alpha} \right) \theta_j - \gamma p \left[ \sum_{i|\theta_i \in \Theta^h} \pi_i \left( 1 - \frac{p^\alpha}{c(\theta_i) T^\alpha} \right) + \sum_{i|\theta_i \in \Theta^l} \pi_i \frac{p^\alpha}{c(\theta_i) T^\alpha} \right] - \frac{1}{2} T^2. 
\]

(11)
The following comment generalizes the corresponding case of Proposition 3 and presents the equilibrium budget, client population, and testing level.

**Comment 4** Costly Goods. If $\theta_j \in \Theta^h$ and (9) does not hold, then at an interior solution:

\[
T^* = \alpha \left( \frac{\pi_j \theta_j - \gamma \sum_{i|\theta_i \in \Theta^h} \pi_i}{\alpha + 1} \right)^{\frac{-1}{\alpha}} \left( \frac{\pi_j \theta_j}{c(\theta_j)} - \gamma \left[ \sum_{i|\theta_i \in \Theta^h} \frac{\pi_i}{c(\theta_j)} - \sum_{i|\theta_i \in \Theta^l} \frac{\pi_i}{c(\theta_j)} \right] \right)^{-\frac{1}{\alpha}}
\]

\[
p^* = \alpha \left( \frac{\pi_j \theta_j - \gamma \sum_{i|\theta_i \in \Theta^h} \pi_i}{\alpha + 1} \right)^{\frac{-1}{\alpha}} \left( \frac{\pi_j \theta_j}{c(\theta_j)} - \gamma \left[ \sum_{i|\theta_i \in \Theta^h} \frac{\pi_i}{c(\theta_j)} - \sum_{i|\theta_i \in \Theta^l} \frac{\pi_i}{c(\theta_j)} \right] \right)^{-\frac{1}{\alpha}}
\]

\[
t^* = \left( \frac{\pi_j \theta_j - \gamma \sum_{i|\theta_i \in \Theta^h} \pi_i}{\alpha + 1} \right)^{-\frac{1}{\alpha}} \left( \frac{\pi_j \theta_j}{c(\theta_j)} - \gamma \left[ \sum_{i|\theta_i \in \Theta^h} \frac{\pi_i}{c(\theta_j)} - \sum_{i|\theta_i \in \Theta^l} \frac{\pi_i}{c(\theta_j)} \right] \right)^{\frac{1}{\alpha}}.
\]

These expressions make clear that compared to the baseline model, when there is a sufficiently high probability weight on types in $\Theta^l$, costly goods will reduce both testing levels and the client population. However, per capita testing increases. The effect of increasing costs can therefore be similar to that of moving from the discrimination to non-discrimination cases in the baseline model.

I note finally that if $\theta_j \in \Theta^l$, then costly goods might induce the principal to offer a budget higher than the minimum seen in Proposition 3. This requires that there be sufficient probability weight on types in $\Theta^l$ besides $\theta_j$, which from the principal’s perspective generates many wasted allocations.

**5 Conclusions**

Theories of political control of the bureaucracy have made considerable progress in recent decades, but much of this work is connected in limited ways to concrete aspects of public administration. Apart from a burgeoning literature on corruption, this gap has impeded theoretically motivated empirical work on the bureaucracy. This paper represents a first attempt at filling this gap.

The model begins with a foundation of incomplete information, bureaucratic expertise, resource constraints, and political control. There is a natural technological trade-off between client populations and per capita spending (or equivalently, testing), but the model provides
guidance as to how these trade-offs are resolved. In particular, the politician’s distributive concerns and the bureaucrat’s ability to discriminate across types play crucial roles in determining whether program implementation tends toward breadth or depth.

While the model addresses a broad set of bureaucratic outputs, it omits some important features. For example, in some environments there are natural limits on the quantity of the good to be distributed. Another important assumption is that the bureaucrat’s individual rationality constraint is automatically satisfied. In reality, principals may need to worry about this constraint not only to address labor market alternatives, but also to avoid side-contracts between clients and bureaucrats. The model also makes somewhat arbitrary choices about what politicians can control. While it is clearly reasonable that politicians control agency budgets, it is less obvious that the size of the client population should be the only other decision variable. Another possibility is that the principal could manipulate the probability of acceptance through judicial appeals or other procedural mechanisms. Finally, in many political systems the role of multiple principals, either as actors who forge compromises in program design or as active overseers, deserves consideration.
APPENDIX

Proof of Lemma 1. As the objective is concave with respect to all $t_i$ and the constraint is linear, first order conditions are sufficient for characterizing the optimal testing levels. The Lagrangian is:

$$\mathcal{L} = \sum_i \pi_i w_i \phi(t_i; \theta_i) + \lambda \left( T - p \sum_i \pi_i t_i \right).$$

Differentiation yields:

$$\frac{\partial \mathcal{L}}{\partial t_i} = \frac{\alpha \pi_i |w_i|}{c(\theta_i)} t_i^{-\alpha-1} - \lambda \pi_i = 0$$

and

$$\frac{\partial \mathcal{L}}{\partial \lambda} = T - p \sum_i \pi_i t_i = 0.$$

Manipulation of $\frac{\partial \mathcal{L}}{\partial t_i}$ produces:

$$\lambda = \frac{\alpha |w_i|}{c(\theta_i)p} t_i^{-\alpha-1}$$

Substituting into $\frac{\partial \mathcal{L}}{\partial t_j}$ then yields:

$$\frac{\alpha \pi_j |w_j|}{c(\theta_j)} t_j^{-\alpha-1} = \frac{\alpha |w_i|}{c(\theta_i)} t_i^{-\alpha-1} \pi_j$$

$$t_j = \left( \frac{c(\theta_i) |w_j|}{c(\theta_j) |w_i|} \right) \frac{1}{1+\alpha} t_i$$

This solution applies for all interior $t_i, t_j$, as claimed. ■

Proof of Lemma 2. Suppose otherwise; i.e., there exists some $t_j^* > t_k^*$ for types $\theta_j$ and $\theta_k$. Under decreasing valuations, it is clear that $u_c(\theta_j; \theta_j) < u_c(\theta_k; \theta_j)$. Thus, type $\theta_j$ would strictly prefer to claim to be type $\theta_k$, contradicting IC. Under increasing valuations we have $u_c(\theta_k; \theta_k) < u_c(\theta_j; \theta_k)$, and so type $\theta_k$ would strictly prefer to claim to be type $\theta_j$, also contradicting IC.

Finally, under decreasing valuations IR is satisfied for all $i$ only if $t^* < t_i^R$ for all $i$. Under increasing valuations, IR is satisfied for any $t_i$. ■

Proof of Proposition 1. If $\theta_i \in \Theta^l$ for all $i$ (i.e., decreasing valuations), then it is easily verified that (8) is increasing in $p$ and decreasing in $T$, thus implying $p = N$ and (since the minimum testing level is 1), $T^* = N$.

If $\theta_i \in \Theta^h$ for all $i$ (i.e., increasing valuations), then observe that the principal’s objective (8) is concave in $p$ and $T$. Thus necessary conditions for an optimum are:

$$\frac{\partial u_p}{\partial T} = \alpha \pi_j \frac{p^{\alpha+1}}{c(\theta_j) T^{\alpha+1}} \theta_j - T = 0$$

$$\frac{\partial u_p}{\partial p} = \pi_j \theta_j - (\alpha + 1) \pi_j \frac{p^\alpha}{c(\theta_j) T^\alpha} \theta_j = 0.$$
Solving yields:

\[ T = \frac{\alpha \pi_j \theta_j c(\theta_j)^{\frac{1}{\alpha}}}{(\alpha + 1)^{\frac{\alpha + 1}{\alpha}}} \]

\[ p = \frac{\alpha \pi_j \theta_j c(\theta_j)^{\frac{2}{\alpha}}}{(\alpha + 1)^{\frac{\alpha + 2}{\alpha}}} . \]

Testing levels \( t^* \) are derived simply by substituting into (3).

**Proof of Lemma 3.** The arguments for why \( t_i^* = t_i^H \leq \min \{ t_i^L \} \) for all \( \theta_i \in \Theta^l \) and \( t_i^* = t_i^H \) for all \( \theta_i \in \Theta^h \) are identical to those in Lemma 2.

To show that \( t_i^L \leq t_i^H \), suppose otherwise. Then for all types \( \theta_j \in \Theta^l \) and \( \theta_k \in \Theta^h \), \( u_c(\theta_j; \theta_j) < u_c(\theta_k; \theta_j) \) and \( u_c(\theta_k; \theta_k) < u_c(\theta_j; \theta_k) \). Thus, all types in \( \Theta^l \) (respectively, \( \Theta^h \)) would strictly prefer to claim to be of a type in \( \Theta^l \) (respectively, \( \Theta^h \)), contradicting IC.

**Proof of Proposition 2.** (i) As the objective is concave with respect to all \( t_i \) and the constraint is linear, first order conditions are sufficient for characterizing the optimal testing levels. Denote by \( \underline{t} \) and \( \bar{t} \) the (uniform) testing levels for types in \( \Theta^l \) and \( \Theta^h \), respectively. The Lagrangian is:

\[ \mathcal{L} = \sum_{i|\theta_i \in \Theta^l} \pi_i w_i (\underline{t}; \theta_i) + \sum_{i|\theta_i \in \Theta^h} \pi_i w_i (\bar{t}; \theta_i) + \lambda \left( T - p \sum_{i|\theta_i \in \Theta^l} \pi_i \underline{t} - p \sum_{i|\theta_i \in \Theta^h} \pi_i \bar{t} \right) . \]

Differentiation yields:

\[ \frac{\partial \mathcal{L}}{\partial \underline{t}} = \alpha \underline{t}^{\alpha - 1} \sum_{i|\theta_i \in \Theta^l} \frac{\pi_i |w_i|}{c(\theta_i)} - \lambda p \sum_{i|\theta_i \in \Theta^l} \pi_i = 0 \]

\[ \frac{\partial \mathcal{L}}{\partial \bar{t}} = \alpha \bar{t}^{\alpha - 1} \sum_{i|\theta_i \in \Theta^h} \frac{\pi_i |w_i|}{c(\theta_i)} - \lambda p \sum_{i|\theta_i \in \Theta^h} \pi_i = 0 \]

\[ \frac{\partial \mathcal{L}}{\partial \lambda} = T - p \sum_{i|\theta_i \in \Theta^l} \pi_i \underline{t} - p \sum_{i|\theta_i \in \Theta^h} \pi_i \bar{t} = 0 . \]

Let \( P^l = \sum_{i|\theta_i \in \Theta^l} \pi_i \), \( P^h = \sum_{i|\theta_i \in \Theta^h} \pi_i \), \( S^l = \sum_{i|\theta_i \in \Theta^l} \frac{\pi_i |w_i|}{c(\theta_i)} \), and \( S^h = \sum_{i|\theta_i \in \Theta^h} \frac{\pi_i |w_i|}{c(\theta_i)} \). Then manipulation of \( \frac{\partial \mathcal{L}}{\partial \underline{t}} \) produces:

\[ \lambda = \frac{\alpha S^l}{p P^l \underline{t}^{\alpha - 1}} \]

Substituting into \( \frac{\partial \mathcal{L}}{\partial \bar{t}} \) then yields:

\[ S^h \bar{t}^{\alpha - 1} = P^h \frac{S^l}{P^l} \underline{t}^{\alpha - 1} \]

\[ \underline{t} = \left( \frac{P^h S^l}{P^l S^h} \right)^{\frac{1}{\alpha + 1}} \bar{t} \]
Substituting into the budget constraint produces:

\[
\begin{align*}
t &= \frac{T}{p \left[ 1 - P^l + P^l \left( \frac{P^h S^l}{P^l S^h} \right)^{1+\alpha} \right]} \\
t &= \frac{T}{p \left[ 1 - P^h + P^h \left( \frac{P^h S^l}{P^l S^h} \right)^{-1+\alpha} \right]}
\end{align*}
\]

Note that these solutions are interior iff \( t \geq 1 \) and \( \bar{t} \geq 1 \).

Finally, consider the IC constraints. By Lemma 3, IC is satisfied for all types if \( \bar{t} > t \), or:

\[
P^l \left[ 1 - \left( \frac{P^h S^l}{P^l S^h} \right)^{1+\alpha} \right] > P^h \left[ 1 - \left( \frac{P^h S^l}{P^l S^h} \right)^{-1+\alpha} \right]
\]

This expression holds iff \( P^h S^l < P^l S^h \), which is equivalent to (9).

(ii) Abusing notation slightly, let \( u_b(t_h, t_l) \) denote the bureaucrat’s objective when all types \( \Theta^h \) receive testing level \( t_h \) and all types in \( \Theta^l \) receive testing level \( t_l \). If (9) does not hold, then \( \bar{t} \leq t \). Suppose that there exists a feasible solution \((\bar{t}, \bar{t}')\) such that \( \bar{t}' > \bar{t} \). Note also that if \( t_h = t_l \), then the optimal is \( T/p \). If \((\bar{t}, \bar{t}')\) gives the bureaucrat higher utility than \( t_i = T/p \) for all \( \theta_i \), then:

\[
\begin{align*}
&u_b(\bar{t}, \bar{t}) > u_b(T/p, T/p) \\
&u_b(\bar{t}, \bar{t}') > u_b(T/p, T/p).
\end{align*}
\]

Now observe that \((\bar{t}', \bar{t}'), (\bar{t}, \bar{t})\), and \((T/p, T/p)\) all lie along the bureaucrat’s budget constraint. Thus, any \((\bar{t}', \bar{t}')\) such that \( \bar{t}' > \bar{t} \) must violate the concavity of the bureaucrat’s objective function: contradiction. The optimal allocation satisfying IC is therefore \((T/p, T/p)\). The IR constraints are satisfied in the same way as in part (i).

Proof of Proposition 3. If \( \theta_j \in \Theta^l \), then \( \hat{u}_c(\theta_i, t_i) \) is decreasing in \( t_i \) and hence \( u_p(p, T; \theta_j) \) is decreasing in \( t_j \). It is easily verified that (5) is maximized at \( p^* = N \) and \( T^* = N \), which implies \( t_j = t^{l*} = t^{h*} = 1 \). This reduces the testing level and budgetary payments to the minimum feasible levels while maximizing the population served.

If \( \theta_j \in \Theta^h \) and (9) does not hold, then by Proposition 2, \( t^{h*} = t^{h*} = t^* \), where \( t^* \) is the budget from the increasing valuations case. The principal’s objective is then given by (8), and the result is identical to that of the increasing valuations case of Proposition 1.

Finally, if \( \theta_j \in \Theta^h \) and (9) does not hold, then \( t^{h*} \) is given by Proposition 2 and the principal’s corresponding objective is given by (10). It is straightforward to verify that this objective is concave in \( p \) and \( T \). Thus necessary conditions for an optimum are:

\[
\begin{align*}
\frac{\partial u_p}{\partial T} &= \alpha \pi_j \frac{m^\alpha p^{\alpha+1}}{c(\theta_j) T^{\alpha+1}} \theta_j - T = 0 \\
\frac{\partial u_p}{\partial p} &= \pi_j \theta_j - (\alpha + 1) \pi_j \frac{m^\alpha p^\alpha}{c(\theta_j) T^{\alpha}} \theta_j = 0.
\end{align*}
\]
Proposition 2, in the absence of the testing restriction, the principal could use the same \( \theta \) and \( t \). This expression is increasing in \( m \) conditional upon whether (9) holds. When it does not, \( T \) is increasing in \( m \) and \( c \), except for \( c(\theta_j) \). Note that \( m \) is decreasing in \( |w_i| \) for \( \theta_i \in \Theta^h \) and \( c(\theta_i) \) for \( \theta_i \in \Theta^l \), and increasing in \( |w_i| \) for \( \theta_i \in \Theta^l \) and \( c(\theta_i) \) for \( \theta_i \in \Theta^h \). Thus when (9) does hold, \( T^* \) and \( p^* \) are increasing in \( |w_i| \) for \( \theta_i \in \Theta^l \) and \( c(\theta_i) \) for \( \theta_i \in \Theta^l \), and decreasing in \( |w_i| \) for \( \theta_i \in \Theta^h \) and \( \theta_i \neq \theta_j \).

Next, consider the the values of \( T^* \) and \( p^* \) when both sides of (9) are equal. This implies \( m = 1 \), and hence \( T^* \) and \( p^* \) are continuous in all \( |w_i| \) and \( c(\theta_i) \). Since (9) is satisfied when \( |w_i| \) for \( \theta_i \in \Theta^h \) or \( c(\theta_i) \) for \( \theta_i \in \Theta^l \) are sufficiently large, or \( |w_i| \) for \( \theta_i \in \Theta^l \) or \( c(\theta_i) \) for \( \theta_i \in \Theta^h \) and \( \theta_i \neq \theta_j \) are sufficiently small, the result follows.

Proof of Comment 2. (i) First consider the values of \( T^* \) and \( p^* \) from Proposition 3 conditional upon whether (9) holds. When it does not, \( T^* \) and \( p^* \) are independent of all \( |w_i| \) and \( c(\theta_i) \), except for \( c(\theta_j) \). Note that \( m \) is decreasing in \( |w_i| \) for \( \theta_i \in \Theta^h \) and \( c(\theta_i) \) for \( \theta_i \in \Theta^l \), and increasing in \( |w_i| \) for \( \theta_i \in \Theta^l \) and \( c(\theta_i) \) for \( \theta_i \in \Theta^h \). Thus when (9) does hold, \( T^* \) and \( p^* \) are increasing in \( |w_i| \) for \( \theta_i \in \Theta^h \) and \( c(\theta_i) \) for \( \theta_i \in \Theta^l \), and decreasing in \( |w_i| \) for \( \theta_i \in \Theta^l \) and \( c(\theta_i) \) for \( \theta_i \in \Theta^h \) and \( \theta_i \neq \theta_j \).

This expression is increasing in \( m \) if (9) holds, and also dependent on \( c(\theta_j) \). Thus it is straightforward to show that the comparative statics are the reverse of those for \( T^* \) and \( p^* \) in part (i).

(iii) By Proposition 2, flexibility occurs when (9) holds. Since (9) is satisfied when \( |w_i| \) for \( \theta_i \in \Theta^h \) or \( c(\theta_i) \) for \( \theta_i \in \Theta^l \) are sufficiently large, or \( |w_i| \) for \( \theta_i \in \Theta^l \) or \( c(\theta_i) \) for \( \theta_i \in \Theta^h \) are sufficiently small, the result follows.

(iv) To show the result for Type I errors, note that \( t^h \) is independent of all \( |w_i| \) and for all \( c(\theta_i) \) excluding \( \theta_i = \theta_j \). By Proposition 2, \( t^h < t^h \) when (9) holds, and thus any combination of the above \( |w_i| \) and \( c(\theta_i) \) parameters such that (9) holds results in a higher probability of passage for types in \( \Theta^l \), thus establishing the result.

The result on Type II errors follows from the expression for \( t^h \) in Proposition 3.

Proof of Comment 3. The argument for why the principal cannot benefit from any restrictions on \( t \) when (9) does not hold is given in the text.

When (9) holds, suppose that under the testing restriction \( t(\theta_i) \in [t_{min}, t_{max}] \) the bureaucrat implements \( t'' \) for types in \( \Theta^l \) and \( t'' \) for types in \( \Theta^h \), where \( t'' \leq t'' \). Clearly, \( t'' \) and \( t'' \) satisfy the bureaucrat’s budget constraint for some population \( p' \) and budget \( T' \). By Proposition 2, in the absence of the testing restriction, the principal could use the same \( p' \).
and $T'$ to induce the bureaucrat to implement $t^{h*} = \frac{T^*}{p^*}$ for types in $\Theta^h$ and $t^{l*}$ satisfying the budget constraint for types in $\Theta^l$. Since the budget constraint binds, there are two possibilities; first, either $t^{h*} > t^{h'}$ and $t^{l*} < t^{l'}$, or second, $t^{h*} < t^{h'}$ and $t^{l*} > t^{l'}$. Under the first, the principal does better without the testing restriction, since she maximizes the acceptance probability of some type in $\Theta^h$. Under the second, the principal does better under the testing restriction, but then the testing restriction clearly does not bind on the bureaucrat and $t^{l*}$ and $t^{h*}$ cannot be the implemented testing level. Since the principal cannot benefit from a testing restriction for any given $T$ and $p$, he cannot benefit from any testing restriction.

**Proof of Comment 4.** Differentiating (11), the necessary conditions for an optimum are:

$$\frac{\partial u_p}{\partial T} = \alpha \pi_j \frac{p^{\alpha+1}}{c(\theta_j)T^{\alpha+1}} \theta_j - \alpha \gamma \left[ \sum_{i | \theta_i \in \Theta^h} \pi_i \frac{p^{\alpha+1}}{c(\theta_j)T^{\alpha+1}} - \sum_{i | \theta_i \in \Theta^l} \pi_i \frac{p^{\alpha+1}}{c(\theta_j)T^{\alpha+1}} \right] - T = 0$$

$$\frac{\partial u_p}{\partial p} = \pi_j \frac{(\alpha + 1)p^\alpha}{c(\theta_j)T^\alpha} \theta_j - \gamma \left[ \sum_{i | \theta_i \in \Theta^h} \pi_i \left( 1 - \frac{(\alpha + 1)p^\alpha}{c(\theta_j)T^\alpha} \right) T \pi_i \right] = 0.$$

Simplifying yields:

$$T = \left( \frac{\alpha \pi_j \theta_j}{c(\theta_j)} - \alpha \gamma \left[ \sum_{i | \theta_i \in \Theta^h} \frac{\pi_i}{c(\theta_j)} - \sum_{i | \theta_i \in \Theta^l} \frac{\pi_i}{c(\theta_j)} \right] \right)^{\frac{1}{\alpha + 2}} \frac{1}{p^\frac{\alpha + 1}{\alpha + 2}}$$

$$p = \left( \frac{\pi_j \theta_j - \gamma \sum_{i | \theta_i \in \Theta^h} \pi_i}{\pi_j \frac{(\alpha + 1)}{c(\theta_j)} \theta_j - \gamma \left[ \sum_{i | \theta_i \in \Theta^h} \pi_i \frac{(\alpha + 1)}{c(\theta_j)} T \pi_i \right] - \sum_{i | \theta_i \in \Theta^l} \pi_i \frac{(\alpha + 1)}{c(\theta_j)} \pi_i} \right)^{\frac{1}{\alpha}} T$$

Solving for this system yields the resulting $T^*$ and $p^*$. By (3), the testing level is simply $t^* = \frac{T^*}{p^*}$. □
REFERENCES


