# Social Learning and Monopolist's Product Launching Strategy*

*Job Market Paper*

Ting Liu† and Pasquale Schiraldi‡

November 12, 2007

**Abstract**

A monopolist launches a new product to distinct markets. The monopolist does not know the quality of the product while consumers in each market receive some private information about the quality. We study how the monopolist may influence consumer learning by manipulating the launching sequence when both the monopolist and consumers can learn about the quality of the product from previous sales. We derive conditions under which the monopolist prefers a sequential launch to a simultaneous launch. The conditions depend on the price of the product and the general reputation of the product. We derive the optimal number of markets in which the monopolist will launch the product in each period. The monopolist's dynamic equilibrium strategy endogenizes informational herding.

*This paper began when Pasquale Schiraldi was a Ph.D. student at Boston University. We are very grateful to Ching-to Albert Ma, Jacob Glazer, Barton Lipman, Christophe Chamley, Alessandro Lizzeri and participants at Microeconomics Theory Workshop at Boston University.

†Ting Liu: Corresponding author, tingl@bu.edu. Boston University, Department of Economics.

‡Pasquale Schiraldi: p.schiraldi@lse.ac.uk. London School of Economics, Department of Economics

# 1 Introduction

This paper studies a monopolist's dynamic product launching strategy when both the monopolist and consumers may learn the quality of the product from its previous launching performances. In this two-sided learning environment, the monopolist can control the information flow and hence the degree of consumer learning by manipulating the launching sequence. For example, a movie studio can launch a new movie globally on the same day or sequentially in different countries. In the global launch, consumers in a country have to decide whether to see the movie without observing the box offices of the movie in other countries. By contrast, in the sequential launch, consumers may first observe the box offices of the movie in countries where the movie has been launched, and then decide whether to see the movie when it is available.

It is well known in the social learning literature that when consumers can learn from predecessors' decisions, an informational cascade arises quickly; that is, the public information will swamp the private information and later consumers will imitate their predecessors' decisions. In this paper we address how a monopolist may influence consumer learning and hence control the herd by determining how many markets in which to launch the product in each period. Our paper is distinct from most early literatures in that it allows the monopolist a more active role in the formation of a herd. In early literatures consumers enter the market one after another in an exogenous order and leave firms a passive role in the formation of a herd.

In our model, the monopolist introduces a product with an unknown quality to many distinct markets. The quality may represent the match between the product's underlying characteristics and consumers' tastes. The monopolist does not know consumers' tastes whereas consumers do not know the product's underlying characteristics. When the product is launched in a market, consumers in this market receive a private signal about the quality. This signal may be generated by the local advertising campaign or by the reports from local media. When consumers' purchase decisions are based on their private information, launching performances in previous markets may influence sales in remaining markets.

We consider the situation in which the monopolist will launch the product in a market at most once. This setting is suitable to products with a short life cycle such as movies, musicals and apparel. When the monopolist determines how many markets to launch the product in the current period, it may face a tradeoff between more informed purchase decisions made by future consumers and less informed purchase decisions made by current consumers. For example, suppose Paramount will launch a new movie to 10 countries. It can launch the movie first in Japan then in 9 other markets. Alternatively, it can launch the movie first in Japan, US and UK on the same date, then in the remaining 7 markets on a later date. Now, US and UK's box offices will generate more information to the remaining consumers. However, consumers in these two markets are making purchase decisions without observing the movie's performance in Japan. How, then, should a firm determine the launching sequence?

We ask the following questions. When does a monopolist prefer a sequential launch to a simultaneous launch? If the monopolist prefers a sequential launch to a simultaneous launch, what is its optimal launching sequence?

We show that when the prior belief is extreme, the monopolist will never launch the product sequentially. When consumers hold extreme beliefs about the product, even if they receive a signal contradicting their prior beliefs, the signal is not strong enough to change their purchase decisions. Hence, early buyers' purchase decisions do not convey any information, and the monopolist cannot improve its profit from a simultaneous launch by a sequential launch. Specifically, if consumers hold an extremely pessimistic belief, the monopolist will not launch the product in any market. If consumers hold an extremely optimistic belief, the monopolist will launch the product simultaneously in all markets.

When the prior belief is less extreme, the monopolist can influence consumer learning by manipulating the launching sequence. Its launching strategy depends upon the prior belief and the price of the product. Consumers will buy the product if and only if the expected value of the product is higher than the price. When the monopolist determines whether to reveal information and, if so, how much information to reveal, it needs to assess what kind of information may be generated by the earlier launches. Intuitively, this depends on whether the product is more likely to be good or bad. We find that the monopolist will launch the product sequentially either when it is more likely to be a good product or when it is more likely to be a bad product but the price is very low.

When the product is more likely to be good, early launches are more likely to reveal favorable information about the product's quality and hence convince later consumers to buy. Allowing some consumer learning is therefore more profitable than suppressing consumer learning all together. This may explain why Sony launched the highly anticipated PlayStation 2 sequentially across countries, even though a sequential launch may disappoint consumers and exposes the PlayStation 2 to the rivals' new generation of game consoles.

Alternatively, when the product is more likely to be bad, early launches are more likely to reveal bad information about the product's quality. However, if the product is very cheap, a little more bad information is not damaging enough to deter later consumers from buying whereas a little more good information may convince later consumers to buy. Hence, the monopolist will launch sequentially but will only reveal the minimum amount of information in the first period, i.e., it will launch the product in one market in the first period. The monopolist does this because it does not want to risk revealing severe bad information and trigger a rejection herd.

In the second period, the monopolist will launch the product in all the remaining markets. A successful launch in the first market will trigger a purchase herd in the remaining markets. Hence, the monopolist will sell to all consumers in the second period. In contrast, a failed launch will reduce consumers' confidence about the good. The monopolist will prevent a future rejection herd by launching it in all the remaining markets, which suppresses any further consumer learning.

Our model yields predictions about the launching duration, the amount of information revealed during the launch and the failure rate of a new product. We find that the launching duration is longer and the monopolist will reveal more information during the launches when the product is more likely to be good than when it is more likely to be bad. Finally, the launching strategy for a more promising product is risker than that of a less promising good and has a higher chance of triggering a rejection herd during the launches.

The paper is organized as follows. Section 2 discusses the related literature. Section 3 introduces the model. Section 4 illustrates the basic idea with a simple model. Section 5 shows the main results. Section 6 concludes.

# 2 Related Literature

Bikhchandani, Hirshlerfer and Welch (1992), and Banerjee (1992) are the pioneer work in social learning literature. They show that herd could be rational and rational herd may contain very little information. In their models, consumers enter the market one after another by an exogenous order and make purchase decisions based on what they observe. Hence, the monopolist is quite passive and cannot influence consumer learning at all. In our model, the monopolist can influence consumer learning by manipulating the launching sequence of the new product.

Bose, Orosel, Ottaviani and Vesterlund (2006, 2007) study a monopolist's pricing strategy when the price affects both the current period profit and the amount of information transmitted to future consumers. In their model, consumers enter the market one after another in an exogenous order and the monopolist affects consumer learning by setting different prices. In contrast, we allow the monopolist to influence learning by manipulating the launching sequence rather than the price. In their model, the main tradeoff is between the current period rent and the valuable information revealed to future consumers. In our model, the tradeoff is between the public information generated by earlier markets and the profit gain (loss) in those earlier markets due to their less informed decisions.

Sgroi (2002) focuses on how a social planner may influence consumer learning by determining the number of offers to make in the first period. He also discusses how a monopolist may affect consumer learning. Our paper differs from his work in various ways. First, in Sgroi the monopolist knows the quality of the product whereas consumers do not know it. Therefore, it is a one-sided learning problem. In our model, both the monopolist and consumers learn the quality of the product over time and it is therefore a two-sided learning problem. Second, in Sgroi the monopolist can only choose the number of units to offer in the first period. After that, consumers enter the market sequentially by an exogenous order. The monopolist is essentially solving a static problem. Our model is more general in that the monopolist can determine the number of units to offer in each period and therefore is solving a dynamic problem. Third, Sgroi studies a special case when the product's prior belief is equal to the price and the probability $\frac{1}{2}$. By contrast, we analyze the monopolist's strategy given any prior belief.

Bergemann and Valimaki (1996, 1997, 2000) study various models in which the uncertainty of a product's quality is resolved over time by consumers' experimentations. They focus on firms' pricing strategies whereas we focus on the monopolist's launching strategy. In their model, the number of consumers in each period remains constant. In contrast, in our model, the number of consumers decreases over time. Lastly, in Bergemann and Valimaki consumers' purchases reveal ex post information such as their experience. In our model, consumers' purchases reveal their ex ante private information.

Bar-Isaac (2003) studies a dynamic learning model where a seller is privately informed about the quality of the product whereas buyers learn the quality of the product over time. In contrast, in our paper buyers are privately informed about the product's quality and both the seller and buyers learn the quality of the product over time. Besides, Bar-Issac focuses on the seller's trading decision in each period. Once the seller decides to trade, he will sell to all consumers in that period. In contrast, we focus on the seller's launching decision in each period. The seller not only decides whether to trade but also decides how many offers to make in each period.

# 3 The Model

A risk-neutral monopolist would like to sell a new product to $N$ markets. These markets are indexed by $i \in \{1, 2, ..., N\}$. Each market has a continuum of risk neutral consumers of mass 1. We assume that these markets are disjoint so that arbitrage between consumers in different markets is impossible.

The monopolist considers selling the product to these markets over time, which we take as discrete. The index $t \in \{1, 2, ...\}$ denotes the periods in which the monopolist may sell the good. We say the monopolist launches the good to market $i$ in period $t$. The monopolist may launch the product in a market at most once. This may be due to an entry cost or to a time constraint. If the product is launched to market $i$, consumers in this market then decide whether to buy the product. For example, a new movie is usually in theaters of a locale for a few weeks. Consumers in this locale may see the movie within this period.

The price of the good is fixed at $p$, which is commonly known. Let $n_t \in \{1, 2, ..., N\}$ be the number of markets in which the product will be launched in period $t$. The monopolist chooses $n_t$ in period $t$. For example, the monopolist may decide to launch the good to markets 1 to 3 in period 1, and then to markets 4 to $N$ in period 2. Alternatively, the monopolist may launch the good to one market in each period.

The monopolist's objective is to maximize the sum of the profits. To simplify the analysis, we normalize the production cost to zero and assume no discounting. Relaxing the zero marginal cost will not change our results as long as the marginal cost is smaller than the price. Our results hold when the discount factor is sufficiently close to one. [1]

The value of the good $v$ is either low or high, $v \in \{L, H\}$. Without loss of generality, we choose $L = 0$ and $H = 1$. The value $v$ is unknown to the monopolist and consumers. The value may represent the match between the good and consumers' taste. The monopolist does not know consumers' tastes, whereas consumers do not know the product's underlying characteristics. In period $t$, the public belief of a high value product is $\lambda_t$, $\lambda_t \in (0, 1)$. A consumer buys at most one unit of the good. His utility is $v - p$ if he purchases the good at price $p$ and 0 otherwise.

When the product is launched to market $i$, a random signal $s_i$ will be generated and observed by all consumers in this market. This signal remains unknown to other markets. Conditional on the true value $v$, signals in different markets are independent and identically distributed. Let $s_i \in \{L, H\}$ be the possible values of the signal, where $s_i = L$ indicates a "bad" signal, while $s_i = H$ indicates a "good" signal, respectively:

$$\text{Prob}(s_i = H|v = 1) = \text{Prob}(s_i = L|v = 0) = q, \forall i,$$

where $q \in (0.5, 1)$. That is, when consumers receive signal $H$, the value of the good is more likely to be high than when they receive signal $L$. The parameter $q$ is common knowledge and usually called the precision of a signal. Note that $q$ does not depend on $i$, so the signals generated by each market are equally precise.

The sequence of events in period $t$ is as follows. First, the monopolist and consumers in the remaining markets observe the monopolist's previous launching decisions including how many markets in which to launch the product in each period and the identity of those markets. In addition, they observe the purchase decisions made by consumers in previous

[1]Our results hold when the discount factor is greater than a threshold. When the discount factor is smaller than the threshold, the monopolist prefers a simultaneous launch to a sequential launch.

markets. Second, the monopolist decides $n_t$, the number of markets in which to launch the product in period $t$. Third, if the monopolist launches the product in market $i$, consumers in this market receive a private signal, $s_i$, about the good's value. Fourth, consumers make purchase decisions if the product is launched in their market.

A consumer will make a purchase decision at most once and therefore his decision rule is very simple. When he is offered the opportunity to buy the product, the consumer's purchase decision is a function of the public belief in that period and the private signal of the market in which he is located. The consumer will buy if and only if his posterior belief is greater than or equal to the price.

The monopolist can observe everything happened in previous periods, but in equilibrium, bases its strategy only on the public belief and the number of the remaining markets in the current period. Let $n^*(\lambda_t, N_t)$ denote the monopolist's equilibrium strategy in period $t$ and $v(\lambda_t, N_t)$ denote the monopolist's value function. Given consumers' strategies, $n^*(\lambda_t, N_t)$ maximizes the monopolist's expected profits.

We assume that when the monopolist is indifferent between launching the product in a market in the current period or in the future, it will launch the product in the current period. This assumption can be justified by a small waiting cost for the monopolist. We assume that when the monopolist's profit from launching the product in a market is zero, it does not enter the market. This assumption can be justified by a small entry cost.

We define some notation here. Given a prior belief $\lambda$, consumer $j$'s posterior belief after receiving a bad signal is

$$\lambda^L(\lambda) \equiv \Pr(v = 1 | \lambda, s_j = L) = \frac{\lambda(1-q)}{\lambda(1-q) + (1-\lambda)q}. \tag{1}$$

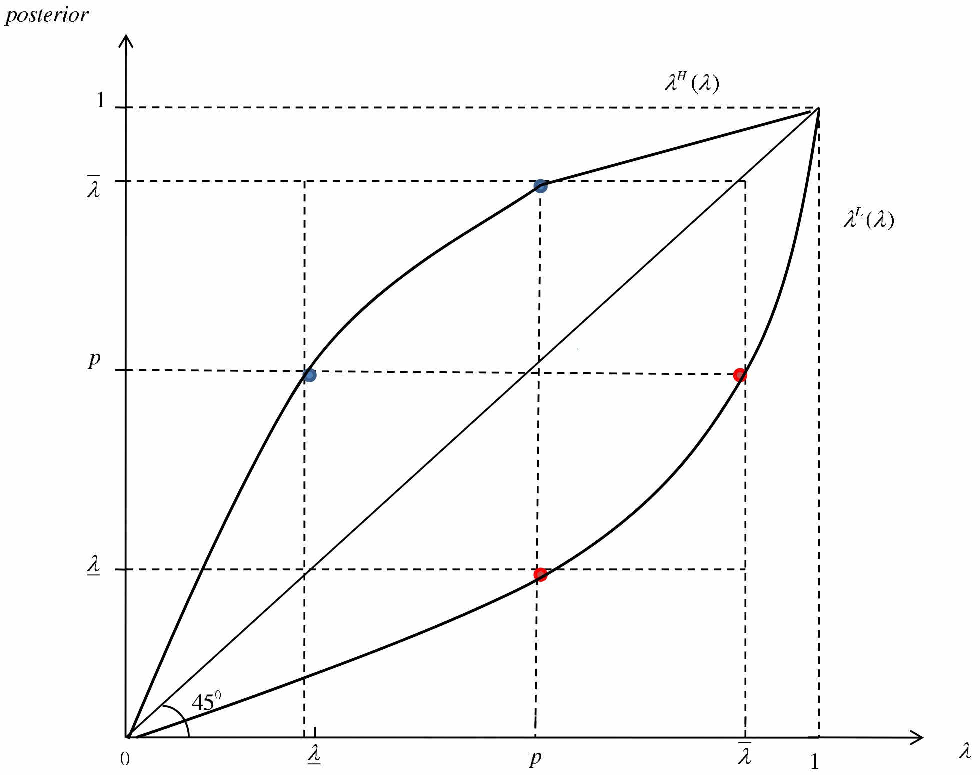


Figure 1: posterior beliefs

Similarly, the consumer's posterior belief upon receiving a good signal is

$$\lambda^H(\lambda) \equiv \Pr(v = 1|\lambda, s_j = H) = \frac{\lambda q}{\lambda q + (1-\lambda)(1-q)}. \tag{2}$$

Figure 1 characterizes a consumer's posterior beliefs as functions of his prior belief. The concave curve represents $\lambda^H(\lambda)$ and the convex curve represents $\lambda^L(\lambda)$. Because the value of the product is $v \in \{0,1\}$, the consumer's willingness to pay after receiving a good signal is $\lambda^H(\lambda)$ and his willingness to pay after receiving a bad signal is $\lambda^L(\lambda)$.

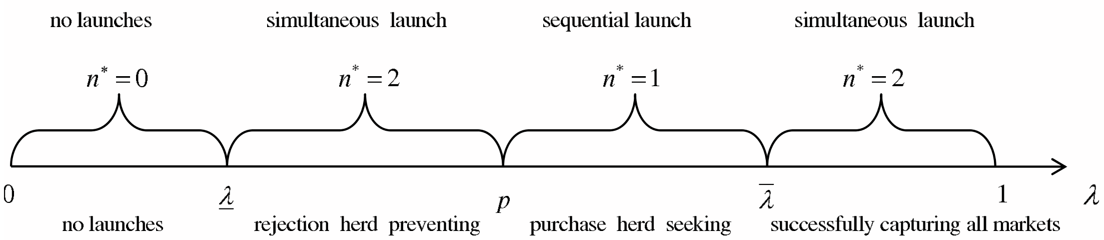


Figure 2: an example with two markets

# 4 An Example with Two Markets

In this section, we use a simple example with two markets to illustrate when the monopolist prefers a sequential launch to a simultaneous launch, and, if so, how much it gains by a sequential launch. The monopolist's launching strategy critically depends on consumers' prior expectation of the good's value and the price.

In the example we drop the subscript $t$. Let $\lambda$ denote the prior belief. We divide the discussion into four cases according to the level of the prior belief (see Figure 2). The cases are ordered by extremely pessimistic consumers ($\lambda < \underline{\lambda}$), pessimistic consumers ($\underline{\lambda} < \lambda < p$), optimistic consumers ($p < \lambda < \bar{\lambda}$) and extremely optimistic consumers ($\bar{\lambda} < \lambda$). The monopolist will launch the good sequentially, i.e., encourage consumer learning, only when consumers are optimistic about the good.

To begin, we define two threshold beliefs

$$\bar{\lambda} \equiv \frac{pq}{pq + (1-p)(1-q)}, \tag{3}$$

$$\underline{\lambda} \equiv \frac{p(1-q)}{p(1-q) + (1-p)q}. \tag{4}$$

At belief $\underline{\lambda}$, a consumer is indifferent between purchasing and not purchasing the product upon receiving a good signal; [2] at belief $\bar{\lambda}$, a consumer is indifferent between purchasing and

[2] $\underline{\lambda}$ is derived by setting expression (2) to $p$.

not purchasing the product upon receiving a bad signal. [3]

## 4.1 No Launches ($\lambda < \underline{\lambda}$)

When consumers are extremely pessimistic about the product, the monopolist will never launch the product.

At the prior belief $\underline{\lambda}$, a consumer is indifferent between purchasing and not purchasing the product upon receiving a good signal, i.e., $\lambda^H(\underline{\lambda}) = p$. Since the posterior belief $\lambda^H(\lambda)$ increases in the prior belief, when a consumer's prior belief is below $\underline{\lambda}$, he will not buy the product even if he receives a good signal, i.e., $\lambda^H(\lambda) < p$ (see Figure 1). Hence, the monopolist's profit from a simultaneous launch is zero.

In a sequential launch, consumers in the first market will never buy the product. Consumers in the second market therefore cannot infer any information from the first market. Consequently, the monopolist's profit from the sequential launch is the same as the simultaneous launch.

## 4.2 Preventing a Rejection Herd($\underline{\lambda} < \lambda < p$)

Pessimistic consumers think the product is overpriced before receiving any additional information. When the product is launched in a market, consumers in this market will buy if and only if they receive a good signal, i.e., $\lambda^L(\lambda) < p < \lambda^H(\lambda)$ (see Figure 1). The monopolist will launch the product simultaneously to prevent a rejection herd.

A sequential launch differs from a simultaneous launch in the profit gained in the second market. This is because in a sequential launch the purchase decisions by consumers in the first market convey their private information. Hence, consumers in the second market make purchase decisions based on what they observe from the first market and their own private

[3] $\bar{\lambda}$ is derived by setting expression (1) to $p$.

signal.

If the second market receives a bad signal, the monopolist's expected profit in a sequential launch is the same as in a simultaneous launch. This is because in a sequential launch, at best, consumers in the second market will observe a successful launch and hence infer a good signal in the first market. Since both markets' signals are equally precise, the good signal in the first market will cancel the bad signal in the second market, and consumers in the second market again hold their prior belief. [4] Because consumers think the product is overpriced ex ante, they will not purchase it.

Alternatively, if the second market receives a good signal, a sequential launch differs from a simultaneous launch only when the first market receives a bad signal. Specifically, the monopolist loses $q(1-q)p$ from a sequential launch, where $q(1-q)$ is the probability that the first market receives a bad signal and the second market receives a good signal. In a sequential launch, if consumers in the second market observe a failure in the first market, they will not buy even if their own signal is good. This is again because the bad news in the first market cancels the good news in the second market. When consumers think the product is overpriced ex ante, a little more bad information can easily convince consumers that the product is not worth buying whereas a little more good information is not strong enough to convince them to buy.

## 4.3 Seeking a Purchase Herd ($p < \lambda < \bar{\lambda}$)

If consumers are optimistic, they think the product is a bargain before receiving any additional information. Again, when the product is launched in a market, consumers in this market will buy if and only if they receive a good signal (see Figure 1). In this case, the monopolist will launch the product sequentially to seek a purchase herd.

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[4] $\lambda^L(\lambda) = \frac{\lambda(1-q)}{\lambda(1-q)+(1-\lambda)q}$, $\lambda^H(\lambda^L(\lambda)) = \frac{\lambda^L(\lambda)q}{\lambda^L(\lambda)q+(1-\lambda^L(\lambda))(1-q)} = \lambda$

The analysis is similar to the Preventing a Rejection Herd subsection because the purchase decisions by consumers in the first market also convey their private information. If the second market's private information is bad, a sequential launch differs from a simultaneous launch only when the first market receives a good signal; hence, the monopolist gains $q(1-q)p$ from a sequential launch. In a sequential launch, if consumers in the second market observe a success in the first market, they will buy even if their own signal is bad. This is because the good signal in the first market cancels the bad signal in the second market and consumers think the product is a bargain ex ante.

Alternatively, if the second market's private signal is good, the monopolist's expected profit in a sequential launch is the same as in a simultaneous launch. This is because in a sequential launch, the worst news they may hear from the first market is a failure. This failure will cancel the good news in the second market and consumers in this market still think the product is a bargain. When consumers think the product is a bargain ex ante, a little more bad information is not damaging enough to deter them from buying, whereas a little more good information can easily convince them to buy.

## 4.4 Successfully Capturing All Markets ($\bar{\lambda} < \lambda$)

When consumers are extremely optimistic, they think the product is worth buying even if they receive a bad signal, i.e., $p < \lambda^L(\lambda)$ (see Figure 1). Hence, if the monopolist launches the product simultaneously, consumers in both markets will purchase the good. The monopolist's profit is $2p$.

If the monopolist launches the product sequentially, consumers in the first market will always purchase the good. Consequently, consumers in the second market will not infer any information from the first market. The monopolist's profit from the sequential launch is the same as the simultaneous launch.

In summary (refer to Figure 2), the monopolist's launching strategy is not monotone in the prior belief. Specifically, a sequential launch is optimal for the monopolist only when consumers are optimistic but not extremely optimistic about the product. When consumers think the product is overpriced, a failure can easily trigger rejections in the remaining markets. A sequential launch is too risky for the monopolist. In contrast, when consumers think the product is a bargain, a sequential launch may trigger a purchase in the remaining markets but will never trigger a rejection. The sequential launch is therefore more profitable than the simultaneous launch. When consumers are extremely optimistic or extremely pessimistic, they cannot infer anything from a previous launch, hence the monopolist does not gain by launching sequentially.

While the two-market example illustrates the tradeoff between sequential and simultaneous launches, new issues arise with more than two markets. With three or more markets, the monopolist may find it optimal to launch in more than one but less than all the markets, choosing an interior solution to how much information to reveal to future consumers. In addition, the dynamic features of the monopolist's strategy are richer with more markets.

# 5 General Results

Now, we analyze the monopolist's equilibrium strategy in the general case when there are $N$ markets, where $N > 2$. When there are more than two markets, if the monopolist prefers a sequential launch to a simultaneous launch, it needs to decide how much information to reveal. We call a product lemon if $\lambda_t < \frac{1}{2}$, a peach, otherwise. Intuitively, if the product is a lemon, the monopolist does not want to take the gamble of allowing consumers to learn more about the product. If the product is a peach, the monopolist would like to allow consumers to learn more about the product. Therefore, unlike the example with two markets, now the comparison between the public belief $\lambda_t$ and the probability $\frac{1}{2}$ plays a role in the analysis.

The monopolist's equilibrium strategy in period $t$ depends on whether the product is believed to be a bargain or overpriced, and whether it is a lemon or peach. We summarize the monopolist's equilibrium strategy in period $t$, $n^*(\lambda_t, N_t)$, by Table 1. In the table, $k$ and $m$ are natural numbers. The monopolist's choice in period t will determine the distribution of $\lambda_{t+1}$ and hence we can derive $n_{t+1}$ accordingly.

Table 1

| | Extremely pessimistic ($\lambda_t < \underline{\lambda}$) | Pessimistic ($\underline{\lambda} < \lambda_t < p$) | Optimistic ($p < \lambda_t < \bar{\lambda}$) | Extremely optimistic ($\bar{\lambda} < \lambda_t$) |
|---|---|---|---|---|
| Lemon ($\lambda_t < \frac{1}{2}$) | 0 | $N_t$ | 1 | $N_t$ |
| Peach ($\frac{1}{2} < \lambda_t$) | 0 | $2k < N_t$ | $2m - 1 < N_t$ | $N_t$ |

The above table summarizes what we will show in sections 5.1 and 5.2. Given a pair of $(\lambda_t, N_t)$, we examine $n_t$ in subsection 5.1 and we characterize the monopolist's equilibrium strategy in subsection 5.2.

## 5.1 Preliminary Results

In the model, consumers make purchase decisions at most once. When the product is launched in a market, consumers in this market make purchase decisions regardless of the number of the remaining markets. Hence, in period $t$, if the public belief is above the threshold $\bar{\lambda}$ defined in expression (3), consumers will always purchase the product; if the public belief is below the threshold $\underline{\lambda}$ defined in expression (4), consumers will not purchase the product. The analysis is the same as in the example with two markets.

**Lemma 1.** *Given $\bar{\lambda}$ and $\underline{\lambda}$ as defined in (3) and (4), $n^*(\lambda_t, N_t) = N_t$ if $\lambda_t > \bar{\lambda}$; $n^*(N_t, \lambda_t) = 0$ if $\lambda_t < \underline{\lambda}$.*

Next, we analyze the monopolist's strategy when $\underline{\lambda} < \lambda_t < \bar{\lambda}$.

**Lemma 2.** *Suppose* $\underline{\lambda} < \lambda_t < min\{p, \frac{1}{2}\}$*, then* $n^*(\lambda_t, N_t) = N_t$.

To see the intuition, suppose the monopolist launches the product in $n_t$ markets, $n_t < N_t$. Let $\Delta_{n_t}$ denote the difference between the number of successful launches and the number of failed launches among the $n_t$ markets. Since signals are equally precise, having observed the purchase decisions made by consumers in period $t$, consumers in the remaining markets will update the belief according to $\Delta_{n_t}$.

If $\Delta_{n_t} \geq 2$, consumers will update the belief above $\bar{\lambda}$ and accordingly always purchase the product. If $\Delta_{n_t} = 1$ or $\Delta_{n_t} = 0$, consumers will update their belief to $\lambda^H(\lambda_t)$, with $p < \lambda^H(\lambda_t) < \bar{\lambda}$, or hold the same belief $\lambda_t$, respectively. As a consequence, consumers will purchase the product if and only if they receive a good signal. If $\Delta_{n_t} \leq -1$, consumers will update their belief below $\underline{\lambda}$ and therefore never purchase the product (see Figure 1).

The monopolist needs the successful launches to outnumber the failed launches by at least two in order to trigger a purchase herd. In contrast, if the failed launches outnumber the successful launches just by one, the monopolist will trigger a rejection herd. Hence, when consumers are not biased ($\lambda_t = \frac{1}{2}$), it is easier for the monopolist to trigger a rejection herd than a purchase herd. Intuitively, when consumers think the product is a lemon ($\lambda_t < \frac{1}{2}$), allowing consumers to learn from each other's purchase decisions will expose the monopolist to a big risk of a rejection herd while bringing it a small chance of a purchase herd. Hence, the monopolist's optimal strategy is to suppress consumer learning and launch the product in all markets.

**Lemma 3.** *Suppose* $max\{\underline{\lambda}, \frac{1}{2}\} < \lambda_t < p$ *and* $N_t > 2$*, then* $n^*(\lambda_t, N_t)$ *is an even number less than* $N_t$.

Given the public belief $\lambda_t$, $\lambda_t \in (\underline{\lambda}, p)$, consumers' decision rule and the rule of triggering a purchase (rejection) herd is the same as in the discussion for Lemma 2.

We first give the intuition for why the monopolist will not launch the product in $N_t$

markets. Let's compare the monopolist's profit from a simultaneous launch to profit from a sequential launch where the monopolist launches the product in two markets in period $t$ and then in $N_t - 2$ markets in period $t + 1$.

In the sequential launch, two successful launches in period $t$ will trigger a purchase herd in the remaining markets. One successful launch and one failed launch in period $t$ will not change consumers' beliefs. Finally, two failed launches will trigger a rejection herd. In the first case, the monopolist gains profit relative to a simultaneous launch in the $N_t - 2$ markets due to consumer learning. In the second case, its profits in $N_t - 2$ markets remain the same as with a simultaneous launch because in the sequential launch, consumers in those markets cannot infer anything from previous launches. In the last case, it loses profit relative to a simultaneous launch in the $N_t - 2$ markets due to consumer learning. When $\lambda_t > \frac{1}{2}$, the probability of two successes is greater than two failures in period $t$.

Now, we explain why the monopolist will not launch the product in an odd number of markets. The monopolist's profit from launching the product in an odd number of markets, $2k + 1$, is lower than in $2k$ markets. When entering market $j$ in period $t + 1$ instead of period $t$, the monopolist's profit changes in two ways. First, the monopolist's expected profit in market $j$ changes because now consumers in this market will make purchase decisions not only based on their private signal but also on the sales in previous $2k$ markets. Second, the monopolist's expected profits in the remaining $N_t - 2k - 1$ markets change. This is because exiting one market in period $t$ will change the probability of herding in the remaining markets.

When $\lambda_t > \frac{1}{2}$, consumers in market $j$ are more likely to hear good news from the $2k$ markets and hence the monopolist's expected profit in this market increases by delaying the launch. The monopolist's expected profit in the remaining $N_t - 2k - 1$ markets increases as well. This is because moving the market $j$ from period $t$ to $t + 1$ will never break a purchase herd in the $N_t - 2k - 1$ markets but may either trigger a purchase herd or break a rejection herd in these markets.

To see this, note that if market $j$ has a good signal, moving it from period $t$ to $t+1$ will reduce the sales in period $t$ and hence transmit less convincing news to subsequent consumers. However, this change is never damaging enough to break a purchase herd. If the monopolist has triggered a purchase herd by launching the product in the $2k+1$ markets, the difference $\Delta_{2k+1}$ must be at least three. Hence, exiting one market will at worst reduce the difference between the successful launches and failed launches to two, which will still trigger a purchase herd in period $t+1$. The change in launching strategy will reduce the monopolist's profit in the remaining $N_t - 2k - 1$ markets only in the marginal case where the successful launches outnumber failed launches just by one in period $t$. In this case the monopolist's loss in the $N_t - 2k - 1$ markets is $v(\lambda^H(\lambda_t), N_t - 2k - 1) - v(\lambda_t, N_t - 2k - 1)$.

If market $j$ has a bad signal, moving it from period $t$ to $t+1$ will transmit more convincing news to subsequent consumers. This change will greatly increase the monopolist's profit in the remaining $N_t - 2k - 1$ markets in two marginal cases: $\Delta_{2k+1} = 1$ and $\Delta_{2k+1} = -1$. In the first case, reducing one bad signal will trigger a purchase herd in period $t+1$; in the second case, it will break a rejection herd in period $t+1$. The gain from entering market $j$ in period $t+1$ instead of period $t$ outweighs the loss and the monopolist will never launch the product in $2k+1$ markets, with $1 < 2k+1$.

Lastly, we explain why the monopolist will never launch the product in one market in period $t$. This is because it can always do better by launching the product in two markets in period $t$ since this will raise the chance of a purchase herd while reducing the chance of a rejection herd.

**Lemma 4.** *Suppose* $p < \lambda_t < min\{\bar{\lambda}, \frac{1}{2}\}$, *then* $n^*(\lambda_t, N_t) = 1$.

To understand the intuition behind Lemma 4, we need to characterize consumers' belief updating rules when $p < \lambda_t < \min\{\bar{\lambda}, \frac{1}{2}\}$. Suppose the monopolist launches the product in $n_t$ markets, with $n_t < N_t$. If $\Delta_{n_t} \geq 1$, the subsequent consumers will update the belief above $\bar{\lambda}$ and they will therefore always purchase the product. If $\Delta_{n_t} = 0$ or $\Delta_{n_t} = -1$, the subsequent consumers will not update the belief or update the belief to $\lambda^L(\lambda_t)$, with $\underline{\lambda} < \lambda^L(\lambda_t) < p$. Accordingly, they will purchase the product if and only if they receive a good signal. If $\Delta_{n_t} \leq -2$, the subsequent consumers will update the belief below $\underline{\lambda}$; hence, they will not purchase the product (see Figure 1).

We first give the idea for why allowing the minimum amount of consumer learning, i.e., $n^*(\lambda_t, N_t) = 1$, is more profitable than no learning, i.e., $n^*(\lambda_t, N_t) = N_t$. In a simultaneous launch, the monopolist will lose those consumers with a bad signal. In the sequential launch, if the first launch is successful, the monopolist will capture the remaining $N_t - 1$ markets even if consumers in those markets receive a bad signal. Although consumers think the product is a lemon, since the price is so low, all the remaining consumers will purchase it after hearing some good news from the previous market.

Now, we give the intuition for why the monopolist does not want consumers to learn more about the product. When $\lambda < \frac{1}{2}$, allowing consumers to learn more may at best slightly increase the probability of a purchase herd in period $t + 1$ but will expose it to a larger risk of a rejection herd. Suppose, for example, that $\lambda = 1/4$ and $q = 2/3$. If the monopolist launches the product in one market, the probability of triggering a purchase herd is $5/12$; the probability of triggering a purchase herd is zero. If the monopolist launches the product in two markets, the probability of triggering a purchase herd is $7/36$; the probability of triggering a rejection herd is $13/36$.

**Lemma 5.** *Suppose max*$\{p, \frac{1}{2}\} < \lambda_t < \bar{\lambda}$ *and* $N_t > 2$, *then* $n^*(\lambda_t, N_t)$ *is an odd number less than* $N_t$.

Given the belief $\lambda_t$, $p < \lambda_t < \bar{\lambda}$, a consumer's purchase decision and the rule of triggering a purchase (rejection) herd is the same as in the discussion for Lemma 4.

First, allowing some consumer learning is more profitable than no learning. The logic is the same as in the discussion for Lemma 4. Next, the monopolist will never launch the product in an even number, $2k$, of markets. Specifically, it can always do better by launching the product in $2k-1$ markets in period $t$. The argument is analogous to that for Lemma 3.

## 5.2 The Equilibrium Strategy

Combining Lemmas 2 to 5, we can summarize the monopolist's equilibrium path induced by the optimal strategy by the following 4 propositions. Propositions 1 to 4 characterize the monopolist's equilibrium path when $\bar{\lambda} < \frac{1}{2}$, $\frac{1}{2} \in (p, \bar{\lambda})$ ,$\frac{1}{2} \in (\underline{\lambda}, p)$ and $\frac{1}{2} < \underline{\lambda}$, respectively. Each case is summarized by a diagram.

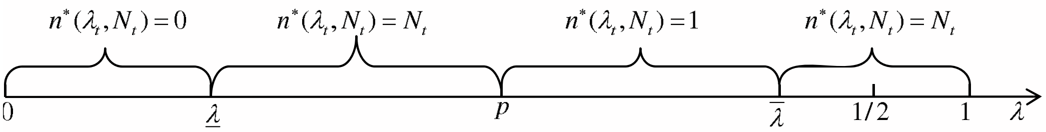


Figure 3: $\bar{\lambda} < \frac{1}{2}$

**Proposition 1.** *When $\bar{\lambda} < \frac{1}{2}$, the monopolist's equilibrium strategy is the following: (see Figure 3)*

- $n^*(\lambda_t, N_t) = 0, \forall \lambda_t \in (0, \underline{\lambda})$
- $n^*(\lambda_t, N_t) = N_t, \ \forall \lambda_t \in (\underline{\lambda}, p)$
- $n^*(\lambda_t, N_t) = 1, \ \forall \lambda_t \in (p, \bar{\lambda})$
- $n^*(\lambda_t, N_t) = N_t, \ \forall \lambda_t \in (\bar{\lambda}, 1)$.

The monopolist's equilibrium strategy is presented by Figure 3. Let $\lambda_0$ denote the prior belief and $N_0$ denote the total number of markets. When $\lambda_0 < \underline{\lambda}$, Lemma 1 shows that the monopolist will not launch the product. In the following propositions, the monopolist's strategy is the same as here as long as the belief drops below $\underline{\lambda}$.

If consumers think the product is an overpriced lemon, i.e., $\underline{\lambda} < \lambda_0 < p$, a little more bad information conveyed by previous launches may trigger a rejection herd. Hence, the monopolist will suppress learning by launching the product in all markets in the first period.

If consumers think the product is a cheap lemon, $p < \lambda_0 < \bar{\lambda}$, the monopolist will allow the minimum amount of learning. This is because a little more good information from the previous market will convince consumers to buy. In contrast, a little more bad information is not damaging enough to deter consumers from buying. To be more specific, the monopolist will launch the product in one market in period 1. If the first launch is successful, the public belief is updated above $\bar{\lambda}$. Then, the monopolist will launch the product in the remaining $N_0 - 1$ markets and all consumers will buy. Alternatively, if the first launch fails, the public belief is updated to a level between $\underline{\lambda}$ and $p$. The monopolist again will launch the product in the remaining $N_0 - 1$ markets. However, consumers in the remaining markets will purchase the good if and only if they receive a good signal. The game ends after two periods.

If consumers are extremely optimistic about the product, $\bar{\lambda} < \lambda_0$, by Lemma 1, the monopolist will launch the product in all the markets and all consumers will buy. The game ends after the first period. In the following propositions, the monopolist's equilibrium strategy when $\bar{\lambda} < \lambda_0$ is the same as here.

When $\bar{\lambda} < \frac{1}{2}$, the launching period is fairly short and consumers learn very little from each other. In addition, the monopolist will never trigger a rejection herd.

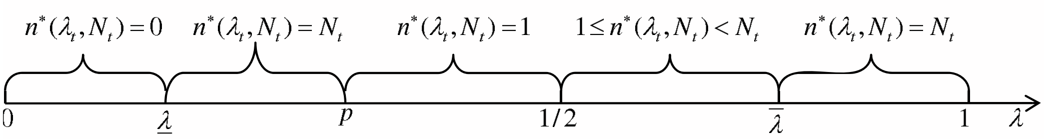


Figure 4: $p < \frac{1}{2} < \bar{\lambda}$

**Proposition 2.** *When $p < \frac{1}{2} < \bar{\lambda}$, the monopolist's equilibrium strategy is the following: (see Figure 4)*

- $n^*(\lambda_t, N_t) = 0, \forall\lambda \in (0, \underline{\lambda})$

- $n^*(\lambda_t, N_t) = N_t$, $\forall\lambda \in (\underline{\lambda}, p)$

- $n^*(\lambda_t, N_t) = 1$, $\forall\lambda \in (p, \frac{1}{2})$

- $n^*(\lambda_t, N_t) \in \{1, 3, 5, ..., N_o(N_t)\}$, *where* $N_o(N_t)$ *is the largest odd number less than* $N_t$, $\forall\lambda_t \in (\frac{1}{2}, \bar{\lambda})$

- $n^*(\lambda_t, N_t) = N_t$, $\forall\lambda_t \in (\bar{\lambda}, 1)$.

As before, if consumers think the product is an overpriced lemon, i.e., $\underline{\lambda} < \lambda_0 < p$, the monopolist will launch the product in all markets, and consumers will buy if and only if they receive a good signal.

If consumers think the product is a cheap lemon, i.e., $p < \lambda_0 < \frac{1}{2}$, the monopolist's strategy is the same as before. In period 2, it will always launch the product in all the remaining markets. The game ends after two periods.

If consumers think the product is a cheap peach, i.e., $\frac{1}{2} < \lambda_0 < \bar{\lambda}$, the monopolist may launch the product in more than one market in the first period to encourage more learning. To be specific, by Lemma 5, the monopolist will launch the product in an odd number of markets, $2k + 1$, where $2k + 1 < N_t$. If successful launches outnumber failed launches by at least one, the consumers' belief is updated above $\bar{\lambda}$ (see Figure 1). The monopolist will launch the product in all the remaining markets, and all consumers will buy.

If failed launches outnumber successful launches by one, consumers' belief is updated to a level between $\underline{\lambda}$ and $p$. The monopolist again will launch the product in the remaining markets in period 2, and consumers will buy if and only if they receive a good signal.

If failed launches outnumber successful launches by more than one, consumers become extremely pessimistic, and the monopolist will not enter any market in period 2. When consumers believe that the product is a cheap peach, the monopolist may allow more learning.

However, the product may completely fail in the second market.

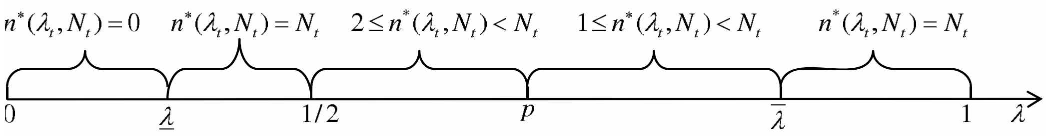


Figure 5: $\underline{\lambda} < \frac{1}{2} < p$

**Proposition 3.** *When $\underline{\lambda} < \frac{1}{2} < p$, the monopolist's equilibrium strategy is the following: (see Figure 5)*

- $n^*(\lambda_t, N_t) = 0, \forall \lambda \in (0, \underline{\lambda})$
- $n^*(\lambda_t, N_t) = N_t$, $\forall \lambda_t \in (\underline{\lambda}, \frac{1}{2})$
- $n^*(\lambda_t, N_t) \in \{2, 4, 6, ..., N_e(N_t)\}$, *where $N_e(N_t)$ is the largest even number less than $N_t$,* $\forall \lambda_t \in (\frac{1}{2}, p)$
- $n^*(\lambda_t, N_t) \in \{1, 3, 5, ..., N_o(N_t)\}$, *where $N_o(N_t)$ is the largest odd number less than $N_t$,* $\forall \lambda_t \in (p, \bar{\lambda})$
- $n^*(\lambda_t, N_t) = N_t$, $\forall \lambda \in (\bar{\lambda}, 1)$.

If consumers believe the product is an overpriced lemon, as before, the monopolist will launch the product in all markets in period 1.

If consumers think the product is an overpriced peach, i.e., $p < \lambda_0 < \bar{\lambda}$, the monopolist will encourage learning by launching the product in an even number of markets in period 1. If more than half the launches are successful, the public belief is updated above $\bar{\lambda}$, and the monopolist will launch the product in all the remaining markets in period 2. If more than half the launches are failures, the public belief is updated below $\underline{\lambda}$, and the monopolist will not enter any market in period 2. If half the launches succeed while half the launches fail, the monopolist and the remaining consumers do not learn anything from previous launches. Hence, again, the monopolist will launch the product in an even number of markets which is not necessarily equal to the previous even number. The monopolist repeats the same pattern until there are no remaining markets or there is only one market left.

When consumers believe the product is a cheap peach, as before, the monopolist will encourage consumer learning by launching the product in an odd number of markets in period 1. If successful launches outnumber failed launches by at least one, the monopolist will trigger a purchase herd and launch the product in all the remaining markets in period 2.

If failed launches outnumber successful launches by one, consumers update the belief downward and either think the product is an overpriced lemon ($\underline{\lambda} < \lambda_1 < \frac{1}{2}$) or an overpriced peach ($\frac{1}{2} < \lambda_1 < p$). If they think the product is an overpriced lemon, the monopolist will launch the product in all the remaining markets in period 2. Alternatively, if they think the product is an overpriced lemon, then in period 2 the monopolist will repeat the pattern characterized in Proposition 2. The launching period can be more than two periods if consumers keep seeing conflicting results in the previous period's launches.

If failed launches outnumber successful launches by at least two, consumers become extremely pessimistic, and the monopolist will not enter any market in period 2.

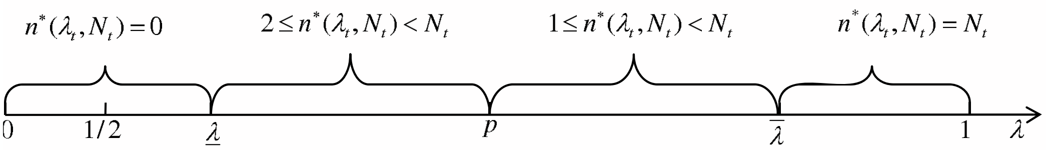


Figure 6: $\frac{1}{2} < \underline{\lambda}$

**Proposition 4.** *When $\frac{1}{2} < \underline{\lambda}$, the monopolist's equilibrium strategy is the following: (see Figure 6)*

- $n^*(\lambda_t, N_t) = 0, \forall \lambda_t \in (0, \underline{\lambda})$

- $n^*(\lambda_t, N_t) \in \{2, 4, 6, ..., N_e(N_t)\}$, *where* $N_e(N_t)$ *is the largest even number less than* $N_t$, $\forall \lambda_t \in (\underline{\lambda}, p)$

- $n^*(\lambda_t, N_t) \in \{1, 3, 5, ..., N_o(N_t)\}$, *where* $N_o(N_t)$ *is the largest odd number less than* $N_t$, $\forall \lambda_t \in (p, \bar{\lambda})$

- $n^*(\lambda_t, N_t) = N_t$, $\forall \lambda \in (\bar{\lambda}, 1)$.

In this case, consumers will think the product is either an overpriced peach or a cheap peach. In the former case, the monopolist will launch the product in an even number of markets and repeat the pattern characterized in Proposition 3. In the latter case, the monopolist will launch the product in an odd number of markets. If the successful launches outnumber the failed launches by at least one, the monopolist will launch the product in all the remaining markets in period 2. If failed launches outnumber successful launches just by one, consumers believe the product is an overpriced peach; hence, the monopolist's strategy is the same as before. If failed launches outnumber successful launches by more than one, the monopolist will not enter any market in period 2.

When consumers believe the product is a peach, the monopolist will always allow some consumer learning. The launching period will extend more than two periods when at some stage, consumers believe the product is an overpriced peach and keep seeing the conflicting results from the previous period's launches. Also, there is a positive probability that the product will be rejected by all markets in the second period.

# 6 Conclusion

We study a monopolist's product launching strategy when both the monopolist and consumers learn the quality of the good from previous sales. When consumers' beliefs are extreme, the monopolist cannot influence consumer learning by manipulating the launching sequence and therefore will always launch the product simultaneous in all markets.

When Consumers' beliefs are less extreme, the monopolist can control consumer learning by manipulating the launching sequence. The optimal launching sequence depends upon the price and the probability of a good product. When the product is more likely to be good, the monopolist will always launch sequentially. When the product is more likely to be bad but the price is low, the monopolist will launch sequentially as well. The amount of information revealed in a sequential launch is different in the above two cases. The monopolist will reveal more information when the product is more likely to be good than when it is more likely to be bad.

We have predictions about the duration of a launch and the probability of triggering a rejection herd during the launch. The launching duration of a product is longer when it is more likely to be good than when it is more likely to be bad. The monopolist will risk triggering a rejection herd when the product is more likely to be good. In contrast, it will not risk triggering a rejection herd when the product is more likely to be bad.

In this paper, we assume the price is fixed and markets are homogeneous ex ante. We plan to study the monopolist's launching strategy when it can choose both the price and the number of markets to launch in each period. In addition, we will examine the monopolist's launching strategy when markets are heterogenous.

## APPENDIX

In the appendix $G \equiv \lambda q + (1-\lambda)(1-q)$. Similarly, $G^H \equiv \lambda^H q + (1-\lambda^H)(1-q)$ and $G^L \equiv \lambda^L q + (1-\lambda^L)(1-q)$.

*Proof of Lemma 2*

The proposition is proved by induction. In the first step we need to prove Lemma 6 which will be used in the induction proof. Define $n^*(N, \lambda)$ as the optimal number of markets in which to launch the product when the public belief is $\lambda$ and the number of the remaining markets is $N$.

**Lemma 6.** *When the public belief is $\lambda$, with $\lambda > \frac{1}{2}$, probability $Pr(\Delta_{2k-1} \geq 1)$ increases in $k$. When $\lambda < \frac{1}{2}$, probability $Pr(\Delta_{2k-1} \geq 1)$ decreases in $k$.*

*Proof:*

$$
\begin{aligned}
\Pr(\Delta_{2k-1} \geq 1) &= \sum_{i=k}^{2k-1} (\lambda q^i (1-q)^{2k-1-i} + (1-\lambda) q^{2k-1-i}(1-q)^i) C_{2k-1}^i \\
&= \lambda \Pr(t \geq k | 2k-1) + (1-\lambda)(1 - \Pr(t \geq k | 2k-1)) \\
&= (1-\lambda) + \Pr(t \geq k | 2k-1)(2\lambda - 1),
\end{aligned}
$$

where, $\Pr(t \geq k | 2k-1) = \sum_{i=k}^{2k-1} q^i (1-q)^{2k-1-i} C_{2k-1}^i$.

$$
\begin{aligned}
\Pr(\Delta_{2k+1} \geq 1) - \Pr(\Delta_{2k-1} \geq 1) &= (2\lambda - 1)(\Pr(t \geq k+1 | 2k+1) - \Pr(t \geq k | 2k-1)) \\
&= (2\lambda - 1) q^k (1-q)^k (2q-1) C_{2k-1}^k.
\end{aligned}
$$

Since $q \in (\frac{1}{2}, 1)$, the difference $\Pr(\Delta_{2k+1} \geq 1) - \Pr(\Delta_{2k-1} \geq 1)$ is positive when $\lambda > \frac{1}{2}$ and negative when $\lambda < \frac{1}{2}$. Q.E.D.

*Induction Proof:*

1. We show that when $N = 2$, $n^*(N, \lambda) = 2$. The expected profit from launching the

product in two markets is $\pi(n_1 = 2) = 2Gp$. The expected total profit from launching the product in one market is $\pi(n_1 = 1) = Gp + GG^H(\lambda))p$. The difference $\pi(n_1 = 2) - \pi(n_1 = 1)$ is $q(1-q)p > 0$.

2. We propose that $n^*(N-1, \lambda) = N-1$.

3. We show $n^*(N, \lambda) = N$. In this step, we first show that $\pi(n_1 = N) > \pi(n_1 = 2n+1)$, where $n = 0, 1, ..., n^{max}$, with $2n+1 < N$. The profit from a simultaneous launch is $\pi(n_1 = N) = NGp$. The profit from launching the product in $2n+1$ markets is

$$\pi(n_t = 2n+1) = (2n+1)Gp + \Pr(\Delta_{2n+1} \geq 3)(N-2n-1)p + \Pr(\Delta_{2n+1} = 1)v(\lambda^H, N-2n-1).$$

Because $q \in (\frac{1}{2}, 1)$ and $\lambda \in (0, \frac{1}{2})$, the value $v(\lambda^H, N-2n-1) < (N-2n-1)p$. Hence, $\pi(n_t = 2n+1) < (2n+1)Gp + \Pr(\Delta_{2n+1} \geq 1)(N-2n-1)p$. By Lemma 6, $\Pr(\Delta_{2n+1} \geq 1)$ decreases in $n$ when $\lambda < \frac{1}{2}$. Therefore,

$$\begin{aligned} \pi(n_t = 2n+1) &< (2n+1)Gp + \Pr(\Delta_1 \geq 1)(N-2n-1)p \\ &= (2n+1)Gp + (N-2n-1)Gp \\ &= NGp. \end{aligned}$$

Next, we show $\pi(n_1 = N) > \pi(n_1 = 2n)$, where $n = 0, 1, ..., n^{max}$, with $2n < N$. The profit from launching the product in $2n$ markets, is

$$\pi(n_t = 2n) = 2nGp + \Pr(\Delta_{2n} \geq 2)(N-2n)p + \Pr(\Delta_{2n} = 0)v(\lambda, N-2n).$$

By the hypothesis $v(\lambda, N-2n) = (N-2n)Gp$. Hence, $\pi(n_t = 2n)$ becomes

$$2nGp + \Pr(\Delta_{2n} \geq 2)(N-2n)p + \Pr(\Delta_{2n} = 0)(N-2n)Gp.$$

The difference $\pi(n_t = N) - \pi(n_t = 2n)$ is

$$(N - 2n)p(G(1 - \Pr(\Delta_{2n} = 0)) - \Pr(\Delta_{2n} \geq 2)).$$

Next, we prove $G(1 - \Pr(\Delta_{2n} = 0)) - \Pr(\Delta_{2n} \geq 2) > 0$.

Let $\Pr(t = T|N, q) \equiv \frac{N!}{T!(N-T)!} q^T (1-q)^{N-T}$, the probability of $T$ successes out of $N$ trials with the probability of success equal to $q$. Since $\Pr(\Delta_{2n} = 0) = \Pr(t = n|2n, q)$, the probability $\Pr(\Delta_{2n} \geq 2)$ becomes

$$(1 - \lambda) + (2\lambda - 1)\Pr(t \geq n + 1|2n, q) - (1 - \lambda)\Pr(t = n|2n, q).$$

Substitute $G$, $\Pr(\Delta_{2n} = 0)$ and $\Pr(\Delta_{2n} \geq 2)$, the expression
$G(1 - \Pr(\Delta_{2n} = 0)) - \Pr(\Delta_{2n} \geq 2)$ becomes

$$(2\lambda - 1)((1 - \Pr(t = n|2n, q))q - \Pr(t \geq n + 1|2n, q)).$$

Since $2\lambda - 1 < 0$, to show $G(1 - \Pr(\Delta_{2n} = 0)) - \Pr(\Delta_{2n} \geq 2) > 0$ is equivalent to show $(1 - \Pr(t = n|2n, q))q - \Pr(t \geq n + 1|2n, q) < 0$. We prove
$(1 - \Pr(t = n|2n, q))q - \Pr(t \geq n + 1|2n, q) < 0$ by induction.

When $n = 1$, $(1 - \Pr(t = n|2n, q))q - \Pr(t \geq n + 1|2n, q) = -q(1 - q)(2q - 1) < 0$.

Suppose $(1 - \Pr(t = n|2n, q))q < \Pr(t \geq n + 1|2n, q)$ for $n$, which is equivalent to

$$\frac{1 - q}{q} > \frac{\Pr(t \geq n + 1|2n, 1 - q)}{\Pr(t \geq n + 1|2n, q)}. \tag{5}$$

We show that

$$\frac{1 - q}{q} > \frac{\Pr(t \geq n + 2|2n + 2, 1 - q)}{\Pr(t \geq n + 2|2n + 2, q)}. \tag{6}$$

The probability $\Pr(t \geq n+2|2n+2, 1-q)$ and $\Pr(t \geq n+2|2n+2, q)$ can be rewritten as the following:

$$\begin{aligned}
\Pr(t \geq n+2|2n+2, 1-q) &= \Pr(t \geq n+1|2n, 1-q) + \Pr(t = n|2n, q)\Pr(t = 0|2, q) \\
&\quad - \Pr(t = n-1|2n, q)\Pr(t = 2|2, q) \\
\Pr(t \geq n+2|2n+2, q) &= \Pr(t \geq n+1|2n, q) + \Pr(t = n|2n, q)\Pr(t = 2|2, q) \\
&\quad - \Pr(t = n+1|2n, q)\Pr(t = 0|2, q).
\end{aligned}$$

Substituting $\Pr(t \geq n+2|2n+2, 1-q)$ and $\Pr(t \geq n+2|2n+2, q)$, condition (6) becomes

$$L < R,$$

where

$$\begin{aligned}
L &= q\Pr(t \geq n+1|2n, 1-q) + q\Pr(t = n|2n, q)\Pr(t = 0|2, q) - q\Pr(t = n-1|2n, q)\Pr(t = 2|2, q) \\
R &= (1-q)\Pr(t \geq n+1|2n, q) + (1-q)\Pr(t = n|2n, q)\Pr(t = 2|2, q) \\
&\quad -(1-q)\Pr(t = n+1|2n, q)\Pr(t = 0|2, q).
\end{aligned}$$

By the hypothesis $q\Pr(t \geq n+1|2n, 1-q) < (1-q)\Pr(t \geq n+1|2n, q)$, a sufficient condition for $L < R$ is

$$\begin{aligned}
& q(\Pr(t = 0|2, q) - \frac{\Pr(t = n-1|2n, q)\Pr(t = 2|2, q)}{\Pr(t = n|2n, q)}) \\
< \ & (1-q)(\Pr(t = 2|2, q) - \frac{\Pr(t = n+1|2n, q)\Pr(t = 0|2, q)}{\Pr(t = n|2n, q)}). \qquad (7)
\end{aligned}$$

Substitute

$$\begin{aligned}
\frac{\Pr(t=n-1|2n,q)}{\Pr(t=n|2n,q)} &= \frac{n(1-q)}{(n+1)q} \\
\frac{\Pr(t=n+1|2n,q)}{\Pr(t=n|2n,q)} &= \frac{nq}{(n+1)(1-q)} \\
\Pr(t=2|2,q) &= q^2 \\
\Pr(t=0|2,q) &= (1-q)^2,
\end{aligned}$$

condition (7) becomes

$$\begin{aligned}
1-q-\frac{nq}{n+1} &< q-\frac{n(1-q)}{n+1} \\
\frac{1}{2} &< q.
\end{aligned}$$

Hence, $(1-\Pr(t=n|2n,q))q - \Pr(t \geq n+1|2n,q) < 0$. As a consequent, $n^*(\lambda, N) = N$. Q.E.D.

*Proof of Lemma 3:*

The proof is divided into three steps. In the first step we prove Lemma 7 which will be used in the next step. In the second step, we show $\pi(n_1 = 2k) > \pi(n_1 = 2k+1)$. In the third step, we show $\pi(n_1 = 1)$ is never optimal.

**Lemma 7.** *When the public belief is $\lambda$, with $\lambda > \frac{1}{2}$, probability $Pr(\Delta_{2n} \geq 2)$ increases in $n$.*

*Proof:*

$$\begin{aligned}
\Pr(\Delta_{2n} \geq 2) &= \sum_{i=n+1}^{2n} (\lambda q^i (1-q)^{2n-i} + (1-\lambda) q^{2n-i} (1-q)^i) C_{2n}^i \\
&= (1-\lambda) + (2\lambda - 1)\Pr(t \geq n+1|2n) - (1-\lambda)\Pr(t=n|2n).
\end{aligned}$$

Similarly,

$$\Pr(\Delta_{2n+2} \geq 2) = (1-\lambda) + (2\lambda - 1)\Pr(t \geq n+2|2n+2) - (1-\lambda)\Pr(t = n+1|2n+2).$$

The difference

$$\begin{aligned}\Pr(\Delta_{2n+2} \geq 2) - \Pr(\Delta_{2n} \geq 2) &= (2\lambda-1)(\Pr(t \geq n+2|2n+2) - \Pr(t \geq n+1|2n)) \\ &\quad -(1-\lambda)(\Pr(t = n+1|2n+2) - \Pr(t = n|2n)) \\ &= (2\lambda-1)\Pr(t = n|2n)q(q - \frac{n(1-q)}{n+1}) \\ &\quad +(1-\lambda)\Pr(t = n|2n)(1 - \frac{2q(1-q)(2n+1)}{n+1}).\end{aligned}$$

Since $(\frac{2q(1-q)(2n+1)}{n+1}) < 4q(1-q) < 1$, $(1-\lambda)\Pr(t = n|2n)(1 - \frac{2q(1-q)(2n+1)}{n+1}) > 0$. When $\lambda > \frac{1}{2}$, $(2\lambda-1)\Pr(t = n|2n)q(q - \frac{(1-q)n}{n+1}) > 0$. Hence $\Pr(\Delta_{2n+2} \geq 2) > \Pr(\Delta_{2n} \geq 2)$. Q.E.D.

Step 2, we show $\pi(n_1 = 2k) > \pi(n_1 = 2k+1)$.

Suppose the number of the remaining market is $N$ and the public belief is $\lambda$. Consider the monopolist launches the product in $i$ markets in the current period, with $i \leq N$. If the difference $\Delta_i \geq 2$, the posterior belief is greater than $\bar{\lambda}$; consequently, the monopolist triggers a purchase herd; if the difference $\Delta_i = 1$, the posterior belief is updated to $\lambda^H$, where $p < \lambda^H < \bar{\lambda}$, and the monopolist's equilibrium expected profit is $v(\lambda^H, N-i)$; if the difference $\Delta_i = 0$, the public belief remains $\lambda$ and the monopolist's equilibrium expected profit is $v(\lambda, N-i)$; if the difference $\Delta_i \leq -1$, the public belief is smaller than $\underline{\lambda}$; as a consequence, the monopolist triggers a rejection herd.

The monopolist's expected profit from launching the product in an even number of markets $2k$ is

$$\pi(n_1 = 2k) = 2kGp + \Pr(\Delta_{2k} \geq 2)(N-2k)p + \Pr(\Delta_{2k} = 0)v(\lambda, N-2k).$$

Its expected profit from launching the product in an odd number of markets $2k+1$ is

$$\pi(n_1 = 2k+1) = (2k+1)Gp + \Pr(\Delta_{2k+1} \geq 3)(N-2k-1)p + \Pr(\Delta_{2k+1} = 1)v(\lambda^H, N-2k-1).$$

The term $\frac{\pi(n_1=2k)-\pi(n_1=2k+1)}{\Pr(\Delta_{2k}=0)}$ is

$$\frac{\Pr(\Delta_{2k} \geq 2) - G}{\Pr(\Delta_{2k} = 0)}p + \frac{\Pr(\Delta_{2k} \geq 2)p - \Pr(\Delta_{2k+1} \geq 3)}{\Pr(\Delta_{2k} = 0)}(N-2k-1)p + v(\lambda, N-2k)$$
$$-\frac{\Pr(\Delta_{2k+1} = 1)}{\Pr(\Delta_{2k} = 0)}v(\lambda^H, N-2k-1).$$

Substituting

$$\begin{aligned}
\Pr(\Delta_{2k} \geq 2) &= (1-\lambda) + (2\lambda-1)\Pr(t \geq k+1|2k, q) - (1-\lambda)\Pr(t = k|2k, q) \\
\Pr(\Delta_{2k+1} \geq 3) &= (1-\lambda) + (2\lambda-1)\Pr(t \geq k+2|2k+1, q) - (1-\lambda)\Pr(t = k+1|2k+1, q) \\
&\quad -(1-\lambda)\Pr(t = k|2k+1, q) \\
\Pr(\Delta_{2k+1} = 1) &= C_{2k+1}^{k+1}q^k(1-q)^k G \\
\Pr(\Delta_{2k} = 0) &= C_{2k}^{k}q^k(1-q)^k,
\end{aligned}$$

$\frac{\pi(n_1=2k)-\pi(n_1=2k+1)}{\Pr(\Delta_{2k}=0)}$ becomes

$$\frac{\Pr(\Delta_{2k} \geq 2) - G}{\Pr(\Delta_{2k} = 0)}p + \frac{kG}{k+1}(N-2k-1)p + v(\lambda, N-2k) - \frac{(2k+1)G}{k+1}v(\lambda^H, N-2k-1).$$

When the public belief is $\lambda$ and the number of the remaining markets is $N$, the monopolist can choose to launch the product in one market in the current period and follow the equilibrium launching strategy in the next period. Hence, its equilibrium expected profit $v(\lambda, N-2k)$ is greater than $G + Gv(\lambda^H, N-2k-1)$. Because $\lambda \in (\frac{1}{2}, 1)$ and $q \in (\frac{1}{2}, 1)$, when the public belief is $\lambda^H$ and the number of the remaining markets is $N-2k-1$, the monopolist's expected equilibrium profit $v(\lambda^H, N-2k-1)$ is smaller than $(N-2k-1)p$. Accordingly,

$$\frac{\pi(n_1 = 2k) - \pi(n_1 = 2k+1)}{\Pr(\Delta_{2k} = 0)} > \frac{\Pr(\Delta_{2k} \geq 2) - G}{\Pr(\Delta_{2k} = 0)} + G.$$

By Lemma 7, when $\lambda > \frac{1}{2}$, $\Pr(\Delta_{2k} \geq 2)$ increases in $k$. It is easy to show $\Pr(\Delta_{2k} = 0)$ decreases in $k$. Hence $\frac{\Pr(\Delta_{2k} \geq 2) - G}{\Pr(\Delta_{2k}=0)} + G$ increases in $k$. When $k = 1$,

$$\frac{\Pr(\Delta_2 \geq 2) - G}{\Pr(\Delta_2 = 0)} + G = \frac{\lambda q^2 + (1 - \lambda(1-q)^2 - G)}{2q(1-q)} = q > \frac{1}{2}.$$

Therefore, $\frac{\Pr(\Delta_{2k} \geq 2) - G}{\Pr(\Delta_{2k}=0)} + G > 0$ and $\pi(n_1 = 2k) > \pi(n_1 = 2k+1)$.

Step 3, we prove $\pi(n_1 = 1|\lambda)$ is never optimal by showing $\pi(n_1 = 2) > \pi(n_1 = 1)$.

$$\begin{aligned}
\pi(n_1 = 2) &= 2Gp + (\lambda q^2 + (1-\lambda)(1-q)^2)(N-2)p + 2q(1-q)v(\lambda, N-2) \\
\pi(n_1 = 1|\lambda) &= Gp + Gv(\lambda^H, N-1).
\end{aligned}$$

The difference $\pi(n_1 = 2) - \pi(n_1 = 1)$ is

$$Gp + (\lambda q^2 + (1-\lambda)(1-q)^2)(N-2)p + 2q(1-q)v(\lambda, N-2) - Gv(\lambda^H, N-1).$$

Because $\lambda q^2 + (1-\lambda)(1-q)^2 = G - q(1-q)$, the difference can be rewritten as

$$Gp + (G - q(1-q))(N-2)p + 2q(1-q)v(\lambda, N-2) - Gv(\lambda^H, N-1).$$

The expected equilibrium profit $v(\lambda, N-2)$ is at least the profit from a simultaneous launch in $N-2$ markets, which is $(N-2)Gp$. When $\lambda > \frac{1}{2}$, $G > \frac{1}{2}$. Consequently, $v(\lambda, N-2) \geq \frac{N-2}{2}p$. Together with $(N-1)p > v(\lambda^H, N-1)$, $\pi(n_1 = 2) > \pi(n_1 = 1)$. Q.E.D.

*Proof of Lemma 4*

Lemma 4 is shown by induction.

1. We show when $N = 2$, $n^*(2, \lambda) = 1$. The profit from a simultaneous launch is $\pi(n_1 = 2) = 2Gp$. The profit from launching the product in one market is
$\pi(n_1 = 1) = 2Gp + (1-G)G^L p$. The difference $\pi(n_1 = 1) - \pi(n_1 = 2) = q(1-q)p > 0$.

2. We propose that $n^*(N-1, \lambda) = 1$.

3. We show $n^*(N, \lambda) = 1$.

The profit from launching the product in one market is

$$\pi(n_1 = 1) = GNp + (1-G)v(N-1, \lambda^L).$$

When $p < \lambda < \min(\bar{\lambda}, \frac{1}{2})$, the posterior belief after receiving a bad signal is $\lambda^L$, where $\underline{\lambda} < \lambda^L < \min(p, \frac{1}{2})$. According to Lemma 2, $v(N-1, \lambda^L) = (N-1)G^L p$. Therefore

$$\begin{aligned} \pi(n_1 = 1) &= GNp + (1-G)G^L(N-1)p \\ &= GNp + q(1-q)(N-1)p. \end{aligned}$$

First, we show that $n_1 = 1$ is more profitable than $n_1 = 2n + 1$, for $n = 1, 2, ..n^{max}$, with $2n + 1 \leq N$.

The profit from launching the product in $2n+1$ markets is

$$\begin{aligned}\pi(n_1 = 2n+1) &= (2n+1)Gp + \text{Prob}(\Delta_{2n+1} \geq 1)(N-2n-1)p \\ &\quad + \text{Prob}(\Delta_{2n+1} = -1)v(N-2n-1, \lambda^L) \\ &= (2n+1)Gp + \text{Prob}(\Delta_{2n+1} \geq 1)(N-2n-1)p \\ &\quad + \text{Prob}(\Delta_{2n+1} = -1)(N-2n-1)G^L p.\end{aligned}$$

The difference $\pi(n_1 = 1) - \pi(n_1 = 2n+1)$ is

$$(N-2n-1)(G - \Pr(\Delta_{2n+1} \geq 1) - \Pr(\Delta_{2n+1} = -1)G^L) + q(1-q)(N-1)p.$$

Substituting

$$\begin{aligned}\text{Prob}(\Delta_{2n+1} \geq 1) &= (1-\lambda) + \Pr(t \geq n+1|2n+1, q)(2\lambda - 1) \\ \Pr(\Delta_{2n+1} = -1) &= C^n_{2n+1} q^n (1-q)^n (1-G),\end{aligned}$$

$\pi(n_1 = 1) - \pi(n_1 = 2n+1)$ becomes

$$(N-2n-1)((1-2\lambda)(\Pr(t \geq n+1|2n+1) - q) - C^n_{2n+1} q^{n+1}(1-q)^{n+1}) + q(1-q)(N-1).$$

When $\lambda \in (0, \frac{1}{2})$ and $q \in (0.5, 1)$, $(1-2\lambda)(\Pr(t \geq n+1|2n+1) - q) > 0$. Hence

$$\begin{aligned}\pi(n_1 = 1) - \pi(n_1 = 2n+1) &> q(1-q)(N-1) - C^n_{2n+1} q^{n+1}(1-q)^{n+1}(N-2n-1) \\ &= q(1-q)(N-2n-1)(\frac{N-1}{N-1-2n} - C^n_{2n+1} q^n (1-q)^n).\end{aligned}$$

The term $C^n_{2n+1} q^n (1-q)^n)$ is equal to $\frac{2n+1}{n+1}\Pr(t = n|2n, q)$ which is less than $2\Pr(t = n|2n, q)$. Since $\Pr(t = n|2n, q)$ decreases in $n$, $\Pr(t = n|2n, q) \leq 2q(1-q)$. Accordingly, $C_{2n+1}^{n}q^n(1-q)^n < 4q(1-q) < 1$. Because $\frac{N-1}{N-1-2n} > 1$, $\pi(n_1 = 1) - \pi(n_1 = 2n+1) > 0$.

Next, we show that $n_1 = 1$ is more profitable than $n_1 = 2n$, for $n = 1, 2, ..n^{max}$, with $2n \leq N$. The profit from $n_1 = 2n$ is

$$\pi(n_1 = 2n) = 2nGp + \text{Prob}(\Delta_{2n} \geq 2)(N-2n)p + \text{Prob}(\Delta_{2n} = 0)v(\lambda, N-2n).$$

The difference $\pi(n_1 = 1) - \pi(n_1 = 2n)$ is

$$(N-2n)(G - \Pr(\Delta_{2n} \geq 2))p + q(1-q)(N-1)p - \Pr(\Delta_{2n} = 0)v(N-2n, \lambda).$$

By the hypothesis in step 2,

$$\begin{aligned} v(N-2n, \lambda) &= G(N-2n)p + (1-G)G^L(N-2n) \\ &= G(N-2n)p + q(1-q)(N-2n). \end{aligned}$$

Substituting $v(N-2n, \lambda)$

$$\begin{aligned} \pi(n_1 = 1) - \pi(n_1 = 2n) &= (N-2n)p(G(1 - \Pr(\Delta_{2n} = 0)) - \Pr(\Delta_{2n} \geq 2)) \\ &\quad + q(1-q)(N-2n)p(\frac{N-1}{N-2n} - \Pr(t = n|2n, q)). \end{aligned}$$

We have shown in the proof of Lemma 2 that $G(1 - \Pr(\Delta_{2n} = 0)) - \Pr(\Delta_{2n} \geq 2) > 0$. The ratio $\frac{N-1}{N-2n} > 1$ for $n = 1, 2, ..., n^{max}$. The probability $\Pr(t = n|2n, q)$ decreases in $n$, which implies $\Pr(t = n|2n, q) \leq 2q(1-q) < 1$. Therefore $\frac{N-1}{N-2n} - \Pr(t = n|2n, q) > 0$. Consequently, $\pi(n_1 = 1) - \pi(n_1 = 2n) > 0$. Q.E.D.

*Proof of Lemma 5:*

Consider the monopolist launches the product in $n$ markets simultaneously, with $n \leq N$. If the difference $\Delta_n \geq 1$, the posterior belief is greater than $\bar{\lambda}$. Accordingly, the monopolist triggers a purchase herd; if the difference $\Delta_n = 0$, the posterior belief remains $\lambda$ and the monopolist's expected future profit is $v(\lambda, N-n)$; if the difference $\Delta_i = -1$, the prior belief is updated to $\lambda^L$, with $\underline{\lambda} < \lambda^L < p$, and the monopolist's expected future profit is $v(\lambda^L, N-n)$; if the difference $\Delta_n < -1$, the posterior belief is less than $\underline{\lambda}$. Consequently, the monopolist triggers a rejection herd.

The monopolist's expected profit from launching the product in an odd number of markets $2k+1$ is

$$\pi(n_1 = 2k-1) = (2k-1)Gp + \Pr(\Delta_{2k-1} \geq 1)(N-2k+1)p + \Pr(\Delta_{2k-1} = -1)v(\lambda^L, N-2k+1).$$

Its expected profit from launching the profit in an even number of markets $2k$ is

$$\pi(n_1 = 2k) = 2kGp + \Pr(\Delta_{2k} \geq 2)(N-2k)p + \Pr(\Delta_{2k} = 0)v(\lambda, N-2k).$$

The term $\frac{\pi(n_1=2k-1)-\pi(n_1=2k)}{\Pr(\Delta_{2k-1}=-1)}$ is

$$\begin{aligned} &\frac{\Pr(\Delta_{2k-1} \geq 1) - G}{\Pr(\Delta_{2k-1} = -1)} + \frac{\Pr(\Delta_{2k-1} \geq 1) - \Pr(\Delta_{2k} \geq 2)}{\Pr(\Delta_{2k-1} = -1)}(N-2k) + v(\lambda^L, N-2k+1) \\ &- \frac{\Pr(\Delta_{2k} = 0)}{\Pr(\Delta_{2k-1} = -1)} v(\lambda, N-2k). \end{aligned} \tag{8}$$

Substituting

$$\begin{aligned} \Pr(\Delta_{2k} = 0) &= C_{2k}^k q^k (1-q)^k \\ \Pr(\Delta_{2k-1} = -1) &= C_{2k-1}^k (q(1-q))^{k-1}(1-G) \\ \Pr(\Delta_{2k} \geq 2) &= (1-\lambda) + (2\lambda - 1)\Pr(t \geq k+1 | 2k, q) - (1-\lambda)\Pr(t = k | 2k) \\ \Pr(\Delta_{2k-1} \geq 1) &= (1-\lambda) + \Pr(t \geq k | 2k-1, q)(2\lambda - 1), \end{aligned}$$

the term $\frac{\pi(n_1=2k-1)-\pi(n_1=2k)}{\Pr(\Delta_{2k-1}=-1)}$ becomes

$$\frac{\Pr(\Delta_{2k-1}\geq 1)-G}{\Pr(\Delta_{2k-1}=-1)}+\frac{q(1-q)}{1-G}(N-2k)+v(\lambda^L,N-2k+1)-\frac{2q(1-q)}{1-G}v(\lambda,N-2k).$$

When the public belief is $\lambda^L$ and the number of the remaining markets is $N-2k+1$, the monopolist can choose to launch the product in one market in the current period and follow the optimal strategy in the next period. Hence,

$$v(\lambda^L,N-2k+1)\geq G^Lp+G^Lv(\lambda,N-2k).$$

Because $\lambda\in(0,1)$ and $q\in(\frac{1}{2},1)$, when the public belief is $\lambda$ and the number of the remaining markets is $N-2k$. The monopolist's expected profit $v(\lambda,N-2k)$ is less than $(N-2k)p$. As a result

$$\begin{aligned}\frac{\pi(n_1=2k-1)-\pi(n_1=2k)}{\Pr(\Delta_{2k-1}=-1)} &> \frac{\Pr(\Delta_{2k-1}\geq 1)-G}{\Pr(\Delta_{2k-1}=-1)}+G^L+(G^L-\frac{q(1-q)}{1-G})v(\lambda,N-2k)\\ &= \frac{\Pr(\Delta_{2k-1}\geq 1)-G}{\Pr(\Delta_{2k-1}=-1)}+G^L.\end{aligned}$$

By Lemma 6, $\Pr(\Delta_{2k-1}\geq 1)\geq\Pr(\Delta_1\geq 1)$. Hence, $\Pr(\Delta_{2k-1}\geq 1)-G\geq 0$. Accordingly $\pi(n_1=2k-1)>\pi(n_1=2k)$. Q.E.D.

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