REPUTATION AND ALLOCATION OF OWNERSHIP*

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Abstract

We show that allocation of ownership matters even in a long-term relationship where problems of opportunism are less severe unless agents are very patient. Ownership structure is chosen to give the agents best incentives to cooperate. The optimal control structure of the static game restricts the gain from deviation to be the lowest but also the punishment will be minimal. The worst ownership structure of the one-shot game is good in the repeated setting, because it provides the highest punishment but bad, because the gain from deviation is also the highest. We show that when investment costs are very elastic partnership and a hostage-type solution arise in equilibrium. While when costs are moderately elastic the results of the one-shot game apply.

Keywords: Ownership, reputation, allocation, one-shot game, investment costs, agents, optimal control structure.

1 Introduction

Partnership, the main organizational form in professional services industries, has long been a puzzle for the economists. According to the present theory there is a more efficient way to organize production: no sharing rule can implement efficient actions in a partnership while efficiency can be attained by hiring a third party to "break the budget" (Holmstrom (1982)). Further, in the incomplete contracting theory of the firm (Grossman and Hart (1986) and Hart and Moore (1990)) joint ownership is a dominated structure. We show that when the agents care about their reputation partnership can be not only efficient but better than any other organizational form.[1]

The incomplete contracting theory of the firm is based on agents' opportunistic behaviour – self-interest seeking with guile in Williamson's (1985) terminology.[2] When there are quasi rents from the relationship due to firm-specific investments and high transaction costs prevent writing a complete contract holdup problem typically arises. An agent pays full cost of the investment but part of the value is expropriated in ex post bargaining. According to this theory ownership rights should be allocated to minimize the holdup problem. However, the behaviour we observe in the real world is not always opportunistic: workers are loyal to their employers, firms offer good quality products to their customers etc. Macaulay (1963) finds in his survey on contractual relations in business that:

> *"Businessmen often prefer to rely on 'man's word' in a brief letter, a handshake, or 'common honesty and decency' – even when the transaction involves exposure to serious risks."*

This kind of behaviour can be explained by reputation concerns. When the one-shot gain from opportunistic behaviour is outweighed by the loss of trust in the future we should observe loyalty and honesty. The situations that involve firm-specific investments are exactly the ones where we would expect long term relationships to predominate and where reputation effects should matter. Can we avoid holdup problem under any ownership structure? Is

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[1] In Radner (1986) partnership can be efficient in a repeated game but it is not explained why partnership would be better than any other organizational form.

[2] Williamson (1975) and (1985), Klein, Crawford and Alchian (1978), Grossman and Hart (1986), Hart and Moore (1990) and Bolton and Whinston (1993).

there any scope for allocation of ownership in the long term relationship? These are the issues raised in this paper. Repeated game is a natural way to analyse reputation effects.

We show that allocation of ownership indeed matters even in a repeated relationship unless agents are very patient. Two types of equilibria exist: one where partnership and a hostage solution are optimal and second where the results of the static game apply.

According to the incomplete contracting theory ownership increases an agent's bargaining power by raising her outside option. The ownership rights – outside options – should be allocated so that the agents have best incentives to invest in firm-specific human capital. Now if under some allocation the ownership of an asset does not affect its owner's outside option (at the margin) then we should reallocate this asset to an agent to whom it is useful; an asset should improve its owner's incentives. This principle gives some nice results on the optimal ownership structure in the static setting.[3] One important determinant is the degree of complementarity between the assets; strictly complementary assets should be owned together while for economically independent assets independent control is optimal. The second result relates to the importance of agent's investment; if only one agent has an investment she should own all assets. Lastly, if an agent is very important as a trading partner, i.e. indispensable to an asset, then he should own this asset.

In the static game we allocate ownership to give the agents the highest outside options. In the dynamic game the ownership structure is chosen to give the agents best incentives to cooperate. The best ownership structure is such that the gain from cheating is lowest relative to the punishment. The worst ownership structure of the one-shot game (no outside options) has the advantage in the repeated game that it provides the highest punishment; the joint surplus is the lowest in the punishment path. However, the highest punishment does not imply that cooperation would be most sustainable. It is also true that when the punishment is highest so is the gain from deviation. When an agent cheats in investment the cooperation breaks down immediately: the surplus will be divided noncooperatively. When there are no outside options the bargaining will result in an even split of the surplus; the deviant gets half of the surplus generated by the opponent's first-best in-

[3]These are the results of Hart and Moore (1990).

vestment and gains a lot from deviation. While when the agents have outside options then the deviant cannot extract as much as half of the value of the efficient investment in bargaining and therefore gains less from cheating. The trade-off present in the repeated game is the following. Ownership structure with no outside options is good because it provides the highest punishment but bad because the gain from cheating is also the highest. The optimal control structure of the static game restricts the gain from deviation to be the lowest but also the punishment will be minimal.

An ownership structure that maximizes punishment is optimal when the investment costs are very elastic. Partnership with a unanimity clause guarantees maximal punishment. The agents cannot use the assets unless they reach a unanimous agreement; cheating would lead to an outcome with very low surplus in the future. Partnership is optimal if compared to the optimal structure of the one-shot game the relative increase in the punishment is greater than the relative increase in the gain from deviation. When investment costs are very elastic the gain from cheating is high; the agents can make a big cost saving by cheating. In the same time punishment is relatively low; the first-best surplus is not much greater than the noncooperative one when the high investment is expensive. When the gain is high and the punishment is low it is easier to obtain a higher *relative* change in punishment. It is optimal to put all the weight in maximizing punishment although then also the gain from deviation will be the highest; partnership is optimal. Reputation effects thus provide a new explanation for partnerships.

Separating strictly complementary assets or giving all the control rights to a noninvesting agent are equally good ways of providing maximal punishment. These are hostage type solutions to prevent opportunism as discussed in Klein (1980) and Williamson (1983) and (1993). Franchising provides an example of hostages: sometimes the franchisor requires franchisees to rent from them short term the land on which their outlet is located (Klein (1980)). Dnes (1992) found that five out of the fifteen franchisors in his sample control the leases of franchisees. A similar practice can be found in the petroleum coke industry: the producers both sell coke to the calciner, and own and lease the land on which the plant of the calciner is built (Goldberg and Erickson (1982)). Also in some petrol stations oil companies own the pumps and the underground tanks while the retailer owns the land and the building.[4]

[4]This equilibrium has not always been sustained. It has happened in Finland that after

When investment costs are moderately elastic the results of the one-shot game broadly apply. These results were given at the beginning of the introduction. The optimal control structure of the static game gives the agents the highest possible outside options and therefore best restricts the gain from deviation. For not very elastic investment costs the gain from deviation is low and the punishment is high and it is easier to induce higher relative change in the gain; thus it is optimal to minimize the gain from deviation. Interestingly the predictions of the one-shot and the repeated game do not fully coincide in this parameter range: integration is less likely in the repeated game. Klein (1980) and Coase (1988) suggest that reputation and integration are substitutes in dealing with the problem of opportunism. They refer to models where the benefit of integration is reduced holdups and the costs of integration are something else (for example arising from bureaucracy). Clearly then reputation concerns make integration less likely; the benefits are lower and the costs have not changed. In our model both the benefits and costs of integration change and it is not a priori clear which way the reputation effect goes. It turns out that integration is less likely which is consistent with Coase's and Klein's conjecture.

The rest of the paper is organized as follows. A numerical example is presented in Section 2. Section 3 introduces our main model where only one agent has an investment. Section 4 briefly discusses the results of the one-shot game. The repeated game is analysed in Section 5. In Section 6 we extend the model to include investment by both agents. Section 7 discusses related literature.

2 An Example

We start with a simple numerical example. There is one asset, a, and two agents, 1 and 2. Ex ante agent 1 makes an investment in specific human capital. The investment can take three values: 0, 150, or 300. The cost of the investment is 0, 90, or 40γ respectively where $5 < \gamma < 6$. Accordingly the highest investment, 300, maximizes the joint surplus.[5] Asset a is essential to agent 1: her investment has no value unless she has access to the

disagreements oil companies have torn away their pumps and dug out their tanks.

[5]The assumption $\gamma < 6$ guarantees that the highest investment is efficient and $\gamma > 5$ is required for the costs to be convex.

asset. Agent 2 does not have an investment but he is important as a trading partner: without his contribution agent 1 can realize only $\frac{1}{3}$ of the value of her investment (that is, either 0, 50, or 100). Due to high transaction costs ex ante contracts can be written only on the allocation of ownership. We compare two ownership structures: joint ownership and agent 1 control. In ex post bargaining the agents split the difference.

Under joint ownership the agents have to reach a unanimous agreement on the use of the asset. Therefore in the one-shot game agent 1 receives only half of the value of her investment which does not cover its cost (both $\frac{1}{2}150 < 90$ and $\frac{1}{2}300 < 40\gamma$). 1 will not invest and the joint surplus is equal to zero. When agent 1 owns the asset she receives $\frac{2}{3}$ of the value ($\frac{2}{3} = \frac{1}{2}(1 + \frac{1}{3})$). Then 1 will choose the medium investment, 150, since $\frac{2}{3}150 > 90$ and $\frac{2}{3}300 < 40\gamma$. The joint surplus is equal to 60 and 1's share of it is equal to 10. The prediction of the one-shot game is that the only investing agent should own the asset.

When the trading relationship is repeated rather than one-shot the agents may support first best by the following trigger strategy. Agent 1 implicitly agrees to make the first-best investment and agent 2 in turn agrees to pay a transfer equal to 1's payoff in the one-shot game plus the cost of the first-best investment. Any deviations will trigger the outcome of the one-shot game as punishment: splitting the difference in bargaining and underinvestment by agent 1. The only way agent 2 can punish is by not paying the promised transfer but instead split the difference. If 1 cheats in investment, 2 observes it and starts the punishment already in the second half of the same period. Agent 1 chooses the cheating investment taking into account that the surplus will be divided by the split-the-difference rule. Therefore the cheating investment and the payoff from cheating equal to their levels in the one-shot game. While if agent 1 makes the first-best investment, 2 pays her the one-shot payoff plus her investment cost. Accordingly agent 1 has no incentive to cheat.

Under *joint ownership* if cooperation is sustained agent 2's payoff is (300-40γ) per period: he pays the investment cost (40γ) and 1's one-shot payoff (0) and receives the rest of the surplus. However, by refusing to pay the promised transfer to agent 1 he can extract more surplus. In fact, he can obtain half of the value of 1's investment, that is 150. But if 2 cheats in this period, from next period on 1 will not invest and 2 receives zero payoff. Agent 2 does not cheat if and only if the discounted payoff stream from the efficient behaviour exceeds the payoff stream from the deviation path:

$$\frac{1}{1-\delta}(300-40\gamma) \geq 150 \tag{1}$$

where δ is the discount factor. Equation (1) is equivalent to:

$$\delta \geq (40\gamma - 150)/150. \tag{2}$$

When *agent 1 owns the asset* agent 2 has to pay her (40γ +10) to implement first-best investment. This leaves him with (290 - 40γ). By cheating agent 2 could obtain 100 ($=\frac{1}{3}300$) but would get only 50 ($=\frac{1}{3}150$) in the following periods. Agent 2 will not cheat if and only if:

$$\frac{1}{1-\delta}(290-40\gamma) \geq 100 + \frac{\delta}{1-\delta}50 \tag{3}$$

which is equivalent to

$$\delta \geq (40\gamma - 190)/50. \tag{4}$$

Accordingly, joint ownership – the worst structure of the static game – implements first best for a greater range of discount factors than agent 1 control if and only if the right-hand-side of (2) is smaller than the right-hand-side of (4), that is iff $\gamma > 5\frac{1}{4}$. While agent 1 control is optimal for $\gamma < 5\frac{1}{4}$.

To understand this result we have constructed Table 1 which gives the gain (G) and the loss (L) from deviation under the two structures. The gain shows how much more agent 2 can obtain by cheating than by cooperating: he does not have to pay the investment cost but receives a smaller share of the value of the investment. The loss describes how much lower the payoff is in the punishment path relative to cooperation: the gross surplus is lower due to underinvestment but there is a cost saving

	joint ownership	*agent 1 control*
G	$\frac{1}{2}300-(300-40\gamma)=40\gamma-150$	$\frac{1}{3}300-(290-40\gamma)=40\gamma-190$
L	$300-40\gamma$	$(290-40\gamma)-\frac{1}{3}150=240-40\gamma$

Table 1

The best ownership structure is such that the gain from cheating is lowest relative to the loss. We can see from Table 1 that joint ownership implements a higher loss for the cheater but the gain from deviation will be higher too. The loss is greater since the investment drops to zero rather than 150 and the gain is greater since agent 2 can extract half rather than $\frac{1}{3}$ of the value of 1's efficient investment.

How does γ affect the incentives to cooperate? It is easy to see from Table 1 that the gain from deviation is increasing in γ. When the first-best investment becomes more expensive (γ increases) agent 2 has to pay a higher transfer to agent 1 to implement efficient investment. Since the value of the investment has not changed 2's payoff is now lower under cooperation. On the other hand 2's deviation payoff is unchanged since it is not related to investment costs. Therefore the gain from deviation is higher. Table 1 also shows that the loss from deviation is decreasing in γ. The drop in surplus after deviation is smaller when the efficient surplus is not very high in the first place.

Since the best ownership structure is such that the gain from cheating is lowest relative to the loss joint ownership is optimal if moving from agent 1 control to joint ownership increases the punishment relatively more than the gain from deviation. In this example the relative increase in the punishment is $60/(240\text{-}40\gamma)$ while the relative increase in the gain is $40/(40\gamma\text{ -}190)$. When γ is high the gain is high and the punishment is low. Therefore it is easier to obtain a higher relative change in the punishment. (Note that the absolute changes do not in fact depend on γ.) Then joint ownership which maximizes punishment is optimal. When γ is low the opposite is true: the gain is low and the punishment is high. Then it is optimal to put all the weight in minimizing the gain since higher relative changes are easier to obtain there and agent 1 control is optimal.

In the rest of the paper we show that the prediction of this example is general. For continuous investment γ has the interpretation of cost elasticity. When the investment costs are very elastic joint ownership is optimal and when the costs are moderately elastic the predictions of the static game hold. This result applies also when both agents have an investment.

3 The Model

Our stage game is a simplified version of Hart and Moore (1990). We analyse a setup where worker 1 uses asset a_1 to supply worker 2 who in turn uses asset a_2 to supply consumers. Ex ante worker 1 makes an investment in human capital which is specific to asset a_1. We model the investment as agent 1 directly choosing the value of the investment, v. The investment makes the worker more productive in using the asset. The worker for example learns to know better the properties of the asset or the environment the firm operates and can therefore generate more surplus. The investment can be either cost reducing or value enhancing. The cost of the investment to worker 1 is $c(v)$. We make the following assumptions about the cost of investment:

Assumption 1. $c(0) = 0$, $c'(v) > 0$ *and* $c''(v) > 0$.

For simplicity we assume that agent 2 does not have an investment. His contribution to the joint surplus is a fixed value, V. This assumption is relaxed in Section 6 where we analyse investments by both agents. Accordingly the joint surplus is equal to:

$$V + v - c(v). \tag{5}$$

Investment in human capital is assumed to be too complex to be described adequately in a contract. It is observable to both agents but not verifiable to third parties like the court. Therefore agent 1 chooses the investment noncooperatively. We also assume that it is very difficult to describe the required input characteristics or worker's duties ex ante. As a result the input trade and wage are also ex ante noncontractible. We also rule out profit-sharing agreements.[6] Ex ante contracts can only be written on the allocation of ownership. The possible ownership structures are *nonintegration* (each asset is owned by its worker), *integration by agent i* (agent i owns both assets), *joint ownership* (the agents jointly own both assets) and *cross ownership* (agent 1 owns asset a_2 and 2 owns a_1).

The assets do not necessarily fully rely on each other but there can be other suppliers/customers available. When each agent owns the asset she works with, worker 1 can produce the input and sell it to an outsider and

[6] See Hart and Moore (1990) for the justification of these assumptions.

worker 2 can buy input from an outsider and produce final good from it. The value of this trade to agents 1 and 2 is assumed to be μv and μV respectively. The value of μ depends on the relationship between the assets. When the assets are strictly complementary, then $\mu = 0$. This can be a physical property – e.g. a building and the land on which it locates are strictly complementary – or an economic relationship: there are no alternative suppliers/customers. The assets are economically independent when agent i can realize the full value of the investment without asset a_j and agent j. In this case $\mu = 1$. When the assets are economically independent 1's input is in no way specific for 2 who can obtain equally good input from alternative suppliers. Also 1 has alternative customers who value the input as much as agent 2.

If an agent owns both assets she can work alone with them and sell the final good to the customers. If agent 1 is the owner the value of the trade without agent 2's contribution is $\lambda_2(v + A_2)$ and if 2 is the owner the value is $\lambda_1(V + A_1)$. A_i is related to the value of asset a_i without its worker. The value of λ_i depends on the importance of agent i as a trading partner. If agent i is indispensable to asset a_i so that giving the control of a_i to agent j (who already owns a_j) does not enhance the surplus he can generate on his own, then $\lambda_i = \mu$. If agent i is dispensable so that agent j could replace her by an outsider without loss of value, then $\lambda_i = 1$. μ is the lowerbound for λ_i; an agent cannot do worse when she owns both assets than when she owns only one.

When an agent does not control any asset on her own (nonowning worker of an integrated firm or a partner in joint ownership) she has an outside option to work for another firm. We assume that asset a_1 is essential to worker 1 (or 1's investment is fully specific to a_1) so that the outside wage does not depend on her investment. Without loss of generality we normalize this fixed wage to zero.

Under cross ownership agent i can use asset a_j for outside trade which has value μA_j. This value does not depend on 1's investment because she does not have access to her essential asset.

We summarize the agents' outside options in Assumption 2. We denote by $v(i, A)$ the value agent i can generate on her own when she controls a set A of assets.

Assumption 2. *$v(i, \emptyset) = 0$, $v(1, \{a_1\}) = \mu v$, $v(2, \{a_2\}) = \mu V$, $v(i, \{a_j\}) = \mu A_j$, $v(1, \{a_1, a_2\}) = \lambda_2(v + A_2)$ and $v(2, \{a_1, a_2\}) = \lambda_1(V + A_1)$ for $i, j = 1, 2$ and $i \neq j$.*

Assumption 3. *$0 \leq \mu \leq \lambda_i \leq 1$ for $i = 1, 2$, $0 \leq A_1 \leq \overline{A}$ and $0 \leq A_2 \leq V$.*

Assumption 3 says that the marginal value of investment is increasing in the number of agents and assets. The assumption furthermore ensures superadditivity: under any ownership structure the joint surplus is at least as great as the sum of the agents' outside options.

Ex post the uncertainty is resolved and the agents negotiate a spot contract on the input trade or the services of non-owning workers. The investment is observable to both agents at the time of bargaining and therefore efficient bargaining solution will be reached. The only source of inefficiency in this model arises from the possible underinvestment. The incentive of the bargaining parties to reach agreement is driven by the risk of breakdown of negotiation. This will result in the "split-the-difference" rule where each agent gets half of the gains from trade.[7] Finally, production occurs and the final good is sold to the customers. This completes the description of the stage game.

In our dynamic model the stage game described above is always repeated one more period with high probability. At date 0 the agents write a contract on the allocation of ownership to maximize the joint surplus. The contract can give the ownership of an asset to the same agent(s) for all the game or induce changes in ownership. Given our assumptions about contractibility the only event this contract can be contingent on is time. Skills depreciate and the environment changes and further investments can be made in the beginning of each period. We make the extreme assumption that the investment depreciates fully before the next period begins. In the second half of the period the gains from trade are realized and the spot contract on the division of surplus is written.

[7] Binmore, Rubinstein and Wolinsky (1986) and Sutton (1986).

4 One-Shot Game

In this section we briefly examine the static game. Equation (6) gives the joint surplus maximizing investment, v^*:

$$1 - c'(v^*) = 0 \tag{6}$$

Since ex ante contracts on input trade or wage cannot be written, the bargaining takes place after the investment is made. Agent 1 foresees that part of the surplus she generates by her investment is expropriated in ex post bargaining while she pays the full cost of investment. Therefore under-investment (holdup) typically arises. Ownership is allocated to induce the highest investment. Below we give the outcome of the bargaining game and the incentives to invest under each ownership structure.

Under *nonintegration* (NI) the owners negotiate a two-part tariff on the input trade. The unit price is equal to marginal cost and the bargaining is over the fixed fee. The bargaining will result in the following division of surplus:

$$P_1^{NI} = \mu v + \frac{1}{2}(1-\mu)(v+V) - c(v) \tag{7}$$

$$P_2^{NI} = \mu V + \frac{1}{2}(1-\mu)(v+V) \tag{8}$$

Accordingly the incentive for investing is:

$$\frac{1}{2}(1+\mu) - c'(v) = 0 \tag{9}$$

It is easy to see from equation (9) that the investment is the greater the less complementary the assets are (the higher is μ). When the assets are economically independent ($\mu = 1$) agent 1 has first-best incentives.

Under integration by agent 1 (1I) the owner of both assets can unilaterally decide to transfer input at marginal cost but she has to bargain with agent 2 for his services. The payoffs for the agents are:

$$P_1^{1I} = \lambda_2(v + A_2) + \frac{1}{2}\left[(1-\lambda_2)v + V - \lambda_2 A_2\right] - c(v) \tag{10}$$

$$P_2^{1I} = \frac{1}{2}\left[(1-\lambda_2)v + V - \lambda_2 A_2\right] \tag{11}$$

The investment is given by:

$$\frac{1}{2}(1+\lambda_2) - c'(v) = 0 \tag{12}$$

The investment is the greater the more dispensable the worker is (the higher is λ_2). In the limit when the worker is fully dispensable ($\lambda_2 = 1$), then the owner has first-best incentives. When the worker is indispensable ($\lambda_2 = \mu$), then the investment is equal under nonintegration and agent 1 control. Assumption 3 ensures that agent 1's investment is at least as great when she owns both assets than when she owns only one ($\lambda_2 \geq \mu$).

Under *integration by agent 2* (2I), *cross ownership* (CO) and *joint ownership* (JO) agent 1 can realize the value of her investment only by reaching an agreement with agent 2; her investment has no value if she does not have access to her essential asset. Under integration by 2 and cross ownership agent 2 owns asset a_1 and under joint ownership the agents have to reach a unanimous agreement to use the assets. Therefore agent 1 receives only half of the value of her investment at the margin and the investment is given by:

$$\frac{1}{2} - c'(v) = 0 \tag{13}$$

Since any fixed outside options do not affect the incentives the size of the surplus is equal in these three structures – the division of surplus differs in general.

In this setup the ownership decision is very simple. We should allocate ownership to give the investing agent 1 the highest incentives, that is to give her the highest outside option related to investment. Since by assumption $\lambda_2 \geq \mu$, concentrating ownership of both assets in 1's hands gives her the best incentives and generates the highest surplus (see first order conditions (9), (12) and (13)). If the assets are economically independent ($\mu = 1$) or agent 2 is indispensable ($\lambda_2 = \mu$), then nonintegration and integration by agent 1 are equally good. Furthermore, joint ownership, cross ownership and integration by agent 2 are strictly dominated for any μ and $\lambda_2 > 0$.[8]

[8] When $\lambda_2 = \mu = 0$ all ownership structures are equally good.

5 Repeated Game

When the agents are in a long term relationship and care about the future, the holdup problems described in the previous section should not be so severe. In this section we analyse when the efficient investment can be supported using the trigger strategy and reversion to the Nash equilibrium of the static game as punishment. Obviously if the agents are very patient (discount factor is close to one) first best can be supported under any ownership structure. We are interested in situations when the agents are not completely patient and our aim is to find an ownership structure that guarantees first best for the greatest range of discount factors.[9]

Agent 1 implicitly agrees to make the efficient investment and both agents implicitly agree to share the surplus according to (P_1^*, P_2^*). (The sharing rule will be determined later.) Deviation from either investment or sharing rule will trigger punishment from the opponent for the rest of the game. In particular, if agent 1 cheats in investment the cooperation breaks down already in the second half of the day[10]: the surplus will be divided as in the static game, not according to the efficient sharing rule. Also if there is no deviation in investment but an agent does not agree to follow the sharing rule (P_1^*, P_2^*), then bargaining will result in the split-the-difference rule. The trigger strategy for agent 1 is:

- in period 1 choose v^* and follow (P_1^*, P_2^*)
- if (P_1^*, P_2^*) in 1,2,...,t-1, then choose v^* and follow (P_1^*, P_2^*) in t
- if not (P_1^*, P_2^*) in t-1, then choose v^N and apply (P_1^N, P_2^N) in t,t+1,... where superscript N refers to the Nash equilibrium of the static game
- if not (P_1^*, P_2^*) in t, then apply (P_1^N, P_2^N) in t,t+1,... and choose v^N in t+1,t+2,...

[9]Klein (1988) also suggests that reputation effects can limit holdup problems. The agents write such an initial contract that they are likely to be within the "self-enforcing range" where reputation effects work. We show that the self-enforcing range depends on the ownership structure.

[10]Note that by cooperation we refer to efficient behaviour: first-best investment and sharing rule. Of course this is a noncooperative game. Note also that even during punishment the agents get together and make the deal but the investment is lower and the division of surplus is different.

and for agent 2:

- if $v = v^*$ in 1,2,...,t and (P_1^*, P_2^*) in 1,2,...,t-1, then follow (P_1^*, P_2^*) in t
- if either $v \neq v^*$ in t or not (P_1^*, P_2^*) in t-1 or t, then apply (P_1^N, P_2^N) in t,t+1,...

Note that the only relevant information about the previous period when a new period begins is whether there was or was not deviation. Whether the deviation was in investment or sharing rule does not matter. This also means that the extensive form and the outcome of the bargaining game for the static model (as proposed in Sutton (1986)) is appropriate also here for the punishment phase. Whether the agents reach an agreement or fail to do so and have to take the outside option this period does not change the rest of the game. The next period starts from the same node.

It is easy to see that cheating in investment dominates cheating in sharing rule for agent 1. When 1 deviates in investment, she chooses her investment taking into account that the surplus will be divided with the split-the-difference rule. (The deviation investment is thus equal to the investment in the one-shot game.) By definition this is more than making the first-best investment and then switching to the split-the-difference rule. Only when agent 1 does not have an incentive to cheat in investment (she has first-best incentives even in the one-shot game) might she choose to deviate in sharing rule. Obviously agent 2 can cheat only in sharing rule since he does not have an investment.

First best will be supported in equilibrium if and only if the discounted payoff stream from efficient behaviour exceeds the payoff stream from the deviation path for both agents.

$$\frac{1}{1-\delta}\left[T - c(v^*)\right] \geq P_1^d + \frac{\delta}{1-\delta}P_1^p \tag{14}$$

$$\frac{1}{1-\delta}\left[V + v^* - T\right] \geq P_2^d + \frac{\delta}{1-\delta}P_2^p \tag{15}$$

where δ is the discount factor, T is the transfer agent 1 receives from 2 under cooperation, P_i^d is i's one-shot deviation payoff and P_i^p is i's payoff in the punishment path. If agent 1 deviates in investment, agent 2 observes it already in the same period and he will not pay T to agent 1. Agent 1 saves in investment costs but receives now a share of the surplus that is determined by the split-the-difference rule. Since agent 2 can punish only by sharing rule and the punishment starts in the same period $P_1^d = P_1^p$ and there is in fact no trade-off from gain today versus punishment tomorrow for agent 1. Equation (14) simplifies to:

$$T - c(v^*) \geq P_1^p \tag{16}$$

Equation (16) is the incentive compatibility constraint for agent 1 and it does not depend on the discount factor – not because future would not matter but because it affects both sides of (16) equally. For example under nonintegration agent 1 chooses efficient investment if and only if:

$$T \geq \frac{1}{2}(1+\mu)v^{NI} + \frac{1}{2}(1-\mu)V - c(v^{NI}) + c(v^*) \tag{17}$$

where v^{NI} is the punishment investment under nonintegration.

Agent 2's incentive constraint (15) depends on the discount factor. The higher share of the surplus goes to agent 1 under cooperation, the more likely it is that agent 2 will cheat, that is 2 will not pay T. Therefore the best we can do is to choose T^* such that (16) is just satisfied. This proves that:

Proposition 1 *When only agent 1 has an investment the optimal sharing rule is:*
$T^* = P_1^p + c(v^*)$
$P_1^* = P_1^p$
$P_2^* = V + v^* - c(v^*) - P_1^p$.

Note that this arrangement gives agent 1 the same surplus as in the one-shot game and the non-investing agent 2 gets all the benefits from 1's higher investment. Agent 2 in effect pays agent 1's investment cost and thus implements efficient investment for sure. However, agent 2 may not always be willing to pay T^* to agent 1. If agent 2 chooses to cheat in sharing rule, he does not have to pay T^* but the transfer is determined in noncooperative bargaining. His gain from deviation under nonintegration is:

$$\begin{aligned} G^{NI} &= \left[\frac{1}{2}(1+\mu)V + \frac{1}{2}(1-\mu)v^*\right] - (V + v^* - T^*) \\ &= \left[\frac{1}{2}(1+\mu)v^{NI} - c(v^{NI})\right] - \left[\frac{1}{2}(1+\mu)v^* - c(v^*)\right] \end{aligned} \tag{18}$$

where $G \equiv P_2^d - P_2^*$. This expression is strictly positive for any $v^{NI} < v^*$ since v^{NI} is chosen to maximize the first term is square brackets. The same is true for any ownership structure and therefore agent 2 can gain by cheating; he can extract more of the value of 1's efficient investment in bargaining than by paying T^*. If agent 1 has first-best incentives in the one-shot game ($v^{NI} = v^*$) there is no reason to deviate for agent 2 either since then he cannot extract any value of 1's investment in bargaining (see equation (8)).

If agent 2 cheats in sharing rule he gains in this period but from the next period on the payoff will be lower because agent 1 punishes by choosing lower investment. Under nonintegration the loss from deviation is equal to:

$$\begin{aligned} L^{NI} &= (V + v^* - T^*) - \left[\frac{1}{2}(1+\mu)V + \frac{1}{2}(1-\mu)v^{NI}\right] \\ &= [v^* - c(v^*)] - \left[v^{NI} - c(v^{NI})\right] \end{aligned} \tag{19}$$

where $L \equiv P_2^* - P_2^p$. The loss is strictly positive for any $v^{NI} < v^*$ since v^* maximizes the first term in square brackets. L shows how much lower the joint surplus will be in the punishment path. If agent 2 is patient enough the one-shot gain from cheating is outweighed by lower payoff in the future. Agent 2's incentive constraint (15) simplifies to:

$$\delta \geq G/(G+L). \tag{20}$$

The main focus of this paper is on equation (20). The gain and loss from deviation will differ in general for different ownership structures. Define $\underline{\delta} \equiv G/(G+L)$. In what follows we concentrate on finding the control structure that guarantees first best for the greatest range of discount factors, that is gives the lowest $\underline{\delta}$. The best ownership structure is such that the gain from deviation is lowest relative to the loss. Now it becomes clear that the optimal allocation gives the ownership to the same agent(s) for all the game. For example giving ownership of the assets to agent 1 for the first t periods and then making agent 2 the owner for the rest of the game does not improve the incentives to cooperate in any way (it may do no harm either if $\underline{\delta}$ is equal under both control structures).

It is quite obvious that if agent 1 has first-best incentives even in the one-shot game ($\delta = 0$) under some ownership structure this must be the optimal structure also for the repeated game. This gives our first results on the optimal control structure:

Proposition 2 *(i) If the assets are economically independent ($\mu = 1$), then nonintegration is (weakly) optimal.*

(ii) If the non-investing agent 2 is dispensable ($\lambda_2 = 1$), then integration by agent 1 is (weakly) optimal.

In these cases agent 2 does not have any holdup power over the investing agent and cannot get any share of the value of 1's investment in bargaining; therefore agent 1 has always first-best incentives (see equations (9) and (12)). Next we turn to analyse the optimal ownership structure when there is underinvestment problem in the one-shot game, that is μ and $\lambda_2 < 1$. We know that G and L are strictly positive in this case. Then it is appropriate to determine the optimal control structure by minimizing the right-hand-side of (20).[11] Furthermore $0 < \underline{\delta} < 1$; if agent 2 is very patient first best can be supported under any ownership structure and if 2 is very impatient underinvestment will occur.

It turns out that as in the one-shot game any fixed values do not affect the incentives. (V cancels out in equations (18) and (19).) Only outside options related to investments and consequently the punishment level of investment are important. Therefore we can obtain the gain and loss from deviation for cross ownership, joint ownership and agent 2 control from equations (18) and (19) by setting μ equal to zero and changing the punishment investment to be appropriate. This proves that:

Lemma 1 $\underline{\delta}^{CO} = \underline{\delta}^{JO} = \underline{\delta}^{2I}$.

Not only are cross ownership, joint ownership and agent 2 control equivalent but from the point of view of the static game these are the structures one would not expect to be useful. The common element in these structures is that they do not give outside option related to investment to agent 1.

Since only the level of punishment investment affects $\underline{\delta}$ it is clear that:

Lemma 2 *(i)* $\underline{\delta}^{1I}(\lambda_2) = \underline{\delta}^{NI}(\mu)$ *if* $\lambda_2 = \mu$.
(ii) $\underline{\delta}^{1I}(0) = \underline{\delta}^{NI}(0) = \underline{\delta}^{CO} = \underline{\delta}^{JO} = \underline{\delta}^{2I}$.

[11]When $\mu = 1$ then both the numerator and denominator of equation (20) are equal to zero under nonintegration and the same is true under integration by agent 1 when $\lambda_2 = 1$.

When agent 2 is indispensable ($\lambda_2 = \mu$) nonintegration and agent 1 control are equivalent since owning both assets rather than only a_1 does not improve 1's incentives to invest in the punishment path. When the assets are strictly complementary ($\mu = 0$) and agent 2 is indispensable ($\lambda_2 = 0$) neither nonintegration or agent 1 control provides any outside option to agent 1. Agent 2 has the maximal holdup power: agent 1 cannot do anything without agent 2 or asset a_2. Then all the ownership structures are equivalent.

Therefore we are left with the question: is $\underline{\delta}$ minimized by removing agent 1's outside option (joint or cross ownership or agent 2 control) or by giving her an outside option (nonintegration or integration by agent 1)? We can derive the optimal control structure by examining how the lowerbounds for the discount factor under nonintegration, $\underline{\delta}^{NI}(\mu)$, and under integration, $\underline{\delta}^{1I}(\lambda_2)$, move with μ and λ_2. Since Lemma 2 shows that $\underline{\delta}^{NI}(\mu)$=$\underline{\delta}^{1I}(\lambda_2)$ when $\mu = \lambda_2$, it is sufficient to concentrate on $\underline{\delta}^{NI}(\mu)$ only. Examining how μ affects $\underline{\delta}^{NI}$ is like comparing different ownership structures.

We start by analysing the gain and loss from cheating.

Proposition 3 *Both the gain and loss from deviation are decreasing in μ under nonintegration.*

Proof. Equation (18) gives the gain from deviation under nonintegration. Total differentiation gives:

$$dG^{NI}/d\mu = \left[\frac{1}{2}(1+\mu) - c'(v^{NI})\right] \partial v^{NI}/\partial\mu + \frac{1}{2}(v^{NI} - v^*) = \frac{1}{2}(v^{NI} - v^*) < 0 \tag{21}$$

The investment effect is negligible and therefore we can ignore the first term in (21). Accordingly, the gain is decreasing in μ. Equation (19) gives the loss from deviation under nonintegration. By total differentiation we obtain:

$$dL^{NI}/d\mu = -\left[1 - c'(v^{NI})\right] \partial v^{NI}/\partial\mu = -\frac{1}{2}(1-\mu)\partial v^{NI}/\partial\mu < 0 \tag{22}$$

The first order condition (9) helps us to determine the sign of this expression and to simplify it. It is easy to see from (9) that $\partial v^{NI}/\partial\mu$ is positive. Therefore (22) is unambiguously negative. ■

High loss and low gain from deviation would guarantee good incentives to cooperate. Proposition 3 tells that removing agent 1's outside option ($\mu = 0$) provides the highest loss. In the punishment path agent 1 receives only half of the value of her investment at the margin and therefore the punishment investment and the joint surplus are the lowest possible. Nonintegration with $\mu = 0$ is like joint ownership, cross ownership and agent 2 control which are the worst structures in the one-shot game. In the repeated game these structures have the advantage that they provide the highest punishment.

However, the highest punishment does not imply that cooperation would be most sustainable. Proposition 3 shows that when the punishment is highest so is the gain from deviation. When agent 2 deviates in sharing rule the spot contract will be written with the split-the-difference rule. When there are no outside options the agents simply split the gross surplus 50:50; the deviant gets half of the surplus generated by agent 1's first-best investment and therefore gains a lot from deviation. While when agent 1 has an outside option ($\mu > 0$) agent 2 can extract less than half of the value of the efficient investment and therefore gains less from deviation. The optimal ownership structure of the one-shot game gives the highest possible outside option and consequently the highest share of the surplus to the investing agent 1 and therefore best restricts the gain from deviation for agent 2. In the same time punishment will be minimal because 1's incentive to invest in the punishment path is maximized. While the worst structure of the static game is good in the repeated game because it provides the highest punishment but bad because the gain from deviation is also the highest.

Proposition 3 tells that the gain and loss from deviation move to the same direction as we change μ and it is not immediately clear what is the effect on $\underline{\delta}^{NI}$. The change in $\underline{\delta}^{NI}$ is given by:

$$\frac{\partial \underline{\delta}^{NI}}{\partial \mu} \stackrel{s}{=} \frac{|\, \partial L^{NI}/\partial \mu \,|}{L^{NI}} - \frac{|\, \partial G^{NI}/\partial \mu \,|}{G^{NI}} =$$

$$\frac{\frac{1}{2}(1-\mu)\partial v^{NI}/\partial \mu}{[v^* - c(v^*)] - [v^{NI} - c(v^{NI})]} - \frac{\frac{1}{2}(v^* - v^{NI})}{\left[\frac{1}{2}(1+\mu)v^{NI} - c(v^{NI})\right] - \left[\frac{1}{2}(1+\mu)v^* - c(v^*)\right]} \qquad (23)$$

where $\stackrel{s}{=}$ denotes that the expressions have the same sign. The sign of (23) depends on the difference between the *relative* changes in the gain and loss. Both changes are negative and to ease the discussion we have chosen to use their absolute values. The first term gives the relative change in the loss from cheating and the second term is the relative change in the gain. If the gain decreases relatively more than the punishment then $\underline{\delta}^{NI}$ is decreasing in μ while when the punishment effect is greater than the gain effect then $\underline{\delta}^{NI}$ is increasing in μ. Analysing such a difference is very subtle. Therefore we introduce an explicit functional form for the investment cost

Assumption 1'. $c(v) = v^{\gamma}$ *where* $\gamma > 1$.

Lemma 3 determines the sign for equation (23).

Lemma 3 $\underline{\delta}^{NI}$ *is* $\left\{ \begin{array}{c} \textit{decreasing in} \\ \textit{independent of} \\ \textit{increasing in} \end{array} \right\}$ μ *if and only if* $\gamma \left\{ \begin{array}{c} < \\ = \\ > \end{array} \right\} 2.$

Proof. In the Appendix.

Lemmata 1 to 3 help us to construct Figure 1. We also include the implication of Proposition 2: $\underline{\delta}^{NI}(1) = 0$. The Figure compares the lowerbounds for the discount factor under different ownership structures for various values of the outside option parameters μ and λ_2. This Figure proves to be very useful in examining the optimal ownership structure.

When costs are very elastic ($\gamma > 2$) the punishment effect dominates; a small increase in μ will lower the punishment more than the gain and cooperation becomes more difficult ($\underline{\delta}^{NI}$ increases). While when costs are moderately elastic ($\gamma < 2$) the gain effect is more important; a higher μ will lower the gain more than the punishment and cooperation is easier ($\underline{\delta}^{NI}$ decreases). This is in line with the numerical example in Section 2 where the investment was discrete.

The result is the same for both discrete and continuous investment but the effects behind the continuous case are more complex. In the discrete case the levels of the first-best and punishment investment are fixed and only the cost of the first-best investment changes in γ. For high values of γ the

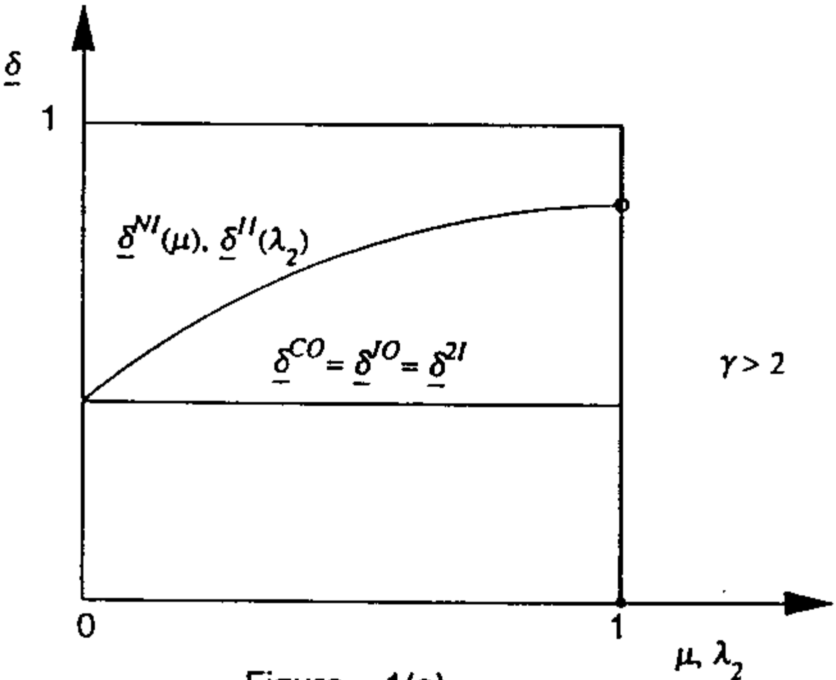

Figure 1(a)

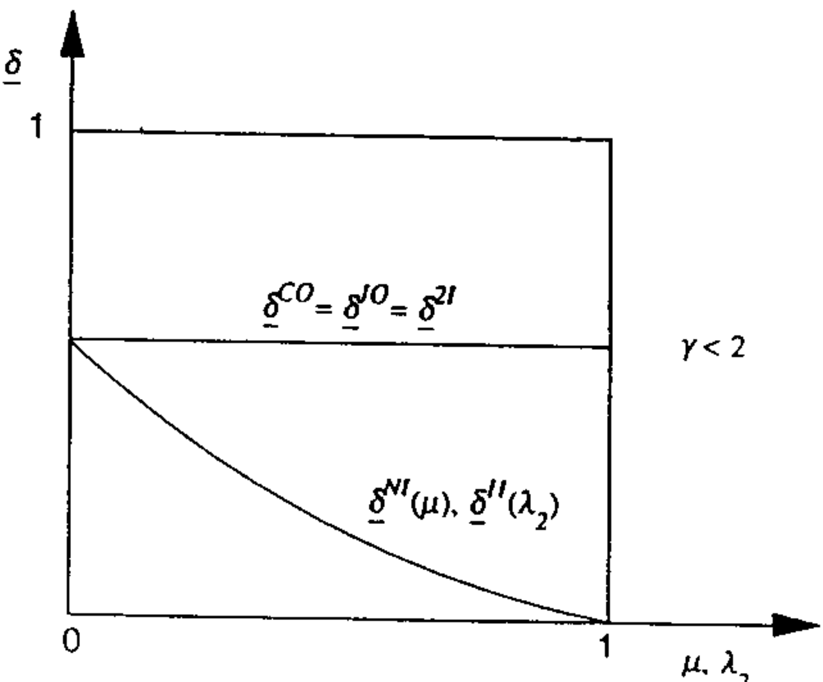

Figure 1(b)

gain from deviation is high and the punishment is low. Since the absolute changes do not depend on γ moving from joint ownership to agent 1 control will result in a higher relative decrease in the punishment than in the gain; $\underline{\delta}^{NI}$ increases. In the continuous case the levels of investment are not fixed but adjust to changes in the cost elasticity. When the costs become more elastic the gap between the efficient and the punishment investment becomes smaller. Now the important effects are the absolute changes in the gain and loss due to higher μ. We rewrite these changes from equations (21) and (22).

$$\begin{aligned}\left|\frac{\partial G}{\partial \mu}\right| &= \frac{1}{2}(v^* - v^{NI}) = \frac{1}{2}\left[g(1) - g(\eta)\right] \\ &= \frac{1}{2}\left[(1-\eta)g'(\eta) + \frac{1}{2}(1-\eta)^2 g''(p)\right] \qquad (24)\end{aligned}$$

$$\left|\frac{\partial L}{\partial \mu}\right| = \frac{1}{2}(1-\mu)\frac{\partial v^{NI}}{\partial \mu} = \frac{1}{2c''}(1-\eta) = \frac{1}{2}(1-\eta)g'(\eta) \qquad (25)$$

where $g(\cdot)$ is the inverse of the marginal cost function, $\eta \equiv \frac{1}{2}(1+\mu)$ and $p \in (\eta, 1)$.[12] Higher μ increases punishment investment by $\partial v^{NI}/\partial\mu = 1/2c''$ and 2's share of it is $\frac{1}{2}(1-\mu) = (1-\eta)$; thus the total effect on the loss is given in equation (25). Higher μ also decreases 2's deviation payoff by $\frac{1}{2}v^*$ and decreases 2's payoff from cooperation by $\frac{1}{2}v^{NI}$ since 2 has to compensate agent 1 the increase in her punishment payoff. Equation (24) gives thus the net effect on the gain from deviation.

Both (24) and (25) in fact describe the change or the rate of change in investment due to higher slope of the value. In (24) the slope changes from $\frac{1}{2}(1+\mu)$ to 1 and in (25) the slope increases a little from $\frac{1}{2}(1+\mu)$. This is why γ= 2 is the critical value. Lemma 3 in fact tells that $\partial\underline{\delta}^{NI}/\partial\mu$ has the same sign as c''' (or $-g''$). In (24) the change in the slope and accordingly the change in the investment is not marginal and we need "a correction term" (the second term). For a convex function the slope tends to underestimate the increase in the value of the function and the correction term is positive. Therefore if g'' is positive, that is c''' is negative, the change in the gain is greater than the change in the punishment. While the slope

[12]We use Taylor series and Lagrange form of the remainder to re-express (24) and the inverse function differentiation rule for (25).

of a concave function tends to overestimate the increase in the value of the function and the correction term is negative. Accordingly for positive c''' the change in the punishment is greater. It turns out that when the absolute change in gain is greater, so is the *relative* change.[13] Accordingly, for $\gamma > 2$ the punishment effect dominates ($\underline{\delta}^{NI}$ is increasing in μ) and for $\gamma < 2$ the gain effect dominates ($\underline{\delta}^{NI}$ is decreasing in μ). This leads to our main results.

Proposition 4 *Ownership structure that does not give any outside option related to investment to agent 1 is optimal if and only if $\gamma > 2$, $\mu < 1$ and $\lambda_2 < 1$. This can be implemented by joint ownership, integration by the noninvesting agent 2, cross ownership, or nonintegration with strictly complementary assets.*

Proof. It is immediately clear from Figure 1(a) that for $\gamma > 2$ $\underline{\delta}^{JO} = \underline{\delta}^{CO} = \underline{\delta}^{2I} \leq \underline{\delta}^{NI}$ and $\underline{\delta}^{JO} = \underline{\delta}^{CO} = \underline{\delta}^{2I} \leq \underline{\delta}^{1I}$ when $\mu < 1$ and $\lambda_2 < 1$. Further $\underline{\delta}^{NI}(0) = \underline{\delta}^{JO} = \underline{\delta}^{CO} = \underline{\delta}^{2I}$. ■

When costs are very elastic it becomes important to ensure that the punishment is maximal. Then joint ownership is optimal. The agents have to reach a unanimous agreement to use the assets. If not they can work for another firm at zero wage. Therefore the joint surplus is the lowest possible in the punishment path and cheating would lead to a very bad equilibrium.

Hostage solution is equally good in providing maximal punishment. By hostage we refer to agent 2 control which removes all the control rights from the only investing agent. Under agent 2 control 1 does not have access to her essential asset without the consent of agent 2. The second hostage solution is separation of strictly complementary assets. Neither agent can walk away with her asset since the assets are useful only together.

Also cross ownership is equivalent to the above structures in our model. However, if agent 1's investment is not fully specific to asset a_1 but is somewhat useful also for working with asset a_2 cross ownership does not guarantee maximal punishment. Then if 1 owns a_2 she has an outside option related to her investment while joint ownership and agent 2 control remove 1's outside option; cross ownership is not optimal.

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[13] It is also true that now the level of gain is greater than the level of punishment. But the absolute change in the gain is so much greater than in the punishment that also the relative change in the gain is greater.

Proposition 5 *Integration by the only investing agent is (weakly) optimal if* $\gamma < 2$.

Proof. Now Figure 1(b) where $\gamma < 2$ is appropriate. Since by assumption $\lambda_2 \geq \mu$, $\underline{\delta}^{1I}$ is the lowest as the Figure illustrates. ■

Proposition 5 tells that when the investment costs are moderately elastic, the prediction of the one-shot game holds. In this parameter range it is more important to ensure that the gain from deviation is the smallest possible although then also punishment is minimal. This will be guaranteed by giving the ownership of both assets to the investing agent.

Proposition 6 *Ownership does not matter if (i)* $\mu = \lambda_2 = 0$ *or (ii)* $\gamma = 2$, $\mu < 1$ *and* $\lambda_2 < 1$.

Proof. (*i*) Follows straightforward from Lemma 2. (*ii*) If $\gamma = 2$ $\underline{\delta}$ is equal for all ownership structures as Lemmata 2 and 3 show (if $\mu < 1$ and $\lambda_2 < 1$). ■

Proposition 6 gives the only two cases when the ownership structure does not matter. First, if all ownership structures are equivalent in the static game they will be equivalent in the dynamic game as well. This is the case when no allocation gives an outside option to agent 1; assets are strictly complementary and agent 2 is indispensable. Second, we have a more interesting equivalence result when the ownership structures differ in the one-shot game but the punishment and gain effect exactly offset each other. On the knife-edge ownership does not matter.

6 Two Investments

We chose a very simple structure for our model to make the main trade-off in the repeated game clear. In this Section we analyse the first natural extension: both agents have an investment. We denote agent 1's and 2's investment by v_1 and v_2 respectively and assume that the cost of investment is $c_i(v_i) = v_i^\gamma/\sigma_i$ where $\gamma > 1$ and $\sigma_i > 0$.

In this setup both agents can cheat and punish by investment and the optimal sharing rule is not as simple as in the main model. Proposition 7 designs a sharing rule such that both agents have best incentives to cooperate.

Proposition 7 *The optimal sharing rule is:*
$P_1^* = sP_1^d + (1-s)(S^* - P_2^d)$
$P_2^* = (1-s)P_2^d + s(S^* - P_1^d)$
where $s = (P_2^d - P_2^p)/(P_1^d + P_2^d - P_1^p - P_2^p)$ *and* $S^* = v_1^* + v_2^* - c_1(v_1^*) - c_2(v_2^*)$.

Proof. When agent i pays a transfer T to agent j for the input or for the contribution of the worker, the payoffs are:

$$P_i^* = v_1^* + v_2^* - T - c_i(v_i^*) \tag{26}$$

$$P_j^* = T - c_j(v_j^*) \tag{27}$$

Then agent i will cooperate if and only if:

$$\delta \geq \frac{P_i^d - v_1^* - v_2^* + T + c_i(v_i^*)}{P_i^d - P_i^p} \tag{28}$$

Likewise agent j cooperates if and only if:

$$\delta \geq \frac{P_j^d - T + c_j(v_j^*)}{P_j^d - P_j^p} \tag{29}$$

Because agent i's incentive to cooperate is decreasing in T while j's incentive is increasing in T, the optimal T gives the agents balanced incentives to cooperate. Setting the right-hand-sides of equations (28) and (29) equal we can solve for T^*:

$$T^* = \frac{(P_i^d - P_i^p)\left[P_j^d + c_j(v_j^*)\right] + (P_j^d - P_j^p)\left[v_1^* + v_2^* - P_i^d - c_i(v_i^*)\right]}{(P_1^d - P_1^p) + (P_2^d - P_2^p)} \tag{30}$$

Inserting T^* in equations (26) and (27) gives the expressions in the Proposition. ∎

Neither agent would have an incentive to deviate if they could get their deviation payoff even under cooperation. Since this is not feasible the best we can do is to give each agent a certain proportion of her deviation payoff. It is like agent 1 gets her deviation payoff with probability s and agent 2 gets his deviation payoff with probability (1 - s) leaving the rest of the surplus,

$(S^* - P_2^d)$, to agent 1. The proportion s is chosen to balance the agent's incentives to cooperate. This weight is related to how close the punishment investment is to the first-best one. For example under nonintegration:

$$s = (v_1^* - v_1^{NI}) / \left[(v_1^* - v_1^{NI}) + (v_2^* - v_2^{NI}) \right]. \tag{31}$$

If agent 1 is better able to punish agent 2 by investment

$$(v_1^* - v_1^{NI}) > (v_2^* - v_2^{NI}) \tag{32}$$

then $s > 1/2$ and agent 1 receives a higher proportion of her deviation payoff than agent 2. In a sense agent 1 is then more important: there is a greater loss from her cheating than from agent 2's deviation. Therefore we should shift more surplus towards her.

Sometimes agent 1 does not have an incentive to deviate in investment: she has first-best incentives even in the one-shot game (she owns both assets and agent 2 is dispensable). Then $P_2^d = P_2^p$ and $s = 0$, that is agent 2 gets his full deviation payoff. This is the same case as we had in our main model; the noninvesting agent could punish only by sharing rule and therefore the investing agent got her full deviation payoff which is equal to her punishment payoff in this case. Here agent 1's punishment investment is equal to its efficient level and therefore it provides no punishment.

Inserting the optimal sharing rule of Proposition 7 in (28) or (29) gives us a lowerbound for the discount factor:

$$\underline{\delta} = (G_1 + G_2)/(G_1 + G_2 + L_1 + L_2) \tag{33}$$

where G_i is the gain from deviation to agent i and L_i is the loss. Now the best ownership structure is such that the aggregate gain from deviation is lowest relative to the aggregate punishment. Since we have equalized the incentives it is the aggregate terms that matter. In the previous section only agent 2 had an incentive to cheat and there we aimed to minimize agent 2's gain from cheating relative to the punishment.

Proposition 8 *Both the aggregate gain from deviation,* $(P_1^d + P_2^d - S^*)$, *and the aggregate loss,* $(S^* - P_1^p - P_2^p)$, *are decreasing in* μ *under nonintegration and decreasing in* λ_j *under integration by agent* i.

Proof. In the Appendix.

The same trade-off is present in this version of the model: an ownership structure that provides maximal punishment will also give the highest gain from deviation. And restricting the gain from cheating lowers the punishment as well.

Now when both agents have an investment obviously agent 2 control is not anymore equivalent to joint ownership and cross ownership; under integration by 2 an investing agent has control rights. The second difference to our main model is that nonintegration and integration by agent i are not equivalent when worker j is indispensable ($\lambda_j = \mu$). Although i's investment is equal under both structures j's investment will differ and therefore $\underline{\delta}$ is not equal. A new property is also the nonmonotonicity of $\underline{\delta}^{iI}$: we can prove that $\underline{\delta}^{iI}(0) = \underline{\delta}^{iI}(1)$. Lemma 4 summarizes the properties for the lowerbounds.

Lemma 4 *(i)* $\underline{\delta}^{JO} = \underline{\delta}^{CO} = \underline{\delta}^{NI}(0) = \underline{\delta}^{iI}(0) = \underline{\delta}^{iI}(1)$.

(ii) $\underline{\delta}^{NI}$ *is* $\left\{\begin{array}{c} \textit{decreasing in} \\ \textit{independent of} \\ \textit{increasing in} \end{array}\right\}$ μ *if and only if* $\gamma \left\{\begin{array}{c} < \\ = \\ > \end{array}\right\} 2$.

(iii) For $0 < \mu = \lambda_j < 1$ $\underline{\delta}^{NI}(\mu) \left\{\begin{array}{c} < \\ = \\ > \end{array}\right\} \underline{\delta}^{iI}(\lambda_j) \left\{\begin{array}{c} < \\ = \\ > \end{array}\right\} \underline{\delta}^{JO}$ *if and only if* $\gamma \left\{\begin{array}{c} < \\ = \\ > \end{array}\right\} 2$.

Proof. In the Appendix.

Lemma 4 helps us to construct Figure 2 which we use to find the optimal ownership structure. The results are in line with our main model.

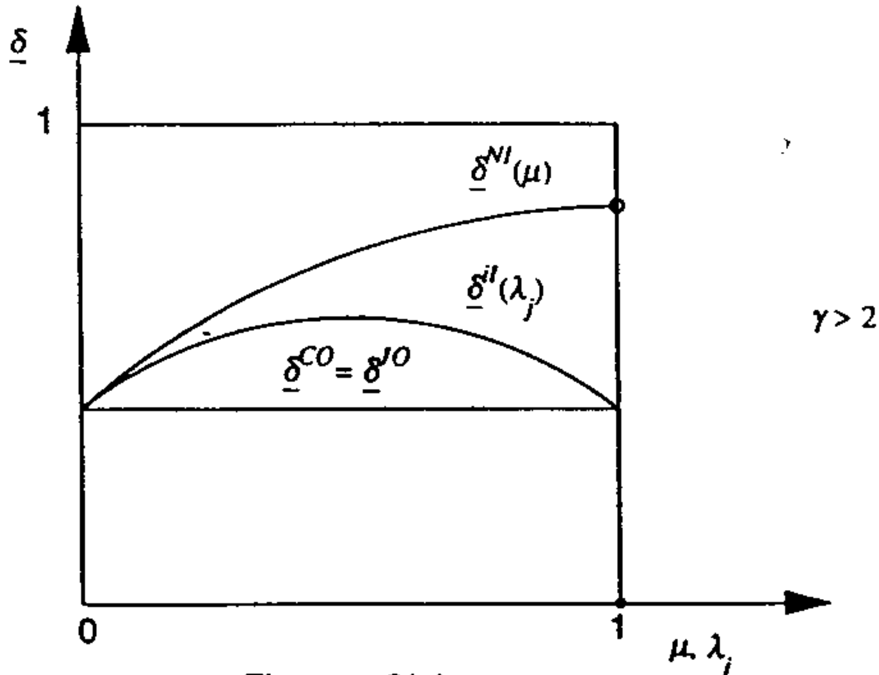

Figure 2(a)

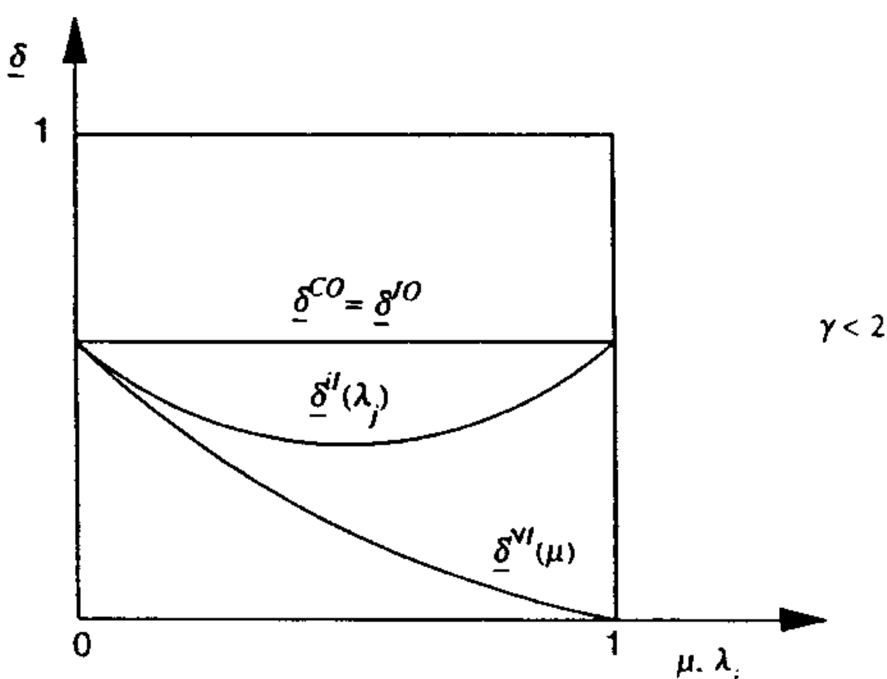

Figure 2(b)

Proposition 9 *When both agents have an investment joint ownership, cross ownership, or separation of strictly complementary assets is (weakly) optimal if and only if $\gamma > 2$ and $\mu < 1$.*

Proof. See Figure 2(a) where $\gamma > 2$. It is immediately clear from the Figure that $\underline{\delta}^{JO} = \underline{\delta}^{CO} \leq \underline{\delta}^{NI}$ and $\underline{\delta}^{JO} = \underline{\delta}^{CO} \leq \underline{\delta}^{iI}$ when $\mu < 1$. Furthermore $\underline{\delta}^{NI}(0) = \underline{\delta}^{JO} = \underline{\delta}^{CO}$. ■

As in the main model maximizing punishment becomes important when costs are very elastic and joint ownership and hostage solution are optimal. The symmetric outcomes (joint ownership and separation of strictly complementary assets) are more natural predictions here than in our main model where the role of the agents was very asymmetric.

Proposition 10 *The following statements are true if $\gamma < 2$:*

(i) Joint and cross ownership are (weakly) dominated by nonintegration and integration.

(ii) If assets are strictly complementary then nonintegration is (weakly) dominated by integration.

(iii) If agent j is indispensable to asset a_j, then integration by agent i is (weakly) dominated by nonintegration.

Proof. See Figure 2(b) where $\gamma < 2$. (i) Under joint ownership and cross ownership $\underline{\delta}$ reaches its maximum. Therefore these structures are dominated. (ii) When assets are strictly complementary ($\mu = 0$), the value for $\underline{\delta}^{NI}$ is given by the intercept in the vertical axis. Therefore $\underline{\delta}^{NI} \geq \underline{\delta}^{iI}$. (iii) When agent j is indispensable to asset a_j, $\lambda_j = \mu$. The Figure shows that then $\underline{\delta}^{NI} \leq \underline{\delta}^{iI}$. ■

As in the main model a control structure that minimizes the gain from deviation is best when investment costs are not very elastic. Proposition 10 gives the same results as Hart and Moore (1990). One important determinant for the optimal ownership structure is the degree of complementarity between the assets; when the assets are strictly complementary they should be owned together. Also, if an agent is indispensable to an asset then he should own this asset. Furthermore, joint and cross ownership are dominated.

In Proposition 10 we considered only the extreme values of the parameters μ and λ_j. It is interesting to examine if the static and repeated game give exactly the same predictions for all parameter values. For this aim we have constructed Figures 3 and 4. The Figures are based on numerical simulations of the model. In Figure 3 the relative importance of the investment is in the vertical axis (when $\sigma_1/\sigma_2 > 1$ agent 1's investment is more important) and the degree of asset complementarity is in the horizontal axis.[14] The predictions for the limit values of parameter are the same: strictly complementary assets should be owned together and for economically independent assets there should be independent control. However, for intermediate values of μ there are some differences and in particular nonintegration is more likely in the repeated game. Figure 3 also shows that the more important an agent's investment is the more likely it is that she owns both assets.

In Figure 4 we have the importance of an agent as a trading partner in the horizontal axis.[15] Here we assume for simplicity that the agents are equally important as trading partners: $\lambda \equiv \lambda_1 = \lambda_2$. In the static game nonintegration dominates when the agents are indispensable (minimum λ) and the more dispensable the agents are the less likely is nonintegration. We know that in the one-shot game an agent should own the asset to which she is indispensable. Therefore when both agents are indispensable agent 1 should own asset a_1 and agent 2 should own asset a_2, in other words the assets should be nonintegrated. The less indispensable the agents are, the more weight is given to the importance of investment and the more likely it becomes that an agent with more important investment owns both assets. In the dynamic game we have the same prediction for the minimum value of as in the static game: nonintegration dominates for indispensable agents. But in the dynamic game λ has a nonmonotonic effect. However, this property is not very robust since it did not occur in our main model. Naturally this effect is driven by the nonmonotonicity of $\underline{\delta}^{iI}$. Also Figure 4 shows that nonintegration is more likely in the repeated game. This observation can be linked to the discussion about integration and reputation being substitutes in dealing with the holdup problem (e.g. Klein (1980) and Coase (1988)). This discussion refers to models where the benefit of integration is reduced holdups and costs are something else (like arising from bureaucracy). There

[14]The Figure is drawn for a given value of λ, where $0 < \lambda < 1$.

[15]The Figure is drawn for a given value of μ, where $0 < \mu < 1$.

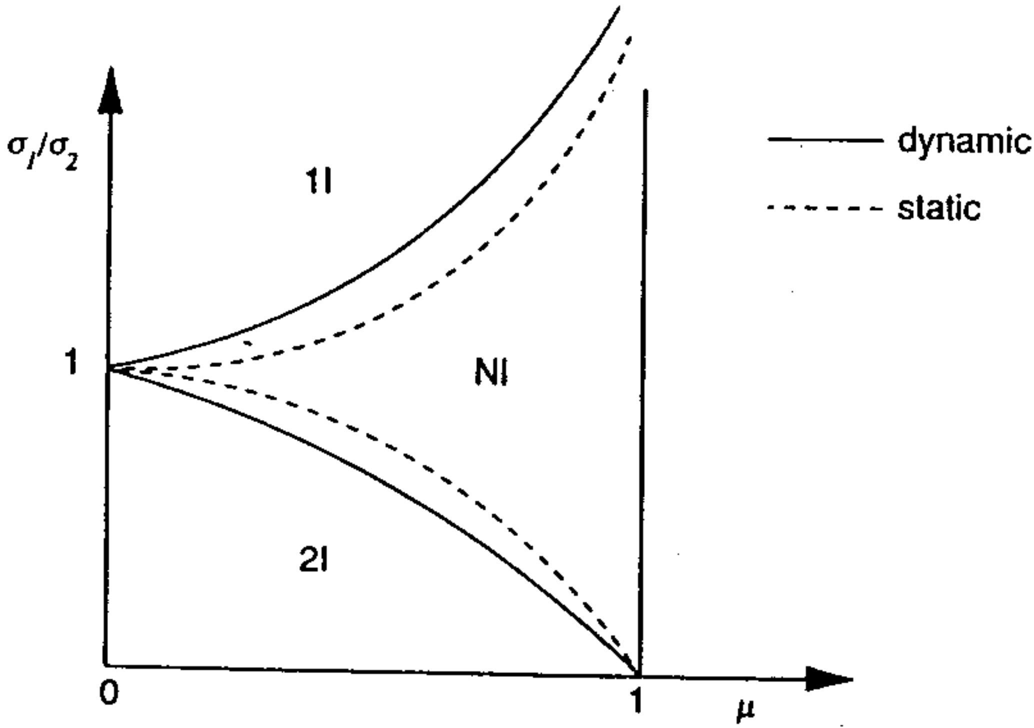

Figure 3

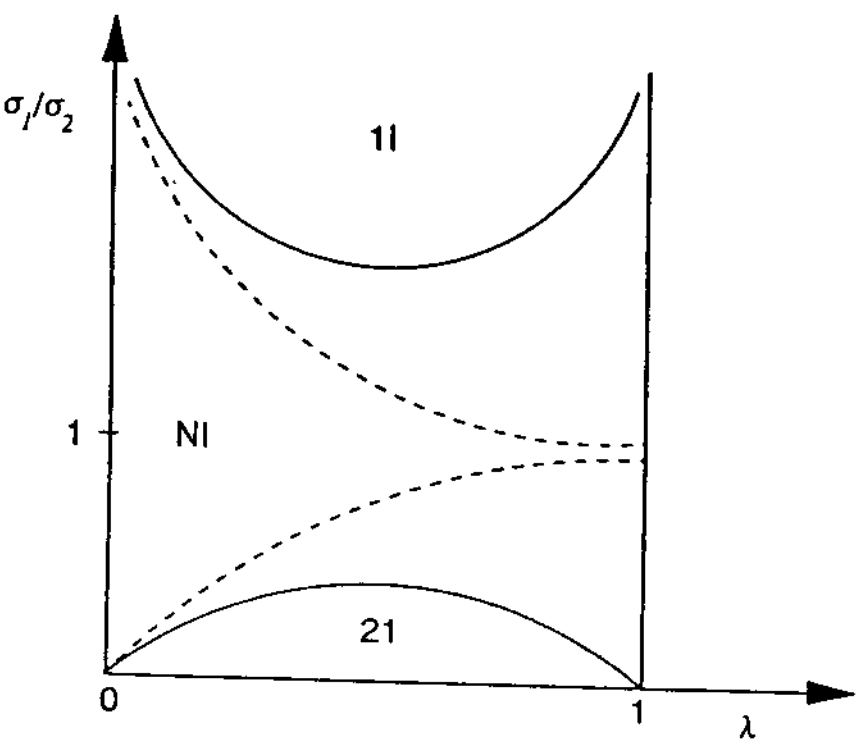

Figure 4

clearly one would expect less integration in a repeated setting; the benefits of integration are lower and costs have not changed. In our model both the benefits and costs of integration change in the repeated game and it is not a priori clear which way the reputation effect goes. As Figures 3 and 4 show nonintegration is more likely when $\gamma < 2$.[16]

7 Related Literature

Garvey (1991) also analyses the effect of reputation on the optimal allocation of ownership rights in a two-agent two-asset setup. He finds that the basic result of Grossman and Hart (1986) holds in the repeated setting: an agent with a much more important investment should own both assets. Garvey takes ownership as a continuous variable and assumes that the asset returns accrue to the owner whereas Grossman and Hart assume that ownership increases a manager's bargaining power only by raising his outside option.[17] Thus his model is not in fact a repeated version of Grossman-Hart. Furthermore, Garvey restricts the division of surplus to be the same on and off the equilibrium path whereas we take into account that other sharing rules than the outcome of the one-shot bargaining game may be supported under cooperation. In addition we examine the role of outside options.

Friedman and Thisse (1993) have a related paper to ours. In their model the firms choose noncooperatively the location in the Hotelling line and then collude in pricing in the repeated game. They find that the firms will locate in the middle of the Hotelling line to maximize punishment. Shapiro (1989) calls this the topsy-turvy principle of tacit collusion: anything that makes more competitive behaviour credible actually promotes collusion. Another paper that obtains an inefficient structure from a static point of view as an equilibrium in a dynamic context is Martimort (1993). He shows that multiprincipals charter acts as a commitment device against principal's incentives to renegotiate long term agreements. Both these papers obtain the reverse structure as the only equilibrium. In our model also the outcome of the static game can be an equilibrium in the dynamic game within some

[16] Of course when $\gamma > 2$ nonintegration is less likely in the repeated setting. In fact, nonintegration occurs only for strictly complementary assets.

[17] Holmstrom and Milgrom (1991) on the other hand make the same assumption about the ownership of asset returns as Garvey.

parameter range.

Klein and Leffler (1981) analyse reputation effects in assuring product quality. They rely on external enforcement: all the customers learn if a firm cheats and therefore a cheater loses all future sales. In our paper enforcement is internal: reputation effects operate only within the contractual parties.[18] When all the customers can observe low quality a question arises why quality is not contractible in the first place. Klein and Leffler conclude that a price premium assures good quality since the loss from deviation will be high. Hostages play a role also in their model but quite a different one from ours: nonsalvageable capital is bought to exhaust supranormal profits and to deter entry in the industry.

[18]The terminology of external and internal enforcement is from Hart and Holmstrom (1987).

Appendix

Proof of Lemma 3: The joint surplus maximizing investment is:

$$v^* = \gamma^{\frac{-1}{(\gamma-1)}} \tag{A.1}$$

and the punishment investment under nonintegration is:

$$v^{NI} = \left[(1+\mu)/2\gamma\right]^{\frac{1}{(\gamma-1)}} \tag{A.2}$$

Inserting these investments in (18) and (19) we obtain:

$$G^{NI} = \gamma^{\frac{-\gamma}{(1-\gamma)}} \left[1 - \left(\frac{1+\mu}{2}\right)^{\frac{\gamma}{\gamma-1}}\right] - \frac{(1+\mu)}{2}\gamma^{\frac{-1}{(\gamma-1)}} \left[1 - \left(\frac{1+\mu}{2}\right)^{\frac{1}{\gamma-1}}\right] \tag{A.3}$$

$$L^{NI} = \gamma^{\frac{-1}{(1-\gamma)}} \left[1 - \left(\frac{1+\mu}{2}\right)^{\frac{1}{\gamma-1}}\right] - \gamma^{\frac{-\gamma}{(\gamma-1)}} \left[1 - \left(\frac{1+\mu}{2}\right)^{\frac{\gamma}{\gamma-1}}\right] \tag{A.4}$$

Therefore the lowerbound for the discount factor under nonintegration is:

$$\begin{aligned} \underline{\delta}^{NI} = & \left\{\left[1 - \left(\frac{1+\mu}{2}\right)^{\frac{\gamma}{\gamma-1}}\right] - \frac{(1+\mu)}{2}\gamma \left[1 - \left(\frac{1+\mu}{2}\right)^{\frac{1}{\gamma-1}}\right]\right\} / \\ & \frac{(1-\mu)}{2}\gamma \left[1 - \left(\frac{1+\mu}{2}\right)^{\frac{1}{\gamma-1}}\right] \end{aligned} \tag{A.5}$$

Differentiating (A.5) with respect to μ we obtain:

$$\begin{aligned} \frac{\partial \underline{\delta}^{NI}}{\partial \mu} \overset{s}{=} & \left[1 - \left(\frac{1+\mu}{2}\right)^{\frac{\gamma}{\gamma-1}}\right] \left\{(\gamma-1) - \left(\frac{1+\mu}{2}\right)^{\frac{1}{\gamma-1}} \left[(\gamma-1) - \frac{(1-\mu)}{(1+\mu)}\right]\right\} \\ & -\gamma \left[1 - \left(\frac{1+\mu}{2}\right)^{\frac{1}{\gamma-1}}\right] \left\{(\gamma-1) - \left(\frac{1+\mu}{2}\right)^{\frac{1}{\gamma-1}} \left[(\gamma-1) - \frac{(1-\mu)}{2}\right]\right\} \end{aligned} \tag{A.6}$$

To simplify notation define $\varepsilon \equiv (\gamma - 2) > -1$ and $\eta \equiv (1+\mu)/2$. Since $0 \leq \mu < 1$, then $1/2 \leq \eta < 1$. Then (A.6) simplifies to:

$$\begin{aligned} F_\eta(\varepsilon) &= \left[1 - \eta^{\frac{2+\varepsilon}{1+\varepsilon}}\right]\left\{(1+\varepsilon) - \eta^{\frac{1}{1+\varepsilon}}\left[(1+\varepsilon) - \frac{(1-\eta)}{\eta}\right]\right\} \\ &\quad - (2+\varepsilon)\left[1 - \eta^{\frac{1}{1+\varepsilon}}\right]\left\{(1+\varepsilon) - \eta^{\frac{1}{1+\varepsilon}}(1+\varepsilon) - (1-\eta)\right\} \end{aligned} \quad \text{(A.7)}$$

Next define $\nu \equiv \eta^{\frac{1}{1+\varepsilon}}$. Substituting ν in (A.7) and simplifying we obtain:

$$\begin{aligned} F_\eta(\varepsilon) &= (1-\nu\eta)\left[(1+\varepsilon)(1-\nu) + \nu(1-\eta)/\eta\right] \\ &\quad -(2+\varepsilon)(1-\nu)\left[(1+\varepsilon)(1-\nu) + \nu(1-\eta)\right] \\ &= (1-\nu\eta)(\nu-\eta)/\eta - \varepsilon(2+\varepsilon)(1-\nu)^2 \end{aligned} \quad \text{(A.8)}$$

From ν's definition we have $\varepsilon = \{[\ln(\eta)/\ln(\nu)] - 1\}$. Substituting this in (A.8) gives:

$$\begin{aligned} F_\eta(\varepsilon) &= (1-\nu\eta)(\nu-\eta)/\eta - \left\{[\ln(\eta)/\ln(\nu)]^2 - 1\right\}(1-\nu)^2 \\ &= \left\{\nu(1-\eta)^2 - \eta(1-\nu)^2[\ln(\eta)/\ln(\nu)]^2\right\}/\eta \\ &= \frac{(1-\eta)^2(1-\nu)^2}{\eta[\ln(\nu)]^2}\left[f(\nu) - f(\eta)\right] \end{aligned} \quad \text{(A.9)}$$

where $f(\nu) = \frac{\nu[\ln(\nu)]^2}{(1-\nu)^2}$. It is straightforward to show that:

$$\begin{aligned} -1 &< \varepsilon < 0 \Leftrightarrow 0 < \nu < \eta < 1 \\ \varepsilon &= 0 \Leftrightarrow 1/2 \leq \nu = \eta < 1 \\ \varepsilon &> 0 \Leftrightarrow 1/2 \leq \eta < \nu < 1 \end{aligned}$$

Lemma 3 says that $F_\eta(\varepsilon) \overset{s}{=} \varepsilon$. It is easy to verify from (A.9) that $F_\eta(0) = 0$. Furthermore $F_\eta(\varepsilon) \overset{s}{=} [f(\nu) - f(\eta)]$. Therefore $f'(\nu) > 0$ for $\nu \in (0,1)$ implies that $F_\eta(\varepsilon) \overset{s}{=} \varepsilon$.

$$f'(\nu) = \frac{(1+\nu)[\ln(\nu)]^2}{(1-\nu)^3} + \frac{2\ln(\nu)}{(1-\nu)^2} = k(\nu)h(\nu) \quad \text{(A.10)}$$

where $k(\nu) = [2(1-\nu)/(1+\nu) + \ln(\nu)]$ and $h(\nu) = (1+\nu)\ln(\nu)/(1-\nu)^3$. $h(\nu) < 0$ for $\nu \in(0,1)$. $k(1) = 0$ and $k'(\nu) = (1-\nu)^2/\nu(1+\nu)^2 > 0$ and therefore $k(\nu) < 0$ for $\nu \in(0,1)$. Accordingly $f'(\nu) = h(\nu)k(\nu) > 0$ for $\nu \in(0,1)$. ■

Proof of Proposition 8: First note that S^* does not depend on μ or λ_j. Agent i's punishment payoff under nonintegration is:

$$P_i^p = \frac{(1+\mu)}{2}v_i^{NI} + \frac{(1-\mu)}{2}v_j^{NI} - c_i(v_i^{NI}) \tag{A.11}$$

Total differentiation gives:

$$dP_i^p/d\mu = \frac{1}{2}(v_i^{NI} - v_j^{NI}) + \frac{(1-\mu)}{2}\frac{\partial v_j^{NI}}{\partial \mu} \tag{A.12}$$

Consequently the aggregate punishment is decreasing in μ:

$$d(S^* - P_1^p - P_2^p)/d\mu = -\frac{(1-\mu)}{2}\left[\frac{\partial v_1^{NI}}{\partial \mu} + \frac{\partial v_2^{NI}}{\partial \mu}\right] < 0. \tag{A.13}$$

Agent i's deviation payoff is:

$$P_i^d = \frac{(1+\mu)}{2}v_i^{NI} + \frac{(1-\mu)}{2}v_j^* - c_i(v_i^{NI}) \tag{A.14}$$

By differentiating totally we obtain:

$$dP_i^d/d\mu = \frac{1}{2}(v_i^{NI} - v_j^*) \tag{A.15}$$

$$d(P_1^d + P_2^d - S^*)/d\mu = \frac{1}{2}(v_1^{NI} + v_2^{NI} - v_1^* - v_2^*) < 0 \tag{A.16}$$

Repeating the analysis for the integrated structure gives:

$$d(S^* - P_1^p - P_2^p)/d\lambda_j = -\frac{(1-\lambda_j)}{2}\frac{\partial v_1^{iI}}{\partial \lambda_j} < 0. \tag{A.17}$$

$$d(P_1^d + P_2^d - S^*)/d\lambda_j = \frac{1}{2}(v_i^{iI} - v_i^*) < 0 \tag{A.18}$$

■

Proof of Lemma 4: (i) $\underline{\delta}^{iI}(0)=\underline{\delta}^{iI}(1)$. The first-best and punishment investments are:

$$v_i^*=(\sigma_i/\gamma)^{\frac{1}{(\gamma-1)}} \tag{A.19}$$

$$v_i^{iI}=[(1+\lambda_j)\,\sigma_i/2\gamma]^{\frac{1}{(\gamma-1)}} \tag{A.20}$$

$$v_j^{iI}=(\sigma_j/2\gamma)^{\frac{1}{(\gamma-1)}} \tag{A.21}$$

Using these equations we obtain the lowerbound for the discount factor under agent i control:

$$\begin{aligned}\underline{\delta}^{iI}(\lambda_j) \;=\; & \{\,\sigma_i^{\frac{1}{\gamma-1}}\left[1-\left(\tfrac{1+\lambda_j}{2}\right)^{\frac{\gamma}{\gamma-1}}\right]+\sigma_j^{\frac{1}{\gamma-1}}\left[1-\left(\tfrac{1}{2}\right)^{\frac{\gamma}{\gamma-1}}\right]-\\ & \tfrac{1+\lambda_j}{2}\gamma\sigma_i^{\frac{1}{\gamma-1}}\left[1-\left(\tfrac{1+\lambda_j}{2}\right)^{\frac{1}{\gamma-1}}\right]-\tfrac{1}{2}\gamma\sigma_j^{\frac{1}{\gamma-1}}\left[1-\left(\tfrac{1}{2}\right)^{\frac{1}{\gamma-1}}\right]\}\,/\\ & \left\{\tfrac{1-\lambda_j}{2}\gamma\sigma_i^{\frac{1}{\gamma-1}}\left[1-\left(\tfrac{1+\lambda_j}{2}\right)^{\frac{1}{\gamma-1}}\right]+\tfrac{1}{2}\gamma\sigma_j^{\frac{1}{\gamma-1}}\left[1-\left(\tfrac{1}{2}\right)^{\frac{1}{\gamma-1}}\right]\right\}\end{aligned} \tag{A.22}$$

We simplify notation by defining $\tilde{\eta}\equiv(1+\lambda_j)/2$, $c_i\equiv\sigma_i^{\frac{1}{\gamma-1}}$ and $\varepsilon\equiv(\gamma-2)$.

$$\begin{aligned}\underline{\delta}^{iI}(\lambda_j) \;=\; & \left\{\begin{array}{c} c_i\left[1-\tilde{\eta}^{\frac{2+\varepsilon}{1+\varepsilon}}\right]+c_j\left[1-\left(\tfrac{1}{2}\right)^{\frac{2+\varepsilon}{1+\varepsilon}}\right]-\tilde{\eta}c_i(2+\varepsilon)\left[1-\tilde{\eta}^{\frac{1}{1+\varepsilon}}\right]\\ -\tfrac{1}{2}c_j(2+\varepsilon)\left[1-\left(\tfrac{1}{2}\right)^{\frac{1}{1+\varepsilon}}\right]\end{array}\right\}/\\ & \left\{(1-\tilde{\eta})c_i(2+\varepsilon)\left[1-\tilde{\eta}^{\frac{1}{1+\varepsilon}}\right]+\frac{1}{2}c_j(2+\varepsilon)\left[1-\left(\frac{1}{2}\right)^{\frac{1}{1+\varepsilon}}\right]\right\}\end{aligned} \tag{A.23}$$

Next define $\tilde{\nu}\equiv\tilde{\eta}^{1/(1+\varepsilon)}$ and $\phi\equiv 2^{-1/(1+\varepsilon)}$

$$\begin{aligned}\underline{\delta}^{iI}(\lambda_j) \;=\; & \left[\begin{array}{c}(1-\tilde{\eta}\tilde{\nu})\,c_i+(1-\phi/2)\,c_j-(2+\varepsilon)\,(1-\tilde{\nu})\,\tilde{\eta}c_i\\ -(2+\varepsilon)\,(1-\phi)\,c_j/2\end{array}\right]/\\ & [(2+\varepsilon)\,(1-\tilde{\nu})\,(1-\tilde{\eta})c_i+(2+\varepsilon)\,(1-\phi)\,c_j/2]\end{aligned} \tag{A.24}$$

For $\lambda_j=0:\tilde{\eta}=1/2$ and $\tilde{\nu}=\phi$ and for $\lambda_j=1:\tilde{\eta}=1$ and $\tilde{\nu}=1$. Substituting these into (A.24) we obtain:

$$\underline{\delta}^{iI}(0) = [(1-\phi/2)(c_i+c_j) - (2+\varepsilon)(1-\phi)(c_i+c_j)/2] / [(2+\varepsilon)(1-\phi)(c_i+c_j)/2]$$
$$= [(1-\phi/2) - (2+\varepsilon)(1-\phi)/2] / [(2+\varepsilon)(1-\phi)/2] \quad (A.25)$$

$$\underline{\delta}^{iI}(1) = [(1-\phi/2)c_j - (2+\varepsilon)(1-\phi)c_j/2] / [(2+\varepsilon)(1-\phi)c_j/2]$$
$$= [(1-\phi/2) - (2+\varepsilon)(1-\phi)/2] / [(2+\varepsilon)(1-\phi)/2] \quad (A.26)$$

This proves that $\underline{\delta}^{iI}(0) = \underline{\delta}^{iI}(1)$.

(*ii*) $\partial\underline{\delta}^{NI}/\partial\mu \stackrel{s}{=} \varepsilon$. The punishment investments under nonintegration are:

$$v_i^{NI} = [(1+\mu)\sigma_i/2\gamma]^{\frac{1}{(\gamma-1)}} \quad (A.27)$$

Substituting these into (33) we obtain (A.5); the lowerbound for nonintegration is equal when one or both agents have an investment. Therefore the proof of Lemma 3 applies here.

(*iii*) $\underline{\delta}^{NI}(\mu) - \underline{\delta}^{iI}(\lambda_j) \stackrel{s}{=} \underline{\delta}^{iI}(\lambda_j) - \underline{\delta}^{JO} \stackrel{s}{=} \varepsilon$ **for** $0 < \mu = \lambda_j < 1$. Since at $\mu = \lambda_j$ $\eta = \tilde{\eta}$ and $\nu = \tilde{\nu}$ we obtain:

$$\underline{\delta}^{NI}(\mu) - \underline{\delta}^{iI}(\lambda_j) = \frac{(1-\eta\nu) - \eta(2+\varepsilon)(1-\nu)}{(1-\eta)(2+\varepsilon)(1-\nu)}$$
$$- \frac{(1-\eta\nu)c_i + (1-\phi/2)c_j - \eta(2+\varepsilon)(1-\nu)c_i}{(1-\eta)(2+\varepsilon)(1-\nu)c_i + (2+\varepsilon)(1-\phi)c_j/2}$$
$$+ \frac{(2+\varepsilon)(1-\phi)c_j/2}{(1-\eta)(2+\varepsilon)(1-\nu)c_i + (2+\varepsilon)(1-\phi)c_j/2}$$
$$\stackrel{s}{=} (1-\eta\nu)(1-\phi) - 2(1-\eta)(1-\nu)(1-\phi/2)$$
$$+ (1-2\eta)(2+\varepsilon)(1-\nu)(1-\phi) \quad (A.28)$$

Note that $\underline{\delta}^{NI}$ is not defined at $\mu = 1$.

Under joint ownership the punishment investments are:

$$v_i^{JO} = (\sigma_i/2\gamma)^{\frac{1}{(\gamma-1)}} \tag{A.29}$$

And we have:

$$\begin{aligned}
\underline{\delta}^{iI}(\lambda_j) - \underline{\delta}^{JO} \;=\;& \frac{(1-\tilde{\eta}\tilde{\nu})\,c_i + (1-\phi/2)\,c_j - \tilde{\eta}\,(2+\varepsilon)\,(1-\tilde{\nu})\,c_i - (2+\varepsilon)\,(1-\phi)\,c_j/2}{(1-\tilde{\eta})\,(2+\varepsilon)\,(1-\tilde{\nu})\,c_i + (2+\varepsilon)\,(1-\phi)\,c_j/2} \\
& - \frac{(1-\phi/2) - (2+\varepsilon)\,(1-\phi)\,/2}{(2+\varepsilon)\,(1-\phi)\,/2} \\
& \stackrel{s}{=} (1-\tilde{\eta}\tilde{\nu})\,(1-\phi) - 2(1-\tilde{\eta})\,(1-\tilde{\nu})\,(1-\phi/2) \\
& + (1-2\tilde{\eta})\,(2+\varepsilon)\,(1-\tilde{\nu})\,(1-\phi)
\end{aligned} \tag{A.30}$$

This proves that $\underline{\delta}^{NI}(\mu) - \underline{\delta}^{iI}(\lambda_j) \stackrel{s}{=} \underline{\delta}^{iI}(\lambda_j) - \underline{\delta}^{JO}$ for $\mu = \lambda_j$. Therefore one of the following has to be true:

(*i*) $\underline{\delta}^{NI}(\mu) < \underline{\delta}^{iI}(\lambda_j) < \underline{\delta}^{JO}$
(*ii*) $\underline{\delta}^{NI}(\mu) = \underline{\delta}^{iI}(\lambda_j) = \underline{\delta}^{JO}$
(*iii*) $\underline{\delta}^{NI}(\mu) > \underline{\delta}^{iI}(\lambda_j) > \underline{\delta}^{JO}$

From Lemmata 2 and 3 we know that $\underline{\delta}^{NI}(0) = \underline{\delta}^{JO}$ and $\partial\underline{\delta}^{NI}/\partial\mu \stackrel{s}{=} \varepsilon$. Therefore for $\mu = \lambda_j = 0$ and/or $\varepsilon = 0$ $\underline{\delta}^{NI} = \underline{\delta}^{JO}$ and (*ii*) holds. For $\mu = \lambda_j \in (0,1)$ and $\varepsilon < 0$ $\underline{\delta}^{NI} < \underline{\delta}^{JO}$ and therefore (*i*) holds. Respectively for $\mu = \lambda_j \in (0,1)$ and $\varepsilon > 0$ $\underline{\delta}^{NI} > \underline{\delta}^{JO}$ and (*iii*) holds. ■

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