Multidimensional poverty and inequality

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Rapidly growing, recent, unconsolidated

Multidimensionality of well-being is now centre stage

- Alkire & Foster “Counting and multidimensional poverty measurement”, Journal of Public Economics 2011

  Multidimensional poverty *has captured the attention of researchers and policymakers alike due, in part, to the compelling conceptual writings of Amartya Sen and the unprecedented availability of relevant data.*

- *Stiglitz-Sen-Fitoussi Report* for French Presidency

- *Europe 2020 strategy*: Five headline targets for national policies:

  Reduction of poverty by aiming to lift at least 20 million people out of the risk of poverty or social exclusion

  Risk of poverty or social exclusion → multidimensional

Does social evaluation be multi-dimensioned? May be not …

- Either a **single variable** can still subsume all dimensions
  - Utility (e.g. revealed by consumption or happiness indicators)
  - Maasoumi's *utility-like function of all the attributes received*
  - income equivalisation

- Or **dimensions kept distinct** on philosophical or practical grounds
  - Walzer's *complex equality*
  - Tobin's *specific egalitarianism*
  - Erikson's intrinsic incommensurability of domains
  - Ravallion's rejection of ad hoc aggregation and unexplained tradeoffs between domains

    → *dashboard approach*

… or maybe yes – and it can be done

- **Intermediate route**: methods for multidimensional measurement of inequality and poverty
  - main motivation: inequalities in different domains cumulate
- **Pattern of association between variables distinguishes multidimensional from unidimensional analysis**
  - empirical vs. normative correlations
- Aim of the paper: unveil underlying measurement assumptions to elucidate their normative content
  - little attention to multivariate statistical techniques

    valuable, but hesitate to entrust mathematical algorithms with essentially normative task such as summarising well-being

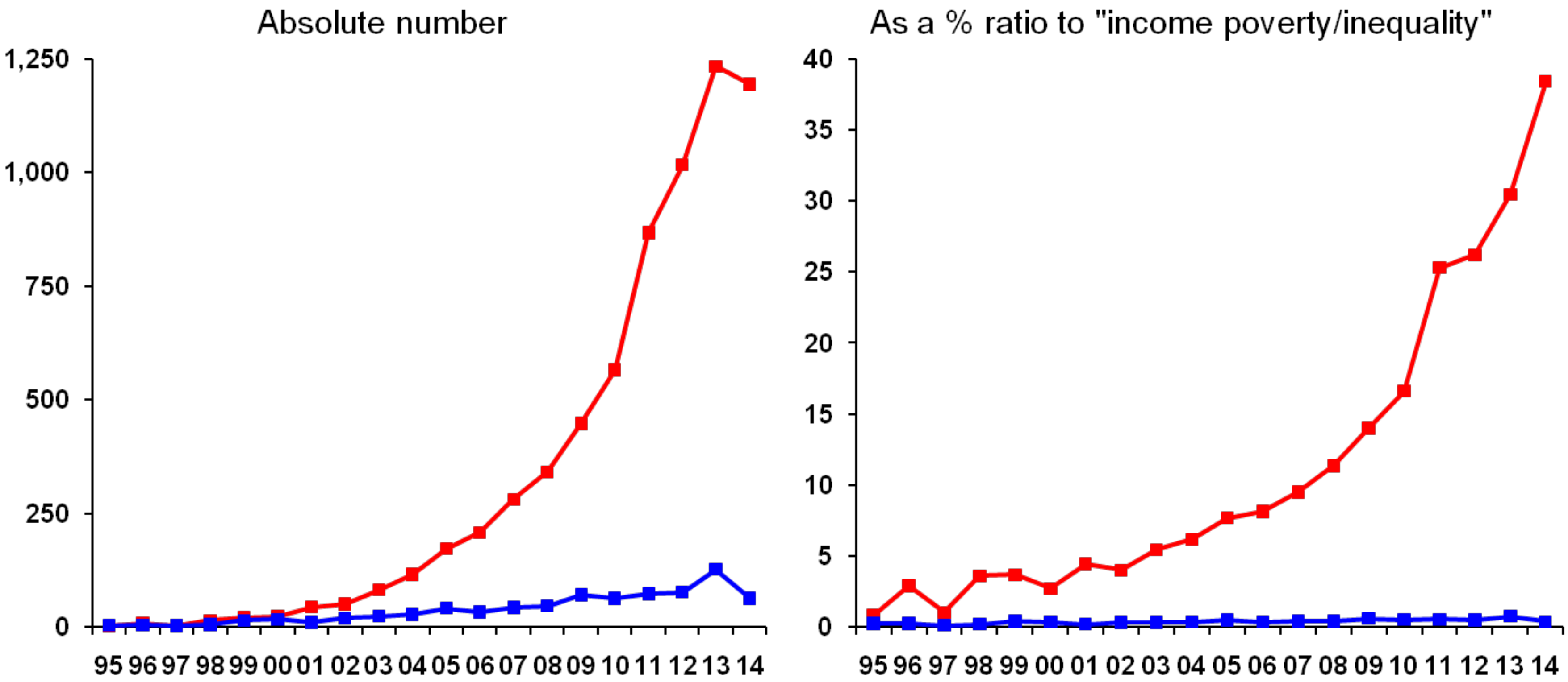
Not a new topic ...

- F M Fisher, “Distribution, Value Judgments, and Welfare”, Quarterly Journal of Economics 1956

It will be best, then, to begin by discussing exactly what we mean by “distribution.” Of course, we are interested in the real, not the money income of the individual or of the society and this makes things somewhat more complex. Real income is not a scalar but a vector whose components are amounts of commodities. Thus, we

6. Of course, no particular significance attaches to the prices as market valuations of the commodities. Any arbitrary set of weights would do as well.

... but with a very recent take-off



multidimensional poverty or deprivation (& ratio to income poverty)

multidimensional inequality (& ratio to income inequality)

Source: authors’ search of exact phrases in *Google Scholar*, 16 November 2014.

State of the art in 2000

Chapter 2

MEASUREMENT OF INEQUALITY

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Outline

- Preliminaries:
  - Selection of dimensions
  - Indicators to measure achievement
  - Weights
- A chronological map of (theoretical) developments
- Counting deprivations
- Poverty measurement with continuous variables
- Dominance criteria
- Conclusions

Preliminaries

Selection of dimensions (1)

- **Material hardship**: inability to consume various socially perceived necessities because of lack of economic resources
- **Social exclusion**: failure in achieving a reasonable living standard, a degree of security, an activity valued by others, some decision-making power, the possibility to draw support from relatives and friends (Burchardt et al 1999)
- **Scandinavian approach to welfare**: health and access to health care; employment and working conditions; economic resources; education and skills; family and social integration; housing; security of life and property; recreation and culture; political resources (Erikson 1993)
- **Capability approach**: life; bodily health; bodily integrity; senses, imagination, and thought; emotions; practical reason; affiliation; other species; play; control over one’s environment (Nussbaum 2003)
- **Sen-Stiglitz-Fitoussi Commission**: ...

Selection of dimensions (2)

- **Wide range and diversity** of domains
- Choice due to:
  - experts – possibly based on existing data, conventions and statistical techniques
  - empirical evidence regarding people's values
  - consultative process involving focus groups or representatives of the civil society or the public at large
- Nature of selected attributes may condition the definition of measurement tools (e.g. transferability of health)

Indicators to measure achievement

- **Different measurement units**
  - continuous variables (income), discrete (number of durable goods owned), categorical (highest school attainment), bounded continuous ordinal variables (numeracy scores), dichotomous (incidence of specific chronic illnesses)
- Problem of multidimensional analysis: **commensurability of indicators** $\rightarrow$ standardization (see Decancq-Lugo 2013)
- In poverty assessments: definition of deprivation thresholds is same problem as in univariate analysis
  - "fuzzy sets approach": continuum of grades of poverty by means of a membership function ranging from 0 to 1

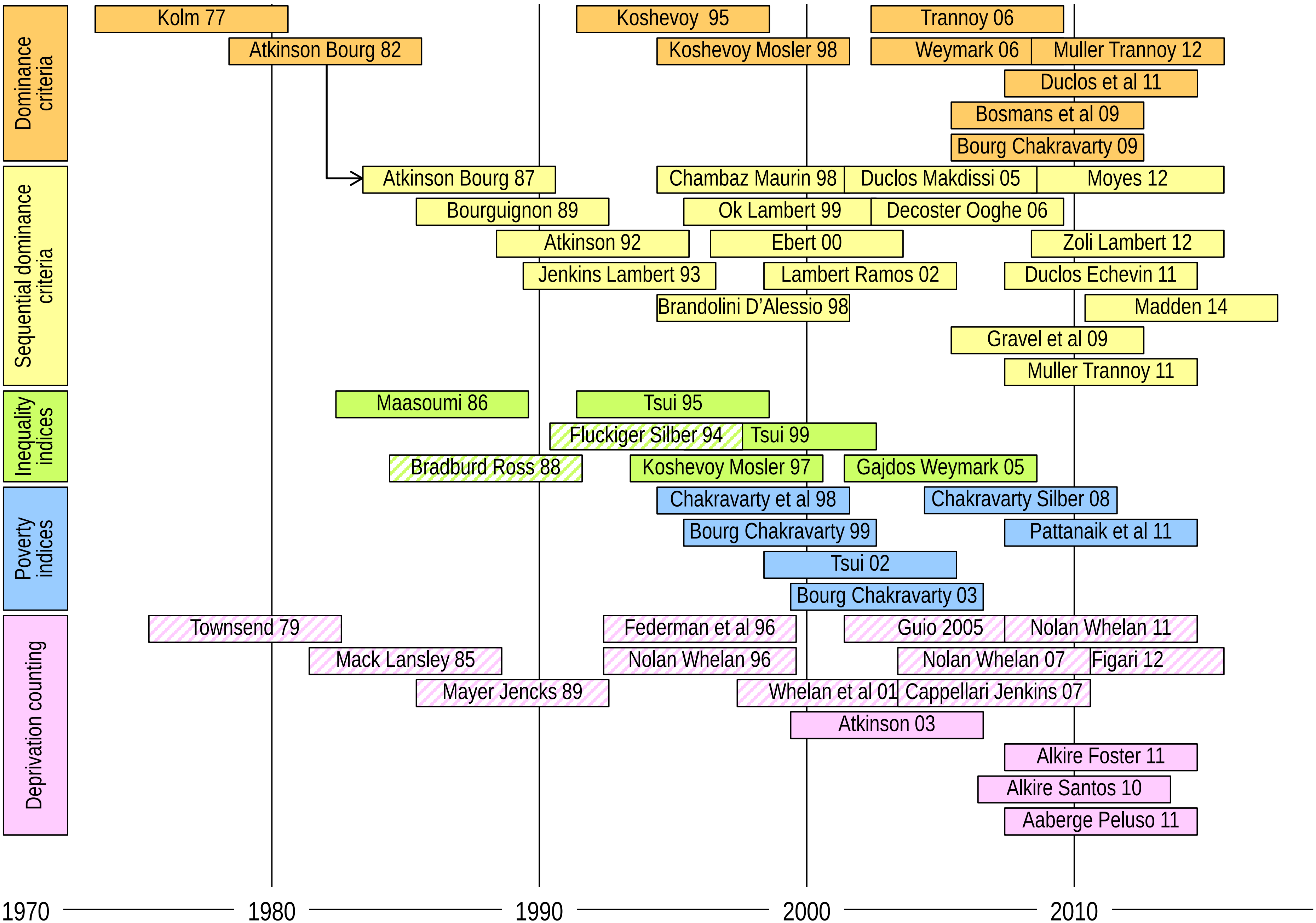
    though largely seen as distinct approach in the multivariate analysis of deprivation, nothing inherently multidimensional in theory of fuzzy sets

Weighting

- Weights determine contribution of attributes to well-being and their degree of substitution
- **Equal weighting**: lack of information about “consensus” view, but no discrimination
- **Consultations**, with experts or public, or **survey responses** (direct questions, indirectly from happiness equations)
- **Users’ own choice** (OECD Better Life Index)
- **Market prices**: non-existing or distorted by market imperfections and externalities, inappropriate for well-being comparisons
- **Data-based weighting**: Frequency-based approaches (weight inversely proportional to share of deprived people) or multivariate statistical techniques

→ ***Different weighting structures reflect different views: normative exercise (Sen: use range of weights)***

A chronological map of (theoretical) developments



Dominance criteria
Sequential dominance criteria
Inequality indices
Poverty indices
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Bradburd Ross 88
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Atkinson 03
Alkire Foster 11
Alkire Santos 10
Aaberge Peluso 11
1970
1980
1990
2000
2010

Counting deprivations

Counting approach (1)

- The **newest (theory)** & the **oldest (empirical practice)**
  - Main poverty statistic adopted by a parliamentary commission of inquiry over destitution in Italy in the early 1950s was a *weighted count of the number of households failing to achieve minimum levels of food consumption, clothing availability, and housing conditions*
- Modern research owes much to Townsend (1979)
  - Townsend's interest largely instrumental:

    "We assume that the deprivation index will not be correlated uniformly with total resources at the lower levels and that there will be a 'threshold' of resources below which deprivation will be marked"
- Huge **impact on social policy debate** in Ireland, UK, EU

Counting approach (2)

- But lack theoretical treatment of **normative bases** until recently
  - see Alkire and Foster (2011), Aaberge and Peluso (2011)
- Atkinson (2003): **difficult reconciliation** with social welfare approach
  - Part of the problem: definition of welfare criteria in terms of the distributions of the underlying continuous variables rather than in terms of the distribution of deprivation scores
  - In counting approach, **distribution of deprivation scores contains all relevant information**, which by construction implies neglecting levels of achievement in original variables

Counting approach (3)

- Indicators of living conditions: ownership of durables, possibility to carry out certain activities (e.g. going out for a meal with friends)
- **Count number of dimensions in which people fail to achieve a minimum standard**
  – simplest way to embed **association** between deprivations at individual level into an index of deprivation
  – aggregation across dimensions for each individual, then across individuals
- Alternative: **composite index of deprivation**
  – aggregation first across people, then across dimensions
  – advantage: combine heterogeneous various sources
  – disadvantage: if suffering from multiple deprivations has more than proportionate effect, cumulative effect is missing

The 2x2 case (1)

	$X_2=0$	$X_2=1$	
$X_1=0$	p_{00}	p_{01}	p_{0+}
$X_1=1$	p_{10}	p_{11}	p_{1+}
	p_{+0}	p_{+1}	1

- Two dimensions (i=1,2)
  - X_i = 1 if person suffers from deprivation in dimension i
  - X_i = 0 if person does not suffer from deprivation in dimension i
- p_{ij}: probability of $X_1 = i$ and $X_2 = j$

The 2x2 case (2)

	X_2=0	X_2=1	
X_1=0	p_{00}	p_{01}	p_{0+}
X_1=1	p_{10}	p_{11}	p_{1+}
	p_{+0}	p_{+1}	1

- Only marginal distributions known
- **Composite poverty index**: $P = g\ (\ p_{1+}\ ,\ p_{+1}\ )$
- Simple average: $P = (p_{1+}+p_{+1})/2$

  Individuals with two deprivations counted twice: suffering from two deprivations is twice as bad as suffering from one deprivation
- *Human Poverty Index*: $HPI = \zeta_1(p_1, p_2, \ldots, p_r) = \left(\sum_{k=1}^{r} w_k p_k^3\right)^{\frac{1}{3}}$

The 2x2 case (3)

	$X_2=0$	$X_2=1$	
$X_1=0$	p_{00}	p_{01}	p_{0+}
$X_1=1$	p_{10}	p_{11}	p_{1+}
	p_{+0}	p_{+1}	1

	$X=X_1+X_2$
$X=0$	$q_0=p_{00}$
$X=1$	$q_1= p_{10}+p_{01}$
$X=2$	$q_2=p_{11}$
	1

- Simultaneous distribution known
- Transform LHS distribution into RHS distribution by computing **deprivation score**: $X=X_1+X_2$ (equal weights)
- Who are the poor?
  - **union**: those who fail in either dimension, $P = g\ (\ 1–p_{00}\ )$
  - **intersection**: those who fail in both dimensions, $P = g\ (\ p_{11}\ )$

General notation

- Deprivation count $$X = \sum_{i=1}^{r} X_i$$

  with cumulative distribution function $$F(k) = \sum_{j=0}^{k} q_j \,, k = 0,1,...,r$$

  and mean $$\mu = \sum_{k=1}^{r} kq_k$$

- Dominance criteria defined in terms of the distribution *F* of univariate discrete variable *X* – not of underlying variables X_i
- Examine:
  1. partial orderings
  2. complete orderings (deprivation indices)

Distribution of material deprivations in some European countries, 2012 (% of total population)

Number of deprivations	France	Germany	Italy	Norway	United Kingdom
None	58.0	60.0	39.6	83.4	49.0
1 item	16.3	16.5	18.3	8.3	19.6
2 items	13.0	12.1	16.9	3.8	14.7
3 items	7.5	6.5	10.7	2.8	8.8
4 items	3.5	3.0	10.1	1.0	5.1
5 items	1.3	1.5	4.0	0.6	1.8
6 items	0.4	0.3	0.3	0.0	0.9
7 items	0.0	0.1	0.1	0.1	0.1
8 items	0.0	0.0	0.0	0.0	0.0
9 items	0.0	0.0	0.0	0.0	0.0
All	100.0	100.0	100.0	100.0	100.0

Source: Eurostat (2014)

First-degree dominance

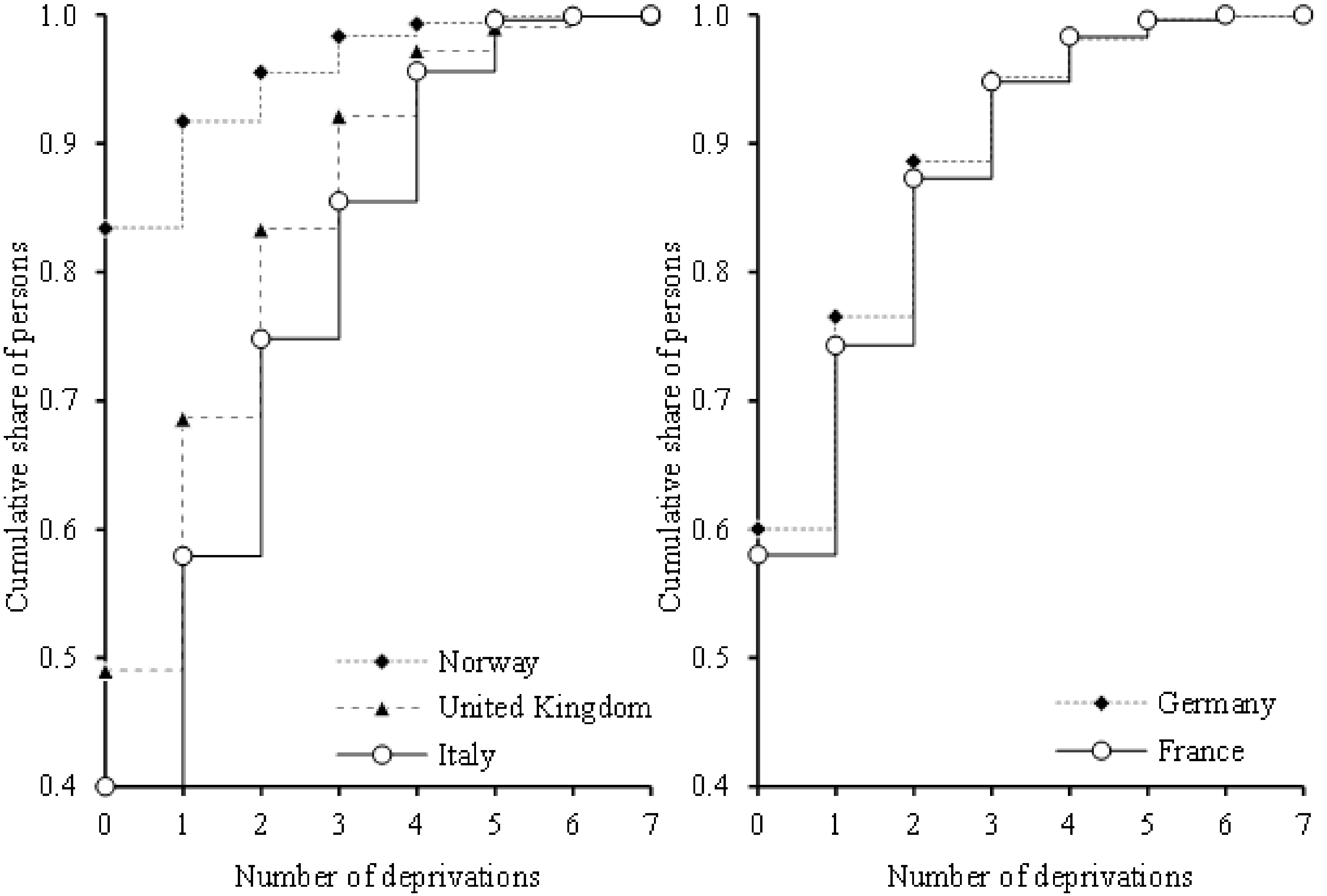
- Definition 1. *A deprivation count distribution* F_1 *is said to first-degree dominate a deprivation count distribution* F_2 *if*

$$F_1(k) \geq F_2(k) \quad for\ all\ k = 0,1,\ldots,r$$

and the inequality holds strictly for some k.

If *F_1* first-degree dominates *F_2*, then *F_1* exhibits less deprivation than *F_2*

Cumulative distributions of material deprivation scores in some European countries, 2012



- **NW first-degree dominates UK, IT**

- **No first-degree dominance**

  UK ahead of IT up to 5 items, but behind IT for 6/7 items

  FR and GE also cross

Second-degree dominance

- First-degree dominance might be too demanding in practice
- Define weaker dominance criteria, i.e. impose stricter conditions on preference ordering of social evaluator
- In counting deprivation account for:
  - **intersection criterion**

    aggregate “from above”, looking first at the proportion of those who are deprived in r dimensions, then adding the proportion of those failing in r–1 dimensions, and so forth
  - **union criterion**

    aggregate “from below”

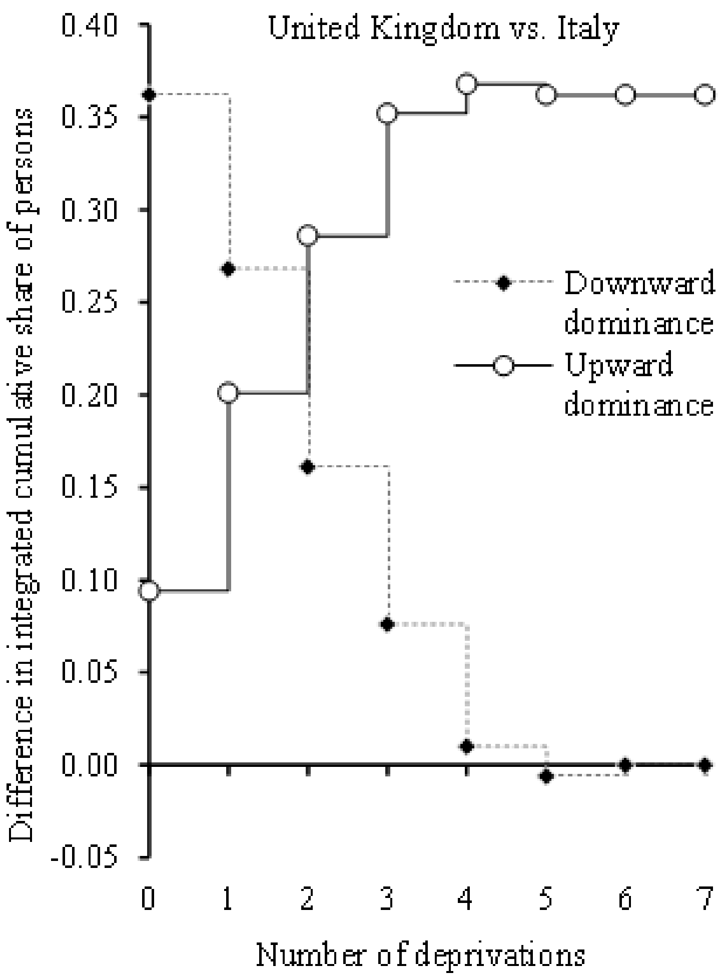
Second-degree dominance

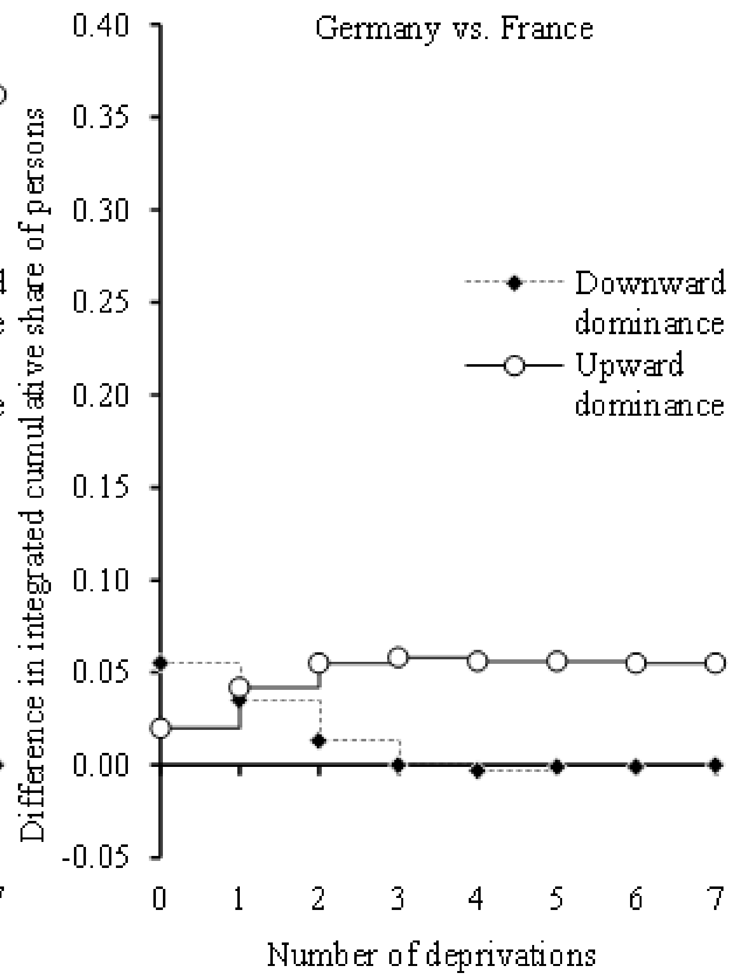
- Definition 2A. *A deprivation count distribution* F_1 *is said to second-degree downward dominate a deprivation count distribution* F_2 *if*
$$\sum_{k=s}^{r} F_1(k) \geq \sum_{k=s}^{r} F_2(k) \ \ for\ all\ s = 0,1,\ldots,r$$
and the inequality holds strictly for some s.

- Definition 2B. *A deprivation count distribution* F_1 *is said to second-degree upward dominate a deprivation count distribution* F_2 *if*
$$\sum_{k=0}^{s} F_1(k) \geq \sum_{k=0}^{s} F_2(k) \ \ for\ all\ s = 0,1,\ldots,r$$
and the inequality holds strictly for some s.

If F_1 second-degree dominates F_2, then F_1 exhibits less deprivation than F_2, but at cost of stricter conditions

Second-degree dominance for material deprivation scores in some European countries, 2012





- **Agreeing on whether *to go up* (union) or *to go down* (intersection) not sufficient**
- If integrate going up, UK/GE second-degree (upward) dominates IT/FR
- If integrate going down, no country second-degree (downward) dominates the other

Complete ordering (1)

- Impose an independence axiom for preference ordering $\succeq$
  $\rightarrow$ roughly, weight differently certain parts of the distributions

- Axiom (Independence). *Let F_1 and F_2 be members of **F**. Then* $F_1 \succeq F_2$ *implies* $\alpha F_1 + (1-\alpha)F_3 \succeq \alpha F_2 + (1-\alpha)F_3$ *for all $F_3 \in$ **F** and* $\alpha \in [0,1]$.

- If overall count deprivation is lower in country 1 than in country 2, so that F_1 is weakly preferred to F_2, the ranking would not change by adding to the population of either country the same group of migrants, whose deprivation distribution is F_3
- **Ordering relation invariant with respect to aggregation of sub-populations across deprivations**
- *NB: alternative Dual Independence axiom*

Complete ordering (2)

- **Independence Axiom** leads to deprivation measures:

$$d_\gamma(F) = \sum_{k=0}^{r} \gamma(k) q_k = \begin{cases} \gamma(\mu) + \delta_\gamma(F) & \text{when } \gamma \text{ is convex} \\ \gamma(\mu) - \delta_\gamma(F) & \text{when } \gamma \text{ is concave} \end{cases}$$

where $$\delta_\gamma(F) = \begin{cases} \sum_{k=0}^{r} \left(\gamma(k) - \gamma(\mu)\right) q_k & \text{when } \gamma \text{ is convex} \\ \sum_{k=0}^{r} \left(\gamma(\mu) - \gamma(k)\right) q_k & \text{when } \gamma \text{ is concave} \end{cases}$$

and $\gamma(k)$, with $\gamma(0)=0$, is a non-negative, non-decreasing continuous function of the number of deprivations k

- **deprivation intensity function $\gamma(k)$: curvature reflects how much we dislike increasingly severe deprivations in <u>convex case</u>, or growingly diffused deprivations in <u>concave case</u>**

Complete ordering (3)

- **Independence Axiom** leads to deprivation measures:

$$d_\gamma(F) = \sum_{k=0}^{r} \gamma(k) q_k = \begin{cases} \gamma(\mu) + \delta_\gamma(F) & \textit{when } \gamma \textit{ is convex} \\ \gamma(\mu) - \delta_\gamma(F) & \textit{when } \gamma \textit{ is concave} \end{cases}$$

- $\gamma(k)=k$ for all $k \rightarrow d_\gamma(F)=\mu$
  only mean matters: social preferences ignore deprivation dispersion; same result as with composite index approach

- **When dispersion matters, judgement depends on whether social preferences give more weight to ...**
  ... *s* people with 1 deprivation each (then concave function *γ*)
  ... or to 1 person with s deprivations (then convex function *γ*)

Complete ordering (4)

- **Independence Axiom** leads to deprivation measures:

$$d_\gamma(F) = \sum_{k=0}^{r} \gamma(k) q_k = \begin{cases} \gamma(\mu) + \delta_\gamma(F) & \text{when } \gamma \text{ is convex} \\ \gamma(\mu) - \delta_\gamma(F) & \text{when } \gamma \text{ is concave} \end{cases}$$

- Inserting $\gamma(k)=2rk-k^2$ (concave) and $\gamma(k)=k^2$ (convex), the term $\delta_\gamma(F)$ equals the variance
- Inserting $\gamma(k)=(k/r)^\theta$, $d_\gamma(F)$ is analogue of ***FGT* measures** and generalises **Atkinson (2003) counting measure** (defined for r=2)

$$A_\theta = 2^{-\theta}\left[p_{1+} + p_{+1} + 2(2^{\theta-1}-1)p_{11}\right] = 2^{-\theta}\left(p_{1+} + p_{+1}\right) + \left(1-2^{1-\theta}\right)p_{11} = 2^{-\theta} q_1 + q_2$$

$A_0=q_1+q_2$ $\rightarrow$ union: all people with at least one deprivation
$A_1=(p_{1+}+p_{+1})/2$ $\rightarrow$ mean of headcount rates (as composite index)
$A_\infty=p_{11}=q_2$ $\rightarrow$ intersection: only people with both deprivations

Indices of material deprivations in some European countries, 2012

Index		Germany	France	Italy	United Kingdom	Norway	Germany vs. France	United Kingdom vs. Italy
Linear indices								
Mean deprivations		0.822	0.877	1.471	1.109	0.320	-6.3	-24.6
Concave indices								
d_{θ}^{GA}	$\theta \to 0$	0.400	0.420	0.604	0.510	0.166	-4.8	-15.6
	$\theta = 0.1$	0.340	0.358	0.523	0.436	0.140	-5.0	-16.6
	$\theta = 0.5$	0.184	0.195	0.303	0.241	0.074	-5.7	-20.4
	$\theta = 0.9$	0.104	0.111	0.184	0.140	0.041	-6.2	-23.8
$d_2^{V,concave}$		12.550	13.399	21.883	16.747	4.914	-6.3	-23.5
Convex indices								
d_{θ}^{GA}	$\theta = 1.1$	0.080	0.086	0.146	0.109	0.031	-6.3	-25.3
	$\theta = 2$	0.028	0.029	0.057	0.040	0.010	-5.9	-30.0
	$\theta = 3$	0.011	0.011	0.023	0.016	0.004	-3.6	-31.6
	$\theta = 4$	0.005	0.005	0.010	0.007	0.002	0.4	-30.1
	$\theta = 8$	0.001	0.001	0.001	0.001	0.000	20.6	-13.5
	$\theta = 9$	0.0003	0.0002	0.0005	0.0005	0.0001	42.8	2.3
	$\theta = 20$	7.6×10^{-06}	1.3×10^{-06}	7.8×10^{-06}	9.4×10^{-06}	6.6×10^{-06}	479.9	20.9
$d_2^{V,convex} = r^2 d_2^{GA}$		2.246	2.387	4.595	3.215	0.846	-5.9	-30.0

Complete ordering (5)

- **Dual Independence Axiom** leads to deprivation measures:

$$D_\Gamma(F) = r - \sum_{k=0}^{r-1} \Gamma(\sum_{j=0}^{k} q_j) = \begin{cases} \mu + \Delta_\Gamma(F) & \textit{when } \Gamma \textit{ is convex} \\ \mu - \Delta_\Gamma(F) & \textit{when } \Gamma \textit{ is concave} \end{cases}$$

where $$\Delta_\Gamma(F) = \begin{cases} \sum_{k=0}^{r-1} \left[\sum_{j=0}^{k} q_j - \Gamma(\sum_{j=0}^{k} q_j) \right] & \textit{when } \Gamma \textit{ is convex} \\ \sum_{k=0}^{r-1} \left[\Gamma(\sum_{j=0}^{k} q_j) - \sum_{j=0}^{k} q_j \right] & \textit{when } \Gamma \textit{ is concave} \end{cases}$$

and $\Gamma(k)$, with $\Gamma(0)=0$ and $\Gamma(1)=1$, is a non-negative, non-decreasing continuous function of the number of deprivations k

- Inserting $\Gamma(t)=2t-t^2$ (concave) and $\Gamma(t)=t^2$ (convex), the term $\Delta_\Gamma(F)$ equals the Gini mean difference

Association rearrangements

- Pattern of association across dimensions – key feature of multivariate case – so far ignored

  How does social welfare respond to change in distribution of deprivations across people, keeping constant mean deprivations?

Marginal-free positive association increasing rearrangement

	X_2=0	X_2=1	
X_1=0	0.35	0.20	0.55
X_1=1	0.20	0.25	0.45
	0.55	0.45	1

	X_2=0	X_2=1	
X_1=0	0.36	0.19	0.55
X_1=1	0.19	0.26	0.45
	0.55	0.45	1

- Attributes are **substitute** (one attribute can compensate for the lack of the other) if deprivation measure increases after a correlation increasing shift; they are **complement** if deprivation measure decreases
- Helps to refine ranking criteria $\rightarrow$ equivalence results

Counting deprivations vs. poverty (1)

- Concern with distribution of deprivation counts $\rightarrow$ focus on “aggregation” more than “identification”, in Sen’s distinction
- Contrast between union and intersection criteria suggests there is some leeway in defining “who is poor”
- Union and intersection are extremes: intermediate cases
  - European Union regards as severally materially deprived all persons who cannot afford at least four out of nine amenities
  - Alkire and Foster’s (2011) **“dual cut-off” approach**: dimension-specific thresholds & threshold identifying minimum number of deprivations to be classified as poor

Counting deprivations vs. poverty (2)

- If a person is poor when deprived in at least *c* dimensions, 0≤*c*≤1, headcount ratio is uniquely determined by count distribution *F*:

$$\tilde{H}(c) = 1 - F(c-1) = \sum_{k=c}^{r} q_k$$

- Previous analysis carries out replacing F with conditional count distribution

$$\tilde{F}(k;c) = \Pr(X \le k \mid X \ge c) = \frac{F(k) - F(c-1)}{1 - F(c-1)} = \frac{\sum_{j=c}^{k} q_j}{\sum_{j=c}^{r} q_j}, \quad k = c, c+1, \ldots, r$$

with mean $\tilde{\mu}(c) = \frac{\sum_{j=c}^{r} j q_j}{\sum_{j=c}^{r} q_j}$

Counting deprivations vs. poverty (3)

- Alkire and Foster (2011) propose to combine the adjusted headcount ratio

$$\tilde{M}_1(c) = \frac{\tilde{H}(c)\tilde{\mu}(c)}{r} = \frac{1}{r}\sum_{j=c}^{r} jq_j$$

Ratio of total number of deprivations experienced by the poor to maximum number of deprivations that could be experienced by entire population

- unequal weights: replace deprivation count for each person by sum of associated weights
- increases if a poor person becomes deprived in an additional dimension (dimensional monotonicity), but indifferent to deprivations of the non-poor as well as to changes in distribution of deprivations across the poor

Counting deprivations vs. poverty (4)

- *FGT* generalisation accounting for distribution of deprivations across the poor

$$\tilde{M}_{\theta}(c) = \frac{1}{r}\sum_{j=c}^{r} j^{\theta} q_j, \quad \theta > 0$$

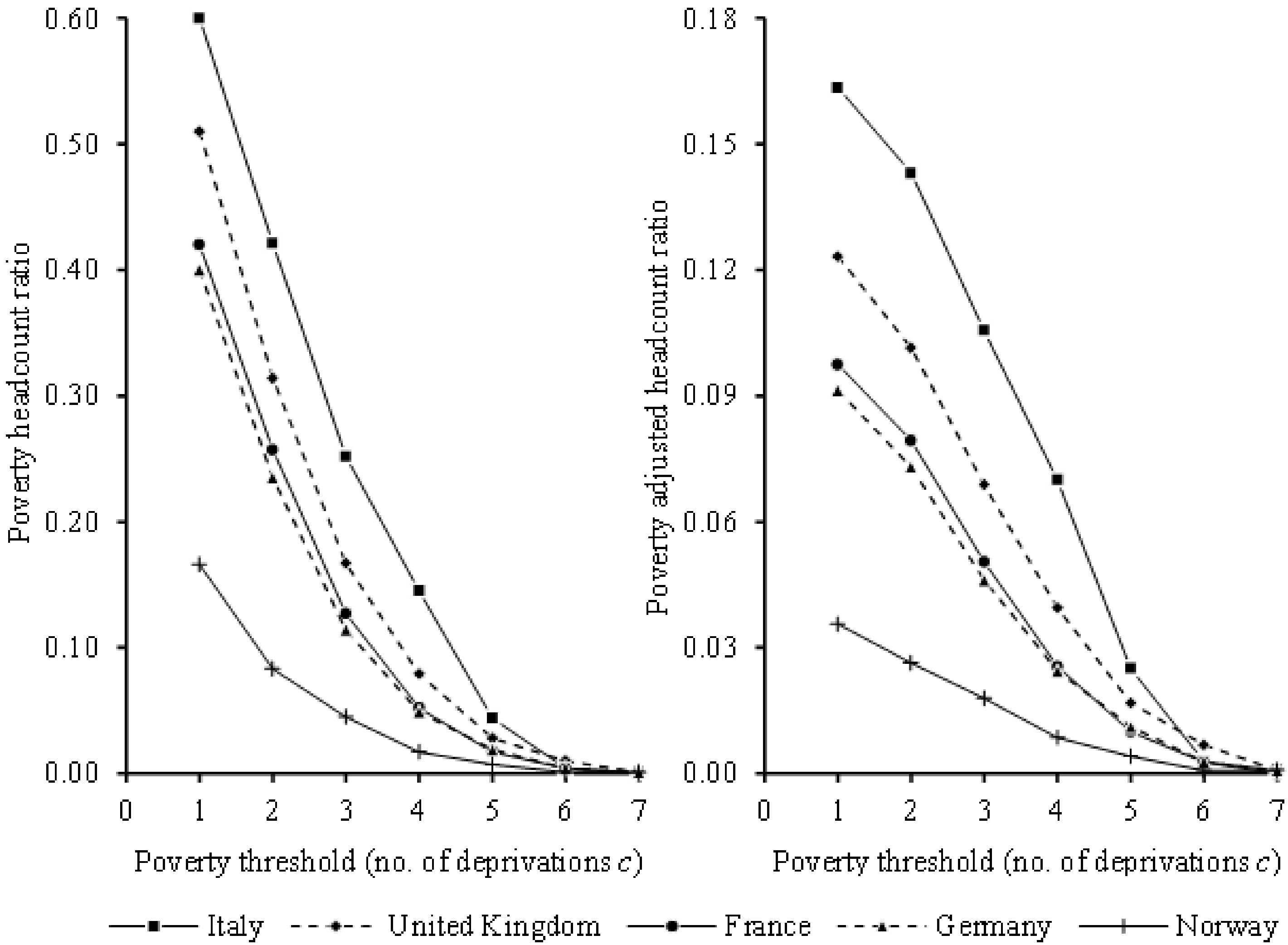
θ=1 gives Alkire and Foster's measure

$\theta \to 0$ ignores cumulative effects of multiple deprivations

As θ rises, greater weight placed on those who suffer from deprivation in several dimensions

- Alkire and Foster's adjusted headcount ratio has great impact on empirical research and provides theoretical basis for Multidimensional Poverty Index (*MPI*) adopted by the United Nations Development Programme since 2010

Poverty adjusted headcount ratios for different poverty cut-offs in some European countries, 2012



- Censoring at 4 implies excluding from measured poverty many people suffering from 1, 2 or 3 deprivations ...

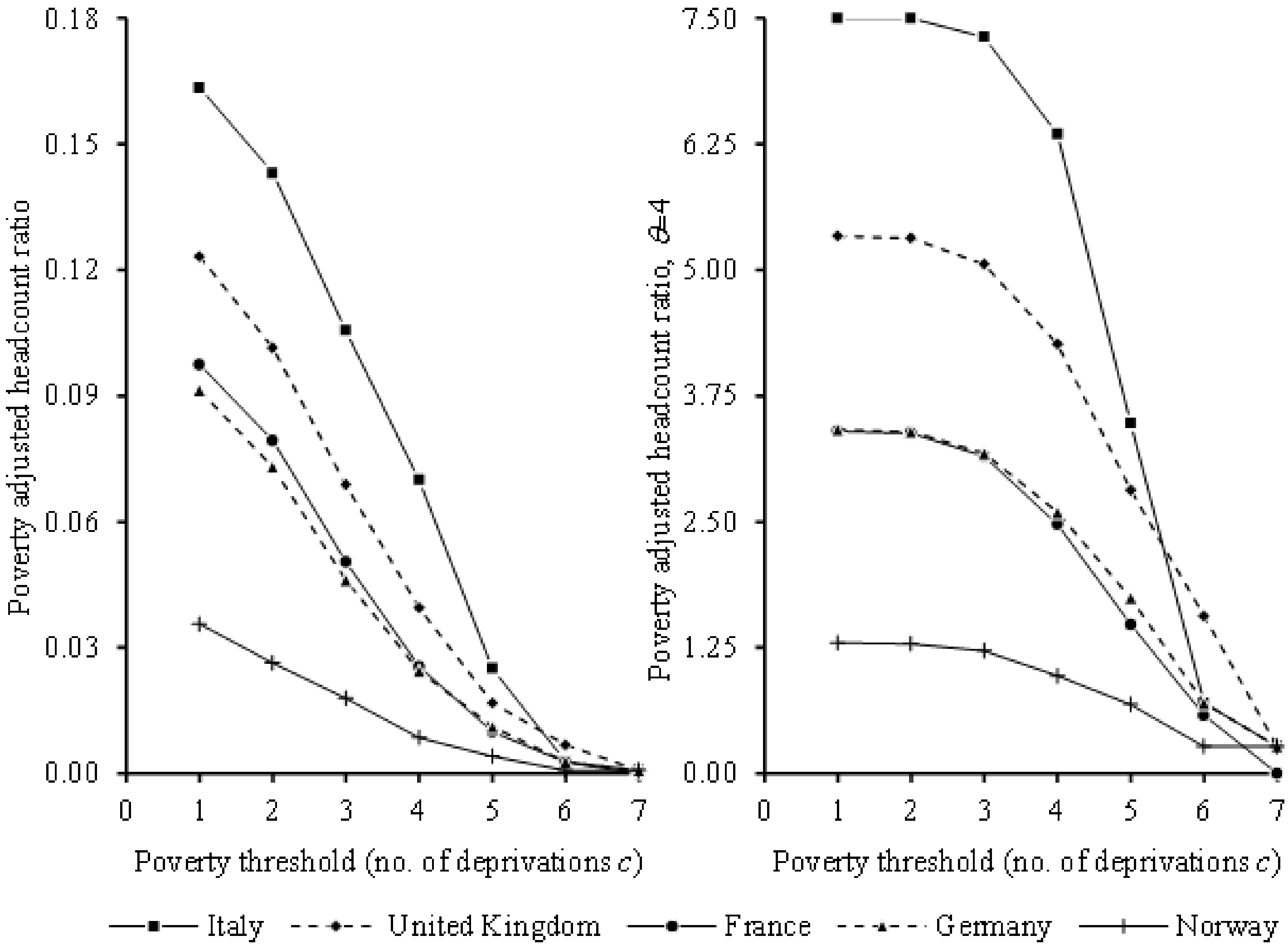
  ... but ranking unchanged

- Ranking changes with cut-off

  at 5: GE and FR reverse order

  at 6: UK country with highest share of poor

Poverty adjusted headcount ratios for different poverty cut-offs in some European countries, 2012



- Varying poverty cut-off has considerable impact on measured poverty
- Adjusting headcount ratio for deprivations experienced by the poor has minor effects, unless their distribution is taken into account

Poverty measurement with continuous variables

Not only counting (1)

- With continuous variables, use measures multidimensional poverty that fully exploit informational richness of available data
  - aggregate first across dimensions, then across individuals → utility-like function
  - axiomatic simultaneous aggregation approach for measuring multidimensional poverty: aggregate individual shortfalls relative to dimension-specific cut-offs
- Bourguignon and Chakravarty (1999)

$$P_\theta(y;z) = \frac{1}{nr}\sum_{i=1}^{n}\sum_{j=1}^{r} a_{ij}\left(1 - \frac{y_{ij}}{z_j}\right)^{\theta_j}, \quad \theta_j > 1$$

where a_{ij} equal to the weight w_j of attribute j if $y_{ij}<z_j$ and 0 otherwise

- Alkire and Foster (2011) also define previous measure but selecting only poor people (deprived in at least c dimensions)

Not only counting (2)

- Not sensitive to association rearrangement interventions
- To account for correlation between attributes Bourguignon and Chakravarty (1999, 2003) introduce a family of non-additive poverty measures for two-dimensional case

$$P^{*}_{\alpha,\beta}(y;z) = \frac{1}{nr}\sum_{i=1}^{n}\left(\sum_{j=1}^{2} a_{ij}\left(1-\frac{y_{ij}}{z_{j}}\right)^{\beta}\right)^{\frac{\alpha}{\beta}}$$

where *α* and *β* are non-negative parameters

Effect of an increasing correlation rearrangement depends on whether the attributes are substitutes (α>β) or complements (α<β)

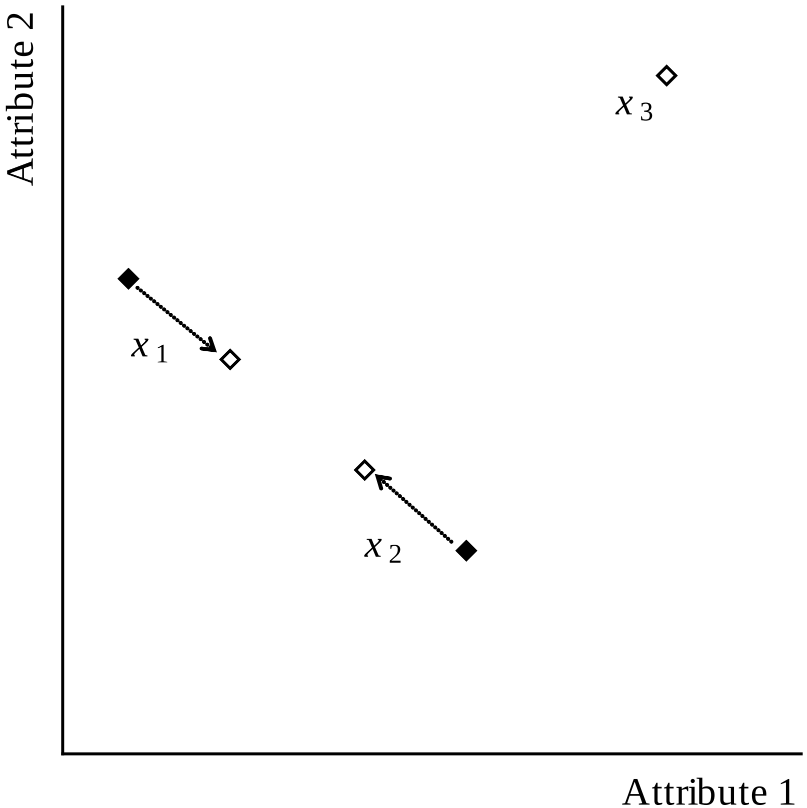
- ***Many more measures***
- ***Then poverty orderings …***

Dominance criteria

Axioms for ranking distributions

- Some axioms easy to extend to multiple dimensions
  - Anonymity principle
- Some extensions less obvious
  - Scale vs. translation invariance for life expectancy
- Extension of Pigou-Dalton transfer principle is not unique
  - Income: inequality falls when income is transferred from a richer to a poorer person
  - Multiple dimensions: many possible “majorization” criteria, i.e forms of averaging of attributes across people (some examples)
  - Many well-being attributes are not transferable: **transferring health** from a healthier individual to a sick one is unfeasible (but for organ transplants) and ethically questionable

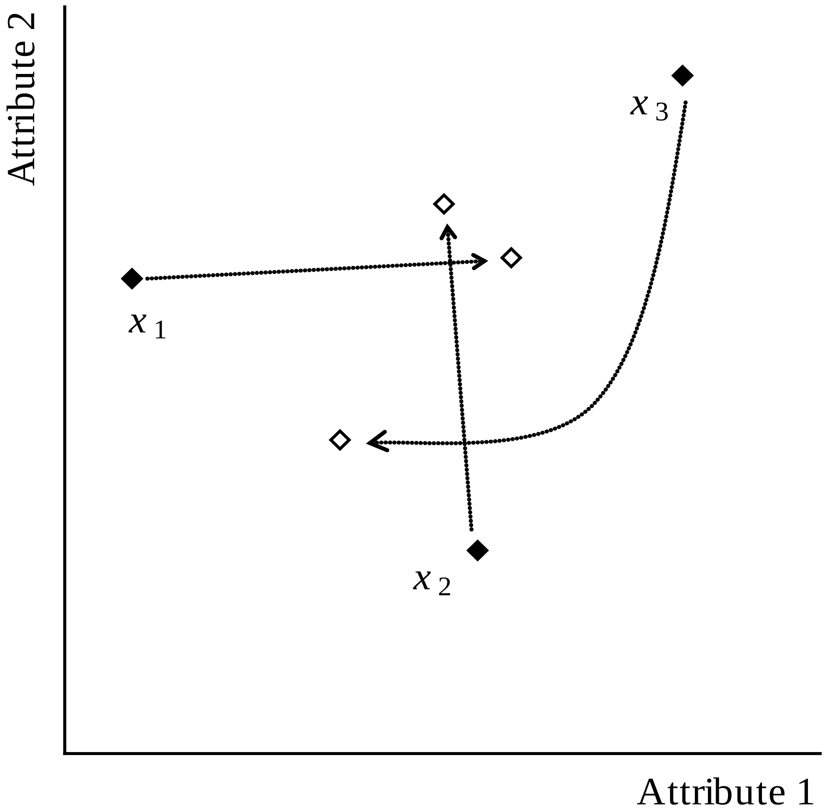
Multidimensional Pigou-Dalton (1)



Uniform Pigou-Dalton majorization or chain majorization

- Transfer involving all attributes simultaneously and identically

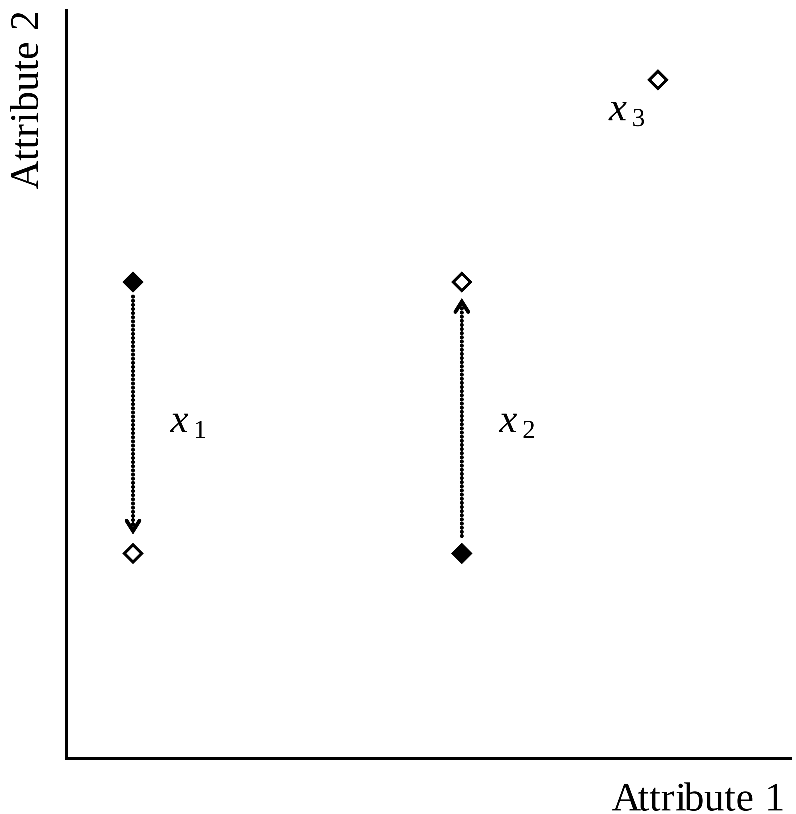
Multidimensional Pigou-Dalton (2)



Uniform majorization

- Form of averaging that makes the distribution less “spread-out”

Multidimensional Pigou-Dalton (3)



Correlation-increasing majorization

- Exchange of all attributes between two individuals after which one individual is left with the lowest endowment and the other with the maximum endowment of each attribute
- By concentrating attributes, this transfer leads to a distribution which is less socially preferable than the original one (if substitute)

Sequential dominance criteria

- Consider income and household composition
  - Standard approach: adjust income by equivalence scales
- Sequential dominance impose weaker assumptions on social preferences
  - equivalisation entails specifying how much a family type is needier than another one (but complete ordering)
  - sequential dominance only require ranking family types in terms of needs (but incomplete orderings)
- More generally: one attribute (e.g. income) can be used to compensate for another non-transferable attribute (e.g. needs, health)

Conclusions

A thriving research area

- Since 1990s, **novel analytical results** accompanied by massive production of applied research
  - **new and rich databases**
  - **new conceptualisations** of well-being – *capability approach*
  - **policy orientation** more inclined to nuances of well-being
- Progress not always coherent
  - applied research sometimes moved from available data unaware of analytical developments
  - theoretical research sometimes ignored applicability of results to real data
- Common when development is rapid

Explain why we enriched our toolbox with many new instruments, but we still disagree on how to use them

Counting deprivations

- Social evaluation analogous to social evaluation of income distributions, though accounting for association among dimensions
  - Concave preferences in income correspond to convex preferences in deprivations counts, which are “bads” (loss in welfare) rather than “goods” (gains in welfare)
- Convex preferences ruled out in income distribution analysis – violate Pigou-Dalton principle of transfers – but concave preferences perfectly legitimate in deprivation counts (→ union criterion)
  - **Multidimensional case brings in new aspects**, unknown to univariate case
  - **Strict connection between value judgements** – who is poor – **and analytical tools** – concavity/convexity of social preferences

Is it really worth it?

- After all, once Sen (1987) remarked:

  “the passion for aggregation makes good sense in many contexts, but it can be futile or pointless in others. ... When we hear of variety, we need not invariably reach for our aggregator”

- Four reasons suggest positive answer

  1. **Pervasive demand by media commentators and policy-makers** $\rightarrow$ avoid that multidimensional analyses left to practitioners that conceive them as bunching together indicators of living standard through some simple averaging or multivariate technique easily available in statistical and econometric packages

Is it really worth it?

2. **Distinct informative value**: theoretical work facilitates interpretation of empirical findings by bringing to the fore implicit measurement assumptions and economic meaning
3. **Difficulties not to be overstated**: choice of degree of poverty or inequality aversion, proper definition of indicators also arise in univariate context

   new problems are weighting structure and degree of substitutability of attributes → no technical hitches but expression of implicit value judgements
4. **Battery of instruments in our toolbox is ample**: if we are reluctant to use a summary poverty or inequality index, we may fruitfully use sequential dominance analysis: it may yield a partial ordering, but it may be sometimes sufficient to evaluate, say, the impact on the distribution of well-being of alternative policies

Thank you for your attention!