Intra-Party Disagreement and Inter-Party Polarization

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Abstract

We develop a theory of legislative competition in which voters care about national party positions and candidates’ quality characteristics when deciding whom to vote for. National party positions, in turn, are determined by the parties’ median legislators, respectively. This leads to the following predictions: (i) as long as election outcomes are predictable enough (from the candidates’ point of view), the only stable equilibria exhibit policy divergence between the parties; (ii) asymmetries in the distribution of district medians can produce situations with a “permanent” majority party that is relatively centrist and a “permanent” minority party that is relatively extremist; (iii) gerrymandering can be used to produce the conditions for (ii) to occur; (iv) as voters place more weight on politicians’ “valence” characteristics—e.g., efficiency in steering government funds to the district or in doing casework, or simply honesty—polarization between the parties decreases; and (v) as the degree of uncertainty about election outcomes increases, polarization between the parties decreases.

Keywords: Legislative competition, polarization, gerrymandering.

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1 Introduction

What is the nature of political parties in representative democracies? In the standard spatial model parties are teams, groups of politicians united to win control of government (Downs, 1957). All politicians within a party have the same induced preferences over the party’s policy positions or ideology.

In practice, however, the politicians in many parties do not exhibit such extreme ideological unity. Dissent frequently arises within parties about the legislative agenda or party platform, in policy proposals, speeches, convention battles, and—at least in the U.S.—roll call votes.\(^1\)

One possible reason for intra-party disagreement is that politicians care explicitly about policy and simply differ in their personal preferences. Another possibility, however, is that politicians care first and foremost about their own careers and re-election chances, and disagreements arise due to differences in the types of districts these politicians represent. Even within the Democratic party, some incumbents represent liberal districts and some represent moderate districts. These politicians try to shift party policy in a direction preferred by their constituents—e.g., Democrats from liberal districts seek to pull their party to the left and those from moderate districts seek to pull the party toward the center.

This paper develops and analyzes a simple model that adopts the second perspective, in which politicians care only about re-election but differ in the types of districts they represent.\(^2\) The model generates a number of interesting predictions, including comparative statics predictions that are relatively easy to test. Among them: (i) as long as election outcomes are predictable enough (from the candidates’ point of view), the only stable equilibria exhibit policy divergence between the parties; (ii) asymmetries in the distribution of district medians can produce situations with a “permanent” majority party that is relatively centrist and a “permanent” minority party that is relatively extremist; (iii) gerrymandering can be used to produce the conditions for (ii) to occur; (iv) as voters place more weight on politicians’ “valence” characteristics—e.g., efficiency in steering government funds to the district or in doing casework, or simply honesty—polarization between the parties decreases; and (v) as the degree of uncertainty about election outcomes increases, polarization between

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\(^1\) As Robertson (1976) documents, this holds even countries with “strong party” systems such as the U.K.

\(^2\) The model borrows heavily from previous models by Snyder (1994); Ansolabehere et al. (2012), and Krasa and Polborn (2015). We describe the similarities and differences below. Those models, in turn, build on the early work by Robertson (1976) and Austen-Smith (1984, 1986). Other papers that examine the electoral consequences of intra-party differences in constituency preferences include Calvert and Isaac (1981); Inghberman and Villani (1993); Snyder and Ting (2002, 2003); Castanheira and Crutzen (2010); Crutzen et al. (2010). For a different approach to intra-party bargaining see Roemer (1999, 2001).
the parties decreases.\textsuperscript{3,4}

Our paper also presents some striking empirical evidence. In particular, we show that data on party polarization from state legislatures and U.S. congress are strongly consistent with prediction (v) from our model. The empirical section also presents anecdotal evidence consistent with predictions (ii), (iii) and (iv). Polarization between Republicans and Democrats is a salient feature of the political landscape in the U.S. today, at the national level and also in many states. According to many measures the degree of polarization has increased over the past few decades. Thus, one important contribution of our model is that it makes clear comparative statics predictions—even better, predictions that comport with some broad patterns found in the relevant data.

The findings regarding prediction (v) are especially interesting, because prediction (v) is exactly the opposite of the prediction generated by standard single-election models in which candidates care about policy as well as winning, such as Wittman (1983); Calvert (1985); Roemer (1994). In those models polarization increases as uncertainty about electoral outcomes increases.\textsuperscript{5}

Thus, although we do not claim that the empirical evidence “supports” our model—since we have not investigated plausible alternatives that might make similar predictions—we do assert that the evidence appears to be inconsistent with the existing single-election models with policy-motivated candidates. In fact, we would go one step further, and suggest that models that take into account the fact that parties compete across diverse constituencies should become a standard element of the political economy toolkit.

\section{The Model}

There is a continuum of districts, indexed by $M$, the ideal policy position of the district median voter. Let $F_M$ be the cumulative distribution function of $M$. We assume that $F_M$ admits a continuous and positive probability density function, denoted $f_M$. Without loss of

\textsuperscript{3}Theorists have identified a variety of factors that may generate platform divergence, including: policy motivation; entry deterrence; incomplete information among voters or candidates; and differential candidate valence. See, e.g., Wittman (1983); Calvert (1985); Loundregan and Romer (1993); Roemer (1994); Martinelli (2001); Gul and Pesendorfer (2009); Palfrey (1984); Callander (2005); Castanheira (2003); Bernhardt et al. (2007); Callander (2008); and Bernhardt and Inghereman (1985); Groseclose (2001); Soubeyran (2009); Krasa and Polborn (2010, 2012). Which factors are most important probably varies depending on factors such as the office(s) sought, the electoral rules, and the nature of the political and policy conflicts in society.

\textsuperscript{4}Predictions (ii) and (iii) are similar in spirit to those in Ansolabehere et al. (2012), but different in important respects. We discuss this further below.

\textsuperscript{5}Wittman (1983) assumes that electoral uncertainty arises because candidates are not perfectly informed about voter preferences and voters are not perfectly informed about candidates’ policy platforms. Calvert (1985) assumes each candidate has a “subjective probability of winning” function. Roemer (1994) derives electoral uncertainty by modeling candidate uncertainty about voter preferences.
generality, we can normalize in a way that $F_M(0) = 1/2$, that is, the median value of $M$ is zero.

The policy preferences of voter $M$ are quadratic, given by $u(x,M) = -(x - M)^2$. We assume that voters care about the national party positions $x_D$ (for Democrats) and $x_R$ (for Republicans) when deciding whom to vote for, rather than the “positions” that their local candidates propose. Fundamentally, voters cannot expect that their local representative will select the policy for the nation at-large. Rather, the parties, made up by the representatives chosen in all districts, are crucial in the process of policy selection in the legislature. Specifically, we assume that $x_D$ and $x_R$ are given by median of the district medians represented by Democrats and Republicans in the legislature, respectively. We will defend this assumption below.

In addition to policy, the voters care about valence. Specifically, if party $k$’s candidate is elected in district $i$, then all voters in that district receive a valence payoff $\alpha v_{k,i}$, where $v_{k,i}$ denotes the elected candidate’s valence and $\alpha$ is a parameter measuring how important valence is for voters relative to policy.\(^6\)

It is useful to define the Republican net valence in district $i$ as $v_i \equiv v_{R,i} - v_{D,i}$, where $v_{R,i}$ and $v_{D,i}$ are the Republican and Democratic candidates’ valences, respectively. The net valence $v_i$ is a random variable that is distributed according to probability density function $\phi$ and cumulative distribution function $\Phi$. Valence shocks in different districts are independent.

**Discussion.** A key feature of our model is that voters care about national party positions, and that those are determined by the median Democrat and median Republican in the legislature, respectively. There is strong evidence that candidates’ party affiliation matters for voters: Districts in which a party’s presidential candidate wins by a substantial margin are usually considered “safe” districts for that party in the Congressional election. This would not be the case if all voters cared about were the positions of their local candidates, which could be freely chosen by the candidates to adjust to the district in which they are running.

Even incumbents whose own position may fit their districts perfectly often have a hard time holding on to a district whose ideological leanings have moved away from their party. For example, Krasa and Polborn (2015) describe the case of Lincoln Chafee, the Republican U.S. senator from Rhode Island from 1999 to 2006, who had taken a number of moderate and liberal positions that brought him in line with voters in his state. While exit polls

\(^6\)The reason why we do not combine $\alpha$ and $v$ into one parameter is that we eventually want to do comparative statics with respect to the importance of valence for voters. For example, one can think of $v_i$ as the overall value of the pork projects that candidate $i$ would be able to attract, and of $\alpha$ as the fraction of those benefits that go to voters in the district (as opposed to voters in other districts).
in the 2006 election gave Chafee a very high 62 percent personal approval rating, “most voters rejected him, many feeling it was more important to give the Democrats a chance at controlling the Senate.” 7 His Democratic challenger Whitehouse “succeeded by attacking the instances in which Chafee supported his party’s conservative congressional leadership (whose personalities and policies were very unpopular, state-wide).” 8

We do not explicitly model the process through which national party positions are determined, but rather assume that each party is identified by the voters with the position of their median representative in the legislature. One possible micro-foundation is that incumbents care only about winning their own seats. Whether a candidate wins also depends on her and her opponent’s valence, which is stochastic. Because voters compare the valence difference between their local candidates with the difference in policy utility that they derive from the two parties’ national positions, each incumbent has induced preferences over her party’s platform that are single-peaked, with the peak located at her district’s median. Therefore, assuming that the Republican and Democratic caucus adopt their national positions by majority vote among their respective caucuses, each party’s position will be equal to the median of the districts represented by that party’s legislators.

Alternatively, the importance of national positions can also be a consequence of voters not learning about the ideological position of their local politician. The first assumption is plausible in the case of “strong” party discipline inside a legislature, under which all legislators within a party are whipped into voting the party line on all important bills. We find the second assumption even more plausible, because of the information environment in most countries. The mass media provides plenty of information about what national party leaders are doing or trying to do—e.g., what “the Democratic” or “the Republican” position is on key issues—but comparatively little information about most rank-and-file legislators. There is ample empirical evidence from surveys that voters are fairly good at distinguishing the relative ideological positions of politicians across parties, but poor at identifying the relative ideological positions of politicians within parties (see, e.g., Snyder and Ting 2002).

Finally, we should ask: what are the parties’ “platforms” really, from the point of view of ordinary voters? As Key (1947, p. 232) argued: “About the best index to party differences is an appeal to the record of this century with admixture of judicious guesses about whether the trend is likely to continue.” That is, a party’s platform is something like a rolling average of what the politicians within that party have been doing—in speeches, legislative proposals, administrative actions, roll call votes, etc.—over the previous decade or so. Under this interpretation, a party’s incumbent officeholders will tend to define the party’s platform,

since their actions and pronouncements are most visible to the voting public.\footnote{One extension is to incorporate the preferences of important outside actors—e.g., interest groups who fund the party, activists who provide free labor, and charismatic or highly visible “outsiders” such as Donald Trump and Ben Carson in the 2016 Republican primaries. We leave this for future work.}

**Related literature.** Relative to the existing literature, our modeling of how national party positions are determined, and how they matter for voters, follows Snyder (1994) and Ansolabehere et al. (2012). In our model, individual races remain competitive between candidates because voters also care about the local candidates’ valence (which is uncertain from an ex-ante perspective); thus, all districts have some positive probability of electing either party’s candidate, though Democrats and Republicans are advantaged in liberal and conservative districts, respectively.

In Snyder (1994) there are no valence shocks, but voters cannot distinguish between the parties’ platforms if they are too close to each other. In Ansolabehere et al. (2012) there is a nationwide valence shock but no race-specific valence shocks. The main focus of our theoretical analysis is how properties of the valence distribution (e.g., the degree of uncertainty about valence, or valence shocks that favor one party) affect the degree of polarization between the parties. Moreover, we test these empirical predictions, while Snyder (1994) and Ansolabehere et al. (2012) are purely theoretical.

Another model that analyzes legislative competition and polarization is Krasa and Polborn (2015). They also analyze a model in which national positions matter for voters and are determined by the median caucus member. Their model differs from ours here in two crucial ways: First, their main focus is on the nomination process in which each candidate’s position is assumed to be determined by policy-motivated primary voters, rather than by the reelection-seeking incumbents in our model. In this framework, they show that the primary voters can exploit the preference of the district median voter (in almost all districts) for one party’s national position, by nominating more extreme candidates than the median voter prefers.

Second, voters in their model choose local candidates calculating how the election of the local Democrat or Republican affects the expected national policy (which is the median of the majority party in the legislature), taking into account both the effect on the likelihood that either party wins a majority, and the position of the majority party. In contrast, our modeling of the voters’ choice is more reduced form, but – we would argue – is behaviorally plausible and much more tractable.
3 Basic Analysis

3.1 District winning probabilities and equilibrium party positions

Given the structure of voter preferences, the median voter in district $M$ prefers the Republican if and only if

$$-(x_R - M)^2 + \alpha v \geq -(x_D - M)^2,$$

where $x_R$ and $x_D$ are the Republican and Democratic platforms, respectively. Rearranging (1), the Republican candidate wins with probability

$$\text{Prob}(\text{R wins in district } M) = 1 - \Phi \left( \frac{2(x_R - x_D) \left[ \frac{x_R + x_D}{2} - M \right]}{\alpha} \right).$$

Thus, in case of a continuum of districts, the density of districts represented by Republicans is given by

$$f_M(M) \left[ 1 - \Phi \left( \frac{2(x_R - x_D) \left[ \frac{x_R + x_D}{2} - M \right]}{\alpha} \right) \right],$$

and the density of districts represented by Democrats is given by

$$f_M(M) \Phi \left( \frac{2(x_R - x_D) \left[ \frac{x_R + x_D}{2} - M \right]}{\alpha} \right),$$

As explained above, the Republican and Democratic positions $x_R$ and $x_D$ are given by the medians of (3) and (4). Setting the expression in (3) equal to 1/2, we get that $x_R$ satisfies

$$\int_{-\infty}^{x_R} f_M(M) \left[ 1 - \Phi \left( \frac{2(x_R - x_D) \left[ \frac{x_R + x_D}{2} - M \right]}{\alpha} \right) \right] dM = \frac{1}{2}$$

and, analogously, $x_D$ satisfies

$$\int_{-\infty}^{x_D} f_M(M) \Phi \left( \frac{2(x_R - x_D) \left[ \frac{x_R + x_D}{2} - M \right]}{\alpha} \right) dM = \frac{1}{2}.$$
There is always an equilibrium in which the parties’ positions are identical. To see this, note that substituting $x_D = x_R = 0$ in (5), and canceling $1 - \Phi(0)$, yields

$$\int_0^0 \frac{f_M(M)}{\int_{-\infty}^{\infty} f_M(t)dt} dM = \frac{1}{2},$$

(7)

which is always satisfied since 0 is the median of $f_M$. An analogous result is obtained when substituting $x_D = x_R = 0$ in (6), and so we have

**Proposition 1.** There exists an equilibrium in which $x_D = x_R = 0$.

Intuitively, if the party positions are identical, then no party’s candidate has an advantage in any district, and therefore the densities of districts won by Democrats and Republicans are indeed identical, so that both parties have the same position. An important caveat to this result is that districts with a median voter literally at 0 can be the median district in both the Democratic and the Republican caucus only if there are multiple such districts. If there is a finite number of districts with distinct median voters, then the no differentiation equilibrium characterized in Proposition 1 is still an equilibrium in a one-shot game in which voters have to calculate with the expected national party positions, which are identical. However, the realized party positions will be different, so a key question with the no differentiation equilibrium is whether it is actually stable. We address this issue in Section 3.2 below.

We now turn to the question whether there exists an equilibrium with policy divergence. In order to increase tractability, we focus on the case that the distribution of districts is uniform on $[-k, k]$ and the distribution $\phi$ is single-peaked and symmetric around 0. This setting allows us to focus on a symmetric profile in which $x_R = x = -x_D$.

**Proposition 2.** Suppose that the distribution of districts is uniform on $[-k, k]$ and the distribution $\phi$ is single-peaked and symmetric around 0. A symmetric equilibrium with policy differentiation, $x_R = x = -x_D$, exists if and only if $\phi_0 > \frac{\alpha}{4k^2}$. Moreover, if such an equilibrium exists, it is unique.

Figure 1 gives an illustration of the type of divergent equilibrium described in Proposition 2. In an equilibrium with policy differentiation, Democrats have higher chances being elected in liberal districts because their national policy position is more popular in those districts, and vice versa for Republicans in conservative districts. Given the higher electoral success rate of Democrats in liberal than in conservative districts, the density of Democratic districts in the legislature is downward-sloping, and thus the median Democrat hails from a relatively liberal district. Conversely, Republicans are concentrated in relatively conservative districts, implying that the median Republican comes from a conservative district, and thus justifying
why voters associate Democrats with liberal positions, and Republicans with conservative ones.

![Figure 1: Republican winning probability by district and the determination of party positions](image)

The existence of an equilibrium with policy differentiation depends on the net valence shock being sufficiently concentrated around zero (i.e., $\phi_0$ is sufficiently large). If the valence shocks are generally small, then most liberal and conservative districts will not experience shocks that are large enough for the median voter to go against her policy preference, and so liberal districts are mostly Democratic, and conservative districts mostly Republican, supporting equilibrium policy differentiation. In contrast, if the valence shocks are too large, then these shocks will be almost all that matters for who gets elected. In this case approximately half of the liberal districts will be won by Republicans, and half of the conservative districts by Democrats, so the medians of the two caucuses will be the same.

### 3.2 Stability

Propositions 1 and 2 characterize equilibria without and with policy divergence between parties. Equilibria are, by definition, profiles where voters in each district choose candidates based on their expectations about the parties’ positions in Congress, and their choices yield party caucuses that exactly justify the voters’ expectations.

For two separate, but interrelated, reasons, it is useful to think about the stability of these equilibria. First, in practice, voters may base their decision whom to vote for in an election not on their rational expectations of party positions (which may be difficult to form),
but rather on their observations of the parties’ positions in the outgoing legislature. In this case, the party positions form is dynamic system, and the decisive question is whether this system converges to one of the equilibria.

Second, in a legislature with finitely many representatives, the composition of the two parties’ caucuses is uncertain. Thus, the system is never exactly at the equilibrium, and therefore the question whether dynamic forces move the system in the direction of an equilibrium, or away from it, is particularly relevant.

Suppose that the legislature is elected based on the belief that the Republican position is at $x$ and the Democratic position is at $-x$. Substituting this in (6), and rearranging, we obtain that an equilibrium is characterized by a zero of the following function $Z(x)$.

$$Z(x) = \int_{-k}^{-x} \frac{1}{2k} \Phi \left( -\frac{4xM}{\alpha} \right) dM - \frac{1}{2} \int_{-k}^{k} \frac{1}{2k} \Phi \left( -\frac{4xM}{\alpha} \right) dM = \frac{1}{2k} \left[ \int_{-k}^{-x} \Phi \left( -\frac{4xM}{\alpha} \right) dM - \int_{-x}^{k} \Phi \left( -\frac{4xM}{\alpha} \right) dM \right].$$

(8)

When $Z(x)$ is positive, then there are more Democrats hailing from districts between $-k$ and $-x$ than from the remaining districts, so that the median Democratic caucus position is to the left of $-x$. Conversely, among Republicans, the median caucus position would be to the right of $x$. Thus, if expectations are adaptive and $Z(x)$ is positive, then we should observe increased policy divergence in the next period, and vice versa.

Therefore, an equilibrium with policy differentiation is stable if and only if $Z'(x) < 0$ at the point of the equilibrium because then a shock that decreases $x$ produces $Z > 0$, and thus leads to an increase of $x$ in the next period; and conversely, a shock that increases $x$ yields $Z < 0$ and thus leads to a decrease of $x$ in the next period.

In the proof of Proposition 2, we show that $Z(\cdot)$ is concave. Therefore, if $Z'(0) > 0$ – such as in the left-hand panel of Figure 2 – so that an equilibrium with $x > 0$ exists, it must be the case that $Z'(x) < 0$ at the interior equilibrium $\bar{x}$. Conversely, if $Z'(0) < 0$, such as in the right-hand panel of Figure 2, then only the equilibrium without policy differentiation exists, and it is stable. Summarizing this discussion, we have

**Corollary 1.** Suppose that the distribution of districts is uniform on $[-k, k]$ and the valence distribution $\phi$ is single-peaked and symmetric around 0. Then, there is a unique stable equilibrium. If $\phi_0 > \frac{\alpha}{2k^2}$, this stable equilibrium involves divergent party platforms, while if $\phi_0 \leq \frac{\alpha}{2k^2}$, the unique stable equilibrium has $x_D = x_R = 0$. 


3.3 Comparative statics

We now show that, as valence becomes less important to voters, the extent of policy differentiation between the parties increases in equilibrium. Intuitively, as valence becomes less important for voters, it becomes less likely that a district votes for a candidate from the ideologically disadvantaged party. This effect leads to ideological stratification of Congress, and that in turn implies that the median party member now comes from a more extreme district.

Proposition 3. Suppose that the distribution of districts is uniform on $[-k, k]$ and the valence distribution $\phi$ is single-peaked and symmetric around 0. Furthermore, suppose that $\phi_0 > \frac{\alpha}{2k^2}$, and consider the symmetric equilibrium with policy differentiation where the parties’ equilibrium positions are $x_R = x = -x_D$. An increase in $\alpha$ leads to less policy differentiation: $\frac{dx}{d\alpha} < 0$.

Proposition 3 predicts that reforming the organization of the legislature in a way that affects how much an individual legislator can do for her constituents also has an effect on polarization. For example, consider a reform that makes it harder for individual legislators to acquire “pork barrel” projects that benefit his district—say, the total amount of pork available for the legislature as a whole is reduced by half, and legislators now compete with each other about this smaller prize. Then, the utility that voters in the district have from a legislator with a given ability to attract pork diminishes. As $\alpha$ decreases, the argument for holding on to an incumbent whose party is a bad ideological fit for the district is diminished.
Consequently, we would expect that a reduction in the importance or availability of pork projects leads to ideological polarization.

Another feature that potentially may affect the importance of valence for voters has to do with the geographical shape of districts. We may think of legislators’ valence being related to their ability to bring local public goods or employment on public projects to their district. However, a part of the benefit of these projects will spill over into other districts – say, a firm in the legislator’s district that gets a federal grant may employ some citizens who are residents in other districts, or residents of other districts may benefit from road construction in the district. Such spillovers, and the concomitant reduced incentive to provide such local public goods, are likely more significant when the ratio of district boundary to district area is high, such as in many skillfully gerrymandered districts.

We now turn to an effect that is very similar from a formal point of view, namely the degree of uncertainty about the shock. One of the main explanations in the literature for why we observe policy divergence in elections is the policy-motivated candidates model, following the seminal contributions of Wittman (1983) and Calvert (1985). In those models, candidates choose their position to trade off an increased probability of winning with a more moderate policy against the lower satisfaction for the candidate that comes from implementing a less preferred policy in case the candidate wins.

However, if the preference of the median voter is known, models with policy-motivated candidates have the same equilibrium as the original Downsian model with office-motivated candidates. Therefore, uncertainty (either about the median voter’s preferred policy, or about his evaluation of the candidates’ valences) is essential in these models in order to generate policy divergence. Intuitively, larger uncertainty implies that it is less likely that the election outcome is very close and therefore that a candidate could affect it by compromising and moving to a more moderate position. Thus, the more uncertainty there is in an election about the voters’ preferences, the greater will be the degree of political polarization between the candidates’ policies in equilibrium in these models of policy-motivated candidates. See, for example, Smirnov and Fowler (2007).

In contrast, we will now show that increased uncertainty about valence in our model of political competition in legislative elections reduces polarization. The intuitive reason is that large valence shocks help some Republicans win in liberal districts, and some Democrats win in conservative districts. These representatives have a strong interest in moving their party to a more moderate position in order to stay competitive in their district. In contrast, when valence shocks are generally small, then the overwhelming majority of liberal and conservative districts are won by Democrats and Republicans, respectively, and the two parties thus represent essentially disjunct sets of districts, so that inter-party polarization is
large.

To formally analyze the effect of a change in the distribution of the valence shock on the equilibrium level of policy divergence, consider the following definition of increased valence uncertainty.

**Definition 2 (Increased risk of the valence shock.).** *We say that \( \beta \) parametrizes an increased risk of the valence shock if the following conditions hold:

1. \( \Phi(v, \beta) \) is symmetric around 0 for all \( \beta \); and
2. \( \Phi(v) - \Phi(\beta_0) \leq 0 \) for \( \beta_1 > \beta_0 \);

Note that the second condition says that \( \Phi(v, \beta_1) > \Phi(v, \beta_0) \) for \( v < 0 \) and \( \Phi(v, \beta_1) < \Phi(v, \beta_0) \) for \( v > 0 \). This increased risk definition is slightly stronger than a standard mean-preserving spread because we assume that the \( \Phi \)-distributions for different values of \( \beta \) intersect only once, at 0. However, many common families of distributions satisfy this assumption. For example, it is satisfied if \( \Phi \) is a uniform distribution on \([-\beta, \beta]\), or if \( \Phi \) is a normal distribution \( N(0, \beta^2) \) with standard deviation \( \beta \).

Proposition 4 shows that an increased risk of the valence shock leads to an equilibrium with less polarization.

**Proposition 4.** Let \( \beta \) parametrize an increased risk of the valence shock, and let \( \beta_1 > \beta_0 \). Suppose that the distribution of districts is uniform on \([-k, k]\) and the distribution \( \phi \) is symmetric around 0 and single-peaked. Furthermore, suppose that \( \phi(0, \beta_0) > \frac{\alpha}{\sqrt{\pi}} \), and consider the symmetric equilibrium with policy differentiation where the parties’ equilibrium positions are \( x_R = x(\beta_0) = -x_D \) for the valence distribution \( \Phi(v, \beta_0) \). Then \( x(\beta_1) < x(\beta_0) \).

Figure 3 illustrates Proposition 4. The increased risk of the valence distribution leads to a flatter cumulative distribution function. The resulting purple regions depict liberal districts that move from being represented by Democrats to being represented by Republicans, and conservative districts that move in the other direction. Both of these switches moderate the two parties.

### 3.4 The Effect of Wave Elections

We now analyze the effects of a valence shock favoring the candidates of, say, the Republicans on the positions of the two parties. Intuitively, the effect of such a shock is that some Democrats will be replaced by Republicans. Moreover, those Democrats who hail from conservative and moderate districts are more endangered by the resulting Republican tide.
than those located in liberal districts. Thus, most freshman Republicans in the new legislature represent relatively moderate districts, and therefore their self-preservation interest is to draw the Republican party position towards moderation. In contrast, the surviving Democrats are, on average, more liberal than before because moderate and conservative Democrats got defeated in disproportionate numbers. We would therefore expect that the Democratic position in the new legislature becomes more liberal.

To analyze this setting formally, we assume that the Republican candidates’ valence is now drawn from cdf $\Phi(\cdot)$ which satisfies $\Phi(v) = \Phi(v - s)$, where $s > 0$ denotes the size of the shift by which every Republican’s valence is shifted to higher values. We focus on an “unexpected shock,” in the sense that voters observe their local Republican candidate’s true valence (which is now, on average, higher), but they still think that the valence shock in other districts is distributed according to $\Phi(\cdot)$, rather than $\tilde{\Phi}(\cdot)$. The only consequence of this assumption is that voters still believe that party positions are given by the old equilibrium positions $x^D_0$ and $x^R_0$.\(^{10}\)

Figure 4 displays the Democratic winning probability by district (rather than the Republican function used in the previous figures). The Democratic winning probability can be shown to be $\Phi(-4\bar{v}M - s) = \Phi\left(-4\bar{v} \left[M + \frac{s}{4}\right]\right)$, for $s = 0$ (i.e., the original equilibrium positions $x^D_0$ and $x^R_0$).

\(^{10}\)Alternatively, voters might simply have adaptive expectations, i.e., consider the parties’ positions in the last legislature (before the shock), rather than a perfect foresight expectation of what the party positions will be after this election. In fact, perfect foresight of the effect of the shock would require a very high degree of sophistication from the voters. Assuming adaptive expectations is therefore probably more reasonable than perfect foresight.
Figure 4: Democratic seat losses and Republican seat gains in liberal, moderate and conservative districts.

without shock) and for $s > 0$. Note that this latter winning probability function is simply shifted to the left by $\frac{x}{\beta}$ at every point. The area between the two curves equals the mass of seats that change from Democrats to Republicans as a consequence of the shock. Some of these changing seats are to the left of the old Democratic median (in orange), or to the right of the old Republican median (in red), but the bulk is in moderate districts.

Clearly, in Figure 4, both parties’ medians in the new legislature will move to the left: Democrats lose more seats to the right of $x_{0}^{D}$ than to the left of it, and Republicans gain more seats to the left of $x_{0}^{R}$ than to the right of it. The following Proposition 5 formally shows that the favored party’s position will always move to a more moderate position. The other party moves to a more extreme position, provided that the size of the shock is not too large.

**Proposition 5.** Assume that districts are uniformly distributed on $[-k, k]$, and that the initial situation is characterized by an equilibrium with divergence, i.e., $x_{0}^{R} = x_{0}^{D} = \bar{x}$, and consider the effect of an unexpected valence shock $s$ in favor of the Republican party: That is, while the utility from the Democrat is still $-(\bar{x} - M)^2$, the utility from the Republican party is now $-(\bar{x} - M)^2 + v + s$, where $s > 0$, and we have normalized $\alpha = 1$ for notational convenience. Let $x_{D}(s)$ and $x_{R}(s)$ denote the median Democratic and Republican position. Then:

1. Republicans gain seats, and the median Republican legislator now has a more moderate position: $x_{R}(s) < \bar{x}$. 

15
2. Democrats lose seats, and a sufficient condition for the median Democratic legislator to be more extreme than before (i.e., \( x_D(s) < -\bar{x} \)) is that \( s \in (0, 8\bar{x}^2) \).

Note that the sufficient condition in the second statement that requires that the shock not be too large is very mild. In particular, if \( s = (2\bar{x})^2 \) (i.e., the shock is equal to the midpoint of the admissible range of shocks), then the winning probability of a Democrat from the median Democratic district is reduced to 50% – this is a very large shock that would wipe out half of the Democratic leadership. The sufficient condition requires that the shock is less than twice as big as this shock. Thus, in all reasonable cases we expect the Democrats’ platform to become more extreme (shift to the left) after the pro-Republican electoral shock.

Finally, note that “wave elections” in our setting may have longer lasting dynamic effects. If voters have adaptive expectations, i.e., they assume that the party positions that are relevant for their choice in this election are given by the party positions in the last period, then a shock that favors the Republicans in period \( t \) also has an effect in later periods. By moving Republicans to a more moderate position, and Democrats to a more extreme one, the earlier shock has an effect that favors the Republicans even after the valence distribution has returned to normal.

3.5 Gerrymandering

Our model also provides a useful framework for analyzing the effects of gerrymandering. Partisan gerrymandering is often described as an attempt to generate a few districts that are packed with as many supporters of the other party as possible, while generating a larger number of districts that are moderately favoring one’s own side. In our model, gerrymandering will not only affect the the expected number of seats each party wins, but also their platform locations.

To capture this intuition formally, suppose that Republicans are in control of the gerrymandering process and that, in the initial situation, all district median voters are located at 0. By gerrymandering, Republicans generate a proportion \( \rho \) of liberal districts where the median is located at \(-2h\), and a proportion \( 2\rho \) of slightly conservative districts where the median is located at \( h \). Note that, this way, the “average median” in society remains at zero – since liberal districts move more towards the left than conservative districts (i.e., they are “packed” with liberal voters), more moderately conservative districts than very liberal districts can be created. In the remaining \( 1 - 3\rho \) of districts, the median voter remains at 0. Evidently, \( 0 < \rho \leq 1/3 \).

What are the effects of this Republican gerrymander? The proposition below shows that
there are essentially three different ranges of $\rho$ with very different implications.

For $\rho$ very small, both parties remain moderate in the sense that their median legislator remains at 0. As $\rho$ increases, we get into a medium range where the gerrymander “backfires.” While the Democrat’s median position remains at moderate at 0, the median Republican position shifts to $h$, and this decreases the Republican’s overall seat share below 50 percent. Finally, for $\rho$ sufficiently large, both parties’ positions shift to the extreme positions, but since the Republican position is more moderate from the point of view of the moderate districts’ median voters (located at 0), the gerrymander is successful for Republicans in terms of increasing their winning probability and seat share.

**Proposition 6.** There exist values $\rho_0 < \rho_1, \rho_2 \leq 1/3$ such that

1. if $\rho \leq \rho_0$, then $x_D = x_R = 0$ is the unique equilibrium;

2. if $\rho \in (\rho_0, \rho_1]$, then $x_D = 0$ and $x_R = h$ is an equilibrium; moreover, $\rho_1 < 1/3$ if and only if $\Phi(5h^2) > 2\Phi(-h^2)$.

3. if $\rho \in (\rho_2, 1/3]$, then $x_D = -2h$ and $x_R = h$ is an equilibrium; moreover, $\rho_2 < 1/3$ if and only if $\Phi(9h^2) > 2/3$.

In the first type of equilibrium, Democrats and Republicans have the same winning probability. In the second class of equilibria, which party wins a majority depends on $h$, $\rho$ and the shape of $\Phi$; a sufficient condition for Democrats winning a majority is $\rho \leq 1/5$. In the third class of equilibria, Republicans win a majority in the legislature.

Proposition 6 shows that there are three different regimes that govern the effectiveness of gerrymandering, starting from a completely homogeneous polity. If the number of gerrymandered districts is small, then it does not affect the position of the median Democrat or Republican, and consequently, the unique equilibrium remains such that both parties are located at 0 and are equally competitive in all districts.

In the second class of equilibria, which obtains for intermediate values of $\rho$, the median Democrat remains moderate, but the median Republican shifts to $h$. Thus, Democrats are advantaged in the moderate districts, i.e., the national median voter prefers the Democratic position to the Republican one. Yet, interestingly, this does not guarantee that Democrats win in every equilibrium of this class. The reason is that, for large $\rho$ there are more conservative-leaning districts than moderate and liberal-leaning ones, and the Republicans advantage in these conservative districts might just be enough to outweigh the Democrats’ advantage in the liberal and moderate districts. If the number of gerrymandered districts is sufficiently small, though, this cannot happen, and in this case, Democrats actually become more likely to win through this Republican gerrymander.
Eventually though, as $\rho$ approaches $1/3$ (i.e., all districts become either liberal-leaning or conservative-leaning), the equilibrium moves to the third class where both parties become extreme, and since the majority of districts then is conservative-leaning, Republicans are guaranteed to win a majority in this class of equilibria.

4 Empirical Analyses

4.1 Electoral Uncertainty and Polarization in State Legislatures

One of the key predictions of the model is that if electoral uncertainty increases, then the degree of polarization between the parties should decrease (Proposition 4). Recall that this is the opposite of the prediction from other models, such as the Calvert-Wittman model. We can test this prediction, at least in a crude fashion, by studying U.S. state legislatures. We find that polarization is strongly and negatively correlated with measures of electoral uncertainty.

McCarty and Shor estimated ideological scores for almost 21,000 state legislators from all 50 states elected over the period 1993 to 2014. They calibrated the scores using the results of a large-scale survey (NPAT) in order to make the scores comparable across states. The scores are oriented so that more conservative legislators have higher scores.

We use this data to construct a polarization measure as follows. Let $x_{Rit}^H$ be the median location of Republicans in the lower house in state $i$ in year $t$, let $x_{Dit}^H$ be the median location of Democrats, and let $Polarization_{it}^H = x_{Rit}^H - x_{Dit}^H$ be the gap between party medians. Define $x_{Rit}^S$, $x_{Dit}^S$ and $Polarization_{it}^S$ analogously for state senates. Finally, let $Polarization_{it} = (Polarization_{it}^H + Polarization_{it}^S)/2$ be the average of the two chamber inter-party gaps.

Updating the data used in Ansolabehere and Snyder (2002) and Hirano and Snyder (2014), we have constructed a dataset of election results for all offices elected statewide in each state—governor, Lt. governor, attorney general, secretary of state, state treasurer, etc.—as well as state-level presidential election results, for the period 1988–2014. Let $Std\ Dev_{it}$ be the standard deviation of the Democratic vote-share across all offices elected in state $i$ between years $t-3$ and $t$. This is a proxy for the electoral uncertainty facing candidates.

---


12The scores range from about -3.5 to 5; the overall mean is 0.02 and the overall standard deviation is 0.89. For Democrats overall, the mean is -0.74 and the standard deviation is 0.53, and for Republicans the mean is 0.75 and the standard deviation is 0.44.

13Nebraska has a unicameral legislature so we set $Polarization_{it} = Polarization_{it}^S$ for Nebraska. For state-years with missing data for one chamber we set $Polarization_{it}$ equal to the polarization score of the non-missing chamber.

14We only include cases with 4 or more races.
When $\text{Std Dev}$ is large, then it is likely that idiosyncratic factors specific to particular races, such as incumbency, candidate attributes and positions on particular issues have a substantial effect on the vote in the district. Furthermore, it could be that the partisan composition of the district electorate exhibits large swings from election to election. In contrast, when $\text{Std Dev}$ is small, then the vote is probably driven more by stable partisan loyalties.\footnote{We investigated alternative measures of electoral uncertainty, such as the standard deviation of the Democratic vote-share across all offices elected in state $i$ in year $t$ alone, and find results quite similar to those reported below.}

Figure 5 shows a scatterplot of Polarization and $\text{Std Dev}$.\footnote{We match $\text{Std Dev}$ for state $i$ and year $t$ to polarization among legislators in state $i$ elected in year $t$ and serving in years $t+1$ and $t+2$.} The correlation between the two variables is -0.451.

Table 1 presents regression results showing that the correlation between the two variables is not only highly significant both substantively and statistically, but also that the correlation remains even after controlling for a time trend. This is true for each chamber separately as well (columns 3–6). We do not claim the results in Table 1 establish causality of any sort, of course, but they do indicate a surprisingly large, negative, correlation.\footnote{For example, the coefficients might reflect causation in the opposite direction—i.e., as the parties become more polarized voters might engage in more purely partisan voting, which would reduce $\text{Std Dev}$.}
Table 1: Polarization vs. Electoral Uncertainty in State Legislatures

<table>
<thead>
<tr>
<th>Variable</th>
<th>House/Senate Average</th>
<th>House Only</th>
<th>Senate Only</th>
</tr>
</thead>
<tbody>
<tr>
<td>Standard Deviation of Vote</td>
<td>-6.221</td>
<td>-6.434</td>
<td>-5.965</td>
</tr>
<tr>
<td></td>
<td>(0.576)</td>
<td>(0.640)</td>
<td>(0.649)</td>
</tr>
<tr>
<td>Year</td>
<td>0.010</td>
<td>0.008</td>
<td>0.008</td>
</tr>
<tr>
<td></td>
<td>(0.003)</td>
<td>(0.004)</td>
<td>(0.004)</td>
</tr>
<tr>
<td># Observations</td>
<td>460</td>
<td>413</td>
<td>417</td>
</tr>
</tbody>
</table>

Dependent variable = Polarization in state legislature or state legislative chamber. Standard errors are in parentheses.

4.2 Electoral Uncertainty and Polarization in the U.S. House

We can perform a similar analysis of the effects of electoral uncertainty on polarization for the U.S. Congress. This has one advantage and one disadvantage relative to state legislatures. The advantage is that we can measure electoral uncertainty at the district level, which corresponds more closely to parameter $\beta$ in the model. The disadvantage is that we can only exploit variation over time.

We focus on the U.S. House for the period 1972 to 2012. For each district $i$ and year $t$, let $\text{Std Dev}_{it}$ be the standard deviation of the Democratic vote-share across all presidential and congressional elections held in the district between years $t-6$ and $t+6$. Let $\text{Std Dev}_t = (1/435) \sum_{i=1}^{435} \text{Std Dev}_{it}$ be the average standard deviation across districts for each year $t$. This is a proxy for the electoral uncertainty facing candidates in year $t$.

We use the well-known DW-NOMINATE scores to measure ideological positions. Let $x_{Rt}^H$ be the median location of Republicans in year $t$, let $x_{Dt}^H$ be the median location of Democrats, and let $\text{Polarization}^H_t = x_{Rt}^H - x_{Dt}^H$ be the gap between party medians.

We find a very strong negative correlation of $-0.94$ between $\text{Polarization}$ and $\text{Std Dev}$ for the U.S. House, which parallels the results for the case of state legislatures.

The reason why we restrict attention to the period 1972-2012 is that measuring $\text{Std Dev}$ accurately for earlier years is problematic. Specifically, (i) prior to 1952, for many districts, we do not have data on presidential election outcomes; (ii) during the 1950s and early 1960s,
a large number of congressional races were uncontested—in particular, almost 60% of U.S. House races were uncontested in the ten states of the “solid south” during the period 1950-1962 (so we would have to rely almost entirely on the the presidential vote for these cases); and (iii) the Supreme Court’s reapportionment decisions of the mid-1960s (Baker v. Carr and Wesberry v. Sanders) produced multiple redistricting episodes for most states for the 1960s.

4.3 Party Positions and Seat Shares

Another prediction of the model is that there should be a positive correlation between a party’s seat share and how “moderate” the party is, even controlling for the underlying average ideology of voters. This is true whether a party’s seat-share advantage is the result of a partisan tide, i.e., a positive party-wide valence shock (as in Proposition 4), or due to a skewed distribution of district medians, possibly due to gerrymandering (as in Proposition 6).

If the Democratic Party has an advantage in seats, then the median position among Democratic legislators should be further to the right. The median position among Republican legislators should also be further to the right. And, the midpoint between the parties should be further to the right. If the Republican Party has an advantage in seats, then the median position among legislators in both parties, and the midpoint between the parties, should be further to the left.\footnote{Note, this is the opposite of the prediction in Smirnov and Fowler (2007), who analyze a dynamic version of the Calvert-Wittman model.}

To check this prediction, we return to the state legislatures. As above, let $x^H_{Rit}$ be the median location of Republicans in the lower house in state $i$ in year $t$, let $x^H_{Dit}$ be the median location of Democrats, and define $x^S_{Rit}$, $x^S_{Dit}$ analogously for state senates. Also, for state $i$ and year $t$, let $Midpoint^j_{it} = (x^j_{Rit} + x^j_{Dit})/2$ be the midpoint between the two parties in chamber $j$.

We constructed a dataset with the partisan composition of each state legislative chamber using Dubin (2007) and data from the National Conference on State Legislatures.\footnote{See, e.g., http://www.ncsl.org/research/about-state-legislatures/partisan-composition.aspx for the last few years of data.} Let $Dem \ Seat \ Share^j_{it}$ be the share of seats held by Democrats in chamber $j$ of state $i$ in year $t$.

We use voter partisanship to proxy for the average voter ideology in each state and year. More specifically, using the data on statewide offices described above, let $Avg \ Dem \ Vote_{it}$ be the average Democratic vote-share across all offices elected in state $i$ between years $t-3$ and $t$.\footnote{Again, we only include cases with 4 or more races.} We also include second-order and third-order polynomial terms of $Avg \ Dem \ Vote$ in the regressions, to capture possible non-linearities in the relationship between ideology and
partisanship. We also include the variables $t$ and $t^2$, to capture the possibility of trends in the national ideolgical “mood” (e.g., Stimson 1991).

Table 2 presents the results of regressions of the chamber party medians or midpoints on the corresponding Democratic seat shares and the variables to control for average voter ideology. The first row is for state lower houses and the second row is for state senates. In the third row we average the dependent and independent variables across the two chambers in each state. For example, in the first column the dependent variable is $(x_D^H + x_D^S)/2$, and the independent variable is $(Dem\ Seat\ Share^H + Dem\ Seat\ Share^S)/2$.

Table 2: Median Party Positions and Midpoints vs. Democratic Seat Shares

<table>
<thead>
<tr>
<th>Case</th>
<th>Democratic Median</th>
<th>Republican Median</th>
<th>Midpoint</th>
</tr>
</thead>
<tbody>
<tr>
<td>State House</td>
<td>1.226 (0.398)</td>
<td>0.055 (0.316)</td>
<td>0.670 (0.249)</td>
</tr>
<tr>
<td>State Senate</td>
<td>1.292 (0.333)</td>
<td>0.310 (0.317)</td>
<td>0.617 (0.253)</td>
</tr>
<tr>
<td>House/Senate</td>
<td>1.447 (0.410)</td>
<td>0.259 (0.309)</td>
<td>0.780 (0.269)</td>
</tr>
</tbody>
</table>

Dependent variable = Party Median or Midpoint. Cell entries are estimated coefficients on Democratic Seat Share. Standard errors are in parentheses. Number of observations = 403 in all cases.

As the model predicts, all of the estimated coefficients are positive. For the Democrat party medians and Midpoints the coefficients are substantively large and statistically significant. For the Republican party, on the other hand, the coefficients are relatively small and not statistically significant. We are not sure why this is the case and it surely deserves further exploration. In any case, we must of course add the caveat that we do not place any causal interpretation on the estimates. We simply note that they are (broadly) consistent with the model.

4.4 Other Evidence

The prediction in Proposition 3 above is broadly consistent with recent work on the U.S. Congress. Hall and Shepsle (2014) argue that as power shifted from committee leaders to party leaders in the House of Representatives beginning in the 1970’s, voters should have started to place less value on the seniority of their representative, because they would have
understood that it was less valuable to have a senior representative serving as—or next in line as—a committee or subcommittee chair. They document that the electoral value of seniority is significantly lower in the “strong party” regime post 1976 compared to the “weak party” regime before 1976. If we take seniority as one of the main components of valence in this context, then Proposition 3 predicts that polarization in the U.S. House should increase after 1976. This is what the standard time-series plots of polarization based on roll call scores, such as NOMINATE, show.

Finally, we can conduct a limited analysis of the correlates of polarization in state legislatures in 1960 using data from LeBlanc (1969). LeBlanc collected roll call data for 26 state senates for 1959-1960, and calculated the Rice index of “Party Likeness” for each case. Since likeness measures the degree to which the parties vote together, we take \( \text{Party Disagreement} = 1 - \text{Party Likeness} \) as a crude measure of polarization.

We consider two independent variables. The first is \( \text{Std Dev} \), the standard deviation of the Democratic vote-share across all offices elected in each state between 1955 and 1958. This is a measure of electoral uncertainty. The second variable is \( \text{High TPO} \), a dummy equal to 1 for states classified by Mayhew (1986) as having strong “traditional party organizations”—i.e., strong, patronage-based, electoral organizations. As argued in Primo and Snyder (2010), it is likely that voters in states with strong party organizations vote more on the basis of party affiliation rather than the candidate characteristics. If so, then states with \( \text{High TPO} = 1 \) will tend to be states where voters place less weight on candidate valence (i.e., states with relatively low values of \( \alpha \)).

The correlation between \( \text{Party Disagreement} \) and \( \text{Std Dev} \) is -0.43. The sign is consistent with the prediction in Proposition 4—i.e., greater electoral uncertainty is associated with less inter-party polarization. The correlation between \( \text{Party Disagreement} \) and \( \text{High TPO} \) is 0.53. The sign is consistent with Proposition 3—i.e., a lower weight on candidate valence is associated with a higher degree of inter-party polarization. Both correlations are statistically significant.

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24 Relatedly, Ansolabehere and Pettigrew (2013) show that in 2009 the job approval ratings and electoral support of incumbents was unrelated to their seniority.


26 For each roll call \( j \), \( \text{Likeness}_j = 1 - |D_j - R_j| \), where \( D_j \) is the percentage of Democrats voting yes and \( R_j \) is the percentage of Republicans voting yes. Averaging across all roll calls yields the Party Likeness index. We drop three states—Connecticut, New Jersey, and Rhode Island—because they had so few roll calls (6, 13 and 12, respectively) that we do not trust their indices.

27 Mayhew (1986, p. 19-20) defines a traditional party organization as a state or local party organization with the following five characteristics: (1) it has substantial autonomy, (2) it lasts a long time, (3) its internal structure has an important element of hierarchy, (4) it regularly tries to bring about the nomination of candidates for a wide range of public offices, and (5) it relies substantially on “material” incentives, and not much on “purposive” incentives, in engaging people to do organization work or to supply organization support. His scores range from 0 to 5. We set \( \text{High TPO} = 1 \) for states with scores of 4 or 5.
significant at the .05 level.

5 Discussion

This paper presents and analyzes a new model of decentralized political parties. Four assumptions are crucial: (i) each party’s policy platform is chosen by its sitting incumbent officeholders; (ii) these officeholders care first and foremost about their own personal re-election; (iii) there is variation in the underlying voter preferences of the various constituencies represented by the incumbents within each party (e.g., some incumbents within a party represent left-wing districts or right-wing districts while others represent centrist districts); and (iv) local election outcomes are uncertain, due to idiosyncratic valence shocks. The model generates testable comparative statics predictions about the degree of inter-party polarization, the effects of electoral tides, and the effects of gerrymandering. We find evidence consistent with key predictions in data from U.S. state legislatures and the U.S. Congress.

The model can be extended in various ways, some of which might prove interesting. First, one could introduce heterogeneity in goals across incumbents—e.g., some incumbents might care strongly about the fortunes of the party as a whole in addition to their own re-election (this might be the case for those in line to be party or committee leaders, if majority party leaders wield much more power than minority party leaders). Second, one could alter the way the party platforms are chosen, moving away from simple majority rule. A weighted majority rule is one possibility, with higher weights for those who are in line to be party leaders. This might give extra weight to incumbents who represent extreme districts, since they are likely to be those with the most seniority in office and therefore the most experience (and it is likely that political parties, like most organizations, value experience). Finally, one could incorporate other key actors in the model, such as interest groups that fund the parties. This might be especially interesting, since it is not clear whether such actors would serve to increase or decrease polarization, or how they would interact with the incumbent politicians in each party.
Appendix

Proposition 2. Suppose that the distribution of districts is uniform on $[-k, k]$ and the distribution $\phi$ is symmetric around 0 and single-peaked. A symmetric equilibrium with policy differentiation, $x_R = x = -x_D$, exists if and only if $\phi_0 > \frac{\alpha}{4k^2}$. Moreover, if such an equilibrium exists, it is unique.

Proof. An equilibrium is characterized by a zero of the function $Z(x)$ as defined in (8). Clearly, $Z(0) = 0$ because then $\Phi(\cdot)$ is constant in both integrals; this corresponds to the equilibrium without divergence. Also, $Z(1) < 0$, since only the second integral is left (and negative). A sufficient (and, as we will show, also necessary) condition for an equilibrium with policy divergence is therefore that $Z'(0) > 0$.

Differentiating (8) with respect to $x$ yields

$$Z'(x) = \frac{1}{2k} \left[ -2\Phi \left( \frac{4x^2}{\alpha} \right) - \int_{-x}^{-x} \frac{4M}{\alpha} \phi \left( -\frac{4xM}{\alpha} \right) dM + \int_x^k \frac{4M}{\alpha} \phi \left( -\frac{4xM}{\alpha} \right) dM \right] =$$

$$\frac{1}{2k} \left[ -2\Phi \left( \frac{4x^2}{\alpha} \right) + \int_{-k}^{k} \frac{4M}{\alpha} \phi \left( -\frac{4xM}{\alpha} \right) dM - \int_{-x}^{-x} \frac{4M}{\alpha} \phi \left( -\frac{4xM}{\alpha} \right) dM + \int_x^k \frac{4M}{\alpha} \phi \left( -\frac{4xM}{\alpha} \right) dM \right] =$$

$$= \frac{1}{k} \Phi \left( \frac{4x^2}{\alpha} \right) + \frac{1}{k} \int_x^k \frac{4M}{\alpha} \phi \left( -\frac{4xM}{\alpha} \right) dM,$$

(9)

where the first and the second equality use the fact that $\int_{-h}^{h} \frac{4M}{\alpha} \phi \left( -\frac{4xM}{\alpha} \right) dM = 0$ for any $h$ because of the symmetry of $\phi(\cdot)$ (for $h = k$ in the first case, and for $h = x$ in the second).

Differentiating (9) again gives

$$Z''(x) = -\frac{8x}{k\alpha} \phi \left( -\frac{4x^2}{\alpha} \right) - \frac{4x}{k\alpha} \phi \left( -\frac{4x^2}{\alpha} \right) - \frac{16}{k\alpha^2} \int_x^k M^2 \phi' \left( -\frac{4xM}{\alpha} \right) dM < 0,$$

(10)

because $\phi'(t) \geq 0$ for all $t \leq 0$ by the assumption that $\phi(\cdot)$ is single-peaked at 0. Thus, a necessary and sufficient condition for an equilibrium with policy differentiation to exist is $Z'(0) > 0$ (remember that $Z(0) = 0$ and $Z(1) < 0$).

Substituting $x = 0$ into (9) yields

$$-\frac{1}{k} \Phi(0) + \frac{1}{k} \int_0^k \frac{4M}{\alpha} \phi(0) dM = \frac{1}{k} \left[ -\frac{1}{2} + \frac{2k^2}{\alpha} \phi_0 \right]$$

(11)

where $\phi_0 = \phi(0)$. Thus, a necessary and sufficient condition for the existence of an equilibrium with policy differentiation is that $\phi_0 > \frac{\alpha}{4k^2}$, i.e., the valence shock is sufficiently concentrated around 0. Furthermore, $Z'' < 0$ guarantees that there is at most one equilib-
Proposition 3. Suppose that the distribution of districts is uniform on \([-k,k]\) and the distribution \(\phi\) is symmetric around 0 and single-peaked. Furthermore, suppose that \(\phi_0 > \frac{\alpha}{4k^2}\), and consider the symmetric equilibrium with policy differentiation where the parties’ equilibrium positions are \(x_R = x = -x_D\). An increase in \(\alpha\) leads to less policy differentiation: \(\frac{dx}{d\alpha} < 0\).

Proof. It is useful to rearrange (8) as follows

\[
Z(x, \alpha) = \frac{1}{2k} \left[ \int_{-k}^{-x} \Phi \left( \frac{-4xM}{\alpha} \right) dM - \int_{-x}^{k} \Phi \left( -\frac{4xM}{\alpha} \right) dM \right]
\]

\[
= \frac{1}{2k} \left[ \int_{x}^{k} \Phi \left( \frac{4xM}{\alpha} \right) dM - \int_{-x}^{x} \Phi \left( -\frac{4xM}{\alpha} \right) dM - \int_{-x}^{k} \left[ 1 - \Phi \left( \frac{4xM}{\alpha} \right) \right] dM \right]
\]

\[
= \frac{1}{2k} \left[ \int_{x}^{k} \left[ 2\Phi \left( \frac{4xM}{\alpha} \right) - 1 \right] dM - 2x \cdot \frac{1}{2} \right]
\]

(12)

To show that the positive solution of \(Z(x)\) decreases in \(\alpha\), it is sufficient to show that \(\frac{\partial Z}{\partial \alpha} < 0\) for \(x > 0\). Differentiating (12) with respect to \(\alpha\) yields

\[
\frac{\partial Z}{\partial \alpha} = \frac{1}{2k} \left\{ \int_{x}^{k} -\frac{4xM}{\alpha^2} \left[ 2\phi \left( \frac{4xM}{\alpha} \right) - 1 \right] dM \right\} = -\frac{4x}{k\alpha^2} \int_{x}^{k} M \phi \left( \frac{4xM}{\alpha} \right) dM < 0, \tag{13}
\]

for all \(x > 0\).

Proposition 4. Let \(\beta\) parametrize an increased risk of the valence shock, and let \(\beta_1 > \beta_0\). Suppose that the distribution of districts is uniform on \([-k,k]\) and the distribution \(\phi\) is symmetric around 0 and single-peaked. Furthermore, suppose that \(\phi_0(0, \beta_0) > \frac{\alpha}{4k^2}\), and consider the symmetric equilibrium with policy differentiation where the parties’ equilibrium positions are \(x_R = x(\beta_0) = -x_D\) for the valence distribution \(\Phi(v, \beta_0)\). Then \(x(\beta_1) < x(\beta_0)\).

Proof. Rewriting the last line in (12), we have (with a slight abuse of notation)

\[
Z(x, \alpha, \beta) = \frac{1}{2k} \left[ \int_{x}^{k} \left[ 2\Phi \left( \frac{4xM}{\alpha} \right) - 1 \right] dM - x \right]
\]

\[
= \frac{1}{k} \int_{x}^{k} \Phi \left( \frac{4xM}{\alpha} \right) dM - \frac{k - x}{2k} - x.
\]

(14)

Since there is an equilibrium with policy divergence for \(\beta_0\), and Proposition 3 shows that there is a unique equilibrium with policy divergence for every distribution \(\Phi(v, \beta)\), showing...
$Z(x(\beta_0), \alpha, \beta_1) < 0$ is necessary and sufficient for the change in the distribution to reduce the equilibrium level of policy divergence.

Integration by substitution, using $\psi(M) = \frac{4x}{\alpha} M$ so that $\psi'(M) = \frac{4x}{\alpha}$, shows that

$$
\int_{x}^{\beta} \Phi\left(\frac{4x}{\alpha} M, \beta\right) \frac{4x}{\alpha} \psi(M) dM = \int_{\psi(x)}^{\psi(\beta)} \Phi(u, \beta) du.
$$

Thus,

$$
Z(x, \alpha, \beta) = \frac{\alpha}{4xk} \int_{\psi(x)}^{\psi(\beta)} [\Phi(t, \beta) - \Phi(t, \beta_0)] dt < 0,
$$

Thus,

$$
Z(x, \alpha, \beta) = \frac{\alpha}{4xk} \int_{\psi(x)}^{\psi(\beta)} [\Phi(t, \beta) - \Phi(t, \beta_0)] dt < 0.
$$

Proposition 5. Assume that districts are uniformly distributed on $[-k, k]$, and that the initial situation is characterized by an equilibrium with divergence, i.e., $x_0^R = -x_0^D = \bar{x}$, and consider the effect of an unexpected valence shock $s$ in favor of the Republican party: That is, while the utility from the Democrat is still $-(\bar{x} - M)^2$, the utility from the Republican party is now $-(\bar{x} - M)^2 + v + s$, where $s > 0$, and we have normalized $\alpha = 1$ for notational convenience. Let $x_D(s)$ and $x_R(s)$ denote the median Democratic and Republican position. Then:

1. Republicans gain seats, and the median Republican legislator now has a more moderate position: $x_R(s) < \bar{x}$.

2. Democrats lose seats, and a sufficient condition for the median Democratic legislator to be more extreme than before (i.e., $x_D(s) < -\bar{x}$) is that $s \in (0, 8\bar{x}^2)$.

Proof. The Democratic candidate wins in district $M$ with probability

$$
Prob(D \text{ wins in district } M) = \Phi\left((\bar{x} - M)^2 - (\bar{x} - M)^2 - s\right) = \Phi\left(-4\bar{x}M - s\right) = \Phi\left(-4\bar{x}\left(M + \frac{s}{4\bar{x}}\right)\right).
$$

Clearly, an increase in $s$ reduces the Democrats’ winning probability in all districts, proving the corresponding claims in the statement.

To prove that the Republican median moves to the left for $s > 0$, we have to show that Republicans gain more seats in districts with $M < x_R^0$ than in those with $M > x_R^0$. In this case, it follows that the Republican median in the new legislature must be to the left of $x_R^0$. 27
Figure 6: Republican seat gains to the left and right of $x_R^0 = \bar{x}$

It is useful to consider Figure 6, and to remember that the new winning probability function is simply shifted to the left by $\frac{s}{4\bar{x}}$ at every point from the old one. The fact that the winning probability function is convex in $M$ at $M = \bar{x}$ implies that area 1 is larger than area 2 in Figure 6.

Integrating vertically rather than horizontally, the figure implies that the Republican seat gains to the left of $\bar{x}$, $RSG_-$, are greater than area 1, plus

$$\frac{s}{4\bar{x}}[\Phi(4\bar{x}k - s) - \Phi(-4\bar{x}^2)],$$

while the Republican seat gains to the right of $\bar{x}$, $RSG_+$, are smaller than area 2, plus

$$\frac{s}{4\bar{x}}[\Phi(-4\bar{x}^2 - s) - \Phi(-4\bar{x}k - s)].$$

Thus, we have

$$RSG_- - RSG_+ > \frac{s}{4\bar{x}}[\Phi(4\bar{x}k - s) - \Phi(-4\bar{x}^2) - \Phi(-4\bar{x}^2 - s) + \Phi(-4\bar{x}k - s)]$$

$$\geq \frac{s}{4\bar{x}}[\Phi(4\bar{x}k - s) + \Phi(-4\bar{x}k - s) - \Phi(-4\bar{x}^2) - \Phi(-4\bar{x}^2 - s)]$$

$$\geq \frac{s}{4\bar{x}}[1 - 2\Phi(-4\bar{x}^2)] > 0,$$

because $\Phi(4\bar{x}k - s) + \Phi(-4\bar{x}k - s) > \Phi(4\bar{x}k) + \Phi(-4\bar{x}k) = 1$ by symmetry of $\Phi$, and $\Phi(-4\bar{x}^2) < 1/2$. This proves the first claim.
For the second claim, it is useful to consider Figure 7. Democratic seat losses to the left of $x^0_D$ are equal to areas 1 and 2 in the figure, while Democratic seat losses to the right of $x^0_D$ are equal to areas 3 and 4.

![Figure 7: Democratic seat losses to the left and right of $x^0_D = -\bar{x}$](image)

The fact that the winning probability function is concave in $M$ at $-\bar{x}$ (which, itself, is a consequence of $\phi$ being single-peaked) implies that area 2 is smaller than area 3. Area 1’s size is

$$s \frac{1 - \Phi(4\bar{x}^2)}{4\bar{x}}$$

while area 4’s size is

$$s\Phi(4\bar{x}^2 - s)$$

Since $\Phi$ is symmetric, it follows that area 4 is larger than area 1 if $s < 8\bar{x}^2$, as claimed.

**Proposition 6.** There exist values $\rho_0 < \rho_1, \rho_2 \leq 1/3$ such that

1. if $\rho \leq \rho_0$, then $x_D = x_R = 0$ is the unique equilibrium;
2. if $\rho \in (\rho_0, \rho_1]$, then $x_D = 0$ and $x_R = h$ is an equilibrium; moreover, $\rho_1 < 1/3$ if and only if $\Phi(5h^2) > 2\Phi(-h^2)$.
3. if $\rho \in (\rho_2, 1/3]$, then $x_D = -2h$ and $x_R = h$ is an equilibrium; moreover, $\rho_2 < 1/3$ if and only if $\Phi(9h^2) > 2/3$.

In the first type of equilibrium, Democrats and Republicans have the same winning probability. In the second class of equilibria, which party wins a majority depends on $h$, $\rho$ and the shape...
of \( \Phi \); a sufficient condition for Democrats winning a majority is \( \rho \leq 1/5 \). In the third class of equilibria, Republicans win a majority in the legislature.

**Proof.** Proposition 1 implies that there is always an equilibrium where \( x_D = x_R = 1 \) (albeit it is unstable if there are other equilibria).

Consider a profile where \( x_D = 0 \) and \( x_R = h \), so that the median voters in the left districts have a policy preference of \((3h)^2 - (2h)^2 = 5h^2\) for the Democrat; the median voters in centrist districts have a policy preference of \(h^2\) for the Democrat; and the voters in the conservative districts have a preference of \(h^2\) for the Republican. For this profile to be an equilibrium, it has to be true that the Democratic median is at 0, so

\[
\rho \Phi(5h^2) < (1 - 3\rho)\Phi(h^2) + 2\rho\Phi(-h^2) \tag{19}
\]

and that the Republican median is at \( h \), so

\[
\rho \Phi(-5h^2) + (1 - 3\rho)\Phi(-h^2) < 2\rho\Phi(h^2). \tag{20}
\]

For both equations to be satisfied, it must be true that

\[
\rho \in \left( \frac{\Phi(-h^2)}{2\Phi(h^2) + 3\Phi(-h^2) - \Phi(5h^2)} , \frac{\Phi(h^2)}{\Phi(5h^2) + 3\Phi(h^2) - 2\Phi(-h^2)} \right) \tag{21}
\]

We have to show that this interval is non-empty. Using symmetry (i.e., \( \Phi(-h^2) = 1 - \Phi(h^2) \) etc.) and using \( t = \Phi(h^2) \) and \( v = \Phi(5h^2) \) for notational convenience, we have to show that

\[
\frac{1 - t}{4 - t - v} < \frac{t}{v + 5t - 2}, \tag{22}
\]

which simplifies to \( 2 - 3t + 4t^2 - z > 0 \). Since \( z \leq 1 \), it is sufficient that \( 1 - 3t + 4t^2 > 0 \), which is always satisfied.

We also need to show that the lower limit of the interval in (21) is smaller than 1/3 in order to guarantee that there are values of \( \rho \) in the relevant range. Note that \( \frac{1 - t}{4 - t - v} \leq \frac{1 - t}{3 - (t + v + 1)} < \frac{1 - t}{3 - 3t} = \frac{1}{3} \), thus proving that the interval on the right-hand side of (21) is non-empty.

Finally, if \( \Phi(5h^2) > 2\Phi(-h^2) \), then the upper limit of the interval in (21), \( \rho_1 = \frac{\Phi(h^2)}{\Phi(5h^2) + 3\Phi(h^2) - 2\Phi(-h^2)} \), is smaller than 1/3. Otherwise, \( \rho_1 = 1/3 \).

Consider now a profile where both parties have medians from the gerrymandered districts \((x_D = -2h, x_R = h)\). For this to be an equilibrium, it must be true that the Democratic median is at \(-2h\),

\[
\rho \Phi(9h^2) < (1 - 3\rho)\Phi(-3h^2) + 2\rho\Phi(-9h^2), \tag{23}
\]
and that the Republican median is at $h$, so

$$\rho \Phi(-9h^2) + (1 - 3\rho)\Phi(3h^2) < 2\rho\Phi(9h^2). \quad (24)$$

For both equations to be satisfied, it must be true that

$$\rho > \max\left(\frac{\Phi(-3h^2)}{3\Phi(-3h^2) + \Phi(9h^2) - 2\Phi(-9h^2)}, \frac{\Phi(3h^2)}{3\Phi(3h^2) + 2\Phi(9h^2) - \Phi(-9h^2)}\right) \quad (25)$$

The second term on the right-hand side of (25) is guaranteed to be smaller than 1/3 because $2\Phi(9h^2) > \Phi(-9h^2)$. For the first term to be smaller than 1/3, it is necessary and sufficient that $\Phi(9h^2) > 2\Phi(-9h^2)$. This simplifies to $\Phi(9h^2) > 2/3$.

It is evident that in the first class of equilibria ($x_D = x_R = 0$), Democrats and Republicans have the same winning probability.

In the third class of equilibria ($x_D = -2h$, $x_R = h$), the Republican seat share is

$$\rho \Phi(-9h^2) + (1 - 3\rho)\Phi(3h^2) + 2\rho\Phi(9h^2) > 2\rho\frac{1}{2} + (1 - 3\rho)\Phi(3h^2) + \rho\Phi(9h^2) > \frac{1}{2}. \quad (26)$$

In the second class of equilibria ($x_D = 0$, $x_R = h$), the Democratic seat share is given by

$$SS_D = \rho\Phi(5h^2) + (1 - 3\rho)\Phi(h^2) + 2\rho\Phi(-h^2) \quad (27)$$

Clearly, if $\rho \leq 1/5$, then the weight on the second term is larger than the weight on the third term, and so $SS_D > 1/2$.

To analyze the case of $\rho > 1/5$, substitute again $t = \Phi(h^2)$ and $v = \Phi(5h^2)$, and differentiate (27) with respect to $\rho$, which yields

$$\frac{\partial SS_D}{\partial \rho} = v - 3t + 2(1 - t) = 2 + v - 5t < 2 + \frac{1}{2} + 5\left[t - \frac{1}{2}\right] = 0, \quad (28)$$

where the inequality follows from the fact that single-peakedness of $\phi$ implies that $\Phi(\cdot)$ is concave in $h$ for $h > 0$. Thus, substituting the highest feasible value of $\rho = \frac{t}{v+3t-2(1-t)}$ for this class of equilibria, the Democratic seat share is greater or equal to

$$\frac{tv}{v+5t-2} + \frac{v - 2(1-t)}{v+5t-2}t + \frac{2t(1-t)}{v+5t-2} = \frac{2tv}{v+5t-2} \quad (29)$$

This will be greater than 1/2 if and only if

$$2 + 4tv - 5t - v > 0. \quad (30)$$
For example, if \( t = \Phi(h^2) = 0.6 \) and \( v = \Phi(5h^2) = 0.8 \), then (30) holds so that Democrats win for every level of \( \rho \). However, if \( t = \Phi(h^2) = 0.7 \) and \( v = \Phi(5h^2) = 0.8 \), then (30) is not satisfied, and consequently Republicans win, at least for high levels of \( \rho \).
References


