

# Winning by Default: Why is There So Little Competition in Government Procurement?

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September 22, 2015

## Abstract

In government procurement auctions, eligibility requirements are often imposed and, perhaps not surprisingly, contracts generally have a small number of participating bidders. To understand the effects of the restrictions of competition on the total cost of government procurement, we develop, identify, and estimate a principal-agent model in which the government selects a contractor to undertake a project. We consider three reasons why restricting entry could be beneficial to the government: by decreasing bid processing and solicitation costs, by increasing the chance of selecting a favored contractor and consequently reaping benefits from the favored contractor, and by decreasing the expected amount of price to the winning contractor. When the participation is costly and bidders are heterogeneous, the expected amount of price to the winning contractor may decrease by excluding ex-ante less efficient contractors. Using our estimates, we quantify the effects of the eligibility restrictions on the total cost of procurement.

## 1 Introduction

In recent ten years, the market for the United States federal government procurement is worth over \$460 billion annually, which constitutes about 18% of the annual federal government spending. Despite its vast size, the extent of competition for a procurement contract is not very intense. Among the procurement contracts awarded during 2004–2012, about 43% of them were awarded under either limited or no competition. Even when a contract was open to full competition, attracting only one bidder for the contract was not uncommon. In this paper, we develop, identify, and estimate a procurement model to better understand the extent of competition observed in the data.

There are two important institutional features of federal government procurement that have received relatively little attention from the literature. First, for each procurement contract, the extent and method by which the contract will be competed is chosen by contracting

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officers who are hired by the government. The regulations allow them to eliminate bidders from consideration, although full and open competition is encouraged. As a result, the number of bids is relatively small; in fact, winning a contract by default is not uncommon in federal government procurement.

Second, a sealed-bid auction is not always a dominant procedure to choose a contractor, depending on the nature of the products or services to be procured.<sup>1</sup> An alternative solicitation procedure is *competitive proposal evaluation*, through which the proposals submitted by contractors are evaluated, negotiated, and selected. After the request for proposals is posted, the qualified contractors can submit their proposals, which will be reviewed in detail to determine which proposals are within a competitive range. Discussions and negotiations may then be carried out with the contractors within the competitive range, and the contractor will be selected whose proposal is found to be most advantageous to the procuring agency. During the discussions and negotiations, the contract terms and prices are considered together, which determines the winner.<sup>2</sup>

In this paper, we construct a principal-agent framework that incorporates these features. In our model, the principal (the procurer) chooses the extent of competition, i.e., the eligibility conditions and the expected number of bidders, and offers a menu of contracts to the participating contractors (the agents) with a hidden type (cost). A contract chosen by a contractor is interpreted as a bid, which reflects the competitive proposal evaluation procedure. This procedure can be more profitable to the procurer than a sealed-bid auction when the number of bidders is very small.

The procurer may choose to decrease the extent of competition for three reasons. First, bid processing and solicitation costs may increase as the number of bidders increases. These costs include the time cost of waiting to receive more bids and the cost of reading the proposals and assessing various attributes of the contractors. Second, decreasing competition may increase the chance that a favored contractor will win the contract. If the procurer receives positive benefits from hiring a contractor favored for reasons that cannot be formally measured or verified, such as better quality or long-term relationship that could potentially be related to corruption, the procurer may want to decrease the extent of competition. Third, an unrestricted competition does not necessarily guarantee a lower expected amount of price to the winning contractor. By excluding ex-ante less efficient contractors, the procurer may incentivize the remaining bidders to bid more aggressively.

Using the federal procurement contracts awarded during 2004–2012, we nonparametrically estimate the model following our identification argument. Using the estimates, we conduct counter-factual analyses to decompose the effects of the three sources of entry restrictions. We also quantify the effects of the eligibility restrictions on the total cost of procurement.

The rest of the paper is organized as follows. We describe the model in Section 2, and then introduce our data and present descriptive analyses in Section 3. The identification of the model follows in Section 4, which we closely follow to nonparametrically estimate our

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<sup>1</sup>According to the Federal Acquisition Regulation, a sealed-bid auction is not considered appropriate to use if the award will be made on the basis of factors other than price; if it is necessary to conduct discussions with the responding contractors about their bids; or if there is not a reasonable expectation of receiving more than one sealed bid.

<sup>2</sup>Contracting officers typically decide on the contract type prior to issuing a solicitation. However, particularly in negotiated procurements, selection of the contract terms can be a matter for negotiation between the procuring activity and the contractor. (48 C.F.R. 16.103(a)).

model. We show the estimation results in Section 5. Section 6 concludes.

## 1.1 Related Literature

Our paper is related to the large literature on procurement and auctions. One strand of the literature explains why less competition does not necessarily lower the payoff of the auctioneer in independent private value auctions. Li and Zheng (2009) show that when the number of bidders is endogenously determined, the equilibrium bidding behavior can become less aggressive as the number of potential bidders increases. Krasnokutskaya and Seim (2011) study a bid preference program, and Athey et al. (2013) compare a set-asides program and the bid subsidy program. Both papers show the importance of allowing endogenous entry when assessing restrictive competition policies.

An important contribution of our paper is that we build and estimate a model where the procurer is assumed to optimally choose the extent of competition, considering favoritism and bid processing and solicitation costs. Using the estimates, we quantify the effects of favoritism and bid processing and solicitation costs on the procurement outcomes. In this regard, Bandiera et al. (2009) and Coviello et al. (2014) are closely related to our paper. Bandiera et al. (2009) develop a formal framework for distinguishing active waste and passive waste in the total government cost of procurement, and separately estimate them exploiting a policy experiment in Italy's public procurement system. Active waste entails utility for the public decision makers, part of which is related to favoritism in our paper, while passive waste does not, such as bid processing and solicitation costs. Coviello et al. (2014) study government discretion on public goods provision in terms of whether or not to impose entry restrictions, and document the casual effect of increasing such discretion on procurement outcomes using a database for public procurement in Italy.

Another strand of the literature studies nonstandard contractor selection procedures, such as scoring auctions (Asker and Cantillon (2010)) or multi-attribute auctions (Krasnokutskaya et al. (2013)), where the price is not the only factor in selecting a contractor. We consider an optimal direct revelation mechanism in a competitive environment, studied by Laffont and Tirole (1987), McAfee and McMillan (1987), and Riordan and Sappington (1987). We extend their models by allowing the procurer to choose the optimal extent of competition.

Lewis and Bajari (2014) and Bajari et al. (2014) are related to our paper in that they study the price adjustments after the winning contractor is chosen and the project initiates. We distinguish the price adjustments into two categories: those incurring due to the cost changes related to the unknown type of the contractors and the rest. Lewis and Bajari (2014) study the former type of the price adjustments, and Bajari et al. (2014) study the latter. We consider both types of the price adjustment, and this is possible because we observe the reasons for contract modifications. Furthermore, although they take the contract type as exogenously given, we allow that the contract type and the winning contractor are endogenously determined.

Our paper also belongs to a literature on the identification of a principal-contractor model, for example, Perrigne and Vuong (2011). In their paper, an implicit identifying assumption is that the optimal contracts are linear in costs. Their theoretical model provides the payoffs of the principal and the contractor, but it does not provide a particular contractual form that an optimal contract must follow. In our identification result, we do not impose any functional form assumptions on the optimal contracts.



Upon a realization of a project, the procurer determines the extent of competition, i.e., whether to exclude non-favored contractors from competition and how many bids to receive on average. The realized number of bidders follows a shifted Poisson distribution with the average arrival rate of choice,  $\lambda$ , with support  $1, 2, \dots, \infty$ . Because we do not observe contracts that received no bids in the data, we assume that the risk of drawing no bids is zero.

When determining the extent of competition, the procurer minimizes the total expected cost of procurement, consisting of (i) the expected transfer to the winning contractor, (ii) the expected bid processing cost, and (iii) the cost of holding an exclusive competition as opposed to an open competition. Here, we consider per-bidder bid processing cost, denoted by  $\kappa_e > 0$  for an exclusive competition and  $\kappa_o > 0$  for a non-exclusive competition, which include the cost of reading the proposals, making sure that the language and terms of the proposals are unambiguous, and assessing various attributes of the contractors, as well as the time cost of delaying the initiation of the project. Furthermore, the last component of the cost, denoted by  $\eta$ , represents the risk of bid protest or the administrative or political burdens to justify exclusion of sources.

Given the realized number of bidders by type, the procurer announces a menu of contracts, and the participating contractors simultaneously choose an item from the menu if the expected rent from doing so is nonnegative. When submitting their contract, the bidders do not know other bidders' efficiency and have the same belief on the distribution as the procurer's. However, they know whether or not each of other bidders is favored by the procurer. Given the submitted contracts, the procurer selects a contractor, who undertakes the project.

A typical contract in the menu has two components, a base price,  $p$ , and a schedule of ex-post price adjustments,  $r(\cdot, \cdot)$ , which are contingent on the realization of the signal and the cost shock  $\epsilon$ . Given the realized value of  $s$  and  $\epsilon$ , the payoff to an inefficient contractor under the contract is

$$p - (c + \beta) + \psi(r(s, \epsilon) - \epsilon),$$

where  $\psi(\cdot)$  is a continuous function, with  $\psi(0) = 0$ ,  $\psi'(0) = 1$ ,  $\psi' > 0$ , and  $\psi'' < 0$ . Note that  $p - (c + \beta)$  is fixed while  $r(s, \epsilon) - \epsilon$  is variable. Due to liquidity concerns and potential adjustment costs, the variable part of the payoff is discounted, which is represented by  $\psi(\cdot)$ .

Note that the procurer does not have liquidity concerns nor bear adjustment costs. Furthermore, the cost shock is independent of the efficiency of a contractor. Therefore, the procurer minimizes her expected price by fully insuring the contractors against the cost shock. Therefore, we focus on the schedule  $r$  such that

$$r(s, \epsilon) = q(s) + \epsilon,$$

and solve for  $q(\cdot)$ . We assume that there exists a maximal penalty that the government can legally impose on contractors,  $M < 0$ . Therefore, the procurer is constrained by

$$q(s) \geq M,$$

for any signal  $s$ . In the following, we first characterize the optimal selection mechanism which induces a truth-telling Bayesian Nash equilibrium, given the number of bidders by favor type. Then we solve the optimal extent of competition to minimize the expected total cost of procurement. The proofs for theorems and corollaries are in Appendix.

## 2.2 Optimal Selection Mechanism

**Symmetric Bidders** Suppose  $n \geq 1$  bidders of the same favor type participate, and the probability that a bidder is efficient is  $\pi \in (0, 1)$ . If the signal is uninformative, i.e.,  $\underline{F}(s) = \overline{F}(s)$  for any  $s$  in its support, then we show that offering two fixed-price contracts, where the cheaper contract is associated with a higher chance of winning and the more expensive one is associated with a lower chance of winning.

**Theorem 2.1** *Suppose  $\underline{F}(s) = \overline{F}(s)$  for any signal  $s$  in the support. The optimal menu is to offer bidders a choice from two fixed price contracts, first priority going to contractors submitting a price of:*

$$c + \frac{\pi(1-\pi)^{n-1}}{1-(1-\pi)^n}\beta$$

and second priority to those who submit  $c + \beta$ .

Note that when  $n = 1$  this menu degenerates to a single fixed price contract of  $c + \beta$ .

We show that if signals are informative, it is always optimal to offer a menu of two contracts, one fixed-price contract,  $\underline{p}$ , and one variable-price contract,  $(\overline{p}, q(\cdot))$ . Given this menu, contractors will reveal their efficiency type by their contract choices; an efficient contractor will choose the fixed-price contract while an inefficient one will choose the variable-price contract.

Given this separation, procurer will select a participant that accepts the fixed-price contract, if there's one. Otherwise, a random participant will be selected. Therefore, the probability that the fixed-price contract is selected is  $1 - (1 - \pi)^n$ . The resulting expected transfer to a winning contractor is:

$$[1 - (1 - \pi)^n] \underline{p} + (1 - \pi)^n \left[ \overline{p} + \int q(s) \overline{f}(s) ds \right].$$

The procurer chooses  $\underline{p}$  and  $\{\overline{p}, q(\cdot)\}$  to minimize the amount of the above expected transfer. Given the selection rule, a bidder believes that the probability of winning if he chooses  $\underline{p}$ , the fixed-price contract in the menu is:

$$\underline{\phi}(n) = \sum_{k=0}^{n-1} \binom{n-1}{k} \frac{\pi^k (1-\pi)^{n-1-k}}{k+1} = \frac{1 - (1-\pi)^n}{n\pi}, \quad (1)$$

assuming that other participating contractors follow the equilibrium strategy. If the contractor chooses the variable-price contract instead, the winning probability,  $\overline{\phi}(n)$ , becomes:

$$\overline{\phi}(n) = \frac{(1-\pi)^{n-1}}{n}. \quad (2)$$

An implementable menu of contracts,  $\underline{p}$  and  $\{\overline{p}, q(\cdot)\}$ , satisfies the individual rationality and incentive constraints for each efficiency type. For the inefficient, these constraints are

$$\overline{p} + \int \psi[q(s)] \overline{f}(s) ds \geq c + \beta, \quad (3)$$

$$\bar{\phi}(n) \left\{ \bar{p} + \int \psi[q(s)] \bar{f}(s) ds \right\} \geq \underline{\phi}(n) \{ \underline{p} - c - \beta \}, \quad (4)$$

where  $\underline{\phi}(n)$  and  $\bar{\phi}(n)$  denote the winning probabilities conditional on the choice of contract,  $\underline{p}$  or the  $\{\bar{p}, q(\cdot)\}$ , as defined in equations (1) and (2), respectively. Similarly, the constraints for the efficient are

$$\underline{p} \geq c, \quad (5)$$

$$\underline{\phi}(n) \{ \underline{p} - c \} \geq \bar{\phi}(n) \{ \bar{p} + \int \psi[q(s)] \underline{f}(s) ds - c \}. \quad (6)$$

We show that the participation constraint for the inefficient, or (3), and the incentive compatibility constraint for the efficient, (6), hold in equality at the optimum. An optimal menu of contracts satisfy the incentive compatibility constraint for the inefficient, or (4), with strict inequality. The participation constraint for the efficient, or (5), however, may or may not be binding at the optimum. We show in Appendix that the participation constraint for the efficient binds if and only if  $\pi$  is greater than or equal to a certain threshold,  $\bar{\pi} \in (0, 1)$ , which is the unique root of the following expression:

$$\tilde{H}(\pi) = \beta - \int_{l(s) < \tilde{l}(\pi, M)} \psi \left( h \left[ \frac{1 - \pi}{1 - \pi l(s)} \right] \right) [1 - l(s)] \bar{f}(s) ds + \psi(M) \int_{l(s) \geq \tilde{l}(\pi, M)} [1 - l(s)] \bar{f}(s) ds,$$

where  $l(s)$  is the likelihood ratio at signal  $s$ , i.e.,  $\underline{f}(s) / \bar{f}(s)$ ,  $h(\cdot)$  is the inverse function of  $\psi'(\cdot)$  defined by  $h[\psi'(q)] \equiv q$ , and the cutoff  $\tilde{l}(\pi, M)$  is defined as:

$$\tilde{l}(\pi, M) \equiv \frac{1}{\pi} - \frac{1 - \pi}{\pi \psi'(M)}.$$

These help us characterize the optimal menu of contracts in the following theorem.

**Theorem 2.2** *Suppose there are  $n$  symmetric bidders for a project. Assume that signals are informative, i.e.,  $\underline{F}(s) \neq \bar{F}(s)$  for some  $s$  in its support. Then, there are two contracts in the optimal menu: one fixed-price contract, denoted by  $\underline{p}(\pi, n)$ , and one variable-price contract, denoted by  $\{\bar{p}(\pi), q(\cdot, \pi)\}$ . The base price of the variable-price contract is:*

$$\bar{p}(\pi) = c + \beta - \int \psi(q(s, \pi)) \bar{f}(s) ds. \quad (7)$$

If  $\pi < \bar{\pi}$ , then  $q(\cdot, \pi)$  is:

$$q(s, \pi) = \begin{cases} h \left[ \frac{1 - \pi}{1 - \pi l(s)} \right] & \text{if } l(s) < \tilde{l}(\pi, M), \\ M & \text{otherwise.} \end{cases} \quad (8)$$

The price of the fixed-price contract is:

$$\underline{p}(\pi, n) = c + \frac{\pi (1 - \pi)^{n-1}}{1 - (1 - \pi)^n} \left[ \beta - \int \psi(q(s, \pi)) [\bar{f}(s) - \underline{f}(s)] ds \right]. \quad (9)$$

Note that  $\underline{p}(\pi, n) = c$  for any  $\pi \geq \bar{\pi}$ . Given the characterization of the menu of contracts, the following corollary summarizes the relationship between the optimal contracts and the

number of bidders.

**Corollary 2.1** *The expected transfer to a winning contractor decreases as the number of bidders increases.*

It can be seen that the expected transfer to a winning contractor decreases as the number of bidders increases through the following two channels. One channel is that the probability of hiring an efficient contractor,  $1 - (1 - \pi)^n$ , increases as the number of bidders increases. Because hiring an efficient contractor is cheaper than hiring an inefficient contractor, the more likely an efficient contractor wins, the lower the expected price becomes. The other channel is through competition: the rent that an efficient contractor receives decreases as more contractors participate.

The following comparative statics of the optimal menu of contracts with respect to  $\pi$  is useful for the identification of the model, which is discussed in Section 4.

**Corollary 2.2** *The volatility of the ex-post adjustment schedule of the variable-price contract is non-decreasing in  $\pi$  for any signal  $s$ :*

$$\frac{\partial |q(s, \pi)|}{\partial \pi} \begin{cases} > 0 & \text{if } l(s) < \tilde{l}(\pi, M) \text{ and } \pi < \bar{\pi}, \\ = 0 & \text{otherwise.} \end{cases}$$

Furthermore, the price of the fixed-price contract,  $\underline{p}(\pi, n)$ , is non-increasing in  $\pi$ :

$$\frac{\partial \underline{p}(\pi, n)}{\partial \pi} \begin{cases} > 0 & \text{if } \pi < \bar{\pi}, \\ = 0 & \text{otherwise.} \end{cases}$$

**Asymmetric Bidders** Suppose  $n_f$  favored contractors and  $n_u$  non-favored ones participate. We assume that the procurer is not allowed to choose a favored, inefficient contractor if a non-favored, efficient contractor participates.<sup>3</sup> Therefore, her selection rule is lexicographic: the first criterion is efficiency and the second criterion is favoritism. As a result, if at least one favored contractor participates, inefficient and non-favored contractors never win a project.

This leads to a stark result that non-favored contractors will receive a take-it-or-leave-it offer of the fixed-price contract of  $c$ . On the other hand, favored contractors will be offered a menu of two contracts, which induces the separation of efficiency types as a Bayesian Nash equilibrium. Given the lexicographic selection rule and the revealed efficiency types of the favored contractors, the probability that a favored contractor is hired, denoted by  $w(\pi_f, \pi_u, n, n_f)$ , is:

$$w(\pi_f, \pi_u, n, n_f) = 1 - (1 - \pi_f)^{n_f} [1 - (1 - \pi_u)^{n - n_f}]. \quad (10)$$

The following theorem characterizes the menus of optimal contracts when both favor types participate.

**Theorem 2.3** *Suppose there are  $n_f \geq 1$  favored bidders and  $n_u \geq 1$  unfavored ones. To the unfavored contractors, a fixed-price contract,  $c$ , is offered. To the favored contractors, a menu*

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<sup>3</sup>This assumption limits the extent to which the favoritism negatively affects efficiency. Alternatively, Burguet and Che (2004) and Celentani and Ganuza (2002) allow the procurer to manipulate the quality assessment in exchange for a bribe.

of two contracts is offered. If the signals are not informative, there will be two fixed-price contracts as in Theorem 2.1. The cheaper fixed-price contract is

$$c + \frac{\pi_f (1 - \pi_f)^{n_f - 1}}{1 - (1 - \pi_f)^{n_f}} \beta$$

and the other is  $c + \beta$ . On the other hand, if the signals are informative as defined in Theorem 2.2, the menu will consist of one fixed-price contract and one variable-price contract. The latter contract consists of the base price,  $\bar{p}(\pi_f)$ , and the ex-post payment adjustment,  $q(\cdot, \pi_f)$ , both of which are defined in Theorem 2.2. The fixed-price contract for the favored, denoted by  $\underline{p}_f(\pi_f, \pi_u, n, n_f)$  is:

$$\underline{p}_f(\pi_f, \pi_u, n, n_f) = c + \frac{\pi_f (1 - \pi_f)^{n_f - 1} (1 - \pi_u)^{n - n_f}}{1 - (1 - \pi_f)^{n_f}} \left[ \beta - \int \psi[q(s, \pi_f)] \{ \bar{f}(s) - \underline{f}(s) \} ds \right]. \quad (11)$$

### 2.3 Optimal Extent of Competition

The procurer chooses (i) whether to allow non-favored contractors to participate and (ii) the rate at which bidders arrive. The optimal bid arrival rate if favored contractors are allowed only,  $\lambda_e$ , minimizes the expected total procurement cost.

$$V_e(\lambda, \pi_f) = \sum_{n=1}^{\infty} \frac{(\lambda - 1)^{n-1} e^{-\lambda+1}}{(n-1)!} T_e(\pi_f, n) + \kappa_e \lambda + \eta, \quad (12)$$

where  $T_e(\pi_f, n)$  denotes the expected transfer to a winning bidder when  $n$  favored bidders participate:

$$T_e(\pi_f, n) = [1 - (1 - \pi_f)^n] \underline{p}_f(\pi_f, n) + (1 - \pi_f)^n \left[ \bar{p}(\pi_f) + \int q(s, \pi_f) \bar{f}(s) ds \right]. \quad (13)$$

As  $\lambda$  increases, the expected transfer to a winning contractor decreases as shown in Corollary 1 while the total bid processing cost increases. Balancing this trade-off guarantees one unique  $\lambda_e$  that minimize the expected total procurement cost under eligibility restrictions based on the favor type.

Without such eligibility restrictions, both favored and non-favored contractors are allowed to participate. Consider the expected transfer to a winning bidder if there are  $n$  bidders,  $n_f \geq 1$  of whom are favored. Let us denote it by  $T_o(\pi_f, \pi_u, n, n_f)$ . Note that when  $n = 1$ ,  $T_o(\pi_f, \pi_u, 1, 1) = T_e(\pi_f, 1)$  and  $T_o(\pi_f, \pi_u, 1, 0) = T_e(\pi_u, 1)$ . When  $n > 1$ ,  $T_o(\pi_f, \pi_u, n, n_f)$  can be written as follows:

$$\begin{aligned} T_o(\pi_f, \pi_u, n, n_f) &= [1 - (1 - \pi_f)^n] \underline{p}_f(\pi_f, \pi_u, n, n_f) + (1 - \pi_f)^{n_f} [1 - (1 - \pi_u)^{n - n_f}] c \\ &\quad + (1 - \pi_f)^{n_f} (1 - \pi_u)^{n - n_f} \left[ \bar{p}(\pi_f) + \int q(s, \pi_f) \bar{f}(s) ds \right]. \end{aligned} \quad (14)$$

The expected total procurement cost without eligibility restrictions includes the expected

transfer as well as other administrative costs:

$$V_o(\lambda, \pi_f, \pi_u) = \sum_{n=1}^{\infty} \sum_{n_f=0}^n \Pr(n_f, n; \lambda) T_o(\pi_f, \pi_u, n, n_f) + \kappa_o \lambda, \quad (15)$$

where  $\Pr(n_f, n; \lambda)$  denotes the probability that  $n$  contractors participate and  $n_f$  of whom are favored:

$$\Pr(n_f, n; \lambda) = \frac{(\lambda - 1)^{n-1} e^{-\lambda+1}}{(n-1)!} \binom{n}{n_f} \rho^{n_f} (1 - \rho)^{n-n_f}. \quad (16)$$

In sum, the procurer chooses to impose eligibility restrictions to prevent non-favored contractors from participating if and only if the expected total procurement cost is lower with the restrictions than without them. Corollary 2 implies that it can be optimal to impose eligibility restrictions if the additional cost of doing so,  $\eta$ , is small enough.

**Corollary 2.3** *Consider two procurement auctions with  $n > 1$  bidders. The bidders in one auction are all favored, i.e.,  $n = n_f$ , while the other auction has both favored and non-favored bidders. If  $\pi_u \geq \pi_f$ , the expected transfer to a winning contractor is always lower in the auction with asymmetric bidders.*

If non-favored contractors are on average more efficient than favored ones, i.e.,  $\pi_u \geq \pi_f$ , then holding a non-exclusive auction lowers the expected transfer. In this case, a large  $\eta$  rationalizes the choice of having an exclusive auction. In the opposite scenario, on the other hand, allowing non-favored contractors to participate will increase the expected transfer in two ways: (i) by increasing the chance of employing an inefficient contractor, i.e.,  $(1 - \pi_f)^{n_f} (1 - \pi_u)^{n-n_f} > (1 - \pi_f)^n$ ; and (ii) by increasing the informational rent of the favored, efficient contractors. These two sources of benefits may outweigh the price reduction from non-favored, efficient contractors when holding a non-exclusive auction. Therefore, even without favoritism, the procurer may prefer an exclusive competition to the alternative.

### 3 Identification

Each contract may differ by unobserved type,  $\pi \in [\pi_L, \pi_U] \subset (0, 1)$ , which is drawn from  $F_\pi(\cdot)$ . The unobserved type affects the probability that a contractor is efficient,  $\pi_f$  and  $\pi_u$ :

$$\pi_f(\pi) = \pi,$$

$$\pi_u(\pi) = 1 - \zeta(1 - \pi).$$

We also allow the bid processing costs ( $\kappa_e$  and  $\kappa_o$ ) vary with  $\pi$ . The remaining components of the model, on the other hand, are assumed not to vary with  $\pi$ .

The model is deterministic in the prediction of whether to exclude non-favored bidders given  $\pi$ . To allow that for any  $\pi$  the probability that eligibility restrictions are imposed is not always degenerate, we assume that the administrative cost of imposing eligibility restrictions,  $\eta$ , is stochastic, drawn from  $F_\eta(\cdot)$ . Thus, the model primitives can be summarized as  $[F_\pi(\cdot), F_\eta(\cdot), \bar{F}(\cdot), \underline{F}(\cdot), \kappa_e(\cdot), \kappa_o(\cdot), \psi(\cdot), c, \beta, \rho, \zeta, \delta]$ .

Identification investigates whether the model primitives can be uniquely recovered from the observed data generating process. Let us introduce two variables for future references:

$d$  indicating that the contract is variable-price and  $e$  indicating that only the favored are allowed to participate. Suppose we observe (i) the probability that a contract is exclusively competed,  $\Pr(e = 1)$ , (ii) the probability that a contract is variable-price,  $\Pr(d = 1|e)$ , (iii) the distribution of the number of bids,  $\Pr(n|e, d)$  for any  $n \geq 1$ , (iv) the distribution of signals,  $G_s(\cdot|e, d)$ , (v) the distribution of the price of fixed-price contracts,  $G_p(\cdot|e)$ , (vi) the distribution of the base price of variable-price contracts,  $G_{\bar{p}}(\cdot|e)$ , and (vii) the distribution of the ex-post price adjustment of variable-price contracts conditional on signal:  $G_q(\cdot|s, e)$ . Notice that the identity of the bidders that are favored by the procurer is not observed.

We make the following four assumptions for identification.

**Assumption 3.1** *The signals are independent of  $\pi$  and  $\eta$  conditional on the efficiency type.*

**Assumption 3.2** *For all procurement auctions, at least one favored contractor participates.*

**Assumption 3.3** *For all  $s$  in the support,  $l(s) < \tilde{l}(\bar{\pi}, M)$ .*

**Assumption 3.4**  *$\eta$  is independent of  $\pi$ .*

If we observe variable-price contracts in the data, then it implies that the signals are informative and that the efficient type chooses a fixed-price contract and the inefficient type chooses a variable-price contract. In other words, the observed type of a contract informs us of the efficiency type of the contractor. Therefore, we identify  $\underline{F}(\cdot)$  from the observed distribution of the signal for fixed-price contracts and  $\overline{F}(\cdot)$  from that for variable-price contracts.

**Proposition 3.1** *Under Assumption 3.1, we identify  $\underline{F}(\cdot)$  and  $\overline{F}(\cdot)$  by*

$$\underline{F}(s) = G_s(s|d = 0),$$

$$\overline{F}(s) = G_s(s|d = 1).$$

Identification of the remaining components of the model proceeds as follows. We first focus on the exclusively competed contracts. We recover the unobserved type by exploiting Corollary 2.2. Given the recovered unobserved type, we identify  $\psi(\cdot)$  off variable-price contracts. Then we identify the cost parameters,  $c$  and  $\beta$ , from the base prices. Then we recover  $\pi$  for variable-price, non-exclusively competed contracts. By exploiting the variation in the number of bidders, we identify  $\rho$  and  $\zeta$  from the average base price of non-exclusively competed contracts. Using the identified parameters and the observed bidder arrival rate given  $\pi$ , we identify  $\kappa_e(\pi)$  and  $\kappa_o(\pi)$ . Lastly, we identify the distribution of  $\eta$  from the observed probability that a variable-price contract is exclusively competed. We discuss each of the identification argument below in order.

### 3.1 Recovering Unobserved Types for Exclusively Competed Contracts

To recover the unobserved types for exclusively competed contracts, we first identify the following three objects directly from the data. First, we identify the cumulative density function of the price of fixed-price contracts with  $n$  bidders for any  $n$ , and denote it by  $G_p(\cdot|n)$ . Second, we identify the cumulative density function of the absolute value of the ex-post price adjustment of variable-price contracts with  $n$  bidders as a response to signal  $s$

for any  $n$  and  $s$ , and denote it by  $G_q(\cdot|s, n)$ . Both  $G_p(\cdot|n)$  and  $G_q(\cdot|s, n)$  are clearly identified and can be estimated using standard nonparametric methods. Third, we identify  $\varphi_n$ , defined as the quotient of the probability that an exclusively competed contract with  $n$  bidders is fixed-price and the probability that it is variable-price. This is identified and has a sample analogue.

Consider the following cutoff,  $\tilde{\pi}$ :

$$\tilde{\pi} = \sup\{\pi \in (\pi_L, \min\{\bar{\pi}, \pi_U\}) : l(s) < \tilde{l}(\pi, M), \text{ for all } s\}, \quad (17)$$

and  $\bar{\pi}$  is defined in Theorem 2.2. We show that we identify the distribution of  $\pi$  for exclusive contracts with  $n$  bidders truncated above this cutoff.

**Proposition 3.2** *Consider the  $k^{\text{th}}$  percentile type for exclusive contracts with  $n$  bidders, denoted by  $\pi_{n,k}$ , which solves  $F_\pi(\pi_{n,k}|e = 1, n) = k$ , for any  $n$  and  $k$  such that  $\pi_{n,k} < \tilde{\pi}$  where  $\tilde{\pi}$  is defined in equation (17). Under Assumption 3.1,  $\pi_{n,k}$  is identified by:*

$$\pi_{n,k} = 1 - \left[ 1 - \frac{\varphi_n g_{\underline{p}}(\underline{p}_{n,k} | n)}{\varphi_n g_{\underline{p}}(\underline{p}_{n,k} | n) + \int g_q(q_{s,n,k} | s, n) \frac{dq_{s,n,k}}{d\underline{p}_{n,k}} \bar{f}(s) ds} \right]^{\frac{1}{n}},$$

where  $\underline{p}_{n,k}$  solves  $k = 1 - G_{\underline{p}}(\underline{p}_{n,k} | n)$  and  $q_{s,n,k}$  solves  $k = G_q(q_{s,n,k} | s, n)$ . Hence, we identify the distribution of  $\pi$  truncated above  $\tilde{\pi}$  for exclusively competed contracts with  $n$  bidders for any  $n$ , denoted by  $G_\pi^*(\cdot|e = 1, n)$ .

The Appendix provides a detailed proof. The key insight is that by Corollary 2.2, for any  $\pi_{n,k} < \tilde{\pi}$ , the following equations hold:

$$F_\pi(\pi_{n,k}|e = 1, d = 0, n) = 1 - G_{\underline{p}}(\underline{p}_{n,k} | n),$$

$$F_\pi(\pi_{n,k}|e = 1, d = 1, n) = \int G_q(q_{s,n,k} | n) \bar{f}(s) ds.$$

The above one-to-one mappings between the observables and the unobserved type, coupled with the independence between the signal distribution and the unobserved type distribution, identify the distribution of  $\pi_k$  for exclusive contracts conditional on the number of bids.

### 3.2 Liquidity Cost Function and Cost Parameters

Exploiting these one-to-one mappings, we recover  $\pi$  for each exclusively competed contract if  $\pi < \tilde{\pi}$ . Given this, we identify the ex-post price adjustment schedule,  $q(\cdot, \pi)$ , and the base prices,  $\underline{p}(\pi, n)$  and  $\bar{p}(\pi)$ , using standard nonparametric methods. Using equation (8) in Theorem 2.2, we identify  $\psi'(\cdot)$ , and with the assumption of  $\psi(0) = 0$ ,  $\psi(\cdot)$  is identified. Then we identify  $c$  and  $\beta$  off the base prices of the fixed-price and the variable-price contracts, using equations (7) and (9) in Theorem 2.2.

**Proposition 3.3** *Under Assumption 3.1, we identify  $q(\cdot, \pi)$ ,  $\underline{p}(\pi, n)$ , and  $\bar{p}(\pi)$  for any  $n$*

and  $\pi < \tilde{\pi}$ . Given  $q(\cdot, \cdot)$ , we identify  $\psi'(\cdot)$  by

$$\psi'[q(s, \pi)] = \frac{1 - \pi}{1 - \pi \underline{f}(s) / \bar{f}(s)},$$

for the domain of  $(M, q_U)$ , where  $q_U \equiv \sup_s q(s, \tilde{\pi})$ . The assumption of  $\psi(0) = 0$  identifies the constant of the integration of  $\psi'(\cdot)$  for the domain of  $(M, q_U)$ . The cost parameters are identified by

$$\beta = \int \psi[q(s, \pi)] d\bar{F} + \frac{[1 - (1 - \pi)^n] [\bar{p}(\pi) - \underline{p}(\pi, n)] + \pi (1 - \pi)^{n-1} \int \psi[q(s, \pi)] d\bar{F}}{1 - (1 - \pi)^{n-1}}, \quad (18)$$

for any  $n$ , and

$$c = \bar{p}(\pi) + \int \psi[q(s, \pi)] \bar{f}(s) ds - \beta. \quad (19)$$

Note that  $c$  and  $\beta$  are over-identified in the sense that the right hand side of equations (18) and (19) depends on  $\pi$ .

### 3.3 Ratio of Favored Contractors and Relative Likelihood that a Non-favored Contractor is Inefficient

Under Assumption 3.2, all variable-price contracts from non-exclusively competed auctions are designed for the favored contractors. This allows us to identify the distribution of  $\pi$  for variable-price, non-exclusively competed contracts with  $n$  bidders. For a given variable-price contract in the data with a realized signal  $s$  and the corresponding  $q$ , the unobserved type  $\pi$  can be solved, given the characterization of  $q(\cdot, \pi)$  in equation (8), as follows.

$$\pi = \frac{\psi'(q) \underline{f}(s) / \bar{f}(s) - 1}{\psi'(q)},$$

if  $l(s) < 1/\tilde{\pi}$ . Hence we have the following lemma.

**Lemma 3.1** *Under Assumptions 3.1 and 3.2, we identify the the cumulative density function of  $\pi$  truncated above  $\tilde{\pi}$  for variable-price, non-exclusively competed contracts with  $n$  bidders for any  $n$ . Let us denote this identified density by  $G_\pi^*(\cdot | e = 0, d = 1, n)$ .*

Next we identify  $G_\pi^*(\cdot | e = 0, n)$  as a function of  $\rho$  and  $\zeta$ . Notice that the following equality holds for any  $n$  and  $\pi$ :

$$\begin{aligned} \Pr(d = 1, \pi | e = 0, n) &= f(\pi | d = 1, e = 0, n) \Pr(d = 1 | e = 0, n) \\ &= f(\pi | e = 0, n) \Pr(d = 1 | e = 0, n, \pi). \end{aligned}$$

Hence the the probability density function of  $\pi$  truncated above  $\tilde{\pi}$  for non-exclusively competed contracts with  $n$  bidders for any  $n$ , denoted by  $g_\pi^*(\cdot | e = 0, n, \rho, \zeta)$  can be written as

$$g_\pi^*(\pi | e = 0, n, \rho, \zeta) = \frac{g_\pi^*(\pi | e = 0, d = 1, n) \Pr(d = 1 | e = 0, n)}{\sum_{n_f=1}^n (1 - \pi)^n \zeta^{n-n_f} \binom{n-1}{n_f-1} \rho^{n_f-1} (1 - \rho)^{n-n_f}}. \quad (20)$$

We have identified  $g_\pi^*(\cdot|d = 1, e = 0, n)$  in the previous lemma, and  $\Pr(d = 1|e = 0, n)$  is identified directly from the data. Given the model prediction,  $\Pr(d = 1|e = 0, n, \pi)$  can be written as a function of  $\rho$ ,  $\zeta$ , and  $\pi$ . This allows us to identify  $g_\pi^*(\pi|e = 0, n, \rho, \zeta)$  for  $\pi < \tilde{\pi}$  as a function of  $\rho$  and  $\zeta$ , and accordingly  $G_\pi^*(\cdot|e = 0, n)$ .

Now, to identify  $\rho$  and  $\zeta$ , we exploit the variation in the number of bidders as well as the model prediction on the base prices of the fixed-price, non-exclusively competed contracts: either  $c$  or  $\underline{p}_f(\pi_f, \pi_u, n, n_f)$  as defined in equation (11). With a slight abuse of notations, let us denote the expected value of the base price for type  $\pi$  fixed-price contracts that are competed without exclusion among  $n$  bidders when  $n_f$  of them are favored by  $\mathbb{E}(p|e = 0, \pi, n, n_f, \zeta)$ .

$$\mathbb{E}(p|e = 0, n, n_f, \pi, \zeta) = c + \pi(1 - \pi)^{n-1}\zeta^{n-n_f} \left[ \beta - \int \psi(q(s; \pi))[\bar{f}(s) - \underline{f}(s)]ds \right].$$

In the data, we observe the expected value of the base price for fixed-price, non-exclusively competed contracts with  $n$  bidders,  $\mathbb{E}(p|e = 0, n)$ . Given that we have identified  $G_\pi^*(\cdot|e = 0, n, \rho, \zeta)$  and the stochastic process for  $n_f$  is a function of  $\rho$ , we identify  $\rho$  and  $\zeta$ .

**Proposition 3.4** *Suppose assumptions 3.1, 3.2, and 3.3 hold.  $\rho$  and  $\zeta$  uniquely solve the following system of  $N \geq 2$  equations with  $n = 2, 3, \dots, N$ :*

$$\begin{aligned} \mathbb{E}(p|e = 0, n) &= \sum_{n_f=1}^n P_f(\rho, n, n_f) \left[ \int_{\pi \leq \bar{\pi}} \mathbb{E}(p|e = 0, n, n_f, \pi, \zeta) g_\pi^*(\pi|e = 0, n, \rho, \zeta) d\pi \right. \\ &\quad \left. + \int_{\pi > \bar{\pi}} \mathbb{E}(p|e = 0, n, n_f, \bar{\pi}, \zeta) [1 - G_\pi^*(\bar{\pi}|e = 0, n, \rho, \zeta)] d\pi \right], \end{aligned}$$

where  $P_f(\rho, n, n_f) \equiv \binom{n-1}{n_f-1} \rho^{n_f-1} (1 - \rho)^{n-n_f}$ . Hence, we identify  $G_\pi^*(\pi|e = 0, n)$  for any  $n$  and  $\pi < \bar{\pi}$ .

### 3.4 Per-bidder Bid Processing Cost

Having identified the primitives of the model related to the contracts, we identify  $\kappa_e(\cdot)$  off the observed distribution of number of bids for exclusively competed contracts. In doing so, we exploit the optimality of the procurer that he chooses the arrival rate,  $\lambda_e(\pi)$ , to minimize the expected total procurement cost,  $V_e(\lambda, \pi)$  in equation (12). The first order condition with respect to  $\lambda$  identifies  $\kappa_e(\pi)$ .

**Proposition 3.5** *Suppose Assumption 3.1 holds. Then we identify the distribution of number of bids for exclusively competed contracts conditional on unobserved type,  $\Pr(n|e = 1, \pi)$  for all  $n \geq 1$  and all  $\pi < \tilde{\pi}$ , and hence the bid arrival rate for exclusively competed contracts,  $\lambda_e(\pi)$ . The per-bidder bid processing cost,  $\kappa_e(\pi)$ , for all  $\pi < \tilde{\pi}$  is identified by*

$$\kappa_e(\pi) = \sum_{n=1}^{\infty} \frac{[\lambda_e(\pi) - n][\lambda_e(\pi) - 1]^{n-2} e^{-\lambda_e(\pi)+1}}{(n-1)!} T_e(\pi, n), \quad (21)$$

where  $T_e(\pi, n)$  is defined in equation (13).

To identify  $\kappa_o(\cdot)$ , we first identify  $\Pr(n|e = 0, \pi)$  and accordingly  $\lambda_o(\pi)$ . The challenge in identifying  $\lambda_o(\pi)$  lies in the fact that we cannot recover  $\pi$  for each fixed-price, non-exclusively competed contract because we do not know whether the winner is a favored contractor or not. However, under Assumption 3.2, we can recover  $\pi$  for each variable-price, non-exclusively competed contract. This helps the identification of  $\lambda_o(\pi)$ , and we exploit the optimality of the procurer in choosing the bid arrival rate,  $\lambda_o(\pi)$ , to identify  $\kappa_o(\pi)$ .

**Proposition 3.6** *Suppose assumptions 3.1, 3.2, and 3.3 hold. We identify  $\Pr(n|d = 1, e = 0, \pi)$  and  $\Pr(\pi, d = 1|e = 0)$  for any  $n$  and  $\pi < \bar{\pi}$ . Given the identified  $\rho$  and  $\zeta$ ,  $\Pr(d = 1|n, e = 0, \pi)$  for  $\pi < \bar{\pi}$  is identified by:*

$$\Pr(d = 1|n, e = 0, \pi) = \sum_{n_f=1}^n \binom{n-1}{n_f-1} \rho^{n_f-1} (1-\rho)^{n-n_f} (1-\pi)^n \zeta^{n-n_f}.$$

Then we identify  $\Pr(n|e = 0, \pi)$  for  $\pi < \bar{\pi}$  by:

$$\Pr(n|e = 0, \pi) = \frac{\Pr(n|d = 1, e = 0, \pi) \Pr(\pi, d = 1|e = 0)}{\Pr(d = 1|n, e = 0, \pi) g_{\pi}^*(\pi|e = 0)}.$$

Hence, we identify  $\lambda_o(\pi)$  for  $\pi < \bar{\pi}$ . The per-bidder bid processing cost,  $\kappa_o(\pi)$ , for all  $\pi < \bar{\pi}$  is identified by

$$\kappa_o(\pi) = \sum_{n=1}^{\infty} \frac{[\lambda_o(\pi) - n][\lambda_o(\pi) - 1]^{n-2} e^{-\lambda_o(\pi)+1}}{(n-1)!} T_o(\pi, n), \quad (22)$$

where  $T_o(\pi, n)$  is defined in equation (14).

### 3.5 Benefit from Selecting a Favored Contractor and the Distribution of the Political Cost of Exclusion

To identify the distribution of  $\eta$ ,  $F_{\eta}(\cdot)$ , we exploit the model prediction on  $\Pr(e = 1|\pi)$  for any  $\pi$ . We first identify the probability that a contract with type  $\pi$  is competed with eligibility restrictions,  $\Pr(e = 1|\pi)$  for  $\pi < \bar{\pi}$  as follows:

$$\Pr(e = 1|\pi) = \frac{g_{\pi}^*(\pi|e = 1) \Pr(e = 1)}{g_{\pi}^*(\pi|e = 1) \Pr(e = 1) + g_{\pi}^*(\pi|e = 0) \Pr(e = 0)}.$$

The procurer chooses to impose eligibility restrictions for type  $\pi$  procurement if the total expected procurement cost of doing so is cheaper than otherwise, i.e.,

$$V_e(\lambda_e(\pi), \pi) \leq V_o(\lambda_o(\pi), \pi),$$

where  $V_e(\lambda, \pi)$  and  $V_o(\lambda, \pi)$  are defined in equations (12) and (15). Given these equations, we have the following equation:

$$\begin{aligned} \Pr(e = 1|\pi) &= \Pr[V_e(\lambda_e(\pi), \pi) \leq V_o(\lambda_o(\pi), \pi)] \\ &= \Pr[\eta \leq \mathbb{E}\{T_o|\lambda_o(\pi), \pi\} - \mathbb{E}\{T_e|\lambda_e(\pi), \pi\} + \kappa_o(\pi)\lambda_o(\pi) - \kappa_e(\pi)\lambda_e(\pi)], \end{aligned} \quad (23)$$

Table 1  
*Summary statistics: Federal procurement contracts, FY 2004–2012*

	Median	Mean	Std. Dev.	Min	Max
<i>Prices (\$K, 2010)</i>					
Base price	578.5	4,027.8	72,462.7	1.0	13,892,030
Final price	712.9	6,340.0	50,001.6	-8,789.3	4,126,789
Price change: Work <sup>a</sup>	0.0	2,569.2	30,653.6	-196,352.9	2,944,045
Price change: Admin. <sup>b</sup>	0.0	1,063.6	65,649.7	-13,392,210	2,129,391
<i>Duration (Days)</i>					
Base duration	364	428.1	385.0	15	3,652
Final Duration	528	718.2	623.5	0	3,953
Duration change: Work <sup>a</sup>	0	186.2	393.1	-3,896	3,390
Duration change: Admin. <sup>b</sup>	0	89.8	285.8	-2,397	3,186
<i>Contract type</i>					
Fixed price	1	0.5	0.5	0	1
<i>Competition</i>					
Eligibility restrictions	1	0.5	0.5	0	1
# of bids	1	6.6	28.0	1	730
# of potential bidders <sup>c</sup>	5	21.7	42.86	1	328

*Note:* The number of observations is 45,743. The amount of prices is CPI-adjusted, where CPI of December 2010 is 100. a. This adjustment is related to work requirement changes or other significant contract modifications. b. This adjustment includes all other adjustments. c. The potential bidders denote the contractors in the data that have provided a product or service of the same code at least once and are in the same 4-digit NAICS industry of the winning contractor.

where  $\mathbb{E}\{T_e|\lambda_e(\pi), \pi\}$  and  $\mathbb{E}\{T_o|\lambda_o(\pi), \pi\}$  denote the expected value of transfer to a winning contractor with and without exclusion restrictions. Note that these arguments in the right hand side of the inequality have already been identified. Therefore, with exogenous variation of  $\pi$  for any given  $\eta$ , we identify the distribution of  $\eta$ .

**Proposition 3.7** *Suppose Assumptions 3.1–3.4 hold. Then we identify the distribution of  $\eta$  for the domain of  $(\eta_L, \eta_U)$ , where*

$$\eta_L = \inf_{\pi \in (\pi_L, \bar{\pi})} \{\mathbb{E}\{T_o|\lambda_o(\pi), \pi\} - \mathbb{E}\{T_e|\lambda_e(\pi), \pi\} + \kappa_o(\pi)\lambda_o(\pi) - \kappa_e(\pi)\lambda_e(\pi)\},$$

$$\eta_U = \sup_{\pi \in (\pi_L, \bar{\pi})} \{\mathbb{E}\{T_o|\lambda_o(\pi), \pi\} - \mathbb{E}\{T_e|\lambda_e(\pi), \pi\} + \kappa_o(\pi)\lambda_o(\pi) - \kappa_e(\pi)\lambda_e(\pi)\}.$$

## 4 Data

The theory predicts the optimal extent of competition and contract terms. In the remainder of this paper, we analyze data from contracts let by the U.S. federal government. Our dataset provides a very detailed information on the extent of competition and the contract terms for each procurement contract.

The data is a selected set of contracts let by the U.S. federal government during the period of FY 2004–2012. It is sourced from Federal Procurement Data System - Next Generation,

which collects information on contracts and their modifications. We restrict attention to contracts with specified terms and conditions of an expected size of \$300,000 or more.<sup>4</sup> This size threshold is chosen because the contracts of an anticipated size greater than \$300,000 are not normally expected to be reserved exclusively for small business concerns.<sup>5</sup> We further narrow down our sample that satisfy the following criteria: (i) available for competition but commercially unavailable, (ii) competed under the competitive proposal evaluation procedure or other similar procedures, (iii) not set-aside for small business concerns, (iv) initiated and completed during the period of study, (v) performed in the U.S., (vi) expected to take longer than two weeks for completion, and (vi) without any inconsistent records on the contract. There are 45,743 contracts that satisfy all of the above criteria, totaling \$290 billion. For each contract, we construct the variables in Table 1.

The base price of a contract is defined as the total value of the contract plus all options that have been exercised at the time of award. On the other hand, the final price is the total amount of funds obligated to the government. The base prices and the final prices often differ from each other, and the final prices are on average larger than the base prices. These differences are due to the contract terms that allow the final price to vary with the observed outcomes of the project. Some outcomes are correlated with the efficiency of the contractor, and others are not. Those correlated with the efficiency are considered as  $\epsilon$  in the model, denoted as ‘shock’ in Table 1, and those uncorrelated with the efficiency are considered as signal,  $s$ , in the model. We consider the work requirement changes are in the former category and the rest are in the latter.

The contract types are grouped into two broad categories: fixed-price and variable-price. The specific contract types range from firm-fixed-price, in which the contractor has full responsibility for the performance costs and resulting profit or loss, to cost-plus-fixed-fee, in which the contractor has minimal responsibility for the performance costs and the negotiated fee is fixed. In between are the various incentive contracts, we consider firm-fixed-price contracts as fixed-price, and the rest as variable-price. In our sample, about half of the contracts are fixed-price.

The extent of competition, which can be determined by the contracting officers with discretion, is observed in two dimensions. One is whether or not there was a full and open competition, under which about half of the contracts in the sample were let. The other dimension is the number of bidders. The average number is 7, but the median is 1. In our sample, 53% of the contracts were awarded to a single bidder, and this ratio is even larger, 73%, when the entry restrictions are imposed. Putting it differently, \$145 billion, about half of the total amount, was obligated to contractors that won a contract by default during the period of the study. This trend is not limited to our sample. For example, about 44% of the total obligated amount during in FY 2010, \$238 billion, is associated with contracts with a single bidder and the average number of bidders during the period is 3.8 with median 2.

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<sup>4</sup>There are contracts without specified terms and conditions, i.e., indefinite delivery, indefinite quantity contracts (IDIQ). These are used when the government cannot determine the precise quantities of supplies or services that the government will require during the contract period. They only comprise 6 percent of the total obligated amount by the federal government in FY 2010, \$33 billion out of \$540 billion. Compared to the contracts with definite delivery and quantity, the extent of competition is more or less similar.

<sup>5</sup>According to the FAR (see 13.003(b)(1)), “acquisitions of supplies or services that have an anticipated dollar value exceeding \$3,000 but not exceeding \$150,000 are reserved exclusively for small business concerns and shall be set aside.” The upper limit can be \$300,000 for certain supplies or services (see part 2.101 and look for the definition of “simplified acquisition threshold”).

## 5 Results

### 5.1 Nonparametric Estimator

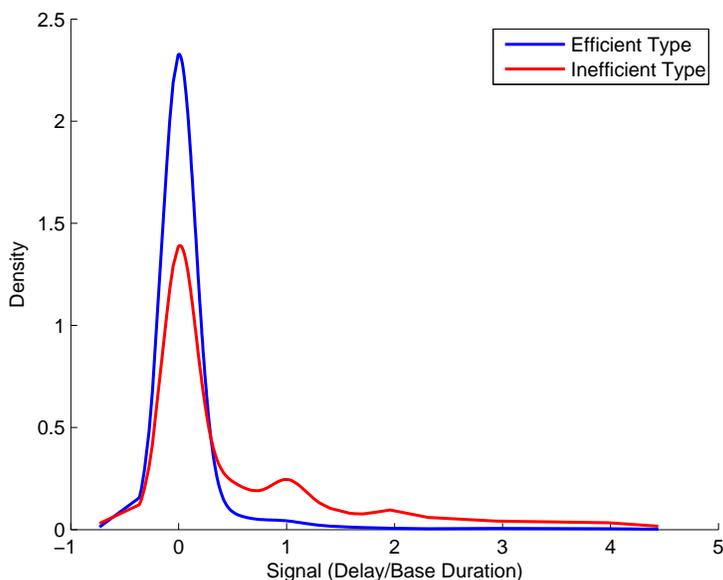
We closely follow the identification arguments to construct the nonparametric estimator. We assume that all components of the model to be estimated depend on the observed characteristics of a procurement project. These characteristics include (i) the number of unique bidders that won any of the procurement contracts of the industry in the state during the period of study, (ii) the average base price of the contracts of the industry in the state during the same period, and (iii) the base duration of the project. See Appendix for the estimator.

### 5.2 Estimation Results

We provide the estimation results conditional on the median value of the observed characteristics of a procurement project. The median values of these characteristics are (i) 5 unique bidders that won any of the procurement contracts of the industry in the state during the period of study, (ii) \$0.7 million of the average base prices of the contracts of the industry during the same period, and (iii) 364 days of base duration. The average size of the total payment of such contracts is \$2.7 million. This can be divided into two parts: one is the sum of the base price and the ex-post price adjustment related to signal, and the other is the ex-post adjustment related to cost shocks. The average size of the former is \$2.2 million and that of the latter is \$0.5 million. Note that the findings here are confined to these contracts. The results are preliminary and the standard errors will be provided in the next version of this draft.

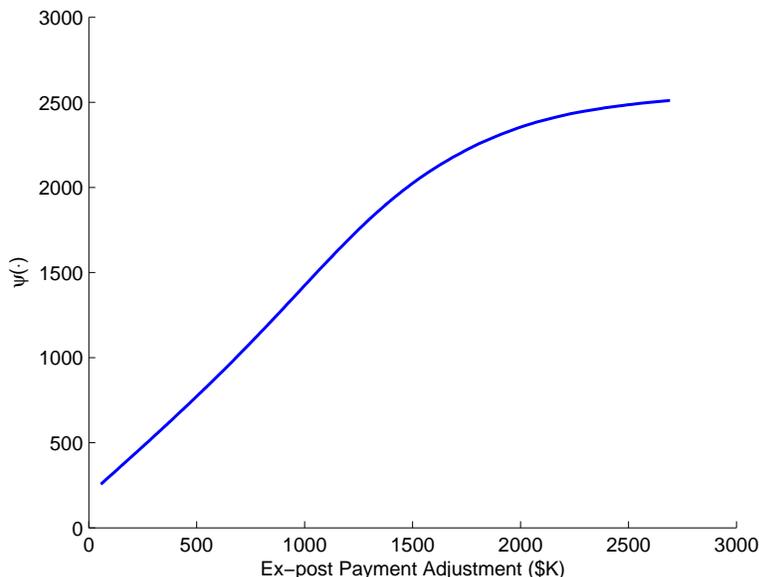
The distribution of the delay divided by the base duration varies by type, as shown in Figure 2. The estimated liquidity cost function,  $\psi(\cdot)$ , can be found in Figure 3. The liquidity

Figure 2  
Signal Distributions by Type



cost function is estimated to be increasing and concave. The unconditional distribution of

Figure 3  
Liquidity Cost Function,  $\psi(\cdot)$



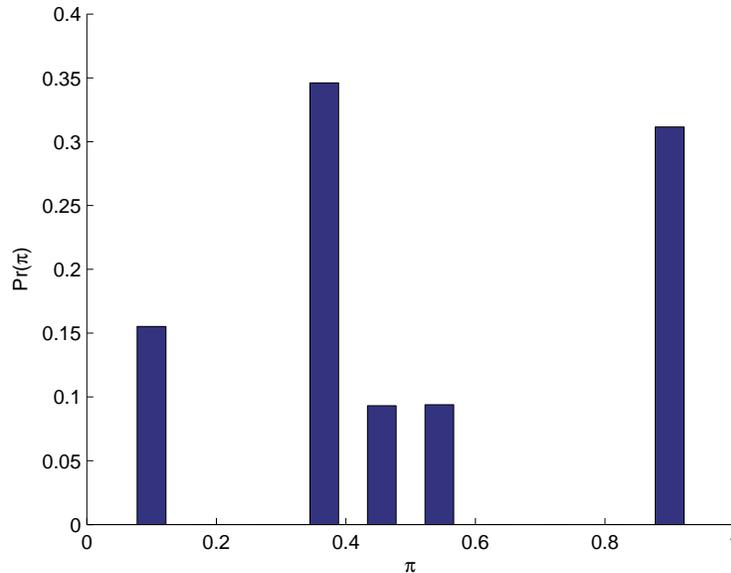
the unobserved type  $\pi$ , or the ratio of efficient bidders among favored contractors is shown in figure 4. The average ratio is 0.47, and that among non-favored contractors is estimated to be slightly smaller, 0.45. This is an important finding because this implies that favored contractors are very similar to or slightly better than non-favored ones in terms of ex-ante efficiency. Our estimates indicate that absent cost shocks, it takes \$0.8 million for efficient contractors to complete a procurement project, while it takes \$1.8 million for inefficient contractors, more than twice of the cost of efficient contractors.

We find that the per-bidder bid processing and solicitation cost is \$30,237, and the resulting average bid processing cost per a contract is \$50,327. This is about 2% of the average size of the total payment. The average number of bidders for exclusively competed contracts is 3, while that for non-exclusively competed ones is 7.3. This implies a relatively large administrative cost of holding a full and open competition.

### 5.3 Counterfactual Analyses

The estimation results point to a conclusion that imposing eligibility restrictions can be cost-effective to the government. To see this, we consider a counter-factual scenario where only favored contractors are allowed to bid. We find that by imposing eligibility restrictions to the non-exclusively competed contracts, the government can reduce the expected payment to contractors by 29%. By limiting the entry of non-favored contractors, who are ex-ante slightly less likely to be efficient than the favored counterparts, the government can induce a lower price from the efficient, favored contractors than otherwise. At the same time, the government forgoes a lower price from the efficient, non-favored contractors. Which of these opposing effects dominates is partially determined by how likely it is for a non-favored contractor to

Figure 4  
 Distribution of the Ratio of the Efficient among Favored Contractors



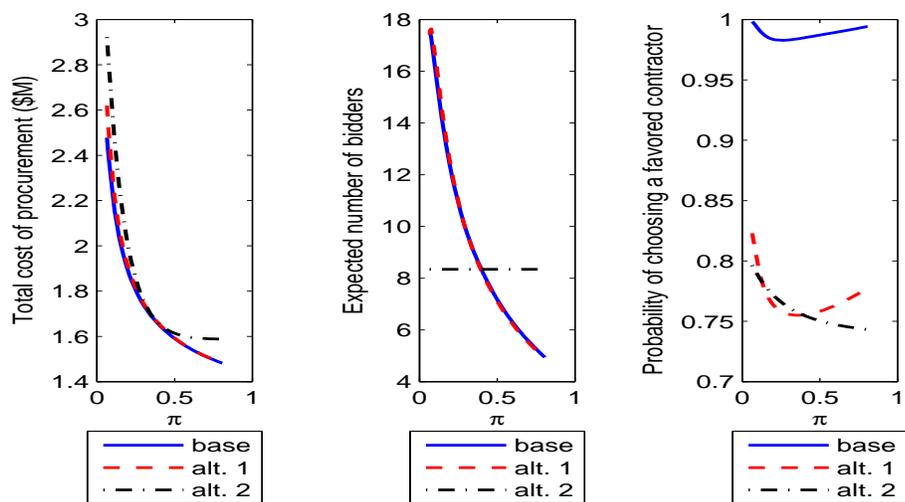
win a full and open competition. We find that although the ratio of favored contractors is estimated to be 71%, the probability that a favored contractor wins a non-exclusively competed contract is 97%. Therefore, the potential cost reduction from hiring an efficient, non-favored contractor does not realize very often, which further supports the use of entry restrictions. Although the procurement cost can decrease by imposing entry restrictions, the government holds a full and open competition because of various administrative or political costs that are associated with limiting competition, which is captured by  $\eta$  in the model. The nature of such costs is to be further investigated.

We study the value of discretion by contracting officers by comparing the government cost of the current regime with two alternative regimes where competition for contracts is open to all contractors. In one regime (Alternative 1 in Figure 5), the contracting officers are not allowed to offer different menus of contracts depending on the favor type but they are allowed to choose the bid arrival rate and the menu of contracts based on the unobserved type,  $\pi$ . In the other regime (Alternative 2 in Figure 5), no discretion is allowed to contracting officers so that the menu of contracts and the bid arrival rate are invariant to type  $\pi$ . We find that the current regime where the contracting officers exploit their expertise of  $\pi$  and practice favoritism yields the lowest procurement cost to the government.

## 6 Conclusion

In this paper, we study the determinants of eligibility requirements and the number of participating bidders in government procurement auctions. To understand the effects of the restrictions of competition on the total cost of government procurement, we develop, identify, and estimate a principal-contractor model in which the government selects a contractor

Figure 5  
Value of Discretion



to undertake a project. We consider three reasons why restricting entry could be beneficial to the government: by decreasing bid processing and solicitation costs, by increasing the chance of selecting a favored contractor and consequently reaping benefits from the favored contractor, and by decreasing the expected amount of price to the winning contractor. Using our estimates, we decompose the effects of these three sources of entry restrictions, and quantify the effects of the eligibility restrictions on the total cost of procurement.

# Appendix

## A. Proofs

### A.1. Proof of Theorem 2.1

To show the role of signaling, we begin with a result from the theory of optimal auctions, that when there is no signal to differentiate between different types of contractors, the procurer can extract some of the surplus an efficient contractor would receive, because of competition posed by other contractors who might be also efficient. Accordingly, suppose there are  $n$  bidders and two contracts on the menu, called high and low priority, such that a contractor bidding on one contract is only successful if no one selects the other contract. Let  $\bar{\phi}(n)$  denote the probability a contractor wins by selecting the low priority contract, that is the quotient of every bidder selecting the same contract and the number of bidders:

$$\bar{\phi}(n) \equiv \frac{(1 - \pi)^{n-1}}{n} \quad (24)$$

Similarly, let  $\underline{\phi}(n)$  denote a contractor's probability of winning with a high priority choice, found by integrating the probability over  $k \in \{0, \dots, n-1\}$  of winning when there are  $k$  other contractors selecting the same contract:

$$\begin{aligned} \underline{\phi}(n) &\equiv \sum_{k=0}^{n-1} \binom{n-1}{k} \frac{\pi^k (1 - \pi)^{n-1-k}}{k+1} \\ &= \frac{1}{\pi} \sum_{k+1=0}^n \binom{n-1}{k+1} \frac{\pi^{k+1} (1 - \pi)^{n-1-k}}{k+1} - \\ &= \frac{1 - (1 - \pi)^n}{n\pi}. \end{aligned} \quad (25)$$

Theorem 2.1 derives the contract menu in this baseline case.

To ensure the project is undertaken, the procurer must meet the individual rationality (henceforth IR) constraint of the inefficient contractor, and the cheapest fixed price contract meeting this constraint is  $c + \beta$ . To meet the incentive compatibility (IC) constraint of an efficient contractor, the procurer must offer terms that are at least as profitable as  $\bar{\phi}(n)\beta$ , which are the expected profits to an efficient contractor from selecting  $c + \beta$ . Letting  $p$  denote any price that solves the IC constraint:

$$\underline{\phi}(n)(p - c) \geq \bar{\phi}(n)\beta$$

Appealing to (24) and (25), this inequality can be expressed as:

$$p - c \geq \frac{\pi(1 - \pi)^{n-1}}{1 - (1 - \pi)^n} \beta$$

which is minimized by setting  $p = c + \frac{\pi(1 - \pi)^{n-1}}{1 - (1 - \pi)^n} \beta$ .

## A.2. Proof of Theorem 2.2

The following five lemmas collectively prove Theorem 2.2. The first lemma shows that variable price contracts are only offered in conjunction with fixed price contracts, not by themselves.

**Lemma 6.1** *The equilibrium contract menu includes a fixed price contract.*

**Proof.** The proof is by a contradiction argument. Suppose to the contrary that every contract on the menu is a variable price contract. Denote by  $\{p, q(s)\}$  one of the contracts on the menu. There are three cases to consider.

First, suppose  $\bar{\mathbb{E}}\{\psi[q(s)]\} \equiv \int \psi[q(s)]f(s)ds > \underline{\mathbb{E}}\{\psi[q(s)]\} \equiv \int \psi[q(s)]\underline{f}(s)ds$ . Then, the procurer can offer an additional, fixed price contract of  $p' = p + \bar{\mathbb{E}}\{\psi[q(s)]\}$ . The inefficient contractor would accept the contract, but the efficient contractor will not. By strict concavity of  $\psi(\cdot)$ , we have  $\bar{\mathbb{E}}\{\psi[q(s)]\} < \bar{\mathbb{E}}\{q(s)\}$ . Therefore, the expected payoff of the procurer increases when the inefficient contractor accept the fixed price contract with any positive probability.

Second, suppose  $\bar{\mathbb{E}}\{\psi[q(s)]\} < \underline{\mathbb{E}}\{\psi[q(s)]\}$ . The procurer can offer an additional, fixed price contract of  $p' = p + \underline{\mathbb{E}}\{\psi[q(s)]\}$ . The efficient contractor would accept the contract, but the inefficient contractor will not. Since  $\underline{\mathbb{E}}\{\psi[q(s)]\} < \underline{\mathbb{E}}\{q(s)\}$ , the expected payoff of the procurer increases when the efficient contractor to accept the new contract with any positive probability.

Lastly, suppose  $\bar{\mathbb{E}}\{\psi[q(s)]\} = \underline{\mathbb{E}}\{\psi[q(s)]\}$ . The procurer can offer instead an fixed price contract of  $p' = p + \underline{\mathbb{E}}\{\psi[q(s)]\}$ . Both types of contractor would accept the contract. Since  $\underline{\mathbb{E}}\{\psi[q(s)]\} < \underline{\mathbb{E}}\{q(s)\}$  the expected payoff of the procurer increases when either or both contractor types to accept the new contract with any positive probability. ■

Given Lemma 6.1, an optimal menu of contracts includes at least one fixed price contract. We show that efficient contractors never selects a variable-price contract.

**Lemma 6.2** *It is optimal for the procurer to offer a menu of contracts that induces the efficient contractor to select a fixed-price contract with probability one.*

**Proof.** Suppose not; i.e., the efficient contractors select a variable-price contract with positive probability. Then by Lemma 6.1, the menu must include a fixed-price contract that is selected by inefficient contractors. In that case, the fixed-price must be  $c + \beta$  so that the individual rationality constraint for the inefficient contractor is satisfied. Notice that the individual rational constraint for the efficient is satisfied with strict inequality; otherwise, the efficient contractor will select the fixed-price contract instead. Given this, the procurer's problem boils down to choosing the terms of the variable-price contract,  $p$  and  $q(\cdot)$  to minimize expected total transfer:

$$\phi(n) \{p + \underline{\mathbb{E}}[q(s)]\} + (1 - \phi(n)) (c + \beta), \quad (26)$$

where  $\phi(n)$  is the probability that a contractor that chooses a variable-price contract becomes a winner, subject to the incentive compatibility constraint for the efficient contractor, which is:

$$\underline{\phi}(n) (p - c + \underline{\mathbb{E}}\{\psi[q(s)]\}) \geq \bar{\phi}(n) \beta, \quad (27)$$

where  $\underline{\phi}$  and  $\bar{\phi}$  denote the subjective probability that a contractor that chooses the variable-price contract (or the fixed-price contract) wins. Since the individual rationality constraint

is satisfied with strict inequality, the incentive compatibility constraint must bind. Solving for  $p$  when (27) holds with equality,

$$p = \frac{\bar{\phi}(n)}{\underline{\phi}(n)}\beta + c - \mathbb{E}\{\psi[q(s)]\}$$

Substituting for  $p$  in (26) and simplifying we obtain:

$$c + \phi(n)\mathbb{E}\{q(s) - \psi[q(s)]\} + \beta \left(1 - \phi(n) + \phi(n)\frac{\bar{\phi}(n)}{\underline{\phi}(n)}\right),$$

which is minimized with respect to  $q(s)$  for each  $s \in S$ . Since  $q(s) \geq \psi[q(s)]$  when  $q(s) \leq 0$  and  $q(0) = \psi[q(0)]$ ,  $q(s) = 0$  for all  $s \in S$ . This leads to a contradiction. ■

This leaves us two generic possibilities on the optimal menu of contracts. Either two fixed price contracts characterized in Theorem 2.1 comprise the optimal menu, or it consists of a fixed price contract designed for efficient contractors and one or more variable contracts designed for the inefficient contractors. If the signal was of very high quality and most of the contractors were efficient, we might expect the procurer to extract all the rent from efficient contractors, and limit his losses to the risk premium paid to inefficient contractors. As proved in Theorem 2.2, this is indeed the case.

In preparation for that theorem we now define the expression:

$$H(\pi) \equiv \int_{l(s) < \tilde{l}(\pi, M)} \psi \left( h \left[ \frac{1 - \pi}{1 - \pi l(s)} \right] \right) [\bar{f}(s) - \underline{f}(s)] ds + \psi(M) \int_{l(s) \geq \tilde{l}(\pi, M)} [\bar{f}(s) - \underline{f}(s)] ds, \quad (28)$$

where  $l(s) \equiv \underline{f}(s) / \bar{f}(s)$  as the likelihood ratio at signal  $s$ , and  $h(\cdot)$  the inverse function of  $\psi'(\cdot)$ ; that is  $h[\psi'(q)] \equiv q$ . The cutoff  $\tilde{l}(\pi, M)$  is defined as:

$$\tilde{l}(\pi, M) \equiv \frac{1}{\pi} - \frac{1 - \pi}{\pi \psi'(M)}. \quad (29)$$

Because  $\psi'(q) > 1$  for any  $q < 0$ ,  $\tilde{l}(\pi, M) > 1$  for any  $\pi \in (0, 1)$  and  $M < 0$ . Since  $\psi'(q)$  is monotone decreasing in  $q$  with  $\psi'(\infty) = \varsigma$  for some  $\varsigma > 0$  and  $\psi'(-\infty) = \infty$ ,  $h(x)$  is well defined on  $x \in (\varsigma, \infty)$  and decreasing in  $x$ , with  $h(\varsigma) = \infty$  and  $h(\infty) = -\infty$ . Also  $h(1) = 0$  because  $\psi'(0) = 1$ . Lemma 6.3 shows that if signals are informative, then the expression  $\beta - H(\pi)$  has a unique root, denoted by  $\bar{\pi}$ .

**Lemma 6.3** *A unique probability denoted by  $\bar{\pi} \in (0, 1)$  solves  $\beta = H(\pi)$  if there exists  $\epsilon > 0$  such that  $\gamma \equiv \Pr\{s | l(s) \leq 1 - \epsilon\}$  is positive.*

**Proof.** Note from equation (28) that  $H(0) = 0$ . For  $\pi \in (0, 1)$ ,  $H(\pi) > 0$ . This is because we have the following two inequalities. First,

$$H_1(\pi, s) \equiv \psi \left( h \left[ \frac{1 - \pi}{1 - \pi l(s)} \right] \right) [\bar{f}(s) - \underline{f}(s)] > 0,$$

for any  $s$  such that  $l(s) < \tilde{l}(\pi, M)$ . This is because  $\underline{f}(s) > \bar{f}(s)$  if and only if  $h \left[ \frac{1-\pi}{1-\pi l(s)} \right] < 0$ . Second,

$$\psi(M)[\bar{f}(s) - \underline{f}(s)]ds > 0,$$

for any  $s$  such that  $l(s) \geq \tilde{l}(\pi, M) > 1$  because  $\psi(M) < 0$  and  $\underline{f}(s) > \bar{f}(s)$ . Now, it can be seen that  $H(\cdot)$  is strictly increasing in  $\pi$ . First, notice that

$$\frac{\partial}{\partial \pi} \tilde{l}(\pi, M) = \frac{1}{\pi^2} \left( -1 + \frac{1}{\psi'(M)} \right) < 0.$$

Second, the following inequality holds:

$$\frac{\partial}{\partial \pi} H_1(\pi, s) = -\psi' \left( h \left[ \frac{1-\pi}{1-\pi l(s)} \right] \right) h' \left( \frac{1-\pi}{1-\pi l(s)} \right) \left[ \frac{l(s)-1}{1-\pi l(s)} \right]^2 > 0,$$

Therefore, as  $\pi$  increases,  $H(\pi)$  also increases, which guarantees that  $\beta - H(\pi)$  has at most one root.

Now, if there exists  $\epsilon > 0$  such that  $\Pr \{s \mid \underline{f}(s) \leq \bar{f}(s)(1-\epsilon)\} = \gamma$ , then we can bound  $H(\pi)$  as follows:

$$\begin{aligned} H(\pi) &\geq \int 1 \{ \underline{f}(s) \leq \bar{f}(s)(1-\epsilon) \} \psi \left( h \left[ \frac{1-\pi}{1-\pi l(s)} \right] \right) [\bar{f}(s) - \underline{f}(s)] ds \\ &\geq \int 1 \{ \underline{f}(s) \leq \bar{f}(s)(1-\epsilon) \} \psi \left( h \left[ \frac{1-\pi}{1-\pi(1-\epsilon)} \right] \right) [\bar{f}(s) - \underline{f}(s)] ds \\ &= \psi \left( h \left[ \frac{1-\pi}{1-\pi(1-\epsilon)} \right] \right) \int 1 \{ \underline{f}(s) \leq \bar{f}(s)(1-\epsilon) \} [\bar{f}(s) - \underline{f}(s)] ds \\ &\geq \psi \left( h \left[ \frac{1-\pi}{1-\pi(1-\epsilon)} \right] \right) \epsilon \int 1 \{ \underline{f}(s) \leq \bar{f}(s)(1-\epsilon) \} \bar{f}(s) ds \\ &= \epsilon \gamma \psi \left( h \left[ \frac{1-\pi}{1-\pi(1-\epsilon)} \right] \right). \end{aligned}$$

Taking the limit as  $\pi \rightarrow (1-\zeta)/[1-\zeta(1-\epsilon)] < 1$  yields:

$$\lim_{\pi \rightarrow (1-\zeta)/[1-\zeta(1-\epsilon)]} \epsilon \gamma \psi \left( h \left[ \frac{1-\pi}{1-\pi(1-\epsilon)} \right] \right) = \epsilon \gamma \psi(h[\zeta]) = \epsilon \gamma \psi(\infty) = \infty$$

Since  $0 < (1-\zeta)/[1-\zeta(1-\epsilon)] < 1$  and  $0 < \beta < \infty$ , the lemma is proved. ■

Now we characterize the optimal menu of contracts when it consists of one fixed-price contract and one variable-price contract.

**Lemma 6.4** *Suppose the optimal menu of contracts consists of one fixed-price contract, denoted by  $\underline{p}(\pi, n)$ , and one variable-price contract, denoted by  $\{\bar{p}(\pi), q(\cdot, \pi)\}$ . The base price of the variable-price contract is:*

$$\bar{p}(\pi) = c + \beta - \int \psi(q(s, \pi)) \bar{f}(s) ds. \quad (30)$$

If  $\pi < \bar{\pi}$ , then the ex-post price adjustment schedule,  $q(\cdot, \pi)$ , is:

$$q(s, \pi) = \begin{cases} h \left[ \frac{1-\pi}{1-\pi l(s)} \right] & \text{if } l(s) < \tilde{l}(\pi, M), \\ M & \text{otherwise,} \end{cases} \quad (31)$$

where  $\tilde{l}(\pi, M)$  is defined in equation (29). If  $\pi \geq \bar{\pi}$ , then  $q(s, \pi) = q(s, \bar{\pi})$  for all  $s$ . The price of the fixed-price contract is:

$$\underline{p}(\pi, n) = c + \frac{\pi(1-\pi)^{n-1}}{1-(1-\pi)^n} \left[ \beta - \int \psi(q(s, \pi)) [\bar{f}(s) - \underline{f}(s)] ds \right]. \quad (32)$$

**Proof.** The principal designs a menu of two contracts that minimizes the expected transfer:

$$[1 - (1 - \pi)^n] \underline{p} + (1 - \pi)^n [\bar{p} + \mathbb{E}(q(s))] . \quad (33)$$

subject to the constraints that efficient contractors select the fixed price contract, inefficient contractors select the variable price contract, and the winning contractor never declares bankruptcy. A necessary condition of the optimal menu is that the IR constraint the inefficient contractors holds with equality (otherwise the base price  $\bar{p}$  could be further reduced, reducing the price and strengthening the IC constraint for efficient contractors). Solving for  $\bar{p}$  yields (30). The IC constraint for efficient contractors is:

$$\underline{\phi}(n)(\underline{p} - c) \geq \bar{\phi}(n) \{ \bar{p} + \mathbb{E}(\psi(q)) - c \},$$

Substituting for  $\bar{p}$  using equation (30),  $\bar{\phi}(n)$  using (24), and  $\underline{\phi}(n)$  using (25), the IC inequality simplifies to:

$$\underline{p} \geq c + \frac{\pi(1-\pi)^{n-1}}{1-(1-\pi)^n} \left( \beta - \int \psi(q(s)) [1 - l(s)] \bar{f}(s) ds \right) \quad (34)$$

Note that the IC for the inefficient will be satisfied with strict inequality at the optimum by Lemma 6.2. Therefore, at least one of the two remaining constraints, IR and IC for the efficient contractors, must bind. Otherwise, the price of the fixed-price contract could be reduced, earning the procurer higher revenue. This leads us to consider the following three cases separately.

**Case 1: IC binds but IR does not** Solving for  $\underline{p}$  from the IC constraint, and substituting the resulting expressions for  $\underline{p}$  and  $\bar{p}$ , obtained from equations and (30) and (32), into the expected total cost for the procurer, we obtain:

$$\begin{aligned} & [1 - (1 - \pi)^n] \left\{ c + \frac{\pi(1-\pi)^{n-1}}{1-(1-\pi)^n} \left( \beta - \int \psi(\bar{q}(s)) [1 - l(s)] \bar{f}(s) ds \right) \right\} \\ & + (1 - \pi)^n \left\{ c + \beta + \int [q(s) - \psi(q(s))] \bar{f}(s) ds \right\} \\ = & c + (1 - \pi)^{n-1} \left\{ \beta - \pi \int \psi(\bar{q}(s)) [1 - l(s)] \bar{f}(s) ds + (1 - \pi) \left\{ \int [q(s) - \psi(q(s))] \bar{f}(s) ds \right\} \right\} . \end{aligned}$$

The (scaled) Lagrangian for the cost minimization problem can now be expressed as:

$$L = -\pi \int \psi(q(s))[1-l(s)]\bar{f}(s)ds + (1-\pi) \int [q(s)-\psi(q(s))]\bar{f}(s)ds - \int \varkappa_1(s) [q(s) - M] \bar{f}(s)ds,$$

where  $\varkappa_1(s) \geq 0$  denotes the Kuhn Tucker multiplier for the linear constraint  $q(s) \geq M$ . The first order condition for  $q(s)$  is:

$$-\pi\psi'(q(s))[1-l(s)] + (1-\pi) [1-\psi'(q(s))] - \varkappa_1(s) = 0.$$

Rearranging terms we obtain:

$$\psi' [q(s)] = \frac{1-\pi-\varkappa_1(s)}{1-\pi l(s)}. \quad (35)$$

Note that  $q(s, \pi)$  as defined in equation (31) satisfies the above first order condition. If  $l(s) < \tilde{l}(\pi, M)$ , then  $q(s) = h \left[ \frac{1-\pi}{1-\pi l(s)} \right] > M$  and  $\varkappa_1(s) = 0$  solve equation (35). If  $l(s) \geq \tilde{l}(\pi, M)$ , then  $\varkappa_1(s) > 0$  and  $q(s) = M$  solve the equation.

**Case 2: IR binds but IC does not** When IR binds,  $\underline{p} = c$ . Substituting for  $\underline{p}$  and  $\bar{p}$ , using equation (30), the expected total transfer (33) simplifies to:

$$c + (1-\pi)^n \left\{ \beta + \int \{q(s) - \psi[q(s)]\} \bar{f}(s)ds \right\} \quad (36)$$

Substituting for  $\underline{p}$  in inequality (34) yields:

$$\beta \leq \int \psi(q(s))[1-l(s)]\bar{f}(s)ds \quad (37)$$

Notice the solution to this problem depends on neither  $\pi$  nor  $n$ . If IR binds but IC does not, then the first order condition for the Kuhn Tucker formulation is:

$$1 - \psi'(q(s)) = \varkappa_1(s)$$

If  $q(s) > M$ , then the complementary slackness condition requires  $\varkappa_1(s) = 0$ , and hence  $1 = \psi'(q(s))$  or  $q(s) = 0$ . Therefore, either  $q(s) = M$ , and the marginal benefit of imposing a harsher signal would exceed its cost were it not for the bankruptcy constraint, or  $q(s) = 0$ . Let us define  $S_M$  as the set of signals such that  $q(s) = M$  and let  $\mu$  denote  $\Pr(s \in S_M)$ . Note that for any  $\mu \in [0, 1]$ , both IR constraints and the IC constraint for the inefficient are satisfied. The total expected transfer can now be written as

$$c + (1-\pi)^n \{ \beta + [M - \psi(M)]\mu \}.$$

Notice that the above transfer is increasing in  $\mu$ , while  $\mu = 0$  does not satisfy the IC condition for the efficient, or inequality (37). This implies that when both IR constraints bind, the IC for the efficient must bind.

**Case 3: Both IR and IC bind** If (37) holds with equality, the (scaled) Lagrangian for the minimization problem can be written as:

$$L = \int (q(s) - \psi[q(s)]) \bar{f}(s) ds - \int \varkappa_1(s) [q(s) - M] \bar{f}(s) ds + \varkappa_2 \left\{ \beta - \int \psi[q(s)] [1 - l(s)] \bar{f}(s) ds \right\}.$$

The first order condition with respect to  $q(s)$  is:

$$1 - \psi'(q(s)) - \varkappa_1(s) - \varkappa_2 \psi'[q(s)] [1 - l(s)].$$

This can be written as:

$$\psi'[q(s)] = \frac{1 - \varkappa_1(s)}{1 + \varkappa_2 [1 - l(s)]} \quad (38)$$

Substituting for  $\varkappa_2 = \bar{\pi}/(1 - \bar{\pi})$  in equation (38) follows that the solution for  $q(s)$  in this case can be obtained by setting  $\pi = \bar{\pi}$  in equation (31) as required.

We have ruled out the second case, implying that the IC for the efficient always binds at the optimum. The IR constraint for the efficient contractors does not always bind, i.e.

$$\beta - \int \psi[q(s)] [1 - l(s)] \bar{f}(s) ds = \beta - H(\pi) \leq 0,$$

where  $H(\pi)$  is defined in equation (28). As shown in Lemma 6.3,  $\beta - H(0) = \beta > 0$ ,  $H(\cdot)$  is increasing in  $\pi$ , and there always exists a unique root of  $\beta - H(\pi)$ ,  $\bar{\pi}$ . Therefore, if  $\pi < \bar{\pi}$ , then the IR does not bind; otherwise, it binds. This completes the proof. ■

We now show that the menu of contracts characterized in Theorem 2.1 is always preferred to that of Lemma 6.4 if the signals are informative. In other words, the procurer is better off exploiting the signals.

**Lemma 6.5** *Suppose  $\bar{F}(s) \neq \underline{F}(s)$  for some signal  $s$  in the support. Then the menu of contracts characterized in Lemma 6.4 minimizes the total expected transfer.*

**Proof.** Given that there are two efficiency types, the optimal menu includes two contracts. By Lemma 6.1, we have shown that at least one of them must be fixed-price, and it is optimal to induce the efficient contractors to choose a fixed-price contract in the menu, as shown in Lemma 6.2. There are two possibilities: one is to offer two fixed-price contracts as characterized in Theorem 2.1, and the other is to offer one fixed-price contract for the efficient and one variable-price contract for the inefficient, as characterized in Lemma 6.4. We show that the latter is cheaper than the former.

The expected total cost of offering two fixed-price contracts of Theorem 2.1, denoted by  $T^F(\pi, n)$ , is:

$$T^F(\pi, n) = (1 - \pi)^n (c + \beta) + [1 - (1 - \pi)^n] c + \pi (1 - \pi)^{n-1} \beta = c + (1 - \pi)^{n-1} \beta.$$

Denoting by  $T^V(\pi, n)$ , the total cost of offering the menu of contracts of Lemma 6.4 is:

$$\begin{aligned} T^V(\pi, n) &= [1 - (1 - \pi)^n] \left[ c + \frac{\pi(1 - \pi)^{n-1}}{1 - (1 - \pi)^{n-1}} \left\{ \beta - \int \psi[q(s)] [\bar{f}(s) - \underline{f}(s)] ds \right\} \right] \\ &\quad + (1 - \pi)^n \left[ c + \beta + \int \{q(s) - \psi[q(s)]\} \bar{f}(s) ds \right] \\ &= c + (1 - \pi)^{n-1} \left\{ \beta + \pi \int \psi[q(s)] \underline{f}(s) ds + (1 - \pi) \int q(s) \bar{f}(s) ds \right\}. \end{aligned}$$

Thus  $T^V(\pi, n) < T^F(\pi, n)$  if and only if:

$$\pi \int \psi[q(s)] \underline{f}(s) ds + (1 - \pi) \int q(s) \bar{f}(s) ds < 0$$

This condition is satisfied if and only if:

$$T^V(\pi, 1) < T^F(\pi, n) = c + \beta$$

We complete the proof by showing that the above inequality holds. When there is only one bidder, the procurer can offer one fixed-price contract,  $c + \beta$ , or two contracts, one fixed-price contract and one variable-price contract. The proof is done by construction that it is less profitable to offer one fixed-price contract than a menu of two contracts.

For some  $\epsilon > 0$ , we define  $S \equiv \{s : \bar{f}(s) - \underline{f}(s) > \epsilon\}$ . Let the probability that a signal is in  $S$  conditional on that the contractor is efficient as  $\gamma_1$  and that conditional on that the contractor is inefficient as  $\gamma_2$ . If  $\bar{F}(s) \neq \underline{F}(s)$  for some signal  $s$  in the support, there exists  $\epsilon > 0$  such that  $\gamma_1 \neq 0$  and  $\gamma_2 \neq 0$ . Note that  $\gamma_2 > \gamma_1$ . For any  $\Delta > 0$  choose  $\mu(\Delta)$  for a two-part variable contract in which  $\bar{p} = c + \beta$  and:

$$q(s) = \begin{cases} \Delta & \text{if } s \in S, \\ \mu(\Delta) & \text{if } s \notin S, \end{cases}$$

where

$$\gamma_2 \psi(\Delta) + (1 - \gamma_2) \psi(\mu(\Delta)) = 0.$$

Note that the above equation implies that  $\mu(\Delta) < 0$ . Because  $\psi(\cdot)$  is strictly increasing,  $\mu(\Delta)$  is uniquely defined by the equation:

$$\mu(\Delta) = \psi^{-1} \left[ \frac{-\gamma_2}{1 - \gamma_2} \psi(\Delta) \right],$$

and is twice differentiable with:

$$\mu'(\Delta) = \frac{-\gamma_2}{1 - \gamma_2} \frac{\psi'(\Delta)}{\psi'(\mu(\Delta))},$$

where  $\mu(0) = 0$ . The fixed contract takes the form:

$$\begin{aligned}\underline{p} &= c + \beta + \gamma_1\psi(\Delta) + (1 - \gamma_1)\psi(\mu(\Delta)) \\ &= c + \beta + \gamma_1\psi(\Delta) - (1 - \gamma_1)\left(\frac{\gamma_2}{1 - \gamma_2}\right)\psi(\Delta).\end{aligned}$$

Note that the incentive compatibility constraint is satisfied with equality by the efficient contractor and strict inequality by the inefficient contractor because  $\gamma_1 < \gamma_2$ . Similarly, the participation constraint is satisfied with equality by the inefficient contractor and strict inequality by the efficient contractor as long as  $\Delta > 0$  is small enough. The expected price to the procurer is:

$$\begin{aligned}\mathbb{E}(T|\Delta) &= c + \beta + \pi[\gamma_1\psi(\Delta) + (1 - \gamma_1)\psi(\mu(\Delta))] + (1 - \pi)[\gamma_2\Delta + (1 - \gamma_2)\mu(\Delta)], \\ &= c + \beta + \pi\left[\gamma_1\psi(\Delta) - \frac{(1 - \gamma_1)\gamma_2}{1 - \gamma_2}\psi(\Delta)\right] + (1 - \pi)[\gamma_2\Delta + (1 - \gamma_2)\mu(\Delta)].\end{aligned}$$

We now show this expression is decreasing in the neighborhood of  $\Delta = 0$ . Differentiating with respect to  $\Delta$  yields:

$$\frac{\partial\mathbb{E}(T|\Delta)}{\partial\Delta} = \pi\left[\gamma_1\psi'(\Delta) - \frac{(1 - \gamma_1)\gamma_2}{1 - \gamma_2}\psi'(\Delta)\right] + (1 - \pi)\left[\gamma_2 - \gamma_2\frac{\psi'(\Delta)}{\psi'(\mu(\Delta))}\right].$$

Evaluating  $\frac{\partial\mathbb{E}(T|\Delta)}{\partial\Delta}$  at  $\Delta = 0$  gives us:

$$\frac{\partial\mathbb{E}(T|\Delta = 0)}{\partial\Delta} = \pi\frac{\gamma_1 - \gamma_2}{1 - \gamma_2} < 0,$$

which shows that a fixed-price contract fails to meet a first order necessary condition. ■

### A.3. Proof of Theorem 2.3

We now extend our model to the case in which bidders are treated differentially according to whether they are favored or not. The procurer minimizes the expected price to a winning contractor given  $n_f$  favored bidders and  $n_u$  unfavored bidders. Given that unfavored contractor will receive a fixed-price contract,  $c$ , the procurer's problem is to minimize the expected price to favored contractors. Note that this problem is identical to the procurer's problem with symmetric bidders except that the probability that the winning contractor is inefficient and favored depends not only the average efficiency and the number of favored bidders,  $(\pi_f, n_f)$ , but also those of unfavored bidders,  $(\pi_u, n_u)$ . Therefore, the optimal menu of contracts to the favored contractors consists of one fixed-price contract,  $\underline{p}_f$ , and one variable-contract,  $\{\bar{p}, q(\cdot)\}$ . The expected transfer is:

$$[1 - (1 - \pi_f)^{n_f}]\underline{p} + (1 - \pi_f)^{n_f}(1 - \pi_u)^{n_u}\{\bar{p} + \mathbb{E}[q(s)]\}. \quad (39)$$

As shown in Lemma 6.4, the IR condition must be binding for inefficient contractors must

bind at the optimum, which characterizes  $\bar{p}$ .

$$\bar{p} = c + \beta - \bar{\mathbb{E}}[\psi(q(s))].$$

We also have shown that the IC condition for the efficient contractors binds at the optimum:

$$\underline{\phi}(n_f)\{\underline{p} - c\} = \bar{\phi}(n_f, n_u)\{\bar{p} + \mathbb{E}(\psi(q)) - c\},$$

where  $\bar{\phi}(n_f)$  (or  $\underline{\phi}(n_f, n_u)$ ) denotes the probability of winning for a favored contractor if he selects the fixed-price (or variable-price) contract.

$$\begin{aligned} \underline{\phi}(n_f) &= \sum_{k=0}^{n_f-1} \binom{n_f-1}{k} \frac{\pi_f^k (1-\pi_f)^{n_f-1-k}}{k+1} = \frac{1 - (1-\pi_f)^{n_f}}{n_f \pi_f}, \\ \bar{\phi}(n_f, n_u) &= \frac{(1-\pi_f)^{n_f-1} (1-\pi_u)^{n_u}}{n_f}. \end{aligned}$$

Using these definitions of  $\bar{\phi}(n_f)$  and  $\underline{\phi}(n_f, n_u)$ , the price of the fixed-price contract,  $\underline{p}_f$ , is characterized as follows:

$$\underline{p}_f = c + \frac{\pi_f (1-\pi_f)^{n_f-1} (1-\pi_u)^{n_u}}{1 - (1-\pi_f)^{n_f}} \left[ \beta - \int \psi(q(s)) [\bar{f}(s) - \underline{f}(s)] ds \right].$$

The objective function (39) can now be written as:

$$\begin{aligned} & [1 - (1-\pi_f)^{n_f}] \left\{ c + \frac{\pi_f (1-\pi_f)^{n_f-1} (1-\pi_u)^{n_u}}{1 - (1-\pi_f)^{n_f}} \left[ \beta - \int \psi(q(s)) [\bar{f}(s) - \underline{f}(s)] ds \right] \right\} \\ & + (1-\pi_f)^{n_f} (1-\pi_u)^{n_u} \left\{ c + \beta + \int \{q(s) - \psi[q(s)]\} \bar{f}(s) ds \right\}. \end{aligned}$$

The ex-post payment adjustment  $q(\cdot)$  that minimizes the above also minimizes the following:

$$-\pi_f \left[ \int \psi(q(s)) [1 - l(s)] \bar{f}(s) ds \right] + (1-\pi_f) \int [q(s) - \psi[q(s)]] \bar{f}(s) ds.$$

Note that  $\pi_u$  does not enter the above expression, and it can be seen that the optimal  $q(s)$  is  $q(s, \pi_f)$  of Lemma 6.4 for all  $s$ .

#### A.4. Proof of Corollary 2.1

We show that the expected transfer to a winning contractor decreases as the number of bidders increases. First, the price of the fixed-price contract,  $\underline{p}(\pi, n)$ , is non-increasing in the number of bidders. Second, the variable price contract is invariant to the number of bidders. Third, the fixed-price contract costs strictly less than the variable-price contract in expectation for any given number of bidders; i.e.,  $\bar{p}(\pi) + \bar{\mathbb{E}}[q(s, \pi)] > \underline{p}(\pi, n)$  for any  $n$  and  $\pi$ .

We start with equation (9) which characterizes  $\underline{p}$ . Note that the last term in the equation,

$$\beta - \int \psi(q(s))[\bar{f}(s) - \underline{f}(s)]ds,$$

must be nonnegative; otherwise, the individual rationality constraint for the efficient contractors will not be satisfied. Because  $q(\cdot)$  is invariant to the number of bidders, it suffices to show that the derivative  $\frac{\pi(1-\pi)^{n-1}}{1-(1-\pi)^n}$  with respect to  $n$  is always negative.

$$\frac{\partial}{\partial n} \frac{\bar{\phi}(n)}{\underline{\phi}(n)} = \log(1-\pi) \frac{\pi(1-\pi)^{n-1}(2-(1-\pi)^n)}{(1-(1-\pi)^n)^2}.$$

The above expression is always negative as long as  $p \in (0, 1)$ . This proves that  $\underline{p}$  is non-increasing in the number of bidders. On the other hand, the number of bidders does not affect  $\bar{p}$  nor  $q(\cdot)$ , as can be seen in equations (7) and (8) in Theorem 2.2.

To compare the expected transfer under the fixed-price contract and that under the variable-price contract, we consider the one-participant case. Because  $\underline{p}$  is non-increasing in the number of bidders, if  $\underline{p} < \bar{p} + \bar{\mathbb{E}}(q)$  when only one contractor participates, the proof is done. Let us start by considering the incentive constraint for the inefficient contractor. Given that the individual rationality constraint holds at equality, the incentive constraint can be written as:

$$\underline{p} - c - \beta \leq 0.$$

Plugging in equation (9), the above inequality can be written as:

$$-\beta + \frac{\pi(1-\pi)^{n-1}}{1-(1-\pi)^n} \left[ \beta - \int \psi(q(s))[\bar{f}(s) - \underline{f}(s)]ds \right] \leq 0.$$

Now, the difference in the expected prices,  $\bar{p} + \int q(s)\bar{f}(s)ds - \underline{p}$ , can be rearranged as:

$$\left( \beta - \frac{\pi(1-\pi)^{n-1}}{1-(1-\pi)^n} \left[ \beta - \int \psi(q(s))[\bar{f}(s) - \underline{f}(s)]ds \right] \right) + \int [q(s) - \psi(q(s))]\bar{f}(s)ds.$$

The first term in parentheses are nonnegative and the second term is positive because  $q(s) > \psi(q(s))$  for all  $s \neq 0$ . Therefore,  $\bar{p} + \int q(s)\bar{f}(s)ds > \underline{p}$ .

### A.5. Proof of Corollary 2.2

We first show the dispersion of  $q(s, \pi)$ . Recall the first order condition is:

$$\psi' [q(s, \pi)] [1 - \pi l(s)] = 1 - \pi.$$

This holds if  $l(s) < \tilde{l}(\pi, M)$ . Note that  $q(s, \pi) = 0$  if  $l(s) = 1$  and  $q(s, \pi) > 0$  if  $l(s) < 1$ . Similarly  $q(s, \pi) < 0$  if  $l(s) > 1$ . Totally differentiating the first order condition with respect to  $\pi$  yields:

$$\psi'' [q(s, \pi)] \frac{\partial q(s, \pi)}{\partial \pi} [1 - \pi l(s)] - \psi' [q(s, \pi)] l(s) = -1.$$

Rearranging to make  $\partial q(s, \pi) / \partial \pi$  the subject of the equation gives:

$$\frac{\partial q(s, \pi)}{\partial \pi} = \frac{l(s) - 1}{\psi'' [q(s, \pi)] [1 - \pi l(s)]^2}.$$

Noting  $\psi''(\cdot) < 0$  it follows that  $\partial q(s, \pi) / \partial \pi > 0$  when  $l(s) < 1$  and  $\partial q(s, \pi) / \partial \pi < 0$  when  $l(s) > 1$ . In other words, if  $l(s) < \tilde{l}(\pi, M)$ , then

$$\begin{aligned} \frac{\partial q(s, \pi)}{\partial \pi} &> 0 \text{ if } q(s, \pi) > 0, \\ &= 0 \text{ if } q(s, \pi) = 0, \\ &< 0 \text{ if } q(s, \pi) < 0. \end{aligned}$$

as was to be proved.

Now we show that  $\underline{p}(\pi, n)$  is decreasing in  $\pi$  for any given  $n$ .

$$\underline{p} = c + \frac{\pi(1 - \pi)^{n-1}}{1 - (1 - \pi)^n} \left[ \beta - \int \psi(q(s)) [\bar{f}(s) - \underline{f}(s)] ds \right].$$

To show that  $\underline{p}'(\pi) < 0$  we consider the two expressions involving  $\pi$  separately. First:

$$\frac{\partial}{\partial \pi} \ln \left[ \frac{\pi(1 - \pi)^{n-1}}{1 - (1 - \pi)^n} \right] = \frac{1 - n\pi - (1 - \pi)^n}{\pi(1 - \pi)[1 - (1 - \pi)^n]}$$

Note that the derivative is zero at  $n = 1$  and that at  $n = 2$  is  $-\pi^2$ , which is negative. Now suppose it is negative for all  $n \in \{2, \dots, n_0\}$ . Then for  $n_0 + 1$  the denominator is clearly positive and the numerator is:

$$1 - (n_0 + 1)\pi - (1 - \pi)(1 - \pi)^{n_0} < \pi(1 - \pi)^{n_0} - \pi < 0.$$

The first inequality follows from an induction hypothesis, and the second one from the inequalities  $0 < \pi < 1$ . Therefore  $\pi(1 - \pi)^{n-1} / \pi(1 - \pi)^{n-1}$  is decreasing in  $\pi$  for all  $n > 1$ .

Second, we note that:

$$\begin{aligned} &\frac{\partial}{\partial \pi} \int \psi[q(s, \pi)] [1 - l(s)] \bar{f}(s) ds = \int \psi'[q(s, \pi)] \frac{\partial q(s, \pi)}{\partial \pi} [1 - l(s)] \bar{f}(s) ds \\ &= \int (1 - \pi) \frac{\partial q(s, \pi)}{\partial \pi} \left[ \frac{1 - l(s)}{1 - \pi l(s)} \right] \bar{f}(s) ds = \int \frac{(\pi - 1) [1 - l(s)]^2}{\psi'' [q(s, \pi)] [1 - \pi l(s)]^3} \bar{f}(s) ds > 0. \end{aligned}$$

The second equality follows from using the first order condition to substitute out  $\psi'[q(s, \pi)]$ , and the third equality uses the expression we derived for  $\partial q(s, \pi) / \partial \pi$ . The last inequality appeals to the concavity of  $\psi(\cdot)$ . Finally note that since the participation constraint is satisfied with an inequality for the efficient bidder if  $\pi < \bar{\pi}$ :

$$\beta - \int \psi[q(s, \pi)] [1 - l(s)] \bar{f}(s) ds > 0.$$

Hence:

$$\begin{aligned} \frac{\partial}{\partial \pi} \underline{p}(\pi) &= \frac{\partial}{\partial \pi} \left[ \frac{\pi(1-\pi)^{n-1}}{1-(1-\pi)^n} \right] \left\{ \beta - \int \{\psi[q(s, \pi)]\} [1-l(s)] \bar{f}(s) ds \right\} \\ &+ \frac{\pi(1-\pi)^n}{1-(1-\pi)^n} \int \frac{[1-l(s)]^2}{\psi''[q(s, \pi)] [1-\pi l(s)]^3} \bar{f}(s) ds < 0, \end{aligned}$$

as claimed.

## B. Simulation

Given unobserved type  $\pi$ , we solve for various moments regarding the menu of contracts, the contract terms, and the extent of competition. Then, we take the expectation of these equilibrium objects over  $\pi$  given its distribution.

One set of moments are based on the variable-price contracts. Note that the variable-price contracts do not vary by the extent of competition, i.e., the number of bidders and the entry restrictions. Equation (7) characterizes the base price of the variable-price contract:

$$\bar{p}(\pi) = c + \beta - \int \psi(q(s, \pi)) \bar{f}(s) ds,$$

where

$$q(s, \pi) = h \left( \frac{1-\pi}{1-\pi \underline{f}(s)/\bar{f}(s)} \right).$$

A second set of moments are based on the price of fixed-price contracts with  $n$  bidders. Under entry restrictions, the price of a fixed-price contract is characterized by equation (9) for any  $n \geq 1$ :

$$\underline{p}_{e,n}(\pi) = c + \frac{\pi(1-\pi)^{n-1}}{1-(1-\pi)^n} \left[ \beta - \int \psi(q(s, \pi)) [\bar{f}(s) - \underline{f}(s)] ds \right].$$

When  $n = 1$ , the price of the fixed-price contract does not depend on the entry restrictions due to the assumption that at least one participating bidder is favored. When  $n > 1$ , however, some of the bidders may be non-favored when entry restrictions are not imposed. The realized fixed-price contract depends on the composition of favored and non-favored contractors among the participating bidders. Because we do not observe the identity of favored contractors, we take the expectation.

$$\mathbb{E}(\underline{p}_{o,n}(\pi)) = \begin{cases} \underline{p}_{e,1}(\pi) & \text{if } n = 1, \\ \sum_{n_f=1}^n \frac{\Pr(n_f, n) \left\{ (1-\pi)^{n_f} [1 - ((1-\pi)\zeta)^{n-n_f}] c + [1 - (1-\pi)^{n_f}] \underline{p}_{o,n_f,n}(\pi) \right\}}{1 - (1-\pi)^n \zeta^{n-n_f}} & \text{if } n > 1, \end{cases}$$

where  $\Pr(n_f, n)$  is:

$$\Pr(n_f, n) = \binom{n-1}{n_f-1} \rho^{n_f-1} (1-\rho)^{n-n_f},$$

and  $\underline{p}_{o,n_f,n}(\pi)$  is defined in equation (11):

$$\underline{p}_{o,n_f,n}(\pi) = c + \frac{\pi(1-\pi)^{n_f-1}((1-\pi)\zeta)^{n-n_f}}{1-(1-\pi)^{n_f}} \left[ \beta - \int \psi(q(s,\pi))[\bar{f}(s) - \underline{f}(s)]ds \right].$$

Third, the probability that a contract is variable-price depends on the extent of competition, as follows:

$$\begin{aligned} \Pr(d=1|e=1, n, \pi) &= (1-\pi)^n, \\ \Pr(d=1|e=0, n, \pi) &= \sum_{n_f=1}^n \Pr(n_f, n)(1-\pi)^n \zeta^{n-n_f}. \end{aligned}$$

Now, we can solve the expected payment to a contractor given the extent of competition,  $\mathbf{E}(t|n, e)$ . When the competition is exclusive,

$$\mathbf{E}(t|n, e=1) = [1 - (1-\pi)^n] \underline{p}_{e,n}(\pi) + (1-\pi)^n \left( \bar{p}(\pi) + \int q(s,\pi) \bar{f}(s) ds \right).$$

Without entry restrictions,  $\mathbf{E}(t|n=1, e=0) = \mathbf{E}(t|n=1, e=1)$ . If  $n > 1$ ,

$$\begin{aligned} \mathbf{E}(t|n, e=0) &= \Pr(n_f, n) \left\{ (1-\pi)^{n_f} [1 - ((1-\pi)\zeta)^{n-n_f}] c + [1 - (1-\pi)^{n_f}] \underline{p}_{o,n_f,n}(\pi) \right. \\ &\quad \left. + (1-\pi)^{n_f} ((1-\pi)\zeta)^{n-n_f} \left( \bar{p}(\pi) + \int q(s,\pi) \bar{f}(s) ds \right) \right\}. \end{aligned}$$

A fourth set of moments are on the number of bidders. The average number of bidders for an exclusively competed contract is:

$$\mathbf{E}(n|e=1, \pi) = \lambda_e(\pi),$$

where  $\lambda_e(\pi)$  minimizes  $V_e(\lambda, \pi)$ , which is defined as in (12), i.e.,

$$V_e(\lambda, \pi) = \left[ \sum_{n=1}^{\infty} \frac{(\lambda-1)^{n-1} e^{-\lambda+1}}{(n-1)!} \mathbf{E}(t|n, e=1) \right] + \kappa_e(\pi) \lambda + \eta.$$

Similarly, the average number of bidders for a non-exclusively competed contract is:

$$\mathbf{E}(n|e=0, \pi) = \lambda_o(\pi),$$

where  $\lambda_o(\pi)$  minimizes  $V_o(\lambda, \pi)$ , which is defined as in (15), i.e.,

$$V_o(\lambda, \pi) = \sum_{n=1}^{\infty} \frac{(\lambda-1)^{n-1} e^{-\lambda+1}}{(n-1)!} \mathbf{E}(t|n, e=1) + \kappa_o(\pi) \lambda.$$

In the above equation,  $P_f(n_f, n, \pi)$  is the probability that a favored contractor to win if entry

restrictions are not imposed, and is defined as:

$$P_f(n_f, n, \pi) = \begin{cases} 1 & \text{if } n_f = n, \\ 1 - (1 - \pi)^{n_f} \{1 - ((1 - \pi)\zeta)^{n - n_f}\} & \text{otherwise.} \end{cases}$$

Lastly, the probability that a contract is competed exclusively,  $\Pr(e = 1|\pi)$ , is written as:

$$\Pr(e = 1|\pi) = \Pr(V_e(\lambda_e(\pi), \pi) - V_o(\lambda_o(\pi), \pi) \leq 0).$$

### C. Estimator

Note that all estimated objects are conditional on observed characteristics of a procurement project, denoted by  $\mathbf{x}$ , although it is omitted for notational easiness.

**Step 1** We first estimate the distribution of the base price of exclusively competed fixed-price contracts,  $\underline{p}_{e,n}$ , conditional on the number of bidders and  $\mathbf{x}$  using a kernel density estimator:

$$\hat{g}_{\underline{p}}(\underline{p}|n) = \frac{\sum_i e_i(1 - d_i)1\{n_i = n\}K_{\underline{p}}\left(\frac{\underline{p}_i - \underline{p}}{h_p}\right)K_x\left(\frac{\mathbf{x}_i - \mathbf{x}}{h_x}\right)}{h_p \sum_i e_i(1 - d_i)1\{n_i = n\}K_x\left(\frac{\mathbf{x}_i - \mathbf{x}}{h_x}\right)},$$

where  $K_{\underline{p}}(\cdot)$  and  $K_x(\cdot)$  are the Gaussian kernel functions and  $(h_x, h_p) > 0$  are bandwidths based on the “normal reference rule-of-the-thumb” approach as suggested by Silverman (1986). We then estimate the distribution of the absolute value of the ex-post price adjustment to a given value of signal  $s$ , conditional on  $\mathbf{x}$ :

$$\hat{g}_{\bar{q}}(\bar{q}|s, n) = \frac{\sum_i e_i d_i 1\{n_i = n\}K_{\bar{q}}\left(\frac{\bar{q}_i - \bar{q}}{h_q}\right)K_s\left(\frac{s_i - s}{h_s}\right)K_x\left(\frac{\mathbf{x}_i - \mathbf{x}}{h_x}\right)}{h_q \sum_i e_i d_i 1\{n_i = n\}K_s\left(\frac{s_i - s}{h_s}\right)K_x\left(\frac{\mathbf{x}_i - \mathbf{x}}{h_x}\right)},$$

where  $K_s(\cdot)$  is the Gaussian kernel function and  $h_s$  is a bandwidth. The odds ratio of having a fixed-price contract as opposed to a variable-price one conditional on the number of bidders and  $\mathbf{x}$  can be estimated using a sample analogue:

$$\hat{\varphi}_n = \frac{\sum_i (1 - d_i)e_i 1\{n_i = n\}K_x\left(\frac{\mathbf{x}_i - \mathbf{x}}{h_x}\right)}{\sum_i e_i 1\{n_i = n\}K_x\left(\frac{\mathbf{x}_i - \mathbf{x}}{h_x}\right)}.$$

Now let us consider a grid,  $M$ , where each point in the grid is in  $(0, 1)$ . We find  $\underline{p}_{n,m}$  such that  $\hat{G}_{\underline{p}}(\underline{p}_{n,m}|n) = 1 - m$  for any given  $m \in M$ . Similarly, we find  $\bar{q}_{s,n,m}$  such that  $\hat{G}_{\bar{q}}(\bar{q}_{s,n,m}|s, n) = m$  for any given  $s$  and  $m$ . Using  $\underline{p}_{n,m}$  and  $\bar{q}_{s,n,m}$ , we calculate  $\hat{r}_{n,m}$  for any  $m \in M$ :

$$\hat{r}_{n,m} = \frac{\hat{\varphi}_n \hat{g}_{\underline{p}}(\underline{p}_{n,m}|n)}{\hat{\varphi}_n \hat{g}_{\underline{p}}(\underline{p}_{n,m}|n) + \sum_j \hat{g}_{\bar{q}}(\bar{q}_{s_j,n,m}|s_j, n) \frac{\bar{q}_{s_j,n,m} - \bar{q}_{s_j,n,m-1}}{\underline{p}_{n,m} - \underline{p}_{n,m+1}} \hat{f}(s_j) \Delta s_j}.$$

In the above equation,  $\hat{f}(s_j)$ , the signal distribution of an inefficient contractor, can be estimated exploiting that the equilibrium is always separating as follows:

$$\hat{f}(s) = \frac{\sum_i d_i K_s \left( \frac{s_i - s}{h_s} \right) K_x \left( \frac{\mathbf{x}_i - \mathbf{x}}{h_x} \right)}{h_s \sum_i d_i K_x \left( \frac{\mathbf{x}_i - \mathbf{x}}{h_x} \right)}.$$

By taking a weighted average of  $1 - (1 - \hat{r}_{n,m})^{\frac{1}{n}}$  over  $n$ , we obtain an estimate of  $\pi_m$ , the  $m^{\text{th}}$  quantile of unobserved type  $\pi$  given that a procurement auction is competed with entry restrictions:

$$\hat{\pi}_m = \sum_n \left\{ 1 - (1 - \hat{r}_{n,m})^{\frac{1}{n}} \right\} \left\{ \frac{\sum_i e_i \{n_i = n\} K_x \left( \frac{\mathbf{x}_i - \mathbf{x}}{h_x} \right)}{\sum_i e_i K_x \left( \frac{\mathbf{x}_i - \mathbf{x}}{h_x} \right)} \right\}.$$

**Step 2** Using estimated signal distributions, we calculate the right hand side of the following equation for each exclusively competed contract  $i$ :

$$\tilde{\psi}_i = \frac{1 - \hat{\pi}_i(s_i, q_i, n, \mathbf{x})}{1 - \hat{\pi}_i(s_i, q_i, n, \mathbf{x}) \hat{f}(s_i, \mathbf{x}) / \hat{f}(s_i, \mathbf{x})},$$

where  $\hat{\pi}_i(s_i, q_i, n, \mathbf{x})$  is obtained from Step 1. Given  $\{(q_i, \tilde{\psi}_i, \mathbf{x})$  for each exclusively competed contract  $i$ , we estimate  $\hat{\psi}'(\cdot; \mathbf{x})$  using a Nadaraya-Watson estimator:

$$\tilde{\psi}'(q; \mathbf{x}) = \frac{\sum_i \tilde{\psi}_i e_i d_i K_q \left( \frac{q_i - q}{h_q} \right) K_x \left( \frac{\mathbf{x}_i - \mathbf{x}}{h_x} \right)}{\sum_i e_i d_i K_q \left( \frac{q_i - q}{h_q} \right) K_x \left( \frac{\mathbf{x}_i - \mathbf{x}}{h_x} \right)}.$$

We allow that observed price adjustment  $q_i$  may include some part of the actual base price, denoted by  $p_0$ . Exploiting our assumption that  $\psi'(0) = 1$ ,

$$\hat{p}_0 = (\hat{\psi}')^{-1}(1).$$

Lastly, the assumption  $\psi(0) = 0$ , we have

$$\hat{\psi}(q) = \int_0^q \hat{\psi}'(u - \hat{p}_0) du.$$

**Step 3** From Step 1, we associate each exclusively competed contract  $i$  with a unique unobserved type,  $\hat{\pi}_i$ . While doing so, we have estimated  $\hat{p}_{e,n}(\pi)$  and  $\hat{q}(s, \pi)$ . As for  $\hat{p}(\pi)$ , a sample analogue is:

$$\hat{p}(\pi_m) = \frac{\sum_i (p_i + \hat{p}_0) e_i d_i \mathbb{1}\{\hat{\pi}_i = \pi_m\} K_x \left( \frac{\mathbf{x}_i - \mathbf{x}}{h_x} \right)}{\sum_i e_i d_i \mathbb{1}\{\hat{\pi}_i = \pi_m\} K_x \left( \frac{\mathbf{x}_i - \mathbf{x}}{h_x} \right)}.$$

Although the distribution of  $\pi$  is continuous, we discretize the support of  $\pi$  into a grid of  $M$  points in the estimation. We denote the  $m^{\text{th}}$   $\pi$  in the grid by  $\pi_m$ .

A weighted average of a sample analogue of equation (19) over  $\pi$  gives us an estimator of  $c + \beta$ :

$$\widehat{c + \beta} = \sum_m \hat{g}(\pi_m | e = 1) \left( \hat{p}(\pi_m) + \sum_j \hat{\psi}[\hat{q}(s_j, \pi_m)] \hat{f}(s_j) \Delta s_j \right),$$

where the weights are based on the sample distribution of  $\pi$  for exclusively competed contracts,  $\hat{g}(\pi_m | e = 1)$ , which is obtained as follows:

$$\hat{g}(\pi_m | e = 1) = \frac{\sum_i e_i 1\{\hat{\pi}_i = \pi_m\} K_x\left(\frac{\mathbf{x}_i - \mathbf{x}}{h_x}\right)}{\sum_i e_i K_x\left(\frac{\mathbf{x}_i - \mathbf{x}}{h_x}\right)}.$$

An estimator of  $q(s_j, \pi_m)$  is based on the Nadaraya-Watson estimator:

$$\hat{q}(s_j, \pi_m) = \frac{\sum_i e_i d_i q_i 1\{\hat{\pi}_i = \pi_m\} K_s\left(\frac{s_i - s_j}{h_s}\right) K_x\left(\frac{\mathbf{x}_i - \mathbf{x}}{h_x}\right)}{\sum_i e_i d_i 1\{\hat{\pi}_i = \pi_m\} K_s\left(\frac{s_i - s_j}{h_s}\right) K_x\left(\frac{\mathbf{x}_i - \mathbf{x}}{h_x}\right)} - \hat{p}_0.$$

To separately estimate  $\hat{c}$  and  $\hat{\beta}$ , we take a weighted average of a sample analogue of equation (18) over  $\pi$  and  $n > 1$ .

$$\begin{aligned} \hat{\beta} = & \sum_m \hat{g}(\pi_m | e = 1) \left\{ \sum_j \hat{\psi}[\hat{q}(s_j, \pi_m)] \hat{f}(s_j) \Delta s_j + \sum_{n>1} \frac{\Pr(n|\widehat{e=1}, \pi_m)}{1 - \Pr(1|\widehat{e=1}, \pi_m)} \right. \\ & \left. \times \frac{[1 - (1 - \pi_m)^n] [\hat{p}(\pi_m) - \hat{p}_{e,n}(\pi_m)] + \pi_m (1 - \pi_m)^{n-1} \sum_j \hat{\psi}[\hat{q}(s_j, \pi_m)] \hat{f}(s_j) \Delta s_j}{1 - (1 - \pi_m)^{n-1}} \right\}, \end{aligned}$$

where  $\Pr(n|\widehat{e=1}, \pi_m)$  is defined as:

$$\Pr(n|\widehat{e=1}, \pi_m) = \frac{\sum_i 1\{n_i = n\} e_i 1\{\hat{\pi}_i = \pi_m\} K_x\left(\frac{\mathbf{x}_i - \mathbf{x}}{h_x}\right)}{\sum_i e_i 1\{\hat{\pi}_i = \pi_m\} K_x\left(\frac{\mathbf{x}_i - \mathbf{x}}{h_x}\right)}.$$

**Step 4** We solve for  $\hat{\rho}$  and  $\hat{\zeta}$  from  $N \geq 2$  equations of a sample analogue of (21) for  $n = 1, \dots, N$ , by minimizing the weighted average of the squared distances as follows:

$$\begin{aligned} (\hat{\rho}, \hat{\zeta}) = & \arg \min_{\rho, \zeta} \sum_n \Pr(n|\widehat{e=0}) \left[ \mathbb{E}(p|\widehat{e=0}, n) \right. \\ & \left. - \sum_m \sum_{n_f=1}^n \binom{n-1}{n_f-1} \rho^{n_f-1} (1 - \rho)^{n-n_f} \mathbb{E}(p|e = \widehat{0}, n, n_f, \pi_m) \hat{g}(\pi_m | e = 0, n) \Delta \pi_m \right]^2, \end{aligned}$$

where  $\Delta\pi_m$  denotes the distance between  $\pi_m$  and its nearby point in the grid for unobserved types. In the above equation,  $\Pr(\widehat{n|e=0})$ ,  $\mathbb{E}(\widehat{p|e=0, n})$ ,  $\mathbb{E}(\widehat{p|e=0, n, n_f, \pi_m})$  and  $\hat{g}(\pi_m|e=0, n)$  are defined as follows. First,  $\Pr(\widehat{n|e=0})$  can be obtained from a sample average,

$$\Pr(\widehat{n|e=0}) = \frac{\sum_i 1\{n_i = n\}(1 - e_i)K_x\left(\frac{\mathbf{x}_i - \mathbf{x}}{h_x}\right)}{\sum_i (1 - e_i)K_x\left(\frac{\mathbf{x}_i - \mathbf{x}}{h_x}\right)}.$$

Similarly, a sample analogue of  $\mathbb{E}(p|e=0, n)$  is defined below:

$$\mathbb{E}(\widehat{p|e=0, n}) = \frac{\sum_i \{p_i + \hat{p}_0 d_i\} (1 - e_i) 1\{n_i = n\} K_x\left(\frac{\mathbf{x}_i - \mathbf{x}}{h_x}\right)}{\sum_i (1 - e_i) \{n_i = n\} K_x\left(\frac{\mathbf{x}_i - \mathbf{x}}{h_x}\right)}.$$

Because we do not know which contractors are favored, we borrow a model prediction to define  $\mathbb{E}(p|e=0, n, n_f, \pi_m)$  for each  $\pi_m$  as a function of  $\rho$  and  $\zeta$ :

$$\mathbb{E}(p|e=0, n, n_f, \pi_m) = \hat{c} + (1 - \pi_m)^{n-1} \zeta^{n-n_f} \left\{ \hat{\beta} - \sum_j \hat{\psi}(\hat{q}(s_j; \pi_m)) [\hat{f}(s_j) - \pi_m \hat{f}(s_j)] \Delta s_j \right\}.$$

Lastly,  $\hat{g}(\pi_m|e=0, n)$  is a sample analogue of the right hand side of equation (20), and it is a function of  $\rho$  and  $\zeta$ :

$$\hat{g}(\pi_m|e=0, n) = \frac{\hat{g}(\pi_m|d=1, e=0, n) \Pr(d=1|e=0, n)}{\sum_{n_f=1}^n (1 - \pi_m)^n \zeta^{n-n_f} \binom{n-1}{n_f-1} \rho^{n_f-1} (1 - \rho)^{n-n_f}}.$$

To obtain  $\hat{g}(\pi_m|d=1, e=0, n)$  for each  $\pi_m$ , we recover  $\pi$  for non-exclusively competed variable-price contracts. For each such contract  $i$ , we can solve for  $\hat{\pi}_i$ :

$$\hat{\pi}_i = \frac{\hat{\psi}'(q_i - \hat{p}_0) \hat{f}(s_i) / \hat{f}(s_i) - 1}{\hat{\psi}'(q_i)}.$$

Given this, we can estimate  $g(\pi_m|d=1, e=0, n)$ :

$$\hat{g}(\pi_m|d=1, e=0, n) = \frac{\sum_i 1\{\hat{\pi}_i = \pi_m\} (1 - e_i) d_i K_x\left(\frac{\mathbf{x}_i - \mathbf{x}}{h_x}\right)}{\sum_i (1 - e_i) d_i K_x\left(\frac{\mathbf{x}_i - \mathbf{x}}{h_x}\right)}.$$

**Step 5** To estimate the per-bidder bid processing cost of exclusive competition,  $\kappa_e(\pi_m)$ , for each  $m^{\text{th}}$  unobserved type in the grid, we obtain a sample analogue of the average number of bidders,  $\lambda_e(\pi)$ .

$$\hat{\lambda}_e(\pi_m) = \frac{\sum_i n_i e_i 1\{\hat{\pi}_i = \pi_m\} K_x\left(\frac{\mathbf{x}_i - \mathbf{x}}{h_x}\right)}{\sum_i e_i 1\{\hat{\pi}_i = \pi_m\} K_x\left(\frac{\mathbf{x}_i - \mathbf{x}}{h_x}\right)}.$$

Using  $\hat{\lambda}_e(\pi_m)$ , we take a sample analogue of the right hand side of equation (21) to derive an estimator for  $\kappa_e(\pi_m)$  for each  $\pi_m$ .

$$\begin{aligned} \hat{\kappa}_e(\pi_m) &= \sum_n \{ [1 - (1 - \pi_m)^n] \hat{p}_{e,n}(\pi_m) + (1 - \pi_m)^n [\hat{p}(\pi_m) + \sum_j \hat{q}(s_j, \pi_m) \hat{f}(s_j) \Delta s_j] \} \\ &\quad \times \frac{(\hat{\lambda}_e(\pi_m) - n)(\hat{\lambda}_e(\pi_m) - 1)^{n-2} e^{-\hat{\lambda}_e(\pi_m)+1}}{(n-1)!}. \end{aligned}$$

To estimate the per-bidder processing cost of non-exclusive competition,  $\kappa_o(\pi_m)$ , for each  $m^{\text{th}}$  unobserved type in the grid, we estimate the optimal bid arrival rate for non-exclusively competed contracts,  $\hat{\lambda}_o(\pi_m)$ , for each  $\pi_m$ . To do so, we obtain the sample analogue of  $\Pr(n|e=0, \pi)$ :

$$\Pr(n|\widehat{e=0}, \pi_m) = \frac{\Pr(n|\widehat{d=1}, e=0, \pi_m) \Pr(\pi_m, \widehat{d=1}|e=0)}{\Pr(\widehat{d=1}|n, e=0, \pi_m) \hat{g}(\pi_m|e=0)}.$$

Given  $\Pr(n|\widehat{d=1}, e=0, \pi)$ 's for each  $n$ , we derive an estimator of  $\hat{\lambda}_o(\pi_m)$  as follows:

$$\hat{\lambda}_o(\pi_m) = \sum_n n \Pr(n|\widehat{e=0}, \pi_m).$$

Using  $\hat{\lambda}_o(\pi_m)$ , we take a sample analogue of the right hand side of equation (22) to derive an estimator for  $\kappa_o(\pi_m)$  for each  $\pi_m$ .

**Step 6** We first obtain  $\Pr(\widehat{e=1}|\pi_m)$  for each  $\pi_m$ :

$$\Pr(\widehat{e=1}|\pi_m) = \frac{\hat{g}(\pi_m|e=1) \Pr(\widehat{e=1})}{\hat{g}(\pi_m|e=1) \Pr(\widehat{e=1}) + \hat{g}(\pi_m|e=0) \Pr(\widehat{e=0})}.$$

All arguments in the right hand side of the above equation can be estimated using respective sample analogues. By employing a sample analogue of equation (23), we recover the distribution of  $\eta$ .

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