

# Electoral Control and the Human Capital of Politicians\*

PRELIMINARY

Peter Buisseret<sup>†</sup>      Carlo Prato<sup>‡</sup>

First draft: September 13, 2013; this version: December 16, 2013

## Abstract

Expertise and experience are frequently held to be universally desirable political attributes. But under what conditions can voters benefit from improvements in the human capital of their politicians? We address this question in a model of electoral control with two districts, built upon three premises. First, both human capital (quality) and effort are key inputs of a politician's performance. Second, ensuring that these inputs are employed to the service of constituents requires the provision of electoral incentives. Third, the reward to politicians from the pursuit of alternative activities to direct constituency service - in particular, pursuing the internal goals of a party or political faction - depends on the human capital and effort decisions of *both* legislators. Our main result is that an increase in the quality of *either* politician can worsen electoral control for *both* voters. As a consequence, voters may be better served by "outsiders," or amateur candidates with lower levels of expertise and prior experience.

**JEL Classification:** D72, D78, D83.

**Keywords:** Electoral Control, Competence, Political Representation, Multidistrict Elections.

---

\*A previous version of this paper circulated under the name "A Demand Side Theory for Bad Politicians." We would like to thank Dan Bernhardt, as well as seminar and conference participants at Georgetown University, Washington University in St Louis and the LSE Workshop on Positive Theory in Comparative Politics for helpful comments.

<sup>†</sup>Department of Economics, Warwick University, *Email:* p.buisseret@warwick.ac.uk

<sup>‡</sup>Edmund A. Walsh School of Foreign Service, Georgetown University, *Email:* cp747@georgetown.edu

# 1 Introduction

How does the human capital of politicians affect the quality of political representation? Conventional scholarly wisdom suggests that citizens with higher human capital should be more productive in service to their voters, and thus a fundamental desiderata for a political system is to draw these individuals into political careers.<sup>1</sup> This idea is founded on a presumption that when the human capital of politicians increases, voters are able to appropriate some - or indeed all - of the associated benefits.

We show that this presumption is often false. Despite the existence of conditions under which voters are indeed better off, we find that the presence of politicians with high levels of human capital can exacerbate problems of electoral control in a legislative body. These problems can be so severe that voters might not only fail to capture the gains associated with the improved quality of their representatives, but may even be worse off as a result.

Our analysis hinges on the relationship between the decentralized accountability mechanism of local elections and the centralized political structures in which politicians operate once they are elected. In contrast with virtually all existing accounts of political agency (e.g., Ferejohn, 1986), in which policy choices and outcomes are the provenance of a single politician, we conceive of a legislature as a set of bilateral agency relationships between constituency voters and their representatives. Within the legislature, however, politicians work in teams - such as political parties or factions within parties - with their own distinct objectives and agendas. The problem faced by each representative is to allocate his time between activities that provide direct benefits to his voters and activities that advance factional (or partisan) goals. A politician's human capital is measured by the effective time at his disposal. In order to ensure that representatives act in their interests, voters make demands on their representatives' time by offering re-election conditional on the provision of a sufficient degree of constituency service.

If each politician's decision about how to allocate his time were unaffected by the decisions of those around him, then the agency relationship between each voter and her corresponding politician could be studied in isolation, as if the legislature effectively consisted of a single politician. We, however, adopt an alternative perspective that the value to a politician of pursuing factional (or partisan) goals

---

<sup>1</sup>See, for example, Ferraz and Finan (2011), Merlo and Mattozzi (2007, 2008).

within the legislature depends positively on the human capital and effort supplied by other politicians to the same activity. In other words, partisan goals involve elements of *team production* (Alchian and Demsetz, 1972), whose associated outputs is inherently the work of a group of legislators (Downs, 1953).

We show that the complementarity between politicians' efforts in pursuing partisan goals creates an externality between voters in their efforts to control their representatives' decisions. This is because each voter's re-election strategy affects the time allocation of his representative and, as a consequence, the opportunity cost of working on factional activities rather than providing constituency service for *each* of his "legislative team mates." As one voter demands less from her representative, she frees some increment of her representative's time in order to pursue partisan goals, which in turn raises the return on pursuing partisan goals for *all* of his fellow representatives. This forces voters to compete implicitly with one another with respect to the relative leniency of the demands made on their representative. Strikingly, we show that the choices made by a voter in one district about her own demands may partially *remove* the ability of another district's voter to control her own politician through electoral incentives.

The presence of politicians distinguished by high levels of human capital exacerbates these externalities in electoral control. Despite their greater intrinsic capacity to provide services to their constituents, politicians with high human capital are also more attractive potential partners for their legislative teammates in the pursuit of factional goals. As such, their presence raises the opportunity cost of providing constituency services for *all* team members.

One consequence of this observation is that voters always suffer when the human capital of representatives from other districts rises. But we also show that voters may suffer from an increase in the human capital of their *own* representative. To see how such an effect might arise, consider a home district in which a politician becomes more productive. This forces voters in other districts to lower their demands in order to re-balance their representatives' increased return on pursuing factional goals. But this adjustment adversely affects the same trade-off for the politician in the home district, who now faces more engaged legislative teammates; this forces a subsequent reduction in the demands of his voter. We show that the welfare costs of this contractual adjustment may dominate the direct effect of having a higher quality politician. As a result, there is no guarantee that voters will be the residual

claimants of improvements in the aggregate human capital of the legislature, or even of their own politicians.

Our model unearths a new form of political failure, induced by the existence of competing claims on the attention of constituency representatives, in the form of legislative factions and political parties. But the wider significance of our contribution is at least three-fold. First, our analysis provides a novel explanation for why voters sometimes demonstrate a revealed preference for politicians with comparatively little experience or demonstrated expertise. Even if voters believe these individuals to be inherently less productive, they may also rationally expect them to be more reliable in serving their constituents, rather than pursuing objectives that are mostly valuable to fellow insider politicians. We later provide some contemporary examples of this phenomenon and relate them to the core mechanics of our model.

Second, we provide a microfoundation for a widely employed modeling shorthand in the political economy literature known as *valence*, used to describe those non-policy attributes of a politician which are universally valued by voters regardless of their ideological disposition.<sup>2</sup> Our analysis describes a qualified set of circumstances under which this approach is valid, and our critique holds whenever a given valence characteristic increases, or is correlated with, the general skills of a politician.

Finally, our results raise a positive and normative challenge to the contemporary study of representation and accountability: what does it mean for a politician to be ‘good’ or ‘bad’? How do politicians’ inherent ‘types,’ or their capacities to produce benefits to voters, actually translate into valuable political outputs? We hope to catalyze new forms of theoretical and empirical enquiry into the relationship between the characteristics of politicians and their consequences for voters.

Our depiction of the trade-off that politicians face between serving voters or their political team in the legislature builds upon a literature in political science which conceives of parties and voters as *competing principals* (Carey, 2007; Hix, 2001). Scholars of legislative organization observe that, on the one hand, voters expect their representatives to serve as tireless advocates, generating constituency

---

<sup>2</sup>The introduction of the concept of valence is attributed to Stokes (1963), and plays a crucial role in many seminal contributions, including Banks and Sundaram (1993 and 1996), Rogoff and Siebert (1988), Ansolabehere and Snyder (2000).

benefits both through direct service and also by influencing the focus of national programs to the benefit of local interests. On the other hand, politicians are invariably part of a team of legislators, such as political parties or factions within parties, which have their own mission and demands. These demands can be ideological or material, and it is not uncommon for them to be orthogonal or even inimical to their constituents' interests, even if solely through the fact that they take representatives' attention away from the latter. Carey (2007) notes:

National legislatures in all democratic systems are organized by parties, and almost all legislators are organized by members of party groups within their assemblies. To varying degrees, the leaders of these groups control resources [...] thus virtually all legislators are subject to influence by at least one principal: their legislative party leadership. [...] Legislators' electoral connection to voters might pull them in directions contrary to the demands of legislative party leaders [...] Where voters exercise relatively more control over legislators' electoral prospects and party leaders less, legislators may face demands from their electoral principals that compete with those of party leaders.

A novelty and indeed an irony of our analysis, however, is that voters also act as competing principals for control of their politicians, in spite of the fact that each holds direct preferences only over the actions of their own politician. Our approach is inspired by a recent interest in modeling parties as teams of politicians (Dewan and Hortala-Vallve, 2013; Krasa and Polborn, 2013), and, more generally, within-party interactions as a source of distortions in the supply of competent politicians (Mattozzi and Merlo, 2011), as well as moral hazard (Zudenkova, 2012). Nevertheless, we are the first to use that approach in an environment where each politician is subject to the oversight of distinct constituencies. The interplay between competing demands and legislative teams allows us to identify and describe previously unknown spillover effects between districts in the control problems faced by their respective voters.

The determinants of the objective quality of politicians have been the subject of a substantial scholarly and policy interest; for example, Morelli and Caselli (2004) and Merlo and Mattozzi (2007 and 2008) show how outside options from the private sector that are correlated with underlying human capital could crowd out competence and intrinsic motivations from the pool of available candidates.

Our focus, however, is somewhat broader. Even if one could improve the quality of the candidate pool through higher salaries or other perks, it is crucial - and, hitherto, presumed - that voters are able to capture the gains associated with such an improvement. We provide conditions under which improvements in politicians' human capital are appropriate that can be related to various aspects of the institutional environment in which legislators operate, such as the strength of the party system.

We assume that human capital is both *observable* and that it enhances the *broad capabilities* of a politician. The first assumption is faithful to a burgeoning empirical literature (Ferraz and Finan, 2011; Fisman et al., 2013; Gagliarducci and Nannicini, 2013; Merlo et al., 2009) which attempts to measure the human capital of politicians, using their educational and professional backgrounds, as well as their legislative effort via roll-call participation and speeches, and which confirms the centrality of both to the production of political outputs. The second assumption could be relaxed to allow for different forms of human capital that relate to differentiated activities, so long as there is sufficient positive correlation in these attributes. But in this respect, also, we are consistent with much of the existing literature. For example, the supposition that more able politicians have better outside options in the private sector presumably arises from a postulate that the same core set of skills foster productivity in many different careers or tasks. Moreover, we believe that our formulation is the most neutral manner in which to introduce human capital into a multi-task setting.

The remainder of the paper is organized as follows: Section 2 introduces the model. Section 3 sets out some preliminary results, including the analysis of a single-district version of the model (where improvements in the quality of their representatives cannot hurt voters). Section 4 characterizes equilibrium, and Section 5 reconsiders the welfare consequences of politicians' human capital. In Section 6, we describe some historical political events that can be interpreted in light of our theory. Section 7 concludes.

## 2 The Model

There are two districts,  $A$  and  $B$ , each of which is composed of a voter and a representative: with a slight abuse of notation, we denote by  $J \in \{A, B\}$  the voter in district  $J$ , and by lowercase letter  $j \in \{a, b\}$  the representative from district  $J$ . While in office, each politician simultaneously chooses

an *effort allocation*,  $e^j \in [0, 1]$ , where  $e^j$  is the time which she devotes to constituency service and  $1 - e^j$  is the time which she devotes to activities which benefit her legislative faction. Each politician is characterized by an observable *quality*  $q^j$ . The voter benefits solely from her representative's constituency service, which is a function of the latter's quality and effort:

$$w^J = q^j e^j.$$

Each voter chooses a *re-election strategy*:

$$r^J : \mathbb{R}_+ \rightarrow \{0, 1\}$$

which is the probability with which she re-elects her representative as a function of the observable level of constituency service. Politicians value holding office, as well as the outcome from party service (which can be thought of as 'rewards,' given by cabinet positions and/or resources that can be diverted from the public budget). Without loss of generality, we normalize to one the discounted value from being re-elected. We also assume that the production of party rewards is a team problem between the two representatives: the benefit from party rewards depend on the politicians' quality, their effort towards party service, and a stochastic (and unobservable to the voters) productivity shock,  $\theta$ :

$$w(\mathbf{e}, \mathbf{q}, \theta) = \theta(1 - e^a)(1 - e^b)q^a q^b.$$

The random variable  $\theta$  captures an array of factors, i.e., 'windows of opportunity' for a faction or a party to achieve their goals, affecting the return of effort on the outcome of the joint activity, and its value vis à vis holding office. For example, if an incumbent party secretary is threatened by a potential scandal, representatives from a minority faction might be willing to sacrifice some of their re-election chances in order to take over the internal organization of the party. What is crucial is that these opportunities cannot be perfectly anticipated by voters at the time of election. In order to keep the problem analytically tractable, we assume that  $\theta$  is drawn from a logistic distribution with location and scale parameters  $\mu$  and  $s$ , respectively (we also denote its cdf by  $F(x)$ ). Each representative's payoff is then given by

$$w(\mathbf{e}, \mathbf{q}, \theta) + r^J.$$

The timing of the game is as follows:

1. Nature determines each representative's quality  $q^j$ , which is publicly observed.
2. Each voter chooses a re-election strategy (a function of observed constituency service  $u^J$ ).
3. Nature determines  $\theta$ , privately observed by each representative.
4. Each representative simultaneously chooses his effort allocation  $e^j$ .
5. Each voter's payoff  $u^J$  is realized, and her re-election decision  $r^J$  is made.
6. Each representative's payoff is realized.

### 3 Preliminary Analysis

#### Cut-off strategies

We begin by showing that it is without loss of generality to restrict attention to equilibria in which each voter uses a *cut-off* strategy: that is, we show that it is possible to span all pure strategy equilibrium payoffs and outcomes with cut-off strategies.

DEFINITION 1 *A voter  $J$  uses a cut-off strategy if there exists  $\underline{u}^J \in \mathbb{R}_+$  such that*

$$r^J(u^J) = \begin{cases} 1 & \text{if } u_j \geq \underline{u}^J \\ 0 & \text{if } u_j < \underline{u}^J \end{cases}$$

An equilibrium in cut-off strategies is an equilibrium in which each voter's strategy is a cut-off strategy.<sup>3</sup>

---

<sup>3</sup>If, in the representatives' subgame, a representative  $j$  is otherwise indifferent between two strictly positive levels of effort,  $e^j$  and  $\tilde{e}^j < e^j$ , and strictly prefers either of these efforts to every other feasible effort level, we assume that she chooses  $\tilde{e}^j$ . Note that for any tuple of re-election strategies, we show that each representative chooses an action  $e^{j'} < 1$  with positive probability. Thus, the set of  $\theta$  realizations for which  $j$  is indifferent between any two levels of effort has measure zero, so this does not affect our argument.

CLAIM 1 *For any equilibrium in pure voting strategies, there exists an equilibrium in cut-off strategies which induces the same distribution over the action profile of the representatives, and the same distribution over all players' payoffs.*

Henceforth, we refer to a voter  $J$ 's strategy simply as  $\underline{u}^J$ .

## The single district case: full appropriability

We begin by examining a benchmark setting in which there is a single politician. The most natural way to do so in the context of our model is to assume that representative  $-j$  is not a strategic player, and fix his effort allocation to an exogenous  $\bar{e}^j$ . Representative  $j$ 's return from pursuing partisan goals is then  $\theta(1 - e^j)q^j\rho$ , where  $\rho = (1 - \bar{e}^{-j})q^{-j}$  is a constant. So,  $j$ 's optimal effort choice is

$$e^j = \mathbb{I}_{\{\theta < (\underline{u}^J \rho)^{-1}\}}.$$

As a consequence, the expected value to voter  $J$  is  $V^J = \max_{\underline{u}^J} \underline{u}^J F[1/(\underline{u}^J \rho)]$ . Notice that  $\underline{u}^J$  enters the voter's payoff both through the services delivered conditional on the representative working for the voter, but also through the probability with which such services are delivered. The next result immediately follows.

### PROPOSITION 1 (APPROPRIABILITY THEOREM)

*The voter's expected value is weakly increasing in the representative's quality.*

*Proof.* The optimal utility cutoff  $\underline{u}^J$  must be such that  $F[1/(\rho \underline{u}^J)] - f[1/(\rho \underline{u}^J)]/(\rho \underline{u}^J) \geq 0$ . Denote by  $\underline{e}$  the optimal threshold cutoff (i.e.,  $\underline{u}^J = \underline{e}q^j$ ) chosen by the voter. We can then write  $V^J = q^j \underline{e} F[1/(\rho q^j \underline{e})]$ , and verify that

$$\frac{\partial V^J}{\partial q^j} = F[1/(\rho q^j \underline{e})] - \frac{f[1/(\rho q^j \underline{e})]}{\rho q^j \underline{e}} = F[1/(\rho \underline{u}^J)] - \frac{f[1/(\rho \underline{u}^J)]}{\rho \underline{u}^J} \geq 0$$

■

Proposition 1 implies that, *holding the behavior of other voters and representatives constant*, the voter is always able to appropriate a weakly positive share of the surplus generated by an increase in her

representative's quality. For this reason, we call it the *Appropriability Theorem*. We will show that this is a partial equilibrium result: when the strategic interaction between representatives and voters across districts is properly accounted for, the Appropriability Theorem might fail.

## The representatives' subgame

We now solve the model beginning in the subgame in which each representative chooses his effort allocation. Assume without loss of generality that  $\underline{u}^J q^{-j} \in \min\{\underline{u}^A q^b, \underline{u}^B q^a\}$ . We also assume that when there are multiple equilibria of this subgame, politicians are able to coordinate on the weakly Pareto dominant one.<sup>4</sup>

CLAIM 2 *In the equilibrium of the representatives' subgame:*

(i) (“Party Equilibrium”) when  $\theta \geq (\underline{u}^J q^{-j})^{-1}$ , both representatives work exclusively on the partisan activity:

$$e^j = e^{-j} = 0;$$

(ii) (“Asymmetric Equilibrium”) when  $\theta \in [(\underline{u}^{-J} q^j - \underline{u}^J \underline{u}^{-J})^{-1}, (\underline{u}^J q^{-j})^{-1})$ , one representative meets his re-election threshold and the other works exclusively on the partisan activity:

$$e^j = \underline{u}^J / q^j, \quad e^{-j} = 0;$$

(iii) (“Constituency Equilibrium”) when  $\theta < \min\{(\underline{u}^{-J} q^j - \underline{u}^J \underline{u}^{-J})^{-1}, (\underline{u}^J q^{-j})^{-1}\}$ , both representatives satisfy their re-election thresholds:

$$e^j = \underline{u}^J / q^j, \quad e^{-j} = \underline{u}^{-J} / q^{-j}.$$

In order for the asymmetric equilibrium to be feasible, there must be a sufficient asymmetry in the stringency of the voters' relative re-election demands. Denote by  $\lambda^J \equiv q^j / \underline{u}^J > 1$  the *leniency* of

---

<sup>4</sup>Notice that this criterion produces, in this environment, a complete ranking.

voter  $J$ 's standard. We will say that voter  $J$  is *relatively demanding* if  $\lambda^{-J} - \lambda^J > 0$  and that  $J$  is *very demanding* if  $\lambda^{-J} - \lambda^J > 1$ . Thus, the existence of an asymmetric equilibrium requires that one voter is being very demanding.

## 4 Equilibrium Analysis

We now focus on voter  $J$ 's problem, keeping fixed the other voter's strategy  $\underline{u}^{-J}$ . We can define  $J$ 's payoff  $U(u)$  as a function of her own standard,  $u$ . An equilibrium is a profile  $(\underline{u}^J, \underline{u}^{-J})$  such that

$$\underline{u}^J \in \arg \max_{u \in [0, q^j]} \{U(u; \underline{u}^{-J})\}; \quad \underline{u}^{-J} \in \arg \max_{u \in [0, q^{-j}]} \{U(u; \underline{u}^J)\}.$$

Using Claim 2, we can write  $U(u)$  as follows:

$$U(u) = \left\{ \begin{array}{ll} uF[(uq^{-j})^{-1}] & \text{if } u \leq \frac{q^j}{q^{-j}} \underline{u}^{-J} \\ uF[(\underline{u}^{-J}q^j)^{-1}] & \text{if } u \in \left( \frac{q^j}{q^{-j}} \underline{u}^{-J}, \frac{q^j}{q^{-j} - \underline{u}^{-J}} \underline{u}^{-J} \right] \\ uF[(u(q^{-j} - \underline{u}^{-J}))^{-1}] & \text{if } u > \frac{q^j}{q^{-j} - \underline{u}^{-J}} \underline{u}^{-J} \end{array} \right\}.$$

Figure 1 graphs an example of the payoff function. To understand the construction, notice that there are three relevant portions of the domain of voter  $J$ 's strategy, holding fixed the strategy of voter  $-J$ . When  $J$  is relatively lenient, a trade-off exists between accountability and control: since the standard  $u$  affects her representative's decision to engage in constituency service, a higher standard generates a higher payoff *conditional on the representative's choice to serve the voter* (higher accountability), but also increases the set of  $\theta$  realizations for which the representative chooses to shirk (lower control). This corresponds to the first portion of the payoff function depicted in Figure 1. On the other hand, when voter  $J$  is being very demanding, the representatives' subgame admits an asymmetric equilibrium. In this case,  $J$  once again faces a trade-off between accountability and control, but with a more responsive probability of shirking than when she is relatively lenient. The reason is that now her representative chooses to shirk even when the other does not. This is shown in the third portion of the payoff function depicted in Figure 1.

Suppose, however, that  $J$  chooses a strategy which renders her more demanding than  $-J$ , but not

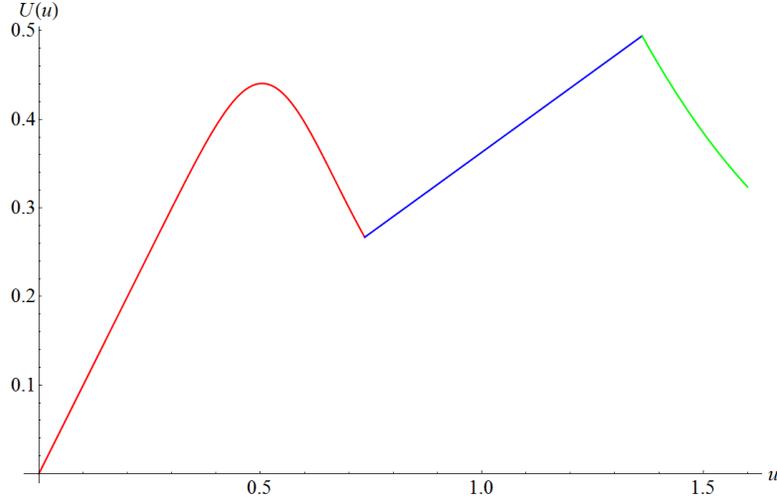


Figure 1: Example of voter  $J$ 's payoff  $U(u)$  as a function of her re-election standard,  $u$ .

significantly more. Voter  $J$  is relatively demanding, but not to such an extent that an asymmetric equilibrium can be triggered in the representatives' subgame. In this case, her representative  $j$  is prepared to work her if and only if the other representative  $-j$  chooses to work for her corresponding voter. This can be obtained by inspection of  $U(u)$  on this interval, in which case  $u$  enters the voter's payoff only through the direct effect of service provision: it does not determine the pivotal event that determines whether or not such services are provided. As such,  $J$  effectively cedes control over the incentives of her representative to voter  $-J$ . Thus, voter  $J$ 's payoff is linear and strictly increasing on the second interval of Figure 1. We record this as a Proposition.

**PROPOSITION 2 (ELECTORAL CONTAMINATION)**

*So long as  $\underline{u}^{-J} < q^j$ , there exists an interval:*

$$\left( \frac{q^j}{q^{-j}} \underline{u}^{-J}, \min \left\{ q^j, \frac{q^j}{q^{-j} - \underline{u}^{-J}} \underline{u}^{-J} \right\} \right] \quad (1)$$

*in which the voter  $J$  cannot affect the probability with which her representative provides constituency service.*

We learn from Proposition 2 that if the voter were to choose a relatively demanding strategy, she would face no trade-off between accountability and control, and so she would always prefer the highest possible demand on this sub-interval. But what about other possible best responses? Our next step is

to identify potential solutions for the case in which the voter chooses to be relatively lenient.

To motivate our subsequent analysis, suppose that the remaining voter has chosen to hold her politician to the highest possible standard ( $\underline{u}^{-J} = q^{-J}$ ). In that case, voter  $J$ 's problem is:

$$\max_{u \in [0, q^J]} uF[(uq^{-j})^{-1}] \quad (2)$$

In the Appendix, we show that this problem has at most two solutions: one in which  $J$  likewise chooses to hold her representative to the highest possible standard ( $u = q^j$ ), or one in which she picks an interior solution  $u^*$  which satisfies the first-order condition associated with (2). This first-order condition equates the marginal benefit from increasing  $u$  to the voter's payoff - conditional on the representative working for her - with the marginal cost to the probability with which the representative chooses to work for her. We show that the solution of this condition can be written in the form  $u^* = \frac{K}{q^{-j}}$ . That is, the interior solution  $u^*$  is related to the quality parameter  $q^{-j}$  through the simple relationship  $u^*q^{-j} = K$ , where  $K$  is the smaller solution of:

$$\frac{\exp(\mu/s)}{\exp(\mu/s) + \exp((Ks)^{-1})} - sK = 0$$

Thus,  $K$  identifies a point at which the voter's trade-off between control (the probability of being served) and accountability (the degree of service) is balanced. In Figure 1, it corresponds to the peak of the voter's payoff on the first interval of the strategy domain.

We are now able to fully characterize equilibrium. We show that, when it exists, the equilibrium is unique and takes one of three possible forms. We first provide the characterization, then explain the associated conditions.

**PROPOSITION 3 (EQUILIBRIUM)**

*There exists at most one (up to a permutation in the districts) pure strategy equilibrium for any value of the parameters:*

(i) (“Demanding Equilibrium”) if and only if:

$$KF[(K)^{-1}] \leq F[(q^j q^{-j})^{-1}]q^j q^{-j}, \quad (3)$$

both voters choose the highest possible standard  $\underline{u}^J = q^j$ ,  $J \in \{A, B\}$

(ii) (“Semi-Demanding Equilibrium”) if and only if:

$$2K > q^j q^{-j}, \quad (4)$$

one voter chooses the interior solution  $\underline{u}^J = K/q^{-j}$ , and the other chooses the highest possible standard:  $\underline{u}^{-J} = q^{-j}$

(ii) (Lenient Equilibrium) if and only if  $2K < q^j q^{-j}$  and:

$$KF(K^{-1}) \geq \begin{cases} q^j q^{-j} F\left(\frac{1}{q^j q^{-j}} \frac{q^j q^{-j} - K}{K}\right) & \text{if } q^j q^{-j} < 3K \\ \max \left\{ \frac{q^j q^j K}{q^j q^{-j} - 2K} F\left(\frac{1}{K} \frac{q^j q^{-j} - K}{q^j q^{-j}}\right), q^j q^{-j} F\left(\frac{1}{q^j q^{-j}} \frac{q^j q^{-j} - K}{q^j q^{-j} - 2K}\right) \right\} & \text{otherwise} \end{cases},$$

one voter chooses the interior solution  $\underline{u}^J = K/q^{-j}$ , and the other chooses the highest standard conditional on avoiding an asymmetric equilibrium in the representatives’ subgame:  $\underline{u}^{-J} = \frac{K}{q^j - K/q^{-j}}$ .

Recall that the scale and location of the distribution of  $\theta$  determines the value  $K$  at which the trade-off between accountability and control is balanced at an interior solution. The form of the equilibrium depends on how these parameters relate to the individual productivities  $q^j$  and  $q^{-j}$ . In the next section, we analyze this relationship.

In a Demanding Equilibrium, each voter imposes the most exacting standard on her representative. Though an accountability / control trade-off still exists, it is insufficiently responsive to either voter’s strategy for her to select an interior standard. The condition (3) is the formal expression of this insight. Note that for either the Semi-Demanding or Lenient equilibrium to exist, (3) must fail.

We start by analyzing the additional constraint imposed by the Semi-Demanding Equilibrium. Suppose that one voter has chosen the interior solution:  $u^{-J} = \frac{K}{q^j}$ , and consider the problem faced by the remaining voter,  $J$ : she can either choose an interior solution that balances the induced trade-off between accountability and control for her own representative, or eschew this trade-off and pick her most demanding standard. Recall from Proposition 1 that on the interval in which  $J$  is relatively demanding:

$$\left( \frac{q^j}{q^{-j}} \underline{u}^{-J}, \min \left\{ q^j, \frac{q^j}{q^{-j} - \underline{u}^{-J}} \underline{u}^{-J} \right\} \right] \quad (5)$$

$J$ 's demand on her representative affects her payoff solely through the amount of services provided: it has no effect on the probability with which they are delivered. Thus, on the interval (5), the only possible solution is the corner. Observe that:

$$q^j = \min \left\{ q^j, \frac{q^j}{q^{-j} - \underline{u}^{-J}} \underline{u}^{-J} \right\} \iff 2K > q^j q^{-j} \quad (6)$$

where we have substituted in  $u^{-J} = \frac{K}{q^j}$ . Substantively, this condition means that the other voter's relatively lenient interior solution,  $\underline{u}^{-J}$ , is not too lenient, relative to the most exacting demand that  $J$  could make of her own representative. It implies that voter  $J$  can never be very demanding. In that case, the best that  $J$  can do is to demand the highest possible level of service, which characterizes the Semi-Demanding Equilibrium.

When, on the other hand,  $2K < q^j q^{-j}$ , it is possible for voter  $J$  to pick a standard which induces an asymmetric equilibrium in the representatives' subgame. Thus,  $J$  has a non-trivial decision to make: either she will pick the highest point on the interval (5), or some even more demanding threshold at which the trade-off between accountability and control arises, once again, since  $J$ 's representative would be prepared to shirk on her voter regardless of the other representative's decision. We show that the solution to this problem is for  $J$  to choose the highest possible standard for which this trade-off does not emerge, i.e. the point which delineates the second and third sub-interval of  $J$ 's strategy. The more demanding requirement, which generates the additional equilibrium conditions given in the Proposition, is to ensure that the remaining voter still prefers to choose the interior solution  $u^{-J} = \frac{K}{q^j}$ , rather than reverting to the most demanding standard, herself. The two additional conditions on  $q^j q^{-j}$  delineate the relevant part of the domain of  $J$ 's strategy on which her alternative possible corner solution  $\underline{u}^{-J} = q^{-j}$  might fall, and the corresponding conditions that rule out the profitability of such an action. These additional conditions require an even greater degree of acuteness in the accountability / control trade-off.

## Comparative Statics on Equilibrium Conditions

Recall that  $K$  is the interior standard (relative to the the other politician's productivity) that allows the relatively more lenient voter to fully balance accountability and control. Condition (3) states that the

loss in control of the relatively more lenient voter of going from  $K$  to the most demanding standard should not be too large. Condition (4) implies that, when  $K$  is *too small*, the relatively less lenient voter can no longer be fully demanding because her politician starts being willing to shirk *regardless of what his colleague does*. We provide comparative statics on  $K$  and  $KF((K)^{-1})$  and relate these to the equilibrium conditions, above.

CLAIM 3 (i)  $K$  is decreasing in  $\mu$ , decreasing in  $s$ . (ii)  $KF((K)^{-1})$  is decreasing in  $\mu$ , increasing in  $s$ .

When a voter chooses an interior standard ( $u^j = K/q^{-j}$ ), she balances the trade-off between accountability and control. This trade-off becomes more favorable to when either the average return on factional activities decreases, or its volatility increases. Notice that the latter result *does not follow from risk aversion*, but from the observation that  $K^{-1} < \mu$ . Intuitively, when this trade-off is very unfavorable to the voter (i.e., (3) holds, for example due to a large  $\mu$ ), both voters prefer giving up control because their politicians are not reactive enough to lessening in their standards. Conversely, when this trade-off is very favorable to the voter (i.e., (4) holds, for example due to a small  $\mu$ ), the only stable configuration is asymmetric: one voter balances this trade-off, while the other is able to be fully demanding without inducing his representative to work ‘solo’ on party goals. In the remaining case, the least lenient voter once again must worry that her representative would be prepared to shirk on constituency service, unilaterally, if she is too exacting in her demands. Thus, she also chooses an interior solution, albeit one that is still relatively demanding.

## 5 The Consequences of Human Capital

We now turn to the analysis our motivating question. Recall from Proposition 1 that in a world with no between-legislator interactions, voters always benefit from improvements in the quality of their politicians. We show that this conclusion need not hold once proper account is taken of a legislator’s external environment. The following proposition formally establishes this finding.

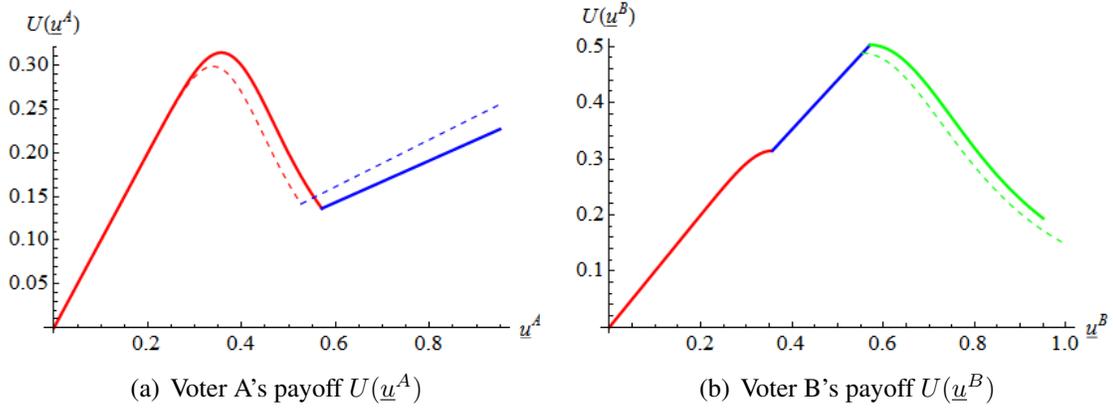


Figure 2: Illustration of Lenient Equilibrium under Benchmark Parameters, before (thick) and after (dashed) an increase in the quality of representative  $b$ .

**PROPOSITION 4 (CONDITIONAL APPROPRIABILITY THEOREM)**

- (i) *In the Demanding Equilibrium: each voter benefits from a more competent representative in her district and is hurt by a more competent representative in the other voter's district;*
- (ii) *in the Semi-Demanding Equilibrium: an increase in the competence of the representative who is subject to the more demanding standard benefits the less lenient voter and hurts the more lenient voter; neither voter benefits from an increase in competence in the other representative;*
- (iii) *in the Lenient Equilibrium: both voters are hurt by an increase in the competence of the representative subject to the more demanding standard, while an increase in competence in the other representative does not benefit the more lenient voter and hurts the less lenient voter.*

A voter can never benefit from an improvement in the human capital of the other district's politician. Such an improvement always increases the incentive of her own representative to shirk, which worsens her problem of electoral control. But how could she be strictly worse off even when *her own* politician's quality increases? To make the discussion concrete, we illustrate with a numerical example of the Lenient Equilibrium.<sup>5</sup> Figure 2 shows the payoff of each voter  $A$  and  $B$  in a Lenient Equilibrium under two sets of parameters. In both cases, we fix  $\mu = 2.25$ ,  $s = .35$  and  $q^a = .95$ . The thick line shows the payoffs for  $q^b = .95$ , whilst the dashed lines show the corresponding payoffs under  $q^b = 1$ . Under  $q^b = .95$ , the Lenient Equilibrium induces re-election strategies  $\underline{u}^A = .357$ ;  $\underline{u}^B = .571$ , and associated equilibrium payoffs  $U(\underline{u}^A) = .314$  and  $U(\underline{u}^B) = .503$ .

<sup>5</sup>In what follows, all figures are approximated to the third decimal.

Now consider an increase of the quality of representative  $b$  to  $q^b = 1$ . Since politician  $b$  has more effective time at her disposal, this raises the attractiveness of working on factional objectives from the perspective of representative  $a$ , which forces her voter  $A$  to respond with a greater degree of leniency. As a result,  $a$  is able to devote more effort to the pursuit of the factional goal. But this adjustment affects politician  $b$ 's trade-off in precisely the same way: her legislative team mate now has additional time to work with her as a result of her new contract. Voter  $B$  recognizes that, at the old contract, her representative  $b$  would now be prepared to shirk regardless of the behavior of representative  $a$ . In order to re-balance her own representative's trade-offs, voter  $B$  is forced to become correspondingly more lenient. This spiral of adjustment continues until we reach a new equilibrium in which  $\hat{u}^A = .338$  and  $\hat{u}^B = .554$ , with associated equilibrium payoffs  $U(\hat{u}^A) = .299$  and  $U(\hat{u}^B) = .488$ . Clearly, both voters are strictly worse off.

More generally, Proposition 4 shows that the progressive erosion of electoral control that takes place in each district renders even voter  $B$  strictly worse off after the increase in her representative's human capital, in spite of the fact that her politician has become inherently more productive. This result shows that when (i) representatives face a trade-off in allocating resources between partisan goals and constituency service, and (ii) partisan goals exhibit complementarities in efforts, the seemingly innocuous claim that "politician quality [should be] ... a valence issue", or something of which "everyone wants more" (Besley, 2005) may not apply.

## 6 Historical Perspective

The essence of our model lies in the competition between voters and legislative factions for the benefits of politicians' skills and experience. In particular, Proposition 4 suggests that there are conditions under which voters may be better served by politicians with less demonstrable experience and expertise, or by political amateurs whose pedigree is indeed the lack of affiliation to political factions or power structures. This is because these politicians are less likely to focus their attention solely on the parochial interests of their fellow politicians and their parties and in doing so forsake the interest of their constituency voters.

Such sentiment played heavily in the defeat of Republican Senator Jeremiah Denton from Alabama

in 1986 when his competitor, Richard Shelby, publicized a recording of Senator Denton telling an audience that he was too busy with national issues to “pat babies’ bottoms” in Alabama. As Herrick and Fisher suggest, “[...] Shelby successfully painted a picture of a disconnected representative more concerned about national issues than those state and local issues pertinent to Alabamians. At issue was a member focusing on matters important inside the Capitol at the expense of the concerns of the district’s citizens.” (Herrick and Fisher, 2007, 37).

The Massachusetts Special Senate Election of 2010 is also illustrative of the mechanisms we have described. Following the death of Senator Edward Kennedy of Massachusetts in August of 2009, a special election took place in January 2010 to fill his Senate seat. The election held elevated national stakes: a Democratic loss would have deprived the party of a filibuster-proof majority in the U.S. Senate during a period in which the Democratic healthcare reform (‘Obamacare’) was under consideration in Congress. Moreover, Massachusetts is traditionally a bastion of the Democratic Party, with registered Democrat voters outnumbering Republicans by three to one.

The Democrat candidate, Martha Coakley, was formerly a District Attorney and then Massachusetts Attorney General, having accumulated over twenty-three years of experience in office, during the course of which she successfully lead a number of high-profile prosecutions on behalf of the state.<sup>6</sup> Her Republican rival, Scott Brown, was a member of the Massachusetts legislature with “few legislative accomplishments on which to build a campaign”<sup>7</sup> and was significantly less experienced than a number of alternative potential contenders for the Republican nomination, including former White House Chief of Staff Andy Card.<sup>8</sup> During the campaign, Coakley made a number of comments which led to her portrayal in the media as being remote from Massachusetts voters, and an instrument of the Democratic Party.<sup>9</sup> For example, the Boston Globe reported that when criticized for not engaging more directly with voters during the course of her campaign, she responded:

“As opposed to standing outside Fenway Park? In the cold? Shaking hands” she fires

---

<sup>6</sup>A notable example was the prosecution of English au-pair Louise Woodward in 1997 on the charge of involuntary manslaughter relating to the death of eight-month-old Matthew Eappen.

<sup>7</sup>“The Curious Case of Scott Brown”, *Campaigns and Elections*, <http://www.campaignsandelections.com/print/175687/the-curious-case-of-scott-brown.shtml>

<sup>8</sup>LeBlanc, Steve (September 9, 2009). “Card says he is considering run for Kennedy’s seat”, Associated Press

<sup>9</sup>A closed-door campaign fundraising dinner in Washington that was attended by a number of health-care lobbyists was also widely reported; see, for example “Coakley’s Saviors: The health-care industry rides to the Democratic rescue”, *The Wall Street Journal*, January 13 2010.

back, in an apparent reference to a Brown online video of him doing just that. “This is a special election. And I know that I have the support of Kim Driscoll [Democratic Mayor of Salem]. And I now know the members of the [Salem] School Committee, who know far more people than I could ever meet.”<sup>10</sup>

Curt Schilling, a famed former pitcher for the Boston Red Sox - whom Coakley subsequently incorrectly identified as a supporter of the New York Yankees - retorted:

“Aside from the apparent feeling that the seat belongs to her just by virtue of her party, she just admitted that she doesn’t need to bother meeting with constituents because she’s meeting people like Kim Driscoll, and political leaders, and Democrat activists [...] Acting as if she doesn’t need to give her constituents the time of day is ludicrous. She can make all the snide remarks about Scott Brown shaking hands with people in the cold that she wants, but that’s what you’re supposed to do when you’re trying to get elected. She seems to have forgotten that she’s trying to get elected in Massachusetts, and not in Washington D.C. - if she remembered that, maybe she’d spend more time trying to impress Massachusetts voters and less time rubbing elbows with the Democrat establishment. . .”

Relatively late in the campaign, the Democrats recognized the threat posed by Scott Brown, and Obama made an emergency visit to Massachusetts to campaign with Coakley. The decision was risky precisely because the basis of Scott Brown’s campaign had been to run against the Democratic machine: “Now, the Democrats were bringing in the machine’s boss. On stage with Coakley, there would be Obama flanked by the congressional delegation and Democratic leaders from Beacon Hill - all those cast as villains to Brown’s hero.”<sup>11</sup> Ultimately, the attempt to rescue the Democratic campaign failed: Brown defeated Coakley to become the first Republican Senator from Massachusetts in over thirty years.

Even beyond the contribution of the candidates’ campaigning, subsequent analysis of the race attributed much of Brown’s victory to discontent amongst independent voters in Massachusetts - who

---

<sup>10</sup>*Boston Globe*, January 13 2010

<sup>11</sup>“The Curious Case of Scott Brown”, *Campaigns and Elections*, <http://www.campaignsandelections.com/print/175687/the-curious-case-of-scott-brown.shtml>

outnumber both Democrats and Republicans - over the perceived undue focus in Washington on Obama's healthcare policy. A poll shortly before the election showed that forty-four percent of Massachusetts voters cited the economy and jobs as their top concern.<sup>12</sup> We interpret healthcare as the 'partisan' issue in our model: Massachusetts voters perceived Coakley to be the Democrats' 'sixtieth senator,' whose main priority was to ensure the successful passage of their healthcare reform rather than focus on the priorities of voters within the state.

## 7 Conclusion

In this paper, we provide a framework to study the relationship between the skills and experience of politicians, and their performance towards voters. Our account is founded on the notion that politicians must balance the demands of "competing principals" (Carey, 2007), which we capture using a novel multi-principal, multi-agent theoretical framework. While the accountability mechanism between voters and politicians is inherently decentralized, the provision of incentives for politicians in legislatures is centralized by virtue of their pursuit of partisan goals. Building upon this tension, we show that when a common set of underlying skills may be applied to the satisfaction of these competing demands, the value of these skills to voters may be severely attenuated. Not only do voters often fail to capture any benefit from the presence of more able politicians: they may even suffer from it. Indeed, our results suggest that voters may be better served by less experienced or demonstrably qualified candidates.

At a deeper level, our results raise a positive and normative challenge to contemporary theories of political economy: what does it truly mean for a politician to be 'better' or 'worse'? How do politicians' inherent capacity to produce benefits to voters actually translate into valuable political outputs? Even in the relatively simple environment presented here, these questions do not admit straightforward answers. We are hopeful, nonetheless, that the tractable framework we have presented can generate new avenues of theoretical and empirical research into the role of alternative institutional arrangements - in particular with respect to electoral institutions - in order to better understand the problematic but necessary externalities faced by voters when attempting to control the incentives of their representa-

---

<sup>12</sup>"Brown wins Massachusetts Senate race", CNN, January 20, 2010.

tives in multi-district environments. With respect to the normative analysis of political institutions, our theory shows that the internal organization of parties - in particular, the strength and career implications of their internal power structures - can act as a fundamental constraint on the extent to which the human capital of politicians can be used effectively for the greater good.

## References

- [1] Timothy Besley. Political selection. *Journal of Economic Perspectives*, 19(3):43–60, 2005.
- [2] John Carey. Competing principals, political institutions, and party unity in legislative voting. *American Journal of Political Science*, 51(92-107):1, 2007.
- [3] Nicholas Carnes. *White Collar Government*. University of Chicago Press, 2013.
- [4] Francesco Caselli and Massimo Morelli. Bad politicians. *Journal of Public Economics*, 88:759–782, 2004.
- [5] Torun Dewan and Rafael Hortala-Vallve. Teams of rivals: Learning about a cabinet and its shadow. *working paper*, 2013.
- [6] John Ferejohn. Incumbent performance and electoral control. *Public Choice*, 50:5–25, 1986.
- [7] Claudio Ferraz and Frederico Finan. Motivating politicians. *working paper*, 2011.
- [8] Raymond Fisman, Florian Schulz, and Vikrant Vig. The private returns to public office. *working paper*, 2013.
- [9] Vincenzo Galasso and Tommaso Nannicini. Competing on good politicians. *American Political Science Review*, 2011.
- [10] Rebekah Herrick and Samuel Fisher III. *Representing America: the citizen and the professional legislator in the House of Representatives*. Lexington Books, 2007.
- [11] Simon Hix. Parliamentary behavior with two principals: Preferences, parties, and voting in the european parliament. *American Journal of Political Science*, 46(3):688–698, July 2002.
- [12] V.O. Key. *American State Politics: An Introduction*. Alfred Knopf, 1956.
- [13] Stefan Krasa and Mattias Polborn. Parties as teams and the nomination of legislative candidates. *working paper*, 2013.
- [14] Cesar Martinelli. Accountability, authority and media. *working paper*, 2013.

- [15] Andrea Mattozzi and Antonio Merlo. Political careers or career politicians? *Journal of Public Economics*, 92(3-4):597–608, 2008.
- [16] Hannah Pitkin. *The Concept of Representation*. University of California Press, 1967.
- [17] Galina Zudenkova. A political agency model of coattail voting. *Journal of Public Economics*, 95(11-12):1652–1660, 2011.
- [18] Galina Zudenkova. A rationale for decentralized parties. *working paper*, 2013.

# Appendix

## Proof of Claim 1

Define

$$U_J = \{k \in \mathbb{R} : \mu_J(k) = 1\}$$

CLAIM 4 *If  $k, k' \in U_J$  and  $k < k'$ ,  $e^j = \frac{k'}{q^j}$  is played with probability zero by  $j$  in an equilibrium.*

*Proof.* The payoff to  $j$  from choosing  $e^j \in [0, 1]$  is

$$\theta q^j q^{-j} (1 - e^{j'}) (1 - e^j) + \mathbf{1}[q^j e^j \in U_J] \quad (7)$$

Suppose  $e^{j'} < 1$ . Then, for any  $\theta > 0$  (i.e., generically) it is immediate that a choice of  $e^j = \frac{k'}{q^j}$  is strictly dominated by  $\tilde{e}^j = \frac{k}{q^j}$ , since in both cases she collects the payoff of 1, but in the latter, she obtains a higher payoff on the partisan activity. If  $e^{j'} = 1$ , the representative is indifferent over all effort levels, and by our earlier selection, she plays  $e^j = \frac{k'}{q^j}$  with probability 0.

CLAIM 5 *For any  $k \notin U_J$ ,  $k > 0$ ,  $e^j = \frac{k}{q^j}$  is played with probability 0 by  $j$ .*

*Proof.* Straightforward extension of the previous argument.

Since the politician cannot feasibly deliver  $u_j > q^j$ , these claims imply that it is without loss of generality to restrict attention to strategies which specify  $\mu(k)$  for  $k \in [0, q^j]$ . Define  $\underline{u}_J$  such that

$$\{u \in U_J | u < \underline{u}_J\} = \emptyset$$

First, we claim that  $\underline{u}_J$  exists in an equilibrium. Suppose not. This implies  $0 \notin U_J$ , and for every  $u \in U_J$ , there exists  $u' \in (0, u)$  such that  $u' \in U_J$ . This implies that whenever  $\theta$  is realized such that

$$\theta(1 - e^{j'}) \left(1 - \frac{u}{q^j}\right) q^j q^{j'} < 1 \quad (8)$$

for at least one  $u \in U_J$ , the best response of  $j$  does not exist, by Claim 1, which means that we cannot have an equilibrium.

By Claims 1 and 2, for any  $\theta \geq 0$ , except a set of  $\theta$  having measure zero,  $j$  has a strict best response which is drawn from the set of actions  $\left\{0, \frac{\underline{u}_J}{q^j}\right\}$ . Similarly, representative  $J'$  generically has a strict best response, which is drawn from the set of actions  $\left\{0, \frac{\underline{u}_{J'}}{q^{j'}}\right\}$ . Consider the following alternative strategy profile: for  $J \in \{J, J'\}$ , set

$$\mu_J(u_j) = \begin{cases} 1 & \text{if } u_j \geq \underline{\mu}_j \\ 0 & \text{otherwise.} \end{cases}$$

This induces the same probability distribution over representative's actions in the representatives' subgame, and thus the same payoffs for all players. Since this is an equilibrium in cut-off strategies, the argument is complete.

## Proof of Claim 2

Since  $j$  faces lower standards, whenever  $j$  prefers allocating all effort towards the partisan activity, so does  $-j$ . As a consequence, there are three possible pure strategy equilibria. First,  $e^j = e^{-j} = 0$ , which requires  $j$ , who faces lower standards, to prefer full effort towards the party activity:

$$\theta q^j q^{-j} \geq \theta q^j q^{-j} (1 - \underline{u}^J / q^j) + 1.$$

Second,  $e^j = \underline{u}^J / q^j$  and  $e^{-j} = 0$ , which requires  $j$ , who faces lower standards, to prefer re-election and  $-j$ , who faces very high standard, to prefer full effort towards the party activity despite  $j$ 's relative disengagement:

$$\begin{aligned} \theta q^j q^{-j} &< \theta q^j q^{-j} (1 - \underline{u}^J / q^j) + 1 \\ \theta q^j q^{-j} (1 - \underline{u}^J / q^j) &\geq \theta q^j q^{-j} (1 - \underline{u}^J / q^j)^{-j} (1 - \underline{u}^{-J} / q^{-j}) + 1. \end{aligned}$$

Third,  $e^j = \underline{u}^J/q^j$  and  $e^{-j} = \underline{u}^{-J}/q^{-j}$ , which requires both representatives to prefer re-election (and, depending on the difference between the re-election standard, either condition can be more stringent):

$$\begin{aligned} \theta q^j q^{-j} (1 - \underline{u}^J/q^j) &\leq \theta q^j q^{-j} (1 - \underline{u}^J/q^j)^{-j} (1 - \underline{u}^{-J}/q^{-j}) + 1 && \text{if } \underline{u}^J q^{-j} < \underline{u}^{-J} q^j - \underline{u}^J \underline{u}^{-J} \\ \theta q^j q^{-j} &< \theta q^j q^{-j} (1 - \underline{u}^J/q^j) + 1 && \text{otherwise.} \end{aligned}$$

## Proof of Proposition 3

### Properties of the Voter's payoff functions

CLAIM 6  $uF[(uq^{-j})^{-1}]$  is first strictly concave, then convex, with an inflection point at  $u = (\mu q^{-j})^{-1}$

*Proof.*

$$\begin{aligned} uF[(uq^{-j})^{-1}] &\equiv H(u) = u [1 + \exp(\mu/s - (uq^{-j}s)^{-1})]^{-1} \\ &\Rightarrow \frac{dH(u)}{du} = [1 + \exp(\mu/s - (uq^{-j}s)^{-1})]^{-1} + \\ &\quad - \frac{\exp(\mu/s - (uq^{-j}s)^{-1})}{uq^{-j}s} [1 + \exp(\mu/s - (uq^{-j}s)^{-1})]^{-2} \\ &\Rightarrow \frac{d^2H(u)}{d^2u} \propto 2 \exp(\mu/s - (uq^{-j}s)^{-1}) [1 + \exp(\mu/s - (uq^{-j}s)^{-1})]^{-1} - 1 \\ &\Rightarrow \frac{d^2H(u)}{d^2u} > 0 \Leftrightarrow \exp(\mu/s - (uq^{-j}s)^{-1}) > 1 \Leftrightarrow \mu > (uq^{-j})^{-1} \end{aligned}$$

CLAIM 7 When  $\mu < 2s$ ,  $uF[(uq^{-j})^{-1}]$  is strictly increasing.

*Proof.* We want to show the following: when  $\mu < 2s$ , the voter's payoff function is monotonic on the interval  $[0, q^j u^{-j} (q^{-j})^{-1}]$ . A sufficient condition for monotonicity of the voter's payoff with respect to  $u$  on the domain is:

$$q^{-j} u s - \frac{\exp\left[\frac{\mu}{s} - \frac{1}{q^{-j} s u}\right]}{1 + \exp\left[\frac{\mu}{s} - \frac{1}{q^{-j} s u}\right]} > 0 \quad (9)$$

We make the following change of variables:  $\mu = 2ks$ , and  $u = \frac{1}{q^{-j} s p}$ , where no restrictions on  $p$  or  $k$  are imposed. The inequality may then be written

$$\frac{1}{p} - \frac{e^{2k-p}}{1 + e^{2k-p}} > 0 \quad (10)$$

for which a necessary and sufficient condition is

$$e^{2k}(1-p) + e^p > 0 \quad (11)$$

By inspection, for any  $k \geq 0$ , the condition is satisfied for  $p \leq 1$ . Thus, it is without loss of generality to focus on  $p > 1$  in the remainder of the argument. The derivative of the last expression with respect to  $p$  is

$$e^p - e^{2k} \quad (12)$$

and by inspection, the second derivative is always positive, so the function is strictly convex. Thus, the expression (3) achieves a minimum at  $p = 2k$ . Substituting in yields the condition:

$$2e^{2k}(1-k) > 0 \quad (13)$$

which is true if and only if  $k < 1$ , which was to be shown.

**COROLLARY 1** *When  $\mu < 2s$ ,  $U(u)$  is strictly increasing in  $[0, q^j \underline{u}^{-J} (q^{-j} - \underline{u}^{-J})^{-1}]$ .*

Define the following function

$$G(u) \equiv \frac{\exp(\mu/s - 1/(uq^{-j}s))}{1 + \exp(\mu/s - 1/(uq^{-j}s))}$$

**CLAIM 8** *When  $\mu \geq 2s$ ,*

$$\arg \max_{u \in [0, q^j \underline{u}^{-J}/q^{-j}]} U(u) \subseteq \{u^*, q^j \underline{u}^{-J}/q^{-j}\},$$

where  $u^*(\mu, s, q^{-j})$  is the smaller positive solution of  $G(u)/q^{-j}s - u = 0$  in  $[0, (\mu q^{-j})^{-1}]$ .

*Proof.* When  $\mu \geq 2s$ , then we cannot guarantee the monotonicity of  $uF[(uq^{-j})^{-1}]$  in  $[0, q^j \underline{u}^{-J}/q^{-j}]$ . We also know that  $uF[(uq^{-j})^{-1}] > 0 \forall u \in (0, q^j \underline{u}^{-J}/q^{-j}]$  and that, by Claim 6, is strictly quasiconcave in the interval  $[0, (\mu q^{-j})^{-1}]$ . In  $[(\mu q^{-j})^{-1}, q^j \underline{u}^{-J}/q^{-j}]$  (assuming the interval exists),  $uF[(uq^{-j})^{-1}]$

is strictly convex. As a consequence, we know that

$$\begin{aligned} \arg \max_{u \in [0, (\mu q^{-j})^{-1}]} u F[(u q^{-j})^{-1}] &= u^* > 0 \\ \arg \max_{u \in [(\mu q^{-j})^{-1}, q^j \underline{u}^{-J}/q^{-j}]} u F[(u q^{-j})^{-1}] &\in \{(\mu q^{-j})^{-1}, q^j \underline{u}^{-J}/q^{-j}\} \end{aligned} \quad (14)$$

where  $u^*$  is implicitly defined by the FONC  $1 - \frac{G(u)}{u q^{-j} s} = 0$ . Equivalently,  $u^*$  must be the smaller positive fixed point of  $G(u)/q^{-j} s$ . Such point exists by Brouwer's Theorem. By strict quasiconcavity of  $G'(u)$ , there are at most two points, and it is immediate to verify that only the smaller satisfies the second order condition of the maximization problem (14).

**COROLLARY 2** *When  $\arg \max_{u \in [0, q^j \underline{u}^{-J}/q^{-j}]} U(u) = u^*$ ,  $u^* q^{-j}$  is equal to a constant  $K(\mu, s)$  that only depends on the scale and location of the distribution of  $\theta$ .*

*Proof.*  $u$  enters the condition  $G(u) - s u q^{-j} = 0$  only multiplied by  $q^{-j}$ . The condition defining  $u^*$  can be then reexpressed as the smaller<sup>13</sup> positive solutions of  $\hat{G}(K) - sK = 0$ , where

$$\hat{G}(K) \equiv \frac{\exp(\mu/s)}{\exp(\mu/s) + \exp(1/(Ks))}.$$

**CLAIM 9** *When  $\mu \geq 2s$  and*

$$K F[(K)^{-1}] \leq F[(q^j \underline{u}^{-J})^{-1}] q^j \underline{u}^{-J}$$

*then  $q^j \underline{u}^{-J}/q^{-j} \in \arg \max_{u \in [0, q^j \underline{u}^{-J}/q^{-j}]} U(u)$  (and the set is a singleton if the above inequality is strict).*

*Proof.* By Claim 8 the maximizer of  $U(u)$  in  $[0, q^j \underline{u}^{-J}/q^{-j}]$  is either the upper bound of the interval, or interior. As a consequence, for the interior to be the maximizer, we need  $U(u^*) \geq U(q^j \underline{u}^{-J}/q^{-j})$ , that is  $u^* F[(K)^{-1}] \geq F[(q^j \underline{u}^{-J})^{-1}] q^j \underline{u}^{-J}/q^{-j}$ , which simplifies to the condition.

<sup>13</sup>The strict quasiconcavity of  $G'(u)$  implies that  $\hat{G}'(K)$  is also strictly quasiconcave. As a consequence,  $\hat{G}(K) - sK = 0$  has at most two positive solutions.

## The Demanding Voter Equilibrium

CLAIM 10 *When*

$$KF[(K)^{-1}] \leq F[(q^j q^{-j})^{-1}]q^j q^{-j} \quad (15)$$

*there is an equilibrium where both voters set the least lenient standard:  $\underline{u}^J = q^j$ ,  $J \in \{A, B\}$ .*

*Proof.* By Corollary 1, and the fact that at most one voter (the relatively less lenient) can set his standard in  $\left(\frac{q^j}{q^{-j}}\underline{u}^{-J}, \frac{q^j}{q^{-j}-\underline{u}^{-J}}\underline{u}^{-J}\right]$ , we know that at least one voter has a strictly dominant strategy: setting  $\underline{u}^J = q^j$ . But then the other voter's domain is also restricted to  $[0, \frac{q^{-j}}{q^j}\underline{u}^J]$ , since  $\frac{q^{-j}}{q^j}\underline{u}^J = q^{-j}$ . Again, by Corollary 1, she also has a strictly dominant strategy  $\underline{u}^{-J} = q^{-j}$ .

Define by  $V^J(q^j, q^{-j})$  voter  $J$ 's equilibrium value from the electoral game.

CLAIM 11  $\forall J \in \{A, B\}$ ,  $V^J(q^j, q^{-j})$  is increasing in  $q^j$  and decreasing in  $q^{-j}$ .

*Proof.*

$$V^J(q^j, q^{-j}) = q^j F[(q^j q^{-j})^{-1}]$$

It is immediate to verify that  $V_2^J = -f[(q^j q^{-j})^{-1}]/q^j q^{-j} < 0$ . Moreover,  $V_1^J = F[(q^j q^{-j})^{-1}] - f[(q^j q^{-j})^{-1}]/q^j q^{-j} > 0$  by the equilibrium condition  $KF[(K)^{-1}] \leq F[(q^j q^{-j})^{-1}]q^j q^{-j}$ .

## The Semi-demanding Voter Equilibrium

When

$$2K > q^{-j} q^j. \quad (16)$$

there is a semi-demanding equilibrium where one voter has interior solution and the other is demanding:

$$\underline{u}^J = K/q^{-j}; \underline{u}^{-J} = q^{-j}.$$

This equilibrium requires that (1)  $J$  prefers his interior solution, that is a failure of (15):  $KF[(K)^{-1}] > F[(q^j q^{-j})^{-1}]q^j q^{-j}$ , and (2) that  $-J$  prefers to set the highest possible standard. There are two cases to consider: (a) when  $q^{-j} \leq \frac{u^J q^{-j}}{q^j - u^J} = \frac{q^{-j} K}{q^j q^{-j} - K}$  the objective function can be only maximized at the

interior point before  $u^J q^{-j}/q^j$ , or at the corner solution  $q^{-j}$ : it is immediate to verify that, since  $\frac{K}{q}F[(K)^{-1}] > q^{-j}F[(K)^{-1}]$ ,  $-J$  will choose the corner. As a consequence, a sufficient condition is  $2K > q^{-j}q^j$ . When, instead,  $q^{-j} > \frac{q^{-j}K}{q^j q^{-j} - K}$ , the objective function has a kink at  $\frac{q^{-j}K}{q^j q^{-j} - K}$ , and is convex afterwards. As a consequence, the relevant comparison is between  $\frac{q^{-j}K}{q^j q^{-j} - K}$  and  $q^{-j}$ :

$$\frac{q^{-j}K}{q^j q^{-j} - K} F[(K)^{-1}] < q^{-j} F[(q^j q^{-j} - K)^{-1}],$$

which is equivalent to

$$KF[(K)^{-1}] < (q^j q^{-j} - K)F[(q^j q^{-j} - K)^{-1}]. \quad (17)$$

Notice that, we know that  $KF[(K)^{-1}] > F[(q^j q^{-j})^{-1}]q^j q^{-j}$ . Since the function  $yF(y^{-1})$  is strictly increasing, with a peak at  $K$ , then decreasing, and then strictly convex,  $(q^j q^{-j} - K)F[(q^j q^{-j} - K)^{-1}] > F[(q^j q^{-j})^{-1}]q^j q^{-j}$  implies that at the point  $q^j q^{-j} - K$ ,  $yF(y^{-1})$  is strictly decreasing from  $K$ , which makes (17) impossible. As a consequence, (16) is both necessary and sufficient.

*CLAIM 12 In the Semi-demanding voter equilibrium  $V^J(q^j, q^{-j})$  is constant in  $q^j$  and strictly decreasing in  $q^{-j}$ ;  $V^{-J}(q^j, q^{-j})$  is constant in  $q^j$  and strictly increasing in  $q^{-j}$ .*

*Proof.* In this equilibrium the value to each voter is given by

$$\begin{aligned} V^J(q^j, q^{-j}) &= KF[(K)^{-1}]/q^{-j} \\ V^{-J}(q^j, q^{-j}) &= q^{-j}F[(K)^{-1}]. \end{aligned}$$

Simple inspection leads to the conclusion.

## The Lenient Voter Equilibrium

Where neither voter is demanding, the relatively more lenient has interior solution and the relatively less lenient chooses  $\underline{u}^{-j} = \frac{q^{-j}}{q^j - \underline{u}^{-j}} \underline{u}^j$ :

$$\underline{u}^j = K/q^{-j}; \underline{u}^{-j} = \frac{K}{q^j - K/q^{-j}}.$$

To simplify the analysis, we define the voter's payoff conditional on the other voter's standard  $U(u|K/q^{-j})$  and  $U(u|K/(q^j - K/q^{-j}))$ , and characterize a few important properties

CLAIM 13 (i)  $U(u|u^*)$  is strictly quasiconvex in  $[K/(q^j - K/q^{-j}), q^{-j}]$

(ii)  $U(u|K/(q^j - K/q^{-j}))$  is strictly convex in  $[K/(q^{-j} - K/q^j), q^j]$ .

*Proof.* (i) In the interval  $[K/(q^j - K/q^{-j}), q^{-j}]$ , the standard of voter  $-j$  affects the probability of his politician working for him, because the politicians' subgame exhibits an asymmetric equilibrium. As a consequence, and given  $u^* = K/q^{-j}$  we have

$$U(u|u^*) = uF((u(q^j - K/q^{-j}))^{-1}).$$

Differentiating the function and computing it at  $u = K/(q^j - K/q^{-j})$  yields

$$U'(K/(q^j - K/q^{-j})|u^*) = F(K^{-1}) - \frac{f(K^{-1})}{K} = 0.$$

Moreover, by the previous analysis and the optimality of  $u^*$ , we also know that  $U'(u)$  is decreasing at  $K/(q^j - K/q^{-j})$ , and then increasing (strictly quasiconvex). As a consequence,  $U(u|u^*)$  is first decreasing, and then potentially increasing in  $[K/(q^j - K/q^{-j}), q^{-j}]$ , hence strictly quasiconvex.

(ii) Analogously, the standard of voter  $j$  affects the probability of his politician working for him. As a consequence, we have  $U(u|u^{-j}) = uF((u(q^{-j} - u^{-j}))^{-1})$ . After differentiating twice with respect to  $u$  and going through some tedious but straightforward algebra, we obtain the following condition:

$$\text{sign}\{U''(u|u^{-j})\} = \text{sign}\{u(q^{-j} - u^{-j}) - 1/\mu\}$$

but, since we are restricting  $u$  to be above  $q^j u^{-j} (q^{-j} - u^{-j})^{-1}$ , we obtain  $u(q^{-j} - u^{-j}) \geq q^j u^{-j}$ , which, by assumption, is larger than  $K$ . But we know that  $1/K < \mu$ , which yields  $U''(u|u^{-j}) > 0$ .

CLAIM 14 *The Lenient Voter Equilibrium requires*

$$q^j q^{-j} > 2K$$

$$KF(K^{-1}) \geq \begin{cases} q^j q^{-j} F\left(\frac{1}{q^j q^{-j}} \frac{q^j q^{-j} - K}{K}\right) & \text{if } q^j q^{-j} < 3K \\ \max\left\{\frac{q^j q^j K}{q^j q^{-j} - 2K} F\left(\frac{1}{K} \frac{q^j q^{-j} - K}{q^j q^{-j}}\right), q^j q^{-j} F\left(\frac{1}{q^j q^{-j}} \frac{q^j q^{-j} - K}{q^j q^{-j} - 2K}\right)\right\} & \text{otherwise} \end{cases}$$

*Proof.* This equilibrium requires that (i) the point  $\frac{K}{q^j - K/q^{-j}}$  be feasible for the less lenient voter; (iii) the relatively less lenient prefers  $\frac{K}{q^j - K/q^{-j}}$  to any potential interior solution; (ii) the more lenient voter has an interior solution. Part (i) is ensured by having  $\frac{K}{q^j - K/q^{-j}} < q^{-j}$ , which is equivalent to  $2K < q^j q^{-j}$ .

Part (ii) requires that  $\frac{K}{q^j - K/q^{-j}} = \arg \max U(u|K/q^{-j})$ . By the previous Claim, we know that  $U(u|K/q^{-j})$  is increasing in  $[0, K/q^j]$ , then strictly quasiconvex in  $[K/q^j, \frac{K}{q^j - K/q^{-j}}]$ , and then convex beyond  $\frac{K}{q^j - K/q^{-j}}$ . As a consequence, we need  $\frac{K}{q^j - K/q^{-j}}$  to dominate both  $K/q^j$  and  $q^{-j}$ . The first condition is verified by simply looking at payoffs:

$$\frac{K}{q^j} F(K^{-1}) < \frac{K}{q^j - K/q^{-j}} F\left(\frac{1}{\frac{K}{q^j - K/q^{-j}} (q^j - K/q^{-j})}\right) = \frac{K}{q^j - K/q^{-j}} F(K^{-1}),$$

the second condition simplifies to

$$KF(K^{-1}) \geq (q^j q^{-j} - K) F((q^j q^{-j} - K)^{-1}), \quad (18)$$

which is implied by (16), since  $(q^j q^{-j} - K) F((q^j q^{-j} - K)^{-1}) > q^j q^{-j} F((q^j q^{-j})^{-1})$  can only happen, as argued in the analysis of the Semi-demanding equilibrium, when (18) also holds.

Part (iii) requires that  $\frac{K}{q^{-j}} = \arg \max U\left(u|\frac{K}{q^j - K/q^{-j}}\right)$ . By the previous Claim, we also know that the set of possible maximizers of  $U\left(u|\frac{K}{q^j - K/q^{-j}}\right)$  is given by  $\frac{K}{q^{-j}}$ ,  $\frac{q^j u^{-j}}{q^{-j} - u^{-j}} = \frac{q^j K}{q^j q^{-j} - 2K}$ , and  $q^j$ , but since we do not whether  $\frac{q^j K}{q^j q^{-j} - 2K}$  is above or below  $q^j$ , we need to distinguish between two cases. If

$3K > q^j q^{-j}$ , then we simply need to ensure that

$$KF(K^{-1}) \geq q^j q^{-j} F\left(\frac{1}{q^j q^{-j}} \frac{q^j q^{-j} - K}{K}\right),$$

which, since is stronger than (18), and therefore implies it. If, instead,  $3K > q^j q^{-j}$ , we need to guarantee that

$$KF(K^{-1}) \geq \max\left\{\frac{q^j q^j K}{q^j q^{-j} - 2K} F\left(\frac{1}{K} \frac{q^j q^{-j} - K}{q^j q^{-j}}\right), q^j q^{-j} F\left(\frac{1}{q^j q^{-j}} \frac{q^j q^{-j} - K}{q^j q^{-j} - 2K}\right)\right\},$$

which, since

$$q^j q^{-j} F\left(\frac{1}{q^j q^{-j}} \frac{q^j q^{-j} - K}{q^j q^{-j} - 2K}\right) > q^j q^{-j} F\left(\frac{1}{q^j q^{-j}}\right)$$

is stronger than (16), also implies (18). Putting together these cases yields the result.

**CLAIM 15** *In the lenient voter equilibrium  $V^J(q^j, q^{-j})$  is constant in  $q^j$  and strictly decreasing in  $q^{-j}$ ;  $V^{-J}(q^j, q^{-j})$  is strictly decreasing in both arguments.*

*Proof.* In this equilibrium the value to each voter is given by

$$\begin{aligned} V^J(q^j, q^{-j}) &= KF[(K)^{-1}]/q^{-j} \\ V^{-J}(q^j, q^{-j}) &= KF[(K)^{-1}]/(q^j - K/q^{-j}). \end{aligned}$$

Simple inspection leads to the conclusion.

## Uniqueness of Equilibrium

So far we have determined that there are three possible class of equilibria (each class contains two potential equilibria): Demanding (D), Semi-demanding (S), and Lenient (L). A parameters' set for this electoral game is a vector of politicians' qualities, location and scales of the payoff from the joint activity:  $\{\mu, s, q^a, q^b\} \in [\mathbb{R}^+]^4$ ; denote by  $\Pi^E$  the subset of  $[\mathbb{R}^+]^4$  where equilibrium  $E \in \{D, L, S\}$  exists.

**CLAIM 16**  $\Pi^L \cap \Pi^D = \emptyset$

*Proof.*  $D$  requires:

$$KF[(K)^{-1}] \leq F[(q^j q^{-j})^{-1}]q^j q^{-j},$$

$L$  requires either

$$KF(K^{-1}) \geq q^j q^{-j} F\left(\frac{1}{q^j q^{-j}} \frac{q^j q^{-j} - K}{K}\right), \quad (19)$$

when  $3K > q^j q^{-j}$ , or

$$KF(K^{-1}) \geq \max\left\{\frac{q^j q^j K}{q^j q^{-j} - 2K} F\left(\frac{1}{K} \frac{q^j q^{-j} - K}{q^j q^{-j}}\right), q^j q^{-j} F\left(\frac{1}{q^j q^{-j}} \frac{q^j q^{-j} - K}{q^j q^{-j} - 2K}\right)\right\}, \quad (20)$$

otherwise.  $\Pi^L \cap \Pi^D \neq \emptyset$  therefore implies that (3) and at least one of (19) or (20) holds. Suppose the former obtains. Then:

$$F[(q^j q^{-j})^{-1}]q^j q^{-j} \geq KF(K^{-1}) \geq q^j q^{-j} F\left(\frac{1}{q^j q^{-j}} \frac{q^j q^{-j} - K}{K}\right) \quad (21)$$

which is a contradiction since  $q^j q^{-j} > 2K$  is necessary for  $L$ , and  $F(\cdot)$  is strictly increasing. Suppose, instead, the latter obtains. Then:

$$F[(q^j q^{-j})^{-1}]q^j q^{-j} \geq KF(K^{-1}) \geq q^j q^{-j} F\left(\frac{1}{q^j q^{-j}} \frac{q^j q^{-j} - K}{q^j q^{-j} - 2K}\right) \quad (22)$$

which is a contradiction since  $\frac{q^j q^{-j} - K}{q^j q^{-j} - 2K} > 1$ .

CLAIM 17  $\Pi^S \cap \Pi^L = \emptyset$

*Proof.*  $S$  requires  $q^j q^{-j} < 2K$ ,  $L$  requires  $q^j q^{-j} > 2K$ .

CLAIM 18  $\Pi^D \cap \Pi^S = \emptyset$

*Proof.*  $D$  requires (3),  $S$  requires a failure of (3).

### Proof of Claim 3

(i)  $K$  is the solution of

$$H(-y, \mu) = F(y^{-1}) - f(y^{-1})y^{-1} = 0,$$

which is equivalent to the FONC of the problem  $\arg \max q'uF((q'u)^{-1})$ . To show that  $K$  and  $u^*$  are decreasing in  $\mu$ , we just need to show that  $H$  has increasing differences. Since

$$\begin{aligned}\frac{\partial H}{\partial(-y)} &= f(y^{-1})/y - F(y^{-1}) \\ \frac{\partial^2 H}{\partial(-y)\partial\mu} &= \frac{\partial}{\partial\mu} \left\{ \frac{1}{1 + \exp\left(\frac{\mu-y^{-1}}{s}\right)} \left[ \frac{1}{y^s} \frac{\exp\left(\frac{\mu-y^{-1}}{s}\right)}{1 + \exp\left(\frac{\mu-y^{-1}}{s}\right)} - 1 \right] \right\} = \\ &= \frac{\partial}{\partial\mu} \left\{ \frac{1}{1 + \exp\left(\frac{\mu-y^{-1}}{s}\right)} \right\} \left[ \frac{1}{y^s} \frac{\exp\left(\frac{\mu-y^{-1}}{s}\right)}{1 + \exp\left(\frac{\mu-y^{-1}}{s}\right)} - 1 \right] \\ &\quad + \frac{\partial}{\partial\mu} \left\{ \frac{\exp\left(\frac{\mu-y^{-1}}{s}\right)}{1 + \exp\left(\frac{\mu-y^{-1}}{s}\right)} \right\} \frac{1}{y^s} \left[ \frac{1}{1 + \exp\left(\frac{\mu-y^{-1}}{s}\right)} \right]\end{aligned}$$

Since we can write

$$\frac{\partial}{\partial\mu} \left\{ \frac{\exp\left(\frac{\mu-y^{-1}}{s}\right)}{1 + \exp\left(\frac{\mu-y^{-1}}{s}\right)} \right\} \equiv \delta > 0; \quad \frac{\partial}{\partial\mu} \left\{ \frac{1}{1 + \exp\left(\frac{\mu-y^{-1}}{s}\right)} \right\} = -\delta,$$

then the sign of  $\frac{\partial^2 H}{\partial(-y)\partial\mu}$  is the sign of

$$\frac{1}{y^s} \frac{1}{1 + \exp\left(\frac{\mu-y^{-1}}{s}\right)} + 1 - \frac{1}{y^s} \frac{\exp\left(\frac{\mu-y^{-1}}{s}\right)}{1 + \exp\left(\frac{\mu-y^{-1}}{s}\right)}.$$

While the above expression is in principle ambiguous, we know that when  $y = K$ , by definition of  $K$

$$1 - \frac{1}{K^s} \frac{\exp\left(\frac{\mu-K^{-1}}{s}\right)}{1 + \exp\left(\frac{\mu-K^{-1}}{s}\right)} = 0.$$

As a consequence, there must be an interval  $[-K, -\underline{y}]$ , with  $\underline{y} < K$ , where  $H$  has increasing differences, which implies that a marginal increase in  $\mu$  must always generate a decrease in both  $u^*$  and  $K$ . To see why the opposite must hold for  $s$ , proceed analogously as above, and notice that the condition determining the sign of  $\frac{\partial^2 H}{\partial(-y)\partial s}$  is the same as before, with the difference that, since

$\frac{\partial}{\partial s} \left\{ \frac{1}{1 + \exp\left(\frac{\mu - y^{-1}}{s}\right)} \right\} = -\frac{\partial}{\partial s} \left\{ \frac{\exp\left(\frac{\mu - y^{-1}}{s}\right)}{1 + \exp\left(\frac{\mu - y^{-1}}{s}\right)} \right\} > 0$ , in the interval  $[-K, -\underline{y}]$ , a marginal increase in  $s$  must always generate an increase in both  $u^*$  and  $K$ . (ii) Follows directly from applying the envelope theorem to  $u^* F((u^* q^{-j})^{-1})$ .