Elections and Durable Governments in Parliamentary Democracies

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Abstract

This paper provides a theory of a parliamentary government system with proportional representation elections and policy-motivated parties and voters. In a symmetric, spatial model governments are majoritarian, they and their policies are durable, and voters elect minority parliaments in every period. A continuum of (Markov) political equilibria exist with policies that represent concessions to centrist voters. In these equilibria the parties in a majoritarian government are equal partners. The greater the concession the more politically patient the parties must be for an equilibrium to exist. If one party is more centrally-located in the space of voter preferences, it can receive a majority and choose its ideal policy. If officeholding benefits are available, the policy of a durable government favors the head of government and changes when the head changes. In the elections the out party loses half its vote share to one of the government parties because rather than waste their votes. If crises can occur, governments can fall, but a new government forms after the next election. If crises are sufficiently frequent, no political equilibrium exists.

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1 Introduction

Political systems in which the parliament chooses the executive are prevalent in Europe and in a number of countries outside Europe. The institutions of parliamentary systems vary considerably among the countries, and abstracting from the institutional variation, this paper presents a dynamic theory of parliamentary systems with proportional representation electoral systems. The theory provides straightforward predictions about representation, government formation, and policy choice. The paper examines the incentives for policy-motivated parties to form and break governments and policy-motivated voters to support or punish parties for the policies they implement. The policy space is multidimensional, and the policies considered are continuing in the sense that they remain in place until changed by the parliament. The theory thus is dynamic. The focus is on durable governments that along with their policies continue from one period to the next. In the absence of crises these governments continue indefinitely. Governments can fall in a crisis, however, and if crises are sufficiently likely, the political system is unstable in the sense that there is no dynamic political equilibrium.

The model of a parliamentary system is simple with little institutional detail. Voters elect parliaments, the parties in parliament form governments, and governments choose policies. The model is neutral with respect to which governments form and for which party a voter votes. Voters base their votes on the governments and policies they anticipate in the current and future periods. The political equilibria characterized are Markov perfect equilibria in which strategies are conditioned on a limited history of past actions and neither parties nor voters can commit to their future actions.

A continuum of equilibria exists with minority parliaments and durable majoritarian governments, and the equilibrium policies can involve policy concessions to centrist voters. The equilibria are supported by threats. One threat is that if a government party withdraws or defects and the government falls, a new government formation round commences in the next period and the defector risks being left out of the next government. Equilibria are also supported by the threat that voters could give the out party a majority if the government parties deviate from their policy promises. These threats are collective in the sense that they punish all the government parties. Equilibria supported by the threat of a new government formation round can have Pareto inefficient policies. Equilibria supported by a threat from voters, however, have only Pareto efficient policies, since otherwise voters give a majority to the out party which chooses a Pareto efficient policy. Consensus governments cannot be supported by the threat of a new government formation round, but they can be supported by the threat of voters giving another party a majority if a party defects from the consensus government.

Durable governments are supported by political equilibria when parties are politically patient, and in a certain world the political equilibria have infinitely durable governments and policies. Durable governments, however, are not immune to crises that alter the government policy. If crises can occur, durable, but not infinitely so, governments form when crises are infrequent, and those governments continue until a crisis occurs. The government then falls, and a new government forms in the next period. If the risk of a crisis is sufficiently high, i.e., crises are frequent, a political equilibrium does not exist, since the stake the parties have in preserving a government when it is likely to be ended by a crisis is insufficient to withstand the temptation to take a short-term gain and risk a new government formation round.

The governments predicted by the theory are minimal winning and include the formateur. Martin and Stevenson (2001) present empirical evidence that governments in parliamentary systems tend to be minimal-winning and include the formateur. They also find strong evidence (Tables 2 and 3) that the incumbent coalition is included in the next government.

Voters are assumed to be policy-motivated and forward-looking and hence anticipate the government formation bargaining and the policies than result from that bargaining. Voters thus may not vote for their preferred party. Empirical studies of parliamentary systems find evidence of such behavior. For example, Kedar (2005, p. 185) "demonstrate[s] that voters are concerned with policy outcomes and hence incorporate the way institutions convert votes to policy into their choices. Since policy is often the result of institutionalized multiparty bargaining and thus votes are watered down by power-sharing, voters often compensate for this watering down by supporting parties whose positions differ from (and are often more extreme than) their own."[1] The theory presented here shows that such behavior is present in a political equilibrium. More specifically, the theory predicts that for majoritarian governments the out party has a larger vote share than either government party, which is consistent with Kedar's "compensational voting."

The formateur party exercises bargaining power by excluding one of the parties from a majoritarian government, but it has no bargaining power within the government. That is, the government parties are equal partners with the equilibrium policies equidistant from the ideal policies of the government parties. The policies can be coalition-efficient; i.e., at the midpoint of the contract curve of the government parties, but there is a continuum of equilibria in which policies are not coalition efficient and represent concessions to centrist voters. These equilibria retain the property that the government parties are equal partners, but the policies are closer to the center of voter preferences. The closer the policies are to the center of voter preferences the more politically patient the parties must be to sustain a political equilibrium.

The result that the equilibrium policies in majoritarian governments are equidistant from the ideal policies of the government parties is reminiscent of Gamson's (1961) Law, which typically is stated as the propostion of ministries held by a government party is proportional to its seat shares in parliament. Parties here are policy-motivated rather than office-motivated and the analogue to Gamson's Law is that the policy utilities of coalition parties are proportional to their seat shares. In the political equilibria supporting majoritarian governments with equidistant policies, the government parties have equal vote shares, so the utilities of the parties are directly proportional to their seat shares.

If reallocable officeholding benefits are available, there are equilibria in which the policy favors the head

[1] Kedar (2012) provides an assessment of the research on strategic voting in parliamentary elections.

of government and the government partner is compensated with additional benefits. If the head of a durable government changes over time, the policy changes as well; i.e., the government reshuffles the cabinet from time to time. The utilities of the parties comprising a majoritarian government remain approximately proportional to the parties' seat shares. The availability of reallocable benefits also has electoral consequences. Voters located near the out party's ideal policy have an incentive to vote for the government party with a policy closest to their own ideal policy so as to increase the probability that the closer policy results and decrease the probability that the more distant policy of the other government party results. The out party loses half its vote share all of which goes to the incumbent government partner. This is reminiscent of Duverger's Law that two-party competition results in first-past-the-post electoral systems, but the institutional context is quite different. Here, for half the voters located near the out party a vote for the out party is a wasted vote, and they prefer instead to vote for the government party that would select a policy closer to their ideal policies.

This prediction is consistent with Bargstad and Kedar's (2009) argument and evidence from the 2006 Israeli elections that the anticipated post-election bargaining led to a form of Duverger's Law. They (p. 308) "identify Duvergerian behavior of voters targeted at the *postelectoral* stage of coalition formation. When voters perceive their favorite party as having little chance of participating in the governing coalition, they often desert it and instead support the lesser of evils among those they perceive as viable coalition partners." Bawn (1999) finds that German voters are rational and strategic despite the complexity of the electoral system. She concludes (p. 489), "Not only do [voters] avoid wasting their district votes as Duverger predicted, they are more likely to give district votes to incumbents and to candidates from parties that are expected to be in power." The theory presented here predicts such behavior when reallocable officeholding benefits are available to ease the bargaining over government formation but not when the benefits cannot be easily reallocated.

Majoritarian governments can be supported by the threat of a new government formation round when voters are forward-looking and either myopic or fully strategic, taking into account the subsequent continuation equilibria. Voter-enforced equilibria with majoritarian governments and policy concessions to centrist voters also require forward-looking voters, and the sets of party-enforced or voter-enforced equilibria are the same if voters are either myopic or fully strategic. That is, in the absence of crises the policies of durable governments are absorbing states, so myopic preferences agree with preferences along the equilibrium path. For any political equilibrium with fully strategic voters there thus is a political equilibrium with forward-looking but myopic voters with the same governments and policies.

Majoritarian governments are generic, but majority, or single-party, governments can form and if they form, they are durable. A single-party government can form in response to a deviation by a government party in a voter-enforced equilibrium. If there is a single-party government, it chooses a policy with concessions to centrist voters. The majority government is durable because voters continue to give it a majority in expectation that it will choose a policy providing concessions to centrist voters.

If one party is sufficiently more centrally-located in the space of voter preferences than are the other parties, voters give that party a majority, and it forms a single-party government and chooses its ideal policy. A less centrally-located party would also choose its ideal policy if it had a majority in parliament, but voters strategically elect a minority parliament to prevent it from doing so.

Political equilibria exist when parties are politically patient, and the more intense are their policy preferences the less patient the parties need be to support a political equilibrium. Polarization of a party may have no effect on the equilibrium governments and their policies. That is, if one party is more extreme than the others, it must make policy concessions to the other two parties to be in a government, resulting in equilibrium policies that are the same as those in the absence of polarization.

Theories of government formation can be categorized into those based on office-motivated parties as initiated by Riker (1962) and those in which parties have policy preferences with or without reallocable officeholding benefits to ease the bargaining. Examples of the former include Ansolabehere, Snyder, Strauss, and Ting (2005), Carroll and Cox (2007), Diermeier, Eraslan, and Merlo (2003)(2007), and Snyder, Ting, and Ansolabehere (2005), and examples of the latter include Austen-Smith and Banks (1988), Axelrod (1980), Baron (1991), Baron and Diermeier (2001), Jackson and Moselle (2002), Schofield and Sened (2006), and Volden and Wiseman (2007). Government formation has been the focus of considerable empirical analysis (Budge and Keman (1990), Diermeier, Eraslan, and Merlo, Laver and Schofield (1991), Laver and Shepsle (1996), Schofield and Sened (2006), Strom (1990), Warwick (1994), among others), and Christensen, Georganas, and Kagel (2013) conduct an experiment based on the Jackson and Moselle model.

The theory is related to recent research on dynamic legislative bargaining (Acemoglu, Egorov, and Sonin (2012), Anesi and Seidmann (2013), Baron (1996), Baron and Bowen (2013), Battaglini and Palfrey (2012), Bowen and Zahran (2012), Deirmeier, Egorov, and Sonin (2013), Kalandrakis (2004)(2010), Nunarri (2014), and Zápal (2013)). For a one-dimensional policy space and no elections, Baron (1996) shows convergence to the median legislator's ideal policy, and Dzuida and Loeper (2011) show that shocks to preferences and discrete policies can lead to disparate policies over time. Cho (2014) also considers a unidimensional policy space with three parties with the median voter located at the ideal policy of the middle party and finds that policies change with the formateur but in the long-run rotate among three policies. The present model has stable policies and governments that are reached in one-step, and governments change only as a result of a crisis.

In a dynamic model of government formation and elections with sincere voting, Fong (2011) adopts the proto-coalition perspective used by Diermeier and Merlo (2004) and Baron and Diermeier (2001). Parties have preferences over policy and officeholding benefits, and voters give equal seat shares to each party. Fong finds that the policies chosen by majoritarian governments are Pareto dominated, since the government parties prefer an inefficient policy that disadvantages the out party in the next government formation stage so that a formateur party can obtain a cheaper partner.[2] Baron, Diermeier, and Fong (2012) consider a two-period dynamic model in which voters in elections are strategic, and Pareto-dominated policies can also result. In these models parties act opportunistically and replace governments, whereas in the present model the incumbent government parties have an interest in preserving their government and resist the temptation to replace it with a government that is no better.

The following section presents the model of the political system and an example with a discrete policy set. Section 3 introduces a continuous policy space, and Section 4 characterizes a continuum of party-enforced political equilibria with concessions to centrist voters. Section 5 characterizes equilibria in which voters discipline the government parties if they deviate from the equilibrium policy. Section 6 considers extensions including polarization, a centrally-located party, and policy risk aversion. Section 7 introduces officeholding benefits, Section 8 considers crises, and Section 9 offers conclusions.

2 The Model

2.1 The Political System

The model includes as actors voters and political parties, all of which are policy-oriented. The political system is a parliamentary democracy with a proportional representation electoral system, where representation is determined by the proportion of the votes received by a party. A government is formed around a policy agreed to by a majority of the parliament, and the parties that agree to the policy constitute the government. That is, the government is of cabinet form in which all government parties must accept the policy.[3] Goodhart (2013) finds that parliamentary governments act collectively in choosing policies rather than delegating authority to a party or minister to choose a policy in a particular domain.[4] The parliamentary democracy is assumed to continue for an infinite number of periods $t = 1, 2, \ldots$, and a government is said to be durable if it continues from one period to the next. Governments cannot dissolve parliament and call for an early election.[5]

A period in the model has two stages. The first consists of an election that determines representation in parliament. The second is legislative in which governments are formed and policy is established.[6] Once a parliament is seated, with probability equal to its seat share a party is selected as formateur to make a policy proposal. Diermeier and Merlo (2004) present evidence for selection probabilities that are proportional to representation in parliament, with somewhat higher probability for the head of the previous government.

[2]Cho (2011) studies a similar model that separates the bargaining over policy from the formation of a government and finds no inefficiency when a confidence procedure is in place.

[3]The rules by which the cabinet operates can affect not only policy outcomes but the governments that form, as shown by Huber and McCarty (2001). Variations in cabinet procedures are not considered here.

[4]Diermeier and Merlo (2000) provide a proto-coalition model in which policy is delegated to ministries.

[5]Strom (2002) provides evidence on the dissolution of parliament and early elections.

[6]In some parliamentary democracies parties form pre-electoral coalitions with the expectation that those parties will form the government if they receive a majority of the seats. See Golder (2006) for empirical evidence and Carroll and Cox (2007) for theory and empirical evidence with parties motivated by officeholding benefits in the form of ministries.

Bäck and Dumont (2008) find that the largest party is the most likely to be selected as formateur, although other institutional features also play a role. If a party has a majority in parliament, it is selected with probability one.

The party selected as formateur makes a take-it-or-leave-it policy proposal, which if accepted by parties with a majority of seats becomes the policy in that period and the majority constitutes the government. The formateur thus has bargaining power, although the policy in a majoritarian government may not favor the formateur over its government partner.[7] Bäck and Dumont find that the party of the formatuer is the most important predictor of the government that forms.

Policies are continuing, so the policy adopted becomes the status quo for the next period. If the policy proposal is rejected, the prior status quo remains in effect. Most policies, including tax policy, trade policy, defense policy, regulations, and entitlements, continue until changed by the legislature. Much of government spending is also in effect continuing.

The interpretation of the legislative process depends on whether there is an incumbent government from the previous period. If there is no incumbent government, the legislative stage is a government formation round in which a party is selected as the formateur to proposes a policy, and acceptance of its proposal forms a new government that enacts the proposed policy. Acceptance can be interpreted as the installation of a government. If there is an incumbent government, the party selected, which will also be referred to as the formateur, makes a proposal. The result may be continuation of the incumbent government, a new government replacing the incumbent government, or the incumbent government falling. For example, if there is an incumbent majoritarian government and the out party is selected as the formateur, it can propose a replacement government. This corresponds to a constructive confidence procedure as in Germany in which a new government is proposed to replace the incumbent government without conducting a new election. The proposal by the out party could also be an attempt to induce a government party to defect, causing the government to fall and resulting in a new government formation round in the next period. In a political equilibrium the out party fails to form a replacement government or induce a defection, so the current policy remains in place, supported by the incumbent government parties.

The political system is assumed to have three parties, and the closed policy space is denoted $X \subseteq \Re^2$, where the dimensions could represent economic policy and social policy. The Manifesto Project (https://manifesto-project.wzb.eu) provides estimates of the "political preferences of parties" over time and across countries, and factor analysis indicates that party preferences are located in a space with no more than two dimensions.[8] Each party has an ideal policy $x^i \in X, i = 1, 2, 3$, which could represent the preferences of party stalwarts or the party leader. So that no party has an advantage in government formation, assume that the ideal policies are located at the vertices of an equilateral triangle, where $x^1 = (0, 0)$, $x^2 = (1, 0)$, and

[7] Ansolabehere, Snyder, Strauss, and Ting show that in a static, weighted voting game where parties allocate officeholding benefits, the formateur receives a disproportionate share. They also provide empirical evidence supporting the prediction.

[8] If the policy space is one dimensional, the political equilibria are driven by the median party. See Baron (1996), Cho (2011)(2014), and Zápal (2013).

$x^3 = \left(\frac{1}{2}, \frac{\sqrt{3}}{2}\right)$. So that no party has an electoral advantage, assume that voters have ideal policies that are uniformly distributed on a disk Z with center at the centroid $\bar{x} = \left(\frac{1}{2}, \frac{1}{2\sqrt{3}}\right)$ of the Pareto set of party preferences, and assume that the disk strictly contains the ideal policies of the parties.[9]

The electoral system is proportional representation, and since the number of parties is fixed, the parties are symmetrically located in the space of voter preferences, and voters are symmetrically distributed, no threshold for representation is assumed. All voters are assumed to vote, so the seat share s_i^t of party i in period t is the proportion of votes party i receives in the period t election, where $\sum_{i=1}^{3} s_i^t = 1$. A minority parliament has seat shares $s_i^t \leq \frac{1}{2}, i = 1, 2, 3$, and a majority parliament has a party i with $s_i^t > \frac{1}{2}$. If a set of voters with mass at least one-half and its closure votes for party i or parties i and j, that party or pair of parties receives a majority in parliament.

The status quo policy entering period t is denoted q^{t-1}, where q^0 is the initial status quo. The formateur in period t makes a policy proposal $y^t \in X$, which is then accepted or rejected by the parties. If a majority accepts the proposal, y^t is implemented. If a majority rejects the proposal, the status quo policy q^{t-1} remains in place. The policy in place in period t and at the beginning of period $t+1$ then is q^t; i.e.,

$$q^t = \begin{cases} y^t & \text{if } \sum_{i=1}^{3} \hat{s}_i^t > \frac{1}{2} \\ q^{t-1} & \text{otherwise} \end{cases},$$

where $\hat{s}_i^t$ denotes the seat share of the parties that accept the proposal y^t where $\hat{s}_i^t = 0$ if party i does not accept.

2.2 Preferences and Strategies

Parties have preferences represented by a quadratic utility loss function $u_i(x), i = 1, 2, 3$, and the utility function is normalized so that $u_i(x^i) = 0$.[10] Parties maximize the discounted sum of their period utilities; i.e., $U_i^t = \sum_{\tau=t}^{\infty} \delta^{\tau-t} E^{\tau} u_i(q^{\tau}), i = 1, 2, 3$, where $\delta \in [0, 1)$ is a common discount factor representing the political patience of parties and E^{τ} denotes expectation with respect to the selection of a formateur in period τ, any randomization in strategies, and the outcomes of elections. Parties are assumed to use stage undominated acceptance strategies.[11] If it is indifferent between a proposal y^t and the status quo policy q^{t-1}, a party is assumed to reject the proposal. This indifference rule allows government coalitions to be durable.[12] The number of parties is fixed and they have no instruments to compete for votes, so election promises by parties are meaningless unless they describe what a party will do once it has the opportunity to act. Neither parties nor voters can commit to how they will act in the future.

[9] This structure is closely related to that in static model of Baron and Diermeier.
[10] Other specifications of preferences are considered in Section 6.2.
[11] See Baron and Kalai (1993).
[12] This rule is used in Baron and Bowen (2013) and Anesi (2010), and a probabilistic version is used in Battaglini and Palfrey (2012).

Voters are policy-motivated, and a voter is identified by her ideal policy $z \in Z$. Voters have quadratic loss utility functions $w_z(q^t)$ with ideal policy z and intertemporal preferences with a discount factor $\xi \in [0,1)$. Voters are assumed to be forward-looking and to anticipate the policies that would be adopted by the various governments that could form in the future. Voters are considered under two behavioral assumptions. Fully strategic voters anticipate the policies of the possible governments that could be in office after each election; i.e., voters look along the entire equilibrium path. A strategic voter z in period t thus maximizes $W_z^t = \sum_{\tau=t}^{\infty} E_z^{\tau} \xi^{\tau-t} w_z(q^{\tau})$, where E_z^{τ} denotes expectation with respect to which policy q^{τ} will be in place in period τ. Voters, however, may be less sophisticated and behave myopically; i.e., they take into account only the current period policy. In the period t election a myopic voter z maximizes $E_z^t w_z(q^t)$. As shown below the set of equilibria with strategic voters is the same as the set of equilibria with myopic voters.

Voters are assumed to use stage-undominated voting strategies, which require a voter to vote for the party that yields the greatest discounted expected utility. If a voter is indifferent among a collection of parties, she votes according to an indifference rule included in the specification of strategies. For example, if a voter is indifferent between retaining the incumbent government and giving the out party a majority, she is assumed to vote for an incumbent party. If a voter is indifferent between two parties in an incumbent government, she votes for the closer party and randomizes if equidistant from their ideal policies. If a voter is indifferent among all three parties, she is assumed to vote for the closest party and to randomize if all the parties are equidistant. Voting for the closest party when indifferent can be justified by indifferent voters preferring to vote for the party that could best represent their interests in a new government formation round.

Strategies are assumed to be Markov. The state variable at the beginning of the period is the status quo policy $q^{t-1} \in X$, and the state variable at the beginning of the legislative stage is (q^{t-1}, s^t), where $s^t = (s_1^t, s_2^t, s_3^t)$. Strategies are assumed to be stationary, so a party chooses the same proposal and acceptance set if $(q^{\tau-1}, s^{\tau}) = (q^{t-1}, s^t)$, $\tau \neq t$ and $t, \tau \geq 1$. Continuation values thus do not depend on t.

In the legislative stage the proposal strategy of party ℓ is denoted $\rho^{\ell}(q^{t-1}, s^t)$. Parties have an acceptance set $A^{\ell}(q^{t-1}, s^t)$ of policies for which they accept a proposal y^t rather than leave q^{t-1} in place. That is, let $a_i(y^t, q^{t-1}) = 1$ denote the acceptance of a proposal y^t and $a_i(y^t, q^{t-1}) = 0$ denote rejection, so $A_i(q^{t-1}, s^t) = \{y^t | a_i(y^t, q^{t-1}) = 1\}$.

A government is a decisive set of parties that prefer that it and its policy continue to the next period rather than fall with a new government formation round commencing in the next period. The parties that compose the government are identified by the policy implemented; i.e., a policy $q^t \in A^i(q^{t-1}, s^t) \cap A^j(q^{t-1}, s^t)$ and $q^t \notin A^k(q^{t-1}, s^t)$ implies that the government in period t is $G^t = \{i, j\}$. If q^{t-1} is not in the intersection of the acceptance sets of a decisive set of parties, a caretaker government can be thought of as being in place. The following period is a new government formation round in which the selected formateur has the opportunity to form a new government.

2.3 Equilibrium

The solution concept is Markov perfect equilibrium (MPE) with voter strategies required to satisfy the strong Nash equilibrium criterion in which a set of voters can deviate from the equilibrium strategies. Since the model is symmetric, the focus is on symmetric equilibria. Equilibria are identified by first conjecturing representation and governments and the profiles of strategies that support them. The continuation values corresponding to the conjectured strategies are then determined, and the incentives for parties to defect are checked to verify that the conjectured strategies along with voter strategies form a MPE. Existence is thus shown constructively. An equilibrium in the sequence of election and legislative stages is referred to as a political equilibrium. The focus is on political equilibria that have simple rather than complex strategies.

Durable governments persist because a defection by a party results in a new government formation round with an uncertain outcome due to the random selection of a formateur. This serves as a collective punishment because the party that defects from the current government could be excluded from the new government that forms in the next period. Voters can also discipline a party or government by shifting their votes based on the strategies they expect parties to use once representation in parliament has been determined.

2.4 A Discrete Example

As an example consider a discrete policy set $X = \{\bar{x}, x^1, x^2, x^3, x^{12}, x^{23}, x^{31}\}$ composed of the centroid of the policy space, the ideal policies of the parties, and the coalition-efficient policies x^{ij}, which are the midpoints of the contract curve for parties i and j. That is, $x^{12} = \left(\frac{1}{2}, 0\right)$, $x^{23} = \left(\frac{3}{4}, \frac{\sqrt{3}}{4}\right)$, $x^{31} = \left(\frac{1}{4}, \frac{\sqrt{3}}{4}\right)$, and $\bar{x} = \left(\frac{1}{2}, \frac{1}{2\sqrt{3}}\right)$. Let $X^i = \{x^{ij}, x^{ki}\}, j, k \neq i$, denote the set of coalition-efficient policies for party i, where $u_i(x^{ij}) = u_i(x^{ki})$.

The (conjectured) proposal strategy of a party i if selected as the formateur in period t for both a minority and a majority parliament is

$$\rho^i(q^{t-1}, s^t) = \begin{cases} q^{t-1} & \text{if } q^{t-1} \in X^i \\ y^t \in X^i \text{ with probability } \frac{1}{2} \text{ each} & \text{if } q^{t-1} \notin X^i. \end{cases} \tag{1}$$

The formateur thus proposes the status quo if it is favorable and otherwise proposes a favorable policy at random from its coalition-efficient policies X^i. The acceptance set of party i in a minority parliament is

$$A^i(q^{t-1}, s^t) = \begin{cases} q^{t-1} & \text{if } u_i(q^{t-1}) + \delta v_i(q^{t-1}) \geq u_i(y^t) + \delta v_i(y^t) \\ y^t & \text{if } u_i(q^{t-1}) + \delta v_i(q^{t-1}) < u_i(y^t) + \delta v_i(y^t), \end{cases} \tag{2}$$

where y^t is the proposed policy and $v_i(x)$ is the continuation value given policy x. Note that the acceptance set incorporates the indifference rule that a party rejects a proposal y^t if it is indifferent between the proposal and the status quo; i.e., $A^i(q^{t-1}, s^t) = \{q^{t-1}\}$ if $u_i(q^{t-1}) + \delta v_i(q^{t-1}) = u_i(y^t) + \delta v_i(y^t)$. The (conjectured) equilibrium acceptance set in a new government formation round is $A^i(q^{t-1}, s^t) = X^i$.

A government is organized around a policy, and any defection from that policy results in the government falling. This can be interpreted as the government dissolving if a party breaks its agreement to the government policy. An incumbent government $G^{t-1} = \{i, j\}$ with policy $q^{t-1} = x^{ij}$ persists in period t if

$$u_\ell(x^{ij}) + \delta v_\ell(x^{ij}) \geq u_\ell(x) + \delta v_\ell(x), \forall x \in X, \ell = i, j.$$

That is, the incumbent government persists if its member parties prefer that the government continue rather than be replaced or fall.

2.4.1 Exogenous Elections

To illustrate the government formation and policy choice aspects of the theory, assume that the election outcome is exogenous and the vote shares are $s_i^t = \frac{1}{3}, i = 1, 2, 3$, in every election. The set of policies supported by the (conjectured) political equilibrium is $\cup_{\ell=1}^3 X^\ell = \{x^{12}, x^{23}, x^{31}\}$, and the corresponding governments are majoritiarian. The continuation values for a party i satisfy

$$\begin{aligned}
v_i(q^{t-1}) &= u_i(q^{t-1}) + \delta v_i(q^{t-1}), \quad \text{if } q^{t-1} \in \cup_{\ell=1}^3 X^\ell \\
v_i(q^{t-1}) &= \frac{1}{3}\Bigg(\frac{1}{2}\left(u_i(x^{12}) + \delta v_i(x^{12})\right) + \frac{1}{2}\left(u_i(x^{31}) + \delta v_i(x^{31})\right) + \frac{1}{2}\left(u_i(x^{23}) + \delta v_i(x^{23})\right) \\
&\quad + \frac{1}{2}\left(u_i(x^{12}) + \delta v_i(x^{12})\right) + \frac{1}{2}\left(u_i(x^{31}) + \delta v_i(x^{31})\right) + \frac{1}{2}\left(u_i(x^{23}) + \delta v_i(x^{23})\right)\Bigg), \text{ if } q^{t-1} \notin \cup_{\ell=1}^3 X^\ell,
\end{aligned}$$

and the continuation values then are

$$\begin{aligned}
v_i(q^{t-1}) &= \frac{1}{1-\delta} u_i(q^{t-1}) \text{ if } q^{t-1} \in \cup_{\ell=1}^3 X^\ell \\
v_i(q^{t-1}) &= v^* \equiv \frac{1}{3(1-\delta)}\left(u_i(x^{12}) + u_i(x^{23}) + u_i(x^{31})\right) \text{ if } q^{t-1} \notin \cup_{\ell=1}^3 X^\ell.
\end{aligned}$$

The difference $v_i(q^{t-1}) - v^*$ represents a collective punishment for the government parties if one of them defects from the agreement and a new government formation round commences in the next period.

Establishing that the strategies in (1) and (2) constitute a political equilibrium requires showing that no party has an attractive deviation from the proposal and acceptance strategies. For $G^{t-1} = \{i, j\}$ and $q^{t-1} \in X^i$ if party i is selected as the formateur, its most attractive deviation is to propose (and accept) its ideal policy $y^t = x^i$. The out party also accepts the proposal, since it prefers that the government fall, resulting in a new government formation round in which it could be in the next government. The proposal

$y^t = x^i$ yields a payoff $u_i(x^i) + \delta v^*$, and party i has no incentive to deviate if

$$\frac{u_i(q^{t-1})}{1-\delta} \geq u_i(x^i) + \delta v^*, \tag{3}$$

which is satisfied for $\delta \geq \frac{3}{5}$, where $v^* = -\frac{5}{12(1-\delta)}$, $u_i(x^{ij}) = u_i(x^{ki}) = -\frac{1}{4}$, and $u_i(x^i) = 0$. Also, for all $\delta \geq 0$ the formateur has no incentive to deviate to $y^t = \bar{x}$ or to $x^{jk}, j, k \neq i$. The expected utility v^* from a new government formation round serves as a collective punishment for the members of the government if one of them deviates from the government policy.

If $q^{t-1} = x^{ij}$, suppose that party k is selected as the formateur and proposes a replacement government $G^t = \{j, k\}$ with policy $y^t = x^{jk}$. Party i rejects the proposal, and party j is indifferent between x^{ij} and x^{jk} and the corresponding governments, so it rejects the proposal under the indifference rule.[13] That is, party j can do no better in a government with k than in a government with i and hence stays with the current government. The out party cannot sweeten its offer, because the policy would not be supported by an equilibrium agreement identified in (1) and (2). A constructive vote of confidence might appear to have no effect on the equilibrium, but the threat of a replacement government is what causes the policy to be equidistant from the ideal policies of the government parties, even for a continuous policy space as in Section 4.

Next, consider $q^{t-1} \notin \cup_{\ell=1}^{3} X^\ell$, and suppose that party i is selected as the formateur. If $q^{t-1} = x^i$, party i could propose $y^t = x^i$, and it would be implemented in period t with a new government formation round commencing in period $t+1$. Party i, however, prefers to propose $y^t \in X^i$ for $\delta > \frac{3}{5}$ as in (3), where the strict inequality is due to the indifference rule. Similarly, if $q^{t-1} = \bar{x}$, party i prefers to propose $y^t \in X^i$ for all δ.

Consequently, no party has an incentive to deviate from the conjectured equilibrium strategies, so those strategies constitute a political equilibrium. The government is majoritarian, and the policy is coalition-efficient; i.e., it maximizes the aggregate utility of the government parties. If the initial status quo $q^0 \in \cup_{i=1}^{3} X^i$, the policy q^0 is implemented in every period by a government composed of the two parties i and j such that $q^0 \in X^i \cap X^j$. If $q^0 \notin \cup_{i=1}^{3} X^i$, the equilibrium policy is determined in one-step by the selection of the period 1 formateur i and its randomization among the policies in X^i. The policy persists thereafter; i.e., the set $\cup_{i=1}^{3} X^i$ is absorbing. Governments are durable as are their policies if the parties are sufficiently politically patient ($\delta > \frac{3}{5}$).

The indifference rule incorporated in (2) is necessary for the durability of a government. The rule may be interpreted as the incumbent government parties recognizing that neither can do better by participating in a replacement government. If a government party defects whenever made an equivalent policy offer by the out party, policies could rotate among the three majoritarian governments.[14] The continuation values in

[13] Any other proposal by party k is rejected by parties i and j.

[14] Kalandrakis (2004)(2010) shows the existence of a rotating dictator equilibrium in a pure distribution, dynamic legislative

that case are $-\frac{5}{12(1-\delta)}$, which is strictly lower than $-\frac{1}{4(1-\delta)}$ for the incumbent majoritarian government with a coalition-efficient policy. Forward reasoning thus supports the indifference rule of accepting the status quo if indifferent between it and another policy.

A consensus government with policy $\bar{x}$ accepted by all three parties is not supported by a political equilibrium, because there is no collective punishment if the consensus government falls. If the conjectured equilibrium strategies call for $y^t = \bar{x}, \forall q^{t-1} \in X$, a party i as formateur has an incentive to deviate from the conjectured equilibrium strategies for some $q^{t-1} \in X$. For example, if $q^{t-1} = x^i$, the formateur i can propose $y^t = x^i$ and its payoff is $u_i(x^i) + \delta v_i(\bar{x}) = \frac{\delta u_i(\bar{x})}{1-\delta}$, which is greater than the payoff $\frac{u_i(\bar{x})}{1-\delta}$ from proposing $y^t = \bar{x}$.

2.4.2 Myopic Voters

Parties cannot commit to the policies they propose if selected as the formateur nor to the proposals they accept, so forward-looking voters base their votes on what the parties are anticipated to do; i.e., on the (conjectured) equilibrium strategies. With forward-looking voters a majority parliament is possible, so proposal strategies must be specified for both minority and majority parliaments, where in the latter acceptance by another party of a majority party proposal is not required.

For a minority parliament, conjecture the proposal strategies in (1) with a formateur i proposing coalition-efficient policies in X^i. For a majority parliament, conjecture that the majority party also chooses $y^t \in X^i$ each with equal probability. Myopic voters base their votes on the payoffs $w_z(y^t)$ and $w_z(q^{t-1})$ from the (anticipated) proposal and the status quo, respectively, and if indifferent between the two, the voter votes for the status quo and randomizes if equidistant from the ideal policies of parties.

It is straightforward to demonstrate as in the previous section that there is no incentive for parties to deviate from the equilibrium strategies if voters are anticipated to elect a minority parliament. Myopic voters consider the current period policies that would be implemented by the governments that would form in minority and majority parliaments. Those policies depend on the status quo. Consider first a $q^{t-1} \notin \cup_{i=1}^{3} X^i$, so there is no incumbent government, and in the new government formation round the formateur selected randomizes as in (1). The expected utility Ew_z of a myopic voter z is

$$Ew_z = \bar{s}_1 \left(\frac{1}{2} w_z(x^{12}) + \frac{1}{2} w_z(x^{31}) \right) + \bar{s}_2 \left(\frac{1}{2} w_z(x^{12}) + \frac{1}{2} w_z(x^{23}) \right) + \bar{s}_3 \left(\frac{1}{2} w_z(x^{23}) + \frac{1}{2} w_z(x^{31}) \right),$$

where the (conjectured) equilibrium expected vote (and seat) shares are denoted $\bar{s}_i$.

Without loss of generality consider a set Δz of voters located closer to party 2 than to party 1, as illustrated in Figure 1A, and who shift their votes from party 2 to party 1. Their expected utility $Ew_{\Delta z}$ is

$$Ew_{\Delta z} = (\bar{s}_1 + \Delta s) \left(\frac{1}{2} w_z(x^{12}) + \frac{1}{2} w_z(x^{31}) \right) + (\bar{s}_2 - \Delta s) \left(\frac{1}{2} w_z(x^{12}) + \frac{1}{2} w_z(x^{23}) \right) + \bar{s}_3 \left(\frac{1}{2} w_z(x^{23}) + \frac{1}{2} w_z(x^{31}) \right), \tag{4}$$

bargaining game.

where Δs is their vote and seat share. Shifting their votes results in a change in their expected utility given by

$$\begin{aligned} \frac{\Delta E w_{\Delta z}}{\Delta s} &= \frac{1}{2}\left(w_z(x^{31}) - w_z(x^{23})\right) \\ &= \frac{1}{2}\left(\frac{1}{2} - z_1\right), \end{aligned} \tag{5}$$

which is negative since $z_1 > \frac{1}{2}$ for all $z \in \Delta z$. Similar analysis for other sets of voters shows that no set of voters has an incentive to deviate, so the expected vote shares are $\bar{s}_i = \frac{1}{3}, i = 1, 2, 3$. Parties thus have equal (expected) representation in a period with no incumbent government.

Consider next $q^{t-1} \in X^i \cap X^j$, so that the incumbent government is $G^{t-1} = \{i, j\}$. If a minority parliament is elected, voters anticipate that the government parties will propose $y^t = q^{t-1}$ and reject any proposal $y^t \neq q^{t-1}$, so the government continues. Voters could give the out party k a majority, in which case it would implement x^{ki} or x^{jk} with equal probability.[15] To determine how the electorate votes, consider $q^{t-1} = x^{12}$ and a voter $z = (z_1, \frac{1}{2\sqrt{3}})$ on the median line illustrated in Figure 1B, where voters above the line strictly prefer the out party 3 and those below the line strictly prefer one of the government parties. Voter z votes for party 3 if and only if

$$\frac{1}{2}w_z(x^{23}) + \frac{1}{2}w_z(x^{31}) > w_z(x^{12}), \tag{6}$$

and evaluation indicates that voter z on the median line is indifferent between voting for party 3 giving it a majority and voting for party 1 or party 2 at random, so (6) is not satisfied. Under the indifference rule voters on the median line vote for party 1 or 2 resulting in a minority parliament and continuation of the incumbent government and its policy.[16] Representation thus yields a majority for the government parties split evenly in expectation between the two parties, and this denies the out party a majority. The expected and actual vote shares are $\bar{s}_1 = \bar{s}_2 = \frac{1}{4}$ and $\bar{s}_3 = \frac{1}{2}$. The party with the largest vote share is not in government when there is an incumbent government, but the causation runs not from having the largest vote share to not being in government but from not being anticipated to be in government to having half the votes.

2.4.3 Strategic Voters

Strategic voters take into account the subgame equilibrium resulting from an election outcome, but they support the same durable coalition governments and policies as do myopic voters. Moreover, there are no other political equilibria. To show that the strategies in (1) and (2) constitute a political equilibrium with strategic voters, first note that policies in $\cup_{\ell=1}^{3} X^\ell$ are absorbing states. If voters elect minority parliaments

[15] Given the conjectured equilibrium strategies, a majority party has no incentive to choose its ideal policy, as shown in (3).

[16] If the strategy of a majority party 3 is to propose its ideal policy x^3, all voters below the line $z = \left(z_1, \frac{\sqrt{3}}{4}\right)$ vote for party 1 or party 2 rather than for party 3, resulting in a minority parliament.

in every election, the analysis in Section 2.4.2 for myopic voters is sufficient to demonstrate that for $\delta > \frac{3}{5}$ a formateur i has no incentive to deviate to $y^t = \bar{x}$ or $y^t = x^i$. Similarly, the out party cannot make a proposal that a government party accepts.

Because a policy $q^{t-1} \in \cup_{\ell=1}^3 X^\ell$ is an absorbing state, the analysis for myopic voters implies that strategic voters elect minority parliaments in every period with in expectation the out party having half the votes and government parties having half the vote and a majority. That is, in (4) the utilities $w_z(x^{ij})$ are divided by $1-\xi$. The condition when strategic voters take into account all future periods is thus equivalent to the condition in (6), since the policies chosen by majoritarian governments are absorbing states. That is, the condition corresponding to (6) is

$$\frac{1}{2(1-\xi)} w_z(x^{23}) + \frac{1}{2(1-\xi)} w_z(x^{31}) > \frac{w_z(x^{12})}{1-\xi}.$$

Similarly, for $q^{t-1} \notin \cup_{\ell=1}^3 X^\ell$, the comparison in (5) implies that strategic voters elect minority parliaments with equal expected vote shares, because the policies resulting in a new government formation round are absorbing states and are symmetrically located in the policy and voter spaces.

With an incumbent government strategic voters could give the out party a majority, however. As shown in Section 2.4.2, if voters deviate from electing a minority parliament and give the out party a majority, it would form a majoritarian government with a coalition-efficient policy, and under the conjectured equilibrium strategies that policy would persist because minority parliaments are elected thereafter. As in Section 2.4.2, however, voters elect a minority parliament with the government parties having a majority.

3 A Continuous Policy Space

Political equilibria are supported by the threat of collective punishment, and the threat can come from two sources. In a party-enforced equilibrium the threat is that a party in an incumbent government that falls might not be in the new government formed in the next period. In a voter-enforced equilibrium voters give the out party a majority if the government falls as a result of a defection from the equilibrium policy, and the resulting single-party government can be durable. In the discrete example in Section 2.4, when parties form a majoritarian government with a coalition-efficient policy, the out party receives a minority vote share, since if it received a majority it would choose a coalition-efficient policy and a majority of voters (weakly) prefer the policy of the incumbent government. With a continuous policy space the out party could make policy concessions to centrist voters to obtain a majority in the next election. As shown in the following two sections, there is a continuum of equilibria of each type where both majoritarian and single-party governments make policy concessions to centrist voters.

To represent policy concessions, let the policy of a majoritarian government $G^t = \{1,2\}$ be $\hat{x}^{12} = (\frac{1}{2}, \hat{x}_2^{12})$

for $\hat{x}_2^{12} \in \left[0, \frac{1}{2\sqrt{3}}\right)$, so $\hat{x}_2^{12}$ represents the extent of the concession to centrist voters. The corresponding policy $\hat{x}^3$ for a majority party 3 is $\hat{x}^3 = \left(\frac{1}{2}, \frac{1}{\sqrt{3}} - \hat{x}_2^{12}\right)$. Similarly, $\hat{x}^{23} = \left(\frac{3}{4} - \frac{\sqrt{3}}{2}\hat{x}_2^{12}, \frac{1}{2}\left(\frac{\sqrt{3}}{2} - \hat{x}_2^{12}\right)\right)$, $\hat{x}^1 = \left(\frac{1}{4} + \frac{\sqrt{3}}{2}\hat{x}_2^{12}, \frac{1}{4\sqrt{3}} + \frac{1}{2}\hat{x}_2^{12}\right)$, $\hat{x}^{31} = \left(\frac{1}{4} + \frac{\sqrt{3}}{2}\hat{x}_2^{12}, \frac{1}{2}\left(\frac{\sqrt{3}}{2} - \hat{x}_2^{12}\right)\right)$, and $\hat{x}^2 = \left(\frac{3}{4} - \frac{\sqrt{3}}{2}\hat{x}_2^{12}, \frac{1}{4\sqrt{3}} + \frac{1}{2}\hat{x}_2^{12}\right)$. These policies are illustrated in Figure 2. Voters on a median line are indifferent between the policies $\hat{x}^{ij}$ and $\hat{x}^k$, where, for example, for $\hat{x}^{12}$ and $\hat{x}^3$ the median line is $z = \left(z_1, \frac{1}{2\sqrt{3}}\right)$. Let $\hat{X} \equiv \{\hat{x}^1, \hat{x}^2, \hat{x}^3\}$ and $\hat{X}^i \equiv \{\hat{x}^{ij}, \hat{x}^{ki}\}, j \neq k, k, j \neq i, i = 1, 2, 3$.

4 Party-Enforced Political Equilibria

4.1 Majoritarian Governments

Policy concessions to centrist voters by majoritarian governments could be self-supporting without the threat of a majority party. Consider strategic voters and the simple (Markov) proposal strategies for $\hat{x}_2^{12} \in \left[0, \frac{1}{2\sqrt{3}}\right)$:

$$\rho^i(q^{t-1}, s^t) = \begin{cases} q^{t-1} & \text{if } q^{t-1} \in \hat{X}^i \\ y^t \in \hat{X}^i & \text{each with probability } \frac{1}{2} \text{ if } q^{t-1} \notin \hat{X}^i. \end{cases} \tag{7}$$

In a minority parliament a formateur i proposes the status quo if it is in $\hat{X}^i$, and otherwise randomizes among the policies in that set. If there is a majority parliament, the majority party k also proposes as in (7). Parties use stage-undominated acceptance strategies as in (2). On the equilibrium path with a government $G^{t-1} = \{i, j\}$ the acceptance set $\hat{A}^i(q^{t-1}, s^t)$ of party i is shown to be $\hat{A}^i(q^{t-1}, s^t) = \hat{X}^i \cap \hat{X}^j$.

If there is an incumbent government G^{t-1}, the parties can continue, replace, or dissolve the government. A government $G^t = \{k, i\}$ replaces a government $G^{t-1} = \{i, j\}$ if a proposal $y^t = \hat{x}^{ki}$ is accepted by parties holding a majority of seats in parliament. A government G^{t-1} falls when $q^t \notin \cup_{\ell=1}^3 \hat{X}^\ell$, and a new government formation round commences in period $t+1$. If a party k receives a majority of the votes in an election, it forms a single-party government with a policy in $\hat{X}^k$ chosen at random.

Given the conjectured equilibrium strategies and voters electing minority parliaments in every period, once $q^{t-1} \in \cup_{\ell=1}^3 \hat{X}^\ell$ the majoritarian government parties accept the status quo in every period, so the government is durable. Voters could give the out party a majority, but the voters on the relevant median line are indifferent between giving that party a majority and retaining the current government, and under the indifference rule they vote for one of the government parties. The expected vote shares are thus $\bar{s}_i = \bar{s}_j = \frac{1}{4}$ and $\bar{s}_k = \frac{1}{2}$, where k is the out party.

Kedar (2005) refers to this as compensational voting where a voter whose vote is watered-down by the anticipated government formation votes for a more extreme party. That is, voters located slightly above the median line $z = \left(z_1, \frac{1}{2\sqrt{3}}\right)$ know that party 1 and 2 will choose $\hat{x}^{12}$ and hence vote for party 3 even though they are closer to party 1 or party 2. Kedar finds that The Netherlands and Norway with proportional representation systems exhibit compensational voting, whereas Canada and Britain which have first-past-the-post electoral systems exhibit representational voting (for closest parties).

In the policies $q^{t-1} \in \cup_{\ell=1}^{3} \hat{X}^{\ell}$ the government parties are equal partners, since the government policy is equidistant from their ideal policies. The policies in $\cup_{\ell=1}^{3} \hat{X}^{\ell}$ are absorbing states and the corresponding continuation value $\hat{v}_i(q^{t-1})$ for a party i is $\hat{v}_i(q^{t-1}) = \frac{u_i(q^{t-1})}{1-\delta}$. Voters on the median line are indifferent and vote for one of the incumbent government parties, so they have a majority when their expected vote shares in a government $G^{t-1} = \{i, j\}$ are $\bar{s}_i = \bar{s}_j = \frac{1}{4}$. No set of voters can gain by shifting their votes from a government party to the out party giving it a majority, because the policies it would implement are farther from their ideal policies than is the policy of the government. If $q^{t-1} \notin \cup_{\ell=1}^{3} \hat{X}^{\ell}$, a new government formation round commences in period t, and the continuation value $\hat{v}^*$ is given in (3) with $\hat{x}^{ij}$ substituted for x^{ij}. Since the proposal strategies of the parties are symmetric, the expected vote shares in a new government formation round are $\bar{s}_\ell = \frac{1}{3}, \ell = 1, 2, 3$. The equilibrium is formalized in the following proposition, and all proofs are presented in the Appendix.[17]

Proposition 1. *(i) For each* $\hat{x}_2^{12} \in \left[0, \frac{1}{2\sqrt{3}}\right)$*, there exists a* $\hat{\delta}(\hat{x}_2^{12}) < 1$ *given by*

$$\hat{\delta}(\hat{x}_2^{12}) = \frac{\frac{1}{4} + (\hat{x}_2^{12})^2}{\frac{5}{12} - \frac{1}{2\sqrt{3}}\hat{x}_2^{12} + (\hat{x}_2^{12})^2}, \tag{8}$$

such that for all $\delta > \hat{\delta}(\hat{x}_2^{12})$ *the proposal strategies in* (7)*, stage undominated acceptance strategies with the indifference rule, and stage undominated voting strategies with the indifference rule constitute a political equilibrium with minority parliaments, majoritarian governments* $G^t = \{i, j\}$*, and policy* $\hat{x}^{ij}$ *in every period.*
(ii) The bound $\hat{\delta}(\hat{x}_2^{12})$ *is strictly increasing in the concession* $\hat{x}_2^{12}$ *to centrist voters, and* $\hat{\delta}(x_2^{12}) \in \left[\frac{3}{5}, \frac{4}{5}\right)$*.*
(iii) If $G^{t-1} = \{i, j\}$ *with policy* $\hat{x}^{ij}$*, the expected representation is* $\bar{s}_i = \bar{s}_j = \frac{1}{4}, \bar{s}_k = \frac{1}{2}$*, and if there is no incumbent government, the expected representation is* $\bar{s}_\ell = \frac{1}{3}, \ell = 1, 2, 3$*.*

The key to proving Proposition 1 is that for all policies except those in $\cup_{\ell=1}^{3} \hat{X}^{\ell}$ the continuation value is $\hat{v}^*$. Checking all possible deviations from the equilibrium path thus involves a comparison among single-period payoffs. Also, the out party cannot sweeten the pot with a policy proposal more favorable to a government party than is the government policy, because the policy proposed is not in $\cup_{\ell=1}^{3} \hat{X}^{\ell}$. That is, if the pot is sweetened, the government falls, and a new government formation round commences in the next period.[18]

The policies $\hat{x}^{ij}$ and $\hat{x}^k$ involve policy concessions to centrist voters, and the extent of the concessions depends on the political patience of the parties. That is, the bound $\hat{\delta}(\hat{x}_2^{12})$ is strictly increasing in $\hat{x}_2^{12}$

[17] There are also asymmetric equilibria with policies close to $\hat{x}^{12}$, $\hat{x}^{23}$, and $\hat{x}^{31}$. For example, for $x_1 \in \left(\frac{1}{3}, \frac{1}{2}\right]$ the policies $x^a = (x_1, 0)$, $x^b = \left(1 - \frac{x_1}{2}, \frac{\sqrt{3}}{2}x_1\right)$, $x^c = \left(\frac{1}{2}(1 - x_1), \frac{\sqrt{3}}{2}(1 - x_1)\right)$ can be supported in a political equilibrium with minority parliaments for some $\delta < 1$.

[18] Sweetening the pot is considered in Section 7, where reallocable officeholding benefits are considered.

for $\hat{x}_2^{12} \in \left[0, \frac{1}{2\sqrt{3}}\right)$, so greater policy concessions to centrist voters require more politically patient parties. Greater political patience is required because the incentive to defect to a party's ideal policy is stronger.

In summary, for $\delta > \hat{\delta}(\hat{x}_2^{12})$ there exists a political equilibrium in which formateurs propose policies $y^t \in \hat{X}^i$ and the proposals are accepted by parties $G^t = \{i, j\}$ identified by $y^t \in \hat{X}^i \cap \hat{X}^j$. For $\delta > \frac{4}{5}$ there exists a continuum of political equilibria corresponding to policies $\hat{x}^{12} = \left(\frac{1}{2}, \hat{x}_2^{12}\right)$ for all $\hat{x}_2^{12} \in \left[0, \frac{1}{2\sqrt{3}}\right)$. The policy of a durable majoritarian government is equidistant from the ideal policies of the government parties. An equilibrium with coalition-efficient policies ($x_2^{12} = 0$) exists for $\delta > \hat{\delta}(0) = \frac{3}{5}$, which is the bound for the discrete example in Section 2.4. In a political equilibrium with $\hat{x}_2^{12} > 0$, parties make policy concessions to centrist voters, and they do so because otherwise they would enter a new government formation round in which they might not be in the next government. The political equilibria thus are enforced by the actions of the parties. When an incumbent government is present, the government parties receive a bare majority of the vote with equal expected vote shares, since the government policy is at least as good for half the voters as the policies if the out party were the formateur. The equilibrium is consistent with Gamson's Law in that the policy is equidistant from the ideal policies of the government parties and their expected seat shares are equal.[19]

4.2 No Party-Enforced Consensus Government

A consensus government with policy $\bar{x}$ cannot be supported in a party-enforced political equilibrium because a party that as formateur deviates is not punished nor is a party that accepts its proposal. That is, after a deviation the parties return to proposing $y^t = \bar{x}$, so a party as formateur can deviate to a majoritarian coalition-efficient policy, for example, without punishment. More formally, conjecture that on the equilibrium path all parties propose $y^t = q^{t-1}$ if $q^{t-1} = \bar{x}$ and propose $y^t = \bar{x}$ for all $q^{t-1} \in X \setminus \{\bar{x}\}$. A formateur i has an incentive to deviate to $y^t = x^{ij}$ for all $\delta \geq 0$ if voters anticipate that in period $t+1$ all parties will propose $y^{t+1} = \bar{x}$ and voters will elect a minority parliament. Formateur i thus is not punished in the next government formation round and hence deviates in period t, so there is no political equilibrium with a consensus government. In contrast, a consensus government exists in a voter-enforced political equilibrium.

5 Voter-Enforced Political Equilibria

5.1 Majoritarian and Single-Party Governments

Majoritarian governments that make policy concessions to centrist voters could be supported by strategic voters giving the out party a majority if a government party defects causing the government to fall. A

[19] Gamson's Law is typically interpreted as pertaining to officeholding benefits such as appointments as minister, but as Goodhart finds, policies are not delegated to ministers but instead are collectively determined by the cabinet. The policy of a government thus is a better measure of the returns to governing than is the number of ministers. The party of a minister, however, is easier to identify than is the policy implemented by the government.

majority party as a single-party government then has an incentive to choose a policy that results in a majority in future elections. For example, if a government party 1 or 2 deviates from $\hat{x}^{12}$ to x^{12} and voters give party 3 a majority, it can choose $\hat{x}^3 = \left(\frac{1}{2}, \frac{1}{\sqrt{3}} - \hat{x}_2^{12}\right)$. Anticipating this policy, voters on the median line $z = (z_1, \frac{1}{2\sqrt{3}})$ strictly prefer giving it a majority to electing a minority parliament with the selected formateur in the new government formation round proposing $y^t \in \hat{X}^\ell$, each with equal probability.

Consider the following proposal strategies for $\hat{x}_2^{12} \in \left[0, \frac{1}{2\sqrt{3}}\right)$:

$$\rho^i(q^{t-1}, s^t) = \begin{cases} q^{t-1} & \text{if } s_j^t \leq \frac{1}{2}, j = 1, 2, 3, \text{ and } q^{t-1} \in \hat{X}^i \\ y^t \in \hat{X}^i & \text{with probability } \frac{1}{2} \text{ each if } s_j^t \leq \frac{1}{2}, j = 1, 2, 3, \text{ and } q^{t-1} \notin \hat{X}^i \\ \hat{x}^i & \text{if } s_i^t > \frac{1}{2} \text{ and } q^{t-1} \in X. \end{cases} \tag{9}$$

The formateur i in a minority parliament proposes the status quo if it is favorable and otherwise chooses a policy in $\hat{X}^i$ at random. A majority party forms a single party government and chooses $\hat{x}^i$. As in Section 4, parties use stage-undominated acceptance strategies and accept the status quo when indifferent.

To punish a government, voters must recall which parties comprised the government that fell. More precisely, if $q^{t-2} \in \hat{X}^i \cap \hat{X}^j$, so $G^{t-2} = \{i, j\}$, and $q^{t-1} \notin \cup_{\ell=1}^3 \hat{X}^\ell$, voters know that the government G^{t-2} fell and was not replaced by a new government, so a new government formation round occurs after the period t election. Strategic voters base their votes on what they anticipate the parties will do in the present and future periods as well as on the history $h^{t-1} = (q^{t-2}, q^{t-1})$. Voters then can give the out party k a majority if $q^{t-2} \in \hat{X}^i \cap \hat{X}^j$ and $q^{t-1} \notin \cup_{\ell=1}^3 \hat{X}^\ell$. Let $\hat{H}^{t-1}$ denote the set of such histories for the three parties. If h^{t-1} is such that $q^{t-2} \in \hat{X}^i \cap \hat{X}^j$ and $q^{t-1} \in \hat{X}^i \cap \hat{X}^k$, i.e., a government $G^{t-1} = \{i, k\}$ replaced $G^{t-2} = \{i, j\}$, and voters do not punish party i.

If $q^{t-1} \in \hat{X}^i$, voters anticipate that for any minority parliament elected the policy will be $q^t = q^{t-1}$. As with party-enforced equilibria, voters with an ideal policy on one side of the relevant median line prefer to vote for the out party and those on the other side prefer to vote for the closer of the two government parties. Voters on the median line are indifferent, and vote for one of the incumbent government parties. The out party thus has an expected vote share equal to one-half and the other two parties have expected vote shares of one-quarter and a majority. A minority parliament thus is elected.

If $q^{t-1} = \hat{x}^i$, a voter who is indifferent between a majority party i and a minority parliament in which the formateur $\ell \neq i$ selected proposes $y^t \in \hat{X}^\ell$ votes for the incumbent majority party i. The voters indifferent between a majority party with policy $\hat{x}^3$ and a new government formation round are $z = \left(z_1, \frac{1}{2\sqrt{3}}\right)$, and all other voters have a strict preference for party 3 or for electing a minority parliament. If $q^{t-2} \in \hat{X}^i \cap \hat{X}^j$ and $q^{t-1} \notin \cup_{\ell=1}^3 \hat{X}^\ell$ indicating that a government $G^{t-2} = \{i, j\}$ fell, a voter indifferent between $\hat{x}^k$ and a minority parliament with a new government formation round votes for party k as punishment of the previous government. As indicated in the following lemma, voters on the median line are indifferent between giving party k a majority and electing a minority parliament in which the formateur selected in the new government formation round forms a majoritarian government.

Lemma 1. *For $\hat{x}_2^{12} \in \left[0, \frac{1}{2\sqrt{3}}\right)$, voters $z = \left(z_1, \frac{1}{2\sqrt{3}}\right)$ on the median line are indifferent between voting for a majority party 3 where $q^{t-1} = \hat{x}^3$ and a minority parliament in which the formateur $j \in \{1, 2, 3\}$ selected forms a majoritarian government with policy $y^t \in \hat{X}^j$, each with equal probability.*

Voters thus give the out party k a majority after a government falls, and they anticipate that k will choose $\hat{x}^k$. The following lemma establishes that a majority party k chooses $\hat{x}^k$ for δ sufficiently high.

Lemma 2. *For $\hat{x}_2^{12} \in \left[0, \frac{1}{2\sqrt{3}}\right)$, a majority party k chooses $\hat{x}^k$ if $\delta > \delta^*(\hat{x}_2^{12})$, where*

$$\delta > \delta^*(\hat{x}_2^{12}) \equiv \frac{\frac{1}{12} + \frac{1}{\sqrt{3}}\hat{x}_2^{12} + (\hat{x}_2^{12})^2}{\frac{5}{12} - \frac{1}{\sqrt{3}}\hat{x}_2^{12} + (\hat{x}_2^{12})^2}, \tag{10}$$

and voters give it a majority in the next election. The bound $\delta^*(\hat{x}_2^{12}) \in \left[\frac{1}{5}, 1\right)$ and is strictly increasing in $\hat{x}_2^{12}$ for $\hat{x}_2^{12} \in \left[0, \frac{1}{2\sqrt{3}}\right)$.

A majority party k thus chooses $\hat{x}^k$ because it anticipates that voters will give it a majority in the election, and from Lemma 1 voters give party k a majority because they anticipate that it will choose $\hat{x}^k$.

The policies $\hat{x}^{12}$ and $\hat{x}^3$ involve policy concessions to centrist voters, and the extent of the concessions depends on the political patience of the parties. The bound $\delta^*(\hat{x}_2^{12})$ is strictly increasing in $\hat{x}_2^{12}$ for $\hat{x}_2^{12} \in \left[0, \frac{1}{2\sqrt{3}}\right)$, so greater policy concessions require more politically patient parties. As examples, $\delta^*(0) = \frac{1}{5}$, and $\sup_{\hat{x}_2^{12} \to \frac{1}{2\sqrt{3}}} \delta^*(\hat{x}_2^{12}) = 1$.

In a voter-enforced equilibrium, voters need enforce only certain deviations by an incumbent government party, because other deviations are party-enforced. If $q^{t-1} \in \cup_{\ell=1}^3 \hat{X}^\ell$, a government is in place, and if $q^t = q^{t-1}$, the incumbent government persists and voters elect a minority parliament in period $t+1$. If $q^t \neq q^{t-1}$, $q^{t-1} \in \cup_{\ell=1}^3 \hat{X}^\ell$, and $q^t \in \cup_{\ell=1}^3 \hat{X}^\ell$, a replacement government is in office and a minority parliament is elected. If an incumbent government deviates to a policy not in the supported set $\cup_{\ell=1}^3 \hat{X}^\ell$, the period $t-1$ government falls, i.e., $h^{t-1} \in \hat{H}^{t-1}$, and voters punish the government parties by giving the out party a majority. If $q^{t-1} \notin \cup_{\ell=1}^3 \hat{X}^\ell$ so there is no government in period $t-1$ and $q^t \in \cup_{\ell=1}^3 \hat{X}^\ell$, a new government is in office and voters elect a minority parliament. If there is no government in period $t-1$ and $q^t \notin \cup_{\ell=1}^3 \hat{X}^\ell$, there is also no government in period t, a minority parliament is elected, and a new government formation round commences in period $t+1$. The following proposition identifies the voter-enforced equilibria.

Proposition 2. *The proposal strategies in* (9)*, stage undominated acceptance strategies with the indifference rule, and stage undominated voting strategies with the indifference rule constitute a political equilibrium for $\delta > \max\{\hat{\delta}(\hat{x}_2^{12}), \delta^*(\hat{x}_2^{12})\}$ for $\hat{x}_2^{12} \in \left[0, \frac{1}{2\sqrt{3}}\right)$.*

Let $\bar{\hat{x}}^{12}$ be defined by $\hat{\delta}(\bar{\hat{x}}^{12}) \equiv \delta^*(\bar{\hat{x}}^{12})$. For small concessions, $\hat{x}_2^{12} < \bar{\hat{x}}_2^{12}$, the required political patience is determined by the incentive of a majoritarian government formateur to serve its short-term interests by deviating with a new government formation round commencing in the next period. For larger concessions, $\hat{x}_2^{12} > \bar{\hat{x}}_2^{12}$, the required political patience is determined by the incentives of a majority party to choose its ideal policy rather than $\hat{x}^3$, resulting in a new government formation round. That is, for large concessions, overcoming the incentive of a majority party to deviate to its ideal policy requires more patient parties than required to overcome the incentive for a majoritarian government party to deviate.

The political equilibrium supported by the threat of a majority out party proceeds as follows. For $q^{t-1} \notin \{\hat{x}^{12}, \hat{x}^{23}, \hat{x}^{31}\} \cup \{\hat{x}^1, \hat{x}^2, \hat{x}^3\}$, strategic voters anticipate that in a minority parliament the parties will form majoritarian governments with policies in $\{\hat{x}^{12}, \hat{x}^{23}, \hat{x}^{31}\}$ in the next period, in which case they elect a minority parliament with parties having equal expected vote shares. Then, $q^t \in \{\hat{x}^{12}, \hat{x}^{23}, \hat{x}^{31}\}$, and voters anticipate that the status quo will prevail thereafter if there is a minority parliament. Voters then elect a minority parliament in which the incumbent parties receive the votes of all voters on and to one side of the median line and all voters on the other side of the median line vote for the out party. A minority parliament is elected. Representation is a bare majority for the parties of the incumbent government with the out party receiving almost a majority. This can be interpreted as Kedar's compensational voting. If a government party deviates from the policy $\hat{x}^{ij}$, a majority of voters vote for the out party k. Party k then chooses $\hat{x}^k$, since it anticipates that it will receive a majority in the next election. Voters on the median line are indifferent between giving incumbent party k a majority and electing a minority parliament followed by a new government formation round, so under the indifference rule they give k a majority when $q^t = \hat{x}^k$.

If $q^{t-1} \in \{\hat{x}^1, \hat{x}^2, \hat{x}^3\}$, the status quo policy persists thereafter and a majority parliament is elected in every period. Generically, minority parliaments, majoritarian governments, and policies in $\cup_{\ell=1}^{3} \hat{X}^\ell$ result on the equilibrium path, and majority parliaments, single-party governments, and policies $\hat{x}^i, i = 1, 2, 3$, result if $q^0 \in \{\hat{x}^1, \hat{x}^2, \hat{x}^3\}$.

5.2 A Consensus Government

A consensus government with policy $\bar{x}$ can be supported by voters giving a party a majority if another party defects from the government. The conjectured equilibrium strategies for a minority parliament are to propose $y^t = \bar{x}$ for all $q^{t-1} \in X$, and on the equilibrium path voters are indifferent among the parties and hence elect a minority parliament. If $q^{t-1} \neq \bar{x}$ for $t > 1$ and voters determine, for example, that party 1 deviated by proposing $y^{t-1} \neq \bar{x}$ with parties 1 and 2 accepting the proposal, they can give the out party 3 a majority in the period t election. If party 3 chooses $y^t \neq \bar{x}$, voters anticipate that in period $t+1$ the formateur will propose $y^{t+1} = \bar{x}$, and hence voters elect a minority parliament. Anticipating this, majority party 3 chooses $y^t = x^3$ in period t.

With these strategies no party has an incentive to deviate from proposals $y^t = \bar{x}$. For example, if in period t party 1 as formateur proposes $y^t = x^{12}$ and it is accepted by party 2, its expected utility is $u_1(x^{12}) + \delta \left(u_1(x^3) + \frac{\delta}{1-\delta} u_1(\bar{x}) \right)$, and on the equilibrium path its expected utility is $\frac{u_1(\bar{x})}{1-\delta}$. Party 1 then does not deviate for $\delta \geq \frac{1}{8}$. It is straightforward to check that no party has an incentive to deviate for $\delta > \frac{1}{8}$. Consensus governments and a policy $\bar{x}$ thus are supported in a voter-enforced political equilibrium.

5.3 Myopic Voters

Myopic voters have an incentive to give a majority to an out party k if they anticipate that as formateur it chooses $\hat{x}^k$ in the current period, so myopic voters can punish a majoritarian government that deviates from the equilibrium strategies. This supports the same equilibrium strategies as with strategic voters. Voters, however, consider only their current period utility, but since policies in $\cup_{\ell=1}^{3} \hat{X}^{\ell}$ and $\{\hat{x}^1, \hat{x}^2, \hat{x}^3\}$ are absorbing states, myopic and strategic voters have the same incentives. The equilibria with myopic and strategic voters are thus the same.

6 Extensions

6.1 Pareto Inefficient Equilibria

The proof of Proposition 1 implies that political equilibria with majoritarian governments exist with policies with $\hat{x}_2^{12} < 0$, which are Pareto dominated for the parties. These equilibria are enforced by the threat of a new government formation round with formateurs proposing Pareto dominated policies, and voters electing minority parliaments. The extent of the inefficiency is limited by the incentive to deviate to a Pareto efficient policy.

Pareto inefficient policies cannot, however, be supported in a voter-enforced political equilibrium if voters use the indifference rule of giving the out party a majority if a majoritarian government chooses a Pareto dominated policy. Suppose that a majority party 3 chooses $\hat{x}^3$ for $\hat{x}_2^{12} \in \left[-\frac{1}{2\sqrt{3}}, 0\right)$, and for $\hat{x}_2^{12} < -\frac{1}{2\sqrt{3}}$, it chooses its ideal policy x^3. For $G^{t-1} = \{i, j\}$, $q^{t-1} = \hat{x}^{ij}$, and $\hat{x}_2^{12} \in \left[-\frac{1}{2\sqrt{3}}, 0\right)$, voters on the median line are indifferent between giving the out party k a majority and electing a minority parliament and preserving the incumbent government. Under the indifference rule voters give party k a majority, and it chooses a Pareto efficient policy $\hat{x}^3$. For $\hat{x}_2^{12} < -\frac{1}{2\sqrt{3}}$, a strict majority prefers to vote for 3. There is thus no voter-enforced political equilibrium with Pareto dominated policies.

6.2 Political Policy Risk Aversion

Political policy risk aversion refers to the intensity of preferences for policies different from the ideal policy of a party. The more intense are policy preferences the less political patience is required to sustain a political equilibrium. That is, the parties can be less politically patient and have the proposal strategies in (7) and (9) and the corresponding acceptance strategies constitute a political equilibrium with minority parliaments.

The intensity of preferences can be represented in a variety of manners, and this section presents several examples to illustrate the effects on the bound on the discount factor for party-enforced equilibria.

A generalization of quadratic preferences is a utility loss function of parties of the form $u_i(x) = -(x_1 - x_1^i)^a - (x_2 - x_2^i)^a, a > 1$, where higher values of a correspond to more intense policy preferences. For $a = 4$ the bound $\hat{\delta}$ on the discount factor with coalition-efficient policies is $\hat{\delta} = \frac{24}{61}$, which is less that $\hat{\delta} = \frac{3}{5}$ with $a = 2$. More generally, more intense policy preferences allow parties with less political patience to sustain a party-enforced equilibrium with majoritarian governments and minority parliaments. In contrast, if parties have less intense policy preferences as represented by the distance measure, for example, a party-enforced equilibrium with majoritarian governments and coalition-efficient policies requires more politically patient parties. That is, if $u_i(x) = -\left((x_1 - x_1^i)^2 + (x_2 - x_2^i)^2\right)^{\frac{1}{2}}, i = 1, 2, 3$, the bound is $\hat{\delta} = \frac{3}{2+\sqrt{3}} = 0.804$. As another example, with absolute value preferences of the form $u_i(x) = -|x_1 - x_1^i| - |x_2 - x_2^i|, i = 1, 2, 3$, the bound is $\hat{\delta} = \frac{6}{7+\sqrt{3}} = 0.687$.

More generally, the more intense are policy preferences the less politically patient the parties need be to sustain a party-enforced political equilibrium, including those with policies that represent concessions to centrist voters. The same is true for voter-enforced equilibria with policy concessions to centrist voters.

6.3 Party Polarization

Polarization refers to the location of party ideal policies in the space of voter ideal policies. To provide a specific example, let the ideal policies of parties 1 and 2 be as above and let the ideal policy of party 3 be $x^3 = \left(\frac{1}{2}, x_2^3\right)$ with $x_2^3 \geq \frac{\sqrt{3}}{2}$. Suppose that the proposal and acceptance strategies generate majoritarian governments with coalition-efficient policies in a party-enforced equilibrium, so a government formed is durable. The proposal strategies are as in (7) with x^{31} replaced by $\widehat{x}^{31} = \left(\frac{1}{4}, \frac{1}{2}x_2^3\right)$ and x^{23} is replaced by $\widehat{x}^{23} = \left(\frac{3}{4}, \frac{1}{2}x_2^3\right)$, so the policies proposed on the (conjectured) equilibrium path are $\{x^{12}, \widehat{x}^{23}, \widehat{x}^{31}\}$. Suppose that $q^{t-1} = \widehat{x}^{31}$, so $G^{t-1} = \{1, 3\}$, and suppose that party 1 is selected as the formateur in period t. The (conjectured) equilibrium strategies have party 1 proposing $y^t = \widehat{x}^{31}$, which yields a dynamic payoff of $\frac{u_1(\widehat{x}_{31})}{1-\delta}$. If party 1 instead proposes $y^t = x^{12}$, party 2 accepts, thus forming a replacement government, and the dynamic payoff is $\frac{u_1(x^{12})}{1-\delta}$. Similarly, if $q^{t-1} \notin \{x^{12}, \widehat{x}^{23}, \widehat{x}^{31}\}$ and party 3 is selected as the formateur and it proposes $y^t = \widehat{x}^{31}$ and party 1 accepts, then $q^t = \widehat{x}^{31}$, but in the following period if party 1 or party 2 is selected as the formateur it proposes x^{12} and the other party accepts. Party 1 thus proposes or accepts $y^t = \widehat{x}^{31}$ only if $u_1(\widehat{x}^{31}) \geq u_1(x^{12})$, which requires $\widehat{x}^{31} = x^{31} = (\frac{1}{4}, \frac{\sqrt{3}}{4})$. The same conclusion results for equilibria with policy concessions to centrist voters. Consequently, there is no party-enforced political equilibrium of the form of Proposition 1 with policies $\{x^{12}, \widehat{x}^{23}, \widehat{x}^{31}\}$ when a party is extreme $\left(x_2^3 > \frac{\sqrt{3}}{2}\right)$. The same is true for voter-enforced equilibria.

For sufficiently high political patience the coalition-efficient party-enforced political equilibrium when there is an extreme party has policies $\{x^{12}, x^{23}, x^{31}\}$.[20] The threat of the two more centrist parties forming with certainty a durable government requires the more extreme party to make policy concessions to the other two parties that are equivalent to those in the absence of polarization. Given the concessions, voters elect minority parliaments.

6.4 A Centrally-Located Party

Party polarization has no effect on the equilibrium policy because the two more proximate parties form a majoritarian government that excludes the extreme party. If a party is more centrally-located in the space of voter ideal policies, however, that threat is not present. Instead the more centrally-located party has an electoral advantage, which can result in policy concessions or a majority parliament and a single-party government. A more centrally-located party has two effects on equilibrium incentives. First, that party is more attractive to centrist voters. Second, the policies resulting in a minority parliament are closer to the ideal policy of the more centrally-located party than to the ideal policies of the other parties. If the party is sufficiently centrally-located, it obtains a majority and chooses its ideal policy.

Let party 3 be the more centrally-located party, and denote its ideal policy by $x^3 = \left(\frac{1}{2}, x_2^3\right), x_2^3 \in \left[0, \frac{1}{\sqrt{3}}\right]$. Let party 3 propose its ideal policy and parties 1 and 2 propose the coalition efficient policy x^{12}. Then party 3's ideal policy is closer to the median line than is x^{12}, so forward-looking voters give party 3 a majority. Party 3 obtains its ideal policy in every period, and parties 1 and 2 cannot cause the government to fall.

The case of $x_2^3 = 0$ corresponds to the presence of a median voter, and as $x_2^3 \to 0$, the limit of the equilibrium policy is x^{12}, so the limit policy remains an equilibrium policy. That equilibrium has all three parties proposing $x^3 = x^{12}$, and once on the equilibrium path all three parties accept the status quo $q^{t-1} = x^3$ and minority parliaments are elected.

7 Officeholding Benefits

If reallocable officeholding benefits are available, they can be used in the government formation process to facilitate the bargaining. A formateur prefers a policy that favors its policy interests, and officeholding benefits can be used to compensate its government partner for tolerating a less favorable policy. A political equilibrium thus can have policies that favor the formateur in a durable government and change when the formateur changes. The divergence in government policies has electoral consequences, since voters located near the ideal policy of the out party can have an incentive to vote for the government party with a policy closer to their own ideal policies. That is, a vote for the out party is not just a wasted vote but is also harmful when the status quo policy if farther from the voter's ideal policy than is the policy that would be proposed by the other government party. The result is that the government parties receive between three-quarters

[20]The bound δ^p on the discount factor is $\delta^p = \frac{3u_3(x^{31})}{2u_3(x^{31})+u_3(x^{12})}$, where $u_3(x) = -\left(x_1 - \frac{1}{2}\right)^2 - \left(x_2 - x_2^3\right)^2$.

and all the votes. The out party thus might not be viable, which is analogous to Duverger's Law, although the institutional setting and the policy space are different. Both the equilibrium here and in Duverger's Law are driven by the incentive not to waste a vote.

Reallocable officeholding benefits provide both the means to induce a party to tolerate an unfavorable policy and a source of temptation, because they can be used by the out party to break an incumbent government or may be captured by a formateur when defecting from an incumbent government. An incumbent government thus must choose a policy and an allocation of benefits that deters both a challenge by the out party and a defection by a government member. Party-enforced political equilibria with reallocable benefits exist, since any deviation from the proposal or acceptance strategies results in the government falling and the threat of being out of the next government supports the incumbent government.

Officeholding benefits are allocated among the government parties and not among the voters. As Baron, Diermeier, and Fong (p. 707) state, "These benefits can be thought of as accruing to party activists who would vote for their party regardless of the distribution of officeholding benefits, so electoral outcomes are not affected by the distribution." The officeholding benefits include appointments as ministers, vice-ministers, and ambassadors, seats on the boards of public corporations, appointments on regulatory commissions and as agency heads, allocations of funds to party research arms, and so on. Baron and Diermeier (p. 935) elaborate:

> These benefits include jobs for party stalwarts, board seats on public companies or the national television system, and transfers to interest groups and party foundations. Again consider Germany. All the major parties (as well as interest groups like churches and labor unions) occupy seats on the supervisory boards of the national television system and major corporations (such as Volkswagen). Moreover, each major party receives substantial amounts of public money for its research and educational foundations.

Voters are assumed to care only about policy and not about the benefits or their allocation, since the benefits accrue to the government regardless of which parties comprise it.

The officeholding benefits $B \geq 0$ are assumed to be fully divisible and allocable among the parties.[21] The allocation of officeholding benefits is part of the government agreement and persists until changed. An allocation $b^{t-1} = (b_1^{t-1}, b_2^{t-1}, b_3^{t-1})$, where $\sum_{i=1}^{3} b_i^{t-1} = B$, $b_i^{t-1} \geq 0, i = 1, 2, 3$, in period $t-1$ is a component of the status quo, so the state variable is $Q^{t-1} = (q^{t-1}, b^{t-1})$. Parties are assumed to have quasilinear preferences represented by a utility function $U_i(x, b_i) = u_i(x) + b_i$. The role of the benefits is to allow members of the government to favor their own policy interests when they head the government. In an experiment implementing a static government formation bargaining game with a one-dimensional policy space, Christiansen, Georganis, and Kagel (2013) find that officeholding benefits move policy away from the

[21] The case of $B = 0$ corresponds to a political system in which officeholding benefits are controlled by strict rules, such as civil service rules, and hence cannot be reallocated by the parties.

median ideal point. An analogous result obtains in the present model.

Equilibrium proposal strategies for a majoritarian government include policies and benefits allocations $(B-c,c)$ and $(c,B-c)$ for the formateur and government partner, respectively. Define $\bar{X}^1 = \{(\bar{x}^{12}, B-c, c, 0), (\bar{x}^{31}, B-c, 0, c)\}$ as party 1's proposal set, where $\bar{x}^{12}$ and $\bar{x}^{31}$ are policies proposed by party 1 for governments $G^t = \{1,2\}$ and $G^t = \{1,3\}$, respectively. Party 1's proposal strategy is

$$\rho^1(Q^{t-1}, s^t) = \begin{cases} Q^{t-1} & \text{if } Q^{t-1} \in \bar{X}^1 \\ y^t \in \bar{X}^1 \text{ with probability } \frac{1}{2} \text{ each} & \text{if } Q^{t-1} \notin \bar{X}^1. \end{cases} \tag{11}$$

Party 1's acceptance set $A^1(Q^{t-1}, s^t)$ is

$$A^1(Q^{t-1}, s^t) = \{(\bar{x}^{12}, B-c, c, 0), (\bar{\bar{x}}^{12}, c, B-c, 0), (\bar{x}^{31}, B-c, 0, c), (\bar{\bar{x}}^{31}, c, 0, B-c)\}, \tag{12}$$

where $\bar{\bar{x}}^{12}$ and $\bar{\bar{x}}^{31}$ are policies proposed by parties 2 and 3, respectively, when they are the formateur in a majoritarian government with party 1. The proposal strategies and acceptance sets for parties 2 and 3 are defined in an analogous manner. The strategies are assumed to be symmetric; e.g., $\bar{x}^{12} = \left(\frac{1}{2} - \gamma, \hat{x}_2^{12}\right)$ and $\bar{\bar{x}}^{12} = \left(\frac{1}{2} + \gamma, \hat{x}_2^{12}\right)$, $\gamma \in [0, \bar{\gamma}(\hat{x}_2^{12})]$, where $\bar{\gamma}(\hat{x}_2^{12}) = \frac{1}{2} - \frac{1}{\sqrt{3}}\hat{x}_2^{12}$.[22] A government $G^t = \{1,2\}$ thus has policy $\bar{x}^{12}$ and an officeholding benefits allocation $(B-c, c, 0)$ when party 1 is the formateur and policy $\bar{\bar{x}}^{12}$ and an officeholding benefits allocation $(c, B-c, 0)$ when party 2 is the formateur. There is thus an adjustment in policy and a reallocation of officeholding benefits whenever there is a change in the head (formateur) of government, where the policy adjustment favors the policy interests of the formateur. This can be thought of as a reshuffling of the cabinet.

With officeholding benefits the indifference rule requires modification. The government parties have a continuing interest in preserving the government and can do so even when making policy proposals that favor their own interests. If the status quo favors the government partner 2, i.e., $Q^{t-1} = \left(\bar{\bar{x}}^{12}, c, B-c, 0\right)$, party 2 is assumed to accept a proposal $y^t = \left(\bar{x}^{12}, B-c, c, 0\right)$ from its government partner party 1 if it is indifferent between the proposal and the status quo. This preserves the government and allows the policy and benefits allocation to depend on which party heads the government. A government party that is indifferent between a proposal by the out party and the status quo is also assumed to preserve the government by rejecting the proposal.

On the (conjectured) equilibrium path for a government $G^{t-1} = \{1,2\}$, if $Q^{t-1} = \left(\bar{x}^{12}, B-c, c, 0\right) \in \bar{X}^1$ and party 1 is the formateur in period t, it proposes $y^t = Q^{t-1}$ and both parties accept the status quo, whereas if party 2 is the formateur, it proposes $y^t = (\bar{\bar{x}}^{12}, c, B-c, 0) \in \bar{X}^2$ and both parties accept the proposal. Similarly, if $Q^{t-1} = \left(\bar{\bar{x}}^{12}, c, B-c, 0\right)$ and party 2 is the formateur it proposes $y^t = Q^{t-1}$ and both parties accept the status quo, whereas if party 1 is the formateur it proposes $y^t = \left(\bar{x}^{12}, B-c, c, 0\right)$ and both

[22]The bound $\bar{\gamma}(\hat{x}_2^{12})$ assures that the policies are Pareto optimal.

parties accept the proposal.

To determine the vote shares of the parties, consider myopic voters and a status quo policy $Q^{t-1} \notin \bar{X}^1 \cup \bar{X}^2 \cup \bar{X}^3$, so there is no incumbent government. The period t election leads to a new government formation round, and symmetry implies that the expected seat shares are $\bar{s}_i = \frac{1}{3}, i = 1, 2, 3$. If the status quo $Q^{t-1} \in \bar{X}^1$, however, and the incumbent government is $G^{t-1} = \{1, 2\}$, some myopic or strategic voters located close to the out party 3 strictly prefer to vote for one of the government parties. A vote for the out party 3 is in effect a vote for the status quo, since if party 3 is selected as the formateur, its proposal is rejected on the (conjectured) equilibrium path and the status quo remains in place. If $Q^{t-1} = (\bar{\bar{x}}^{12}, c, B-c, 0)$, voters located close to party 3 with $z_1 < \frac{1}{2}$ strictly prefer to vote for party 1 than for party 2 or 3, since party 1 as formateur would propose a policy closer to their ideal policy than is the status quo. Party 1 thus has an expected seat share $\bar{s}_1 = \frac{1}{2}$ but not a majority. Voters with $z_1 > \frac{1}{2}$ are indifferent between voting for party 2 or 3, and vote for the closer party 3. Voters close to party 3 with $z_1 = \frac{1}{2}$, randomize between party 1 and parties 2 or 3. Party 3 thus has an expected vote share $\bar{s}_3 = \frac{1}{4}$, and the expected vote share of party 1 is one-half and for party 2 is $\bar{s}_2 = \frac{1}{4}$. The expected vote shares given $Q^{t-1} = \left(\bar{x}^{12}, B-c, c, 0\right)$ are determined in an analogous manner. The electoral equilibrium given the (conjectured) equilibrium strategies of the parties is summarized in the following lemma and illustrated in Figure 3.

Lemma 3. *If $Q^{t-1} \notin \bar{X}^1 \cup \bar{X}^2 \cup \bar{X}^3$, the equilibrium expected vote shares are $\bar{s}_i = \frac{1}{3}, i = 1, 2, 3$. If $Q^{t-1} = (\bar{x}^{12}, B-c, c, 0)$, the expected vote shares are $\bar{s}_1 = \frac{1}{4}$, $\bar{s}_2 = \frac{1}{2}$, $\bar{s}_3 = \frac{1}{4}$. If $Q^{t-1} = (\bar{\bar{x}}^{12}, c, B-c, 0)$, the expected vote shares are $\bar{s}_1 = \frac{1}{2}$, $\bar{s}_2 = \frac{1}{4}$, $\bar{s}_3 = \frac{1}{4}$. The expected vote shares for the other status quos on the equilibrium path are analogous. Minority parliaments are elected.*

The out party thus loses half its votes when reallocable officeholding benefits are available. This is consistent with Duverger's law for first-past-the-post electoral systems and consistent with the empirical findings of Bawn and Bargstad and Kedar.

If the government parties reject the proposals by the out party, the continuation values $v_1\left(\bar{x}^{12}, B-c, c, 0\right)$ and $v_1\left(\bar{\bar{x}}^{12}, c, B-c, 0\right)$ for party 1 for $q^{t-1} = \bar{x}^{12}$ and $q^{t-1} = \bar{\bar{x}}^{12}$, respectively, satisfy[23]

$$\begin{aligned}
v_1\left(\bar{x}^{12}, B-c, c, 0\right) &= \frac{1}{2}\left(u_1(\bar{x}^{12}) + B - c + \delta v_1\left(\bar{x}^{12}, B-c, c, 0\right)\right) + \frac{1}{2}\left(u_1(\bar{\bar{x}}^{12}) + c + \delta v_1\left(\bar{\bar{x}}^{12}, c, B-c, 0\right)\right) \\
v_1\left(\bar{\bar{x}}^{12}, c, B-c, 0\right) &= \frac{1}{2}\left(u_1(\bar{\bar{x}}^{12}) + c + \delta v_1\left(\bar{\bar{x}}^{12}, c, B-c, 0\right)\right) + \frac{1}{2}\left(u_1(\bar{x}^{12}) + B - c + \delta v_1\left(\bar{x}^{12}, B-c, c, 0\right)\right).
\end{aligned}$$

The continuation values then are

$$v_1\left(\bar{x}^{12}, B-c, c, 0\right) = v_1\left(\bar{\bar{x}}^{12}, c, B-c, 0\right) = \frac{1}{2(1-\delta)}\left(u_1(\bar{x}^{12}) + u_1(\bar{\bar{x}}^{12}) + B\right).$$

[23]Given the status quo $Q^{t-1} = (\bar{x}^{12}, B-c, c, 0)$, the probability that $Q^t = Q^{t-1}$ is $\bar{s}_1 + \bar{s}_3 = \frac{1}{2}$, and if the status quo is $Q^{t-1} = (\bar{\bar{x}}^{12}, c, B-c, 0)$, the probability that $Q^t = (\bar{x}^{12}, B-c, c, 0)$ is $\bar{s}_1 = \frac{1}{2}$.

The continuation values $v_2\left(\bar{\bar{x}}^{12}, c, B-c, 0\right)$ and $v_2\left(\bar{x}^{12}, B-c, c, 0\right)$ for government party 2 are equal to $v_1(\bar{x}^{12}, B-c, c, 0)$ and $v_1(\bar{\bar{x}}^{12}, c, B-c, 0)$, respectively, and $v_3\left(\bar{\bar{x}}^{12}, c, B-c, 0\right)$ and $v_3\left(\bar{x}^{12}, B-c, c, 0\right)$ for the out party is $v_3(\bar{x}^{12}, B-c, c, 0) = v_3(\bar{\bar{x}}^{12}, c, B-c, 0) = \frac{1}{2}u_3(\bar{x}^{12}) + \frac{1}{2}u_3(\bar{\bar{x}}^{12})$.

In a new government formation round the continuation value $\bar{v}^*$ is, using symmetry to simplify the expression,[24]

$$\bar{v}^* = \frac{1}{3(1-\delta)}\left(u_1(\bar{x}^{12}) + u_1(\bar{\bar{x}}^{12}) + u_1(\bar{x}^{23}) + B\right),$$

where $u_1(\bar{x}^{23}) = u_1(\bar{\bar{x}}^{23})$ is the utility when the government formed is $G^t = \{2, 3\}$.

The following proposition identifies a class of political equilibria with officeholding benefits.

Proposition 3. *For a sufficiently high discount factor the strategies in* (11) *and* (12) *and analogous strategies for parties 2 and 3 and the corresponding indifference rules and myopic voters electing minority parliaments constitute a political equilibrium for all* $\gamma \in \left[0, \bar{\gamma}(\hat{x}_2^{12})\right]$ *and* $x_2^{12} \in \left[0, \frac{1}{2\sqrt{3}}\right)$ *for c satisfying*

$$c \geq c^o \equiv \frac{1}{2}\left(B + u_1(\bar{x}^{12}) - u_1(\bar{\bar{x}}^{12})\right) \geq \frac{1}{2}B. \tag{13}$$

and B satisfying

$$B \geq u_1(\bar{x}^{12}) - u_1(\bar{\bar{x}}^{12}). \tag{14}$$

The formateur obtains a policy that favors its policy interests, and the other government party receives more than half the benefits.

The same strategies constitute an equilibrium with strategic voters.

To trace the equilibrium policy path in a political equilibrium, for an initial status quo Q^0 such that

$$Q^0 \in \left\{(\bar{x}^{12}, B-c, c, 0), (\bar{\bar{x}}^{12}, c, B-c, 0), (\bar{x}^{23}, 0, B-c, c), (\bar{\bar{x}}^{23}, 0, c, B-c), (\bar{x}^{31}, c, 0, B-c), (\bar{\bar{x}}^{31}, B-c, 0, c)\right\},$$

a minority parliament is elected in period 1, and the formateur i selected proposes $y^1 \in \bar{X}^i$, which is accepted by the party receiving c. The next election then has expected vote shares equal to $\frac{1}{2}$ for the government partner and $\frac{1}{4}$ for the other two parties. Regardless of the realized vote shares the government continues with probability one, and with probability $\frac{1}{2}$ the status quo remains in place and with probability $\frac{1}{2}$ it changes to the proposal of the government partner. The government policy and the allocation of benefits thus change when the formateur changes.

A political equilibrium with officeholding benefits results in a weakly higher utility for the government partner than for the head of government. That is, when party 1 heads government in period t and the policy

[24] For example, $u_1(\bar{x}^{12}) = u_1(\bar{\bar{x}}^{31})$.

is $\bar{x}^{12}$, the difference in the utilities of the two government parties is

$$u_1(\bar{x}^{12}) + B - c + \delta v_1(\bar{x}^{12}, B - c, c, 0) - \big(u_2(\bar{x}^{12}) + c + \delta v_2(\bar{x}^{12}, B - c, c, 0)\big) = u_1(\bar{x}^{12}) + B - 2c - u_2(\bar{x}^{12}),$$

which is nonpositive from (13) and $u_2(\bar{x}^{12}) = u_1(\bar{\bar{x}}^{12})$. If the difference in the policies $\bar{x}^{12}$ and $\bar{\bar{x}}^{12}$ is not large, the equilibrium approximates Gamson's Law.

An immediate corollary of Proposition 3 is:

Corollary 1. *If $B = 0$, no durable government in a party-enforced political equilibrium has disparate policies; i.e., for $G^t = \{i, j\}$, $\bar{x}^{ij} = \bar{\bar{x}}^{ij} = \hat{x}^{ij}$.*

Political equilibria with officeholding benefits thus can result in durable governments with policies that differ depending on which party heads government, and the maximal difference in the policies that can be supported in a political system with officeholding benefits B is given by (14), and the difference is strictly increasing in B. This is stated in the following corollary.

Corollary 2. *The maximal difference $||\bar{x}^{ij} - \bar{\bar{x}}^{ij}||$ in policies in (14) for a government $G^t = \{i, j\}$ is strictly increasing in B.*

A durable government thus can have greater variation over time in both policy and the distribution of officeholding benefits the greater are the benefits available.

8 Crises

A government may be affected by exogenous forces such as a financial crisis, large trade imbalances, external threats, natural disasters, or other shocks. Government policies are also affected by implementation problems or by court decisions that affect policies. The governments identified above are vulnerable to such crises, but in their absence a government and its policies are durable. If a crisis leaves the policy off the equilibrium path and the government falls, the disruption is short-lived as a new government is formed in the next period. If crises are frequent, governments and policies can be unstable. The durability of governments thus depends on how frequent crises are, and if crises are too frequent, a political equilibrium does not exist.

A crisis is assumed to be exogenous to the government and its policy. Let the probability a crisis occurs in a period be denoted by $\eta \geq 0$, and assume that its magnitude is uncertain as represented by a continuous random shock $\tilde{\epsilon} = (\tilde{\epsilon}_1, \tilde{\epsilon}_2)$ with a mean of $(0, 0)$ and marginal variances $\sigma_i^2, i = 1, 2$. To simplify the notation, let the covariance be zero. A crisis occurs after the government has chosen a policy, so if x is the policy adopted by the government in period t, the policy after a crisis is $q^t = x + \epsilon$, where $\epsilon = (\epsilon_1, \epsilon_2)$ denotes a realization of $\tilde{\epsilon}$. Payoffs in period t depend on the shocked policy $x + \epsilon$, and the status quo for period $t + 1$ is $q^t = x + \epsilon$. In the absence of a crisis the policy in place is that accepted by the government parties.

To characterize a party-enforced political equilibrium in the presence of crises, consider the strategies in (1) and (2) with coalition-efficient policies so $X^i = \{x^{ij}, x^{ki}\}$, and assume that $B = 0$ and myopic voters elect minority parliaments in every period. Suppose that $q^{t-1} = x^{12}$, which is the policy of the government $G^{t-1} = \{1, 2\}$ in period $t-1$ in which no crisis occurred. If selected as the formateur, party 1 or 2 proposes $y^t = q^{t-1}$, and if party 3 is the formateur, parties 1 and 2 reject its proposal. With probability η the policy q^t in place in period t is $q^t = x^{12} + \tilde{\epsilon}$ and with probability $1-\eta$ the policy is $q^t = x^{12}$. If $q^t \notin \cup_{\ell=1}^{3} X^{\ell}$, a minority parliament is elected in period $t+1$ and the formateur i randomizes among the coalition-efficient policies which are subject to the risk of a crisis.

The continuation values are presented in the following lemma.

Lemma 4. *The continuation values in the presence of crises are*

$$v_1(x^{12}) = \frac{1}{1-\delta}\left[u_1(x^{12}) - \eta(\sigma_1^2 + \sigma_2^2) - \frac{\eta\delta\frac{1}{2}(1-\bar{s}_1)}{1-(1-\eta)\delta}\left(u_1(x^{12}) - u_1(x^{23})\right)\right]$$

$$v_1(x^{23}) = \frac{1}{1-\delta}\left[u_1(x^{23}) - \eta(\sigma_1^2 + \sigma_2^2) - \frac{\eta\delta\frac{1}{2}(1+\bar{s}_1)}{1-(1-\eta)\delta}\left(u_1(x^{12}) - u_1(x^{23})\right)\right]$$

$$v_1(x') = E_{\epsilon}v^* = \frac{1}{1-\delta}\left[\frac{1}{2}(1+\bar{s}_1)\,u_1(x^{12}) + \frac{1}{2}(1-\bar{s}_1)\,u_1(x^{23}) - \eta(\sigma_1^2+\sigma_2^2)\right],\ x' \notin \cup_{\ell=1}^{3} X^{\ell},$$

where by symmetry $u_1(x^{31}) = u_1(x^{12})$ and $v_1(x^{31}) = v_1(x^{12})$.

To characterize the political equilibrium, first suppose that $G^{t-2} = \{1, 2\}$ and a crisis in period $t-1$ results in a status quo policy $q^{t-1} = x' \notin \cup_{\ell=1}^{3} X^{\ell}$. Also, suppose that party 3 is selected as the formateur, and consider the acceptance set of party 1. Suppose party 3 proposes $y^t = x^{31}$, which party 1 is to accept on the equilibrium path. If party 1 accepts, its payoff is

$$(1-\eta)\left(u_1(x^{31}) + \delta v_1(x^{31})\right) + \eta E_{\epsilon}\left(u_1(x^{31}+\tilde{\epsilon}) + \delta v_1(x^{31}+\tilde{\epsilon})\right), \tag{15}$$

and if it rejects its payoff is

$$(1-\eta)\left(u_1(x') + \delta v_1(x')\right) + \eta E_{\epsilon}\left(u_1(x'+\tilde{\epsilon}) + \delta v_1(x'+\tilde{\epsilon})\right). \tag{16}$$

The most attractive status quo for party 1 is $x' = x^1$. Then, party 1 has no incentive to reject $y^t = x^{31}$ if the payoff in (15) is strictly greater than the payoff in (16), which is the case for sufficiently high δ since $v_1(x^{31}) > v_1(x^1) = E_{\varepsilon}v^*$.

Next, consider $q^{t-1} \in X^1$, so if party 1 is the formateur, it proposes $y^t = q^{t-1}$. The most attractive deviation is to $y^t = x^1$, since all proposals off the equilibrium path result in the same continuation value $E_{\varepsilon}v^*$. Party 1 has no incentive to so deviate if the payoff in (15) is (weakly) greater than the payoff in (16) with $x' = x^1$. Similarly, the out party cannot make a proposal that a government party accepts.

The payoffs in (15) and (16) imply that party 1 has no incentive to deviate if

$$\delta > \tilde{\delta} \equiv \frac{1}{(1-\eta)(2-\bar{s}_1)}, \tag{17}$$

where $\bar{s}_1 = \bar{s}_2$ are the expected vote shares of the government parties. For $\eta = 0$ and $\bar{s}_1 = \frac{1}{3}$, the bound is $\tilde{\delta} = \frac{3}{5}$, as in Section 2.4. The bound $\tilde{\delta}$ is increasing in η, since a greater likelihood of a crisis decreases the value of preserving the government and increases the incentive to deviate. More politically patient parties thus are required to withstand a higher probability of a crisis. The bound is also increasing in $\bar{s}_1$, since if party 1 has greater expected representation, a new government formation round is not as costly. The bound $\tilde{\delta} < 1$ for $\eta < \tilde{\eta} \equiv \frac{1-\bar{s}_1}{2-\bar{s}_1}$, where $\tilde{\eta} = \frac{2}{5}$ for $\bar{s}_1 = \frac{1}{3}$ and $\tilde{\eta} = \frac{3}{7}$ for $\bar{s}_1 = \frac{1}{4}$. The bound $\tilde{\eta}$ is decreasing in $\bar{s}_1$, since the less costly is a new government formation round to a government party the smaller is the risk of a crisis that a political equilibrium can tolerate. If $\eta > \tilde{\eta}$, there is no political equilibrium. Thus, if crises are sufficiently likely, there is no durable government.

The following proposition summarizes the result.

Proposition 4. *For $\delta \in (\tilde{\delta}, 1)$ there exists a party-enforced political equilibrium in which parties form majoritarian governments with coalition-efficient policies and myopic voters elect minority parliaments. The bound $\tilde{\delta}$ is strictly increasing in the probability η of a crisis and in the expected seat share of the incumbent government parties. If $\eta > \tilde{\eta}$, crises are too frequent for a political equilibrium to exist.*

A similar argument shows that there are political equilibria in which parties make concessions to centrist voters when the risk of a crisis is not too high.

When there is a small ($\eta < \tilde{\eta}$) risk of a crisis, government coalitions are durable, although not infinitely so, but if the risk is greater than $\tilde{\eta}$, there is no political equilibrium. The political system then is unstable. If a crisis occurs that causes the current government to fall, the probability that the same government $G^{t-1} = \{i, j\}$ is returned to office in the next period is $\frac{\bar{s}_i + \bar{s}_j}{2} = \frac{1}{3}$.

A majoritarian government falls in a crisis, but if crises are not too likely, the incumbent government parties might be able to tolerate minor shocks (small ε) with the government falling only if there is a major shock. For example, a government $G^{t-1} = \{i, j\}$ could tolerate a crisis such that $q^t \in \tilde{X}^{ij}$ for a closed set $\tilde{X}^{ij}$ that contains the coalition-efficient policy x^{ij} and fall if $q^t \notin \tilde{X}^{ij}$. Characterizing such a political equilibrium is a subject for future research.

9 Conclusions

Parliamentary systems vary across countries but share the central feature that the parliament chooses the government. Proportional representation electoral systems also vary across countries, and in most parliamentary systems parties are the principal actors in both elections and governance. This paper provides a theory in which policy-motivated parties form governments and choose policy in a multidimensional policy space, and policy-motivated voters elect parliaments. Policies typically continue from one period to the next unless changed by the legislature, and parties are long-lived, so dynamic considerations are present. Simple dynamic political equilibria exist with voters and parties using Markov strategies. Voters have long-term interests, as do the parties, but myopic and strategic voting strategies are equivalent in the sense that they result in the same set of governments and policies. In a political equilibrium, governments and their policies are durable and once established they persist thereafter in the absence of a crisis. Governments are majoritarian, although non-generic single-party governments can occur. A single-party government can also result if a party is more centrally located in the space of voter preferences than are the other parties.

Government formation is led by a formateur with proposal power, yet the policy adopted by a majoritarian government does not favor one government party over the other. That is, the equilibrium policy is equidistant from the ideal policies of the government parties, which is reminiscent of Gamson's law because the expected vote shares of the government parties are equal when they are expected to choose that policy. The policy can be coalition-efficient, but a continuum of equilibria exist in which the policy is closer to the ideal policies of centrist voters.

A crisis that disturbs the government policy can cause the government to fall, in which case a new government forms in the next period. If crises are too frequent, a political equilibrium does not exist.

If reallocable officeholding benefits are present, a durable government can have policies that change if the government head changes from one government party to another. That policy favors the head, but the government partner is compensated with additional benefits. Government policy thus is responsive to the preferences of the head of government but is limited by the cost of the benefits foregone. Reallocable benefits also result in the out party losing half its votes, since voters close to the out party do not waste their vote, as Duverger argued.

Political equilibria exist because of the threat of collective punishment of the government parties, and that punishment can come from parties or from voters. In a party-enforced equilibrium the punishment results from the fall of the government with a new government formation round commencing in the next period in which the defector might not be in the next government. In a voter-enforced equilibrium if the government parties deviate from the equilibrium policy by, for example, choosing a policy that better serves their interests, voters can give the out party a majority in which case it forms a durable single-party government with a policy that is attractive to centrist voters. Consensus governments can result in voter-enforced equilibria but not in party-enforced equilibria.

Political equilibria require a degree of political patience on the part of parties (but not voters), and greater patience is required the closer the equilibrium policy is to the ideal policies of centrist voters. The political patience required for an equilibrium also depends on the intensity of policy preferences of the parties. The more intense, or politically risk averse, are those preferences, the less political patience is required for a political equilibrium.

The basic model considered is symmetric with respect to both parties and voters, and if a party becomes more extreme, it is forced to propose and accept the same policies that it would propose and accept if it were symmetrically located. Otherwise the other two parties would form the government, and the resulting policy would be distant from the ideal policy of the more extreme party. If a party is sufficiently centrally-located, it receives a majority in the next election, forms a single-party government, and chooses its ideal policy.

Additional equilibria exist in dynamic models such as the one considered here if strategies are based on a richer history of play. For example, if a party defects from a government, in the next period the other two parties could punish the defector by forming a government and choosing a policy far from the defector's ideal policy. Whether parties and voters use such histories or let bygones be bygones and base their actions on the present and future is an open question.

Four extensions of the model are of particular interest. The first is to explore, in all likelihood computationally, equilibria with parties located asymmetrically using country-specific data on party preferences from the Manifesto Project. The second is to incorporate institutional variation in the model, including a variety of confidence procedures and forms of proportional representation electoral institutions used by countries. The third is to explore the resiliency of governments to the frequency and magnitude of crises; i.e., to characterize the policy shocks that cause a government to fall and those that the government can withstand. The fourth is to endogenize the number and locations of parties.

Appendix

Proof of Proposition 1:

Proof. (i) To show that the conjectured strategies form a political equilibrium, the parties are first shown to have no incentive to deviate from the conjectured proposal and acceptance strategies given the conjectured voting strategies of voters, and second no set of voters is shown to have an incentive to deviate from the conjectured voting strategies. For $q^{t-1} \in \cup_{\ell=1}^{3} \hat{X}^\ell$ a government party as formateur could deviate and propose another policy that it and the out party would accept. If $q^{t-1} = \hat{x}^{ij}$ and party i is the formateur in period t, its most attractive deviation is to propose $y^t = x^i$. Similarly, if the out party k is selected as the formateur and wants to cause the incumbent government to fall, the strongest incentive comes from a proposal $y^t = x^i$. In the next period a new government formation round commences, so the continuation values are $\hat{v}^*$ for each party. Given the conjectured voting strategies, the deviation to x^i is not attractive to party i if

$$u_i(\hat{x}^{ij}) + \delta v_i(\hat{x}^{ij}) = \frac{u_i(\hat{x}^{ij})}{1-\delta} \geq u_i(x^i) + \delta \hat{v}^* = u_i(x^i) + \frac{\delta}{3(1-\delta)} \left(u_i(\hat{x}^{12}) + u_i(\hat{x}^{23}) + u_i(\hat{x}^{31}) \right), \tag{A.1}$$

which is satisfied for $\delta \geq \hat{\delta}(\hat{x}_2^{12})$, where $\hat{\delta}(\hat{x}_2^{12})$ is given in (8). The bound $\hat{\delta}(\hat{x}_2^{12}) \in [\frac{3}{5}, \frac{4}{5})$ for $\hat{x}_2^{12} \in [0, \frac{1}{2\sqrt{3}})$, and differentiation shows that $\hat{\delta}(\hat{x}_2^{12})$ is strictly increasing in $\hat{x}_2^{12}$, which establishes (ii). For $\delta \geq \frac{4}{5}$ there is no incentive to deviate for any $\hat{x}_2^{12} \in [0, \frac{1}{2\sqrt{3}})$.

Similarly, if the proposal is $y^t = x^{ij}$ and the status quo is $q^{t-1} = \hat{x}^{ij}$, the payoff for parties i and j from acceptance of the proposal is the right side of (A.1) with $u_i(x^{ij})$ substituted for $u_i(x^i), i = 1, 2$, and the payoff from rejection is the left side. For $\delta \geq \hat{\delta}(\hat{x}_2^{12})$ party i rejects the proposal.

Next consider $q^{t-1} \in \cup_{\ell=1}^{3} \hat{X}^\ell$ and a deviation to another policy $x \in \cup_{\ell=1}^{3} \hat{X}^\ell$. Government party i as formateur in a government $G^{t-1} = \{i, j\}$ could deviate from $q^{t-1} = \hat{x}^{ij}$ to $y^t = \hat{x}^{ki}$ or the out party k could propose $y^t = \hat{x}^{ki}$, both of which result in a replacement government $G^t = \{k, i\}$ if the proposal is accepted. Voters then elect a minority parliament. The utility $\frac{u_i(\hat{x}^{ki})}{1-\delta}$ along the subsequent equilibrium path is no greater than from maintaining the current policy, so under the indifference rule party i rejects y^t, preserving the government and the policy q^{t-1}. Similarly, a formateur could propose $y^t = \hat{x}^{jk}$, but for party i this is dominated by accepting the status quo, and party j is indifferent and rejects the proposal.

If $q^{t-1} \notin \cup_{\ell=1}^{3} X^\ell$, a formateur i proposes $y^t \in \hat{X}^i$ on the equilibrium path. The best status quo for i is $q^{t-1} = x^i$, and as shown in (A.1) party i prefers $y^t \in \hat{X}^i$ for $\delta \geq \hat{\delta}(\hat{x}_2^{12})$. Given the indifference rule party i accepts the proposal for $\delta > \hat{\delta}(\hat{x}_2^{12})$. Consequently, no party has an incentive to deviate from the conjectured equilibrium proposal strategies.

If voters give party k a majority, it can choose $y^t \in \hat{X}^k$, resulting in a durable majoritarian government, or choose $y^t \notin \cup_{\ell=1}^{3} \hat{X}^\ell$ and enter a new government formation round in the next period. The condition in

(A.1) implies that for $\delta > \hat{\delta}(\hat{x}_2^{12})$ party k chooses $y^t \in \hat{X}^k$ when it has a majority. Also, a majority party k strictly prefers $y^t \in \hat{X}^k$ to $y^t \in \hat{X}^i \cap \hat{X}^j$.

Given the proposal strategies in (7) and the acceptance strategies in (2), voters elect minority parliaments in every period. If $q^{t-1} = \hat{x}^{12}$ and a set of voters deviates and gives the out party 3 a majority, that party chooses $y^t \in \hat{X}^3$, and that policy persists thereafter. The voters on the median line $z = \left(z_1, \frac{1}{2\sqrt{3}}\right)$ are indifferent between $q^{t-1} = \hat{x}^{12}$ and randomizing over $y^t \in \hat{X}^3$, and under the indifference rule they vote for the closer incumbent government party. The government parties have expected vote shares of $\frac{1}{4}$ and the out party has an expected vote share of $\frac{1}{2}$. If $q^{t-1} \notin \cup_{\ell=1}^3 \hat{X}^\ell$, voters are indifferent and vote for the closest party and randomize if equidistant, resulting in a minority parliament with equal expected vote shares, establishing (iii). ■

Proof of Lemma 1:

Proof. The period-t utility of a strategic voter $z = \left(z_1, \frac{1}{2\sqrt{3}}\right)$ from giving the out party 3 a majority is $w_z(\hat{x}^3) = -\frac{1}{3} + z_1 - z_1^2 + \frac{1}{\sqrt{3}}\hat{x}_2^{12} - (\hat{x}_2^{12})^2$, and the expected utility Ew_z from a new government formation round in a minority parliament is[25]

$$Ew_z = \frac{1}{1-\xi}\left[\bar{s}_1\left(\frac{1}{2}w_z(\hat{x}^{12}) + \frac{1}{2}w_z(\hat{x}^{31})\right) + \bar{s}_2\left(\frac{1}{2}w_z(\hat{x}^{12}) + \frac{1}{2}w_z(\hat{x}^{23})\right) + \bar{s}_3\left(\frac{1}{2}w_z(\hat{x}^{31}) + \frac{1}{2}w_z(\hat{x}^{23})\right)\right], \tag{A.2}$$

where $\bar{s}_i, i = 1, 2, 3$, denotes the expected vote share of party i in period t. Evaluation yields

$$\begin{aligned}
w_z(\hat{x}^{12}) &= -\frac{1}{3} + z_1 - z_1^2 + \frac{1}{\sqrt{3}}\hat{x}_2^{12} - (\hat{x}_2^{12})^2 \\
w_z(\hat{x}^{23}) &= -\frac{7}{12} + z_1\left(\frac{3}{2} - \sqrt{3}\hat{x}_2^{12}\right) - z_1^2 + \frac{5}{2\sqrt{3}}\hat{x}_2^{12} - (\hat{x}_2^{12})^2 \\
w_z(\hat{x}^{31}) &= -\frac{1}{12} + z_1\left(\frac{1}{2} + \sqrt{3}\hat{x}_2^{12}\right) - z_1^2 - \frac{1}{2\sqrt{3}}\hat{x}_2^{12} - (\hat{x}_2^{12})^2.
\end{aligned}$$

Symmetry implies $w_z(\hat{x}^{12}) - w_z(\hat{x}^{23}) = w_z(\hat{x}^{31}) - w_z(\hat{x}^{12})$ and $\frac{1}{2}w_z(\hat{x}^{23}) + \frac{1}{2}w_z(\hat{x}^{31}) = w_z(\hat{x}^3) = w_z(\hat{x}^{12})$, so voter z prefers to give party 3 a majority rather than elect a minority parliament if and only if

$$\frac{w_z(\hat{x}^3)}{1-\xi} \geq Ew_z \iff \frac{1}{2}(\bar{s}_1 - \bar{s}_2)\left(\frac{1}{2} - \frac{\sqrt{3}}{2}\hat{x}_2^{12}\right)\left(\frac{1}{2} - z_1\right) \leq 0.$$

Voter $z' = (\frac{1}{2}, \frac{1}{2\sqrt{3}})$ is indifferent to how she votes, whereas the preferences of other voters depend on the expected vote shares $\bar{s}_1$ and $\bar{s}_2$. Voters are symmetrically located around $\bar{x}$ as are the ideal policies of the parties and their proposals $\{\hat{x}^{12}, \hat{x}^{23}, \hat{x}^{31}\}$, so party 1 receives neither more or fewer votes than party 2.

[25] The superscript t is omitted where no ambiguity results.

In expectation then $\bar{s}_1 = \bar{s}_2$, and all voters on the median line are indifferent between electing a minority parliament and giving party 3 a majority. ■

Proof of Lemma 2:

Proof. To simplify the notation, without loss of generality let the out party be $k = 3$. To show that party 3 chooses $\hat{x}^3$ when it has a majority, consider two potentially attractive deviations: (i) to form a majoritarian government with policy $\hat{x}^{31}$ or $\hat{x}^{23}$, which would persist thereafter, and (ii) to choose its ideal policy x^3, which in the next period would be followed by a minority parliament and the random formation of majoritarian governments with policies in (9). On the equilibrium path party 3 chooses $\hat{x}^3$, and by Lemma 1 it receives a majority in the next election. The utility of majority party 3 then is $\frac{u_3(\hat{x}^3)}{1-\delta}$, and if it deviates to $\hat{x}^{31}$, its utility is $\frac{u_3(\hat{x}^{31})}{1-\delta}$. Party 3 does not deviate if $u_3(\hat{x}^3) \geq u_3(\hat{x}^{31})$, which is satisfied for all $\hat{x}_2^{12} \in [0, \frac{1}{2\sqrt{3}})$ and for all $\delta \in [0, 1)$.

If $q^{t-1} = x^3$, voters elect a minority parliament in the next period with expected vote shares $\bar{s}_i = \frac{1}{3}, i = 1, 2, 3$, if party 3 proposes $y^t = q^{t-1}$. The expected utility Eu_3 is then

$$\begin{aligned} Eu_3 &= u_3(x^3) + \frac{\delta}{3(1-\delta)} \left(u_3(\hat{x}^{12}) + u_3(\hat{x}^{23}) + u_3(\hat{x}^{31})\right) \\ &= -\frac{\delta}{3(1-\delta)} \left(\frac{5}{4} - \sqrt{3}\hat{x}_2^{12} + 3(\hat{x}_2^{12})^2\right), \end{aligned}$$

and party 3 proposes $y^t = \hat{x}^3$ if $\delta > \delta^*(\hat{x}_2^{12})$, given in (10). Differentiation shows that $\delta^*(\hat{x}_2^{12})$ is strictly increasing in $\hat{x}_2^{12}$ for $x_2^{12} \in \left[0, \frac{1}{2\sqrt{3}}\right)$. Since $\delta^*(\hat{x}_2^{12})$ is strictly increasing in $\hat{x}_2^{12}$, for all $\hat{x}_2^{12} \in [0, \frac{1}{2\sqrt{3}})$ party 3 has no incentive to deviate to its ideal policy for $\delta \in (\delta^*(\hat{x}_2^{12}), 1)$. ■

Proof of Proposition 2:

Proof. To check for deviations from the (conjectured) equilibrium proposal strategies in (9), suppose a formateur in a government $G^{t-1} = \{1, 2\}$ with $q^{t-1} = \hat{x}^{12}$ for $\hat{x}_2^{12} > 0$ deviates to $y^t = x^{12}$. If party 3 and either party 1 or 2 accept the proposal, voters give the out party 3 a majority in the period $t + 1$ election, and by Lemma 2 it chooses $\hat{x}^3$ for $\delta > \delta^*(\hat{x}_2^{12})$. By Lemmas 1 and 2, a single-party government with policy $\hat{x}^3$ persists thereafter. That is, voters on and above, and some below, the median line $z = \left(z_1, \frac{1}{2\sqrt{3}}\right)$ vote for party 3, giving it a majority. The analysis for party-enforced deviations is the same as in the proof of Proposition 1.

From Lemma 2 there is no incentive for a majority party i to deviate from $y^t = \hat{x}^i$, and from Lemma 1 there is no incentive for a set of voters to deviate from the conjectured equilibrium strategies. Similarly, in a minority parliament there is no incentive for a set of voters to deviate as in the analysis in (4) and (5).

The difference between the bound $\delta^*(\hat{x}_2^{12})$ from the incentives to deviate for a majority party and the bound $\hat{\delta}(\hat{x}_2^{12})$ from the incentives to deviate in a minority parliament is

$$\delta^*(\hat{x}_2^{12}) - \hat{\delta}(\hat{x}_2^{12}) = \frac{-\frac{15}{144} + \frac{15}{12\sqrt{3}}\hat{x}_2^{12} - \frac{1}{3}(\hat{x}_2^{12})^2 + \frac{\sqrt{3}}{2}(\hat{x}_2^{12})^3}{(\frac{5}{12} - \frac{1}{2\sqrt{3}}\hat{x}_2^{12} + (\hat{x}_2^{12})^2)(\frac{5}{12} - \frac{1}{\sqrt{3}}\hat{x}_2^{12} + (\hat{x}_2^{12})^2)},$$

which is strictly increasing in $\hat{x}_2^{12}$ with range $[-0.6, 0.2]$. Define $\bar{\hat{x}}_2^{12}$ by $\delta^*(\bar{\hat{x}}_2^{12}) - \hat{\delta}(\bar{\hat{x}}_2^{12}) \equiv 0$, so for $\hat{x}_2^{12} < \bar{\hat{x}}_2^{12}$ majoritarian incentives to deviate are the more restrictive, whereas for $\hat{x}_2^{12} > \bar{\hat{x}}_2^{12}$ the incentives in a majority parliament are the more restrictive. For $\delta > \max\{\delta^*(\hat{x}_2^{12}), \hat{\delta}(\hat{x}_2^{12})\}$ no party has an incentive to deviate from the (conjectured) equilibrium strategies. ■

Proof of Proposition 3:

Proof. The proof involves showing that there is no attractive deviation from the strategies in (11) and (12) and the corresponding strategies for the other parties. Suppose the status quo is $Q^{t-1} = \left(\bar{x}^{12}, B - c, c, 0\right)$, party 2 is selected as the formateur, and it proposes $y^t = \left(\bar{\bar{x}}^{12}, c, B - c, 0\right)$. Party 1 accepts the proposal if

$$u_1(\bar{\bar{x}}^{12}) + c + \delta v_1(\bar{\bar{x}}^{12}, c, B - c, 0) \geq u_1(\bar{x}^{12}) + B - c + \delta v_1(\bar{x}^{12}, B - c, c, 0), \tag{A.3}$$

which is equivalent to (13). The head of government thus must compensate its government partner with more than half the benefits to induce the partner to accept a less favorable policy. An equilibrium requires that there are sufficient benefits that c^o in (13) is no greater than B, which requires (14). More benefits are required the more disparate are the policies implemented by the heads of a durable government.

The out party could propose $y^t = (x^1, B, 0, 0)$ when the status quo is $Q^{t-1} = (\bar{\bar{x}}^{12}, c, B - c, 0)$ in an attempt to break the government. If party 1 accepts the proposal, the government $G^{t-1} = \{1, 2\}$ falls, a minority parliament with $\bar{s}_i = \frac{1}{3}, i = 1, 2, 3$, is elected, and a new government formation round commences the next period. Similarly, if the period t election outcome provides a majority for party 1, it could choose its ideal policy and take all the benefits. Party 1 rejects the proposal and does not deviate if

$$u_1(\bar{\bar{x}}^{12}) + c + \delta v_1(\bar{\bar{x}}^{12}, c, B - c, 0) \geq u_1(x^1) + B + \delta \bar{v}^*, \tag{A.4}$$

which is satisfied for sufficiently high δ, since $v_1(\bar{\bar{x}}^{12}, c, B - c, 0) - \bar{v}^* = \frac{1}{6(1-\delta)}\left(u_1(\bar{x}^{12}) + u_1(\bar{\bar{x}}^{12}) - 2u_1(\bar{x}^{23}) + B\right)$ is positive. Similarly, if $Q^{t-1} \notin \bar{X}^1 \cup \bar{X}^2 \cup \bar{X}^3$, the most attractive status quo for party 1 is $Q^{t-1} = (x^1, B, 0, 0)$. Party 1 chooses $y^t \in \bar{X}^1$ if the strict inequality in (A.4) holds. The inequality in (A.4) is

$$c > -u_1(\bar{\bar{x}}^{12}) + B - \frac{1}{6(1-\delta)}\left(u_1(\bar{x}^{12}) + u_1(\bar{\bar{x}}^{12}) - 2u_1(\bar{x}^{23}) + B\right),$$

where for high δ the right side is negative, so a c satisfying the condition exists. The utility $u_1(x^1) + B$ in (A.4) is the maximal single-period utility for party 1, and all deviations that result in a new government formation round have the same continuation value $\bar{v}^*$, so the deviation considered is the most attractive to party 1. Consequently, there is no proposal by the out party that can cause the government to fall nor does party 1 deviate if it has a majority. From Lemma 3 the expected vote share of one or both the incumbent parties equals one-half.

The out party could also propose a replacement government to entice a government party to defect, but that party is indifferent between the present government and the replacement government and under the indifference rule rejects the proposal. That is, suppose $Q^{t-1} = \left(\bar{\bar{x}}^{12}, c, B - c, 0\right)$ and party 3 proposes $y^t = \left(\bar{\bar{x}}^{31}, B - c, 0, c\right)$ to form a government $G^t = \{1, 3\}$. Party 1 rejects the proposal when (13) and (14) are satisfied, since (A.3) is satisfied with the right side replaced by $u_1(\bar{\bar{x}}^{31}) + B - c + \delta v_1(\bar{\bar{x}}^{31}, B - c, 0, c)$.

For sufficiently high δ, parties thus have no incentive to deviate from the proposal and acceptance strategies in (11) and (12) when B satisfies (14) and c satisfies (13). Given those strategies, Lemma 3 implies that myopic voters anticipate electing minority parliaments with expected vote shares $\bar{s}_1 = \frac{1}{2}$, $\bar{s}_2 = \frac{1}{4}$, $\bar{s}_3 = \frac{1}{4}$ when $Q^{t-1} = \left(\bar{\bar{x}}^{12}, c, B - c, 0\right)$ and $\bar{s}_1 = \frac{1}{4}$, $\bar{s}_2 = \frac{1}{2}$, $\bar{s}_3 = \frac{1}{4}$ when $Q^{t-1} = \left(\bar{x}^{12}, B - c, c, 0\right)$. ■

Proof of Lemma 4:

Proof. For $q^{t-1} = x^{12}$, the continuation value $v_1(x^{12})$ for government party 1 when the out party's proposal is rejected on the equilibrium path satisfies

$$v_1(x^{12}) = (1 - \eta)\left(u_1(x^{12}) + \delta v_1(x^{12})\right) + \eta E_\epsilon \left(u_1(x^{12} + \tilde{\epsilon}) + \delta v_1(x^{12} + \tilde{\epsilon})\right), \tag{A.5}$$

where E_ϵ denotes expectation with respect to $\tilde{\epsilon}$. Similarly, if $q^{t-1} = x^{23}$, the continuation value $v_1(x^{23})$ satisfies

$$v_1(x^{23}) = (1 - \eta)\left(u_1(x^{23}) + \delta v_1(x^{23})\right) + \eta E_\epsilon \left(u_1(x^{23} + \tilde{\epsilon}) + \delta v_1(x^{23} + \tilde{\epsilon})\right). \tag{A.6}$$

Given $q^{t-1} \in \cup_{\ell=1}^3 X^\ell$, a crisis results in a policy $q^t \notin \cup_{\ell=1}^3 X^\ell$ with probability one, since X^ℓ is finite. A new government formation round thus commences in period $t + 1$ in which the formateur i randomizes among the policies in X^i. Consequently, $E_\epsilon v_i(x^{12} + \tilde{\epsilon}) = E_\epsilon v_i(x^{23} + \tilde{\epsilon}) = E_\epsilon v_i(x^{31} + \tilde{\epsilon}), i = 1, 2, 3$, and by symmetry $E_\epsilon v_1(x^{12} + \tilde{\epsilon}) = E_\epsilon v_2(x^{12} + \tilde{\epsilon}) = E_\epsilon v_3(x^{12} + \tilde{\epsilon})$. Denote that continuation value by $E_\epsilon v^*$, which for $q^t \neq \cup_{\ell=1}^{3} X^\ell$ satisfies

$$
\begin{aligned}
Ev_\epsilon^* \quad = \quad & \bar{s}_1 \left[\frac{1}{2} \left[(1-\eta) \left(u_1(x^{12}) + \delta v_1(x^{12}) \right) + \eta E_\epsilon \left(u_1(x^{12} + \tilde{\epsilon}) + \delta v_1(x^{12} + \tilde{\epsilon}) \right) \right] \right. \\
& \left. + \frac{1}{2} \left[(1-\eta) \left(u_1(x^{31}) + \delta v_1(x^{31}) \right) + \eta E_\epsilon \left(u_1(x^{31} + \tilde{\epsilon}) + \delta v_1(x^{31} + \tilde{\epsilon}) \right) \right] \right] \\
& + \bar{s}_2 \left[\frac{1}{2} \left[(1-\eta) \left(u_1(x^{12}) + \delta v_1(x^{12}) \right) + \eta E_\epsilon \left(u_1(x^{12} + \tilde{\epsilon}) + \delta v_1(x^{12} + \tilde{\epsilon}) \right) \right] \right. \\
& \left. + \frac{1}{2} \left[(1-\eta) \left(u_1(x^{23}) + \delta v_1(x^{23}) \right) + \eta E_\epsilon \left(u_1(x^{23} + \tilde{\epsilon}) + \delta v_1(x^{23} + \tilde{\epsilon}) \right) \right] \right] \\
& + \bar{s}_3 \left[\frac{1}{2} \left[(1-\eta) \left(u_1(x^{31}) + \delta v_1(x^{31}) \right) + \eta E_\epsilon \left(u_1(x^{31} + \tilde{\epsilon}) + \delta v_1(x^{31} + \tilde{\epsilon}) \right) \right] \right. \\
& \left. + \frac{1}{2} \left[(1-\eta) \left(u_1(x^{23}) + \delta v_1(x^{23}) \right) + \eta E_\epsilon \left(u_1(x^{23} + \tilde{\epsilon}) + \delta v_1(x^{23} + \tilde{\epsilon}) \right) \right] \right],
\end{aligned}
\tag{A.7}
$$

where $\bar{s}_i, i = 1, 2, 3$, are the expected seat shares. Solving (A.5), (A.6), and (A.7) for the continuation values yields the expressions in the lemma with $\bar{s}_1 = \bar{s}_2$ and $\bar{s}_3 = 1 - 2\bar{s}_1$. ■

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Figure 1A

Election Preceding a New Government Formation Round

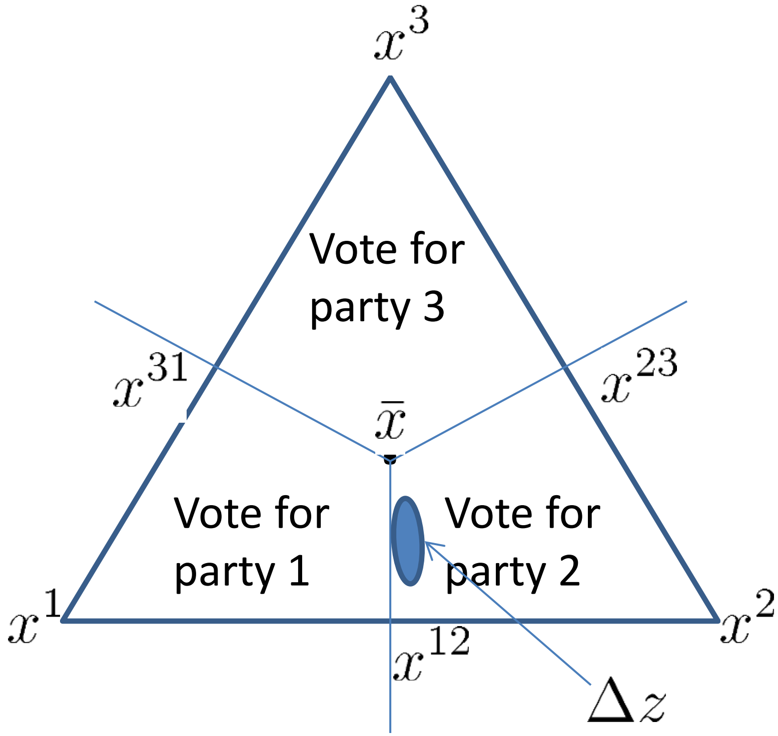


Figure 1B

Election with an Incumbent Government $G^{t-1} = \{1, 2\}$

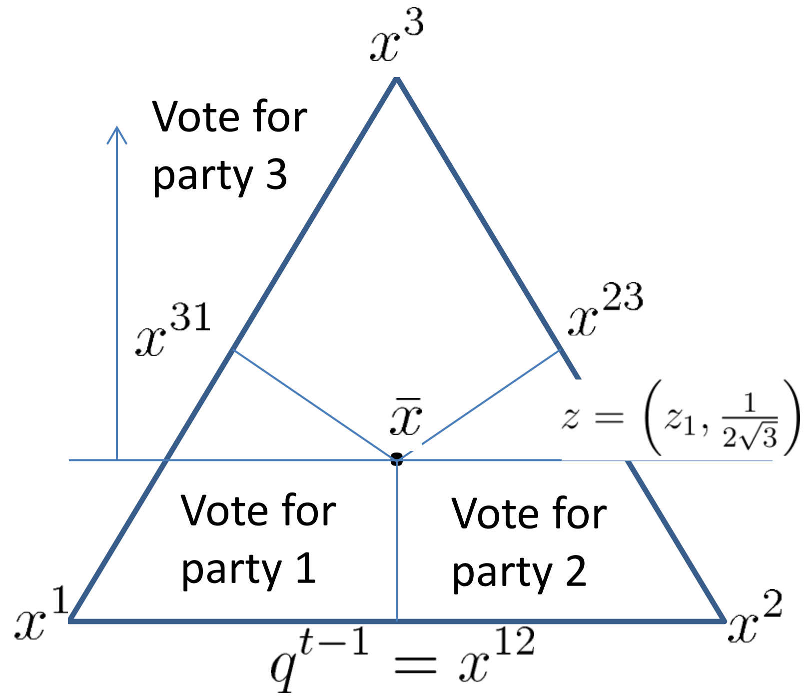


Figure 2
Equilibria with Concessions
to Centrist Voters

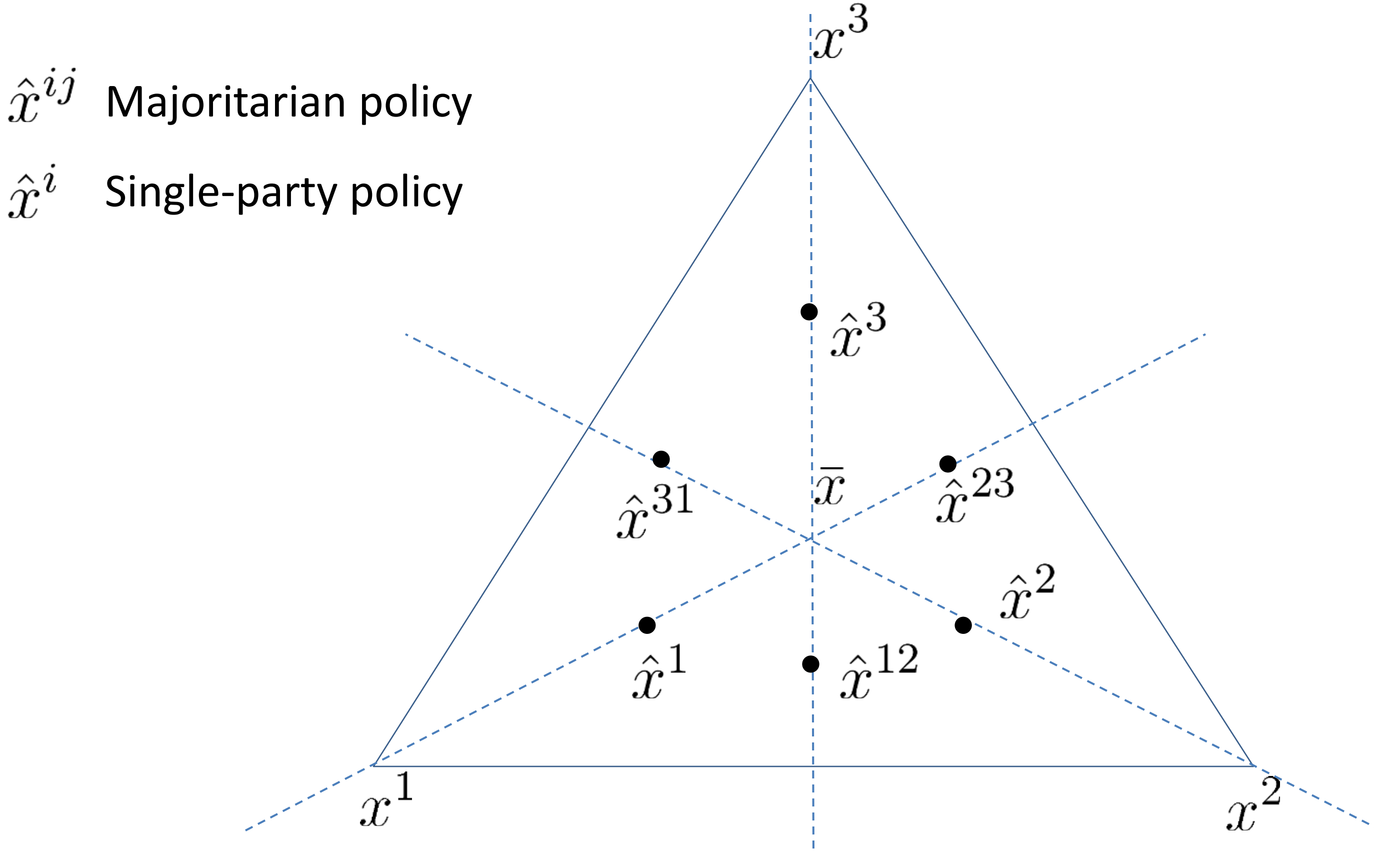
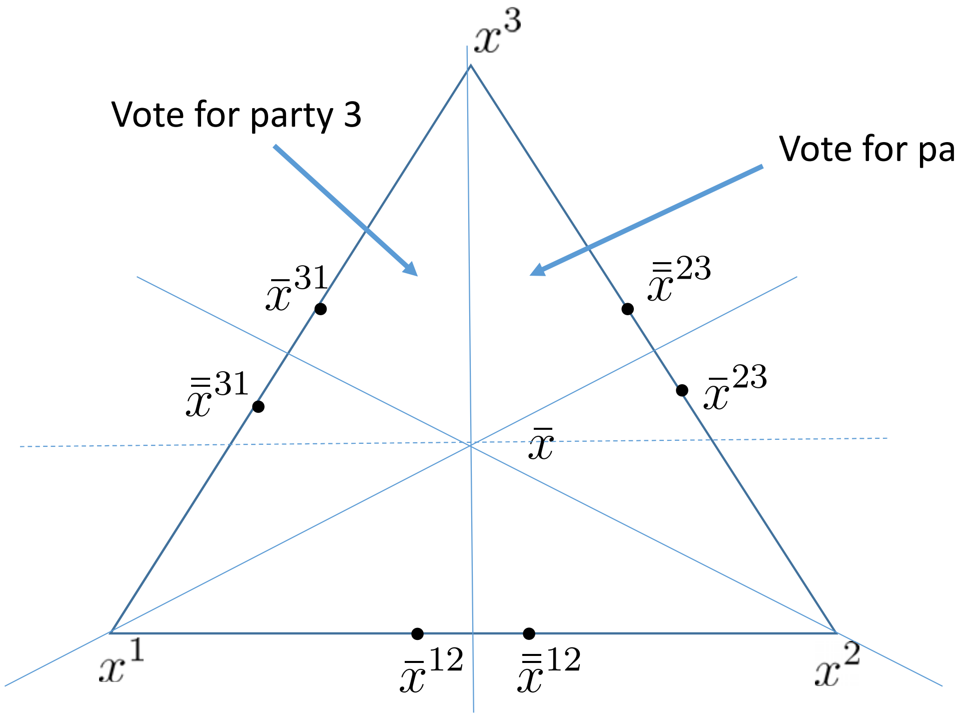


Figure 3

Equilibrium Voting with Officeholding Benefits



$$q^{t-1} = \bar{x}^{12}$$

$$S_1 = \frac{1}{4}$$

$$S_2 = \frac{1}{2}$$

$$S_3 = \frac{1}{4}$$