A Theory of Non-Democratic Redistribution and Public Good Provision*

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Abstract

This paper proposes a new theoretical definition of (non-)democracy based on two "political rights" parameters (η, k) that capture the extensive and intensive margin of the population's ability to replace the incumbent; and an "individual rights" parameter λ that captures the degree to which individual citizens are protected from political retribution. Within the rules of the game specified by (η, k, λ), two office-motivated politicians compete for power by making promises to citizens. The policy space features a trade-off between redistribution and public good provision. I study two types of public good: one that delivers egalitarian benefits, the other that delivers non-egalitarian benefits.

I find that when political rights are stronger, and/or individual rights are weaker, competition drives politicians to treat citizens more equally, and to provide the egalitarian public good more efficiently. Regimes where political and individual rights are perfectly protected give politicians incentives to treat citizens inequitably for political advantage; these regimes provide the non-egalitarian public goods efficiently.

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1 Introduction

A lot of the world's GDP is produced by non-democracies (see Figure 1). Yet, little is known theoretically about the political incentives that shape economic policy in non-democracies, and how these might be different in democracies. I address these two questions here.

To address these questions, I propose an operational definition of non-democracy that nests democracy as a special case. I define a *regime type* based on three parameters: the fraction $\eta \in [0,1]$ of citizens who are *voiceful*, i.e. who have the power, through their collective action, to oust the incumbent politician; the cost k of taking political action against the incumbent; and a parameter $\lambda \in [0,1]$ which captures the extent to which individuals are protected from political retribution. When $(\eta, k, \lambda) = (\approx 0, \infty, 0)$ only a vanishing fraction of the citizens have the power to oust the incumbent, supporting a challenger is infinitely costly, and a citizen who mistakenly "backed the wrong horse" has all his/her welfare (job, home, etc) taken away. A pluralist democracy is a regime $(\eta, k, \lambda) = (1, 0, 1)$ where all the citizens are able to vote, the cost of supporting the challenger instead of the incumbent is zero, and a citizen suffers no material consequences for having supported the losing candidate. The triple (η, k, λ) captures the main criteria used by rankings such as "Polity IV" or *The Economist's* "Democracy Index," to classify political regimes.[1]

Given any triple (η, k, λ), I posit that two office-motivated politicians compete for power by making promises to citizens (or groups thereof). After observing both politicians' promises, every voiceful citizen simultaneously decides whether to support the incumbent or, at a cost k, support the challenger. If enough citizens support the challenger, the incumbent is replaced.

The model's description is completed by specifying the policy space – what politicians can promise. Politicians can promise to redistribute tax revenue across individuals in a targeted fashion. Alternatively, politicians can promise to use the tax revenue to pay for a public good, which is a policy that yields higher social welfare than redistribution, but cannot be targeted either to the voiceful citizens or among them.

The main question I address is the following: given the choice between socially inefficient redistribution and a socially efficient public good, what incentives do politicians have to promise the efficient policy? Obviously, the answer will depend on the rules of the competitive game, i.e., the value of (η, k, λ). The key observation is that, when λ is

[1] The parameters η and k capture the citizens' ability to vote on alternative leaders, whereas λ captures a combination of independent judiciary and civil liberties.

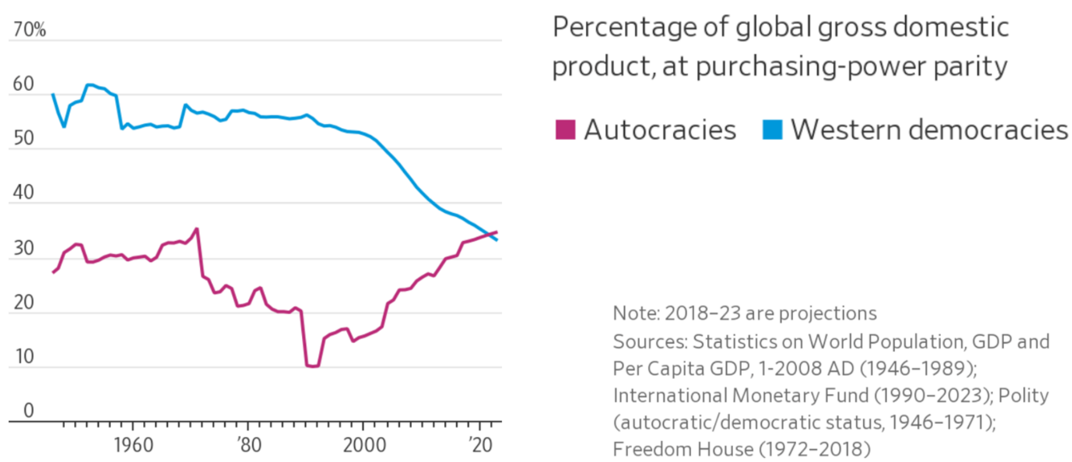

Figure 1: The Growing Economic Power of Autocracies. Reproduced from Foa and Mounk (2019).

small, politicians "compete for coordination." To explain what this means I describe the mechanics of the model next.

Suppose the incumbent promises welfare level α_i to voiceful citizen i, either through redistribution or through public good provision. If the incumbent survives, by assumption citizen i receives α_i *provided she supported the incumbent*; else, she receives welfare level $\lambda\alpha_i$. Therefore, a promise of α_i generates the following incentive to support the incumbent: "if you support me and I hang on to power, you will enjoy the full α_i; but if you supported my opponent you will only enjoy $\lambda\alpha_i$." Whenever $\lambda < 1$, the incumbent's promise to citizen i is effectively conditioned on that citizen's support; similarly, the challenger's promises are also assumed to be conditional. This conditionality creates a coordination game among citizens, as follows: because citizens are assumed to be atomistic, each citizen's political action has a *negligible* impact on the incumbent's survival; however, if $\lambda < 1$, the citizen has *nonnegligible* individual incentives to "back the right horse." Therefore, individual behavior is driven by a fear of supporting the losing politician. Which one of the politicians wins, in turn, depends on how many citizens support either politician. Therefore, as soon as citizens receive promises from both politicians, they are locked in a coordination game. The study of two politicians who "compete for coordination" using redistributive politics is new in the political economy literature, to my knowledge.

I will resolve the equilibrium multiplicity in this coordination game using a "global game" approach, and then ask: what kind of promises would incumbent and challenger make in equilibrium to try and "win the coordination game"? In particular, is there a tendency for promises to be the same for all i?

I find that, when $\lambda < 1$, politicians have incentives to treat all the voiceful citizens equally. This means the following. Suppose a politician promises every voiceful citizen \$1, meaning "\$1 if you support me, and \$$\lambda$ if you don't;" then every citizen has the same incentive $(1 - \lambda)$ to support the politician. If, instead, the politician promises \$2 to half the citizens and zero to the other half, then half the citizens will have a strong incentive $2(1 - \lambda)$ to support the politician, and the other will have zero incentive. It turns out that, in the global game, this kind of unequal incentivization tends not to be helpful: the incentives for the voiceful citizens to coordinate on supporting a politician are maximized if the politician treats these citizens equally. This finding, which is based on the properties of global games and is contingent on k being small enough, is not obvious and is new in the political economy literature. It implies that in any regime with $\lambda < 1$ and k small enough, competition pushes both politicians toward the egalitarian treatment of all voiceful citizens.

The notion that competition for coordination leads to equal treatment of voiceful citizens is not absolute: first, it is limited to voiceful citizens: voiceless citizens will be neglected by politicians, and this is a force against equal treatment. But even voiceful citizens will be treated unequally by the challenger when k is large. This is because incentivizing a citizen to support the challenger requires a promise of at least k (otherwise it is a dominant strategy to support the incumbent), and when k is large the challenger cannot afford such large promises to all voiceful citizens. Instead, the challenger is better off concentrating his relatively limited resources on a few citizens, which leads to inequitable treatment among the voiceful citizens.

In sum, in any regime with $\lambda < 1$ there is a politico-economic force driving toward equality for voiceful citizens, but that force is tempered if k is large. Regimes with $(\eta, k, \lambda) = (1, 0, < 1)$ are the most conducive to equal treatment among all citizens; I call this class of regimes "consensual democracies." The limit point of this class is $(1, 0, 1)$, a regime with special significance because it is the game that is studied in most voting theory; I call it "pluralist democracy."[2] I show that pluralist democracy behaves differently from any system with $\lambda < 1$, because when $\lambda = 1$ voiceful citizens do not face a coordination game, but rather a dominant strategy game. As a result, the objective function of politicians is qualitatively different, and it gives politicians electoral incentives

[2] The terms "consensual" and "pluralist" democracy are terms of art in the political science literature. In a separate paper, I will comment on the connection between my mathematical models and the way these words are used by political scientists.

to treat citizens inequitably. This qualitative "phase change" as λ varies is one of the main conceptual insights from this paper.

Having understood that consensual democracy has a built-in tendency toward egalitarianism, and pluralist democracy has a tendency toward non-egalitarian treatment, it becomes intuitive that consensual democracy is the best regime for efficient provision of egalitarian public goods, and pluralist democracy is the best regime for efficient provision of non-egalitarian public goods. In addition, the analysis will show that in illiberal and non-democratic regimes (low λ, high k) the incumbent can be capable of providing egalitarian public goods (basic health care, education) provided that broad sections of the population have political voice (large η). These are the main applied result in this paper.

In Section 8 I look across countries and ask whether the public good is provided in accordance to the model's prediction, or not. This section is not intended to cover all regimes; rather, it is meant to illustrate how the theory maps into real world applications.

1.1 Related literature

I limit this literature review to theoretical papers that study policy determination in non-democracies. The most related papers are those where the policy space features a trade-off between redistribution and public good provision; the least related are those where the policy space does not feature this trade-off.

McGuire and Olson (1996) model a redistributive democracy that underprovides an egalitarian public good, and compare it to an autocracy that optimally provides it. Their policy space, like mine, features a trade-off between redistribution and public good provision. The conceptual difference lies in the definition of autocracy: McGuire and Olson (1996) assume that the autocrat is a consumption-motivated "stationary bandit" who faces no competition for power. This autocrat owns all the tax receipts, pays for the public good out of tax receipts, and maximizes the leftover, i.e., tax receipts net of public good expenditures. In their setting the tax rate is set independently of public good provision, which means that the autocrat's payoff equals a fixed fraction of social welfare, in turn leading to efficient public good provision. In my theory, in contrast, the incumbent autocrat acts under pressure of replacement by a challenger. The incumbent (respectively, challenger) promises the public good if and only if it reduces (resp., increases) the probability of replacement. The policy that is enacted depends on which of the two politicians prevails, and it need not be efficient. In sum, the two papers are very different, but they share a focus on the tradeoff between targetability of redistribution and efficiency of the public good, and the notion that democracy provides incentives for inefficient targeted redistribution.[3]

Bueno de Mesquita et al. (2002) also study a trade-off between redistribution and public good provision. They classify regimes according to the size of the selectorate S – the set of people who have an institutional say in choosing leaders – and the size of the winning coalition W – the minimal number of selectors whose support the incumbent needs to remain in power. They find that, the larger the ratio W/S, the greater is public good provision. The logic is that only citizens in W need to be bribed in order for the incumbent to survive, and the public good "wastes bribes" unnecessarily on selectors outside of W. The same logic drives the result in this paper that public good provision is increasing in η (refer to Proposition 2 part 2), but the connection is not perfect because in this paper membership in $[0, \eta]$ is exogenous, whereas in Bueno de Mesquita et al. (2002) the membership of W is determined by the autocrat.

Besley and Kudamatsu's (2007) policy space also features redistribution and public good provision. However, crucially, there is no direct trade-off between redistribution and public good because the latter doesn't cost money to provide. Rather, it is a choice between different policy options that is driven by the politicians' personal preference and that, in equilibrium, signals the politician's type to voters. There are many other differences between my model and Besley and Kudamatsu's (2007), including the fact that they only have two groups of voters, whereas I have a continuum. But there are also some common elements including the fact that the incumbent risks being removed from office, and that democracy obtains for certain parameter values of the model. Roemer (1985) studies a sequential game between two office-motivated politicians, an incumbent and a challenger. The focus is on whether the challenger's promises are more egalitarian than the incumbent's. There are many differences with the present paper, including that the incumbent's promises are fixed exogenously. Perhaps the biggest difference is that the policy space is purely redistributive – in my language, there is no public good.

Jia et al. (2021) is, in some respects, the most related paper to this one even though its policy space does not contain a public good. Jia et al. (2021) provide a model where an autocrat seeks to maintain power by promising resources to two groups. As in this paper, the autocrat has the power to "claw back" some of its promises from the citizens that failed to support him. And, as in this paper, they find that the autocrat has an incentive to treat both groups equally. Their paper does not address political competition (the challenger's strategy is fixed exogenously), and their model does not nest democracy

[3] The reasons why democracy gives rise to inefficiency are somewhat different in the two papers, however. I will return to this issue in Section 6.

as a special case. Nevertheless, their paper and this one both study resource allocation under a fragile autocracy in which the political class has "claw-back power." The two papers were developed independently, they focus on different historical phenomena, and they complement each other.

The following papers are interesting models of non-democracy, but their policy space is not "redistribution vs public good," or in some cases even "redistribution." Francois et al. (2015) offer a model of redistribution under the dual threats of revolution and coup. This model is similar in spirit to mine because citizens receive benefits in proportion to their ability to "make trouble." This model does not nest democracy, nor does it consider public goods. Padro i Miquel (2007) highlights that part of the citizens' cost of overthrowing an incumbent may be exclusion from future benefits. Myerson (2008) highlights the commitment problem that an autocrat faces in promising benefits; I have simply assumed away this commitment problem. Guriev and Treisman (2020) develop a theory where the incumbent autocrat survives if the media say good things about her, and so an autocrat will invest resources in state-controlled media. Bidner et al. (2015) focus on "minimal democracies" where incumbents step down after they lose elections, and they ask why incumbents do so even if they have the power to resist the transition. Acemoglu et al. (2008, 2010, 2012, 2015) study relatively unstructured environments where institutions are minimal, and derive the features of "stable" regime types. In contrast, in this paper the regime structure (η, k, λ) is taken as an exogenous parameter in order to focus on policy determination.

Finally, from a purely technical perspective, when $k = 0$ the politicians' payoff functions (the right hand side in eq. 5) are homeomorphic to the payoff function in the "lottery Colonel Blotto" game studied by Friedman (1958), Snyder (1989), and Kovenock and Rojo Arjona (2019). None of these paper derive this functional form from a global game, as I do here; rather, they assume it. In this sense, the present paper may be viewed as a "micro-foundation" of the reduced-form models in this literature. This literature is not concerned with public good provision.

2 Model

Society is a mass one of identical citizens indexed by $i \in [0, 1]$. Two office-motivated politicians, an incumbent (she) and a challenger (he), simultaneously make promises to citizens. Based on these promises, citizens simultaneously choose either $a_i = 0$ ("support incumbent") or $a_i = 1$ ("support challenger"). If enough citizens support the challenger, the incumbent is replaced by the challenger. The game is described next, with the policy space being limited to redistribution only – public goods will be added in later sections. The parameters (η, k, λ) are fixed exogenously.

Stage 1: incumbent and challenger make promises. The incumbent promises $\alpha_i \geq 0$ to citizen i if the citizen supports her, and $\lambda\alpha_i$ otherwise. This promise is only kept if the incumbent retains power. The incumbent's promises must satisfy the budget constraint:

$$\int_0^1 \alpha_i \; di = B_1 > 0. \tag{1}$$

The incumbent maximizes the probability of retaining power.

Simultaneously, the challenger promises $\omega_i \geq 0$ to individual i if the citizen supports him, and $\lambda\omega_i$ otherwise. This promise is kept only if the incumbent is ousted. The challenger's promises must satisfy the budget constraint:

$$\int_0^1 \omega_i \; di = B_2 > 0. \tag{2}$$

The challenger maximizes the probability of replacing the incumbent. It is natural to assume that $B_1 = B_2$, meaning that no politician enjoys an advantage, but that is not necessary for the analysis.

Constraints (1) and (2) mean that the policy space is purely redistributive. Later, this policy space will be expanded to include a public good.

Stage 2: citizens take collective action Every citizen i contemplates the vectors of promises $\boldsymbol{\alpha} = \{\alpha_i\}$ and $\boldsymbol{\omega} = \{\omega_i\}$, and then all citizens simultaneously choose $a_i \in \{0, 1\}$. The cost of supporting the challenger is $k \geq 0$. Citizen i's payoff is as follows:

	Incumbent replaced	**Incumbent survives**
$a_i = 1$ (support challenger)	$\omega_i - k$	$\lambda\alpha_i - k$
$a_i = 0$ (support incumbent)	$\lambda\omega_i$	α_i

(3)

Stage 3: outcome θ is drawn from a Uniform random variable with support $[\underline{\theta}, \overline{\theta}]$. The incumbent is replaced if:

$$a = \int_0^\eta a_i \, di \geq 1 - \theta. \tag{4}$$

The number a measures the political support for the challenger. Citizens with $i \in [0, \eta]$ are said to have political voice: if more than $1 - \theta$ among them choose $a_i = 1$, the incumbent is replaced. Citizens with $i \notin [0, \eta]$ are politically voiceless: their actions do not affect regime survival. The variable θ represents the incumbent's vulnerability (which is increasing in θ). Following Sakovics and Steiner (2012), the support $[\underline{\theta}, \overline{\theta}]$ is a strict superset of $[0, 1]$.

Citizens' information. Citizen i is endowed with a private signal $z_i = \theta + \sigma\varepsilon_i$, where ε_i is i.i.d. independent of θ and has support $[-1/2, 1/2]$, and $\sigma \in (0, 1]$ is a scaling factor that determines the precision of i's signal. Henceforth, I will focus on the limit as $\sigma \to 0$, that is, on the case of very precise signals.

2.1 Discussion of modeling assumptions

Modeling citizens as a continuum allows me to use the law of large numbers, as in Myerson (1993). The index i could refer to a citizen or to an identifiable group of citizens. For example, i could represent "factory workers" or "workers in a given factory" or "the manager in a given factory." For expositional brevity I will henceforth refer to i as a citizen.

The action a_i represents citizen i's contribution to keeping the incumbent in office, or to removing her. The nature of the action will vary depending on the regime type. In authoritarian regimes, $a_i = 1$ represents taking a stand against the regime, including by protesting. In democracy, a_i can be interpreted as voting: but if voting is secret, rewards and punishment cannot be conditioned on a_i, so the voting interpretation requires $\lambda = 1$.[4] A democracy with $\lambda < 1$ is also possible, and in this case a_i represents *observable* forms of political support, including public speech, get-out-the-vote efforts, and financial contributions.

The set of voiceful citizens $[0, \eta]$ represents the fraction of citizens who, collectively, can

[4] I will study the case $\lambda = 1$ in Section 6.

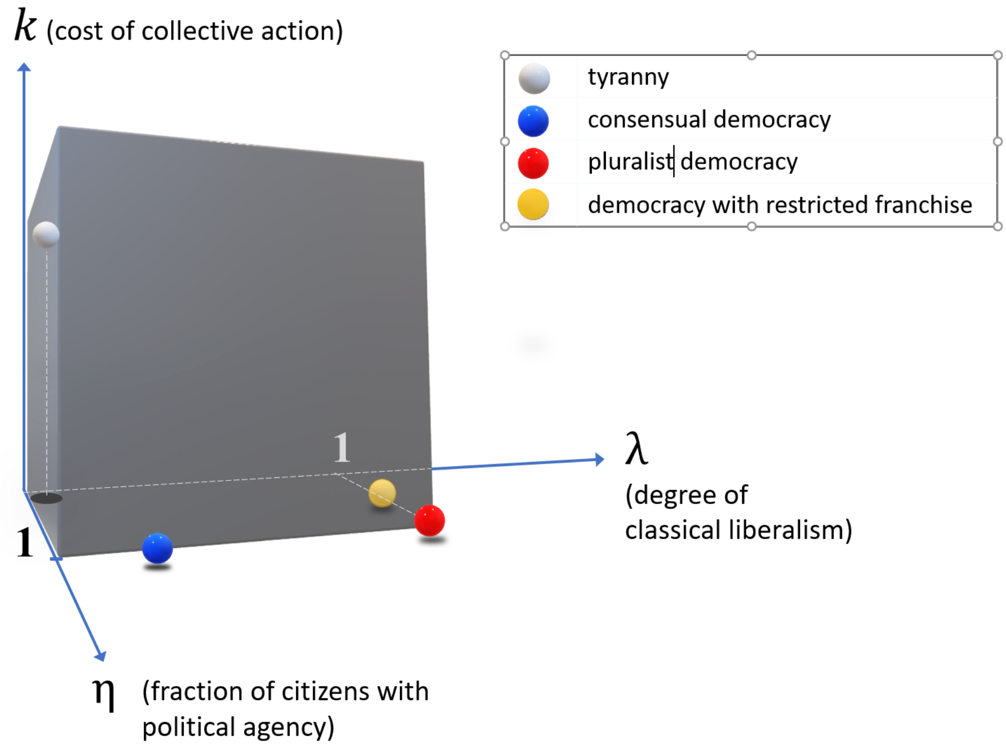

Figure 2: Regime space. Democracies lie at the base of the regime space.

unseat the incumbent. This set becomes empirically observable when the incumbent falls.[5] When incumbents are toppled by bloodless military coups, the set $[0, \eta]$ represents the generals and perhaps the economic elites. When incumbents are toppled by revolutions, the set $[0, \eta]$ represents broad strata of society. In democracy, η represents the voters (which, historically, may have excluded the poor, women, and any slaves).

The parameter k represents the cost of supporting the opposition against the incumbent. In non-democratic regimes k may be large: loss of job, beatings, imprisonment, or worse. In a democracy, it is natural to assume that $k = 0$ because the cost of political activity is the same whether it is exerted in favor or against the incumbent. When $k > 0$ the incumbent has an advantage, so the model is asymmetric. If k is large, the challenger will choose to make inequitable promises to citizens (see Corollary 1 part 2).

The parameter λ represents the fraction of a citizen's economic status that she is

[5]For example, in the 1917 Russian revolution these citizens were: the intelligentsia, the soldiers, and the factory workers – but not the farmers, arguably. (According to my model, this same set of citizens may well have passed up the opportunity to coordinate on replacing the incumbent many times before, due to low previous realizations of θ.) This modeling feature is historically accurate: indications that these citizens could coordinate and overthrow the incumbent had existed long before 1917.

allowed to retain after having supported the politician that lost the struggle for power. The effect of λ is different from k in that: λ is not paid by citizens who support the challenger, if the revolt is successful; and λ is paid by citizens who support the incumbent, if the revolt is successful.[6] When λ is close to zero, citizens are highly vulnerable to political retribution: such a system might be called illiberal because one's welfare is conditioned on one's political behavior. Conversely, when λ is close to 1 one's political activity has almost no effect on one's economic status. We can expect λ to be large in most democracies, but not necessarily to equal 1. I call a democracy "pluralist" when $\lambda = 1$, and "consensual" when $\lambda < 1$.

The triple (η, k, λ) represents, in effect, a set of "rules of the game" under which incumbent and challenger compete, and that are taken as given. I call this set of rules a regime type. Figure 2 depicts the regime type space.

Definition 1 *The triple* (η, k, λ) *is a regime type.*

For any constellation of promises $(\boldsymbol{\alpha}, \boldsymbol{\omega})$, the payoff matrix (3) reveals that every citizen who is promised little by the challenger (specifically, $\omega_i \leq k/(1-\lambda)$) has a dominant strategy to support the incumbent; the rest of the citizens are engaged in a *coordination game*. The two politicians set $\boldsymbol{\alpha}$ and $\boldsymbol{\omega}$ competitively to "win the coordination game."

In equilibrium, citizens who don't have a dominant strategy will coordinate their actions perfectly for almost all values of θ.[7] As a consequence, there are no failed coups in equilibrium. This equilibrium feature captures the idea that, in nondemocracies, the kind of competition that shapes resource allocation may well be subterranean. What I have in mind is a setting where the voiceful citizens (could be the army, or the politburo members, or ethnic tribes) waver between supporting the incumbent or a challenger. The incumbent is perennially at risk but, most of the times, no coup is launched. Thus the risk to the incumbent is not manifest until coordination against the incumbent actually takes place (in which case it is always successful, in the model). In the model, incumbent and challenger seek to decrease or increase this latent risk by making promises strategically.

The realization θ represents the incumbent's capacity to reduce or prevent coordination. For example, a capable chief of police or interior minister may be represented by a low θ. The exact location of the interval-support $\left[\underline{\theta}, \overline{\theta}\right]$ is strategically inconsequential, meaning that it will not affect the politicians' equilibrium promises, although it will affect

[6] For example, when an autocrat exiles political opponents after surviving a power struggle, the cost to the exiled is captured by λ rather than k because they would not have been exiled, had the struggle been successful.

[7] See Section 3.

the ex ante probability that the incumbent is replaced. It is important for the analysis that θ is realized after the promises $(\boldsymbol{\alpha}, \boldsymbol{\omega})$ have been committed to. This requirement means that $(\boldsymbol{\alpha}, \boldsymbol{\omega})$ must be interpreted not as tactical redistribution, that can easily be adjusted within the space of, say, months but, rather, as strategic commitments that are difficult to substantially alter in the short run. For example, the Russian Czar's $\boldsymbol{\alpha}$ might be "to each according to their social position," whereas Lenin's $\boldsymbol{\omega}$ might be "to each according to their needs;" and the $\boldsymbol{\omega}'$ offered by a liberal democratic leader might be "to each according to their ability."

3 Citizens' equilibrium behavior

This section solves for the citizen's equilibrium behavior given a constellation of promises $(\boldsymbol{\alpha}, \boldsymbol{\omega})$. The citizens' behavior will determine the whether the incumbent is replaced. For pedagogical purposes, this section restricts attention to the case $(\eta, \lambda) = (1, 0)$; later on I will remove this restriction.

Given a constellation of promises $(\boldsymbol{\alpha}, \boldsymbol{\omega})$, citizens are engaged in a "global game" with potentially heterogeneous players (heterogeneity arises whenever $(\alpha_i, \omega_i) \neq (\alpha_j, \omega_j)$). In a similar setting, Sakovics and Steiner (2012) show that, in equilibrium, individual i supports the challenger if and only if her signal z_i exceeds a threshold z_i^*. As $\sigma \to 0$, all the thresholds z_i^* converge to a common limit θ^*.[8] Their analysis must be customized to the present setting because, if ω_i is small enough, it is a dominant strategy for citizen i to support the incumbent, and this violates a maintained assumption in Sakovics and Steiner (2012). The following lemma provides the required extension.

Lemma 1 *Suppose* $(\eta, \lambda) = (1, 0)$. *Given a constellation of promises* $\{\boldsymbol{\alpha}, \boldsymbol{\omega}\}$, *as* $\sigma \to 0$ *the equilibrium condition for incumbent survival converges to:*

$$1 - \theta > \underbrace{\int_0^1 \frac{\omega_i - k}{\omega_i + \alpha_i} \cdot \mathbf{1}\left[\omega_i \geq k\right] \; di}_{\textit{incumbent vulnerability index}} . \tag{5}$$

Proof. See the appendix. ■

The right hand side of (5) will be called the *vulnerability index.* In my setting, Sakovics and Steiner's (2012) analysis implies that any citizen i who does not have a dominant strategy to support the incumbent, will support the challenger if $z_i \geq z_i^*$, and that all

[8] Sakovics and Steiner (2012), Proposition 1.

the terms $(1 - z_i^*)$ converge to the vulnerability index as $\sigma \to 0$.[9] The significance of this fact is that, in the limit, i's strategy is independent of α_i and ω_i. Thus, in the limit, an individual citizen's behavior is independent of the promises that the individual received and, instead, is a function of the entire profile of promises $(\boldsymbol{\alpha}, \boldsymbol{\omega})$. Put differently, *individual behavior is determined solely by the collective perception of regime stability.* Also, as is typical in global games, all citizens who do not have a dominant strategy coordinate their actions perfectly as $\sigma \to 0$.

Henceforth I will assume that the incumbent (resp., the challenger) minimize (resp., maximize) the right hand side of expression (5).

A rough intuition for the functional form in (5) is the following. Set $\lambda = 0$ in the payoff matrix (3). For citizen i to be indifferent between supporting either politician, the citizen's expectation of incumbent survival p_i must solve:

$$p_i \cdot \Delta_\alpha + (1 - p_i) \cdot \Delta_\omega = 0,$$

where $\Delta_\alpha = -k - \alpha_i$ (resp., $\Delta_\omega = \omega_i - k$) represents citizen i's incentive to support the challenger over the incumbent in the event that the incumbent survives (resp., is ousted). Solving for p_i yields:

$$p_i = \frac{\Delta_\omega}{\Delta_\omega - \Delta_\alpha} = \frac{\omega_i - k}{\alpha_i + \omega_i}.$$

This functional form is similar to the argument of the integral in equation (5). This intuition is not yet complete because p_i is citizen-specific, but in a global game we expect coordination to be approximately perfect when $\sigma \to 0$, meaning that all the p_i's must converge to the same number for almost all θ's. Sakovics and Steiner (2012) deliver the last part of the intuition, showing that this common number is the average of all p_i's. The integral in equation (5) is exactly this average, where the indicator $\mathbf{1}[\omega_i \geq k]$ ensures that the average is taken only over those citizens who don't have a dominant strategy to vote for the incumbent.

The vulnerability index has the expected properties. The integrand is between zero and one and thus so, too, is the index; this implies that, regardless of the promise profiles $(\boldsymbol{\alpha}, \boldsymbol{\omega})$, the incumbent survives when $\theta < 0$ and is ousted when $\theta > 1$. The mass of citizens who are promised $\omega_i \leq k$ do not contribute to the index, regardless of α_i: this reflects the fact that, for them, supporting the challenger is a dominated (at least weakly) strategy. In the region $\omega_i \geq k$, the integrand is increasing in ω_i and decreasing in α_i. These properties are intuitive: the incumbent is less vulnerable when the incumbent's

[9] Condition (5) reduces to condition (5) in Sakovics-Steiner (2012) when $\omega_i \geq k$ for all i.

promises are more generous and the challenger's promises are less generous. As expected, the index is nonincreasing in k, meaning that incumbent replacement is less likely when the cost of supporting the challenger is high.

4 Politicians' equilibrium promises

This section removes the restriction that $(\eta, \lambda) = (1, 0)$ and shows that, in any regime type (η, k, λ), the incumbent will treat voiceful citizens equally, but the challenger may not. That the incumbent always chooses to treat the voiceful equally is not obvious; the intuition for this result will be developed later in this section, and further strengthened in Section 6. The fact that the challenger may deviate from equal treatment is due to the disadvantage embodied in k: if the challenger treats everyone equally he risks spreading his resources too thin. I will provide more intuition for the challenger's equilibrium strategy after stating this section's result.

A politician's strategy is a set of probability distributions from which the voters' promises are drawn. The probability distributions are allowed to depend on the citizens' identities;[10] however, following Myerson (1993) I will restrict attention to equilibria in symmetric strategies, i.e., in strategies such that the promises to all voiceful voters are drawn from the same distribution.[11] Symmetric strategies are strategically strong because they do not allow the opponent to pick off any citizens who might be particularly sensitive to his/her promises. The formal definition follows.

Definition 2 (symmetric strategy) *A strategy is called symmetric if promises to citizens are drawn from a probability distribution that conditions only on whether a citizen has voice.*

Symmetric strategies require that promises to all voiceful citizens be drawn from a single probability distribution. This does not imply that they will all receive the same promise: indeed, if the probability distribution is non-degenerate, different voiceful citizens will receive different promises.

By assumption, the incumbent (resp., challenger) selects her (resp., his) strategy to minimize (resp., maximize) the vulnerability index subject to the linear constraint (1) (resp., 2). These optimization problems are well-behaved if the vulnerability index is

[10] For example: citizen i may be promised 2 and citizen i' may be promised 4 with probability 1/2, and 6 with probability 1/2.

[11] I am not restricting the strategy space here: I will have to show that deviating to personalized promises is not profitable in equilibrium.

convex in $\boldsymbol{\alpha}$ (resp., concave in $\boldsymbol{\omega}$). Inspecting the right hand side of (5) reveals that it is indeed a convex function of $\boldsymbol{\alpha}$; however, it is not a concave function of $\boldsymbol{\omega}$ if $k > 0$. This observation suggests that the incumbent's strategy should be interior but, if k is large, the challenger's strategy may be extremal. These important observations are developed in the next proposition.

Proposition 1 (egalitarian vs inequitable equilibrium promises) *Assume $\lambda < 1$. For candidate $j = 1, 2$ denote:*

$$\overline{B}_j = \frac{(1-\lambda)}{\eta} B_j,$$

and denote:

$$h(\alpha; k) = k + \sqrt{k\alpha + k^2}. \tag{6}$$

There is a unique equilibrium in symmetric strategies, and it has the following features.

1. *Voiceless citizens are promised zero by both politicians.*
2. *The incumbent promises all voiceful citizens the egalitarian distribution, i.e., $\alpha_i^* = B_1/\eta$ for all $i \in [0, \eta]$.*
3. *If $\overline{B}_2 \geq h(\overline{B}_1; k)$ the challenger promises all voiceful citizens the egalitarian distribution, i.e., $\omega_i^* = B_2/\eta$ for all $i \in [0, \eta]$.*
4. *If $\overline{B}_2 < h(\overline{B}_1; k)$ the challenger promises the voiceful citizens an inequitable distribution: some of them, chosen at random, are offered $h(\overline{B}_1; k)/(1-\lambda)$, the rest are offered zero.*

Proof. See the Appendix. ■

Part 1 is obvious: voiceless citizens get zero because no rational politician would waste resources on citizens with no political power. The rest of the proposition, intuitively, says that the incumbent distributes her budget equally among all voiceful citizens, but the challenger only does so if he "can afford it," meaning that his disadvantage is below some threshold (part 3). The challenger's disadvantage is expressed in terms of the function h, which is increasing in the incumbent's budget and in k. If the challenger's disadvantage exceeds the threshold (part 4), the challenger is better off making disparities: he must give zero to a subset to the voiceful citizens in order to give enough to the rest. I now provide some in-depth intuition for this result.

Suppose $(\eta, \lambda) = (1, 0)$ so that Lemma 1 can be applied directly. Then the incumbent seeks to *minimize* the right hand side in (5) subject to the budget constraint (1). The

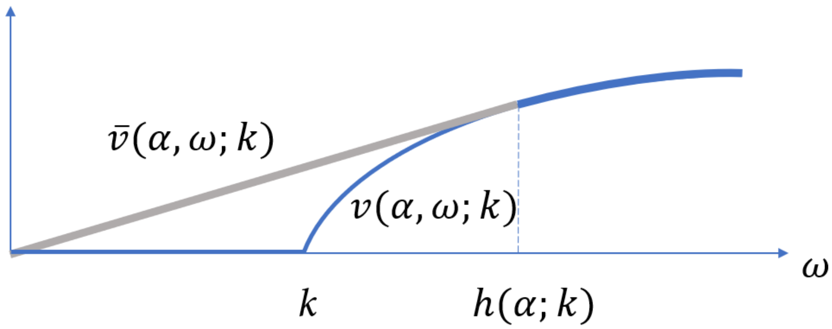

Figure 3: Why the challenger may redistribute inequitably. The incumbent promises α. If the challenger's budget is below $h(\alpha;k)$, the challenger benefits from making inequitable promises: in fact, the incumbent's vulnerability is maximized by promising $h(\alpha;k)$ with some probability, and 0 with complementary probability. If the challenger's budget is greater than $h(\alpha;k)$, the challenger's best response is to treat every voiceful citizen equally.

right hand side in (5) may be written as:

$$\int_0^1 v\left(\alpha_i, \omega_i; k\right) \; di, \tag{7}$$

where I denote:

$$v\left(\alpha, \omega; k\right) = \left(\frac{\omega - k}{\omega + \alpha}\right) \cdot \mathbf{1}\left[\omega \geq k\right]. \tag{8}$$

Since v is a convex function of α, if the challenger treats all citizens symmetrically then (7) is minimized by promising every citizen an equal share of the budget. The challenger's problem is somewhat more complex: he seeks to *maximize* (7), but v is *not* a globally concave function of ω: refer to Figure 3, which plots v as a function of ω. Therefore the challenger may benefit from not to treating all citizens equally. To understand why, suppose the incumbent promises α to all citizens, and form the *concave envelope* $\overline{v}\left(\alpha, \omega\right)$ as a function of ω. Figure 3 plots v and $\overline{v}$: the concave envelope $\overline{v}$ is never smaller than v, and it is strictly greater for small values of ω. If the challenger's available resources b_2 are less than $h\left(\alpha; k\right)$, equal treatment only gets the challenger $v\left(\alpha, b_2; k\right)$; however, the challenger can attain the full $\overline{v}\left(\alpha, b_2; k\right)$ through the following inequitable strategy: each citizen is promised $h\left(\alpha; k\right)$ with some probability, and zero with complementary probability, with the probability being chosen such that the budget constraint is met. Thus it is optimal for the challenger to use an inequitable strategy. Finally, the challenger picks the lucky citizens randomly to prevent the incumbent from "picking off" the most receptive citizens.

The rescaled budgets $\overline{B}_1$ and $\overline{B}_2$ represent "incentive budgets:" they can be interpreted as the amount of *incentives* (as opposed to resources) that the politicians have available to distribute, relative to the size of k. The incentive budgets are more generous when η and λ are small. This is intuitive: when η is small more is left over to distribute to the voiceful citizens, after the voiceless citizens are expropriated; and when λ is small, the incentives available to the politicians are, in effect, more powerful relative to k. The incentive budgets $\overline{B}_1$ and $\overline{B}_2$ determine whether the challenger will promise the egalitarian distribution because they appear in the inequality in part 4 of the proposition. Inspecting this inequality immediately yields the following comparative statics.

Corollary 1 (factors that lead to inequitable promises by the challenger)

1. *If* $k = 0$ *the challenger promises all voiceful citizens the egalitarian distribution.*
2. *If* $k > 0$ *the challenger is more likely to treat the voiceful citizens inequitably if:*
   (a) k *is larger*
   (b) *his budget* B_2 *is smaller relative to the incumbent's budget* B_1.
3. *If* $k > 0$, *the challenger treats the voiceful inequitably for* λ *sufficiently close to one.*
4. *The challenger treats the voiceful equally for* η *sufficiency close to zero.*

Parts 1 and 2a suggest that the egalitarian distribution prevails when k is sufficiently small. This is indeed the case, and it can be seen from Figure 3. Expression (6) reveals that $h(\alpha; k)$ is increasing in k and converges to zero as $k \to 0$. Therefore, for small k the region where v is non-concave is small and, therefore, for most values of b_2 it is optimal for the challenger to treat voiceful citizens equally. Conversely, for large k the region of non-concavity is large and, therefore, inequitable treatment among the voiceful citizens is a best response for many values of b_2. A large k represents an incumbent advantage so large that if the challenger were to spread his resources equally among all the voiceful citizens, he would be spread too thin.

Corollary 1 parts 2b and 3 reveal that the challenger will treat the voiceful inequitably when both politicians' "incentive budget" is small. Conversely, part 4 shows that the challenger will make egalitarian promises to the voiceful when both politicians' "incentive budget" is large.

5 Provision of an egalitarian public good

In this section I enlarge the policy space by adding a policy which I call an *egalitarian public good*. I start by setting $B_1 = B_2 = B$, which means that both politicians have the same amount of resources. I assume that either politician can either invest all of B to produce a public good that gives exactly $G > 0$ to each citizen; or, alternatively, the politician can redistribute B as s/he was free to do in the previous section. If $G > B$ the public good is a potentially expedient platform, but if a politician provides the public good s/he loses the ability to target resources to the voiceful citizens, or among them. Both the benefits of redistribution and those from the public good are decreased by λ if a citizen failed to back the winning politician; this implies that the public good is *excludable*.[12]

I interpret the egalitarian public good as the use of available state capacity (i.e., the tax revenue B) to pay for a policy with broad-based benefits. For example: using coercive state powers to procure grain (B) that is then exported to pay for: agricultural machines that improve the productivity of collective farms (G); or universal education/health care (G); or national defense (G).

Definition 3 (socially efficient vs voiceful-optimal policies) *Fix $B_1 = B_2 = B$. The socially efficient policy is to provide the public good if and only if $G \geq B$. The voiceful-optimal policy is to provide the public good if $G \geq B/\eta$, else to redistribute all the budget to the citizens with voice.*

Voiceful-optimality looks at outcomes from the collective perspective of the citizens who have voice.[13] It selects the highest per-capita value for them, between the public good (G) and the value of redistribution (B/η) that is available for the voiceful citizens *after the voiceless citizens have been expropriated*. Whenever $\eta < 1$, the voiceful-optimal policy provides the public good less often than is socially efficient, meaning that there are some value of G that are socially efficient and not voiceful-optimal, but not vice-versa.[14] This is because the voiceful citizens do not internalize the entire population's benefits of consuming the public good or, put differently, they benefit from redistribution more than is socially optimal.

In equilibrium, no politician will promise the public good if $G < B/\eta$ because it is a dominated strategy: redistribution offers more value among the voiceful, *and* more

[12] In authoritarian regimes, citizens can be excluded from the enjoyment of most public goods through coercion, incarceration, or worse.

[13] This definition abstracts from distributional considerations (inequality). Inequality is addressed separately as an equilibrium outcome throughout this paper.

[14] These are the values in the region $G \in (B, B/\eta)$.

targetability. So in equilibrium the public good is provided no more frequently than in the voiceful-optimal policy and, therefore, weakly less than is socially efficient. In fact, the challenger may even provide the public good *strictly* less than prescribed by the voiceful-optimal policy. The next proposition describes the equilibrium.

Proposition 2 ***(provision of an egalitarian public good)*** *Assume $\lambda < 1$. Suppose $B_1 = B_2 = B$, and denote:*

$$\begin{aligned} \overline{B} &= \frac{(1-\lambda)}{\eta} B \\ \overline{G} &= (1-\lambda) G \\ M &= \max\left[\overline{B}, \overline{G}\right]. \end{aligned}$$

There is a unique equilibrium in symmetric strategies, and it has the following features.

1. *Voiceless citizens are promised zero whenever redistribution is promised.*
2. *The incumbent promises the voiceful-optimal policy and equitable treatment among the voiceful citizens.*
3. *The challenger promises the voiceful-optimal policy and equitable treatment among the voiceful citizens if and only if:*

$$v(M, M; k) \geq \overline{v}\left(M, \overline{B}; k\right). \tag{9}$$

Else, the challenger will promise unequal redistribution among the voiceful citizens.

Proof. See the Appendix. ■

Parts 1 and 2 say that the incumbent will promise whatever is best for the voiceful citizens, be it redistribution (featuring expropriation of the voiceless) or the public good. Therefore, the incumbent is a faithful agent of the voiceful citizens. In contrast, the challenger does not necessarily promise the voiceful-optimal policy. Part 3 says that the challenger will promise inequitable redistribution among the voiceful if condition (9) fails.

As discussed above, the public good is sometimes under-provided, but never over-provided in equilibrium, relative to the social optimum. Therefore, increasing public good provision is socially desirable. The next result speaks to the factors that affect public good (under-)provision in equilibrium.

Corollary 2 (Factors that lead to public good underprovision)

1. *For any given value of* (B, η), *parameters* $G > B/\eta$ *and* $k > 0$ *exist such that the challenger does not promise the public good even though it is voiceful-optimal.*

2. *For both challenger and incumbent, the set of values* (B, G) *such that the public good is promised grows with* η.

3. *The probability that the incumbent promises the public good is independent of* λ. *The challenger promises inequitable redistribution for all* $\lambda > (G - k)/G$.

4. *Given any pair* $\left(M, \overline{B}\right)$, *for any* k *that is small enough the challenger promises the voiceful-optimal policy.*

Proof. See the Appendix. ■

Part 1 establishes that an underprovision problem exists for any pair (B, η), even if $\eta = 1$. This problem arises because the challenger will prefer redistribution for tactical purposes, in order to overcome the disadvantage embodied in k. Part 2 says that, as η increases, the public good is more likely to be provided (which, as discussed above, is socially desirable). This is because as η increases there are fewer voiceless citizens on which, from a strategic perspective, the benefits of the public good are "wasted." Part 3 is subtle. As λ grows, i.e., as citizens become more protected from political retribution, the incumbent's promises don't change: they remain voiceful-optimal (which may still fall short of the socially-efficient provision level); in contrast, for large enough λ the challenger under-provides the public good relative even to the voiceful-optimal level. This effect arises because the challenger's "incentive budget" shrinks relative to k, which increases the challenger's disadvantage relative to the incumbent, leading the challenger to create inequality among the voiceful. Part 4 says that if k is small the challenger is a faithful agent of the voiceful citizens, meaning that the public good will be provided if and only if it is optimal for them.

I have assumed at the beginning of this section that the public good is excludable. If the public good is non-excludable, Proposition 2 does not hold and the public good will not be provided in equilibrium.

Remark 1 (non-excludable public good) *A non-excludable public good will not be promised if* $\lambda < 1$, *because it generates no incentive to support the politician who promised it. This is because the politician cannot "claw back" non-excludable public goods from citizens who failed support him/her.*

Of course, in the context of authoritarian regimes where citizens can be excluded from the enjoyment of most public goods through coercion, incarceration, or worse, practically all public goods may be regarded as excludable.

6 Consensual and pluralist democracies

I define democracy as the special case where $(\eta, k) = (1, 0)$, i.e., full franchise and no challenger disadvantage.[15] I maintain the assumption that the incumbent is replaced according to condition (4).[16] Democracies come in two qualitatively different flavors: a consensual one ($\lambda < 1$) that, as we will see, will generate efficient provision of the egalitarian public good; and a pluralist one ($\lambda = 1$) that will not.

In a pluralist democracy $\lambda = 1$, so the politicians' promises are *not* conditional on a_i. When a_i is observable (public speech, monetary contributions, activism), $\lambda = 1$ captures those rules of liberal society that prevent politicians from disadvantaging the citizens who failed to support them. Alternatively, $\lambda = 1$ may capture the case where a_i is unobservable (as in anonymous voting), in which case political retribution is impossible for technological reasons.

The pluralist scenario $(\eta, k, \lambda) = (1, 0, 1)$ is the setting that is studied in most theoretical voting models. In my model, pluralist democracy happens to be a special limit case because the citizen's incentive to vote vanishes (to check this, plug $k = 0, \lambda = 1$ into matrix (3)). This ambiguity is resolved with a standard assumption: I assume that citizen i votes for the candidate who promises the most. This assumption, which is standard in voting games, turns the coordination game into a dominant strategy game where voters do not seek to coordinate but, rather, choose their favorite candidate without regard to each other's behavior. This is a fundamental point in this paper: the voters' calculus is qualitatively different depending on the value of λ. In a democracy, this difference will give rise to different equilibrium promises, as shown in the next result.

[15] However, ex-ante payoff *levels* need not be the same between the two politicians – one or the other may enjoy a *non-strategic* advantage, meaning that the prior distribution of θ may favor either, ex ante.

[16] This assumption means that replacement occurs in proportion to the challenger's vote share, and not when his vote share exceeds 1/2. This assumption is not uncommon in the voting literature. It is made here to avoid changing the game structure discontinuously in the neighborhood of $(\eta, k) = (1, 0)$. This choice allows me to attribute the cause of the difference in outcomes across political systems to variation in (η, k, λ) alone, as opposed to a concurrent change in the replacement rule. In Remark 2 I show that the results extend to the more natural case where replacement in a democracy occurs when the challenger's vote share exceeds 1/2.

Proposition 3 (provision of egalitarian public good in democracy) *Fix* B, G, *and set* $(\eta, k) = (1, 0)$ *(i.e., full franchise and no challenger disadvantage).*

1. ***(consensual democracy: efficient provision)*** *Suppose* $\lambda < 1$. *Then both politicians promises the socially efficient policy and equal treatment across the entire population.*

2. ***(pluralist democracy: inefficient provision)*** *Suppose* $\lambda = 1$. *Then neither politician promises the public good if* $G \leq B$, *and this is socially efficient. If* $G \in (B, 2B)$ *both politicians promise the public good with probability* $(G - B)/B$, *which is inefficient underprovision. If* $G \geq 2B$ *both politicians promise the public good, and this is socially efficient. When the public good is not promised, inequitable redistribution is promised.*

Proof. Part 1. Follows from Proposition 2 part 2 and Corollary 2 part 4, since $k = 0$ and $\eta = 1$ here.

Part 2. When $\lambda = 1$, by assumption citizen i chooses $a_i = 1$ if and only if $\alpha_i \leq \omega_i$. Then the condition for incumbent replacement (4) rewrites as:

$$\int_0^1 \mathbf{1}(\alpha_i \leq \omega_i)\, di \geq 1 - \theta. \tag{10}$$

This means that the challenger replaces the incumbent if the challenger's vote share exceeds $(1 - \theta)$. Since θ is distributed uniformly, maximizing (or minimizing) the probability of event (10) is the same as maximizing (or minimizing) the vote share. Therefore this game between politicians is exactly equal to the "proportional system" analyzed in Lizzeri and Persico (2001). The relevant result is found in their Theorem 3, which studies the case where politicians maximize their vote share. ∎

The intuition for Proposition 3 is based on comparing the politicians' objective functions in a pluralist and a consensual democracy, i.e., comparing the integral in (10) to the integral in (5) with $(\eta, k) = (1, 0)$. The latter is convex in $\boldsymbol{\alpha}$ and, since $k = 0$, it is also concave in $\boldsymbol{\omega}$; these properties imply that politicians have an incentive to treat all citizens equally. Hence, the fact that the public good is egalitarian turns out to be no binding constraint for the politicians, leading to efficient provision in any consensual democracy (Proposition 3 part 1). In contrast, the integral in (10) is neither concave nor convex in the politicians' control variables. Therefore, in a pluralist democracy politicians have no incentive to treat all citizens equally; in fact, as discussed in Example 1 below and shown in Figure 4, politicians have an incentive to create disparities. Here, the fact

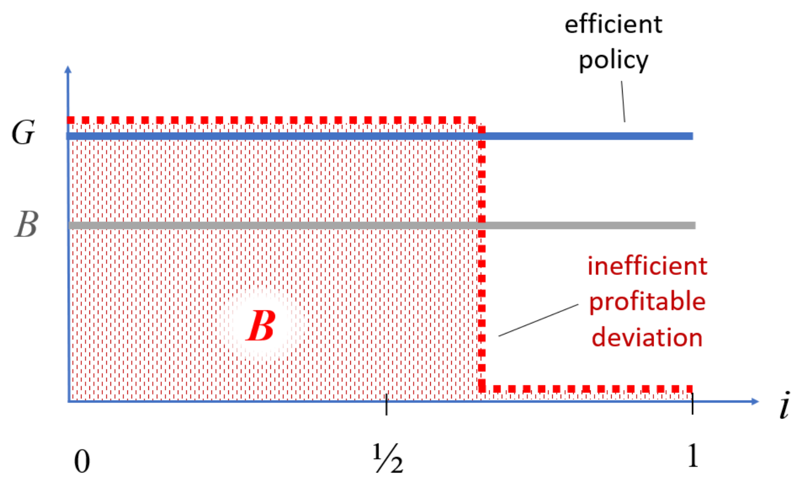

Figure 4: Incentive to underprovide an egalitarian public good in a pluralist democracy. Because $G > B$, the socially efficient policy is to promise the public good, but it is not an equilibrium for both politicians to promise it. If a politician promises the public good (blue line), the opponent benefits from deviating to redistribution (dashed red line). The deviation is carefully targeted to win as many votes as possible, which is more than 1/2 of the votes. The area under the dashed red line equals B as required to meet the budget constraint.

that the public good is egalitarian *is* a binding constraint for the politicians: this is why politicians will promise it less than is efficient (Proposition 3 part 2).

The next example illustrates why the two modes of political competition result in different equilibrium policies.

Example 1 (drivers of egalitarian public good underprovision in pluralist democracy) *Suppose* $B = 1$ *and* $G = 1.5$*, so that G is socially efficient. If both candidates were to promise G for sure, then each of their vote shares would be equal to 1/2. In a liberal democracy, candidate 2 could deviate and promise* $1.5 + \varepsilon$ *to almost* $2/3$ *of the citizens, and zero to the rest. This deviation delivers a vote share of almost 2/3, which is better than 1/2. So "G for sure" is not an equilibrium in a pluralist democracy. Candidate 2's deviation is inefficient because it makes 2/3 of the voters vanishingly happier than* G*, and 1/3 much less happy: but the deviation pays off because the intensity of the voters' preferences does not matter. This scenario is illustrated in Figure 4. In a consensual democracy, the same deviation would deliver candidate 2* less than $1/2$ *when plugged into expression (5), due to the concavity of the expression in* $\boldsymbol{\omega}$.

The difference in the politicians' incentives across democracy types reflects the difference in the mode of competition. In a pluralist democracy politicians target *individual* *voter preferences*, because citizen i votes based on whether politician 1's promise *to herself* is greater than politician 2's. This reasoning on the part of citizens gives rise to the objective function on the left-hand side of (10). In a consensual democracy, instead, the politicians' objective function is the right-hand side of equation (5) with $\eta = 1$ and $k = 0$ which, as observed on page 13, is monotonically related to the *common belief* of the likelihood of regime change.

The next theorem is the first main result: it characterizes the entire regime type space according to the efficiency of egalitarian public good provision.

Theorem 1 (efficiency of egalitarian public good provision across regime types)

1. ***(non-democracies: inefficient provision)*** *Any non-democratic system, i.e., one where* $(\eta, k) \neq (1, 0)$ *, fails to achieve efficient provision for some pair* (B, G) *.*

2. ***(consensual democracy: efficient provision)*** *Consensual democracy, i.e.,* $(\eta, k, \lambda) = (1, 0, < 1)$ *achieves socially efficient provision for any pair* (B, G)*.*

3. ***(pluralist democracy: inefficient provision)*** *Pluralist democracy, i.e.,* $(\eta, k, \lambda) = (1, 0, 1)$ *fails to achieve efficient provision for some pair* (B, G) *.*

Proof. Part 1. Efficient provision requires *both* politicians to promise the efficient policy, because in equilibrium both politicians obtain power with positive probability. Therefore, inefficient provision when $k > 0$ is established in Corollary 2 part 1. Inefficient provision when $\eta < 1$ follows because, as discussed at page 19, the provision level is *at most* the voiceful-optimal one for any triple (B, G, η). Choosing the triple such that $B < G < B/\eta$, ensures that the voiceful-optimal level of public good provision is strictly below the socially efficient one.

Parts 2, 3. These are merely restatements of Proposition 3 parts 1 and 2. ∎

The reason why non-democracies do not achieve efficient provision is the following. Any system with $\eta < 1$ suffers from an under-representation problem, which obviously leads to underprovision of the public good. But even if $\eta = 1$, a non-democratic system $(k > 0)$ suffers from the problem that the challenger will sometimes "go for broke" and focus his promises on a subset of the citizens to cope with the strategic disadvantage created by $k > 0$. In order to do so, the challenger has to pass on the public good even when it is efficient.

With this being said, the theory predicts that there are some values of G such that public good provision is more efficient in a non-democracy with large η and small k, than in a pluralist democracy. This is *not*, as in McGuire and Olson (1996), because in pluralist democracy politicians only care about 50% of the votes. Indeed, in this section I have purposely assumed that politicians maximize the *vote share* (refer back to the discussion in Footnote 16), and the offending deviation in Figure 4 maximizes the *vote share* – and yet it leads to inefficiency. The reason, rather, is that a pluralist democracy rewards the formation of "barely on-board" coalitions, as shown in Example 1.[17]

I close this section by generalizing the condition that triggers incumbent replacement. So far, I have restricted attention to condition (4), which means that the challenger wins if his vote share a exceeds $(1-\theta)$. I have done this for comparability: keeping the incumbent replacement rule fixed as the parameters (η, k, λ) vary allowed me to pinpoint the source of the difference in performance across regime types. However, in a democracy a majoritarian rule of incumbent replacement may be more natural. I address this in the next remark.

Remark 2 (extension to majoritarian rule of incumbent replacement) *In a democracy the condition that triggers incumbent replacement is often $a \geq 1/2$, rather than condition (4) as currently assumed. In Appendix B the results in Proposition 3 part 1 are shown to extend verbatim to a transition rule arbitrarily close to $a \geq 1/2$. The results in part 2 also extend with minor changes to the case $a \geq 1/2$ (cf. Theorem 2 of Lizzeri and Persico 2001, where candidates maximize the probability of winning the election rather than their vote share.) ThereforeTheorem 1 continues to hold essentially verbatim if the replacement rule is $a \geq 1/2$.*

7 The virtue of pluralism: beyond egalitarianism

This section is concerned with the provision of a public good with non-egalitarian benefits. This type of public good is interpreted as the use of available state capacity (i.e., the tax revenue B) to support institutions that produce a welfare improvement with unequally-distributed benefits.

For example, imagine that citizen i can either make a costly investment in the private sector and earns returns γ_i, or make no investment and get zero. Investment can be

[17] The profitability of the "barely on-board" coalition in Example 1 reflects that fact that politicians do not care about the intensity of voter preferences in a pluralist democracy Indeed, the vote share (10) only depends on whether $\alpha_i \leq \omega_i$, but not on the magnitudes of α_i and ω_i In contrast, in a non-democracy the intensity of voter preferences shapes the politicians' objective functions, because expression (5) depends on the magnitudes of α_i and ω_i. This being understood, it makes sense that politicians may promise more of an egalitarian public good when their incentives take into account preference intensity.

interpreted as effort provision or as capital investment. Assume that the cost of investment equals 1. Then, all citizens with $\gamma_i \geq 1$ will make the investment and attain welfare level $\gamma_i - 1$. If the γ_i's are not the same for all citizens, as might be the case for the distribution of scarce skills, abilities, or if capital is unequally distributed in society, then the welfare distribution ends up being non-egalitarian. In reality, attaining $\gamma_i - 1$ depends on two conditions: the existence of a private sector, and the citizen's ability to keep the γ_i after having earned it. These two conditions require capitalist institutions such as a competent bureaucracy, impartial courts to enforce contracts, and strong property rights (capital and IP protection) to prevent others from taking citizen i's earnings. Without these institutions, no citizen has any incentive to exert effort, leading to a decrease in aggregate welfare.

The previous paragraph sketched out a model where state capacity can be used to support institutions that produce a welfare improvement *and* inequality. For my purposes here, that model can be summarized as follows: the government can invest B into a non-egalitarian public good (bureaucracy, courts) that produces a distribution of benefits g_i (corresponding to $\gamma_i - 1$). Going forward, therefore, I assume that each candidate can either offer redistribution, or an excludable non-egalitarian public good that gives $g_i \sim \widetilde{G}$ to citizen i, with the random variable $\widetilde{G}$ representing the (non-equal) distribution of benefits from the policy.

The next theorem makes the convenient assumption that $\widetilde{G}$ is distributed as a Uniform on $[0, \overline{g}]$. Under this assumption, pluralist democracy is shown to produce the voiceful-optimal public good level. The notions of social efficiency and voiceful-optimality are extended in the obvious way to a non-egalitarian public good, i.e., by replacing G with $\mathbb{E}\left(\widetilde{G}\right)$ in Definition 3.

Theorem 2 (efficiency of non-egalitarian public good provision across systems) *Suppose* $\widetilde{G} \sim U[0, \overline{g}]$.

1. *The non-egalitarian public good* $\widetilde{G}$ *is not promised by either politician if it is not voiceful-optimal, i.e., if* $\mathbb{E}\left(\widetilde{G}\right) < B/\eta$.

2. ***(non-pluralist system: parameter region of inefficient provision)*** *Assume* $\lambda < 1$. *If* $\mathbb{E}\left(\widetilde{G}\right) = (cB)/\eta$ *and* $c \in (1, 1.25)$, *the non-egalitarian public good is voiceful-optimal but, for* k *small enough, it is not an equilibrium for both politicians to provide it. In this parameter region both politicians promise egalitarian redistribution among the citizens with voice.*

3. ***(pluralist system: efficient provision)*** *Assume* $(k, \lambda) = (0, 1)$. *Then the non-egalitarian public good is provided if and only if it is voiceful-optimal, i.e., if* $\mathbb{E}\left(\widetilde{G}\right) \geq B/\eta$.

Proof. See the Appendix. ■

Part 1 is obvious: there is no strategic advantage in providing a policy that has lower mean *and* less flexibility than redistribution. Part 2 is intuitive: we know from Proposition 1 that when k is small enough egalitarian redistribution among the voiceful citizens is a best response to itself; therefore, a policy $\widetilde{G}$ that is very unequal cannot be a best response to redistribution unless it is much more efficient than redistribution; the proposition indicates that $\widetilde{G}$ must be more than 25% more efficient than redistribution in order to be appealing. Theorem 2 part 3 concerns a pluralist democracy with a possibly restricted franchise (if $\eta < 1$). In this system, the theorem says that the non-egalitarian public good is provided right up to its theoretical upper bound (refer back to part 1). This is because, in a pluralist system, a policy that creates inequality does not automatically entail a strategic penalty (refer back to the discussion on page 22). The contrast between parts 2 and 3 illustrates the virtue of pluralism. This is the main takeaway from this section.

The assumption that $\widetilde{G}$ is distributed as a Uniform on $[0, \overline{g}]$ is convenient to achieve the stark contrast between parts 2 and 3, but it is not a knife-edge case: approximate versions of Theorem 2 hold if $\widetilde{G}$ is distributed similar to a Uniform.

Voiceful-optimality coincides with social efficiency if the pluralist system has a full franchise ($\eta = 1$). In this case Theorem 2 part 3 yields the following result.

Corollary 3 (most socially efficient system) *Fix any* B. *An egalitarian public good with value* G *is efficiently provided by a consensual democracy. A non-egalitarian public good with value* $\mathbb{E}\left(\widetilde{G}\right)$ *is efficiently provided by a pluralist democracy.*

Proof. Follows from Theorems 1 and 2. ■

8 Examples of regimes with different values of (B, G, η, k, λ)

In this section I look across countries. Taking the parameter constellation (B, G, η, k, λ) in a country as exogenous, I ask whether the public good is provided in accordance to the model's prediction, or not. This section is not intended to cover all countries; rather, it is meant to illustrate how the theory maps into real world applications.

8.1 Relative size of B and G

$B > G$ means that the socially efficient policy is targetable. Conversely, $B < G$ means that the socially efficient policy is non-targetable and egalitarian.

In countries where wealth is created by individual effort, the socially efficient policy is to tax as little as possible (to minimize labor supply distortions), and to invest tax revenue in public goods (roads, health care, education) that raise individual productivity. This policy happens to be non-targetable and egalitarian, at least at a first approximation. In the language of the model, the egalitarian public good is efficient: $G > B$. If, moreover, $G > B/\eta$, then the public good is voiceful-optimal. In this case the public good will be provided in equilibrium at least by the incumbent.[18] This means that even the voiceless benefit from the incumbent's policy: in this sense, policy is inclusive.

In countries where wealth comes mainly from the exploitation of natural resources, the socially efficient (wealth-maximizing) policy happens to be targetable: the revenues from resource extraction can be freely redistributed as lump sums without creating labor supply distortions. In the language of the model, redistribution is efficient: $B > G$. In these countries the public good is never voiceful-optimal ($G < B/\eta$ necessarily holds) and so the theory predicts that will not be provided. Instead, in equilibrium the government will redistribute the revenues from resource extraction to politically voiceful citizens only: the voiceless get nothing.

This argument can account for the observation that non-democracies differ in terms of inclusiveness. In many resource-rich non-democracies, the voiceless don't seem to benefit much from government policy. In contrast, some resource-poor non-democracies (including the Asian tigers and Mao's China) have developed in a reasonably inclusive way by providing productivity-enhancing public goods. The theory can account for this dualism by appealing to the different targetability of the available growth opportunities.[19]

[18] See Proposition 2 part 2.

[19] An additional factor that has been invoked to help explain the Asian tigers' inclusive policy outcomes is the threat to the national interest. Doner et al. (2005) argue that it was easier for the Asian tigers to resist the pull of special interest politics because of the presence of aggressive neighboring states. Redistributing to the voiceful elites within the country would create intense social conflict within the country, and that would facilitate foreign meddling. In the language of this paper, for the Asian tigers G was especially large relative to B due to geopolitical realities. The same argument might apply to Soviet Russia and Mao's China, although Doner et al. (2005) do not make this connection.

8.2 High v. low η

When η is smaller, that is, when fewer citizens have voice, the theory predicts that public goods are less likely to be provided.[20] This prediction seems uncontroversial: the history of European colonial rule, for example, shows that when the colonial subjects have no political voice, colonial policies are extractive in nature. In the language of this paper: when η is small enough, redistribution from the colonial subjects ($i \in [\eta, 1]$) to the colonial masters ($i \in [0, \eta]$) is the voiceful-optimal policy no matter how socially efficient the public good might be (formally, $G < B/\eta$ no matter how large G is). Then, in equilibrium the socially efficient public good will not be provided: the voiceless colonial subjects get nothing.

In a democracy, η can be expanded by law. This happened when the urban poor gained the right to vote in 19th century Britain, or when women gained the franchise in the US. The theory predicts that in both cases socially efficient public goods should be provided more often. This prediction finds support in the literature.[21]

In non-democracies, the size of the set $[0, \eta]$ depends on the technology of political voice. Historically, the citizens' ability to coordinate against the incumbent required coming together physically – typically, moving from the countryside to the city – so only urban dwellers had voice. Focusing on post-colonial Africa, Bates (2014) tells a story of non-democratic regimes[22] that provided welfare-decreasing policies in order to favor highly coordinated city dwellers at the expense of voiceless rural dwellers. This is consistent with this model's predictions that when η is low policy does not maximize social welfare and the voiceless get nothing.

Today, social media have made it easier for citizens to coordinate their political activity, so more people have voice relative to the time when traditional media acted as information gatekeepers. In the model, this corresponds to an increase in η. The model predicts that equilibrium policy should become more equitable, and more geared toward public goods.

[20] This is because a socially efficient public good can only be provided in equilibrium if $G > B/\eta$: refer to the discussion on page 19. When η is smaller this inequality is less likely to hold.

[21] For Britain, see Lizzeri and Persico (2004); for women in the US, see Lott and Kenny (1999) and Miller (2008).

[22] Bates (2010, p. 1136) writes that "the forces that took over the colonial state [...] failed to endorse open political competition and the attendant rights of political expression and public assembly. [... B]y the mid-1990s, authoritarian regimes had become a dominant feature of African political life.

8.3 High v. low k

The magnitude of k does not affect the incumbent's equilibrium strategy: her strategy always delivers exactly what the voiceful citizens prefer.[23] On the other hand, as k increases the challenger's strategy becomes less egalitarian than the incumbent's,[24] and less likely to provide the egalitarian public good.[25] Therefore, in the period of a successful challenge, implemented policy could be less egalitarian than the old incumbent's policy. This is the scenario in which the challenger won the power struggle by promising disproportionate rewards to a subset of voiceful citizens (a revolutionary vanguard), and must now deliver on his promises. After having so delivered, the successful challenger becomes the new incumbent, and we expect her steady-state policy to revert back to the old incumbent's policy (more egalitarian).

8.4 High v. low λ

The model predicts that, as λ becomes smaller, politicians are more likely to adopt egalitarian platforms.[26] This prediction may appear counterintuitive because it means that less liberal regimes tend to deliver more egalitarian policies. However, there is a body of evidence consistent with this observation. Jia et al. (2021) make a powerful case that Imperial China was both less liberal and more egalitarian than pre-modern Europe. Closer to our times, illiberal regimes such as Soviet Russia, Mao's China, and today's North Korea, had egalitarian rhetoric and policies. In Latin America, Albertus (2014, p. 60) argues that land redistribution is only possible under autocracy:

> "large-scale changes in redistributive policy such as land redistribution are more difficult to achieve when there are more institutional constraints to political rule. The opposition of a small number of institutional actors can jeopardize reform: if the executive opposes reform, the legislature cuts off funding, or the bureaucracy is corrupt or unorganized, redistributive land reform efforts will fall flat. Because land redistribution requires significant political concentration and administrative capacity, it is more likely to occur under autocratic rule."

[23] See Propositions 1 and 2.

[24] Corollary 1 parts 1 and 2a.

[25] Corollary 2 parts 3 and 1.

[26] When $(\eta, k) = (1, 0)$, i.e., democracy, we see this from comparing parts 1 and 2 of Proposition 3. When $(\eta, k) \neq (1, 0)$ we see this from Corollary 2 part 3, showing that if λ is large enough the challenger will not provide the egalitarian public good.

For post-war East Asian countries, the same argument is made by Wade (2004, p. 372), who interprets economic development as a broad-based public good:

> "The class structure of many developing countries implies a cruel choice between faster economic development and well-defended civil and political rights. Power and wealth are often concentrated in groups engaged in socially unproductive activities (including renting out of land, moneylending, exploitation of bureaucratic or military office). [...] Most likely [faster economic development] will require some curtailment of the political and civil rights of those who opposed the changes, and of the powers of democratically elected legislatures."

Note that the argument made in these quotes is that inclusive policies are not politically feasible under democracy, but there is no explanation for why inclusive policies might, under some conditions, be feasible under non-democracy. My model provides an explanation.

As concerns the (largely non-democratic) regimes of Africa, Francois et al. (2015) show that the distribution of ministerial posts is egalitarian, i.e., allocated to different ethnicities in proportion to their population shares. This is in implicit contrast with Western democracies, where the losing parties are typically not represented in the government. They argue that equal treatment arises because all citizens have approximately the same power to instigate coups or revolutions. In the language of my model, $\eta = 1$: all ethnic groups have political voice, and so in equilibrium the incumbent redistributes equally.

The point of this section is to argue that illiberal regimes (small λ) may in fact promote more egalitarian policies than liberal ones (large λ), caeteris paribus.

8.5 G v. $\widetilde{G}$

Truly innovative growth cannot be achieved by command and control policies. Instead, it requires government to invest in the institutions that promote and protect the (unequal) appropriation of rents by innovators and capitalists; these include strong IP protection, functioning courts, and weak labor protections. Generating this type of growth requires accepting inequality between innovative elites and the rest of the citizens. This type of growth corresponds to the non-egalitarian public good $\widetilde{G}$ from Section 7. By Corollary 3, pluralist democracy is the best system for providing such growth. This is consistent with the fact that the US, among the most pluralist countries in the world, is at the forefront of technology, and that most truly innovative growth comes from democratic countries.

Imitative (or catch-up) growth relies on importing foreign technology. This type of growth is compatible with equitable allocations because there is no need to reward innovators, and capital can be allocated by the state without allowing capitalists to earn rents. This type of growth corresponds to the egalitarian public good G from Section 5. According to the theory, non-pluralist systems are capable of delivering this type of growth because in such systems the incentives to cater to special interests is more muted than in pluralist democracy.[27]

The theory's prediction that non-democracy can deliver imitative growth but democracy cannot because it is vulnerable to special interests, is consistent with the experience of developing countries that have grown quickly under non-democratic regimes: in the 20th century these include Mao's China, Soviet Russia, South Korea, Singapore, and Taiwan. India is often cited as the counterfactual experiment because it is a vibrant democracy but its growth has been relatively slow, arguably because of the paralyzing influence of the special interests, as predicted by the theory.

9 Regime transition: some observations

A complete theory of regime transition is beyond the scope of this paper. In this section I sketch out some observations which can hopefully help shape a future theory of regime transition.

The first observation is that, whether regime change comes about *in order to shape an ongoing political struggle*, or after the fact as the *fulfillment of a promise* by the winner of the political struggle, it will be shaped by the conflict of interest between the bargaining parties. What are these conflicts of interest? One is between incumbent and voiceful citizens regarding the contestability of the regime in the presence of an egalitarian public good. All else equal, the incumbent's bliss point is $k = \infty$ (no risk of replacement), but the voiceful citizens' bliss point is $k = 0$ (when $k = 0$, egalitarian public good provision is always voiceful-optimal except in the limit case $\lambda = 1$). So, when good performance coincides with provision of an egalitarian public good, we can expect the voiceful citizens to push for a more contestable regime than the incumbent is willing to grant.

The second observation has to do with the nature of public goods. Depending on whether the available public goods are egalitarian or not, different regime types are most suited for optimal provision from the voiceful citizens' perspective. Therefore, we expect

[27] Theorem 2.

the voiceful citizens to seek regime transition when the nature of the available public good changes. In Section 8.5 I interpret the availability of either type of public good as related to the distance from the technology frontier. If we accept that interpretation, it follows that the voiceful citizens will not prefer a pluralist democracy in developmental states, but they will prefer a pluralist democracy in advanced economies, because each regime type efficiently delivers the type of public good that is available given the distance from the frontier. As a corollary, when an economy moves closer to the technological frontier, we expect its voiceful citizens to bargain for a more pluralist system. This seems to be the case in reality, because as countries get wealthier they tend to move toward more pluralist systems.

The third observation has to do with voluntary expansion of political rights. One might conjecture that the voiceful citizens would like η to stay the same, wherease the voiceless citizens would like η to increase. However, as in Lizzeri and Persico (2004), here too the voiceful citizens may have a preference for voluntarily increasing η. The reason is that under certain parameter values the outcome of the political contest is not voiceful-optimal: the challenger fails to promise a voiceful-optimal public good (Corollary 2 part 1), and so the welfare of voiceful citizens is reduced if the challenger prevails. In these circumstances, the voiceful citizens benefit from giving voice to more citizens because doing so increases the set of parameters under which the challenger promises an egalitarian public good (Proposition 2 part 2).

10 Conclusions

Little is known theoretically about the political incentives that shape economic policy in non-democracies, and how these might compare to democracies. This paper proposes a new theoretical definition of (non-)democracy based on two "political rights" parameters (η, k) and one "individual rights" parameter λ. The parameter λ captures the degree to which individual citizens are protected from political retribution. Studying the effect of λ on the outcome of political competition is the most innovative aspect of this paper.

The policy space features a trade-off between redistribution and public good provision. This is a classic theme in the political economy tradition of J. Buchanan, G. Tullock, and M. Olson. A new twist here is the distinction between two types of public good: one that delivers egalitarian benefits, the other that delivers non-egalitarian benefits. To my knowledge, this distinction is new in the literature on redistribution vs public good provision. I interpret public goods as growth opportunities that may, or may not, be enabled by government policy. According to this interpretation, my model distinguishes between egalitarian and non-egalitarian growth opportunities.

I find that in regimes with $\lambda < 1$, politicians compete for coordination. This type of competition pushes politicians to make egalitarian promises, but this force is tempered if η is small and k is large. I find that the citizens' political behavior is qualitatively different in a pluralist democracy, i.e., when $(\eta, k, \lambda) = (1, 0, 1)$, because citizens face a dominant strategy game rather than a coordination game. As a result, the nature of political competition is different and gives politicians incentives to treat citizens inequitably for electoral gain. This qualitative difference is one of the main conceptual insights from this paper.

Because an illiberal regime ($\lambda < 1$) has a built-in (though not absolute) tendency toward egalitarianism, but pluralist democracy has a tendency toward non-egalitarian treatment, an illiberal regime can efficiently provide an egalitarian public good under some conditions (large η, small k), and pluralist democracy efficiently provides a non-egalitarian public goods.

I have argued that the theory maps into real world applications in sensible ways. Future work may illuminate the incentives for political actors, including citizens with political voice, to effect regime transition starting from any given configuration (η, k, λ). Of particular interest, I think, are the incentives to evolve toward larger values of λ, that is, toward stronger protections of individual rights against political retribution.

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A Proofs and ancillary results

Proof of Lemma 1.

Proof. Citizen i's payoff is:

	Regime change	Status quo
Citizen i supports challenger	$\omega_i - k$	$-k$
Citizen i supports incumbent	0	α_i

Subtracting α_i from the right-hand side column does not alter the citizen's incentives, so we get:

	Regime change $(a \geq 1-\theta)$	Status quo $a < 1-\theta$
Citizen i supports challenger	$\omega_i - k$	$-\alpha_i - k$
Citizen i supports incumbent	0	0

For notational convenience we set

$$b_i = (\omega_i + \alpha_i)\,, c_i = (\alpha_i + k)\,,$$

so that we get:

	Regime change $(a \geq 1-\theta)$	Status quo $(a < 1-\theta)$
Citizen i supports challenger	$b_i - c_i$	$-c_i$
Citizen i supports incumbent	0	0

Now partition citizens into equally treated groups, so that all members of a group g receive the same b_g, c_g. In this setting, Sakovics and Steiner (2012, Proposition 1) show that, in equilibrium, group g supports the challenger if and only if $z_i \geq z_g^*$, as $\sigma \to 0$, all thresholds converge to a common limit $\theta^* = \sum_g m_g \frac{c_g}{b_g}$, so that incumbent survives if and only if:

$$\theta < \sum_g m_g \frac{c_g}{b_g}. \tag{11}$$

This formula, however, requires $b_g > c_g$ (this is a maintained assumption in Sakovics and Steiner 2012). If this condition is violated for some group g' then that group supports the incumbent for sure (dominant strategy). Lemma 1 claims that when $b_g \leq c_g$ is permitted, the equilibrium condition for incumbent survival is:

$$\begin{aligned}\theta - 1 &< \sum_g m_g \left(\frac{c_g}{b_g} - 1\right) \mathbf{1}\left[b_g \geq c_g\right] \\ &= \sum_g m_g \cdot \left(\frac{\alpha_g + k}{\omega_g + \alpha_g} - 1\right) \cdot \mathbf{1}\left[\omega_g \geq k\right].\end{aligned}$$

To derive this condition observe that if $b_{g'} \leq c_{g'}$ for some group g' then that group does not revolt for sure. In that case, we can eliminate group g' from the game, and there is a new game with new weights

$$\widetilde{m}_g = \frac{m_g}{\sum_{g \neq g'} m_g}.$$

Let's express the condition on behavior for incumbent survival (same as in the old game) using the new-game notation. The condition on behavior using the old notation is:

$$\begin{aligned}\sum_{g \neq g'} m_g a_g &\leq 1 - \theta \\ \sum_{g \neq g'} \widetilde{m}_g a_g &\leq \frac{1-\theta}{\sum_{g \neq g'} m_g} \\ \sum_{g \neq g'} \widetilde{m}_g a_g &\leq \frac{\sum_{g \neq g'} m_g - \sum_{g \neq g'} m_g + 1 - \theta}{\sum_{g \neq g'} m_g} \\ \sum_{g \neq g'} \widetilde{m}_g a_g &\leq 1 - \frac{\sum_{g \neq g'} m_g - 1 + \theta}{\sum_{g \neq g'} m_g} \\ \sum_{g \neq g'} \widetilde{m}_g a_g &\leq 1 - \frac{-m_{g'} + \theta}{\sum_{g \neq g'} m_g} \\ \sum_{g \neq g'} \widetilde{m}_g a_g &\leq 1 - \widetilde{\theta}.\end{aligned}$$

The condition on behavior for incumbent survival in the new game involves the transformed random variable $\widetilde{\theta}$. Plug into the Sakovics-Steiner condition (11) to get the equilibrium condition (on primitives, not on behavior) for incumbent survival in the new game:

$$
\begin{aligned}
\frac{-m_{g'}+\theta}{\sum_{g\neq g'} m_g} &< \sum_{g\neq g'} \widetilde{m}_g \frac{c_g}{b_g} \\
-m_{g'}+\theta &< \sum_{g\neq g'} m_g \frac{c_g}{b_g} \\
\theta &< m_{g'}\cdot 1 + \sum_{g\neq g'} m_g \frac{c_g}{b_g}
\end{aligned}
$$

So, letting g' index any group such that $b_g \leq c_g$, the equilibrium condition for survival (now back in old game notation) is:

$$
\begin{aligned}
\theta &< \sum_g m_g \cdot \mathbf{1}\left[b_g < c_g\right] + \sum_g m_g \cdot \left(\frac{c_g}{b_g}\right) \mathbf{1}\left[b_g \geq c_g\right] \\
&= \sum_g m_g \cdot \left\{1 - \mathbf{1}\left[b_g \geq c_g\right] + \left(\frac{c_g}{b_g}\right) \mathbf{1}\left[b_g \geq c_g\right]\right\} \\
&= \sum_g m_g \cdot \left\{1 + \left(\frac{c_g}{b_g} - 1\right) \mathbf{1}\left[b_g \geq c_g\right]\right\} \\
&= \left(\sum_g m_g\right) + \sum_g m_g \left(\frac{c_g}{b_g} - 1\right) \mathbf{1}\left[b_g \geq c_g\right] \\
&= 1 + \sum_g m_g \left(\frac{c_g}{b_g} - 1\right) \mathbf{1}\left[b_g \geq c_g\right] \\
&= 1 + \sum_g m_g \cdot \left(\frac{\alpha_g + k}{\omega_g + \alpha_g} - 1\right) \cdot \mathbf{1}\left[\omega_g \geq k\right].
\end{aligned}
$$

Note that the condition reduces to the Sakovics-Steiner condition (11) when $b_g > c_g$. Rearranging the above inequality we get the following expression for the equilibrium condition for survival:

$$
\begin{aligned}
1-\theta &> \sum_g m_g \left(1 - \frac{\alpha_g + k}{\omega_g + \alpha_g}\right) \cdot \mathbf{1}\left[\omega_g \geq k\right] \\
&= \underbrace{\sum_g m_g \left(\frac{\omega_g - k}{\omega_g + \alpha_g}\right) \cdot \mathbf{1}\left[\omega_g \geq k\right]}_{\text{incumbent vulnerability index}}.
\end{aligned}
$$

In my setting there is a continuum of targetable units, so the integral sign must replace the summation sign. With this replacement Lemma 1 is proved. ∎

Lemma 2 ***(characterizing*** $\overline{v}$***)*** *The concave envelope* $\overline{v}$ *of the function* v *defined in (8) has the following form:*

$$\overline{v}(\alpha,\omega;k)=\begin{cases}\frac{1}{\left(\sqrt{(\alpha+k)}+\sqrt{k}\right)^2}\cdot\omega & \text{for } \omega<h(\alpha;k)\\ v(\alpha,\omega;k) & \text{for } \omega\geq h(\alpha;k)\end{cases},$$

where

$$h(\alpha;k)=k+\sqrt{k\alpha+k^2}. \tag{12}$$

Proof. Compute the derivative at any point ω:

$$\frac{dv}{d\omega}(\alpha,\omega;k)=\frac{\alpha+k}{(\omega+\alpha)^2}\cdot\mathbf{1}[\omega\geq k]. \tag{13}$$

Now compute the slope r_g of the ray going through any $v(\alpha,\omega;k)$ with $\omega>k$:

$$r_\alpha=\frac{v(\alpha,\omega;k)}{\omega}=\frac{1}{\omega}\left(1-\frac{\alpha+k}{\omega+\alpha}\right). \tag{14}$$

At the tangency point $\omega=h(\alpha;k)$ it must be $v'=r_\alpha$. Use this condition to solve for $h(\alpha;k)$:

$$\begin{aligned}\frac{\alpha+k}{(h(\alpha;k)+\alpha)^2} &= \frac{1}{h(\alpha;k)}\left(1-\frac{\alpha+k}{h(\alpha;k)+\alpha}\right)\\ \frac{\alpha+k}{(h(\alpha;k)+\alpha)^2} &= \frac{1}{h(\alpha;k)}\left(\frac{h(\alpha;k)-k}{h(\alpha;k)+\alpha}\right)\\ \frac{\alpha+k}{(h(\alpha;k)+\alpha)} &= \frac{1}{h(\alpha;k)}(h(\alpha;k)-k)\\ (\alpha+k)\,h(\alpha;k) &= (h(\alpha;k)-k)(h(\alpha;k)+\alpha)\end{aligned}$$

Solving for $h(\alpha;k)$ yields two solutions: $k\pm\sqrt{k\alpha+k^2}$, but we are looking for the one exceeding k, so the relevant solution is:

$$h(\alpha;k)=k+\sqrt{k\alpha+k^2}.$$

The slope r_g is:

$$\begin{aligned} r_g &= v'\left(h\left(\alpha;k\right);\alpha\right) \\ &= \frac{\alpha+k}{\left(h\left(\alpha;k\right)+\alpha\right)^2} \\ &= \frac{\alpha+k}{\left(\alpha+k+\sqrt{k\alpha+k^2}\right)^2} \\ &= \frac{\alpha+k}{\left(\alpha+k+\sqrt{k}\sqrt{\alpha+k}\right)^2} \\ &= \frac{1}{\left(\sqrt{(\alpha+k)}+\sqrt{k}\right)^2}. \end{aligned}$$

■

Proof of Proposition 1.

Proof. First, suppose $\eta < 1$. No rational politician would make any positive promises to powerless citizens. This proves part 1.

Now I state the politicians' problems when (η, λ) does not necessarily equal $(1, 0)$. Refer back to matrix (3), and subtract $\lambda\alpha_i$ from the right-hand column and $\lambda\omega_i$ from the left-hand one. This does not alter the citizen's incentives, and results in:

	Incumbent replaced	**Incumbent survives**	
$a_i = 1$ (support challenger)	$(1-\lambda)\,\omega_i - k$	$-k$	(15)
$a_i = 0$ (support incumbent)	0	$(1-\lambda)\,\alpha_i$	

This game is strategically equivalent to the case $\lambda = 0$ that was analyzed in Lemma 1 except that here: the politician's control variables are $x_i = (1-\lambda)\,\alpha_i$ and $y_i = (1-\lambda)\,\omega_i$; and, also, the mass of voiceful citizens is $\eta < 1$. We now show that the latter difference is strategically irrelevant for voters.

Consider a game where the set of players is $[0, \eta]$, payoffs are given by (15), and the condition for regime change is (4). Then one can define a strategically equivalent "replica game" where the set of players is $[0, 1]$, payoffs are still given by (15), and the condition for regime change is now:

$$\int_0^1 a_i \, di \geq \frac{1-\theta}{\eta}.$$

Lemma 1 required only two assumptions on the distribution of $1-\theta$ in order to yield the right-hand side in (5). First, the random variable $1-\theta$ must be uniformly distributed; second, the interval $\left[\left(1-\overline{\theta}\right),\left(1-\underline{\theta}\right)\right]$ must be a superset of $[0,1]$. Since these assumptions have been made already, it follows that for any $\eta \in (0,1]$, the random variable $\xi = (1-\theta)/\eta$ is uniformly distributed and, furthermore, ξ's support is a superset of $[0,1]$ being equal to the interval $\left[\left(1-\overline{\theta}\right)/\eta,\left(1-\underline{\theta}\right)/\eta\right]$. Therefore, after replacing ξ for $(1-\theta)$, the replica game satisfies all the conditions required by Lemma 1. It follows that the value of η does not affect the citizens' equilibrium behavior in the replica game, and therefore the politicians' objective function is described by Lemma 1 except that $\{\boldsymbol{\alpha},\boldsymbol{\omega}\}$ are replaced by $\{\mathbf{x},\mathbf{y}\}$.

Therefore in the original (not the replica) game the incumbent seeks to minimize:

$$\int_0^\eta \frac{y_i-k}{y_i+x_i}\cdot \mathbf{1}\left[y_i \geq k\right]\ di. \tag{16}$$

With the change of variables, and taking account of the fact that voiceless citizens must receive zero, the incumbent's budget constraint (1) rewrites as:

$$\int_0^\eta \frac{x_i}{(1-\lambda)}\ di \leq \frac{B_1}{\eta}.$$

Multiplying through by $(1-\lambda)$ yields:

$$\int_0^\eta x_i\ di \leq \overline{B}_1.$$

The challenger's problem is dealt with symmetrically.

Incumbent's best response: In either case 3 or 4, the challenger's strategy may be described as follows. The challenger sets $y_i = y^*$ with probability p independent of i, and $y_i = 0$ with probability $(1-p)$. Using expression (8) for v we may write the incumbent's problem as:

$$\min_{\mathbf{x}} \int_0^\eta p \cdot v\left(x_i, y^*; k\right) \; di. \tag{17}$$

$$\text{s.t.} \int_0^\eta x_i \; di \leq \overline{B}_1.$$

The function v is symmetric and strictly convex in $\mathbf{x}$ because $y^* = h\left(\overline{B}_1; k, \lambda\right) > k$ (refer to expression 8), so the solution to problem (17) is $x_i^* = \overline{B}_1$ for all $i \in H$, or $\alpha_i^* = B_1/\eta$.

Challenger's best response. The challenger maximizes incumbent vulnerability, i.e., expression (16), given $x_i^* = \overline{B}_1$ for all i. Using expression (8) for v we may write the challenger's problem as:

$$\max_{\mathbf{y}} \int_0^\eta v\left(\overline{B}_1, y_i; k\right) \; di \tag{18}$$

$$\text{s.t.} \int_0^\eta y_i \; di \leq \overline{B}_2.$$

Let $\overline{v}\left(\alpha, \omega\right)$ denote the concave envelope of $v\left(\alpha, \omega\right)$ (refer to Figure 3). The following problem

$$\max_{\mathbf{y}} \int_0^\eta \overline{v}\left(\overline{B}_1, y_i; k\right) \; di \tag{19}$$

$$\text{s.t.} \int_0^\eta y_i \; di \leq \overline{B}_2.$$

is a relaxed version of problem (18) because $\overline{v}\left(\overline{B}_1, y; k\right) \geq v\left(\overline{B}_1, y; k\right)$. Because the objective function in problem (19) is symmetric and concave in $\mathbf{y}$, the problem's solution is $y_i = \overline{B}_2$ for all i. Therefore the value of the relaxed problem must be $\overline{v}\left(\overline{B}_1, \overline{B}_2; k\right)$.

In case 3, $\overline{B}_2 \geq h\left(\overline{B}_1; k\right)$ implies $\overline{v}\left(\overline{B}_1, \overline{B}_2; k\right) = v\left(\overline{B}_1, \overline{B}_2; k\right)$ (refer to Lemma 2). Therefore the value of the relaxed problem (19) is achievable in the original problem (18) by setting $y_i^* = \overline{B}_2$ for all $i \in [0, \eta]$. This implies that $y_i^* \equiv \overline{B}_2$, or $\omega_i^* = B_2/\eta$ for all $i \in [0, \eta]$, is the solution to the original problem (18).

In case 4, $\overline{B}_2 < h\left(\overline{B}_1; k\right)$ implies $\overline{v}\left(\overline{B}_1, \overline{B}_2; k\right) > v\left(\overline{B}_1, \overline{B}_2; k\right)$, and so the value of the relaxed problem (19) is *not* achievable in the original problem (18) by setting $y_i^* = \overline{B}_2$ for all i. By construction of the concave envelope we have:

$$\overline{v}\left(\overline{B}_1, \overline{B}_2; k\right) = v\left(\overline{B}_1, h\left(\overline{B}_1; k\right)\right) \cdot \frac{\overline{B}_2}{h\left(\overline{B}_1; k\right)}, \tag{20}$$

where $h(\alpha; k)$ is as in expression (6) in light of Lemma 2. Expression (20) shows that the value of the relaxed problem is achievable in the original problem (18) by promising $y_i^* = h\left(\overline{B}_1; k\right)$, or $\omega_i^* = h\left(\overline{B}_1; k\right) / (1-\lambda)$ to a mass $\overline{B}_2 / h\left(\overline{B}_1; k\right)$ of the voiceful citizens, and $y_i^* = 0$ to the rest.

The proof of uniqueness is deferred to Lemma 3 below. ■

Proof of Proposition 2

Proof. For uniqueness, see Lemma 3.

Part 1. Obvious.

Part 2. Observe that the incumbent does not take advantage of the targetability of redistribution (see Proposition 1), therefore the incumbent will promise the voiceful-optimal policy.

Part 3. Part 2 guarantees that the incumbent's strategy is to promise M, so by the same logic as in the proof of Proposition 1 the challenger's is to promise M to everyone if $v(M, M; k) \geq \overline{v}\left(M, \overline{B}; k\right)$, else he will redistribute the budget to the voiceful citizens, and unequally among them. ■

Proof of Corollary 2

Proof. Part 1. Fix (B, η) and let us look for parameter constellations such the challenger promises inequitable redistribution even though $\overline{G} > \overline{B}$. First, let us set k large enough that $\overline{B} < h\left(\overline{B}, k\right)$. We then have $\overline{B} < h\left(\overline{G}; k\right) = h(M; k)$ for any $\overline{G} > \overline{B}$. Finally, refer to Figure 3: in the region $\omega < h(\alpha, k)$ any two values of ω sufficiently close to each other violate $v \geq \overline{v}$. As any choice of $\overline{G} > \overline{B}$ sufficiently close to $\overline{B}$ lies within the region $\omega < h(M, k)$, this choice of $\overline{G}$ produces a violation of condition (9). This means that the challenger's best response is to redistribute B unequally.

Part 2. The incumbent promises the public good iff $G > B/\eta$, and the pairs (G, B) that satisfy this inequality grows as η increases. The challenger promises the public good if, simultaneously, $G \geq B/\eta$ (else redistribution strategically dominates the public good) and condition (9) holds. When $G \geq B/\eta$ holds, condition (9) is violated if and only if:

$$v\left(\overline{G},\overline{G};k\right) < \overline{v}\left(\overline{G},\overline{B};k\right) = \frac{v\left(\overline{G},h\left(\overline{G};k\right)\right)}{h\left(\overline{G};k\right)}\overline{B}. \tag{21}$$

This inequality depends on η only through $\overline{B} = (1-\lambda)B/\eta$. As η increases the set of pairs (G,B) that satisfy condition (21) , i.e., that violate condition (9), shrinks.

Part 3. The incumbent promises the public good iff $G > B/\eta$, which is independent of λ. Let us now turn to the challenger. The statement is vacuous for $k=0$, so let's focus on the case $k>0$. The challenger promises the public good if, simultaneously, $G > B/\eta$ (which is independent of λ) and if condition (21) fails. The right hand side of (21) is strictly positive for every $\lambda < 1$. The left hand side equals zero whenever $\mathbf{1}\left[\overline{G} \geq k\right] = 0$. Therefore, condition (21) holds whenever $k > \overline{G} = (1-\lambda)G$. This condition rewrites as $\lambda > (G-k)/G$.

Part 4. Given any $\alpha > 0, \omega > 0$, picking k sufficiently small ensures that $\overline{v}(\alpha,\omega;k) = v(\alpha,\omega;k)$. In particular, given $\left(M,\overline{B}\right)$, picking k sufficiently small ensures that $\overline{v}\left(M,\overline{B};k\right) = v\left(M,\overline{B};k\right) \leq v(M,M;k)$, where the inequality follows because v is non-decreasing in ω. Therefore condition (9) holds for any pair $\left(M,\overline{B}\right)$ when k is small. The desired result then follows from Proposition 2 part 3. ■

Lemma 3 *There is a unique equilibrium in symmetric strategies in Propositions 1 and 2.*

Proof. The proof deals with the case $(\eta,\lambda) = (1,0)$. The case $(\eta,\lambda) = (1,0)$ is a straightforward extension.

Take any equilibrium in which the challenger uses the symmetric strategy where promises are drawn from the distribution F_2. Expression (7) reads:

$$\begin{aligned} & \int_0^\eta v(\alpha_i,\omega_i;k)\ di \\ = & \int_0^\eta \int_0^\infty v(\alpha_i,\omega;k)\ dF_2(\omega)\ di \\ = & \int_0^\eta Q(\alpha_i;k)\ di, \end{aligned} \tag{22}$$

where the function

$$Q(\alpha_i; k) = \int_0^{\infty} v(\alpha_i, \omega; k) \ dF_2(\omega)$$

is convex in α_i, and indeed strictly so because rationality requires F_2 placing positive probability on some $\omega > k$. Therefore, the problem of minimizing (22) subject to the incumbent's budget constraint (1) yields $\alpha_i = \alpha$ for all i. Hence, in any equilibrium where the challenger uses a symmetric strategy F_2, the incumbent uses the symmetric strategy which is either to promise the public good, or to promise $\frac{B}{\eta}$ (redistribution) to the voiceful citizens only, depending on which strategy gives the most welfare to the voiceful citizens $\max\left[\frac{B}{\eta}, G\right]$. Now, the challenger's best response to this strategy is unique and symmetric, as shown in the proof of Propositions 1 and 2. Therefore, the equilibrium in Proposition 1 is the unique equilibrium in symmetric strategies. ■

Proof of Theorem 2

Proof. Part 1. Obvious because $\widetilde{G}$ is dominated by redistribution.

Part 2: not an equilibrium for both politicians to provide the voiceful-optimal public good.

Suppose both politicians promise the public good. Then the vulnerability index is:

$$\begin{aligned} & \int_0^{\eta} \int_0^{\overline{g}} \frac{(1-\lambda) g - k}{(1-\lambda) g + (1-\lambda) g} \cdot \mathbf{1}\left[(1-\lambda) g \geq k\right] \ \frac{1}{\overline{g}} dg \ di \\ = \ & \frac{\eta}{\overline{g}} \frac{1}{2} \int_{\overline{k}}^{\overline{g}} \frac{g - \overline{k}}{g} \ dg, \end{aligned} \tag{23}$$

where we denote $\overline{k} = k/(1-\lambda)$ If the incumbent deviates to equal redistribution $\alpha_i \equiv \frac{B}{\eta}$ the vulnerability index is:

$$\begin{aligned} & \int_0^{\eta} \int_0^{\overline{g}} \frac{g - \overline{k}}{g + \left(\frac{B}{\eta}\right)} \cdot \mathbf{1}\left[g \geq \overline{k}\right] \ \frac{1}{\overline{g}} dg \ di \\ = \ & \frac{\eta}{\overline{g}} \int_{\overline{k}}^{\overline{g}} \frac{g - \overline{k}}{g + \left(\frac{B}{\eta}\right)} \ dg. \end{aligned} \tag{24}$$

We seek values of $\overline{g}$ such that the deviation is profitable for the incumbent. Because the incumbent seeks to *minimize* vulnerability, the deviation is profitable for the incumbent if (23) is larger than (24), that is:

$$\frac{1}{2}\int_{\overline{k}}^{\overline{g}} \frac{g-\overline{k}}{g}\, dg > \int_{\overline{k}}^{\overline{g}} \frac{g-\overline{k}}{g+\left(\frac{B}{\eta}\right)}\, dg. \tag{25}$$

For given parameters n, b, the following antiderivative formula is known:

$$\int \frac{x+n}{x+b}\, dx = x + (n-b)\ln|x+b| + \text{constant}.$$

Using this formula, condition (25) can be written as:

$$\frac{1}{2}\left\{\left[\overline{g}-\overline{k}\ln(\overline{g})\right]-\left[\overline{k}-\overline{k}\ln\left(\overline{k}\right)\right]\right\} > \left[\overline{g}+\left(-\overline{k}-\frac{B}{\eta}\right)\ln\left(\overline{g}+\frac{B}{\eta}\right)\right]-\left[\overline{k}+\left(-\overline{k}-\frac{B}{\eta}\right)\ln\left(\overline{k}+\frac{B}{\eta}\right)\right]. \tag{26}$$

Express $\overline{g}$ as the following monotone transformation of the ancillary parameter C:

$$\overline{g} = (C-1)\frac{B}{\eta} + C\overline{k}. \tag{27}$$

Substitute this expression for $\overline{g}$ into (26) and perform some algebra (see Appendix C) to get:

$$2\left(\frac{B}{\eta\overline{k}}+1\right)\left[\ln(C)-\frac{(C-1)}{2}\right] > \ln\left(C+(C-1)\frac{B}{\eta\overline{k}}\right). \tag{28}$$

The term in brackets on the LHS is a single-peaked function of C that is positive if:

$$\ln(2c+1) > c,$$

where I denote $c = (C-1)/2$. This is the case for $c \in (0, 1.2564)$. In this interval, both sides of inequality (28) go to infinity as $\overline{k} \to 0$ or $\eta \to 0$ but the LHS grows faster, so for $\overline{k}$ or η small enough inequality (28) holds. Within this interval the set of c's that make the public good voiceful-optimal is that which satisfies the condition $\overline{g}/2 > \frac{B}{\eta}$; substitute

$$\frac{\overline{g}}{2} = c\left(\frac{B}{\eta}\right) + \left(c+\frac{1}{2}\right)\overline{k} \tag{29}$$

from (27) into the condition and isolate c to get the condition:

$$c > \frac{2B-\eta\overline{k}}{2B+2\eta\overline{k}}, \tag{30}$$

which is satisfied for any $\left(\eta,\overline{k}\right)$ if $c>1$. Therefore, if $c\in(1,1.2564)$ the public good is voiceful-optimal but, for $\overline{k}$ or η small enough, it is not an equilibrium for both politicians to provide it. Use (29) to characterize the values of the public good with the desired property. This is the set of all $\widetilde{G}$'s such that:

$$\mathbb{E}\left(\widetilde{G}\right)=\frac{\overline{g}}{2}=c\left(\frac{B}{\eta}\right)+\left(c+\frac{1}{2}\right)\overline{k}\text{ for }c\in(1,1.2564).$$

For $k\to 0$ this set converges to the set:

$$\left\{\widetilde{G}:\mathbb{E}\left(\widetilde{G}\right)=\frac{cB}{\eta}\text{ for }c\in(1,1.2564)\right\}.$$

Part 2: equilibrium with egalitarian redistribution

Incumbent's best response. Suppose the challenger promises $\omega_i=B/\eta$ for all $i\in H$. The vulnerability index is:

$$\int_0^\eta \frac{\left(\frac{B}{\eta}\right)-\overline{k}}{\left(\frac{B}{\eta}\right)+\alpha_i}\cdot\mathbf{1}\left[\frac{B}{\eta}\geq\overline{k}\right]\,di.$$

This is a symmetric and strictly convex function of $\boldsymbol{\alpha}$ for $\frac{B}{\eta}\geq\overline{k}$. If the incumbent is restricted to using redistribution, then her best response is to set $\alpha_i=B/\eta$ for all $i\in[0,\eta]$. The value of the incumbent's problem assuming $\frac{B}{\eta}\geq\overline{k}$ is:

$$\begin{aligned}&\int_0^\eta \frac{1}{2}\frac{\left(\frac{B}{\eta}\right)-\overline{k}}{\left(\frac{B}{\eta}\right)}\,di\\ =\;&\frac{1}{2}\eta\frac{\left(\frac{B}{\eta}\right)-\overline{k}}{\left(\frac{B}{\eta}\right)}\end{aligned}\qquad(31)$$

Now remove the restriction: would the incumbent benefit from promising $\widetilde{G}$? Assuming $\frac{B}{\eta}\geq\overline{k}$ the vulnerability index after a deviation to $\widetilde{G}$ would read:

$$\int_0^\eta\int_0^{\overline{g}}\frac{\left(\frac{B}{\eta}\right)-\overline{k}}{\left(\frac{B}{\eta}\right)+g}\,\frac{1}{\overline{g}}dg\;di.\qquad(32)$$

After some algebra (see Appendix C) we get that the vulnerability after a deviation is larger than that under the posited equilibrium strategy, i.e., (32) is larger than (31) if and only if:

$$\mathbb{E}\left(\widetilde{G}\right) = \overline{g}/2 < c\left(\frac{B}{\eta}\right) \tag{33}$$

where $c \approx 1.26$ is the solution to $c = \log(1+2c)$.

Challenger's best response. Suppose the incumbent promises $\alpha_i = B/\eta$ for all $i \in [0,\eta]$. The vulnerability index is:

$$\int_0^\eta \frac{\omega_i - \overline{k}}{\omega_i + \left(\frac{B}{\eta}\right)} \cdot \mathbf{1}\left[\omega_i \geq \overline{k}\right] \; di.$$

This is a symmetric and strictly concave function of $\boldsymbol{\omega}$ for $\frac{B}{\eta} \geq \overline{k}$. Suppose the challenger is restricted to using redistribution. Then his best response does no worse than setting $\alpha_i = B/\eta$ for all $i \in [0,\eta]$, and the value of the challenger's problem is greater or equal than (31). Now remove the restriction: would the challenger benefit from promising $\widetilde{G}$? The vulnerability index would read:

$$\int_0^\eta \int_{\overline{k}}^{\overline{g}} \frac{g - \overline{k}}{g + \left(\frac{B}{\eta}\right)} \frac{1}{\overline{g}} dg \; di,$$

which is exactly equal to (24). After some algebra (see Appendix C) we get that (31) is greater than (24), and so the challenger would not benefit from promising $\widetilde{G}$, if and only if

$$\mathbb{E}\left(\widetilde{G}\right) = \overline{g}/2 < c\left(\frac{B}{\eta}\right) \tag{34}$$

where $c \approx 1.26$ is the solution to $c = \ln(1+2c)$.

Part 3. Suppose candidate j plays the prescribed equilibrium strategy, i.e., a uniform $[0,m]$ where $m = \max\left[\overline{g}, 2B/\eta\right]$. I will show that candidate $-j$'s best response is to promise the distribution with the highest possible mean, regardless of the specific shape of that distribution. Therefore a best response to j's prescribed equilibrium strategy is to play the public good or redistribution, whichever is more efficient, which is in fact $-j$'s prescribed equilibrium strategy.

If candidate j draws her promises to the voiceful citizens from $U[0,m]$ and candidate $-j$ from a probability distribution X_{-j}, candidate $-j$'s vote share is:

$$\begin{aligned} S_{-j} &= \eta \int_0^m \frac{x}{m} \, dF_{-j}(x) \\ &\leq \frac{\eta}{m} \mathbb{E}(X_{-j}) \end{aligned}$$

where $F_{-j}(x)$ represents the probability that X_{-j} is less than or equal to x, and equality holds when $X_{-j} \leq m$. This concludes the proof. ■

B Making the consensual democracy model realistic

Citizens perfectly coordinate on the winner in the equilibrium of my model. Perfect coordination is an unappealing feature in a model that seeks to approximate democratic elections. However, this unappealing feature is easily mitigated. Suppose that λ is close to 1, meaning that supporting the losing politician hardly reduces the voter's material benefits. In this political system, irrespective of the probability of incumbent change, ideological voters do not give up much material benefit by voting their ideology. So one can write a model where a fraction of voters is ideological, and the rest are as in my model. When λ is low the ideological voters will vote their ideology (left or right, for example) but the non-ideological voters will perfectly coordinate on the winning politician provided that they are pivotal, i.e., they can decide the election. In such a game we will have votes for both politicians in equilibrium, which is more realistic.

A second dimensions in which the consensual democracy model can be made more realistic is the incumbent replacement condition. The model assumes that the challenger wins if condition (4) holds, but this is not the same as $a = \int_0^1 a_i \; di \geq 1/2$, which is that the challenger wins if his vote share exceeds 1/2. Helpfully, Proposition 1 by Szkup (2020) shows that the equilibrium analysis in Section 3 extends verbatim if I assume that incumbent replacement happens when

$$R(\theta, a) \leq 0,$$

where R is *any smooth and strictly decreasing function in θ and a* with the property that $R\left(\overline{\theta}, 0\right) < 0$ (the regime may change even if no citizen supports the challenger) and $R\left(\underline{\theta}, 1\right) > 0$ (the incumbent may survive even if all citizens protest). This result is helpful because a function R^* with these properties can be found that lies as close as we wish to the function

$$\widehat{R}(\theta, a) = \frac{1}{2} - a,$$

except for a set of arbitrarily small measure. The function $\widehat{R}$ expresses the democratic replacement rule. By Szkup's (2020) result, the voters' equilibrium strategies in my game under R^* are as specified in Section 3. Furthermore, since the function R^* is decreasing in a, in my game under R^* the autocrat will seek to minimize a and the challenger to maximize it. In sum, the entire analyis developed in this paper goes through if we replace condition (4) with the condition $R^*(\theta, a) \leq 0$, which is a rule for incumbent replacement that closely approximates the democratic rule.

C Appendix not for publication: calculations for the proof of Theorem 2

From (26) to (28)

Due to our choice of $\overline{g}$ as in (27) we have

$$\left(\overline{g}+\frac{B}{\eta}\right)=C\left(\overline{k}+\frac{B}{\eta}\right).$$

Substitute into (26) to get:

$$\begin{aligned}
\frac{1}{2}\left(\overline{g}-\overline{k}\right)-\left(\overline{k}+\frac{B}{\eta}\right)\left[\ln\left(\overline{g}+\frac{B}{\eta}\right)-\ln\left(\overline{k}+\frac{B}{\eta}\right)\right] &< \frac{1}{2}\left\{-\overline{k}\left[\ln\left(\overline{g}\right)-\ln\left(\overline{k}\right)\right]\right\}\\
\frac{1}{2}\left(\overline{g}-\overline{k}\right)-\left(\overline{k}+\frac{B}{\eta}\right)\ln\left(C\right) &< \frac{1}{2}\left\{-\overline{k}\ln\left(C+\frac{\left(C-1\right)}{\overline{k}}\frac{B}{\eta}\right)\right\}\\
\frac{1}{2}\left(\left(C-1\right)\frac{B}{\eta}+C\overline{k}-\overline{k}\right)-\left(\overline{k}+\frac{B}{\eta}\right)\ln\left(C\right) &< \frac{1}{2}\left\{-\overline{k}\ln\left(C+\frac{\left(C-1\right)}{\overline{k}}\frac{B}{\eta}\right)\right\}\\
\frac{1}{2}\left(\left(C-1\right)\frac{B}{\eta}+\left(C-1\right)\overline{k}\right)-\left(\overline{k}+\frac{B}{\eta}\right)\ln\left(C\right) &< \frac{1}{2}\left\{-\overline{k}\ln\left(C+\frac{\left(C-1\right)}{\overline{k}}\frac{B}{\eta}\right)\right\}\\
\frac{\left(C-1\right)}{2}\left(\frac{B}{\eta}+\overline{k}\right)-\left(\overline{k}+\frac{B}{\eta}\right)\ln\left(C\right) &< \frac{1}{2}\left\{-\overline{k}\ln\left(C+\frac{\left(C-1\right)}{\overline{k}}\frac{B}{\eta}\right)\right\}\\
\left(\frac{B}{\eta}+\overline{k}\right)\left[\frac{\left(C-1\right)}{2}-\ln\left(C\right)\right] &< \frac{1}{2}\left\{-\overline{k}\ln\left(C+\frac{\left(C-1\right)}{\overline{k}}\frac{B}{\eta}\right)\right\}\\
\left(\frac{B}{\eta\overline{k}}+1\right)\left[\frac{\left(C-1\right)}{2}-\ln\left(C\right)\right] &< -\frac{1}{2}\left\{\ln\left(C+\left(C-1\right)\frac{B}{\eta\overline{k}}\right)\right\}\\
\left(\frac{B}{\eta\overline{k}}+1\right)\left[\ln\left(C\right)-\frac{\left(C-1\right)}{2}\right] &> \frac{1}{2}\left\{\ln\left(C+\left(C-1\right)\frac{B}{\eta\overline{k}}\right)\right\}\\
2\left(\frac{B}{\eta\overline{k}}+1\right)\left[\ln\left(C\right)-\frac{\left(C-1\right)}{2}\right] &> \ln\left(C+\left(C-1\right)\frac{B}{\eta\overline{k}}\right)
\end{aligned}$$

Getting expression (30)

$$
\begin{aligned}
c\left(\frac{B}{\eta}\right)+\left(c+\frac{1}{2}\right)\overline{k} &> \frac{B}{\eta} \\
c\left(\frac{B}{\eta}\right)+c\overline{k} &> \frac{B}{\eta}-\frac{1}{2}\overline{k} \\
c\left(\frac{B}{\eta}+\overline{k}\right) &> \frac{B}{\eta}-\frac{1}{2}\overline{k} \\
c\left(\frac{B+\eta\overline{k}}{\eta}\right) &> \frac{2B}{2\eta}-\frac{\eta\overline{k}}{2\eta} \\
c &> \frac{2B-\eta\overline{k}}{2B+2\eta\overline{k}}
\end{aligned}
$$

Getting to (33)

The vulnerability index after the deviation reads:

$$
\begin{aligned}
& \frac{1}{\overline{g}}\int_0^{\eta}\int_0^{\overline{g}}\frac{\left(\frac{B}{\eta}\right)-\overline{k}}{\left(\frac{B}{\eta}\right)+\alpha}\,d\alpha di \\
= & \frac{1}{\overline{g}}\eta\int_0^{\overline{g}}\frac{\left(\frac{B}{\eta}\right)-\overline{k}}{\left(\frac{B}{\eta}\right)+\alpha}\,d\alpha \\
= & \frac{1}{\overline{g}}\eta\left[\left(\frac{B}{\eta}\right)-\overline{k}\right]\int_0^{\overline{g}}\frac{1}{\left(\frac{B}{\eta}\right)+\alpha}\,d\alpha \\
= & \frac{1}{\overline{g}}\eta\left[\left(\frac{B}{\eta}\right)-\overline{k}\right]\int_{\frac{B}{\eta}}^{\left(\frac{B}{\eta}\right)+\overline{g}}\frac{1}{x}\,dx \\
= & \frac{1}{\overline{g}}\eta\left[\left(\frac{B}{\eta}\right)-\overline{k}\right]\left[\ln(x)\right]_{\frac{B}{\eta}}^{\left(\frac{B}{\eta}\right)+\overline{g}}.
\end{aligned}
$$

So the incumbent prefers equal redistribution to the public good iff the vulnerability at the public good is larger, that is, if

$$\begin{aligned}
\frac{1}{2}\eta\frac{\left(\frac{B}{\eta}\right)-\overline{k}}{\left(\frac{B}{\eta}\right)} &< \frac{1}{\overline{g}}\eta\left[\left(\frac{B}{\eta}\right)-\overline{k}\right]\left[\log\left(x\right)\right]_{\frac{B}{\eta}}^{\left(\frac{B}{\eta}\right)+\overline{g}} \\
\frac{1}{2}\frac{1}{\left(\frac{B}{\eta}\right)} &< \frac{1}{\overline{g}}\left[\ln\left(x\right)\right]_{\frac{B}{\eta}}^{\left(\frac{B}{\eta}\right)+\overline{g}} \\
\frac{\overline{g}}{2}\frac{1}{\left(\frac{B}{\eta}\right)} &< \ln\left(\frac{B}{\eta}+\overline{g}\right)-\ln\left(\frac{B}{\eta}\right)
\end{aligned}$$

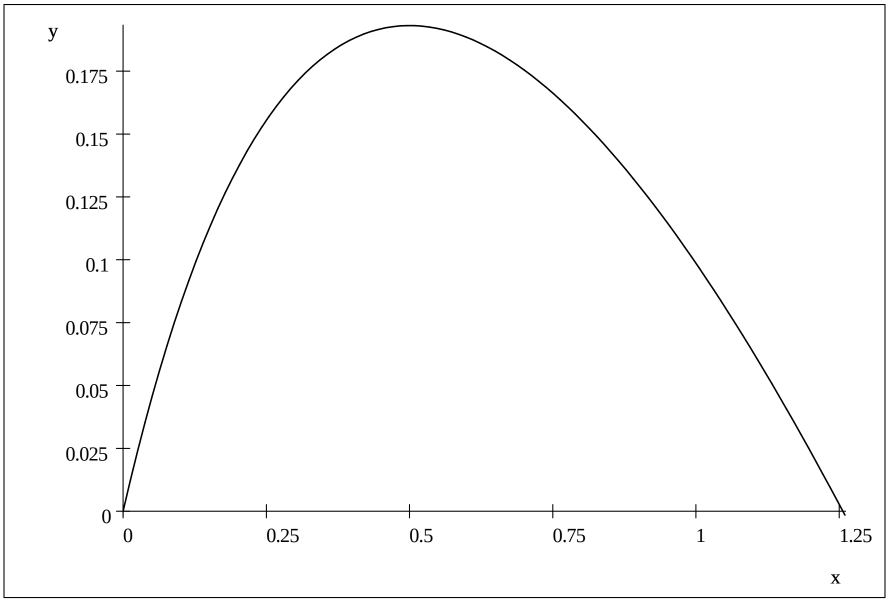
We need

$$\begin{aligned}
\frac{\overline{g}}{2}\frac{1}{\left(\frac{B}{\eta}\right)} &< \ln\left(\frac{B}{\eta}+\overline{g}\right)-\ln\left(\frac{B}{\eta}\right) \\
\frac{\overline{g}}{2} &< \left(\frac{B}{\eta}\right)\ln\left(\frac{B}{\eta}+\overline{g}\right)-\left(\frac{B}{\eta}\right)\ln\left(\frac{B}{\eta}\right) \\
\overline{g} &< 2\left(\frac{B}{\eta}\right)\ln\left(\frac{B}{\eta}+\overline{g}\right)-2\left(\frac{B}{\eta}\right)\ln\left(\frac{B}{\eta}\right) \\
2\left(\frac{B}{\eta}\right)\ln\left(\frac{B}{\eta}\right) &< 2\frac{B}{\eta}\ln\left(\frac{B}{\eta}+\overline{g}\right)-\overline{g}
\end{aligned}$$

Replace $\overline{g}=c2\frac{B}{\eta}$. Then we get:

$$\begin{aligned}
2\left(\frac{B}{\eta}\right)\ln\left(\frac{B}{\eta}\right) &< 2\frac{B}{\eta}\ln\left(\frac{B}{\eta}+c2\frac{B}{\eta}\right)-c2\frac{B}{\eta} \\
2\left(\frac{B}{\eta}\right)\ln\left(\frac{B}{\eta}\right) &< 2\frac{B}{\eta}\ln\left(\frac{B}{\eta}\left(1+c2\right)\right)-c2\frac{B}{\eta} \\
\ln\left(\frac{B}{\eta}\right) &< \left[\ln\left(\frac{B}{\eta}\right)+\ln\left(1+2c\right)\right]-c \\
0 &< \log\left(1+2c\right)-c.
\end{aligned}$$

The function $\log\left(1+2c\right)-c$



is positive if $c \in (0, 1.26)$. After restricting to values of $c \geq 1$ in order to ensure that the public good efficient we get the condition

$$\frac{\overline{g}}{2\left(\frac{B}{\eta}\right)} = \frac{\mathbb{E}\left(\widetilde{G}\right)}{\left(\frac{B}{\eta}\right)} = c \in (1, 1.26).$$

Getting to (34)

We need (31) greater than (24), i.e.:

$$\begin{aligned}
\frac{1}{2}\eta\frac{\left(\frac{B}{\eta}\right)-\overline{k}}{\left(\frac{B}{\eta}\right)} &> \frac{\eta}{\overline{g}}\int_{\overline{k}}^{\overline{g}}\frac{g-\overline{k}}{g+\left(\frac{B}{\eta}\right)}\,dg \\
\frac{1}{2}\eta\frac{\left(\frac{B}{\eta}\right)-\overline{k}}{\left(\frac{B}{\eta}\right)} &> \frac{\eta}{\overline{g}}\left[\overline{g}+\left(-\overline{k}-\frac{B}{\eta}\right)\ln\left(\overline{g}+\frac{B}{\eta}\right)\right]-\left[\overline{k}+\left(-\overline{k}-\frac{B}{\eta}\right)\ln\left(\overline{k}+\frac{B}{\eta}\right)\right] \\
\frac{\overline{g}}{2}\left[\frac{\left(\frac{B}{\eta}\right)-\overline{k}}{\left(\frac{B}{\eta}\right)}\right] &> \overline{g}-\left(\overline{k}+\frac{B}{\eta}\right)\ln\left(\overline{g}+\frac{B}{\eta}\right)-\overline{k}+\left(\overline{k}+\frac{B}{\eta}\right)\ln\left(\overline{k}+\frac{B}{\eta}\right) \\
\frac{\overline{g}}{2}\left[\frac{\left(\frac{B}{\eta}\right)-\overline{k}}{\left(\frac{B}{\eta}\right)}\right] &> \overline{g}-\overline{k}+\left(\overline{k}+\frac{B}{\eta}\right)\ln\left(\frac{\overline{k}+\frac{B}{\eta}}{\overline{g}+\frac{B}{\eta}}\right)
\end{aligned}$$

Due to our choice of $\overline{g}$ as in (27) we have

$$\begin{aligned}\left(\overline{g}+\frac{B}{\eta}\right) &= C\left(\overline{k}+\frac{B}{\eta}\right)\\ \overline{g} &= C\left(\overline{k}+\frac{B}{\eta}\right)-\frac{B}{\eta}\end{aligned}$$

and so the inequality rewrites as:

$$\begin{aligned}
\frac{C\left(\overline{k}+\frac{B}{\eta}\right)-\frac{B}{\eta}}{2}\left[\frac{\left(\frac{B}{\eta}\right)-\overline{k}}{\left(\frac{B}{\eta}\right)}\right] &> \left((C-1)\frac{B}{\eta}+C\overline{k}-\overline{k}\right)+\left(\overline{k}+\frac{B}{\eta}\right)\ln\left(\frac{1}{C}\right)\\
\left[C\left(\overline{k}+\frac{B}{\eta}\right)-\frac{B}{\eta}\right]\left[\frac{\left(\frac{B}{\eta}\right)-\overline{k}}{\left(\frac{B}{\eta}\right)}\right] &> 2\left[(C-1)\frac{B}{\eta}+C\overline{k}-\overline{k}\right]+2\left(\overline{k}+\frac{B}{\eta}\right)\ln\left(\frac{1}{C}\right)\\
\left[C\left(\overline{k}+\frac{B}{\eta}\right)-\frac{B}{\eta}\right]\left[\frac{\left(\frac{B}{\eta}\right)-\overline{k}}{\left(\frac{B}{\eta}\right)}\right] &> 2(C-1)\left(\frac{B}{\eta}+\overline{k}\right)+2\left(\overline{k}+\frac{B}{\eta}\right)\ln\left(\frac{1}{C}\right)\\
\left[C\left(\overline{k}+\frac{B}{\eta}\right)-\frac{B}{\eta}\right]\left[\frac{\left(\frac{B}{\eta}\right)-\overline{k}}{\left(\frac{B}{\eta}\right)}\right] &> 2\left(\frac{B}{\eta}+\overline{k}\right)\left[(C-1)+\ln\left(\frac{1}{C}\right)\right]\\
2\left(\frac{B}{\eta}+\overline{k}\right)\left[\ln(C)-(C-1)\right] &> \frac{B}{\eta}\left[\frac{\left(\frac{B}{\eta}\right)-\overline{k}}{\left(\frac{B}{\eta}\right)}\right]-C\left(\overline{k}+\frac{B}{\eta}\right)\\
\left(\frac{B}{\eta}+\overline{k}\right)\left[2\ln(C)-2(C-1)\right] &> \left[\frac{B}{\eta}-\overline{k}\right]-C\left(\overline{k}+\frac{B}{\eta}\right)\\
\left(\frac{B}{\eta}+\overline{k}\right)\left[2\ln(C)-C+2\right] &> \left[\frac{B}{\eta}-\overline{k}\right]\\
2\ln(C)-C &> -2+\frac{\left[\frac{B}{\eta}-\overline{k}\right]}{\left(\frac{B}{\eta}+\overline{k}\right)}
\end{aligned}$$

Now with the change of variables $c=(C-1)/2$ we get $C=2c+1$:

$$\begin{aligned}
2\ln(C) &> C-2+\frac{\left[\frac{B}{\eta}-\overline{k}\right]}{\left(\frac{B}{\eta}+\overline{k}\right)}\\
2\ln(2c+1) &> 2c-1+\frac{\left[\frac{B}{\eta}-\overline{k}\right]}{\left(\frac{B}{\eta}+\overline{k}\right)},
\end{aligned}$$

which for $\overline{k} \to 0$ reduces to the desired inequality:

$$2 \ln (2c + 1) > 2c.$$