Inequality Measurement and the Rich:
Why inequality increased more than we thought

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Abstract

Many standard inequality measures can be written as ratios with the mean in the denominator. When one income moves away from equality, both the numerator and the denominator may vary in the same direction and such indices may decrease. This anomalous behaviour is not shared by median-normalised inequality measures developed in this paper, where the mean at the denominator is replaced by the median. However, median-normalised inequality measures do not respect the principle of transfers. We show that the absolute Gini and the mean logarithmic deviation, or second Theil index, are the only measures that both avoid anomalous behaviour when one income is varied and also satisfy the principle of transfers. An application shows that the increase in inequality in the United States over recent decades is understated by the Gini index and that the mean logarithmic deviation index should be preferred in practice.

Keywords: inequality measures, median, axiomatic approach

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1 Introduction

When the rich get richer does inequality go up? Perhaps. Many standard inequality measures, such as the Gini and Theil indices, sometimes indicate the opposite. The reason for this behaviour is that these and many other inequality indices are expressed as ratios, where the numerator is an indicator of dispersion and the denominator is the mean. The indicator of dispersion in these indices ensures that they satisfy the principle of transfers; division by the mean ensures that they are scale independent. But nevertheless they have a property that may seem unattractive.

To see this, consider what happens when the income of just one person is changed in a direction "away from equality" so that, if the person's income is above the point representing equality, the income is increased; if it is below this point, it is decreased. In such situations we might want to appeal to the following *principle of monotonicity in distance*:

> if two distributions differ only in respect of one individual's income, then the distribution that registers greater distance from equality for this individual's income is the distribution that exhibits greater inequality.

The principle appears to be attractive, but it is evidently possible that the Gini, the Theil and many other standard inequality indices might behave in a problematic fashion. When the rich get richer both the numerator and the denominator of the index will change in the same direction and, as a result, the value of the index could fall. To illustrate what can happen, suppose income distribution $\mathbf{x}$ changes to $\mathbf{x}'$ where :

$$\mathbf{x} = \{1, 2, 3, 4, 5, 9, 10\} \quad \text{and} \quad \mathbf{x}' = \{1, 2, 3, 4, 7, 9, 10\}. \tag{1}$$

Given that the mean of $\mathbf{x}$ is 4.857 and the mean of $\mathbf{x}'$ 5.143, it is clear that the fifth person's income increase from 5 to 7 represents a move away from equality. However, computing the Gini and the Theil indices, we find *more* inequality in $\mathbf{x}$ than in $\mathbf{x}'$:

$$\text{Gini}(\mathbf{x}) = 0.361 \quad > \quad \text{Gini}(\mathbf{x}') = 0.357 \tag{2}$$

$$\text{Theil}(\mathbf{x}) = 0.216 \quad > \quad \text{Theil}(\mathbf{x}') = 0.214 \tag{3}$$

So, indeed, the Gini and the Theil indices do not respect the principle of monotonicity in distance. More generally, the numerator and the denominator change in the same direction for any income lying above the point representing equality; if an income below that point is reduced, the numerator increases and the denominator decreases. So variations away from equality are attenuated (amplified) when a someone rich (poor) get richer (poorer).

In this paper, we develop a median-normalised class of inequality measures that respects the principle of monotonicity in distance, based on an axiomatic approach. We show that it is closely related to the Generalised-Entropy class of inequality measures, where the mean in the denominator is replaced by the median. However, it does not respect the principle of transfers.

We further show that the Mean Logarithmic Deviation (MLD) index, or second Theil index, which is a limiting case of the mean-normalised Generalized Entropy class of inequality measure, is also a limiting case of our median-normalised class of measures. Thus it shares properties of both mean- and median-normalised inequality measures. In other words, the MLD index is the only relative inequality measure that respects *both* the principle of transfers and the principle of monotonicity in distance. Indeed, we find less inequality in $\mathbf{x}$ than in $\mathbf{x}'$ with the MLD index:

$$\text{MLD}(\mathbf{x}) = 0.254 \quad < \quad \text{MLD}(\mathbf{x}') = 0.263 \tag{4}$$

The lack of the principle of monotonicity in distance may have strong implication for empirical studies. Examining inequality in Great Britain and in the United States over recent decades, we show that the Gini index under-record variations of inequality and the MLD index should be preferred in practice.

The paper is organised as follows. Section 2 sets out a theoretical approach to inequality measurement based on principles that accord with intuition; it characterises the inequality measures that are consistent with these principles and compares the two core principles – the principle of monotonicity in distance and the principle of transfers. Sections 3 and 4 discuss the behaviour of the inequality measures introduced in Section 2 in terms of their sensitivity to different parts of the income distribution and decomposability by population subgroups. Section 5 shows how the alternative approach to inequality developed here affects the conclusions on inequality comparisons in the United States and in Great Britain. Section 6 concludes.

2 Inequality: an approach

What is an inequality measure and what should it do? Technically, inequality measurement can be seen as a way of ordering all possible income distributions. There is potentially a large collection of statistical tools that may appear to do the job of inequality comparisons. The fact that two different inequality measures may rank a pair of distributions in opposite ways may not matter – each of the two measures may respect the same underlying principles, but give different weight to information in different parts of the distribution. What may matter is when two different measures contradict each other in practice because they are founded on different, potentially conflicting, economic principles.

We will first describe a core set of principles that capture three key aspects of inequality measurement. We then show these principles characterise a family of inequality measures. Using this family we show how two fundamental principles of inequality may be in conflict.

2.1 Reference point and principles

The meaning of inequality comparisons can be expressed concisely and unambiguously by adopting three elementary principles. To introduce these we introduce the concept of a *reference point*, a particular income value used as the basis for assessing changes in inequality. There are are several possibilities for specifying this reference point (Cowell and Flachaire 2017). It could be the mean – the income that everyone would have if there were perfect equality and lump-sum income transfers were possible. It could be the median, arguably a more satisfactory way of characterising an "equality reference point". It could even be some income value that is independent of the income distributions being compared.

We use this reference point to give meaning to the first of the three principles, already mentioned in the introduction. The principle of monotonicity in distance means that the movement of any income away from the reference point should be regarded as an increase in inequality. So, if two distributions differ only in respect of person i's income, then the distribution that registers greater individual distance from equality for i is the distribution that exhibits greater inequality.

The second principle amounts to one of decomposability by subgroups. If income changes increase inequality in any one subgroup then, if there are no income changes in other subgroups, inequality overall must also increase. This principle could be applied in two ways: (1) we could insist on this property for arbitrary subgroups (such as by race, gender, region) or (2) restrict this principle only to subgroups that can be strictly ordered by income.

The third principle, scale irrelevance, refers to the rescaling of income distributions and encapsulates two ideas: First, the meaning of inequality comparisons should not change as all incomes grow proportionately, relative to a given reference point. Second, rescaling all income values – the income distribution as well as the reference point – should leave inequality comparisons unaltered. This is analogous to anchoring poverty comparisons to an exogenous poverty line.

2.2 Inequality measures

The three principles just outlined lead directly to a characterisation of a specific class of inequality measures.[1] If we also require the index be anonymous,[2] then, in an n-person society with equality reference point e, the inequality index must either take the form

$$G(\mathbf{x}) := \frac{1}{2n^2}\sum_{i=1}^{n}\sum_{j=1}^{n}|x_i - x_j| \tag{5}$$

(the "Absolute Gini"), or the form

$$I_\alpha(\mathbf{x}; e) := \frac{1}{\alpha[\alpha-1]}\left[\frac{1}{n}\sum_{i=1}^{n} x_i^\alpha - e^\alpha\right], \tag{6}$$

where α is a sensitivity parameter, or by some strictly increasing transformation of (5) or (6).

In order to develop the basic formulation (5), (6) into practical inequality indices, two further things need to be done.

[1] The three principles described in section 2.1 are made precise in Axioms 1-3 set out in the Technical Appendix. The formal result establishing (5) and (6) for non-negative incomes is also in the Technical Appendix. The result also needs the assumption of continuity of the inequality ordering.

[2] Anonymity means that, if all information relevant to inequality is embodied in the income measure, then switching the labels on the individuals must leave inequality unchanged.

First, the reference point e needs to be specified in (6).[3] In the case of income inequality, it is natural to choose mean income as the reference point, $e = \mu$, in that $(\mu, \mu, ..., \mu)$ represents a perfectly equal distribution in the case where lump-sum transfers of x are assumed to be possible; but other specifications may also make sense.

Second, the expression (6) is defined only up to an increasing transformation: we need to determine what type of normalisation is appropriate to make it into a practical index. Two small aspects of this normalisation have already been incorporated in (6), the division by the constant $\alpha [\alpha - 1]$ and the division by population size to insure that the index is independent under population replication. There remains a third normalisation step concerning the way that the index should behave when all incomes change proportionately (the principle of scale irrelevance on inequality comparisons ensures that inequality *comparisons* remain unaffected by such income changes, but says nothing about inequality *levels*). It is common to assume that inequality should remain constant under such proportional changes. But there are several ways of doing this. One could divide through by the reference point – in this case the mean – but it could be some other function of incomes. Here we investigate the use of the median as an alternative normalisation criterion instead of the mean, as in the following examples.

The conventional ("relative") Gini index (Yitzhaki and Schechtman 2013) is found from the absolute Gini (5) after normalising by the mean:

$$G(\mathbf{x}/\mu) = \frac{E(|x_1 - x_2|)}{2\mu} = \frac{\sum_{i=1}^{n} \sum_{j=1}^{n} |x_i - x_j|}{2\mu n^2} \tag{7}$$

As an alternative to the conventional Gini index Gastwirth (2014) proposed replacing the mean by the median in the standard definition of the Gini index,

$$G(\mathbf{x}/m) = \frac{E(|x_1 - x_2|)}{2m} = \frac{\mu}{m} G(\mathbf{x}/\mu) \tag{8}$$

Now consider the family (6). If we use the mean as the reference point and also normalise by the mean we find that this yields the Generalised-Entropy

[3] It is clear from (5) that the absolute Gini $G(\mathbf{x})$ is independent of e.

class of measures given by[4]

$$I_\alpha(\mathbf{x}/\mu;\mu) = \frac{1}{\alpha[\alpha-1]}\left[\frac{1}{n}\sum_{i=1}^{n}\left(\frac{x_i}{\mu}\right)^\alpha - 1\right], \alpha \neq 0,1, \tag{9}$$

$$I_0(\mathbf{x}/\mu;\mu) = -\frac{1}{n}\sum_{i=1}^{n}\log\left(\frac{x_i}{\mu}\right), \tag{10}$$

$$I_1(\mathbf{x}/\mu;\mu) = \sum_{i=1}^{n}\frac{x_i}{\mu}\log\left(\frac{x_i}{\mu}\right). \tag{11}$$

If instead we use the mean as the reference point, but normalise by the median, we find that (6) yields the following:

$$I_\alpha(\mathbf{x}/m;\mu) = \frac{1}{\alpha[\alpha-1]}\left[\frac{1}{n}\sum_{i=1}^{n}\left(\frac{x_i}{m}\right)^\alpha - \left(\frac{\mu}{m}\right)^\alpha\right], \tag{12}$$

$$= \left(\frac{\mu}{m}\right)^\alpha I_\alpha(\mathbf{x}/\mu;\mu), \text{ for all } \alpha \in \mathbb{R}. \tag{13}$$

2.3 Monotonicity in distance and the transfer principle

There is an obvious difficulty with the type of normalisation that we have just discussed. If we normalise by an expression that involves the income vector then the behaviour of the resulting inequality measure may be affected by the specific form of income-normalisation that is adopted.

Monotonicity in distance

This point is easily seen for the Gini index (7) in the case of normalisation by the mean, as was illustrated in the introduction (see equations 1 and 2): if one income is increased the mean increases and so the contribution of each distance between all other incomes in (7) decreases.[5] This problem is further illustrated in the example depicted in Figure 1, which shows the values of the mean-normalised (7) and median-normalised (8) Gini coefficient for a sample

[4] The limiting form as $\alpha \to 0$, the Mean Logarithmic Deviation (10), follows from (9) using l'Hôpital's rule. The limiting form for $\alpha \to 1$, the Theil index (11), follows from (9) by expressing it in the equivalent form $\frac{1}{\alpha[\alpha-1]}\sum_{i=1}^{n}\frac{x_i}{n\mu}\left[(x_i/\mu)^{\alpha-1} - 1\right]$ and again applying l'Hôpital's rule.

[5] When only $x_k\uparrow$, we have $\mu\uparrow$ and so $|x_i - x_j|/\mu\downarrow$ for all $i, j \neq k$.

of 1000 observations, drawn from a standard lognormal distribution, plus 1 additional observation x_i', where $x_i' \in]0, 8]$. The left-hand panel of Figure 1 confirms that the conventional (mean-normalised) Gini is not consistent with the principle of monotonicity in distance: if x_i' is very low – below the median (marked $\circ$) and the mean (marked $*$) – then increasing x_i' reduces inequality; if x_i' lies well above the mean then increasing x_i' increases inequality; both these things seem to accord with common sense – see also Lambert and Lanza (2006). But there is a part of the curve just to the right of $*$ where an increase in an above-average income *reduces* inequality. Note that the median-normalised Gini also violates the principle of monotonicity, as can be seen from the right-hand panel of Figure 1.[6] It is clear that there is a zone where increasing a below-the-mean income *increases* inequality – check the part of the curve just to the left of $*$.

Now consider this issue for the family of inequality measures defined in (9). If we set $e = \mu$ in the non-normalised (6) and differentiate with respect to x_i we find the following impact on inequality:

$$\delta_i(\mathbf{x}) := \frac{\partial I_\alpha(\mathbf{x};\mu)}{\partial x_i} = \frac{1}{n}\,\frac{x_i^{\alpha-1} - \mu^{\alpha-1}}{\alpha - 1}, \tag{14}$$

The expression $\delta_i(\mathbf{x})$ is positive/negative according as $x_i \gtrless \mu$, for all values of α – a property directly inherited from Axiom 1. But if we normalise by the mean to obtain (9) and differentiate we have:

$$\frac{\partial I_\alpha(\mathbf{x}/\mu;\mu)}{\partial x_i} = \mu^{-\alpha}\delta_i(\mathbf{x}) - \frac{\alpha}{\mu n} I_\alpha(\mathbf{x}/\mu;\mu). \tag{15}$$

Clearly, if $\alpha > 0$ and $\delta_i(\mathbf{x}) > 0$ then, for some $\mathbf{x}$, expression (15) will be negative; likewise, if $\alpha < 0$ and $\delta_i(\mathbf{x}) < 0$ then, for some $\mathbf{x}$, expression (15) will be positive. In sum, for mean-normalised inequality measures and any $\alpha \neq 0$, there will always be some income distribution for which the anomalous

[6] In the neighbourhood of the median there is a large change in the index: this is due to a change of the median, which affects every term in the normalised Gini. For $n = 1001$:

$$m = \begin{cases} x_{(500)} & \text{if } x_i' \leq x_{(500)} \\ x_i' & \text{if } x_i' \in [x_{(500)}; x_{(501)}] \\ x_{(501)} & \text{if } x_i' \geq x_{(501)} \end{cases}$$

where $x_{(k)}$ denotes the k^{th}-order income. The median varies in a narrow interval: its values are bounded by two adjacent mid-rank incomes. In our example $m \in [0.9647,\ 0.9659]$.

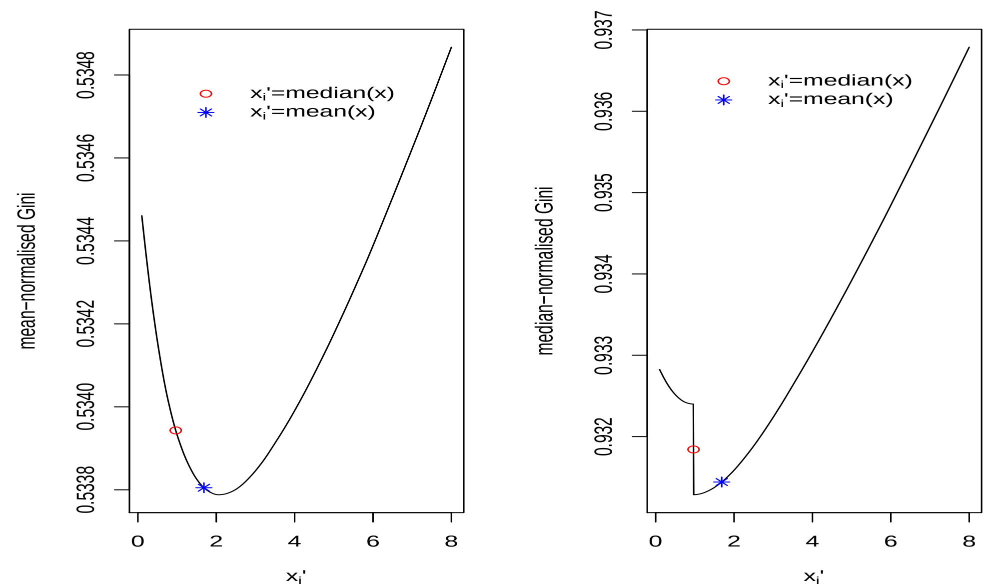

Figure 1: Principle of monotonicity in distance: mean-normalised Gini (left) and median-normalised Gini (right) indices, computed from 1000 observations drawn from a lognormal distribution, plus 1 additional observation x_i', where $x_i' \in]0, 8]$.

behaviour illustrated in the introduction will emerge: the normalised indices must violate the property of monotonicity in distance. By contrast, consider normalising by the median. Differentiating (12) we have

$$\frac{\partial I_\alpha(\mathbf{x}/m;\mu)}{\partial x_i} = m^{-\alpha}\delta_i\left(\mathbf{x}\right) - \frac{\alpha}{m} I_\alpha(\mathbf{x}/m;\mu)\frac{\partial m}{\partial x_i}. \tag{16}$$

For all values of α, the first term on the right-hand side takes the sign of $\delta_i\left(\mathbf{x}\right)$. The second term is zero if inequality is zero, if $\alpha = 0$, or if the change in x_i does not change the median; otherwise it takes the sign of $-\alpha$. So, if $\alpha > 0$ and $x_i < \mu$ both $\delta_i\left(\mathbf{x}\right)$ and expression (16) are negative. If $\alpha \geq 0$ and $x_i > \mu$ and if we confine attention to income distributions for which $\mu \geq m$ then the second term in (16) is zero, so $\delta_i\left(\mathbf{x}\right)$ determines the sign of the whole expression (16). Therefore, for right-skew distributions – such as income or wealth distributions – it is true that the principle of monotonicity

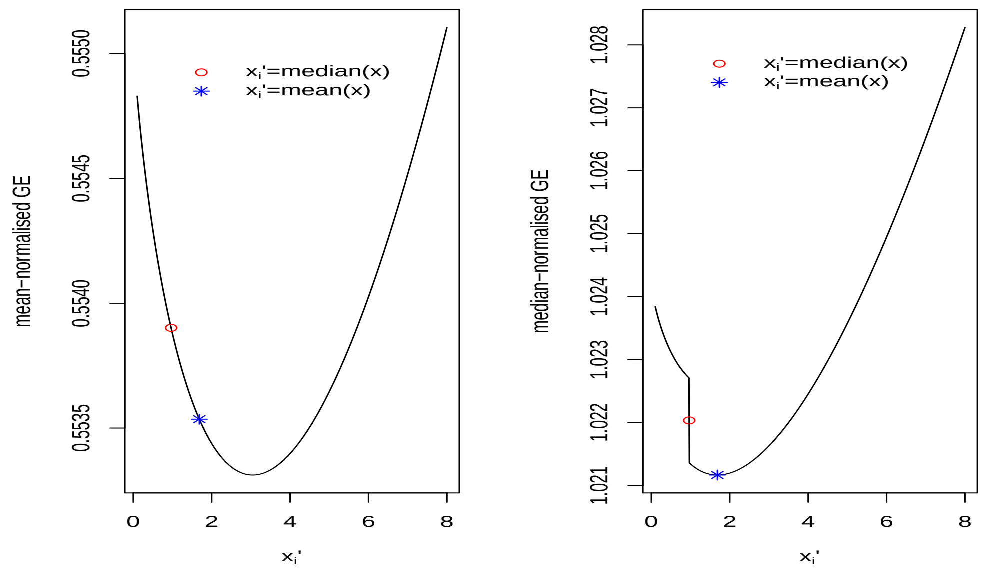

Figure 2: Principle of monotonicity in distance: mean-normalised (left) and median-normalised (right) inequality measures I_α, computed for $\alpha = 1.1$ using the same data as in Figure 1.

in distance is satisfied for the median-normalised inequality indices (12), for $\alpha \geq 0$.[7] Figure 2 is similar to Figure 1, but drawn for mean-normalised (9) and median-normalised (12) inequality measures with $\alpha = 1.1$. The left-hand panel shows that the minimum of the mean-normalised inequality index is not where $x_i' = \mu$ (marked $*$), but where $x_i' = 3.05$: so for any $\mu \leq k_1 < k_2 < 3.05$, the index will exhibit more inequality with $x_i' = k_1$ than with $x_i' = k_2$, which is inconsistent with the principle of monotonicity in distance. The right-hand panel shows that the median-normalised inequality

[7]One might wonder whether measures of the form $I_\alpha(\mathbf{x}/m; m)$ – where the median is both the reference point and the scaling factor – are worthy of consideration. One can see that they are problematic with a simple example. Clearly $I_0(\mathbf{x}/m; m) = I_0(\mathbf{x}/m; \mu) + \log(\mu/m)$. Therefore, in the case where $\alpha = 0$, from (14) and (16), the effect of increasing person i's income is $\frac{1}{nx_i} - \frac{1}{m}\frac{\partial m}{\partial x_i}$. So it would be possible to have a situation where $x_i < m$ and, if the median remains unchanged, the increase in x_i increases measured inequality. In other words the principle of monotonicity is violated. It can be shown that this behaviour is a general problem applying to other values of α.

index is at a minimum when x_i' is equal to the mean and it increases when x_i' moves away from the mean: the principle of monotonicity in distance is respected.

Principle of transfers

The principle of transfers – that a transfer from a poorer person to a richer person should always increase inequality – has long been regarded as the cornerstone of inequality analysis.

As is well known, the absolute Gini coefficient (5) and the regular Gini (7) both satisfy the principle of transfers. The median-normalised Gini (8) will satisfy the principle if the transfers take place strictly above or strictly below the median; in other cases the median may shift and the principle may be violated.

What of the family (6)? Let the income of person i change by an amount $\Delta x > 0$ and the income of person $j \neq i$ by an amount $-\Delta x$. Denote the consequent change in the median by Δm; the change in the mean is zero, by construction. Differentiating (9) and (12), we obtain, respectively:

$$\Delta I_\alpha(\mathbf{x}/\mu;\mu) = \mu^{-\alpha}\left[\delta_i\left(\mathbf{x}\right) - \delta_j\left(\mathbf{x}\right)\right]\Delta x \tag{17}$$

$$\Delta I_\alpha(\mathbf{x}/m;\mu) = m^{-\alpha}\left[\delta_i\left(\mathbf{x}\right) - \delta_j\left(\mathbf{x}\right)\right]\Delta x - \frac{\alpha}{m} I_\alpha(\mathbf{x}/m;\mu)\Delta m \tag{18}$$

where δ_i is defined in (14). The transfer principle requires that each of these expressions be positive/negative according as $x_i \gtrless x_j$. This is obviously true in the case of (17) but, in the case of (18), this can only be true for arbitrary $\mathbf{x}$ if $\alpha = 0$.

So median-normalised inequality measures in (12) do not respect the principle of transfers, but a weaker version of this principle. They respect this principle for any transfers *strictly* above/below the median, that is, as long as the median is unchanged. However, we can easily find a counterexample, where transfer from the median individual to a poorer individual increases $I_\alpha(\mathbf{x}/m;\mu)$. For instance, let us consider the following distributions:

$$\mathbf{x}'' = \{1, 2, 3, 5, 10\} \qquad \text{and} \qquad \mathbf{x}''' = \{1, 2.5, 2.5, 5, 10\}. \tag{19}$$

From the principle of transfers, mean-normalised inequality measures always exhibit more inequality in $\mathbf{x}''$ than in $\mathbf{x}'''$. By contrast, a median-normalised inequality measure may exhibit less inequality in $\mathbf{x}''$ than in $\mathbf{x}'''$: for instance, we have $I_1(\mathbf{x}''/m;\mu) < I_1(\mathbf{x}'''/m;\mu)$ and $G(\mathbf{x}'', m) < G(\mathbf{x}''', m)$,[8] which is *not* consistent with the principle of transfers – see Gastwirth (2014).

In sum, the main difference between the two types of normalisation for I_α is that the median-normalised class of inequality measures respects the principle of monotonicity in distance when $\alpha \geq 0$, while mean-normalised class of inequality measures in (9) respects the (Pigou-Dalton) principle of transfers.

2.4 Two principles in one

As should be clear from equations (15) and (18), apart from the absolute Gini (5) there is exactly one case where both the principle of monotonicity in distance and the principle of transfers are respected. This is where $\alpha = 0$, corresponding to the MLD index. It is immediate from (13) that this case has the following property:

$$I_0(\mathbf{x}/m;\mu) = I_0(\mathbf{x}/\mu;\mu). \tag{20}$$

Figure 3 illustrates the principle of monotonicity using the same data as in Figures 1 and 2. We can see that the index is minimum when x_i' is equal to the mean ($*$) and it changes smoothly as x_i' moves away from the mean.

3 Sensitivity of the inequality measures

An inequality measure implicitly puts different weight on different parts of the income distribution. In the case of the class (13), we can see that this sensitivity to different parts of the distribution is captured by the parameter α as follows:

- $\alpha > 1$ puts more weight on high incomes (when $x_i \gg m$),
- $\alpha < -1$ puts more weight on low incomes (when $x_i \ll m$),
- $-1 \leq \alpha < 0$ puts more weight on incomes close to and above the median,
- $0 < \alpha \leq 1$ puts more weight on incomes close to and below the median.

[8] $I_1(\mathbf{x}''/m;\mu) = 0.3745$, $I_1(\mathbf{x}'''/m;\mu) = 0.4414$, $G(\mathbf{x}'', m) = 0.560$ and $G(\mathbf{x}''', m) = 0.656$.

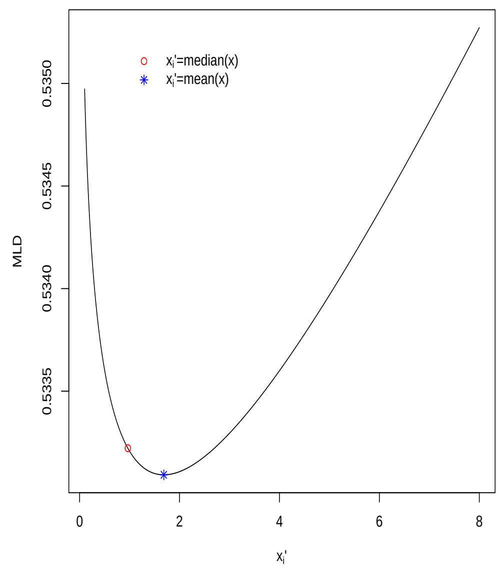

Figure 3: Principle of monotonicity in distance: the mean logarithmic distance I_0, computed for the same data as in Figure 1.

When $\alpha \in [-1, 1]$, the index puts more weight on incomes in the middle of the distribution, rather than in the tails.

The limiting case $\alpha = 0$ is *non-directional*, in the sense that it does not put more weight on incomes above/below the median or above/below the mean. This can be seen by rewriting (10) as follows:

$$I_0(\mathbf{x}/m; \mu) = \log(\mu/g), \tag{21}$$

where g is the geometric mean of incomes. So, the MLD index is then the log differences between the arithmetic mean and the geometric mean and it is clear that in computing arithmetic and geometric means, every income has the same weight.

The relationship between median- and mean-normalised inequality measures is given by (13). For income distributions, skewed to the right ($m < \mu$), median-normalised inequality measures are always greater (less) than mean-normalised inequality measures for $\alpha > 0$ ($\alpha < 0$). To illustrate this feature, Figure 4 plots values of median- and mean-normalised inequality measures,

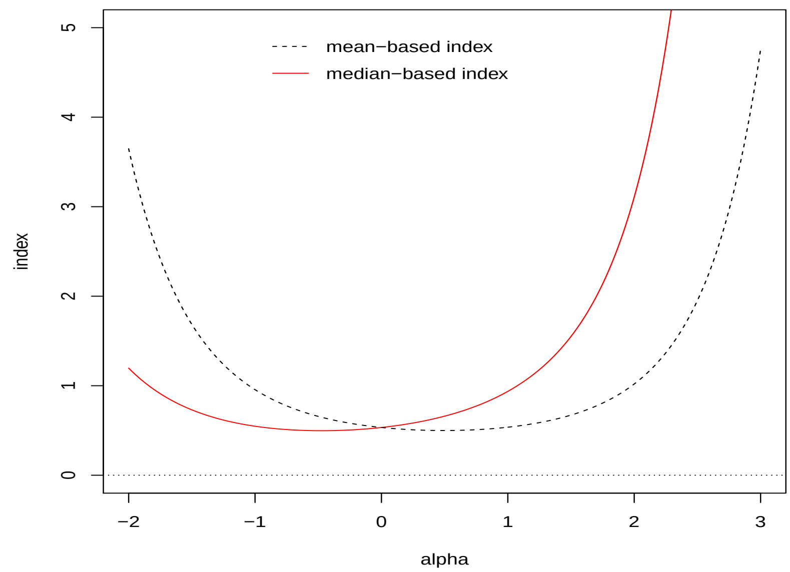

Figure 4: Median- and mean-normalised inequality measures, for $\alpha \in [-2; 5]$.

as defined in (12) and (9), for different values of α, using a sample of 1000 observations drawn from the standard lognormal distribution. In this example, the ratio $\mu/m \approx 1.648$ and it is clear from Figure 4 that median-normalised indices are greater (less) than mean-normalised indices when $\alpha > 0$ ($\alpha < 0$). The two curves intersect at $\alpha = 0$.

Calculating median-normalised inequality measures does not require microdata: (13) suggests that knowing mean-normalised measure with the mean and the median income is enough.

Now consider the Gini family. There is a link between the median-normalised Gini (8) and the median-normalised Generalized Entropy measures in (12). Indeed, the median-normalised measure in (12) with $\alpha = 2$ is equal to

$$I_2(\mathbf{x}/m; \mu) = \frac{\sigma^2}{2m^2} = \frac{E([y - \bar{y}]^2)}{2m^2} = \frac{E([y_1 - y_2]^2)}{4m^2} \tag{22}$$

We can see that $G(\mathbf{x}; m)$ and $[I_2(\mathbf{x}/m; \mu)]^{1/2}$ are two very similar measures, both are ratios of a dispersion measure on twice the median. For the Gini, the dispersion measure is based on Manhattan L_1-distance, while for the

Generalised-Entropy measure it is based on Euclidean L_2-distance. As a consequence, $I_2(\mathbf{x}/m;\mu)$ puts more weight on high incomes, compared with the Gini. It is also true for mean-normalised inequality measures $G(\mathbf{x};\mu)$ and $[I_2(\mathbf{x}/\mu;\mu)]^{1/2}$, since both are ratios of a dispersion measure on twice the mean.

Using L_1-distance, it is clear that the Gini index does not put higher weight on high/low incomes. In the class of inequality measures defined in (12), it is the limiting case $\alpha = 0$ which does not put more weights on high/low incomes (see section 3). Thus, Gini and MLD indices are both quite similar in terms of putting similar weights on every income.

4 Decomposability

Let the population be divided into K mutually exclusive and exhaustive groups and let the proportion of population falling in group k be p_k ; furthermore let m_k and μ_k denote, respectively, the median and the mean in group k and m and μ denote, as before, the corresponding population median and mean. There are two types of decomposability by groups that are of interest: (1) *"non-overlapping"* decomposability where we impose an additional condition that the groups can be unambiguously ordered by income (for example; the richest person in group k has an income that is less than the poorest person in group $k+1$ for all $0 < k < K$); (2) general decomposability, where no additional conditions are imposed.

If the grouping is chosen such that the non-overlapping property is respected then the absolute Gini, the regular Gini and the median-Gini coefficient respectively can be decomposed as follows (Cowell 2016):

$$G(\mathbf{x}) = \sum_{k=1}^{K} p_k^2 G(\mathbf{x}_k) + G^{\mathsf{btw}}(\mathbf{x}) \tag{23}$$

$$G(\mathbf{x}/\mu) = \sum_{k=1}^{K} p_k^2 \frac{\mu_k}{\mu} G(\mathbf{x}_k/\mu_k) + G^{\mathsf{btw}}(\mathbf{x}/\mu) \tag{24}$$

$$G(\mathbf{x}/m) = \sum_{k=1}^{K} p_k^2 \frac{m_k}{m} G(\mathbf{x}_k/m_k) + G^{\mathsf{btw}}(\mathbf{x}/m) \tag{25}$$

where G^{btw} is evaluated by assuming that, in each group k, all incomes are concentrated at the group mean μ_k (for equations 23 and 24) or the group median (for equation 25). In all cases the non-overlapping property of the grouping is important: for other types of grouping the Gini is not decomposable; this applies also to the median-normalised Gini index in (8).

No restriction on grouping is necessary for the class of inequality measures given by (9) or by (12). In the case of scaling by the mean, and scaling by the median, respectively, decomposition can be expressed as:

$$I_\alpha(\mathbf{x}/\mu;\mu) = \sum_{k=1}^{K} p_k \left[\frac{\mu_k}{\mu}\right]^\alpha I_\alpha(\mathbf{x}_k/\mu_k;\mu_k) + \frac{1}{\alpha^2-\alpha}\left(\sum_{k=1}^{K} p_k \left[\frac{\mu_k}{\mu}\right]^\alpha - \left[\frac{\mu}{\mu}\right]^\alpha\right) \tag{26}$$

$$I_\alpha(\mathbf{x}/m;\mu) = \sum_{k=1}^{K} p_k \left[\frac{m_k}{m}\right]^\alpha I_\alpha(\mathbf{x}_k/m_k;\mu_k)+\frac{1}{\alpha^2-\alpha}\left(\sum_{k=1}^{K} p_k \left[\frac{\mu_k}{m}\right]^\alpha - \left[\frac{\mu}{m}\right]^\alpha\right) \tag{27}$$

In particular, the MLD index, the limiting where case $\alpha = 0$, can be decomposed as follows:

$$I_0(\mathbf{x}/m;\mu) = I_0(\mathbf{x}/\mu;\mu) = \sum_{k=1}^{K} p_k\, I_0(\mathbf{x}_k/\mu_k;\mu_k) - \sum_{k=1}^{K} p_k \log\left(\frac{\mu_k}{\mu}\right) \tag{28}$$

Taking a natural special case as an example, this means that we may partition the population into a group of females F and a group of males M and, using an obvious notation, express overall inequality as

$$I_\alpha = w^{\mathsf{F}} I_\alpha^{\mathsf{F}} + w^{\mathsf{M}} I_\alpha^{\mathsf{M}} + I^{\mathsf{btw}} \tag{29}$$

where the weights w^{F}, w^{M} and the between-group inequality component I^{btw} are functions of the income mean (or of the median) for each of the two groups and overall; comparing I_α^{F} and I_α^{M} enables one to say precisely where changes in inequality have taken place.

Shorrocks (1980) argued that the MLD index is the "most satisfactory of the decomposable measures," because it unambiguously splits overall inequality into the contribution due to inequality within subgroups and that due to inequality between subgroups, for arbitrary partitions of the population. This property, called *path independent decomposability* by Foster and Shneyerov (2000), is not shared by (26) and (27), because the weights in the within-subgroup terms are not independent of the between-group term.

Indeed, changing the income subgroup means, μ_k, will also affect the within-subgroup contribution through μ_k/μ in (26), but not in (28). It follows that the inequality that would result from removing differences between subgroups, the inequality within subgroups being unchanged, is given by the first term in (28), not that in (26) or (27).

5 Application

In previous sections, we have seen that the Mean Logarithmic Deviation (MLD) index (21) is a particularly attractive measure, since it is the only one measure that shares properties of both median- and mean-normalised inequality measures and it is decomposable (see sections 2.4 and 4). In this section we examine inequality in Great Britain and in the United States. We compare results obtained with the MLD and with the standard Gini indices, both giving similar weighting schemes to different parts of the distribution (see section 3).

5.1 Inequality in Great Britain

First, we examine inequality in Great Britain, from 1961 to 2015. The values of Gini and MLD indices are given by the Institute for Fiscal Studies (*Tools and resources: “"Incomes in the UK”*).[9] They are based on the Family Expenditure Survey up to and including 1992, and the Family Resources Survey thereafter. We use inequality indices computed on disposable income before housing costs.

Figure 5 (top plot) shows values of the Gini and MLD indices, defined in (21) and (7). Both inequality measures describe quite *similar* patterns. We can see that inequality increased quite a lot during the 1980s and it appears to have fallen slightly during the 1990s, as suggested by Jenkins (2000). However, we can see that inequality is relatively stable since the 1990s: if we estimate linear regressions of Gini and of MLD indices in log against time, over the period 1994-2015, we find slope coefficients not significantly different from zero.[10]

[9] see `https://www.ifs.org.uk/tools_and_resources/incomes_in_uk`

[10] We obtain: $\widehat{\log \text{Gini}}_t = \underset{(1.505)}{-2.879} + \underset{(0.0007511)}{0.0009046}t$ and $\widehat{\log \text{MLD}}_t = \underset{(3.142)}{-7.442} + \underset{(0.001568)}{0.002916}t$.

Figure 5 (bottom plot) shows variations of inequality between two successive years, in percentage, for the Gini and MLD indices. We can see that variations from the MLD index are always greater than variations from the Gini index, with large differences. For instance,

- between 1962 and 1963, inequality increases by 9.15% with the Gini and by 21.44% with the MLD;
- between 1982 and 1983, inequality increases by 2.54% with the Gini and by 7.91% with the MLD;
- between 2009 and 2010, inequality decreases by 5.65% with the Gini and by 10.95% with the MLD.

Such discrepancies are explained by the fact that both the numerator and the denominator of the Gini coefficient will vary in the same direction for any variations of incomes above the mean: any variations of incomes above the mean will be attenuated in terms of the impact on measured inequality. Underlying this behaviour is the fact that the Gini coefficient does not respect the principle of monotonicity in distance. The same is true for any mean-normalised inequality index, such as the Generalised-Entropy measures (9), with the exception of the MLD index. As we have seen, the MLD index is the limiting case $\alpha = 0$ of both mean-normalised indices (9) and median-normalised indices (12), and it respects the principle of monotonicity in distance. By contrast to the Gini, the MLD index does not attenuate variations of income towards the top of the income distribution.

5.2 Inequality in the United States

Second, we examine inequality in the United States, from 1967 to 2016. The values of Gini and MLD indices are given by the U.S. Census Bureau, in the report *Income and Poverty in the United States: 2016* (Table A-3).[11] The sample survey was redesigned in 1994, it is thus not fully comparable over time: for more on data quality issues see McGuiness (1994), Burkhauser et al. (2011).

Figure 6 (top plot) shows values of the Gini and MLD indices. The two inequality measures describe quite *different* patterns. The increase of inequality, since the 1980s is much higher with the MLD index than with the

[11] see `http://www.census.gov/data/tables/2017/demo/income-poverty/p60-259.html`

Gini index. If we estimate linear regressions of Gini and of MLD indices in log against time, over the period 1994-2016, we find slope coefficients significantly different from zero and equal to, respectively, 0.003273 and 0.015427.[12] It means that, inequality increases at an average annual rate of 0.3273% with the Gini index and at an average annual rate of 1.5427% with the MLD index. Thus, over the last 22 years, the rate of growth of inequality with the MLD index is between four and five times that of the Gini index.[13]

Figure 6 (bottom plot) shows variations of inequality between two successive years, in percentage, for the Gini and MLD indices. We can see that variations of the MLD index are mostly greater than variations of the Gini index, with a few opposite results. For instance,

- between 1969 and 1970, inequality increases by 1.13% with the Gini, it increases by 5.65% with the MLD;
- between 1996 and 1997, inequality increases by 0.69% with the Gini, it increases by 5.49% with the MLD;
- between 2013 and 2014, inequality decreases by 0.64% with the Gini, it increases by 2.05% with the MLD.

Such opposite results could be explained by the fact that a shift to the right of incomes greater than the mean can lead to a decrease of mean-normalised inequality measures, such as GE indices and the Gini. It is because these measures do not respect the principle of monotonicity in distance (see sections

[12] We obtain: $\widehat{\log \text{Gini}}_t = \underset{(0.419)}{-7.361} + \underset{(0.000209)}{0.003273}t$ and $\widehat{\log \text{MLD}}_t = \underset{(1.63)}{-31.52} + \underset{(0.000814)}{0.015427}t.$. For the contrast between the regular Gini and the mean-normalised Gini over the period see Gastwirth (2014), page 314.

[13] Clearly growth/change comparisons of inequality depend on the cardinalisation of the inequality indices. In principle any cardinalisation could be used, but it makes sense in practice to confine attention to those that are used in practice. There are no alternative cardinalisations of the Gini coefficient that are used in the literature. However, in the case of the MLD, there is an alternative cardinalisation in current use: the Atkinson inequality index with parameter 1 is given by $A_1(\mathbf{x}) = 1 - \frac{g}{\mu}$, where g is the geometric mean. Using (9) it is clear that $A_1(\mathbf{x}) = 1 - \exp(-I_0)$ and so growth$(A_1) = \lambda$growth(I_0) where $\lambda = I_0 [1 - A_1] / A_1 = I_0 / [\exp(I_0) - 1] = 1 / \left[\frac{1}{2!} I_0 + \frac{1}{3!} I_0^2 + ...\right] < 1$. So the proportional changes in A_1 will be less than those of I_0. However, for our data, this change of growth rate attributable to the change in cardinalisation is relatively modest, as λ ranges from 0.71 to 0.89. In no case is the conclusion that the Gini understates the changes in inequality reversed.

1 and 2.3). By contrast, median-normalised inequality measures respect this principle and so would increase in such a case. The shift of incomes towards top incomes in the United States over the last decades is well documented – see for example Atkinson and Piketty (2010), Krueger (2012) and Piketty (2014).

5.3 United States vs. Great Britain

Finally, we compare inequality in Great Britain and in the United States with Gini and MLD indices. Although the income definitions in the two countries are different, comparisons of inequality changes are instructive.

Figures 7 - top plot - shows values of the Gini index. It is clear that inequality is always much higher in the United States compared to Great Britain – a point that would remain true under alternative definitions of income. However, when we compare trends over the last 22 years, we can see that the increase of inequality is not very different. We have seen that, from 1994 and 2015, the annual rate of growth is not significantly different from zero in Great Britain and is equal to 0.3273% in the United States (see footnotes 10 and 12).

Figures 7 - bottom plot - shows values of the MLD index. It provides a quite different picture: inequality increases are much higher in the United States than in Great Britain over the last 22 years. Indeed, we have seen that, while the annual rate of growth is not significantly different from zero in Great Britain, it is equal to 1.5427% in the United States (see footnotes 10 and 12)

Overall, comparisons based on the Gini index suggest that the increase of inequality over the last 22 years is not very different between Great Britain and the United States, while it is found to be remarkably different based on comparisons of the MLD index.

6 Conclusion

At a first glance the newcomer to the study of income inequality might think that the choice of practical measurement tool is very obvious. The Gini index is probably the most widely used inequality index in the world and is often published by statistical agencies as part of their data reporting. This index has a simple weighting scheme on individual incomes, it respects the principle of transfers, it is independent of income re-scaling and it is also decomposable (but only for groupings that are "non-overlapping" in terms of income). But, as we have discussed, the Gini coefficient does not satisfy the monotonicity in distance property, which can result in some strange behaviour. Measures that are closely related to the Gini also have some drawbacks. Although the absolute Gini satisfies both the monotonicity principle and the transfer principle its value is not independent of income scale – double all the incomes and you double the inequality index. The median-normalised Gini does not satisfy either the principle of monotonicity or the principle of transfers

By contrast the MLD index has all of the attractive properties of the Gini coefficient and more: it also respects the principle of monotonicity in distance and is decomposable for arbitrary partitions with the path-independence property.

As we have seen, the lack of the principle of monotonicity in distance may have strong implications in empirical studies. Indeed, the Gini index may understate variations of inequality. Our application suggests that the increase of inequality in the United States over the last 22 years is significantly understated by the Gini index. By contrast, the MLD index has more desirable properties, estimates variations of inequality more accurately and should be preferred in practice.

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Technical Appendix

Consider an inequality ordering as a complete and transitive relation $\succeq$ on $\mathbb{R}_+^{n+1}$; denote by $\succ$ the strict relation associated with $\succeq$ and denote by $\sim$ the equivalence relation associated with $\succeq$. We define $e \in \mathbb{R}_+$ as an equality-reference point which could be exogenously given or could depend on the income vector $\mathbf{x}$. For any vector $\mathbf{x}$ denote by $\mathbf{x}(\xi, i)$ the vector formed by replacing the ith component of $\mathbf{x}$ by $\xi \in \mathbb{R}_+$. We can characterise the structure of the inequality relation using just four axioms:

Axiom 1 [Monotonicity in distance] *Suppose* $\mathbf{x}, \mathbf{x}' \in \mathbb{R}_+^n$ *differ only in their* i^{th} *component. Then (a) if* $x_i' \geq e$*:* $x_i > x_i' \Longleftrightarrow (\mathbf{x}, e) \succ (\mathbf{x}', e)$*; (b) if* $x_i' \leq e$*:* $x_i < x_i' \Longleftrightarrow (\mathbf{x}, e) \succ (\mathbf{x}', e)$*.*

Axiom 2 [Independence] *For any* i *such that* $1 < i < n$*, if* $\mathbf{x}(\xi, i), \mathbf{x}'(\xi, i) \in \mathbb{R}_+^n$ *satisfy* $(\mathbf{x}(\xi, i), e) \sim (\mathbf{x}'(\xi, i), e)$ *for some* ξ *then* $(\mathbf{x}(\xi, i), e) \sim (\mathbf{x}'(\xi, i), e)$ *for all* $\xi \in [x_{i-1}, x_{i+1}] \cap \left[x_{i-1}', x_{i+1}'\right]$*.*

Axiom 3 [Scale invariance] *For all* $\lambda \in \mathbb{R}_+$*: if* $\mathbf{x}, \mathbf{x}', \lambda\mathbf{x}, \lambda\mathbf{x}' \in \mathbb{R}_+^n$ *and* $e, e', \lambda e, \lambda e' \in \mathbb{R}_+$ *then*
(a) $(\mathbf{x}, e) \sim (\mathbf{x}', e') \Rightarrow (\lambda\mathbf{x}, e) \sim (\lambda\mathbf{x}', e')$*;*
(b) $(\mathbf{x}, e) \sim (\mathbf{x}', e') \Rightarrow (\lambda\mathbf{x}, \lambda e) \sim (\lambda\mathbf{x}', \lambda e')$*.*

Axiom 4 [Continuity] $\succeq$ *is continuous on* $\mathbb{R}_+^{n+1}$*.*

The key result can be stated as follows.

Theorem 1 *Given Axioms 1 to 4* $\succeq$ *is representable either as either the Absolute Gini (5) or as a member of the class (6), where* $\alpha \in \mathbb{R}$*,* $\alpha \neq 1$*, or by some strictly increasing transformation of (5) or (6) involving* e*.*

Proof. From the proof of Theorem 1 in Cowell and Flachaire (2017), Axioms 1, 2 and 4 jointly imply that, for a given e, $\succeq$ is representable by a continuous function $\mathbb{R}_+^{n+1} \to \mathbb{R}$:

$$\sum_{i=1}^{n} d_i(x_i, e), \forall (\mathbf{x}, e) \in \mathbb{R}^{n+1}. \tag{30}$$

The form in (30) is the same as that in Cowell and Flachaire (2018) equation (11) with ϕ replaced by d, u_i replaced by x_i and v_i replaced by e. With this translation of notation Axiom 5 in Cowell and Flachaire (2018) becomes Axiom 3 here. From equation (87) of Cowell and Flachaire (2018) we may conclude that the functions d_i must take the form

$$d_i\left(x_i, e\right) = a_i x_i^{\alpha} + e^{\alpha-1} a_i' x_i + a_i'' e^{\alpha}.$$

It remains to introduce the principle of anonymity. This can be done in two different ways in two basic cases.

Case 1. $\alpha = 1$, $a_i'' = 0$. The index can then be written as

$$\sum_{i=1}^{n} b_i x_{(i)}, \tag{31}$$

where $b_i := a_i + a_i'$ and $x_{(i)}$ denotes the ith component of $\mathbf{x}$ if it is arranged in ascending order (the ith order statistic). Anonymity will be satisfied if the b_i depend on only the rank of i up to a suitable normalisation. Taking the following normalisation

$$b_i = \frac{4i-2}{n^2} - \frac{2}{n} \tag{32}$$

and substituting this in (31) and rearranging gives (5).

Case 2. $a_i' = 0$, $a_i'' = -a_i$. Anonymity requires a_i be independent of i. Up to a suitable normalisation this gives the class (6).

■

Remark 1 *We have $b_i > 0$ if $i > \frac{n+1}{2}$.*

Remark 2 *(a) An inequality index of the form (5) (or a transformation involving e) permits inequality decomposition only by subgroups that can be strictly ordered by income; an inequality index of the form (6) (or a transformation involving e) permits inequality decomposition by arbitrary subgroups. (b) The current version of Axiom 2 restricts income changes to those that preserve the ranking of the income vector. If we were to replace this with a modified form of independence that allows the change in income i to alter the ranking, then the class of inequality measures consists of (6) alone. Such a modified axiom can be stated as: "For any i, if $\mathbf{x}(\xi, i), \mathbf{x}'(\xi, i) \in \mathbb{R}_+^n$ satisfy $(\mathbf{x}(\xi, i), e) \sim (\mathbf{x}'(\xi, i), e)$ for some ξ then $(\mathbf{x}(\xi, i), e) \sim (\mathbf{x}'(\xi, i), e)$ for all $\xi \in \mathbb{R}_+$." See Cowell and Flachaire (2017) for a proof.*

	Great Britain				United States			
Year	Gini	MLD	ΔGini%	ΔMLD%	Gini	MLD	ΔGini%	Δmld%
1961	0.261	0.116	-	-	-	-	-	-
1962	0.248	0.109	-4.870	-5.974	-	-	-	-
1963	0.271	0.132	9.153	21.439	-	-	-	-
1964	0.264	0.126	-2.581	-4.899	-	-	-	-
1965	0.251	0.108	-4.879	-13.694	-	-	-	-
1966	0.261	0.119	3.737	10.062	-	-	-	-
1967	0.251	0.107	-3.719	-9.943	0.362	0.303	-	-
1968	0.250	0.106	-0.571	-1.081	0.351	0.285	-3.039	-5.941
1969	0.257	0.113	2.853	6.059	0.353	0.283	0.570	-0.702
1970	0.259	0.113	0.982	0.170	0.357	0.299	1.133	5.654
1971	0.266	0.120	2.768	5.977	0.359	0.300	0.560	0.334
1972	0.269	0.128	1.119	7.147	0.362	0.302	0.836	0.667
1973	0.259	0.112	-3.977	-12.335	0.360	0.298	-0.552	-1.325
1974	0.251	0.106	-2.886	-5.312	0.354	0.295	-1.667	-1.007
1975	0.243	0.100	-3.248	-6.224	0.359	0.306	1.412	3.729
1976	0.243	0.100	-0.147	0.051	0.359	0.311	0.000	1.634
1977	0.240	0.094	-1.271	-5.718	0.362	0.315	0.836	1.286
1978	0.240	0.095	0.017	0.474	0.363	0.315	0.276	0.000
1979	0.253	0.106	5.648	12.225	0.366	0.322	0.826	2.222
1980	0.257	0.112	1.739	5.455	0.367	0.330	0.273	2.484
1981	0.263	0.118	2.237	5.591	0.373	0.352	1.635	6.667
1982	0.261	0.111	-0.786	-5.734	0.384	0.370	2.949	5.114
1983	0.268	0.120	2.546	7.909	0.389	0.373	1.302	0.811
1984	0.270	0.121	0.796	0.255	0.389	0.366	0.000	-1.877
1985	0.282	0.132	4.471	9.736	0.394	0.369	1.285	0.820
1986	0.291	0.142	3.165	7.128	0.397	0.375	0.761	1.626
1987	0.305	0.155	4.827	9.476	0.399	0.381	0.504	1.600
1988	0.323	0.176	5.757	13.375	0.402	0.380	0.752	-0.262
1989	0.327	0.180	1.318	2.599	0.408	0.393	1.493	3.421
1990	0.339	0.197	3.761	9.137	0.406	0.388	-0.490	-1.272
1991	0.341	0.201	0.502	1.878	0.406	0.402	0.000	3.608
1992	0.340	0.196	-0.227	-2.097	0.413	0.419	1.724	4.229
1993	0.340	0.194	-0.117	-1.332	0.436	0.472	5.569	12.649
1994	0.333	0.186	-2.062	-3.836	0.436	0.474	0.000	0.424
1995	0.333	0.187	0.163	0.235	0.433	0.463	-0.688	-2.321
1996	0.333	0.188	0.048	0.440	0.437	0.474	0.924	2.376
1997	0.341	0.197	2.162	5.195	0.440	0.500	0.686	5.485
1998	0.348	0.206	2.302	4.505	0.439	0.506	-0.227	1.200
1999	0.346	0.203	-0.644	-1.351	0.441	0.492	0.456	-2.767
2000	0.353	0.212	1.962	4.186	0.442	0.501	0.227	1.829
2001	0.349	0.209	-1.226	-1.470	0.446	0.527	0.905	5.190
2002	0.344	0.203	-1.339	-2.868	0.443	0.523	-0.673	-0.759
2003	0.340	0.197	-1.216	-2.724	0.445	0.548	0.451	4.780
2004	0.341	0.197	0.313	0.006	0.447	0.559	0.449	2.007
2005	0.345	0.205	1.337	3.735	0.450	0.571	0.671	2.147
2006	0.352	0.214	1.934	4.540	0.452	0.557	0.444	-2.452
2007	0.358	0.219	1.775	2.193	0.444	0.548	-1.770	-1.616
2008	0.357	0.218	-0.503	-0.112	0.450	0.568	1.351	3.650
2009	0.358	0.221	0.331	1.126	0.456	0.605	1.333	6.514
2010	0.337	0.197	-5.653	-10.952	0.456	0.617	0.000	1.983
2011	0.340	0.197	0.841	0.106	0.463	0.626	1.535	1.459
2012	0.337	0.196	-1.080	-0.570	0.463	0.629	0.000	0.479
2013	0.343	0.203	1.856	3.646	0.467	0.635	0.864	0.954
2014	0.340	0.199	-0.882	-1.963	0.464	0.648	-0.642	2.047
2015	0.347	0.209	2.185	4.851	0.462	0.623	-0.431	-3.858
2016	-	-	-	-	0.464	0.629	0.433	0.963

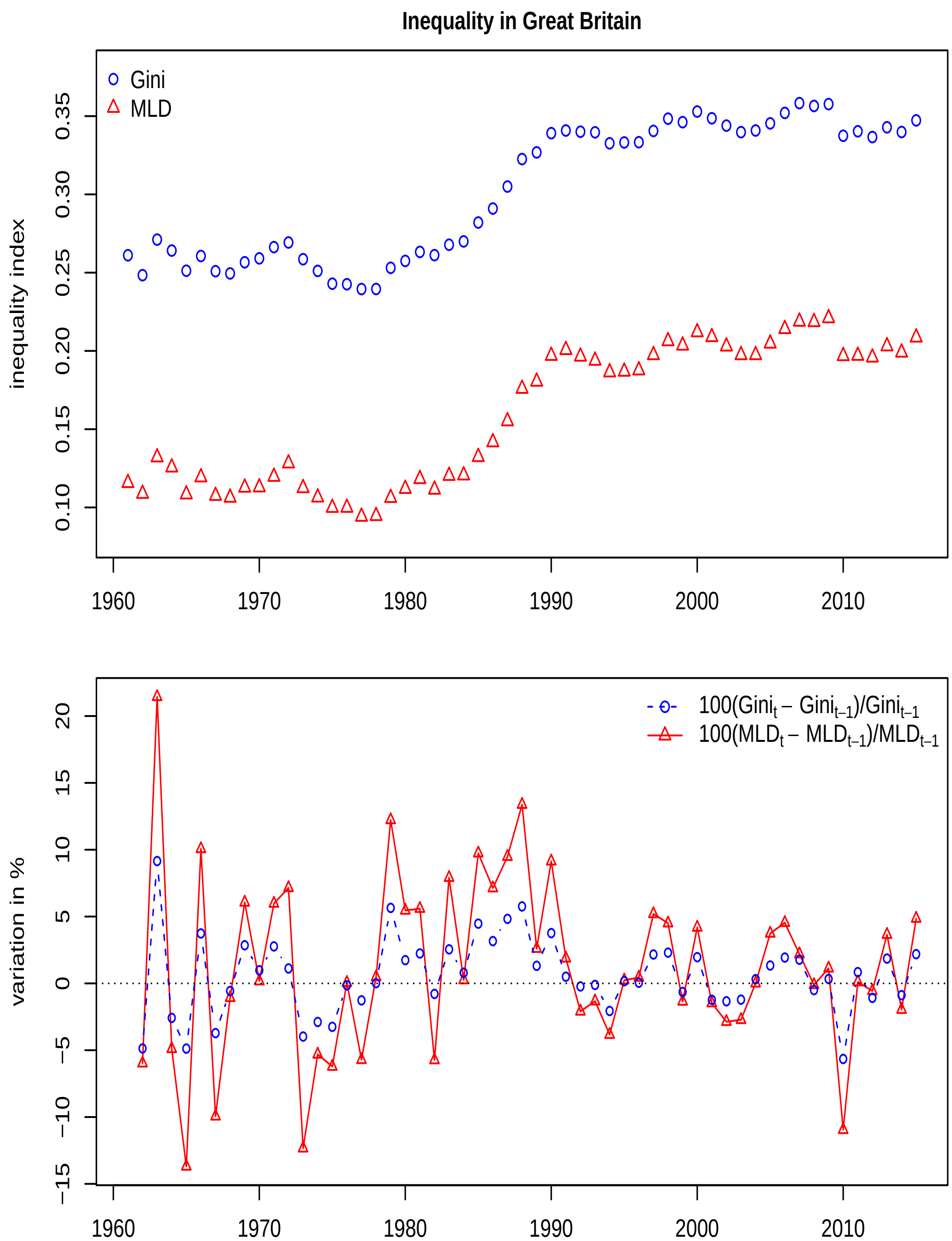
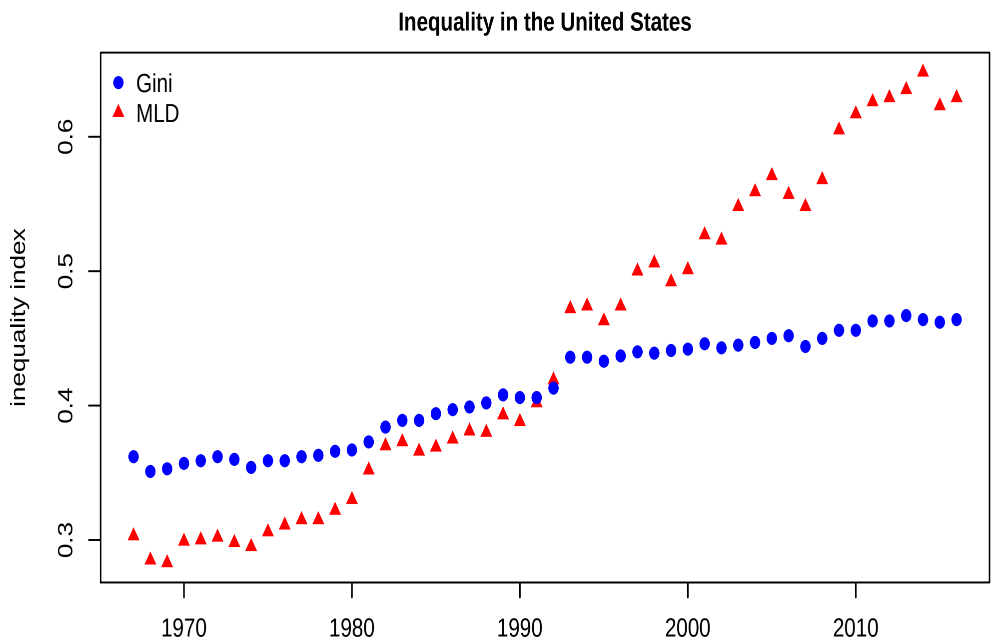


Figure 5: Inequality in Great Britain from 1961 to 2014.



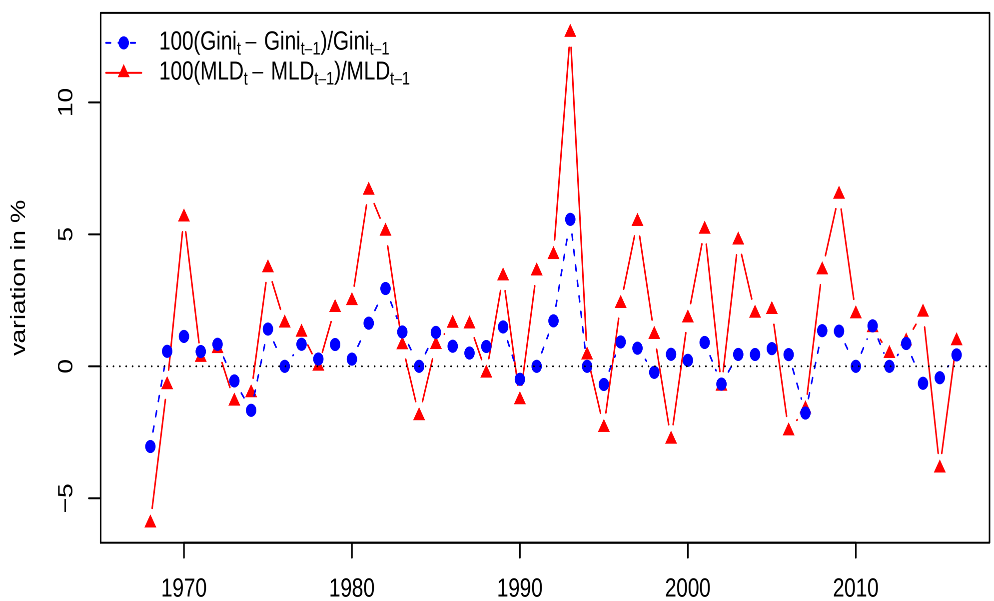


Figure 6: Inequality in the United States from 1967 to 2015.

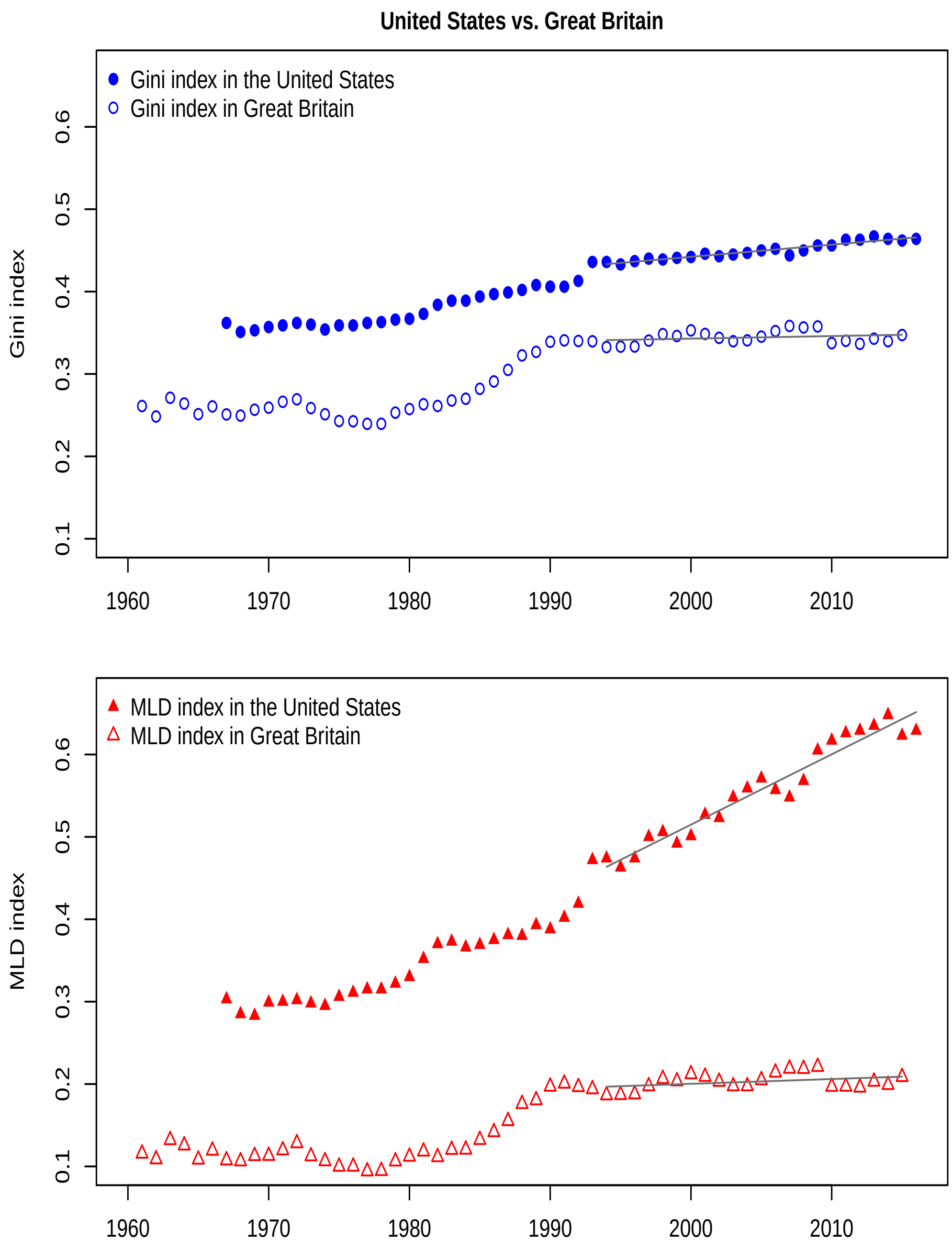


Figure 7: Inequality in the United States and in Great Britain.