Credibility and Commitment in Crisis Bargaining*

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Abstract

Although scholars of international security share a skepticism for the extent to which agreements can be externally enforced, much of the extant game-theoretic work involves strong forms of commitment. We build on the canonical model of crisis bargaining to gain insights about the role of two forms of commitment in bargaining—the ability to commit to a settlement and the ability to commit to end negotiations and initiate war fighting. We show that, contrary to the received wisdom, allowing a proposer to retract their offer after learning of its acceptance need not lead to a series of ever increasing demands. Instead, a rational actor is best off honoring the accepted agreement in crisis bargaining, even though the act of accepting an offer changes the the proposer's beliefs about the distribution of power and the accepting country's resolve. On the other hand, allowing a proposer to continue bargaining in lieu of fighting does change the dynamics of bargaining, although this effect diminishes as players become more patient. Finally, when there is not commitment to offers or fighting after a rejected proposal, the one period play of the ultimatum game remains a perfect bayesian equilibrium to the dynamic bargaining game.

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1 Introduction

If you give in, you will immediately be confronted with some greater demand, since they will think that you only gave way on this point out of fear.
–Pericles, in Thucydides, 1972 (p.402)

Modern theories of war describe international crises as bargaining problems and then explain war as the result of bargaining failure. In this view crises are bargaining situations where a dispute over a divisible good or policy can result in some form of peaceful settlement or war. These theoretical models of crisis bargaining rely heavily on economic bargaining models first created to study different phenomenon. Therefore, two import an questions are: does bargaining between countries differ in important ways from economic bargaining and do these differences imply important changes in the predictions of international bargaining outcomes?

In almost all models of bargaining, whether they are applied to bilateral trade, labor strikes, or war, uncertainty is a key feature of the environment. When there is incomplete information, players learn from the bargaining process, given the proposals that are made and the responses of actors to those proposals. To a great extent the analysis of the learning process in bargaining has been shaped by studying bargaining in economic settings. For example, haggling in markets, buying a house, or sales in a retail store all have a similar structure. In these environments, to the extend that there is learning, proposals are made and sometimes rejected. Rejected offers lead to decreases in prices over time, though there is some cost to delay, and once an agreement is reached the bargaining–and the models of bargaining–end (Ausubel, Cramton and Deneckere 2002). In the crisis bargaining setting many have used similar models of bargaining to analyze the influence of asymmetric information on the probability of war (Brito and Intriligator 1985, Fearon 1995, Fey and Ramsay 2011). The standard economics model also has been extended by many scholars to include new and important features when studying crisis bargaining. For example, Filson and Werner (2002), Slantchev (2003), Smith and Stam (2004), Powell (2004) all add the inside option of fighting costly battles during the bargaining process to allow learning from both equilibrium actions and possibly battlefield outcomes. Common to all these models, the learning happens during the disagreement phase of the bargaining. Because the game ends when an agreement is reached, and the settlement is at a minimum implemented for a single period, this commitment to agreement has ruled out immediate strategic responses for acceptance.

The modern theory of war, largely in a separate literature, has also considered another common cause of bargaining failure: the commitment problem. These arguments focus on the lack of enforcement for peaceful agreements and how concerns about changes in the future preferences or relative military capabilities influence the prospects for peace(Fearon 1995, Powell 2006, Leventoğlu and Slantchev 2007, Tingley 2011). Once the commitment problem is acknowledged, many questions arise regarding existing models. Clearly, standard theoretical models have strong commitment assumptions. In the ultimatum bargaining game, for example, there is a commitment to accepted offers being enforced and to rejection of offers causing war.

Obviously, a game-theoretic model starts with a description of the rules of the game. These rules generate the extensive form to be analyzed and describes which actions are possible at which decision nodes. Implicit in such a structure is the assumption that players are committed to following these rules. For example, it is not possible for players to choose something not permitted by the extensive form at a decision-node. In this light, standard theoretical models have strong commitment assumptions. In the standard ultimatum bargaining game there is a commitment to accepted offers being enforced and to rejection of offers causing war.

Using formal theory to understand commitment simultaneously clarifies and obscures these issues. While ideas like sequential rationality require in game commitments to be credible the choice of extensive form, by definition, presupposes players are committed to that particular set of actions and timing. That is, any game form used to study commitment starts with e assumption that players are committed to the game under analysis.

In the standard bargaining model of war there are two kinds of commitments we will relax. First we are interested in bargaining when there is no commitment to enacting accepted agreements. Second we consider crisis bargaining with no commitment to fighting. Finally we are interested in the case where there is no commitment to agreements or fighting. Unlike Fearon (1995), Leventoğlu and Slantchev (2007), Powell (2006) we focus on how the commitment problem operates in the presence of incomplete information. Wolford, Reiter and Carrubba (2011) consider a similar situation, but our analysis differs from theirs. In the Wolford, Reiter and Carrubba model the commitment problem arises from a change in "long-run" fundamentals–i.e. a between period shift in the known distribution of power. We focus instead on how changes in beliefs, and hence short-run non-fundamental aspects of the crisis environment, might influence bargaining and the probability of war. Continuing with the example of ultimatum bargaining, we ask: if accepting an offers signals weakness of power or high cost of war, does that open an opportunity for the proposer to make further demands? If an offer is rejected, might the proposer want to make a last-ditch effort to avoid a conflict? Does the first dynamic increase the risk of war? Does the second lower the risk of war?

To answer these questions we start with the benchmark of the standard crisis bargaining model from Fearon (1995). One might think, like Schelling (1960, p.93), "[o]ne never quite knows in the course of a diplomatic confrontation how opinion will converge on signs of weakness. One never quite knows what exits will begin to look cowardly to oneself or to the bystanders or to one's adversary. It would be possible to get into a situation in which either side felt that to yield now would create such an asymmetric situation, would be such a gratuitous act of surrender, that whoever backed down could not persuade anybody that he wouldn't yield again tomorrow and the day after." We find, contrary to this concern, learning that the opponent is willing to accept the proposed offer does not create an incentive for the proposer to retract the accepted proposal and try to renegotiate the deal.

An analogue to our concern about commitment to not renegotiate after an offer is accepted surfaces in the contracting literature, but with very different results. Hart and Tirole (1988) consider the distinction between rental and sales contracts. In a rental market the seller can renegotiate the price for rental in subsequent periods, which is analogous to being able to raise the price after learning that the buyer was willing to pay a particular rental fee, modulo a one period flow payoff. In contrast the sales market does not allow such renegotiation because the property right changes once a deal is struck. In analyzing both types of markets, Hart and Tirole assume that following rejection the seller can offer a lower price. Thus their model of the rental market involves relaxing commitment to delivery and to bargaining failure. Put loosely they have relaxed both the commitment to "taking it" and "leaving it."[1] Here a ratchet effect is present. If the buyer accepts an offer/price she reveals information about her valuation and this information can be used against her when the seller announces future rental prices. As a result very little price discrimination occurs in equilibrium. [2]

Fearon (2007) has recently built on the Hart and Tirole framework to allow for infor-

[1]See also Laffont and Tirole (1988).

[2]The paper goes on to show that with a richer set of contracts it is possible to sustain renegotiation proof equilibria in which the seller does get to price discriminate in the rental market, thus establishing an equivalence between seller revenue in rental and sales markets.

mation transmission through the fighting of battles. His model is similar to that of Hart and Tirole, but in every period in which a deal is not reached there is the possibility of an informative battle. As a model of learning by fighting this contribution is interesting and distinct. The starting point, however, of Fearon's paper is the argument that the ratchet effect that surfaces in the rental models will also be present in the security context. On this point, further elaboration is needed. We find that an important distinction between the trade and security contexts leads to very different results about the importance of commitment. The bilateral trade models typically assume that the second mover has private information about her valuation of the item up for sale or rent. In the security context the private information is thought to be about the payoff to fighting. Thus, in the trade models there is private information about the agreement outcome and in the security context there is private information about the disagreement or outside option. In the trade context, when the seller can offer a second price following rejection of her first offer it is possible for her to screen the buyer. The cost of discounting and the fact that a buyer's agreement payoff depends on her type can be sufficient to satisfy a single-crossing condition. In the security context, however, screening can not be supported by type specific variation in the continuation payoffs to accepting offers because the second player's agreement payoff does not depend on her type. Thus if the proposer is to offer different offers across time, weak types will be willing to pretend to be strong types, rejecting the first offer and then accepting a better subsequent offer. Fearon's model yields results that are closer to Hart and Tirole because there is an exogenous source of information (the possibility of battles which provide information about the player's types in any period of delay).

In what follows we develop our analysis to understand the interaction of incomplete information, learning, and commitment in the crisis bargaining model. In the next section we walk through a two period two type model of retractable offers to give a clear intuition for our results. We then analyze the dynamic two period model with no commitment to agreements and a continuum of types. In the third section we give results when there is no commitment to fig hint after a rejection. Here we show that there is a screening equilibrium where offers become more favorable after rejections. Last we analyze the situation where there is renegotiation of accepted offers and where rejected offers can lead to new proposals rather than war. We show that when both these kinds of commitments are relaxed there are new equilibria to this game that are not equilibria to the game where there is a commitment to fight after rejected offers, but no commitment to enact accepted proposals.

2 No Commitment to Agreements

The baseline model for our analysis and comparison is the standard crisis bargaining ultimatum game (Fearon 1995). In this game there is a resource of unit size under dispute. Country 1 makes a proposal of a division of the resource where it keeps a share x, leaving $1 - x$ for the other country. Country 2 can then accept or reject this offer. If accepted there is a peaceful settlement of the dispute and the payoffs are equal to the shares x and $1-x$. If the proposal is rejected, war occurs. In the case of war, country 1 wins with a know probability p and country two wins with the complementary probability, $1 - p$. Each side to the dispute pays a cost c_i if war occurs. Thus the expected payoffs to war are $p - c_1$ for country 1 and $1 - p - c_2$ for country 2.

It is well know that because war is costly option, there exists a set of peaceful agreements that both countries prefer to fighting. In the complete information environment the probability of the dispute turning to war is zero. In the unique subgame perfect equilibrium to this crisis bargaining game country 1 demands the larges share that country 2 will accept, $x^* = p + c_2$.

To see the effect, or lack thereof, of learning that country two is willing to accept an offer x, we consider a model with one-sided incomplete information. We assume that country 1's cost, c_1 is common knowledge and country 2's cost, c_2, is private information. For this example we also assume that country 2 can have two possible cost types. Let $c_2 = c_H$ with probability q, $c_2 = c_L$ with probability $1 - q$, and $c_L < c_H$. Later we will extend our results to the case with a continuum of types. Our equilibrium concept is (strong) Bayesian equilibrium, as defined by Fudenberg and Tirole (1991). The key requirement of this definition is that if country 1 deviates in the first period by making an off-the-equilibrium-path offer, it must still use the strategy of country 2 in response to this offer to calculate its belief (via Bayes' Rule) in period 2.

Under these assumptions the standard model has a unique equilibrium whose nature depends on the value of q. We are going to focus on the equilibrium with a positive probability of war. If $q > q^* = (c_L + c_1)/(c_H + c_1)$, then the proposer demands an equilibrium share $x^* = p + c_H$, the low cost type of country 2 rejects this proposal and the high cost type of country 2 accepts. In the two type case, this is the equilibrium with a risk-reward trade-off. The proposer is willing to risk war with the low cost type in order to get the better settlement when there is peace with the high cost type.

We now make the simplest modification to the standard model to allow for the possibility of retractable offers and, by this change, relax the assumed level of commitment to agreements in crisis bargaining.

In this new model, illustrated in Figure 2, country 1 still gets to make an initial offer $(x_1, 1 - x_1)$, and if it is rejected the game ends with war. If, however, country 2 accepts the offer then country 1 can agree, obtaining the payoffs $(x_1, 1 - x_1)$ or renegotiate. If country 1 renegotiates then it makes a second proposal x_2 at some future time. This second proposal is a true take-it-or-leave-it offer. We assume that time between offers is $\Delta \geq 0$, which is possibily very small, and let $r > 0$ be the players common discount rate. As is standard, we can define the players discount factor $\delta = \exp(-r\Delta)$, which goes to 1 as the time between offers goes to 0. If the second proposal is accepted then the payoffs for country 1 and country 2 are δx_2 and $\delta(1 - x_2)$. If the second proposal is rejected, then war ensues and its payoff is discounted by δ. So rejection in the first period leads to immediate war, but acceptance can be renegotiated.

When $q > q^*$ and $\delta < 1$, there is a unique perfect Bayesian equilibrium outcome without commitment to agreements.

Proposition 1. *In the two type model, assume $q > q^*$ (the screening condition) and $\delta < 1$. Then every strong PBE has the same equilibrium path as the one-shot ultimatum game.*

Before we give the proof of the Proposition, we present two useful lemmas. Let

$$v_1^*(\mu) = \max\{p + c_L, \mu(p + c_H) + (1 - \mu)(p - c_1)\}.$$

This is the equilibrium utility to country 1 of the one-shot ultimatum game with prior μ. In what follows, we will denote the high cost type of country 2 by $2H$ and the low cost type of country 2 by $2L$. We begin our analysis in the second period. Let $\mu(x_1)$ denote the belief of country 1 about the type of country 2 in the second period, after an offer x_1 in the first period. Specifically, $\mu(x_1) = P[c_2 = c_H | x_1 \text{ accepted}]$, the probability that country 1 believes country 2 is the high cost type. As period 2 is just a standard ultimatum game, we can rely on the standard arguments to show that in every PBE, $x_2 \in \{p + c_L, p + c_H\}$. Moreover, these standard arguments yield $x_2 = p + c_L$ if $\mu < q^*$, $x_2 = p + c_H$ if $\mu > q^*$, and country 1 is indifferent between making these two offers if $\mu = q^*$. Finally, we know from standard arguments that if $x_2 = p + c_L$ in equilibrium, then both types of country 2 accept this offer with probability one, and if $x_2 = p + c_H$ in equilibrium, then $2H$ accepts with probability one and $2L$ rejects with probability one. Therefore, it is easy to establish that whatever value μ takes, the (undiscounted) payoff to $2L$ in period 2 is $1 - p - c_L$. Moreover, the (undiscounted)

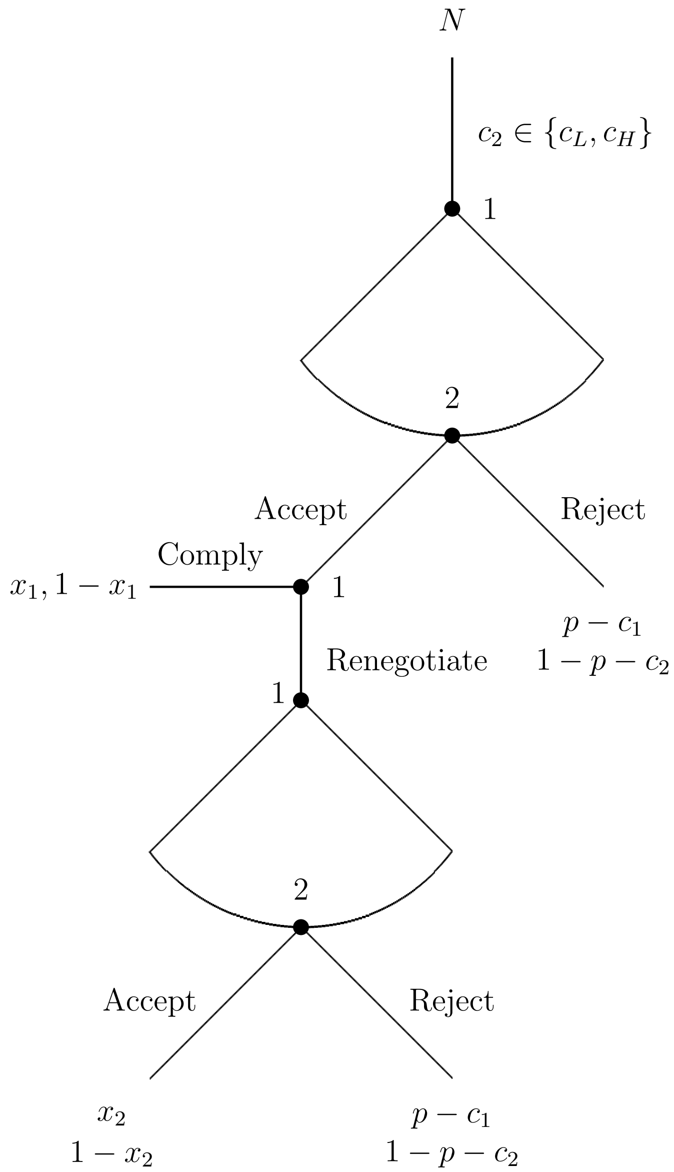

Figure 1: Two period game with retractable offers and two types

payoff to $2H$ is in the interval $[1-p-c_H, 1-p-c_L]$. Likewise, the (undiscounted) payoff to country 1 is $v_1^*(\mu(x_1))$.

Let $r(x_1)$ be the probability that country 1 chooses to renegotiate after an offer x_1 has been accepted. As the payoff of renegotiating is no higher than $\delta(p+c_H)$ and no lower than $\delta(p+c_L)$, it follows that $r(x_1)=0$ for all $x_1 > \delta(p+c_H)$ and $r(x_1)=1$ for all $x_1 < \delta(p+c_L)$.

Our first lemma is a simple consequence of incentive compatibility.

Lemma 1. *Assume $\delta < 1$. In every strong PBE, if $2L$ plays accept with positive probability in response to some offer x_1, then $2H$ must play accept x_1 with probability one.*

Proof. In order for accept to be a best response to x_1 for $2L$, we must have

$$r(x_1)\delta(p_2-c_L)+(1-r(x_1))(1-x_1) \geq p_2-c_L$$
$$(1-r(x_1))(1-x_1) \geq (1-r(x_1)\delta)(p_2-c_L)$$

On the other hand, the minimum payoff for $2H$ for accepting is $r(x_1)\delta(p_2-c_H)+(1-r(x_1))(1-x_1)$. Therefore $2H$ must accept with probability one when

$$r(x_1)\delta(p_2-c_H)+(1-r(x_1))(1-x_1) > p_2-c_H$$
$$(1-r(x_1))(1-x_1) > (1-r(x_1)\delta)(p_2-c_H)$$
.

As $c_L < c_H$, it is clear that the inequality for $2L$ implies the inequality for $2H$. This establishes the lemma. □

So if the low-cost type is not rejecting for sure, then the high-cost type must be accepting for sure. The next lemma shows what must be true if the low-cost type is rejecting for sure.

Lemma 2. *Assume $\delta < 1$. In every strong PBE, if $x_1 < \delta(p+c_H))$ and $2L$ plays fight with probability one in response to the offer x_1, then $2H$ must also play fight with probability one in response to x_1.*

Proof. For a proof by contradiction, suppose there is an equilibrium in which $2L$ fight with probability one and $2H$ plays accept with positive probability after some such offer $x_1 < \delta(p+c_H))$. Then $\mu(x_1)=1$ and thus $x_2 = p+c_H$, which country 1 believes will be accepted with probability one. Therefore, renegotiating gives country 1 a payoff of $\delta(p + c_H)$ while agreeing to acceptance gives a payoff of $x_1 < \delta(p+c_H)$, so $r(x_1) = 1$. But then the utility of accepting for $2H$ is $\delta(1 - p - c_H)$ and so $2H$ strictly prefers to fight in response to x_1. This contradiction proves the lemma. $\square$

We are know ready to prove the proposition.

Proof. Consider an arbitrary offer x_1. For $2L$, accepting this offer will either be agreed to or play will move to period 2. Therefore accepting this offer will give a payoff no larger than $\max\{1 - x_1, \delta(1 - p - c_L)\}$. Therefore, $2L$ will choose fight in response to every offer such that $1-p-c_L > 1-x_1$, which is equivalent to $x_1 > p+c_L$. For $x_1 < \delta(p+c_L)$ we know that $r(x_1) = 1$ and so $2L$ strictly prefers to fight. For $2H$, because $r(x_1) = 0$ for all $x_1 > \delta(p+c_H)$, it follows that $2H$ strictly prefers to fight for all $x_1 > p+c_H$ and strictly prefers to accept for all $x_1 \in (\delta(p+c_H), p+c_H)$. From this, we see that the utility to country 1 of making an offer $x_1 > p+c_H$ is $p-c_1$. On the other hand, for all offers $x_1 \in (\max\{\delta(p+c_H), p+c_L\}, p+c_H)$, the above implies that the utility to country 1 is $qx_1 + (1 - q)(p - c_1)$. In particular, this means that country 1 can achieve a payoff arbitrarily close to $q(p + c_H) + (1 - q)(p - c_1)$. It follows that it cannot be an equilibrium for country 1 to make an offer $x_1 > p + c_H$. In addition, no offer $x_1 \in (\max\{\delta(p + c_H), p + c_L\}, p + c_H)$ can be optimal, as country 1 can deviate to an slightly higher offer and improve its payoff.

Next we show that it cannot be an equilibrium for country 1 to offer $x_1 \leq \max\{\delta(p + c_H), p + c_L\}$. We consider two cases. First, suppose $\delta(p + c_H) > p + c_L$. Then for all $x_1 \in (p+c_L, \delta(p+c_H)]$, $2L$ is fighting with probability one, so the maximum possible utility to country 1 is $q\delta(p + c_H) + (1 - q)(p - c_1)$ which cannot be optimal. For $x_1 \leq p + c_L$, if $2L$ is fighting with probability one then the preceding argument still holds. Otherwise, $2L$ is accepting with positive probability, which by Lemma 1 implies $2H$ is accepting with probability one and country 1 either renegotiates or agrees. If it renegotiates, its highest possible payoff is again $q\delta(p+c_H)+(1-q)(p-c_1)$, and if it agrees, its highest possible payoff is $x_1 \leq p+c_L$, which by the screening condition is strictly less than $q(p+c_H)+(1-q)(p-c_1)$. So no such offer can be optimal.

The seconds case is $\delta(p + c_H) \leq p + c_L$. For all $x_1 \leq p + c_L$, if $2L$ is fighting with probability one, then the maximum possible utility to country 1 is $q(p+c_L)+(1-q)(p-c_1)$ which cannot be optimal. Otherwise $2L$ is accepting with positive probability, which by Lemma 1 implies $2H$ is accepting with probability one and country 1 either renegotiates or agrees. If it renegotiates, its highest possible payoff is $q\delta(p + c_H) + (1 - q)(p - c_1)$, and if it agrees, its highest possible payoff is $x_1 \leq p + c_L$, which by the screening condition is strictly less than $q(p + c_H) + (1 - q)(p - c_1)$. So no such offer can be optimal.

From this we conclude that the only possible equilibrium offer is $x_1 = p + c_H$ which $2L$ rejects and, by standard arguments, $2H$ accepts with probability one. It is clear that country 1 will agree to this acceptance. This proves the Proposition. $\square$

Although the equilibrium path of play in this model is unique, the equilibrium strategies are not unique. In particular, for some offers, there are several strategies that are consistent with equilibrium. For example, suppose $\delta(p + c_H) > p + c_L$ and consider the offer $x_1 = \delta(p + c_H)$. From the above, we know $2L$ will fight in response to this offer. It is easy to see that it sequentially rational for $2H$ to accept this offer and for country 1 to agree. But there are also mixed strategies that are also consistent with equilibrium. As long as $2H$ accepts with positive probability, country 1 believes country 2 is the high type and so will offer $x_2 = p + c_H$, which country 1 expects to be accepted. Therefore country 1 is indifferent between agreeing and renegotiating. Indeed, it is consistent with equilibrium for $2H$ to accept with probability one and country 1 to mix with $r \leq p + c_H$. In fact, if country 1 plays $r \leq p + c_H$, type $2H$ is indifferent between accepting and fighting and it is consistent with equilibrium for $2H$ to arbitrarily mix between these two choices.

It is also true that, while the equilibrium outcome is the same as for a one-shot ultimatum game, the behavior for non-equilibrium offers can be very different than the ultimatum game. For example, the following is a PBE to this game. Country 1 offers $x_1 = p + c_H$, chooses $r(x_1) = 1$ for all $x_1 < \delta(p + c_H)$ and $r(x_1) = 0$ for all $x_1 \geq \delta(p + c_H)$, and chooses $x_2(x_1) = p + c_H$ for all x_1. Type $2L$ fights in response to every x_1 and type $2H$ fights in response to every $x_1 < \delta(p + c_H)$ and to every $x_1 > p + c_H$ and accepts in response to every $x_1 \in [\delta(p + c_H), p + c_H]$. One notable feature of this equilibrium is that $2L$ rejects *all* offers and both $2H$ and $2L$ reject generous offers ($x_1 < \delta(p + c_H)$). The intuition for this is that accepting a generous offer will not be agreed to by country 1 (it is too generous!), but renegotiation is always worse for both types than fighting in the first period.

While the intuition is clear in the two type case as to why there is no incentive to renegotiate in the short-run when there is no commitment to agreements, one might be concerned that the fact that country 1 only can make one of two proposals in equilibrium is driving the results. Maybe the presence of more types and the possibility for a continuum of complete information equilibrium offers might change the results.

Suppose c_2 is distributed according to F with support $[0, 1 - p]$ and let $\delta < 1$. Let x^* be the one-shot ultimatum equilibrium offer with corresponding cutpoint $c^* = x^* - p$.

Proposition 2. *In the continuous type model with $\delta < 1$, but sufficiently large, there is a PBE equivalent to the baseline model in that $x_1^* = x_2^* = x^*$, all types $c_2 < x - p$ reject and all types $c_2 > x - p$ accept in both periods, and the game ends in period one.*

Proof. First we want to show that if country 2 is playing their cut point strategy in the first period, then country 1's optimal offer in the second period equals either the offer it would make in a baseline ultimatum game with their prior belief or the constraint that country 1's share is at minimum $p + c^*$ where $c^* = \{c_2 | xip = c_2\}$.

Clearly, as a function of offers in the second period, the baseline model cut point strategy for country 2 is optimal.

Now suppose country 2 is playing that cut point strategy in the first period, then when making its offer in the second period it is solving the program:

$$\max_x [x(1 - \hat{F}(x - p \mid c_2 \geq x - p)) + \hat{F}(x - p \mid c_2 \geq x - p)(p - c_1)]$$
$$s.t. \quad x \geq p + c^*.$$

where $\hat{F}(c_2)$ is country 1's updated belief given acceptance of x^*.

Applying Bayes's rule and substituting yields

$$\max_x \left[x \frac{\int_{x-p}^{1-p} g(t)dt}{1 - F(c^*)} + (p - c_1) \frac{\int_{\underline{c}}^{x-p} g(t)dt}{1 - F(c^*)} \right]$$

where $g(t) = f(t)$ on $[c^*, 1 - p]$ and 0 otherwise.

By Leibniz's rule we obtain the first order condition

$$\frac{\int_{x-p}^{1-p} g(t)dt}{1 - F(c^*)} - \frac{xg(x - p)}{1 - F(c^*)} + (p - c_1)\frac{g(x - p)}{1 - F(c^*)} = 0$$

The key observation is that the solution to this first order condition also solves the condition

$$\int_{x-p}^{1-p} g(t)dt - xg(x - p) + (p - c_1)g(x - p) = 0$$

which does not depend on c^*. Accordingly, one's optimal offer is invariant to the truncation. Finally the reader can check that the new solution at least weakly satisfies the old and new constraint.

Thus proposing $x_1 = x_2 = x^*$ is an equilibrium for country 1 because x^* is optimal in the one show game. Given these proposals, country 2's cut point strategy at c^* is also optimal describing an equilibrium. □

In the case with a continuum of types for country 2 a second class equilibrate also exist. To see this consider the following example. Again think about the game after the offer x^* has been made. Let δ satisfy $x^* < \delta < 1$. Now define $\bar{c} = (x^*/\delta) - p$.[3] It is easy to check that $\bar{c} \in (c^*, 1-p)$. I claim the following is also consistent with equilibrium: after the offer x^*, the types $c_2 \geq \bar{c}$ accept the offer, and the types $c_2 < \bar{c}$ reject the offer. Following acceptance, country 1 mixes between agreeing (with probability $q = 1 - (x^*/\delta)$) and renegotiating (with probability $1 - q = x^*/\delta$). To show that this is true, we first must evaluate the renegotiation part of the tree. If country 1 sees an acceptance, his belief is truncated at $\bar{c}$. As $\bar{c} > c^*$, country 1's offer in the second period will be $x_2 = p + \bar{c}$.

Now consider the payoff of type $c_2 \geq \bar{c}$ of country 2 from agreeing. All such types will accept the offer x_2 in the second period. So the payoff for such a type is

$$\begin{aligned} u_2(A|x^*) &= q(1-x^*) + (1-q)\delta(1-x_2) \\ &= (1-(x^*/\delta))(1-x^*) + (x^*/\delta)\delta(1-(p+\bar{c})) \\ &= (1-(p+\bar{c}))(1-x^*) + x^*(1-(p+\bar{c})) \\ &= 1 - p - \bar{c}. \end{aligned}$$

As this is weakly better than rejecting for all such types, it is sequentially rational for these types to accept. Now consider the payoff of type $c_2 < \bar{c}$ of country 2 from agreeing. All such types will reject the offer x_2 in the second period. So the payoff for such a type is

$$u_2(A|x^*) = q(1-x^*) + (1-q)\delta(1-p-c_2) = (1-p-\bar{c})(1-x^*) + x^*(1-p-c_2) < 1-p-c_2.$$

This shows it is sequentially rational for these types to reject.

Finally, we must check that country 1 is indifferent. If it agrees to the acceptance, it receives a payoff of x^*. If it renegotiates and offers $x_2 = p + \bar{c}$, under its updated belief it believes this offer will be accepted with probability 1, so its payoff is δx_2. But we have

$$\delta x_2 = \delta(p + \bar{c}) = \delta(x^*/\delta) = x^*.$$

[3]Notice that if $\delta = 1$ then the cut point is the ultimatum game cut point.

So country 1 is indifferent, as required.

Clearly, this arrangement exhibits delay. More seriously, note that country 1 receives a strictly lower payoff here than in the standard equilibrium, because more types are rejecting, but it still only gets x^* from the smaller set of types that are accepting.

It is also true that for any x such that $x^* \leq x < \delta$, we can construct a similar sort of mixing situation: country 2 uses a cutpoint given by $\bar{c} = (x/\delta) - p$ and country 1 mixes with probability $q = 1 - (x/\delta)$. So these off-the-path offers can also exhibit delay. On the path, however, the first period offer is always x^*

3 No Commitment to Fighting

We now examine what happens when we relax our commitment assumption in a different way. We suppose that agreements are binding, but rejecting an offer can lead to continued bargaining. Specifically, in this game country 1 makes an initial offer $x_1 \in [0, 1]$ in period 1, which country 2 either accepts or rejects. If the offer is accepted by country 2, then countries 1 and 2 receive payoffs of x_1 and $1 - x_1$, respectively. If country 2 rejects the offer, then country 1 can either decide to end the bargaining and fight, which gives each side its war payoff $p_i - c_i$, or to move the game to a second period of bargaining, in which the payoff to both sides is discounted by a factor $\delta \in (0, 1)$. Specifically, country 1 makes an offer $x_2 \in [0, 1]$, which country 2 either accepts or rejects. If the offer is accepted by country 2, then countries 1 and 2 receive payoffs of δx_2 and $\delta(1 - x_2)$, respectively. If country 2 rejects the offer, then war ensues and each country i receives its (discounted) war payoff, which is given by $\delta(p_i - c_i)$.

To reduce notation, we make the following definitions. We let $q^* = (c_1 + c_L)/(c_1 + c_H)$. It is clear that $q* \in (0, 1)$. In addition, we let $x_L^* = 1 - \delta(1 - p - c_L)$ and $x_H^* = 1 - \delta(1 - p - c_H)$. It is clear that $0 < x_L^* < x_H^* < 1$. We also let

$$\delta^* = \max \left\{ \frac{1 - q}{1 - q + (q - q^*)(c_H - c_L)}, \frac{p - c_1}{p + c_L} \right\}$$

and, finally,

$$v_1^*(\mu) = \max\{p + c_L, \mu(p + c_H) + (1 - \mu)(p - c_1)\}.$$

This is the equilibrium utility to country 1 of the one-shot ultimatum game with prior μ.

We have the following proposition. It characterizes the generically unique (strong) PBE.

Proposition 3. *Suppose $q > q^*$ (the screening condition) and $\delta > \delta^*$. Then there exists a exists a unique (strong) perfect Bayesian equilibrium to this game. The equilibrium path of play is given by $x_1 = x_H^*$, which is rejected by the low cost type and which the high cost type mixes between accepting and rejecting. Country 1 chooses to continue bargaining and offers $x_2 = p + c_H$. Only the high cost type accepts this offer.*

Proof. In our proof, we will denote the high cost type of country 2 by $2H$ and the low cost type of country 2 by $2L$. We begin our analysis in the second period. Let $\mu(x_1)$ denote the belief of country 1 about the type of country 2 in the second period, after an offer x_1 in the first period. Specifically, $\mu(x_1) = P[c_2 = c_H | x_1 \text{ rejected}]$, the probability that country 1 believes country 2 is the high cost type. As period 2 is just a standard ultimatum game, we can rely on the standard arguments that $x_2 \in \{p + c_L, p + c_H\}$. Moreover, these standard arguments yield $x_2 = p + c_L$ if $\mu < q^*$, $x_2 = p + c_H$ if $\mu > q^*$, and country 1 is indifferent between making these two offers if $\mu = q^*$.

Finally, we know from standard arguments that if $x_2 = p + c_L$ in equilibrium, then both types of country 2 accept this offer with probability one, and if $x_2 = p + c_H$ in equilibrium, then $2H$ accepts with probability one and $2L$ rejects with probability one.

Therefore, it is easy to establish that whatever value μ takes, the (undiscounted) payoff to $2L$ in period 2 is $1 - p - c_L$. Moreover, the (undiscounted) payoff to $2H$ is in the interval $[1 - p - c_H, 1 - p - c_L]$. Likewise, the (undiscounted) payoff to country 1 is $v_1^*(\mu(x_1))$.

We now turn to the decision of country 1 after an arbitrary offer x_1 has been rejected. Given country 1's belief $\mu(x_1)$ at this point in the game, its payoff to continue to bargain is $\delta v_1^*(\mu(x_1)) \geq \delta(p + c_L)$. By choosing to fight at this point, however, country 1 receives a payoff of $p - c_1$. Given our condition on δ, we know that $\delta(p + c_L) > p - c_1$. Therefore, it is never optimal for country 1 to fight after an offer has been rejected. In other words, in every perfect Bayesian equilibrium, country 1 always continues bargaining rather than fighting, regardless of the initial offer x_1.

Now we turn to period 1. Consider an arbitrary offer x_1. For $2L$, rejecting this offer gives a payoff of $\delta(1 - p - c_L)$. Therefore, $2L$ will reject any offer such that $1 - x_1 < \delta(1 - p - c_L)$, which is equivalent to $x_1 > x_L^*$, and will accept any offer such that $x_1 < x_L^*$. For $2H$, because rejecting this offer gives a payoff of at most $\delta(1 - p - c_L)$, $2H$ will also accept any offer such that $x_1 < x_L^*$. On the other hand, because rejecting this offer gives $2H$ a payoff of at least $\delta(1 - p - c_H)$, $2H$ will reject any offer such that $x_1 > x_H^*$. Because both types of country 2 accept any offer $x_1 < x_L^*$, in any perfect Bayesian utility the equilibrium payoff to country 1 satisfies $u_1^*(x_1) = x_1$ for all $x_1 \in [0, x_L^*)$. An immediate consequence of this is that there is no PBE in which country 1 makes such an offer with positive probability.

In any strong perfect Bayesian equilibrium, the belief of country 1 must be derived via Bayes' Rule from the sequentially rational response of country 2 to a given offer, whether this offer is on or off the equilibrium path. Thus, as both types of country 2 reject any offer $x_1 > x_H^*$, in any perfect Bayesian equilibrium $\mu(x_1) = q$. This implies that in any PBE, $u_1^*(x_1) = \delta v_1^*(q)$ for all $x_1 \in (x_H^*, 1]$.

Next, consider an offer $x_1 \in (x_L^*, x_H^*)$. From above, $2L$ rejects this offer. We claim that $2H$ must play a mixed strategy in response to this offer. If $2H$ rejects x_1 with probability one, then $\mu(x_1) = q$ and because $q < q^*$, $x_2 = p + c_H$. This gives $2H$ a payoff of $1 - x_H^*$. But then deviating to accepting x_1 gives a payoff strictly greater than $1 - x_H^*$, so this cannot be an equilibrium. If $2H$ accepts x_1 with probability one, then $\mu(x_1) = 0$ and therefore $x_2 = p + c_L$. So deviating to rejecting x_1 gives a payoff of $1 - x_L^*$, which is strictly greater than $1 - x_1$. Therefore this cannot be an equilibrium. So $2H$ must play a mixed strategy.

In order to calculate this mixed strategy, let $s(x_1)$ be the probability that $2H$ accepts $x_1 \in (x_L^*, x_H^*)$. In order to make $2H$ indifferent between accepting and rejecting x_1, country 1 must mix between $x_2 = p + c_L$ and $x_2 = p + c_H$. So let $r(x_1)$ be the probability that country 1 makes the offer $x_2 = p + c_L$ after offering x_1 in the first period. In order to ensure that $2H$ is indifferent, we must have

$$1 - x_1 = r(x_1)\delta(1 - p - c_L) + (1 - r(x_1))\delta(1 - p - c_H).$$

Solving this linear equation gives the following unique solution:

$$r(x_1) = \frac{1 - \delta(1 - p - c_H) - x_1}{\delta(c_H - c_L)} = \frac{x_H^* - x_1}{\delta(c_H - c_L)}.$$

In order for country 1 to be willing to mix in period 2, it must be indifferent between offering $x_2 = p + c_L$ and $x_2 = p + c_H$. This requires that $\mu = q^*$. From Bayes' Rule we know the belief μ is given by

$$\mu = \frac{q(1 - s(x_1))}{q(1 - s(x_1)) + (1 - q)} = \frac{q - qs(x_1)}{1 - qs(x_1)},$$

and this allows us to solve for $q(x_1)$, which is given by

$$s(x_1) = \frac{q - q^*}{q(1 - q^*)}.$$

It is easy to see that $s(x_1) \in (0, 1)$ and, in fact, $s(x_1)$ is independent of x_1.

Given this unique mixed strategy, we can calculate the expected utility of country 1 making an offer $x_1 \in (x_L^*, x_H^*)$. This utility is given by

$$\begin{aligned} u_1^*(x_1) &= qs(x_1)x_1 + ((1-q) + q(1-s(x_1)))\delta(p+c_L) \\ &= \frac{x_1(q-q^*) + \delta(1-q)(p+c_L)}{1-q^*}. \end{aligned}$$

As $q > q^*$, we see that $u_1^*(x_1)$ is linearly increasing in x_1. A direct implication of this is that there is no PBE in which country 1 puts positive probability on any $x_1 \in (x_L^*, x_H^*)$.

Next, consider an offer $x_1 > x_H^*$. As both types of country 2 will reject this offer, the utility to country 1 of making this offer is

$$u_1^*(x_1) = \delta v_1^*(q) = \delta[(1-q)(p-c_1) + q(p+c_H)].$$

Although we omit the details, it is possible to algebraically to show that

$$\frac{x_H^*(q-q^*) + \delta(1-q)(p+c_L)}{1-q^*} - u_1^*(x_1) = \frac{(1-\delta)(q-q^*)}{1-q^*}.$$

As $q > q^*$, this shows that there exists a value $x_1' < x_H^*$ sufficiently close to x_H^* that gives country 1 a payoff strictly greater than $u1^*(x_1)$. Therefore, there is no PBE in which country 1 puts positive probability on $x_1 > x_H^*$.

By process of elimination, the only two remaining possibilities are $x_1 = x_L^*$ and $x_1 = x_H^*$. Consider the offer $x_1 = x_H^*$. Of course, $2L$ rejects this offer. If country 1 offers $x_2 = p + c_L$ in the second period, then $2H$ will strictly prefer to reject x_1 as well. This implies $\mu = q > q^*$, so country 1 strictly prefers to offer $x_2 = p + c_H$. This argument shows that country 1 must play $x_2 = p + c_H$ with positive probability after $x_1 = x_H^*$. This requires that $\mu(x_H^*) \geq q^*$. This in turn implies that the probability that $2H$ accepts $x_1 = x_H^*$, which is denoted s^*, must satisfy $s^* \geq (q-q^*)/(q(1-q^*))$. Moreover, if $x_1 = x_H^*$ is played with positive probability in equilibrium, we must have $u_1^*(x_H^*) \geq u_1^*(x_1)$ for all $x_1 \in (x_L^*, x_H^*)$. This requires that

$$s^* = \frac{q-q^*}{q(1-q^*)}.$$

In order for $2H$ to be indifferent between accepting and rejecting $x_1 = x_H^*$, it must be that

$x_2 = p + c_H$ with probability one. We conclude that

$$u_1^*(x_H^*) = \frac{x_H^*(q - q^*) + \delta(1-q)(p + c_L)}{1 - q^*}$$

if this offer is played with positive probability. In order for this offer to be an equilibrium, $u_1^*(x_H^*) \geq u_1^*(x_1)$ for all $x_1 < x_L^*$. As the highest possible payoff from $x_1 = x_L^*$ is the limit of $u_1^*(x_1)$ as $x_1 \to x_L^*$, this condition insures there is no profitable deviation to $x_1 = x_L^*$, as well. This condition is satisfied if and only if

$$\frac{x_H^*(q - q^*) + \delta(1-q)(p + c_L)}{1 - q^*} \geq x_L^*$$

Again we omit the details, but this inequality holds when

$$\delta \geq \frac{1 - q}{1 - q + (q - q^*)(c_H - c_L)} = \delta^*.$$

So for all $\delta > \delta^*$, there exists a unique PBE in which country 1 makes the offer $x_1 = x_H^*$. □

The key feature of this result is that agreement does not necessarily occur immediately. The high cost type of country 2 sometimes accepts country 1's initial offer and sometimes delays and accepts the offer in the second round. The low cost type of country 2, on the other hand, rejects both offers and ends up in war, as in the baseline model with commitment. Indeed, although the exact value of the offers made differs from the baseline model, the main intuitions carry over. Country 1 makes demanding offers in both periods and ends up with peaceful agreements with doves and war with hawks. Upon closer examination, we see that the differences, such as they are, are driven by the ability of country 1 to delay war until the second period, which lowers the bargaining leverage inherent in rejecting offers. In support of this, we see that as δ goes to one, the offers of country 1 in both periods converge to the standard screening offer $x = p + c_H$.

As in the previous section, we also investigate this model with a continuum of types. In this case, we suppose that the cost of war to country 2 is drawn from a uniform distribution on the interval $[0, \bar{c}]$. We assume that $0 < c_1 < \bar{c}$.

To reduce notation, we make the following definitions. We let $z = 1 - \delta(1 - p)$. The following property identifies a (strong) PBE to this game.[4]

[4]We conjecture that it is the unique such equilibrium.

Proposition 4. *For sufficiently high δ, the following is the path of play of a (strong) PBE to this game.*

Player 1 makes the offer $x^{1*} = \delta(p + \frac{\bar{c}-c_1}{2})$ in the first period and makes the offer $x^{2*} = p + \frac{\bar{c}-c_1}{2} - \frac{1-\delta}{\delta}$ in the second period. Player 2 responds in one of three ways, depending on her type. If $c_2 \in [0, \hat{c}]$, then she rejects both offers and chooses war. If $c_2 \in [\hat{c}, c^*]$, then she rejects the first offer and accepts the second offer. If $c_2 \in [c^*, \bar{c}]$, then she accepts the first offer. On the equilibrium path, $\hat{c} = \frac{\bar{c}-c_1}{2} - \frac{1-\delta}{\delta}$ and $c^* = \bar{c} - 2(\frac{1-\delta}{\delta})$.

Incomplete Sketch of Proof: We begin with some preliminary points about country 1's initial offer x^1. If $x^1 < z$, then all types of country 2 will accept this offer. If $x^1 > z + \delta\frac{\bar{c}-c_1}{2}$, then all types of country 2 will reject this offer and the resulting offer in the second period. This last option is never optimal for country 1.

Now consider an arbitrary offer $x^1 \in [z, z + \delta\frac{\bar{c}-c_1}{2}]$. We will consider strategies of player 2 such that all types $c_2 < c^*(x^1)$ reject this initial offer and all types $c_2 > c^*(x^1)$ accept this initial offer. Therefore, player 1's belief about the distribution of costs of player 2 in the second period is uniform on the interval $[0, c^*(x^1)]$. As period 2 is just a standard ultimatum game, we can rely on the standard arguments that $x^2 \in [p, p + c^*(x^1)]$. Indeed, these standard arguments yield $x^2 = p + \frac{c^*(x^1)-c_1}{2}$ if $c^*(x^1) \geq c_1$ and $x^2 = p$ if $c^*(x^1) < c_1$. It is easy to see that for sufficiently high δ, country 1 always prefers to continue bargaining into the second period rather than fight.

Now we turn to period 1. How do we find $c^*(x^1)$? The key ingredient in our analysis is that the offers x^1 and $x^2(x^1)$ must make country 2 indifferent between accepting in period 1 and accepting in period 2. To see the intuition for why this must be the case, suppose country 2 strictly prefers to accept the offer in the first round. Then all of the types who reject this initial offer will also reject the second offer, leading to war. But then country 1 should adjust the second offer to get a peaceful settlement from these types, and this profitable deviation should not exist in an equilibrium. On the other hand, suppose country 2 strictly prefers the second offer to the first offer. Then all types will wait to the second period to accept the offer. But because of discounting, country 1 should alter its first period offer to induce some types of country 1 to accept in the first period, and this is another profitable deviation. Therefore, all types of country 2 must be indifferent between accepting x^1 and accepting x^2.

This indifference implies that $1 - x^1 = \delta(1 - x^2(x^1))$. So if $c^*(x^1) \geq c_1$, then we have

$1 - x^1 = \delta(1 - p - \frac{c^*(x^1) - c_1}{2})$. Solving for $c^*(x^1)$ gives

$$c^*(x^1) = 2\left[1 - p - \frac{(1 - x^1)}{\delta} + \frac{c_1}{2}\right].$$

This identifies the cut point between those who reject and accept in the first round. Of the types who reject the first round offer, which reject the second round offer? Let $\hat{c}(x^1)$ be this cut point. Then a type of country 2 with cost $c_2 = \hat{c}(x^1)$ is indifferent between accepting and rejecting $x^2(x^1)$. But accepting $x^2(x^1)$ has the same value as accepting x^1. So the indifferent type satisfies $\delta(1 - p - \hat{c}(x^1)) = 1 - x^1$. Solving this yields

$$\hat{c}(x^1) = 1 - p - \frac{(1 - x^1)}{\delta}.$$

It is possible to show that for all $x^1 \in [z, z + \delta\frac{\bar{c} - c_1}{2}]$, we have $\hat{c}(x^1) < c^*(x^1)$. The interesting thing to note here is that the all types of country 2 with costs $c_2 > \hat{c}(x^1)$ prefer to settle rather than fight, but some of those types settle in the first period and some choose to do so in the second period.

Now all that remains is to find the optimal offer x^1. Given the above, the expected utility of this offer is given by

$$u_1(x^1) = \int_0^{\hat{c}(x^1)} \delta(p - c_1)\,\frac{dc}{\bar{c}} + \int_{\hat{c}(x^1)}^{c^*(x^1)} \delta x^2(x^1)\,\frac{dc}{\bar{c}} + \int_{c^*(x^1)}^{\bar{c}} x^1\,\frac{dc}{\bar{c}}.$$

Maximizing this expression yields

$$x^{1*} = \delta(p + \frac{\bar{c} - c_1}{2}).$$

The result follows.

□

4 No Commitment to Fighting or Agreement

Finally we consider the case where there is neither commitment to enact accepted proposals or to fight when an offer is rejected. In this environment we can see whether there are equilibria that are similar to the case with no commitment to agreements and commitment to fighting after a rejection, commitment to accepted agreements and no commitment to fighting, or a new qualitatively different kind of equilibrium distribution of outcomes. Our results show that the set of possible outcomes and there distributions are a subset of the set of distributions that are possible in the case with no commitment to agreements and a commitment to fight after rejected offers. We can also then conclude that as δ goes to 1 there equilibrium without commitment to fighting or agreements has a distribution over outcomes equal to that of the one-shot ultimatum game.

4.1 Two type case

To start our analysis, let G_1 be the game without commitment to agreeing. We will denote the version of the game without commitment to agreeing or fighting by G_2. In this version of the game, country 1 makes an initial offer $x_1 \in [0, 1]$ in period 1. Country 2 responds by choosing accept, reject, or fight. If country 2 chooses to fight, war ensues and both sides receive its war payoff $p_i - c_i$. If the offer is accepted by country 2, then country 1 can either agree, in which case the two sides receive payoffs of x_1 and $1 - x_1$, respectively, or country 2 can choose to renegotiate, which moves the game to a second round of bargaining. Finally, if country 2 rejects the offer, then country 1 can either decide to end the bargaining and fight, which gives each side its war payoff, or to move the game to a second period of bargaining. In the the second round, the payoff to both sides is discounted by a factor $\delta \in (0, 1)$. Specifically, country 1 makes an offer $x_2 \in [0, 1]$, which country 2 either accepts or rejects. If the offer is accepted by country 2, then countries 1 and 2 receive payoffs of δx_2 and $\delta(1 - x_2)$, respectively. If country 2 rejects the offer, then war ensues and each country i receives its (discounted) war payoff, which is given by $\delta(p_i - c_i)$. So in this version of the model, either acceptance or rejection can be countered with a new offer, but fighting leads to immediate war.

First consider the case where there are two cost types for country 2, as above. We can then prove the following results.

Lemma 3. *Suppose $\delta < 1$. In every equilibrium of the game G_2, if some type of country 2 rejects an offer x_1 with positive probability, then country 1 responds to this rejection by fighting with probability one.*

Proof. Let σ^* is an equilibrium of G_2 in which after an offer x_1, some type rejects this offer with positive probability. Let c_m be equal to c_L is this type rejects x_1 with positive probability, otherwise let c_m be equal to c_H. By standard ultimatum game arguments, in the second period, country 1 will make an offer to country 2 that is no smaller than the war payoff of type c_m of country 2, so that $1 - x_2 \leq p_2 - c_m$. So the maximum payoff achievable by type c_m in the second period is $\delta(p_2 - c_m)$. Now, moving back to country 1's decision on whether to renegotiate or fight after country 2's rejection, let q_F be the probability that country 1 fights. Then the utility of type c_m of country 2 for rejecting is $q_F(p_2 - c_m) + (1 - q_F)\delta(p_2 - c_m)$. As $p_2 > c_m$ and $\delta < 1$, this payoff is strictly less than $p_2 - c_m$ for all $q_F < 1$. But in order for rejecting to be played with positive probability by type c_m, it must be at least as good as fighting, which gives a payoff of $p_2 - c_m$. This establishes that $q_F = 1$. □

For games G_1 and G_2, the outcome of a strategy profile is a probability distribution over fighting in periods one and two and the terms of settlement in periods one and two. In other words, the outcome is defined over terminal nodes where all "fight" terminal nodes in a given period are grouped together and all "peaceful settlement" with given terms in a given period are grouped together.

Proposition 5. *For all $\delta < 1$, the unique equilibrium distribution of outcomes of G_2 is equal to the one-shot ultimatum game.*

Proof. As game G_1 has a unique distribution of equilibrium outcomes, which is equal to the one-shot ultimatum game, it is enough to show that every equilibrium outcome of G_2 is an equilibrium outcome of G_1. Suppose σ^* is an equilibrium of G_2. We will construct an equilibrium σ' of G_1 that has an identical outcome. To begin, consider an arbitrary offer x_1 by country 1 in the first period. If neither type chooses to reject this offer (with positive probability), then the construction of σ' is simple; it is equal to σ^*. As it is optimal for all types to either play accept or fight under σ^*, it is still optimal for these types to take these same actions when the reject option is unavailable. Clearly, the outcome of G_2 under σ^* is identical to the outcome of G_1 under σ'.

The other possibility is that some type rejects the offer x_1 with positive probability. In this case, σ' is equal to σ^* for the type (if any) that does not play reject with positive probability and σ' is equal to the fight action in period one for all types that do reject with positive probability under σ^*. To see that σ' represents equilibrium play, we use the Lemma above to note that the equilibrium outcome of every type that is rejecting under σ^* is fighting in period 1. Therefore these types receive the same utility under σ' by choosing the fight action instead. For those types who are not rejecting under σ^*, since a possible deviation to reject gives the same utility as choosing to fight, there is no change in equilibrium incentives for these types when the reject option is removed. Therefore σ' represents optimal behavior in G_1. Moreover, the outcome of the types in who reject under σ^* is fighting in period 1 and by construction the outcome in G_1 for these types under σ' is also fighting in period 1. This shows that the outcome of G_2 under σ^* is identical to the outcome of G_1 under σ'.

We conclude that for all offers x_1, the outcome of G_2 under σ^* is identical to the outcome of G_1 under σ'. Therefore the optimal choice of offer by country 1 must be the same in σ^* and σ' and so every equilibrium outcome of G_2 is an equilibrium outcome of G_1. □

4.2 Continuum case

We first give the following lemma about equilibrium behavior in game G_2.

Lemma 4. *In every equilibrium of the game G_2, if the set of types of country 2 that reject an offer x_1 has positive measure, then country 1 responds to this rejection by fighting with probability one.*

Proof. Let σ^* is an equilibrium of G_2 in which after an offer x_1, be the set of types that reject this offer, which we denote $C_R(x_1, \sigma^*)$, is non-empty. Let m_R be the infimum of $C_R(x_1, \sigma^*)$. As as $C_R(x_1, \sigma^*)$ is a set of positive measure, it must be that $m_R < 1 - p$. By Bayes' Rule, after observing an rejection, country 1 will place zero probability on the set of types $\{c|c < m_R\}$. Therefore, by standard ultimatum game arguments, country 1 will make an offer to country 2 that is no smaller than the war payoff of type m_R of country 2, so that $1 - x_2 \leq p_2 - m_R$. So the maximum payoff achievable by a type in $C_R(x_1, \sigma^*)$ in the second period is $\delta(p_2 - m_R)$. Now, moving back to country 1's decision on whether to renegotiate or fight after country 2's rejection, let q_F be the probability that country 1 fights. Then the utility of a type $c_2 \in C_R(x_1, \sigma^*)$ of country 2 for rejecting is no more than $q_F(p_2 - c_2) + (1 - q_F)\delta(p_2 - m_R)$. Also, because such a type is choosing to reject instead of choosing to fight, this utility is no less than $p_2 - c_2$. So for every $c_2 \in C_R(x_1, \sigma^*)$, we have

$$q_F(p_2 - c_2) + (1 - q_F)\delta(p_2 - m_R) \geq p_2 - c_2$$
$$(1 - q_F)\delta(1 - p - m_R) \geq (1 - q_F)(1 - p - c_2).$$

To establish the result, suppose that $q_F < 1$. Because $m_R < 1 - p$, this inequality reduces to

$$\delta \geq \frac{1 - p - c_2}{1 - p - m_R}.$$

This must hold for every $c_2 \in C_R(x_1, \sigma^*)$. But because we can find values of c_2 in $C_R(x_1, \sigma^*)$ that are either equal to m_R (if m_R is a minimum of $C_R(x_1, \sigma^*)$) or arbitrarily close to m_R (if not), the right hand side can be made equal to or arbitrarily close to one. This contradiction implies that $q_F = 1$ and this establishes the result. $\square$

For games G_1 and G_2, the outcome of a strategy profile is a probability distribution over fighting in periods one and two and the terms of settlement in periods one and two. In other words, the outcome is defined over terminal nodes where all "fight" terminal nodes in a given period are grouped together and all "peaceful settlement" with given terms in a given period are grouped together. Our main result is that the set of equilibrium outcomes of G_2 is a subset of those in G_1. That is, allowing country one to respond to rejections by country 2 with counter-offers does not add equilibria to the game in rejection leads to war.

Proposition 6. *For all $\delta \leq 1$, every equilibrium outcome of G_2 is an equilibrium outcome of G_1.*

Proof. Suppose σ^* is an equilibrium of G_2. We will construct an equilibrium σ' of G_1 that has an identical outcome. To begin, consider an arbitrary offer x_1 by country 1 in the first period and let $C_A(x_1, \sigma^*)$ be the set of types of country 2 that accept this offer, let $C_R(x_1, \sigma^*)$ be the set of types that reject this offer, and let $C_F(x_1, \sigma^*)$ be the set of types that choose to fight in response to this offer.

If $C_R(x_1, \sigma^*)$ is measure zero, then the construction of σ' is simple; it is equal to σ^*. As it is optimal for all types to either play accept or fight under σ^*, it is still optimal for these types to take these same actions when the reject option is unavailable. Clearly, the outcome of G_2 under σ^* is identical to the outcome of G_1 under σ'.

The other possibility is that $C_R(x_1, \sigma^*)$ has positive measure. In this case, σ' is equal to σ^* for all types not in $C_R(x_1, \sigma^*)$ and σ' is equal to the fight action in period one for all types in $C_R(x_1, \sigma^*)$. To see that σ' represents equilibrium play, we use the Lemma above to note that the equilibrium outcome of every type in $C_R(x_1, \sigma^*)$ is fighting in period 1. Therefore these types receive the same utility under σ' by choosing the fight action instead. For those types outside $C_R(x_1, \sigma^*)$, since a possible deviation to reject gives the same utility as choosing to fight, there is no change in equilibrium incentives for these types when the reject option is removed. Therefore σ' represents optimal behavior in G_1. Moreover, the outcome of the types in $C_R(x_1, \sigma^*)$ is fighting in period 1 and by construction the outcome in G_1 for these types under σ' is also fighting in period 1. This shows that the outcome of G_2 under σ^* is identical to the outcome of G_1 under σ'.

We conclude that for all offers x_1, the outcome of G_2 under σ^* is identical to the outcome of G_1 under σ'. Therefore the optimal choice of offer by country 1 must be the same in σ^*

and σ' and so every equilibrium outcome of G_2 is an equilibrium outcome of G_1. □

5 Conclusion

When considering the commitment problem and how it interacts with learning in crisis bargaining, we have shown a number of things. First, in the two type case and no commitments to agreements every strong perfect Bayesian equilibrium has the same equilibrium path as the one shot ultimatum game. With a continuum of types there are two classes of equilibria. In the first the equilibrium distribution of outcomes is the same as the one-shot ultimatum game. In the second, there is a higher probability of war and with positive probability an offer greater than the ultimatum offer is reached after the retraction of the first period ultimatum proposal. But, as $\delta \to 1$ in this equilibrium the probability of war, the probability of renegotiation, and the the second period equilibrium proposal x_2^* all converge to the distribution in the one-shot ultimatum game.

When we consider relaxing the commitment problem with regards to fighting after rejection rather, but assume accepted proposals are enacted, there exists a strong perfect Bayesian equilibrium where country 1 screens the types of country 2 and makes country 2 a more favorable offer after the first period proposal is rejected. Interestingly, the probability of war in this equilibrium is the same as in the one-shot ultimatum game and as $\delta \to 1$, the proposed offers in periods 1 and 2 converge to the ultimatum offer.

Finally, when we relax both types of commitment the equilibrium set for this new game is no larger than the equilibrium set when we only relax the commitment to enact accepted agreements. In this sense, when it comes to strategic behavior relaxing the commitment to agreements appears to create incentives that dominate the screening incentives of the relaxation of commitments to fight.

This last result is important, however, because when combined with our initial results about relaxing the commitment to agreements, the distribution of equilibrium outcomes converged as $\delta \to 1$, in all the equilibria, to the distribution of outcomes in the one-shot ultimatum game. One can thus conclude that the commitment assumptions implicit in the standard crisis bargaining games are not restrictive with respect to the learning that occurs in that game when country 2 reveals that it is of low resolve, and as a result, is willing to accept the offer on the table. In these environments country 1 has no incentive to renegotiate if if it could.

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