Participation Screen for Collusion in Auctions

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25th October 2010

Abstract

I propose a new statistical method for detecting collusion in auctions that requires data only on entry decisions. The test is robust to unobserved heterogeneity, which I confirm using Monte Carlo analysis. Besides many different auction settings with different collusive schemes, this test can be applied generally in static simultaneous discrete games with flexible information structures. I apply this test to procurement auctions for contracts for snow removal from schools in Helsinki. Two of the bidders seem to participate in a territorial allocation scheme.

Keywords: Auctions; Collusion; Discrete games; Simultaneous equations estimation

JEL Classification: C35; D44; L40; L85

*This research has benefited most from the numerous discussions with Otto Toivanen. I also thank Petter Berg, Estelle Cantillon, Peter Davis, Elias Einiö, Joe Harrington, Ari Hyytinen, Mikael Juselius, Andreas Knabe, Erkki Koskela, Timo Kuosmanen, Tommi Kyntölä, Matti Liski, Eugenio Miravete, Rob Porter, Heikki Pursiainen, Frode Steen, Rune Stenbacka and Douglas Zona for their help and comments. Seppo Stenroos and Petri Alarinne of PALMIA and Kalevi Saarelainen of the City of Helsinki Supplies Department helped with the data and provided industry expertise. Seminar and conference participants at ESEM-09, ESWC-10, CEPR Applied IO-09, CORE-07, EARIE-07, ECARES-07, FDPE-07, HECER-08, FEAAM-07, NORIO-07, RUESG-08 and in particular one industry expert at Jyväskylä provided useful discussions. I gratefully aknowledge the financial support of FDPE, RUESG, the Yrjö Janhsson Foundation and Emil Aaltonen Foundation. This paper was partly written while visiting ECARES at ULB and finished while visiting STICERD at LSE. Email: janne.tukiainen@helsinki.fi. Tel:+358505454490.

1 Introduction

Collusion means that firms coordinate their actions to limit competition. A social planner dislikes collusion because colluding firms profit less from the increased market power than their customers lose due to the increased prices. In a public procurement setting, the social planner prefers the auctioneer's revenue over the bidders', because a public auctioneer needs to collect costly and distortionary taxes to pay the winning bidder. It is possible to hinder the formation of bidding rings by using auction mechanisms that make it hard to form or sustain a cartel. Another way to prevent cartel formation is to use statistical methods to detect collusion. Besides being preventive, these methods can help in detecting and prosecuting illegal cartels.

In this study, I propose a new statistical method to test for collusion in auctions. This new test has five attractive features. First, it uses data only on entry decisions (bid submissions). Data on prices (bids) are not needed. Second, the test is robust to unobserved heterogeneity. Third, it can be used to detect both territorial allocation and phony bidding collusive schemes. Fourth, it is applicable in a wide variety of different auction mechanisms and works also under different assumptions concerning timing and information structures of the auction game. Fifth, the test can be applied in many other institutional settings besides auctions, as long as independent and mutually exclusive markets can be defined, and in addition to possible common shocks, the player's have some private information. In general, this test can be used in simultaneous static discrete games of incomplete information where the choice space is binary. Many of these features are shared by one of the many collusion tests proposed by Porter and Zona (1999, denoted PZ). However, that test is not robust to unobserved heterogeneity. Porter and Zona (1993) propose another collusion test that is robust to unobserved heterogeneity but their test requires price data.

A robust test that uses only entry decision data to detect cartels is useful for several reasons. First, data on bids or prices is often not available or would be very costly to obtain. Second, in some markets there is very little variation in the price data. Franchising chains, for example, often employ nationwide pricing policies. Also in my application, there is not enough variation in the bids. In these cases, tests that use bids are not informative. Third, Lang and Rosenthal (1991) show that in simultaneous first-price sealed-bid auctions with decreasing returns to scale, noncooperative equilibrium price patters emerge that look very much like collusive outcomes. They state that "our two-bidder results suggest that it may be impossible to use price/output data to demonstrate the existence of bid rigging". However, with many contracts it is very unlikely that bid submission patterns that would be interpreted as collusion in entry tests would emerge in the Lang-Rosenthal model.

The central difficulty in detecting collusion is that similar market outcomes can be the result of either collusive or competitive behavior. Territorial allocation can be the result of either an explicit agreement or due to cost advantages that firms have in different areas. In the presence of entry costs, capacity constraints or reservation prices together with transaction costs, firms could decide to submit a bid only in those markets that are near the location of their operations. If firms are located in different places, territorial allocation emerges as a competitive result. It is suspicious if territories overlap, but firms nonetheless systematically avoid bidding for the same contracts. However, this can again be the result of competitive behavior if the contracts are heterogenous. Some firms may have cost advantages in some types of contracts. Therefore, with heterogenous contracts and asymmetric bidders, participation patterns of any kind may emerge in a competitive setting. Nevertheless, if we control for both observed and unobserved bidder and contract heterogeneity, then the identity of other participants should not affect the participation decision of any bidder in a competitive setting. This makes testing for collusion possible. For strategic reasons, bidders would like to avoid bidding for the same contracts, but if the bidders are symmetric, one bidder has no reason to avoid a given competitor any more than any other competitor.

PZ propose several tests for detecting collusion. Although their main focus is on tests that rely on price data, they also use the above idea and propose a test based on estimating the effect of competitors' identity on entry. PZ suggest testing for the correlation of the residuals of single equation participation choice models. Negative correlation between two bidders' residuals implies territorial allocation and positive correlation phony bidding. PZ use it to detect phony bidding. My test is based on solving a fully endogenous simultaneous equations model of participation. I show how this approach makes the test robust to unobservable heterogeneity. I use the estimation strategy proposed by Bresnahan and Reiss (1990 and 1991, denoted BR) and extended by Tamer (2003). Applying these methods that are developped for entry games of complete information in a setting where the information structure includes also private shocks, that is incomplete information, allows me to interpret the complete information competition parameters as incomplete information collusion parameters.

I apply both the PZ test and my new method to school yard snow removal auctions in the City of Helsinki held in the falls of the years 2003 to 2005. In this market, two bidders seem to avoid bidding for the same contracts in a region where they are both otherwise active. I test whether this territorial allocation is a result of collusion or competitive allocation that results from contract and bidder characteristics. I find some evidence of collusion.

I make several contributions to the literature on empirical analysis of auctions and discrete game. First, I propose a new, widely applicable test to detect collusion that is robust to unobserved heterogeneity. I confirm the claimed robustness by Monte Carlo analysis. The other main contribution of this study is to show that it is possible to conduct inference about auctions using only entry data. To my knowledge, Li and Zheng (2010) provide in a simultaneous but independent work, the only other auction study that conducts inference using only the information on entry decision. Typically, only bidding information has been used to test theory or infer auction model parameters, although some approaches have allowed entry to be endogenous. Furthermore, the empirical application is important in itself because it is the first empirical study of a territorial allocation scheme. The minor contribution of this article is the evidence for the competition authority resulting from the empirical application.

This study is related to two different fields of empirical industrial organization. The first is the literature on the empirical estimation of auction models, more specifically the detection of collusion in auctions. The second is the literature on discrete games, especially entry games, because this test can be applied in all simultaneous static discrete games of incomplete information where players make binary choices, since a single auction is simply an analog of a single market in standard entry game. Harrington (2008) provides a recent survey on detecting cartels. Berry and Tamer (2007) provide a survey of empirical analysis of entry models. The existing studies on the detection of collusion in auctions (e.g. Bajari and Ye (2003), Baldwin, Marshall and Richard (1997), Banerji and Meenakshi (2008), Porter (1983), Porter and Zona (1993,1999), Price (2008)) only have applications on phony bidding scenarios and with the exception of a minor part in PZ, use only price information. Moreover, I am aware of no previous studies that propose empirical methods to detect collusion in any discrete games.

This study proceeds a follows. In Section 2, I present my new test, discuss the underlying theory and derive conditions for identification. I conduct Monte Carlo analysis to examine the finite sample properties of both the PZ test and my new test in Section 3. In Section 4, I present the market of the application and analyze its characteristics with respect of collusion. I present the data and descriptive statistics in Section 5 and analyze the results in Section 6. Finally, Section 7 concludes.

2 Testing

In this section, I discuss the extend of different data generating processes to which my test applies. Besides a wide variety of different auction models, these include a large set of interesting static simultaneous discrete games of incomplete information. Furthermore, I show that the allowed information structures in the discrete games to which my test applies are more general than just the classic pure incomplete information setting where only private shocks are allowed. In addition to these private shocks, I show that also common shocks can be allowed. This generalization of the information structure makes the test robust to unobserved heterogeneity. As I proceed, I make some comparisons to the closest counterpart in the literature, that is the PZ collusion test. Their test is discussed in detail in appendix A. Despites my test's generality, I discuss testing here mainly in the light of my application, which is a procurement auction. Due to their simultaneous and often static nature, auctions are particularly well suited for the estimation methods that are applied here. Moreover, in auctions it is relatively easy to argue the independence of the markets under scrutiny.

2.1 Detecting collusion in auctions

First, I assume that there is no collusion and N risk-neutral potential bidders are competitively bidding for a procurement contract. Assume that an affiliated values (from nowon I write costs instead of values due to the procurement setting) auction model as proposed by Milgrom and Weber (1982), with the extension that bidders are allowed to be asymmetric, is a good approximation of the data generating process. This affiliated costs model incorporates both pure common costs and independent private costs models as its special cases. The auctioneer may set a public reservation price. A secret reservation price could also be modelled in this setting as one additional bidder who is known to always enter.

Entry (submitting a bid) in auctions has been typically modelled in two different ways that differ on their assumptions concerning the timing that information is revealed. In the first model, all the bidders learn their costs of providing the contracted service or product, or signals on these costs, only after they pay an entry cost. Bidders enter if their expected profits of entry exceed this entry cost. This modelling framework was initiated by Levin and Smith (1994), and it is used more often than the second model in empirical analyses. The second model, which was first suggested by Samuelsson (1985), assumes that costs, or signals, are known before the entry cost has to be paid. Bidders enter if their private signal exceeds a known screening level. The entry costs in the second model consists only of bid preparation costs, but the first model can also include information acquisition costs. The equilibrium in the first model is that the entry behavior is always randomized but in the second model also pure strategy equilibria exist. In both the models, the entry behavior is determined by a cut-off, which in the first model is in the entry cost and in the second model in the private signal.

Li and Zhang (2010) show that in the Milgrom and Weber (1982) context, both of these different entry models generate the same reduced form binary choice model which they write as $D_{ti} = 1(x_t\beta + z_{ti}\gamma + \eta_t + e_{it} > 0)$. Now, $D_{ti} \in \{0,1\}^N$ denotes the entry decision of bidder $i = 1, ..., N$ in an auction $t = 1, ..., T$. D_{ti} is 1 if potential bidder i enters and 0 if not. x_t denotes a vector of observable auction characteristics and z_{ti} a vector of observable bidder specific characteristics. η_t denotes such auction heterogeneity that is unobserved to the researcher but observed by all the players. e_{it} is the idiosyncratic shock. 1 is an indicator function. By adding also unobserved bidder heterogeneity ξ_{it} and the observed characteristics of all the potential competitors $\sum_j z_{tj}\lambda_{ij}$, $j = 1, ..., N$, $j \neq i$, to the Li and Zhang (2010) model we get

$$(1)\quad D_{ti} = 1(x_t\beta_i + z_{ti}\gamma_i + \textstyle\sum_j z_{tj}\lambda_{ij} + \eta_t + \xi_{it} + e_{it} > 0).$$

In a standard complete information discrete game, the error term would be sum of the two commonly observed error terms η_t and ξ_{it}. e_{it} is the private shock that would be the only shock present in a standard incomplete information discrete game. Therefore, this model can be seen as a generalization of static simultaneous entry game of incomplete information in a sense that also commonly observed shocks are allowed. I assume e_{it} to be independent of all the other variables in the model. However, to capture the possible affiliation either in the private costs or in the entry costs, the e_{it}'s can be correlated with each other. Li and Zhang (2010) base their affiliation test on testing whether the private shocks e_{it} are actually correlated. Unlike my collusion test, their test is not robust to unobservable bidder heterogeneity, although they are able to control for unobservable auction heterogeneity.

I assume that all the three shocks are distributed independently from each other, but they (or their effect) on entry can be correlated between bidders. Since all of these shock are unobservable, only their sum can be included in the estimated model. Let $u_{it} = \eta_t + \xi_{it} + e_{it}\tilde{}F(0, \Sigma_u)$, where $\Sigma_u = \begin{bmatrix} \sigma^2_{1\eta} + \sigma^2_{1\xi} + \sigma^2_{1e} & \rho_{1N\eta} + \rho_{1N\xi} + \rho_{1Ne} \\ & ... & \\ \rho_{N1\eta} + \rho_{N1\xi} + \rho_{N1e} & \sigma^2_{N\eta} + \sigma^2_{N\xi} + \sigma^2_{Ne} \end{bmatrix} = \begin{bmatrix} \sigma^2_{1u} & \rho_{1Nu} \\ & ... & \\ \rho_{N1u} & \sigma^2_{Nu} \end{bmatrix}.$

Thus the reduced form of the competitive model that is observable to the researcher is $D_{ti} = 1(x_t\beta_i + z_{ti}\gamma_i + \sum_j z_{tj}\lambda_i + u_{it} > 0)$. To allow for collusion in the reduced form model, I include the actual participation decision of all the potential bidders in the equation to be estimated. The idea is that if we control for both the observed and unobserved bidder and contract heterogeneity, then the identity of other actual participants as such should not affect the participation decision of any bidder in a competitive setting. All the bidders prefer to face as little competition as possible, which is captured mainly by λ_{ij}, but they do not discriminate between identical competitors. Whereas, in an explicit collusive setting, where the players divide the markets by negotiating with each other, the identity of competitors has an effect on participation. In the case of territorial allocation, the fellow cartel members are avoided. In the case of phony bidding, the fellow cartel members may bid too often to the same contracs. Thus, the reduced form of the collusive entry model can be presented as

(2) $D_{ti} = 1(x_t\beta_i + z_{ti}\gamma_i + \sum_j z_{tj}\lambda_{ij} + \sum_j y_{tj}\delta_{ij} + u_{it} > 0).$

Instead of having D_{tj} in the equation (2), we can just include the actual observed entry decision of competitors y_{tj}, because y_{tj} is observable to both the researcher and the colluding players, sice the cartel members agree on this collusion. Thus the collusion test can be based on estimating the model (2) and then testing whether the δ_{ij}'s differ from zero. Since λ_{ij} and u_{it} (and also β_i to some extent, see below) capture both the observed and unobserved competition effects, δ_{ij}'s contain the essential information about collusion. Whether testing based on δ_{ij}'s solely is enough, depends on the assumptions about the information structure as discussed in Table 1.

The downside of this reduced form approach is that conrafactual analysis, for example on the welfare costs of the possible collusion, is difficult, but the upside of this is that a similar reduced form could be generated by a wide variety of auction models. In addition to applying for the general affiliated information structure and different assumptions about timing in the entry model, this test can be applied regardless of whether the auction mechanism is that of first-price or second-price, or open or sealed-bid, as long as the entry process follows either of the two models described above. This robustness to different mechanisms is gained by analysing only entry decisions and not taking any stand on how the actual bidding decisions are made.

In the auction case, a structural form would be a function of (expected) costs, entry costs, bids and winning probabilities. In a model with entry costs or fees, there is a threshold value below which the bidder enters. It seems natural to assume that this threshold and therefore the entry decision is some function of the observables. How well the reduced forms (1) and (2) approximate reality depends on the particular auction. Similar specifications where the number of entrants or the entry decision is a linear reduced form function of the observables have also been used previously in the empirical auction literature (e.g. Bajari and Hortacsu 2003 and Athey et al. 2004). A fully structural form of the profit function in the general market entry case should be written in terms of prices, quantities and costs. Typically, researchers have had to resort to a similar reduced form because of a lack of data and the endogeneity of price and quantity (Ciliberto and Tamer 2009). Rather than a reduced form of a discrete game, model (1) can also be thought of as a joint estimation of the players' reaction functions. In fact, models similar to (1) are often called structural in the traditional entry literature. In relation to auction theory, (1) is clearly just a reduced form approximation.

2.2 Detecting collusion in static simultaneous discrete games

Next, I compare this auction model to a static simultaneous entry game of pure incomplete information (e.g. Seim 2006 and Bajari et al. 2009a). The competitive entry decision in these games can be written in a following way:

(3) $D_{ti} = 1(x_t\beta_i + z_{ti}\gamma_i + \sum_j E[y_{tj}|x_t, z_t]\theta_{ij} + e_{it} > 0)$.

Typically this model is estimated in two stages. Note that because of private information, y_{tj} is not known to the player i, and therefore it has to form expectations on it. In the first stage, this expected entry of competitors is estimated, for example by using some more flexible form, such as orthogonal polynomials, of this simple linear probability model: $y_{tj} = 1(x_t\varsigma_j + z_t\tau_j + \varepsilon_{jt} > 0)$ with iid shocks ε_{jt}. This estimate of y_{tj} is then inserted into equation (3) to replace $E[y_{tj}|x_t, z_t]$. The strategic elements can left out of the first stage equation, because the equilibrium is only a function of observable state variables and thus a flexible first stage that includes these state variables is sufficient to obtain consistent estimates of these probabilities (e.g. Bajari et al. 2008). Identification of θ_{ij}'s comes from excluding z_{tj} from the second stage equation (3). One advantage of the two-stage approach is that the parametric first stage does not need to be correctly specified for the second stage to be consistent (Bajari et al. 2009a). In order to analyze the differences with the auction version (1), let us insert the variables of this simple linear first stage directly into the second stage instead of the estimate as usual. We get

$$(4)\quad D_{ti} = 1(x_t\beta_i + z_{ti}\gamma_i + \textstyle\sum_j (x_t\varsigma_j + z_t\tau_j)\theta_{ij} + e_{it} > 0)$$
$$= 1(x_t(\beta_i + \textstyle\sum_j \varsigma_j\theta_{ij}) + z_{ti}(\gamma_i + \sum_j \tau_j\theta_{ij}) + \sum_j z_{tj}\tau_j\theta_{ij} + e_{it} > 0).$$

We can see that this standard pure incomplete information entry game (4) is very similar to the reduced form auction game (1), but it sheds some additional light on how to interpret the parameters of the models (1) or (2). This means that when we are estimating the auction model (1) or (2), the parameters for the observed contract and bidder characteristics capture both the effect they have on the player i's costs and the effect they have on the entry of all its competitors j, adjusted by how much that entry effect in turn affects player i's decision θ_{ij}. Since the separate identification of the cost effects from the competition effects is not the purpose of my test, my test can be estimated in one stage. Two stage approach would also be possible, but that has the cost that the estimation would have to be conducted using estimated observations which complicates the statistical inference on those variables.

Let us again assume the more flexible information structure u_{it} instead of e_{it} alone. Then we have to include the unobserved heterogeneity also to the first stage. Similar calculations as done for the equation (4), and including the actual participation decision of all the potential competitors in order to capture the collusive effects, leads to equation (5). This is exactly the model (2) but we now have a better understanding also on the interpretation of the error terms of the model (2).

$$(5)\quad D_{ti} = 1(x_t(\beta_i+\textstyle\sum_j \varsigma_j\theta_{ij})+z_{ti}(\gamma_i+\sum_j \tau_j\theta_{ij})+\sum_j z_{tj}\tau_j\theta_{ij}+\sum_j y_{tj}\delta_{ij}+(1+\sum_j \theta_{ij})\eta_t+(1+\sum_j \theta_{ij})\xi_{jt}+e_{it} > 0)$$
$$= 1(x_t\mu_i + z_{ti}\phi_i + \textstyle\sum_j z_{tj}\lambda_{ij} + \sum_j y_{tj}\delta_{ij} + \upsilon_{it} > 0).$$

The main assumption for my testing purposes is that the identification of the collusion effect hinges on the game being that of incomplete information. Which means that some relevant amount of unonserved information must be private. In that case, the estimate of the expected entry captures the competition effect. Whether all of it or part of it, depends on the information structure of the error terms. If any private information is present, the competitors' characteristics capture some of the competition effect, because when firms have private information, variation in the observed competitors' cost shifters will affect a given players actions, because this variation shifts the expectation about the competitors' entry. In the case of complete information, the actual entry would capture the competition effect, because all shocks are observed, and assuming pure strategies, then the competitors' true entry decisions are observed. Therefore, in the complete information case, variation in competitors' cost shifters do not cause variation in the expectation of entry. To help to understands under what information structures my test works, I present a following table:

Table 1. Parameter restrictions based on the information structure and the form of competition.

Information regime	**Shock structure**	**Collusion**	**Competition**
Pure incomplete information	$\eta_t = 0, \xi_{it} = 0$, $e_{it} = u_{it}$	$\lambda = 0$, $\delta \neq 0$	$\lambda \neq 0$, $\delta = 0$
Incomplete information w. unobserved heterogeneity	$\eta_t + \xi_{it} \neq 0$, $e_{it} \neq 0$	$\lambda = 0$, $\delta \neq 0$	$\lambda \neq 0$, $\delta \neq 0$
Pure complete information	$\eta_t + \xi_{it} = u_{it}$, $e_{it} = 0$	$\lambda = 0$, $\delta \neq 0$	$\lambda = 0$, $\delta \neq 0$

These testable parameter restrictions imply, that if one is willing to assume pure incomplete information, the test can be based solely on testing whether $\delta \neq 0$. In this case, ρ_u captures the affiliation effect. Already this is a generalization of the PZ test, since their test works only if there is no affiliation. In the case of incomplete information, but allowing also for common shocks, that is unobserved heterogeneity, the test needs to be based jointly on $\lambda = 0$ and $\delta \neq 0$. It can be seen that the competition effect θ is not identified from model (5). Nonetheless, important inference on θ, from the point of view of my testing, can be made based on λ. Since $\lambda = \tau\theta$, if own cost shifter $\tau \neq 0$, then the total competitive effect $\lambda = 0$, if and only if the effect of my competitors expected entry on my entry $\theta = 0$. Moreover, it is plausible to assume that $\tau \neq 0$, if $\phi \neq 0$. This is because own cost shifters should dominate the competition effects related to these own cost shifters.

In model (2), ρ_u captures both the possible affiliation effect and the effect of unobserved heterogeneity on entry. Unfortunately, under pure complete information it is not possible to separate competition from collusion since the same parameter captures both the collusion and the competition effects. The main identification assumption is therefore the following:

Identification assumption 1: The game is of incomplete information.

When this is the case, x, z and u in models (2) and (5) capture both the competitive effect, either all of it or part of it, and all of the unobserved heterogeneity. This assumption holds if a relevant amount of u consists of e, the private information component. This means that the game is that of incomplete information but also common shocks allowed. However, private information and the observable cost shifters z should be relevant enough to affect the decision making. The relevance of observed cost shifters can be evaluated my checking whether the own cost shifter is an important determinant of own entry. This identification assumption is more flexible than the similar one that PZ need to make. They need for only the x and z to capture the competition effect totally, whereas I allow also for the unobservable heterogeneity ρ_u to capture part of the competition effect. Moreover, it is not necessary in my test to capture all of it as long as z captures some of it. See Grieco (2010) for more discussion about entry games with both private and common shocks and Toivanen and Tukiainen (2010) for testing between the incomplete and complete information structures.

To highlight the advantage of allowing for correlation in the error term, I present an example related to my school yard snow removal application. There could be differences in production technologies that make it more costly for bidder 1 to provide the service to certain schools, but the characteristics of these particular schools do not hinder bidder 2. For example, firm 1 has larger vehicles and thus trouble getting them through the gates of certain schools. This is unobserved by the econometrician but observed by bidder 2. Then bidder 1 would usually avoid those schools and therefore bidder 2 would bid more often to those schools. This will make ρ_u negative.

The main contribution of this article is the following. I propose to test for collusion by estimating the endogenous simultaneous equations system (2) or (5) and by basing the collusion test on whether the δ_{ij}'s differ significantly from zero. In the case of territorial allocation, δ_{ij}'s are negative and in the case of phony bidding they are positive. The collusion in more general entry games, in which entry requires opening an office or a factory and hiring employers, and thus incurs high entry costs relative to costs of submitting a bid, the collusion parameters would typically be negative. To allow for unobservable heterogeneity, the test needs to be based jointly on λ and δ.

No collusion between players i and j, H_0: $\delta_{ij} = 0$

Collusion between players i and j, H_1: $\delta_{ij} \neq 0$ and $\lambda_{ij} = 0$,

$i = 1, ..., N$ and $j = 1, ..., N$, $j \neq i$.

2.3 Estimation

Ciliberto and Tamer (2009) propose an estimation strategy that can be used to estimate all the parameters of the models (2) or (5), including the parameters of the error term distribution u, especially including all the correlation parameters ρ_{iju} of this error term. Therefore, the δ_{ij}'s are estimated correctly even when there is unobserved bidder or auction heterogeneity, or affiliation in the private shocks. Ciliberto and Tamer (2009) propose their estimation method for analyzing competition effects in a complete information setting. Despite the fact that they have a different interpretation of the δ_{ij}'s, their estimation strategy can be used here. However, the issue of multiple equilibria is different, which I will address later. Unfortunately, Ciliberto and Tamer's (2009) estimation technique is computationally intensive, requires knowledge of statistical coding and inference in partially identified models. Therefore, it could be a too complicated tool for the Competition Authorities that would want to apply my test in their cartel cases. It could also be too cumbersome method for this test to be used to screen different markets for candidates of further investigative work. For this reason, I show how to estimate my test as a 2-player version. The N-player version of the 2-player case can also be conducted using pairwise analysis. Dropping the subscript t, 2-player version of the N-player collusive game (2) or (5) is as follows

$$\begin{aligned} & y_1^* = x\beta_1 + z_1\gamma_1 + z_2\lambda_1 + y_2\delta_1 + u_1, \\ (6)\quad & y_2^* = x\beta_2 + z_2\gamma_2 + z_1\lambda_2 + y_1\delta_2 + u_2, \\ & y_i = 1 \text{ if } y_i^* \geq 0, \text{ otherwise } y_i = 0,\ i = 1, 2. \end{aligned}$$

Now y_i^* denotes the latent continuous variable that determines the participation decision. In an auction setting, y_i^* is the expected profit of bidder i from submitting a bid. Bidder i submits a bid in an auction if $y_i^* > 0$. We observe $y_i = 1$ if bidder i submitted a bid and $y_i = 0$ if it did not. Note that this model nests the PZ test.

Models similar to (6) have been used widely in the entry literature (see Berry and Tamer 2007). The notable exception is that all the previous models are competitive whereas (6) is collusive. In the seminal contributions, BR estimate similar entry models but their approach does not allow for firm heterogeneity. Tamer (2003) provides a more flexible estimation method for models akin to (6) that allows for firm heterogeneity. His main contribution is to show that the parameters of such incomplete[1] model can be identified. Depending on the assumptions about the

[1] In this case, the word incomplete refers to a statistical incompleteness arising from the endogeneity of the system (6), rather than the

data generating process, the parameters are either point or set identified. Tamer (2003) shows that the identification of the parameters in model (6) requires the following four assumptions.

Identification result 1 (Tamer 2003): There is one *unique* continuous regressor in either x_1 or x_2, when $x_i = x + z_i + z_j$, $i, j = 1, 2$, $i \neq j$.

This means that an exclusion restriction is required. In typical competition models, the firm characteristics provide these. For example in Tamer's (2003) model the parameters λ_1 and λ_2 would be zero. The identification of his complete information competition effects δ_1 and δ_2 would require this exclusion restriction. However, interpretation of the δ_i's as collusion requires that these incomplete information competion effects, λ_1 and λ_2, are present. This makes obtaining the exclusion restriction more involved in my collusion test. Its depends on the particular application how this issue can be solved. My solution is following. I assume that there are more than two players present even if the model is estimated pairwise. To capture the competition effects, it is not necessary to include the characteristics of all the competitors, but some functional transformation of these characteristics is enough. I assume that when bidders consider entry, they only consider competing against the most advantageous competitor. PZ make the same assumption in their estimations. Thefore, I replace $z_2\lambda_1$ and $z_1\lambda_2$ in (6), with $\max(z_{j\neq 1})\lambda_1$ and $\max(z_{j\neq 2})\lambda_2$, $j = 1, .., N$ and $N \geq 3$, in my application because the production costs are decreasing in z. In the Monte Carlo, I assume that the production costs are increasing in z and therefore I use a *min* operator. The model that is actually estimated in my application is therefore the model (7).

$$
\begin{aligned}
& y_1^* = x\beta_1 + z_1\gamma_1 + \max(z_{j\neq 1})\lambda_1 + y_2\delta_1 + u_1, \\
(7)\quad & y_2^* = x\beta_2 + z_2\gamma_2 + \max(z_{j\neq 2})\lambda_2 + y_1\delta_2 + u_2, \\
& j = 1, .., N \text{ and } N \geq 3, \\
& y_i = 1 \text{ if } y_i^* \geq 0, \text{ otherwise } y_i = 0,\ i = 1, 2.
\end{aligned}
$$

Identification result 2 (Tamer 2003): Denote $x{+}z_i{+}\max(z_{j\neq i}) = x_i$. We then have an iid sample $\{(y_{1t}, y_{2t}), x_{1t}, x_{2t}\}$ such that $0 < \Pr[y_1, y_2|(x_1, x_2)] < 1$ for all $(y, x_1, x_2) \in Y \times R^{d_1} \times R^{d_2}$ where $x = (x_1, x_2) \in R^d$ and $Y = \{(0,0), (1,1), (0,1)(1,0)\}$.

incompleteness of the infomation structure.

Identification result 3 (Tamer 2003): Let $U = (u_1, u_2)$ be a random vector independent of x with a known joint conditional distribution F_u that is absolutely continuous with mean 0 and unknown covariance matric Ω.

Note that as there are no restrictions on Ω, this assumption implies that the correlation of the error terms is allowed. Therefore, the y's are allowed to be correlated with the error terms. Only the independence of x and z from the u's is required. Moreover, Tamer (2003) argues that even this independency assumption and that the distribution F_u is known can be relaxed. Furthermore, for my testing purposes, it is not necessary to estimate β, γ and λ correctly as long as δ is correct. This also relaxes the need for this independency assumption. When we assume that F_u is bivariate normal, that is $\binom{u_1}{u_2}\tilde{}IIDN\left(\begin{bmatrix}0\\0\end{bmatrix}, \begin{bmatrix}1 & \rho_u\\ \rho_u & 1\end{bmatrix}\right)$, model (6) nests the PZ testing approach. However, any other parametric distribution could be chosen as well. Tamer (2003) argues that as this is a threshold-crossing model, it is possible to normalize the variances in Ω to one. Then the identification of Ω boils down to identifying ρ_u.

Identification result 4 (Tamer 2003): $\delta_1 \times \delta_2 > 0$.

Note that this assumption allows both the territorial allocation case (both δ's negative) and the phony bidding case (both δ's positive). This assumption can be relaxed but that means losing the point identification and having to identify sets. Moreover, my Monte Carlo analysis shows that this is an important restriction that has to be imposed excplicitly on the optimization problem when sample sizes are small.

In a territorial allocation setting that I study in my application, it is natural to assume that the δ_i's are both negative. Then, according to Tamer (2003), it is easy to see that $Pr[(0,0|x)] + Pr[(0,1|x)] + Pr[(1,0|x)] + Pr[(1,1|x)] > 1$. This is an example of an incoherent model. Tamer (2003) argues that although the system (6) is an example of an incoherent model, with some restrictions, namely those in the identification result 4, on the parameters it becomes a coherent model[2]. It remains statistically an incomplete model but Tamer (2003) then shows that this incompleteness does not render identification impossible. Typically, econometricians have imposed a coherency condition $\delta_1 * \delta_2 = 0$. However, this condition changes the model into a recursive one and thus eliminates the simultaneity. BR and Berry (1992) transform their complete information model into one that predicts the joint outcome [(0,1) or (1,0)]. This provides consistent point estimates for the parameters of interest but involves loss of information. Their solution will

[2] A structural model is complete and coherent if for any value of regressors there is a unique value for the responses. Tamer (2003) uses coherency to refer to the existence of a solution to the model and calling the model complete if the solution is unique. An analog in games is the existence of Nash equilibrium and whether it is unique.

be used in my testing approach. Another solution is proposed by Bjorn and Vuong (1985) and Kooreman (1994) who assume that a unique outcome is chosen with known probability in the region of incompleteness. According to Tamer (2003), this method may lead to inconsistent estimates.

According to Tamer (2003), the identification results 1-4, together with assuming that the explanatory variables have full rank, ensure that the parameter vector (β_1(including constant)$, \beta_2$(including constant)$, \gamma_1, \gamma_2, \lambda_1, \lambda_2, \delta_1, \delta_2$) is point identified. Moreover, if the observed variables have rich enough support, all the parameters in the covariance matrix Ω are be point-identified. If the support is not rich enough, the model is no longer point identified but it is still set-identified. Because Tamer (2003) shows that the separate effects of all the parameters are identified from the simultaneous equations model (7), there is no endogeneity problem in this testing approach even though by construction the y's are correlated with the u's whenever $\rho_u \neq 0$.

Tamer (2003) assumes that the firms play only pure strategies but discusses how the estimation could be modified to allow for mixing. Basically, it would again mean set-identification. According to Tamer (2003), his complete information game always has multiple equilibria for large enough supports of the error terms. This needs to be taken into account in his estimation. Since in my collusive model firms agree who enter and who does not, there is no problem of multiple equilibria related to the underlying collusive game. However, the researcher does not observe how the cartel decides which member enters when either of the firms could make profits. To be able to remain agnostic about how the cartel makes this decision, we are back to the problem of multiple equilibria that is mathematically equivalent to the multiple equilibrium problem of competitive entry under complete information.

BR provide one estimation method that is useful in this case. The cost of using the BR idea, that is treating [(0,1) or (1,0)] as the same outcome, is loss of information and thus efficiency. Instead of using an ordered probit as BR do, I use the simultaneous equation formulation of their idea provided by Tamer (2003). Tamer (2003) also proposes a new and more efficient estimator that uses all the information. It is, however, computationally more challenging and conducting Monte Carlo analysis using it would take a lot of time. Moreover, it uses multidimensional kernel smoothing that requires more data points than I have available in the application (given the number of variables in x). Furthermore, for this test to be attractive to the practitioners, I prefer the simpler version. For these two reasons, Tamer's (2003) new estimation method is not used here, but it could be very useful in other applications of my test.

When x denotes all the explanatory variables (also z's), the maximum likelihood estimator presented by Tamer

(2003) that uses the BR idea is defined by the following log likelihood:

$$(8)\quad L_{ML}(b) = \sum_{t=1}^{T} \left[\begin{array}{c} y_{t1}y_{t2}\log(P_1(x_t,b)) + (1-y_{t1})(1-y_{t2})\log(P_2(x_t,b)) \\ +((1-y_{t1})y_{t2} + y_{t1}(1-y_{t2}))\log(1-P_1(x_t,b)-P_2(x_t,b)) \end{array} \right],$$

where $P_1(x_t,b) = \Pr[(y_{t1}=1, y_{t2}=1)|x] = \Pr(u_{t1} \geq -x_{t1}\beta_1 - \delta_1; u_{t2} \geq -x_{t2}\beta_2 - \delta_2)$ and

$P_2(x_t,b) = \Pr[(y_{t1}=0, y_{t2}=0)|x] = \Pr(u_{t1} < -x_{t1}\beta_1; u_2 < -x_{t2}\beta_2)$

There are $t = 1, ..., T$ auctions in the data. y_{t1} gains a value of 1 if bidder 1 submitted a bid in auction t, otherwise it is 0. Assuming that the u_1 have a bivariate normal distribution, the P's are known functions and (8) can be maximized using standard numerical optimization methods. The significance of the δ_i's and the other parameters can be calculated using separate t-tests, but it is also possible to use the Wald test to test for joint significance of the desired variables.

The existing literature on estimating games of complete information thus provides a way to deal with multiple equilibria resulting from not observing how cartels decide which of the members enter. Unfortunately, the potential problem multiple equilibria that may result from the competition game that takes place under incomplete information when firms are not colluding has also to be resolved. In the previous literature on discrete games of incomplete information, several different approaches have been proposed. Easy way out is to assume that the game has unique equilibria or that only one equilibrium is played in the data. This assumption is made for example by Seim (2006) who then uses simulations to show that her assumption of unique equilibrium cannot be rejected atleast, if not verified either. Bajari et al. (2009a) use an algortihm that finds all the equilibria of the game. Grieco (2010) incorporates equilibrium selection into his flexible model that, similar to my shock structure here, allows for both private and common shocks. Toivanen and Waterson (2005) eliminate the possibility of multiple equilibria by assuming that the entry game proceeds as Stackelberg competition. I follow Seim (2006) and assume that only one equilibrium is played. Note that if the firms collude this is indeed always the case. Therefore, incomplete information multiple equilibria would be a problem only if it caused false positives in the test. That would happen when multiple equilibria caused competition effects to be estimated into δ instead of λ. I cannot think of any reason why this would happen.

Recent studies on structural analysis of entry games in auctions have different approaches to addressing the problem of multiplicity of equilibria. For Bajari and Hortacsu (2003), multiple equilibria are not a concern as they only consider the number of bids, not the identity of the entrant, which is the same as BR approach. Athey et al. (2004) abstract from the multiple equilibria problem by arguing that "as is often the case with entry models, there may be many equilibria, as a result, our results compare sets of equilibria across auction methods". Krasnokutskaya and Seim (2010) verify the uniqueness of the equilibrium entry probabilities numerically. Li and Zheng (2009) take the fully structural approach to estimate a model that allows for endogenous entry, an uncertain number of actual bidders, unobserved heterogeneity and mixed strategy entry equilibrium under the independent private values paradigm. Their model requires observations where the number actual bidders $n \geq 2$. Most of the auctions in my application data have only 1 actual bidder. Li (2005) also allows for mixed strategies. Both of these articles assume symmetric bidders. Bajari et al. (2009b) estimate equilibrium selection.

3 Monte Carlo analysis

I conduct Monte Carlo analysis to compare the finite sample properties of the new test (called BR in Tables 3 and 4 due to the estimation logic) with the existing PZ test. This is done by comparing the empirical power and size of these different tests. The Monte Carlo model is chosen so as to reflect the characteristics of the actual application. The variable x can be considered a contract characteristic, such as contract size, and the z variables can be considered bidder characteristics, such as distance. Following the application, I discretize the z variables. I assume that there are three different firms in the markets, and the collusion test is conducted only for firms 1 and 2. The BR estimation is based on model (9) and is estimated using equation (8). The PZ model is estimated using single-equation probits, omiting $y_j\delta_i$ from the model (9).

$$
\begin{aligned}
& y_1^* = \beta_{10} + x\beta_{11} + z_1\beta_{12} + \min(z_2, z_3)\beta_{13} + y_2\delta_1 + u_1 \\
(9) \quad & y_2^* = \beta_{20} + x\beta_{21} + z_2\beta_{22} + \min(z_1, z_3)\beta_{23} + y_1\delta_2 + u_2 \\
& \binom{u_1}{u_2}\tilde{\ } IIDN\left(\begin{bmatrix}0\\0\end{bmatrix}, \begin{bmatrix}1 & \rho\\ \rho & 1\end{bmatrix}\right).
\end{aligned}
$$

I use eight different specifications in the data-generating process. They are summarized in Table 2. To address the multiplicity of equilibria, the data is generated in the following way. First I generate x from a uniform (0,1) distribution. The three z's are discrete variables that gain the values 0,1,2,3,4,5 or 6, each with equal probability. Then I calculate the following cell probabilities for the y's. Let $\beta_{10} + x\beta_{11} + z_1\beta_{12} + \min(z_2, z_3)\beta_{13} = x_1\beta_1$ and

$\beta_{20} + x\beta_{21} + z_2\beta_{22} + \min(z_1, z_3)\beta_{23} = x_2\beta_2$. Next, I assign the multiple region (see Tamer (2003)) an equal chance of being a bid by either bidder, i.e. the outcomes (1,0) and (0,1) have equal probability. This is done by reducing $[(P_{00} + P_{01} + P_{10} + P_{11}) - 1]/2$ from both P_{01} and P_{10}. Then I use these new cell probabilities to randomly and simultaneously assign values for the pair (y_1, y_2).

	$y_2 = 0$	$y_2 = 1$
$y_1 = 0$	$P_{00} = BVN(-x_1\beta_1, -x_2\beta_2, \rho)$	$P_{01} = BVN(-x_1\beta_1 - \delta_1, x_2\beta_2, -\rho)$
$y_1 = 1$	$P_{10} = BVN(x_1\beta_1, -x_2\beta_2 - \delta_2, -\rho)$	$P_{11} = BVN(x_1\beta_1 + \delta_1, x_2\beta_2 + \delta_2, \rho)$

In small and moderate samples, maximizing (8) turns out to be a difficult optimization problem to solve. First, depending on the algorithm used, it converges very slowly or not at all. Second, it has numerous local maxima as starting the optimization routine from different starting values leads to different results. Third, it is difficult to identify the δ_i's and the ρ separately if the equation (8) is unmodified. Fortunately, it is possible to solve all these problems.

The problem of identification is solved by imposing the identification result 4 ($\delta_1 \times \delta_2 > 0$) on the likelihood function. I impose this nonlinear constraint by using a penalty function. I use a distance-based penalty function ($constant \times \delta_1 \times \delta_2$) because that is well suited for use with evolutionary algorithm method (EA, they are also called genetic algorithms). The problem of multiple local maxima is solved by using these EA methods, in particular the "rgenoud" package in R (Mebane and Sekhon 2009). As this algorithm chooses the starting population randomly, the researcher does not have to be concerned with the problem of what starting values to choose. It is sufficient to choose a large enough starting population. The longer the EA is allowed to run, the larger is the probability that it will find the global maximum or at least the slope that leads to the global solution. Unfortunately, EA is notoriously slow at hill climbing. Therefore, I use a Nelder-Mead optimization routine ("optim" command in R) on the best parameter vector after running the EA for some generations. Nelder-Mead seems to work better than BFGS (which is the default option for making the convergence faster in the rgenoud package) in terms of power, size and convergence. Also the Nelder-Mead is fairly slow in this particular problem but always converges given time. This approach is very time-consuming. The computer time is not a big problem in running a single application, but it does take a long time to conduct the Monte Carlo analysis. Running Monte Carlo with 250 observations takes about 1 hour to converge (after first letting the EA run for 1.5 hours) using a 3 Ghz processor, and the computer time seems to be approximately linear in the number of observations. Therefore, even with parallel computing, these Monte Carlo calculations take months.

In Table 3, I present the results of the Monte Carlo experiments with 1000 observations. The PZ results were obtained using a STATA "simulate" routine. For the PZ test, 1000 repetitions were used. To save on computer time, the BR test was repeated only 100 times. I used one-sided t-tests in BR. These worked better in terms of size and power than two-sided t-tests or two-sided Wald tests of joint significance. If either of the δ_i's is significant, the test implies collusion. The one-sided test is consistent with the model, because already the likelihood function formulation assumes that the δ_i's are negative. For models with no unobserved heterogeneity (1.-4.), the PZ test has much better power. It detects guilty firms more often. For models with unobserved heterogeneity (5.-8.), which is generated with ρ, PZ fails utterly as expected. Either there is a large share of type I errors (model 5.) with unobserved heterogeneity that makes firms avoid each other (negative ρ), or very low power (model 8.) with unobserved heterogeneity that makes firms bid for the same contracts (positive ρ). The BR estimation performs roughly as well with unobserved heterogeneity as without it. Therefore, according to this Monte Carlo analysis, in cases with unobserved heterogeneity, the BR method works much better than PZ. On average, the BR method produces accurate estimates for all the parameters (not reported). Because on average also ρ is estimated correctly, the BR test can be used jointly with the PZ test as a way to evaluate whether any variables are missing. A Hausman type test can be used for this purpose.

Even though the results with 1000 observations are very encouraging for the new test, it fares worse with smaller samples. With sample sizes as small as the one I have in the application (258 obs), we need to restrict the model further. When I impose a restriction ($\delta_1 = \delta_2 = \delta$), the BR test works fairly well even with such a small sample although the power is quite low. The power can be improved in some models by imposing box-constraints on the $\delta' s$ and the ρ. I constrained them to be between -1 and 1. For model 4, the constraint on delta was (-1.5,1.5) to allow the mean of the delta estimates to be correct. The results for these two Monte Carlo experiments along with results for the PZ test are reported in Table 4.

4 The school yard snow removal market in Helsinki

I will apply both the PZ test and the new test to detect possible collusion in the school yard snow removal markets in the City of Helsinki in 2003 - 2005. This particular market is interesting because it allows for an analysis of territorial allocation. This is the first empirical application that tries to detect that form of collusion. This is surprising because territorial allocation is a fairly typical collusion scheme. For example, of all the reported Finnish cartels in the period 1959 - 1990, when the cartels were legal, 7.2% of cartels were of this type (Hyytinen, Steen, Toivanen 2010).

In Figure 1, I present the spatial participation pattern in this market. I show on the city map all the schools that each bidder submitted bids for in 2003. The locations of the bidders' and the city's garages are also marked on the map (bold and larger letters). Most firms seem to participate more actively near their garages than further away. The map shows that bidders A and K seem to avoid each other. Moreover, they systematically avoid each other in an overlapping geographical area, as we can see in the lower left corner. This map raises suspicions about collusive territorial allocation. Of the other bidders, bidder R submits bids for all but two contracts and three small bidders T, S and P only a few bids. The maps for 2004 and 2005 are in Appendix A. It is interesting to note that in 2005 the participation pattern no longer implies territorial allocation. I mainly restrict the discussion in this Section to 2003 because that is the year in which I suspect that the collusion took place.

Starting from the autumn of 2003, the City of Helsinki has auctioned snow removal services for school yards. Prior to 2003, the contracts were allocated by the Ministry of Education. All the contracts are auctioned simultaneously. Package bidding is not allowed. The auction format is a first-price sealed-bid auction with a secret reservation price. The lowest bid wins the given contract and the winner is paid their unit bid times the respective size of the contract. For example, the winning bid conserning snow ploughing is in euro per square meter. That amount multiplied by the size of the school yard in square meters is the payment per ploughing operation. After the auction, all the bids are public knowledge. Thus all the bidders detect deviators from collusive agreements easily.

The bidders submit single sealed bids separately for four different types of services for each school. The first service type consists of snow ploughing and sanding. The second service is the transportation of snow from the school to the snow dump. The third service is the transportation of sand from the school to the snow dump and the fourth is washing the yard. The last two services are needed only once every spring. Different services can be allocated to different firms within the same school and providing these services requires different production technologies. Therefore, these different services can be seen as independent of each other in the statistical analysis. I consider only the bidding for the first service type because it is the most important in monetary terms. For the purpose of this study, the chosen service type does not matter. Typically, bidders submitted bids for all of the services, but there are exceptions. For example, one firm participated only in the snow transportation service and bid for all the schools.

There are some minimum quality requirements. For example, snow has to be ploughed whenever there is more than 5 cm of snow on the ground. In 2003, the auction was based on price only but in 2004 and 2005 some scoring rules were introduced. However, the auctioneer admitted that these scoring rules were not introduced to get higher quality, but rather to discourage one unwanted bidder from participating. These scores did not include relevant quality dimensions like the delivery time, but rather had irrelevant things like the equipment used to provide the service. As expected, the introduction of the scoring had no noticable effect on the bidding behavior nor on the actual allocation of the contracts, but it did have the desired effect on the entry of the unwanted bidder. According to the auctioneer, the scores were not even calculated nor did the bidders take the scoring part seriously. The purpose of the scoring as just a potential tool for discrimination seemed to be common knowledge. Therefore, all the three periods can be seen as price only auctions. A more significant variation is evident in the amount of schools that are contracted each year. In 2003, there were 153 schools, in 2004 37 schools and in 2005 65 schools. This number varied according to the proportion of the services the city decided to provide itself. Despite these year-to-year differences, I treat the data as one cross section, but I include time dummies. I also allow the set of potential bidders to vary from year to year.

The invitation to tender states that "the buyer reserves the right to transfer some of the contracts to be serviced by the city itself". In other words, the city is saying that it has set a secret reserve price for the contract. The secret reserve price means that the city will not accept bids that are too high. In this case, too high means a bid higher than the costs that the city would incur by providing the service itself. Based on the participation frequencies, it seems that this secret reserve price may be binding for many firms in most auctions. In 2003, a total of six bidders participated in the snow ploughing and sanding services. Of 153 contracts, there were 2 with 0 bidders, 85 with 1 bidder, 60 with 2 bidders, 5 with 3 bidders and 1 with 4 bidders. If the secret reserve price was not binding, we would expect all the potential bidders that are not capacity-constrained to submit a bid in all the auctions. Another explanation is that entry costs limit participation. However, there is no reason to suspect that bid preparation incurs large costs for the firms in these markets because the bidding process is very simple and they have previous experience of providing the service being tendered. An industry expert explained that it would take him about two minutes to calculate a bid in this sort of market because the costs are so well known. The actual number of submitted bids can still change due to capacity constraints and differing numbers of potential bidders in different areas of the city. PZ observe a similar distribution of actual bidders in their data set of school milk bidding. They suggest that a small number of actual bidders indicates that "there may not be significant firm-specific information in the markets. If bidders knew their costs as well as the costs of the other potential suppliers, then under a set of standard assumptions either one or two bids would be observed. The low cost supplier would submit a bid just below the cost of the next-lowest-cost supplier, and the next-lowest supplier would be indifferent between bidding at its own cost and not bidding". In contrast to the markets analyzed in PZ, there may be more uncertainty about the costs of other bidders in this market. The bidders use somewhat different equipment, their principal activities are different and they may have differences in efficiency. It is also implausible that the asymmetries between bidders would be so large that the cost ordering of the bidders in all the auctions would be common knowledge. I think that the explanation of a binding secret reserve price possibly along with territorial allocation is more plausible. Capacity constrains probably also limit the participation of smaller bidders in particular.

Snow removal is typically a secondary activity for the firms. The main activities of the three larger participants are construction, paving, delivery services and landscaping. For the three smaller firms, real estate maintenance is their main activity. All the firms use the snow removal equipment for these main activities outside the winter period. Capacity is not adjusted for the snow removal market due to its supplementary nature. The larger firms are probably not capacity constrained, but the smaller companies are typically one-man firms with a very limited amount of equipment. Three smaller firms only submitted from three to six bids for schools located near their premises. Another reason to suspect that the large firms are not constrained by capacity is the fact that they have subcontracting deals with each other. Thus they have access to additional capacity beyond their own. The secondary nature of the activity also acts as an entry barrier. It is not profitable to enter the snow removal activity alone. The required equipment is too expensive in relation to the seasonal nature of the activity. On the other hand, there are numerous construction firms in the area that already have the necessary equipment.

Typically, theory predicts that stable demand and homogeneity of the product make collusion more likely (e.g. Harrington 2008). Demand for snow removal services is very inelastic, because the weather is not affected by price. Nor are the conditions in the invitation to the tender regarding the times when the service is needed price-dependent. This property makes collusion more profitable because an increase in prices due to collusion does not reduce demand. On the other hand, the existence of the secret reserve price makes demand elastic. If a cartel bids too high, the contract may not be awarded to anyone. Thus reservation prices reduce incentives to collude. Snow removal is a homogenous product, and thus this market is susceptible to collusion also in this respect. There can be very little quality differences in snow removal. It is either removed or not. However, the production processes can still be different due to differences in the firms' main activities that result in them having different snow removal equipment.

There are several other characteristics in this market that may also facilitate collusion. First, firms compete only on price, which simplifies cartel operations. Thus a cartel needs only to coordinate participation or the level of its bids. Second, announcing all the bids and the identities of the bidders publicly makes it easier for a cartel to detect deviation. Third, separate markets are easily defined, allowing the assignment of territories. Fourth, the set of participating firms is small and there are entry barriers, making it possible to submit higher carter bids. Fifth, subcontracting is typical in this market. This provides an easy way to distribute the cartel's rents and facilitates direct communication and a pretext for the cartel to meet. Sixth, the buyer's representative (Palmia, the City's service center) thinks it is plausible that some of the firms could be colluding. However, there is no legal outside evidence. On the other hand, the simultaneous nature of these auctions makes it more difficult to sustain collusion. Bidders can punish others for deviation only in the following year's auction. However, if bidders meet in some other markets that they are active in, for example construction, they can possibly punish there. Multi-market collusion also raises the cartel's profits without significantly increasing the costs of organizing the cartel. Another way to punish deviators is not to give them subcontracting deals.

As can be seen from the participation maps (Figures 1-3), the behavior of bidder A changes over time. In 2005 it bids for seven of the same schools as bidder K, whereas in 2003 they never bid for the same school. K generally bids for the same schools in 2005 as in 2003. Therefore, in respect of equipment and location, it would probably have been possible for A to compete with K in 2003 too, because I am not aware of any technology or location changes for A. This is further evidence of collusive behavior in 2003.

Job rotation is a similar phenomenon to territorial allocation. In a sequential auction setting, job rotation can exist either as a result of collusion or as a result of an efficient outcome of a competitive bidding process when capacity constraints or decreasing returns to scale matter (Hendricks and Porter 1989). This makes the detection of collusion more difficult in a sequential setting. In contrast to sequential auctions where the winners of previous auctions are observed, in simultaneous auctions the bidders do not observe how much capacity is already committed when deciding to participating in a given auction. Thus there is no backlog. In a simultaneous setting, capacity constraints or decreasing returns to scale only affect the total number of auctions that a bidder participates in. If there is enough uncertainty about other bidders' costs, competitive bidding should not result in cases where certain bidders systematically avoid each other. Assuming that bidders do not know which homogenous auctions their competitors are going to bid for, one could imagine that firms randomly submit bids for contracts up to their capacity. Then it is highly unlikely that some firms would manage to systematically avoid each other when there are many contracts. In sequential auctions, bidders may also signal their preferences to other bidders more easily than in simultaneous auctions. Territorial allocation can be a result of competitive behavior when there are large observable cost differences among bidders. But if these differences are controlled for, we should not observe that identity in itself matters in a competitive setting.

In a simultaneous game it is not possible to know exactly where the others are going to bid. Therefore we should observe that bidders sometimes bid for the same contracts if there is no way of communicating to each other what actions they are going to take. Explicit communication is explicit collusion. If the game is played repeatedly, bidders can perhaps infer each others' future actions from past decisions. If incumbency, for example, explains much of the participation decision, collusion could also be tacit. Unfortunately, the information on contracts in 2002 was not available and thus the effect of incumbency cannot be checked. Therefore, in my particular application, the test detects either form of collusion but it does not distinguish between implicit and explicit collusion. However, if there are enough repetitions of the game, one could simply exclude some early periods from the data and use these data to construct incumbency variables. Tacit collusion would show in the competitors incumbency status and excplicit collusion in the current period decision.

5 Data and modeling choices

The data comprise 258 contracts put out to tender, with 335 bids submitted for them. 28 of these 258 auctions did not receive any bids. 19 of the zero-bidder auctions where held in 2004. Nine bidders participated in these auctions, six in 2003, three in 2004 and six in 2005. Three firms exited the market after 2003, including the largest bidder R and three new bidders entered in 2005. Firm R was not able to provide the service for the contracts it won in 2003 and thus it was barred from the subsequent tenders by introduction of the scoring rule, as explained above.

The data include information on entry, bids, contract characteristics and bidder characteristics. The participation decisions of the bidders are described in Table 5 along with the bid levels. It shows the number of bids submitted, the number of contracts won, the number of contracts won conditional on facing any competition and bid level information for each bidder. It also shows to which city areas a given bidder submitted bids and in which years the bidder submitted any bids. Only three bidders submitted bids in all three years. Looking at the map in Figure 1, we notice that only A and K avoid each other in the same city area. Therefore I conduct the tests only for bidders A and K. Moreover, Table 5 shows that bidder R submitted too many bids in the year it participated and bidders T,S, H and O too few bids to be of much use in discrete choice models. There is too little variation in their decisions to use the tests for them. However, their characteristics are used as controls for the competition effects. I include in the set of potential bidders each year all the bidders that submitted any bids at all in that particular year.

Figure 2 in Appendix B shows the scatter plots of bids in relation to school yard size for each bidder separately. Bidder A participated in smaller auctions than the other bidders. The reason for this could be that they operate only in the center of the city where school yards are typically smaller. It could also be because they specialize in smaller yards because of having different equipment. Note also that unit bids are decreasing in yard size, implying economies of scale. These seem to be decreasing. Thus I include yard size and its square in the econometric analysis. Figure 2 also shows that there is too little variation in the price data for bidder A for me to be able to utilize the information in prices to test for collusion.

Besides the yard size, the contract characteristics include also some measures of yard's shape or "tightness" that are intended to capture how difficult it is to plough the yard. These are the number of walls or fences that surround the yard, the number of permanent obstacles like trees or small buildings in the yard and a dummy for whether the yard includes tight spaces. The shape variables are obtained by studying 1:1250 -maps of the school areas. This allows detail down to a single tree. The contract characteristics also include the distance to the schools from the city's garages to account for possible changes in the secret reservation price.

The information on the bidders is limited. The bidders did not agree to be interviewed. They only answered a few short questions. The most important variable that I am interested in is the location of bidders' garages. I could use that to calculate the distances from the garages to the contracted schools. Flambard and Perrigne (2006) find empirical evidence of asymmetry resulting from firm location in their study of snow removal in Montreal, because in the urbanized part of the city storage costs are prohibitive. This provides additional support for using the distances to the contract site from each firm's location as the relevant cost shifters that my test requires. Unfortunately, I did not receive the precise addresses of the garages but the firms gave their location information at the postal code level. I assume that the garage is located in the middle of a given postal code area. This creates measurement errors in the distance variable. Perhaps an even more important reason to use something other than just the distance to capture the cost shifters of the firms is that, according to an industry expert, another important factor is where the bidders' main activities are located at the time. Bidders prefer schools near their construction sites, for example. This is not observable. Thus I need to construct a proxy variable for the firms' cost shifters.

To proxy all the firm-specific cost shifters, I construct a variable called "ofsix", meaning "to how many of the six nearest schools to a given school a given bidder submitted a bid for a given year". This variable captures not only the distance, but also the overall costs of the bidder in the close proximity of a given school. It should capture also things like the benefits of serving schools along a certain route. There is a possible endogeneity problem with this proxy. A bidder might or might not have bid for some of the nearby schools for reasons of collusion rather than costs. To capture the competitive effect, I calculate for each bidder the maximum of the "ofsix" variable among all of its potential competitors separately for each school. I use this variable instead of including the "ofsix" variables of all competitors, because then the own "ofsix" variable provides the exclusion restriction. I also use four area dummies in the analysis to capture the bidders' possible advantages in a larger area around the school. These area dummies also capture possible changes in the number of potential bidders.

I also include year dummies. It would be more interesting and flexible to conduct the analysis for each year separately. This would allow the model to take into account the possible effects of entry, exit and regime changes, for example changes in the auctioneer's policies. Most importantly, we could see whether possible collusion took place only in some years. Unfortunately, there are not enough yearly observations for separate analysis using the new test. For the PZ test, I can also conduct the analysis separately.

6 Results and discussion

The results of both tests are presented in Table 6. For the control variables, I report only the sign and the significance level. The actual numbers are not interesting because the parameter estimates for the effects of control variables on entry are an unknown mixture of own cost effects and competition effects. In the PZ test, I estimate probit models separately for bidders A and K. I estimate the model using data for all the three years 2003-2005, because it is not possible to conduct the new test (called BR in Table 6) separately for the individual years. The activity variable "ofsix" is significant at all standard levels for both the bidders and has the expected positive sign. This implies that inserting this variable also for competitors should capture the competition effect in the case that firms were competing with incomplete information. Bidder A seems to bid close to the city's garages, more in the southern region of the city and more for difficult yards. This is in line with the fact that they advertise in their website having equipment best suited for difficult yards. Surprisingly, the competition variable, i.e. the maximum of competitors' activity, has a positive and significant sign, which means that bidder A bids more in auctions where tough competition is expected. Bidder K seems to get some returns to scale from yard size. The residuals of these two probit models are negatively correlated. This correlation is significant at the 1% level. This implies that collusion occurred at least in some of the three years in the data. If the years 2003 and 2005 are analyzed separately with the PZ test, there is even stronger evidence of collusion in 2003 and no evidence in 2005. Collusion is also supported by the fact that competitors' characteristics are not important for bidder K and for bidder A the effect of competition is opposite to what it should be under competition. Thus the results of the PZ test suggest collusive behavior. This is assuming that I have not overlooked any important explanatory variable.

The results from estimating the simultaneous equation model (7) are also presented in Table 6. As was shown necessary by the Monte Carlo analysis of the 250 observations, I use the restriction $\delta_1 = \delta_2$. First, I run the EA for 25 generations with a starting population of 5000 and then use the Nelder-Mead algorithm until convergence. I calculate the significances using a well behaving Hessian matrix. I report also the bootstrapped significances in the last two columns. To save on computer time, I only use 100 draws and only use the Nelder-Mead algorithm in the bootstrap. I follow the logic of the k-step bootstrap (Andrews 2002 and Davidson and McKinnon 1999) and use the estimates from the original sample as the starting values for the algorithm. I discuss only the bootstrapped significance results here.

The activity variable "ofsix" is significant and has the expected positive sign for both bidders. The competitors' characteristics are not important. A also bids more for schools with difficult yards. K is significantly more active in 2005 and A in 2004 than in they are in 2003. A bids more in the southern area and seems to bid close to the city's garages. The test statistic is negative for both firms but not significant. The sign of the estimate of rho suggests that there are no unobserved variables that would invalidate the PZ test result. This could possibly be tested formally by using the difference in the estimates of correlation from the BR test and the PZ test for a Hausman-type test for relevant omitted variables. However, the rho is estimated very imprecisely in this specification of the BR test and therefore testing based on that rho would not be very interesting. Also the collusion parameters δ_1 and δ_2 are estimated imprecisely. This reflects the need for more observations in the new test compared to the PZ test. Collusion is again implied by the fact that the strategic elements do not seem to be important for the bidders, because they do not avoid bidding for the contracts that their competitors are likely to bid for. To summarize, the new test also provides some evidence of A and K colluding, but the evidence is weaker than in the PZ test. Moreover, the new test does not imply that unobserved heterogeneity would invalidate the PZ results.

The behavior of firms A and K seems to be more consistent with a collusive than a competitive model, although the evidence is not strong. In Harrington's (2008) terms, this is a screening result, which means that I have identified this market being susceptible to collusion. This can also be thought of as the verification of the cartel because, by its construction, the method identifies the exact model of collusion (territorial allocation). This is not, however, necessarily sufficient for the prosecution of the colluding firms. Screening is useful to fairly quickly analyze a market and detect those markets where more attention should be paid to find legal evidence.

One source of possible endogeneity is that unlike PZ, I do not have a control group. PZ use legal evidence to create a control group made up of non-defendant firms bidding for Ohio school milk contracts. They compare the behavior of this control group with the behavior of defendant firms. PZ use the control group estimates also for the cartel group to address the problem of endogeneity that arises because the participation decisions of cartel firms are affected by collusion. This biases the estimates of the effect of observables on their participation. Assuming that all the bidders are identical, the control group estimates can be used as unbiased estimates also for the treatment group. I do not have any outside evidence or enough bidders to form a control group. There is a trade-off between the endogeneity problem and the need to make the conditional symmetry assumption that all firms react identically to changes in the explanatory variables. In situations where bidders have different production technologies or differ in some other important respects, it could be better not to use a control group at all. Moreover, legal evidence to form the control is not always available. Fortunately, unlike in the PZ test, this possible bias in the parameters of the control variables does not bias the estimates of the test statistics in the new test because it makes a difference only to error term estimation but not the new collusion parameters. Furthermore, the main idea of the new test is that when colluding, the behavior of the firms with respect to observable characteristics is different from when competing. In theory, both the tests could use either the control group approach or the approach used here.

7 Conclusions

I have a proposed a new test to detect collusion that is robust to unobserved heterogeneity and requires only participation data. Using only participation decisions in the estimation increases the applicability of this test for two reasons. Firstly, useful data on bids is harder to obtain than data on entry decisions only. Secondly, tests that use entry data apply in much wider settings and institutions than bidding tests. Tests based on bids can only be used to detect phony bidding but not territorial allocation, whereas entry tests can detect both. Furthermore, this test can be applied to static simultaneous discrete games with flexible information structures as long as independent and mutually exclusive markets can be defined. Auctions are only one potential application environment.

The new test can be conducted by estimating simultaneous discrete choice equations with methods developed for estimation of complete information such as BR, as modified by Tamer (2003), or Ciliberto and Tamer (2009). I have used Monte Carlo analysis to show that this simultaneous equations estimation strategy makes the test robust to unobserved heterogeneity unlike the existing PZ test which has otherwise similar properties. The robustness arises from allowing both private and common shocks in the model. The Monte Carlo analysis also points out the need to impose parameter restrictions on the BR entry model as the sample size gets smaller. It also demonstrates the usefulness of evolutionary algortihms in estimating simultaneous equations discrete choice models with maximum likelihood.

Robustness to unobserved heterogeneity comes at a cost. The PZ test is better than the new test in the sense that it requires no exclusion restriction and needs less observations to work. The PZ test is also computationally more attractive and has better power when there is no unobserved heterogeneity. On the other hand, the new test nests the PZ test and that allows the new test to be used to check whether an application involves any important missing variables that would invalidate the PZ testing approach.

I apply both test to school yard snow removal auctions in the City of Helsinki and find some evidence of collusion. The PZ test suggests collusion and based on the BR results there is little reason to doubt the result. Moreover, the analyzed firms do not behave strategically, as one would expect in a competitive setting. Two bidders seem to participate in a contract allocation scheme. The collusive regime only seems to be in place in 2003. However, there are some potential endogeneity problems in the application and inaccuracies in the estimation of the parameters of interest in the new test. Therefore, these results should be treated with caution. Nevertheless, this empirical analysis should validate a legal investigation to study whether these two companies formed an illegal cartel.

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Table 2. Different model specifications used in the Monte Carlo analysis

Model	β_{10},β_{20}	β_{11},β_{21}	β_{12},β_{22}	β_{13},β_{23}	δ_1,δ_2	ρ
1.	0	0.3	-0.3	0.2	0	0
2.	0	0.3	-0.3	0	-0.3	0
3.	0	0.3	-0.3	0	-0.45	0
4.	0	0.3	-0.3	0	-0.6	0
5.	0	0.3	-0.3	0.2	0	-0.5
6.	0	0.3	-0.3	0	-0.45	-0.5
7.	0	0.3	-0.3	0.2	0	0.5
8.	0	0.3	-0.3	0	-0.45	0.5

Table 3. Power and size results for the Monte Carlo comparison of the tests at 9.75% significance level. 1000 observations and constraint ($\delta_1 \times \delta_2 > 0$) imposed.

Model	Obs	PZ, power	PZ, size	BR, power	BR, size
1.	1000		12%		11%
2.	1000	97%		42%	
3.	1000	100%		60%	
4.	1000	100%		65%	
5.	1000		100%		15%
6.	1000	100%		59%	
7.	1000		0%		6%
8.	1000	2%		56%	

The size is 9.75%, because the BR test is based on either of the δ_i's being significant at the 5% level. The PZ test is adjusted accordingly. 1000 repetitions are used for the PZ and 100 for the BR.

Table 4. Power and size results for the Monte Carlo comparison of the tests at 9.75% significance level. 250 observations and constraint ($\delta_1 = \delta_2$) imposed. BR test with box constraints in the last two columns.

Model	Obs	PZ, power	PZ, size	BR, power	BR, size	BR, power, bc	BR, size, bc
1.	250		5%		3%		7%
2.	250	50%		20%		16%	
3.	250	75%		21%		32%	
4.	250	89%		28%		25%	
5.	250		98%		11%		10%
6.	250	99%		21%		42%	
7.	250		0%		3%		2%
8.	250	8%		15 %		17%	

The size is 9.75%. 1000 repetitions are used for the PZ and 100 for the BR.

Table 5. Descriptive statistics for the bidders in years 2003-2005. Participation and bids.

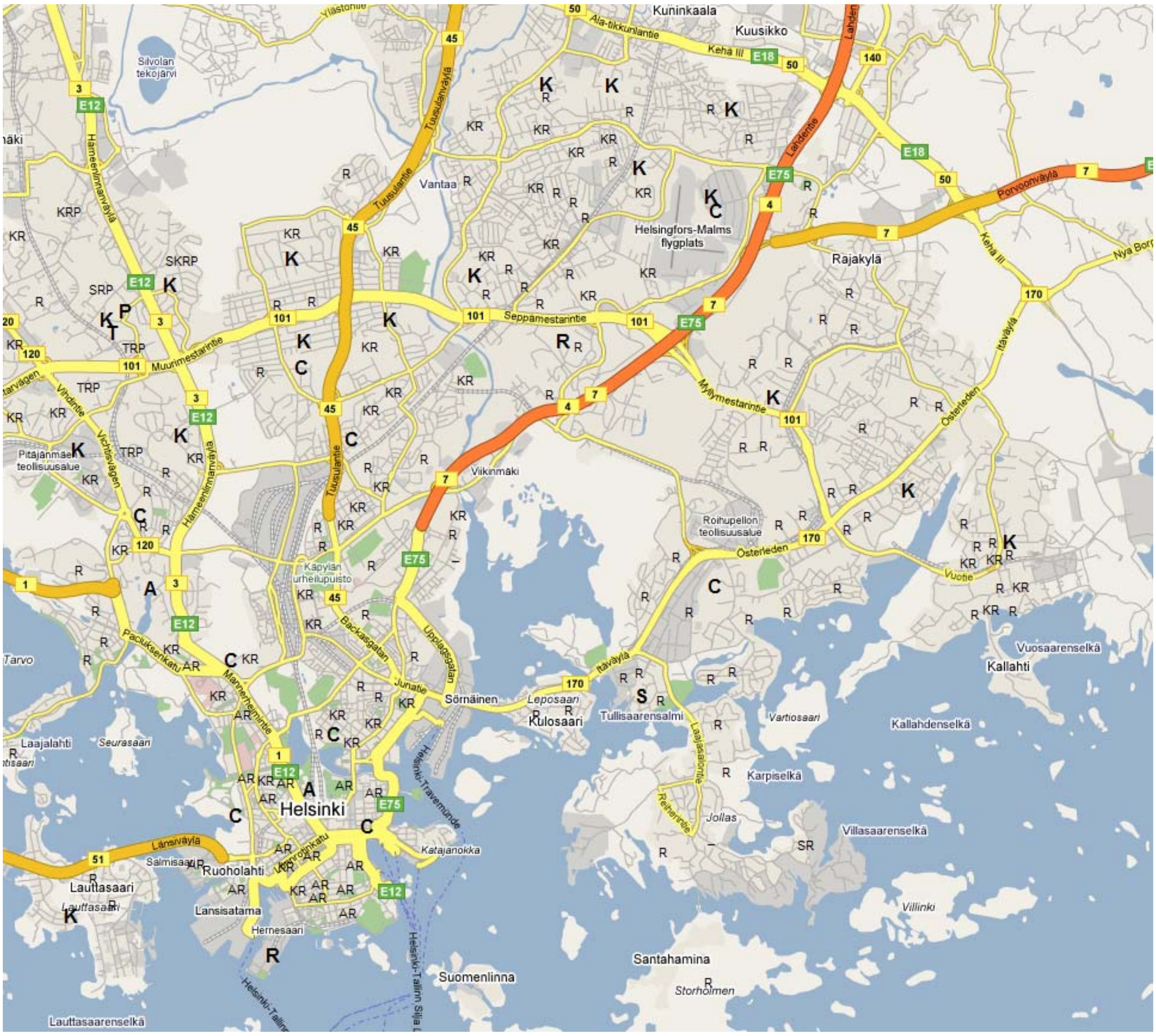
	A	K	R	T	S	P	H	J	O
# of bids	42	98	151	3	3	16	1	19	2
# wins	33	89	97	1	1	4	1	2	2
# of wins|com	16	66	12	1	1	1	0	0	2
mean bid	0.097	0.071	0.089	0.100	0.090	0.107	0.050	0.103	0.097
sd bid	0.005	0.020	0.005	0.017	0.052	0.021	NA	0.022	0.018
min bid	0.082	0.040	0.085	0.080	0.060	0.068	0.050	0.071	0.084
max bid	0.110	0.156	0.980	0.110	0.150	0.135	0.050	0.140	0.110
South(centre)	Yes	Yes	Yes	No	No	No	Yes	No	No
Northwest	Yes	Yes	Yes	Yes	Yes	Yes	No	No	No
North	No	Yes	Yes	No	No	No	No	Yes	Yes
East	No	Yes	Yes	No	Yes	No	No	No	No
Year 03	Yes	Yes	Yes	Yes	Yes	Yes	No	No	No
Year 04	Yes	Yes	No	No	No	Yes	No	No	No
Year 05	Yes	Yes	No	No	No	Yes	Yes	Yes	Yes

Table 6. The estimation results.

	PZ		BR		BR BS Sig.	
	Bidder A	Bidder K	Bidder A	Bidder K	Bidder A	Bidder K
constant	-*	-****	-	-		****
city dist	-****	+	-*	+*	*	
yard size	-	+*	+	-		
yard size sq	+	-	-	-		
walls	+**	+	+	+		
obstacles	+	-	+*	-		
shape	+**	+	+	-	*	
t03	ref group	ref group	ref group	ref group		
t04	+	+	-	+	*	
t05	+	+	+	+**		**
OfsixA	+****	NA	+	NA	*	
Max Ofsix -A	+**	NA	-	NA		
OfsixK	NA	+****	NA	+****		****
Max Ofsix -K	NA	-	NA	-		
Area S	+****	-	+**	-	*	
Area NW						
Area N	ref group	ref group	ref group	ref group		
Area E						
delta	NA	NA	-0.35 (0.85)	-0.35 (0.85)	(0.56)	(0.56)
Rho	-0.165***		0.14 (0.88)		(1.34)	
Log lik	-24.8	-144.2	-139.8			

A unit of observation is a school. n = 258. "city dist" is the distance from the nearest city garage. "yard size" is the yard size of the school and "yard size sq" its square. "walls" is the number of walls surrounding the yard, "obstacles" the number of obstacles in the yard and "shape" a dummy for yards including tight areas. "t03 - t05" are the year dummies and "Area X" the area dummies. "Ofsixi" is the activity of bidder i. It tells how many of the six schools nearest to a given school bidder i has submitted a bid for. "Max Ofsix -i" is the maximum of the Ofsix variable among i's competitors. "delta" denotes the test variable δ. "*" means 10% significance level, "**" means 5% significance level, "***" means 1% significance level and "****" means 0,1% significance level for two-sided tests (one-sided for delta).

Figure 1. Bidder participation in school yard snow removal auctions in Helsinki 2003.



Small capital letters give the location of schools and which bidders (A, K, P, R, S, T) have bid for a given school. "-" means that there were no bids. The approximate location of the bidders' and the city's (C) garages are marked with larger bold capital letters.

Appendix A: The Porter and Zona (1999) entry test

PZ propose several tests to detect collusion in auctions. They utilize both the participation decisions and the bid levels to test whether some bidders submitted phony bids. Their main focus is on using the bid data, and the entry test is incidental. Nevertheless, it is useful to discuss their entry test in detail here, because it is the only other test in the literature that utilizes entry decisions only. The main focus of this discussion is on the implications of unobserved heterogeneity for both the tests. Even though PZ discuss the possible problem caused by unobserved heterogeneity, it is probably not a concern in their application. They have rich panel data in terms of observations, years and observed characteristics and their results were robust to firm and contract fixed effects. However, in many possible applications of these tests unobserved heterogeneity can be a serious concern.

The main difference with the PZ test and my test is that they base their test on testing on the correlation of the error terms whereas I use this correlation only as a control. Therefore, their test works only in auction where no affiliation is present. Moreover, their test applies only in pure incomplete information entry games. This means that any unobservable heterogeneity would make their test to fail.

PZ test for statistical independence in the probability of bidding using a standard pairwise procedure. PZ state: "Under the null hypothesis of independent action based on public information and the maintained specifications of our probit submission model, knowledge of whether one particular firm bids should not help predict whether another firm has also bid. In the case of complementary bidding, if one cartel member bids, then other ring members also bid. In this case the unexplained portion of the competitive bidding equation is positively correlated across cartel firms. In the case of territorial allocation, if a particular cartel member bids, then other cartel members will tend to not bid. Then the unexplained portion of the competitive bidding equation is negatively correlated across cartel firms." They propose using the Spearman correlation coefficients computed using pairs of weighted residuals based on the control group probit models.

The PZ test can be presented by the system of equations (A1). To get (A1) from the new test (6), we need to define $\epsilon_1 = y_2\delta_1 + u_1$ and $\epsilon_2 = y_1\delta_2 + u_2$ and assume a bivariate normal distribution of these new error terms ϵ_i. Note that, unlike in the new test, this does not seem like an innocuous assumption because the y's are discrete variables and yet the error terms are assumed to follow a smooth continuous distribution. The PZ test is a test of correlation between the error terms ϵ_i. This they carry out by estimating the two equations of the system (A1) separately by univariate probit and then by calculating a Spearman correlation between the error terms of these two probit equations.

$$
\begin{aligned}
&y_1^* = x\beta_1 + z_1\gamma_1 + z_2\lambda_1 + \epsilon_1,\\
\text{(A1)}\quad &y_2^* = x\beta_2 + z_2\gamma_2 + z_1\lambda_2 + \epsilon_2,\\
&y_i = 1 \text{ if } y_i^* \geq 0, \text{ otherwise } y_i = 0,\ i = 1, 2.\\
&\binom{\epsilon_1}{\epsilon_2}\tilde{\ } IIDN\left(\begin{bmatrix}0\\0\end{bmatrix}, \begin{bmatrix}1 & \rho_\epsilon\\ \rho_\epsilon & 1\end{bmatrix}\right)
\end{aligned}
$$

If the Spearman correlation is negative and statistically significant, we can conclude that we are missing some variable from the estimation that affects the bidders differently and significantly. If we have no other missing variables, this is the competitor's decision to bid. For example, firm 1 bids for those contracts that are allocated to it in the collusion scheme and firm 2 avoids those contracts as agreed. The benefit of this test is that it does not require many observations and that it can be estimated even when the firms under scrutiny never bid for the same contracts. It is also computationally very fast, easy to implement and has better convergence properties than simultaneous equation methods that are used in the new test. Moreover, the separate estimation of the equations in (A1) means that no exclusion restriction is required. The test hypothesis in the PZ case is:

No collusion, H_0: $Corr(\epsilon_1, \epsilon_2) = 0$,

Collusion, H_1: $Corr(\epsilon_1, \epsilon_2) \neq 0$.

PZ detect positive correlation and thus conclude phony bidding. As the PZ test is a test of endogeneity in a bivariate probit model, there are numerous other ways to test for endogeneity in this model. This includes the standard trio of likelihood ratio, Lagrange multiplier and Wald tests. The first test of endogeneity in a bivariate probit model was introduced to the statistics literature by Kiefer (1982). Monfardini and Radice (2007) survey and compare these tests with a Monte Carlo analysis in a recursive probit framework. The PZ test hypothesis can also be written as H_0: $\rho_\epsilon = 0$, and H_1: $\rho_\epsilon \neq 0$.

The main difference between the PZ test and the new test is that the PZ test hinges on the assumption that $Cov(u_1, u_2) = \rho_u = 0$. Given this assumption, $\rho_\epsilon = \delta_1\delta_2 Cov(y_1, y_2)$, and then a test for the significance of ρ_ϵ can be used as a test for collusion, because it is then essentially a test for the joint significance of the δ_i's in the system (A1). This assumption is the main weakness of the PZ test. The test is not robust to the heterogeneity that is observed by both the firms but unobserved by the econometrician. Any missing variable that is correlated with the participation decision and affects both bidders (directly or indirectly and in the same or in a different direction) will enter the residuals and thus corrupt the test.

Firms would prefer being the only bidder in an auction to competing against other firms. If this is not controlled for in the estimation, it creates negative correlation in the residuals that would lead one to identify innocent firms as guilty of territorial allocation or make it harder to detect phony bidding. For this reason, PZ include the observed competitors' characteristics in the regressions. They seem to implicitly assume that this captures all the strategic reasons for the bidders to avoid each other in a competitive setting. With these assumptions, ρ_ϵ is a measure of collusion because it captures the effect of the δ_i's. It means that knowledge of whether one particular firm bids should not help to predict whether another firm has also bid when firms are not colluding. As in the new test, this assumption seems to be natural in an incomplete information setting where bidders form beliefs on each others' entry decisions based on their observable characteristics. The extend that the observables need to control for the competition effect is relaxed in two ways in the new test compared to the PZ test. First, also unobserved heterogeneity controls for competition effect. Second, all of the competition effect needs not be controlled, as long as the observed competitors' characterics capture some of the competition effect. Because the new test relaxes some of the identification assumptions needed for the PZ test, it can be thought of as a nested structure. However, since the PZ test does not require an exclusion restriction, the new test nests the PZ test in some data sets only.

PZ propose the use of their test in an N-bidder case by conducting pairwise analysis for all the possible pairs of bidders. It should also be possible to extend the PZ test to the more rigorous analysis of the N-bidder case by using multivariate probit analysis. This is similar to extending the new test by following the estimation strategies proposed by Ciliberto and Tamer (2009) but the PZ extension is simpler. The benefits of the new test come with some further costs. This test requires more observations than the PZ test for the numerical optimization. It also requires there to be some auctions in the data where both bidders have submitted a bid. The need to numerically optimize a simultaneous equation means also that this test is also harder to implement, may have convergence problems and can be computationally time-consuming.

Appendix B. Participation patterns

Figure 2. Scatter plots of bids and school yard size for each bidder separately for snow ploughing and sanding contract (bid 1 + bid 2) in 2003.

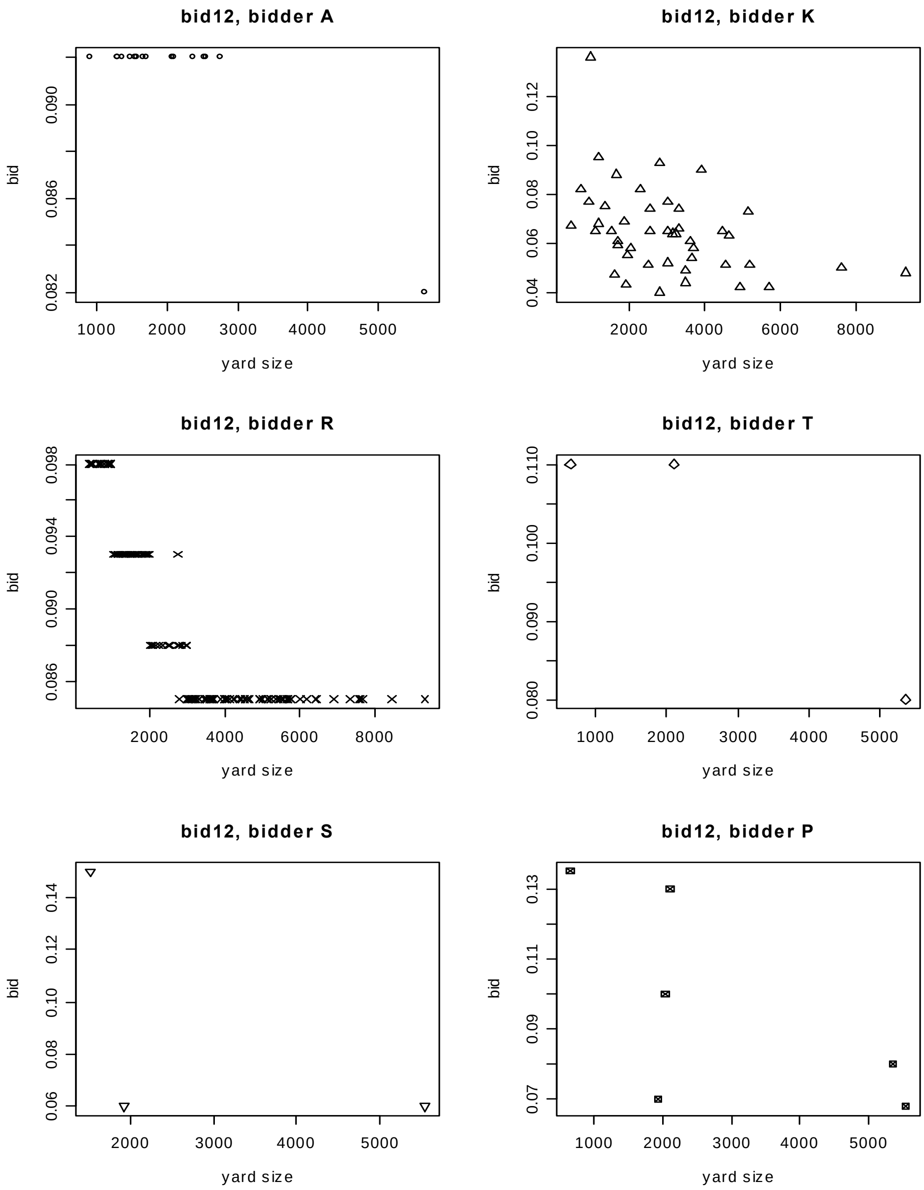
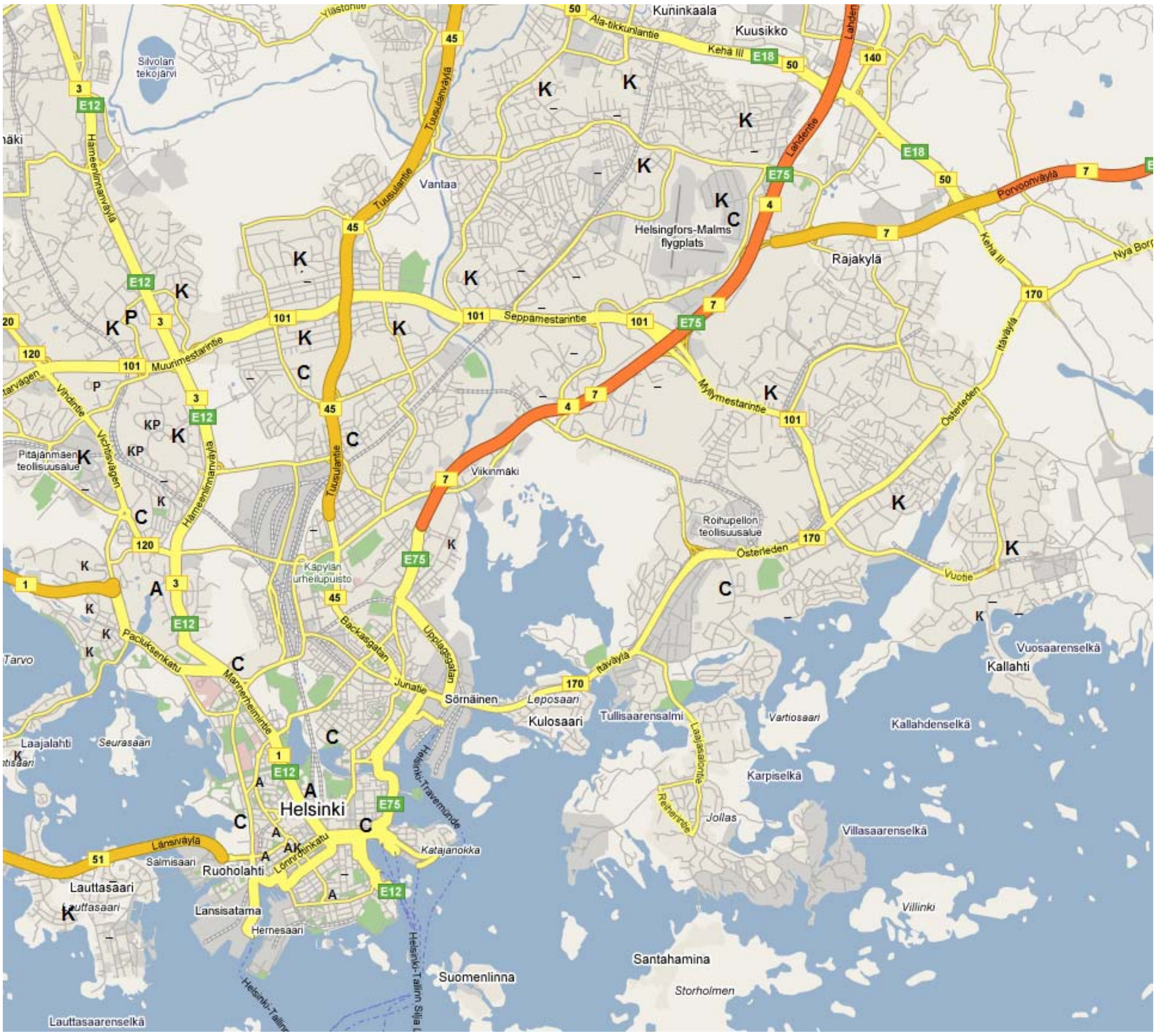
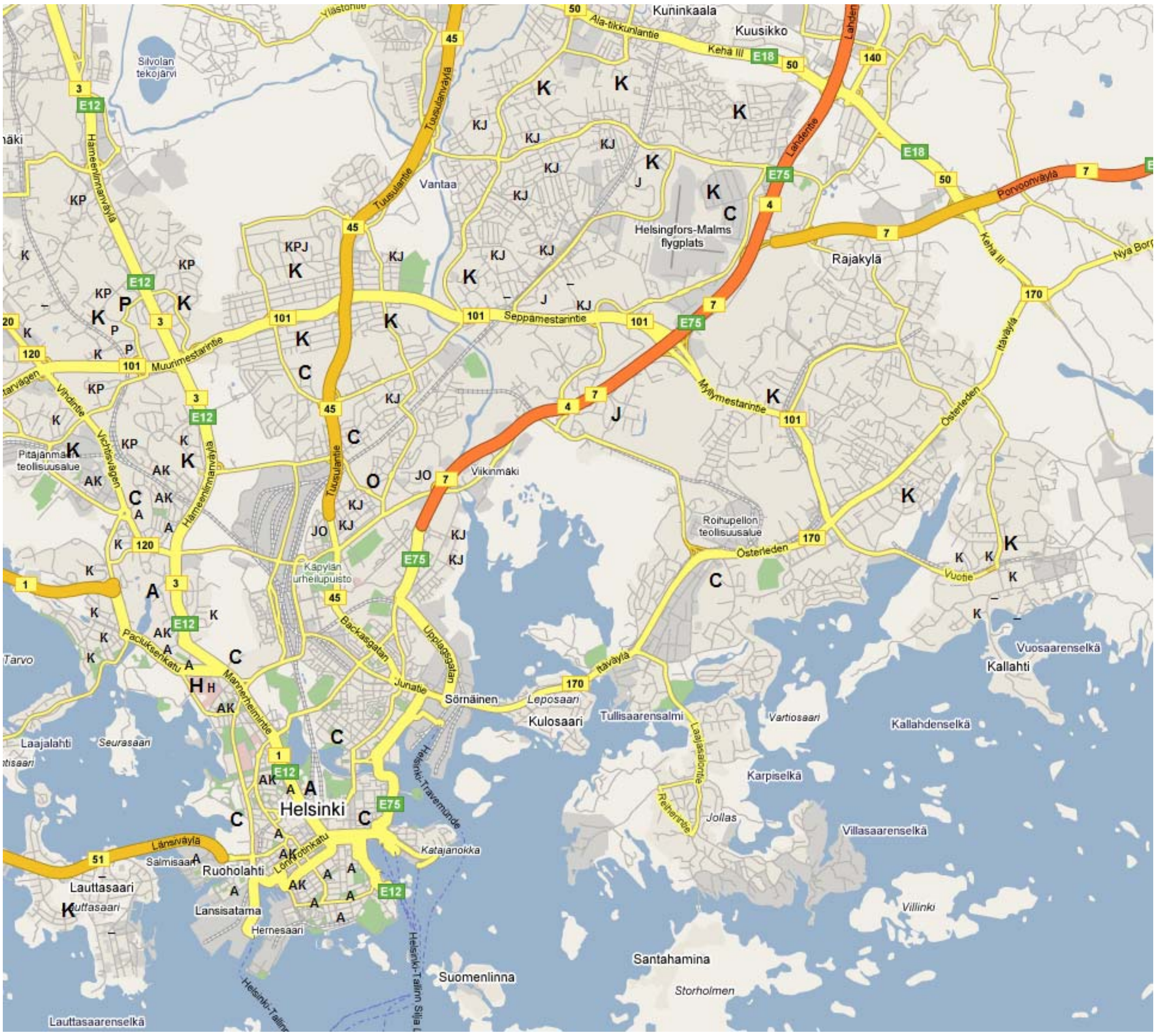


Figure 3. Bidder participation in school yard snow removal auctions in Helsinki 2004.



Small capital letters give the location of schools and which bidders (A, K, P) have bid for a given school. "-" means that there were no bids. The approximate location of the bidders' and the city's (C) garages are marked with larger bold capital letters.

Figure 4. Bidder participation in school yard snow removal auctions in Helsinki 2005.



Small capital letters give the location of schools and which bidders (A, H, J, K, O, P) have bid for a given school. "-" means that there were no bids. The approximate location of the bidders' and the city's (C) garages are marked with larger bold capital letters.