

Estimating a War of Attrition: The Case of the U.S. Movie Theater Industry*

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Abstract

This paper estimates the impact of competition and exogenous demand decline on the exit process of movie theaters in the U.S. from 1949 to 1955. I modify Fudenberg and Tirole (1986)'s model of exit in a duopoly with incomplete information to be used in an oligopoly. I estimate this model with panel data, using variations in TV diffusion across households and other market characteristics to identify the parameters in the theater's payoff function and the distribution of unobservable exit values. Using the estimated model, I show that theaters who are making negative profits choose to remain in the market if they expect to outlast their competitors, because at that point their profits would increase. This creates a significant delay in the exit process. The loss of industry profit due to incomplete information is larger in markets with fewer competitors. This paper also provides a simulation method to address the initial condition problem, which is substantial in estimating games with serially-correlated private information.

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1 Introduction

As demand decreases in an industry, firms often exit sequentially. One example of this is single-screen theaters in the U.S., where a drop in demand caused the number of movie theaters to decrease in the 1950's. The total number of indoor movie theaters decreased by 35% in 10 years from the end of the 1940's. This demand decline was mostly due to exogenous forces, such as the nationwide penetration of televisions (Lev (2003) and Stuart (1976)). Since costs were mostly fixed and capacity adjustments were usually infeasible, theaters had to respond to declining demand by leaving the market. At the same time, since local markets were oligopolies, theaters considered their opponents' behaviors when choosing an optimal exit time. In addition, when theaters do not have exact information on competitors' profitability, they act based on their beliefs on this, which are updated over time. These interactions and demand decline jointly determine the exit process.

There are many examples of when diffusion of new technology creates a nonstationary environment, causing sequential exits from an oligopolistic market. For example, today, on-line movie services are pressuring local video rental stores. Nevertheless, to the best of my knowledge, there are few empirical studies that separate players' interactions, expectations, and learning from exogenous demand factors in a declining industry.¹ This paper contributes to the literature by providing a tractable framework for analyzing such industries.

This paper estimates the effect of competition among theaters and their beliefs about competitors separately from the effect of exogenous demand decline on the devolution of the industry. I assume that competition in the U.S. movie theater industry in the 1950's resembled a "war of attrition." In a war of attrition, some players will eventually exit. As time passes, competition becomes costly, and each player incurs a loss while competing. If a player outlasts its competitors, it earns a positive profit. Therefore, at each moment, players decide if they should remain in the market, for if they do so, there is some chance their competitors will exit. I assume there is asymmetric information about each player's time-invariant profitability. Thus, players learn more about their competitors over time. To separate the strategic interactions between theaters (i.e., outlasting motives) from exogenous demand change, I need to infer theaters' expectations and the learning process, as well as the theaters' payoff functions. This allows me to quantify the delay of exits (and the resulting cost) due to theaters' outlasting motives.

For this purpose, I modify Fudenberg and Tirole (1986)'s model of exit in duopoly with incomplete information to be used in an oligopoly. At each instant, theaters choose whether

¹Nishiwaki (2010) empirically studies the dynamics of an declining industry, but he considers a case without learning and focuses on firms' divestment process and business stealing effects.

to exit or stay in the market. I assume each theater knows its own time-invariant value of exit (scrap value) but not that of its competitors. In equilibrium, theaters get discouraged and exit if competitors stay open long enough.

For a given number of surviving players and the industry's history, I can define a marginal player (i.e., a player whose cost and benefit of waiting are equal). The benefit of waiting when n players are still surviving is the product of the probability that one of the $n - 1$ competitors exits in the next instant and the value of proceeding to a subgame with $n - 1$ players. This way, I can nest an $n - 1$ player game into an n player game, which is further nested into an $n + 1$ player game, and so on. In equilibrium, the value of exit for the marginal player decreases as time passes. The path of this value is given by a set of differential equations, and it serves as a policy function for theaters. When one of the players exits, a new set of differential equations characterizes the survivors' exit policies. As a result, theaters sequentially exit, with the theater with the highest exit value leaving first.

By recursively solving the set of differential equations, I can find equilibrium exit times. I can then repeatedly solve the model in a nested fixed point algorithm (the full solution approach), allowing me to explicitly take theaters' expectations and unobservable market level heterogeneity into account.

Using data from the U.S. movie theater industry, I estimate theaters' payoff functions and the distribution of exit values. I use TV penetration rates, which vary across locations and time, to measure changes in demand. By imposing the equilibrium condition, the model predicts the distribution of theaters' exit times for a given set of parameters and unobservables. I estimate the parameters by matching the predicted distribution with the observed distribution of exit times.

This paper also develops a method to solve the initial condition problem, which is another contribution of the paper to the literature of estimation of dynamic games. In particular, I control for the endogeneity generated by the correlation between the initial number of theaters and the unobservable variables in the model. To achieve this goal, I set up a model of entry at the beginning of my sample period. For a given set of unobservables, the entry game predicts the number of entrants in equilibrium. I restrict the support of market heterogeneity and exit values such that, in equilibrium, the entry game predicts the same number of entrants as is observed. Since it is hard to analytically characterize such a restriction, I use a simulation to approximate the joint distribution of unobservable variables, conditional on the observed number of entrants. This simulated distribution can then be used as an input to simulate and solve the dynamic stage of the game.

Using the estimated model, I quantify the significance of a war of attrition. In a war of attrition with learning, players get discouraged and exit if their competitors stay open long

enough. I show that many theaters delay exiting, even when making negative profits, in hopes that their competitors will exit first. These delays come from asymmetric information. If exit values are common knowledge (complete information), then in equilibrium, a theater exits the game exactly when its profit becomes lower than its exit value. Cumulative market profits in the war of attrition and under complete information differ by 0.8% on average, but they differ significantly in duopoly markets (12.7%) and triopoly markets (3.6%).

To the best of my knowledge, almost no paper in the literature estimates a dynamic game with serially-correlated private values (a notable exception is Fershtman and Pakes (2010)). Two difficulties arise in estimating such models. First, to account for theaters' expectations, the entire history of the game should be included in the state space. It is difficult to do so in the framework of Ericson and Pakes (1995), which is commonly used in the literature. Second, the initial condition problem is more significant with serially-correlated private values, as players at the beginning of the sample period are selected samples. Because I account for these factors, I can estimate a game with serially-correlated private values, in comparison to much of the literature.

Related Literature

Schmidt-Dengler (2006) analyzes the timing of new technology adoption, separately estimating how it is affected by business stealing and preemption, respectively. In the environment he considers, the cost of adopting a new technology declines over time. One important difference is the source of inefficiency in oligopolistic markets. In Schmidt-Dengler (2006), players can delay competitors' adoption times by adopting before those competitors' adoption, even though such an adoption time is earlier than the stand-alone incentive would perceive as optimal. Thus, this preemption motive hastens the industry's adoption of new technology. In the current study, there is asymmetric information, so players have an incentive to delay their exits, hoping that they can outlast their competitors, even if they are currently making a negative profit.

Klepper and Simons (2000) and Jovanovic and MacDonald (1994) investigate the U.S. tire industry, where a large number of firms exited within a relatively short period of time. They assume this market is competitive. In their model, innovation opportunities encourage entry in the early stage of the industry. As the price decreases due to the new technology, firms that failed to innovate exit. Competition affects the devolution of the industry through the market price. In comparison, in the movie theater industry, competition was local, and hence strategic interactions among theaters should be taken into account. Another important difference is that the shakeout in the U.S. tire industry was not due to declining demand.

A number of papers analyze firm exits (Fudenberg and Tirole (1986), Ghemawat and Nalebuff (1985, 1990)). I estimate a modified version of Fudenberg and Tirole (1986), which has asymmetric information between players that delays their exits. A model similar to Ghemawat and Nalebuff (1985), one without asymmetric information, is used for my counterfactual analysis to evaluate the significance of asymmetric information. Ghemawat and Nalebuff (1990) consider a case where firms can continuously divest their capacity in an declining industry. While such a case is more sensible in many settings, as Section 2 will discuss, the current application fits better into a case of binary exit/stay decisions.

Bulow and Klemperer (1999) analyze a general game in which there are $N + K$ players competing for N prizes. My model is different because the value of the prize (operating profits) change over time and are affected by the number of surviving players, which is endogenous.

I use a full solution approach to estimate this dynamic game, instead of the 2-step method that has recently been used (e.g., Aguirregabiria and Mira (2007), Bajari, Benkard, and Levin (2007), Pakes, Ostrovsky, and Berry (2007), and Pesendorfer and Schmidt-Dengler (2008)). In the first stage of the 2-step method, a policy function is calculated for every possible state, which is difficult in a non-stationary environment. Moreover, unobservable variables play an important role in my model, so the first stage estimation in the 2-step method would not be consistent. For these reasons, I construct a tractable game, allowing me to repeatedly solve the equilibrium in the estimation algorithm and thereby account for the learning process.

The remainder of the paper is organized as follows. Section 2 briefly summarizes the background of the U.S. movie theater industry in the 1940's and 1950's. Section 3 modifies the model of Fudenberg and Tirole (1986) to be used in an oligopoly. In Section 4, I describe the data. Section 5 discusses my estimation strategy. I present estimation results and simulation analyses in Section 6. Section 7 concludes. All proofs are shown in the appendices.

2 The U.S. Movie Theater Industry in the 1940's and 1950's

After a big boom starting in the 1920's, the U.S. movie theater industry faced a severe decrease in demand in the 1950's and the 1960's, primarily due to the growth of television broadcasting. In 1950, less than one out of ten households in the U.S. owned a TV set. In 1960, however, the share of households with a television reached almost 90%. In response, demand for theaters decreased. Movie attendance declined most quickly in places where televisions were first available, providing evidence that TV penetration caused a decline in demand. According to Stuart (1976), the addition of a broadcast channel in a market caused an acceleration in the

decline in movie theater attendance.

There were other factors that contributed to the decline in demand. I focus on demand and exit behavior in the classic single-screen movie theater industry. Suburbanization also contributed to the downturn in demand for these movie theaters. Suburb growth and motorization facilitated the growth of drive-in theaters, which in turn further decreased demand for classic movie theaters.

Changes in government policy at the end of the 1940's also contributed to the downturn in demand.² Vertical integrations between producers, distributors, and exhibitors had been wide-spread until the late 1940's. The major movie producers (called "Hollywood majors"³) formed an oligopoly in which they colluded to divide the country. The major producers had control over theaters through exclusive contracts and explicit price management. They owned 3,137 of 18,076 movie theaters (70% of the first-run theatres in the 92 largest cities). However, the Paramount decree (1946), put an end to this vertical integration, resulting in the separation of those producers from their vertical chains of distributors and exhibitors. For example, explicit price management by distributors was prohibited. The government also mandated that the spun-off theater chains would have to further divest themselves of between 25 to 50 percent of their theater holdings.

This breakdown of the dominance of the major producers created a more unstable and risky business environment for movie theaters. For example, movie producers no longer had a strong incentive to produce movies year round. Furthermore, according to Lev (2003), the production companies started to regard television as an important outlet for their movies. In the era of vertical integrations, producers had an incentive to withhold their movies from televisions for the interest of their exhibitor-partners. However, after divestment, movie theaters became just one of the customers for producers, along with the television companies.

Because of all of these factors, demand for incumbent movie theaters shrunk in an arguably exogenous way. Figure 1 shows the average yearly theater attendance and the total number of indoor theaters from 1947 to 1960. Demand, implied by theater attendance, started to decrease in 1949, and kept declining mostly monotonically afterwards. Almost all theaters had only a single screen in those days (e.g., the first twin theater in the Chicago area opened in 1964), and their fixed investments were often heavily mortgaged. Therefore, they could not adjust capacity to deal with declining demand. They could only bear the loss and stay open or exit the market. Thus, the number of indoor movie theaters decreased with demand.

An outstanding fact in this figure is that the decline in the number of indoor theaters is

²The figures and facts in this paragraph are from Chapter 6 of Melnick and Fuchs (2004).

³Majors in this era include the "Big Five" (Loew's/MGM, Paramount, 20th Century-Fox, Warner Bros., and RKO) and the "Little Three" (Universal, Columbia, and United Artist).

slower than that of theater attendance. Since the number of theaters is affected by both entry and exit, new entrants may explain a small portion of this difference. As I discuss in section 4, however, entries of theaters in this time period were much less common than theaters' exits. One way to further explore this difference is to look at the relationship between exits of incumbent theaters and market structure. Figure 2 plots the share of theaters that were open in 1949 and remained open in each year until 1955⁴. As is clear from the figure, the exit rate increases with the number of competitors. One possible explanation is that theaters were trying to outlast their competitors in the declining environment. Since a few theaters could still operate profitably, each theater preferred to stay open as long as it expected some competitors to exit early enough. If there are many competitors, it is highly unlikely for a theater to be one of the few survivors at the end, so the theater may give up and exit early. This situation fits nicely into the framework of a war of attrition. Thus, in this paper, I use the framework of a war of attrition, exploiting the relationship between the number of competitors and exit probability to analyze the exit behavior of movie theaters.

The structure of the U.S. movie theater industry changed significantly in the 1960's, when multiplex theaters emerged. This arguably changed the nature of competition. Once a theater has multiple screens, it can potentially respond to a change in demand by, for example, closing several screens. The structure of the industry became even more different and complicated after the 1980's because of the advent of home videos/DVD and horizontal integrations by big theater chains. Thus, I focus on the period between 1949 and 1955.

3 The Model

I assume that theaters play an exit game in each local market. Markets are independent from each other. I suppress the market subscript in this section for notational simplicity. I modify the model of exit in duopoly developed by Fudenberg and Tirole (1986) to be used in an N-player oligopoly setting. I detail the three player case, because this setting is general enough in order to show the essence of this modification. I also focus on a symmetric equilibrium. Throughout this section, I use t to denote time, τ to denote a player's choice of exit time, and T to denote a player's strategy. I use the terms "player" and "theater" interchangeably in the rest of the paper.

⁴Figure 2 was constructed by my dataset, which contains theater information from 1949 to 1955 only.

3.1 Setup

Three theaters start the game at $t = 0$.⁵ Time is continuous. At each instant, each theater chooses whether to exit the market or to stay open. Once a theater exits the market, it cannot re-enter. At time t , if $n \leq 3$ theaters are operating, each theater receives the common value $\Pi_n(t)$. I assume that this instantaneous profit is deterministic and decreases in the number of surviving theaters; i.e., $\Pi_1(t) > \Pi_2(t) > \Pi_3(t)$ for all $t \in [0, \infty]$.

When a theater exits, it receives its value of exit θ , which is private information. At the start of the game, theater i draws θ_i from the common distribution G , which has a density g on the support $[\underline{\theta}, \bar{\theta}]$, and $g > 0$ everywhere. θ can also be interpreted as the fixed cost that a theater incurs at each instant. Each theater discounts the future at a common rate r .

The game proceeds as follows. At $t = 0$, each theater decides when to exit, conditional on none of the competitors having exited by then. In Online Appendix A, I show that this decision is equivalent to choosing to exit or to remain open at every instant. When one theater exits, other surviving theaters revise their exit times, based on the currently available information. A strategy is a mapping from the state space of each subgame to the real line (exiting time), $T : S \rightarrow [0, \infty]$, where S denotes the domain of the mapping.⁶ The domain of the strategy depends on the current number of survivors. I let T_1 , T_2 , and T_3 denote a player's strategy when there are one, two, and three surviving theaters in the game, respectively.

The value of being a monopolist is embedded into the two player game as the prize of winning the two player game. The value of playing the two-player game is in turn embedded into the three-player game. Because of the nested structure of the game, I begin with a monopolist's problem, then solve the two-player game, and finally characterize the three-player game. Throughout this section, let $t_{\mathcal{M}}$ and $t_{\mathcal{D}}$ denote the time when the one-player game and the two-player game start, respectively.

3.1.1 The One Player Case

Define the value of being a monopolist from time $t_{\mathcal{M}}$ onward as

$$V_1(\theta_i, t_{\mathcal{M}}) \equiv \max_{\tau \in [t_{\mathcal{M}}, \infty]} \left[\int_{t_{\mathcal{M}}}^{\tau} \Pi_1(s) e^{-r(s-t_{\mathcal{M}})} ds + \frac{\theta_i}{r} e^{-r(\tau-t_{\mathcal{M}})} \right]. \quad (1)$$

Thus, the strategy is a mapping from the type space and the current time to a real number; i.e., $T_1 : [\underline{\theta}, \bar{\theta}] \times \mathbb{R}_+ \rightarrow [t_{\mathcal{M}}, \infty]$. That is,

$$T_1(\theta_i, t_{\mathcal{M}}) \in \arg \max_{\tau \in [t_{\mathcal{M}}, \infty]} \left[\int_{t_{\mathcal{M}}}^{\tau} \Pi_1(s) e^{-r(s-t_{\mathcal{M}})} ds + \frac{\theta_i}{r} e^{-r(\tau-t_{\mathcal{M}})} \right], \quad (2)$$

⁵I use 1949 as the starting year of games, but for ease of exposition, I use $t = 0$ throughout the discussion of the model.

⁶I focus on a symmetric equilibrium, so the strategy does not have a theater subscript i .

which is simply a single-agent optimal stopping problem.

3.1.2 The Two Player Case

Suppose that one theater exited at $t = t_{\mathcal{D}}$. Theaters i and j are the remaining players in this subgame. The only information that each player has about its opponent is the fact that the opponent has survived up until $t_{\mathcal{D}}$. As I will show in Section 3.1.3, there exists the highest possible value of θ of surviving opponents, denoted by $\tilde{\theta}$, which is a sufficient statistic for the history of the game up until $t_{\mathcal{D}}$. For ease of exposition, I keep dependence of $\tilde{\theta}$ on other factors implicit. In the rest of this subsection, I take $(t_{\mathcal{D}}, \tilde{\theta})$ as given. The domain of the strategy of the two-player subgame is the product of the type space, the current time ($t_{\mathcal{D}}$), and the highest possible value of θ of surviving opponents; i.e., $S = [\underline{\theta}, \tilde{\theta}] \times \mathbb{R}_+ \times \mathbb{R}_+$. The strategy of this subgame T_2 is defined as $T_2 : [\underline{\theta}, \tilde{\theta}] \times \mathbb{R}_+ \times \mathbb{R}_+ \rightarrow [t_{\mathcal{D}}, \infty]$.

If theater i , with exit value θ_i , chooses stopping time τ , given that theater j follows some strategy T_2 , the present discounted value of i 's expected profit over time is

$$\begin{aligned} W_2 \left(\tau, T_2(\cdot, t_{\mathcal{D}}, \tilde{\theta}), t_{\mathcal{D}}, \tilde{\theta}; \theta_i \right) &= \Pr \left(T_2(\theta_j, t_{\mathcal{D}}, \tilde{\theta}) \geq \tau \right) \left[\int_{t_{\mathcal{D}}}^{\tau} \Pi_2(s) e^{-r(s-t_{\mathcal{D}})} ds + \frac{\theta_i}{r} e^{-r(\tau-t_{\mathcal{D}})} \right] \\ &+ \int_{\{\theta_j | T_2(\theta_j, t_{\mathcal{D}}, \tilde{\theta}) < \tau\}} \left[\int_{t_{\mathcal{D}}}^{T_2(\theta_j, t_{\mathcal{D}}, \tilde{\theta})} \Pi_2(s) e^{-r(s-t_{\mathcal{D}})} ds \right. \\ &\left. + e^{-r(T_2(\theta_j, t_{\mathcal{D}}, \tilde{\theta})-t_{\mathcal{D}})} V_1 \left(\theta_i, T_2(\theta_j, t_{\mathcal{D}}, \tilde{\theta}) \right) \right] g \left(\theta_j | \theta_j \leq \tilde{\theta} \right) d\theta_j. \end{aligned}$$

Notice that the value of being a monopolist $V_1(\theta, t)$ from equation (1) is embedded into this value function.

Now I define the equilibrium in the 2-player subgame.

Definition 1 For a given pair of $(t_{\mathcal{D}}, \tilde{\theta})$, $(\hat{T}_2(\theta_i, t_{\mathcal{D}}, \tilde{\theta}), \hat{T}_2(\theta_j, t_{\mathcal{D}}, \tilde{\theta}))$ is a symmetric Bayesian equilibrium of the two-player subgame if for all $l \in \{i, j\}$, $\theta_l \in [\underline{\theta}, \tilde{\theta}]$, and $\tau \geq t_{\mathcal{D}}$

$$W_2 \left(\hat{T}_2(\theta_l, t_{\mathcal{D}}, \tilde{\theta}), \hat{T}_2(\cdot, t_{\mathcal{D}}, \tilde{\theta}), t_{\mathcal{D}}, \tilde{\theta}; \theta_l \right) \geq W_2 \left(\tau, \hat{T}_2(\cdot, t_{\mathcal{D}}, \tilde{\theta}), t_{\mathcal{D}}, \tilde{\theta}; \theta_l \right).$$

I make the following assumption throughout this paper, unless otherwise stated.

Assumption 1 (i) $\Pi_n(t)$ decreases over time for $n = 1, 2, 3$. (ii) $\lim_{t \rightarrow \infty} \Pi_3(t) > \underline{\theta}$. (iii) $\Pi_1(0) < \tilde{\theta}$.

Section 3.2 discusses how to apply the argument of Fudenberg and Tirole (1986) to characterize the equilibrium. For the moment, assume that there exists a unique symmetric equilibrium. Let $(\hat{T}_2(\theta_i, t_{\mathcal{D}}, \tilde{\theta}), \hat{T}_2(\theta_j, t_{\mathcal{D}}, \tilde{\theta}))$ be the equilibrium strategies. Then, define

$$V_2(t_{\mathcal{D}}, \tilde{\theta}; \theta_i) \equiv W_2 \left(\hat{T}_2(\theta_i, t_{\mathcal{D}}, \tilde{\theta}), \hat{T}_2(\cdot, t_{\mathcal{D}}, \tilde{\theta}), t_{\mathcal{D}}, \tilde{\theta}; \theta_i \right).$$

That is, $V_2(t_{\mathcal{D}}, \tilde{\theta}; \theta_i)$ is i 's value of entering the two-player subgame with $\tilde{\theta}$ at $t = t_{\mathcal{D}}$ when theaters i and j both follow the equilibrium strategy $\hat{T}_2(\cdot, t_{\mathcal{D}}, \tilde{\theta})$. It is important to note that this value for the “worst” type (when $\theta_i = \tilde{\theta}$) can be calculated as

$$V_2(t, \tilde{\theta}; \tilde{\theta}) = \int_t^{\Pi_2^{-1}(\tilde{\theta})} \Pi_2(s) e^{-r(s-t)} ds + \frac{\tilde{\theta}}{r} e^{-r(\Pi_2^{-1}(\tilde{\theta})-t)}. \quad (3)$$

That is, if a player is the worst type at the moment, then the value of entering the subgame is simply the sum of the following two terms: duopoly profits earned until $\Pi_2(t)$ becomes equal to $\tilde{\theta}$ and the discounted sum of exit value from time $\Pi_2^{-1}(\tilde{\theta})$ on.

3.1.3 The Three Player Case

Since the three-player game starts at $t = 0$, no information has been updated at the start of the game. The strategy T_3 only has the type space as its domain; $T_3 : [\underline{\theta}, \bar{\theta}] \rightarrow [0, \infty]$. Suppose theaters i , j , and k start the game at $t = 0$. Let $t_{jk}^*(\theta_j, \theta_k) = \min \{T_3(\theta_j), T_3(\theta_k)\}$ for some strategies $T_3(\cdot)$. Let $T_{-i} = (T_3(\cdot), T_3(\cdot))$ denote a set of strategies followed by i 's opponents. Then, for $\tau \geq 0$, let

$$\begin{aligned} W_3(\tau, T_{-i}, \theta_i) = & \Pr(t_{jk}^*(\theta_j, \theta_k) \geq \tau) \left[\int_0^\tau \Pi_3(s) e^{-rs} ds + \frac{\theta_i}{r} e^{-r\tau} \right] \\ & + \int_{\{\theta_j, \theta_k | t_{jk}^*(\theta_j, \theta_k) < \tau\}} \left[\int_0^{t_{jk}^*(\theta_j, \theta_k)} \Pi_3(s) e^{-rs} ds \right. \\ & \left. + e^{-rt_{jk}^*(\theta_j, \theta_k)} V_2(t_{jk}^*(\theta_j, \theta_k), \tilde{\theta}; \theta_i) \right] g(\theta_j) g(\theta_k) d\theta_j d\theta_k. \end{aligned} \quad (4)$$

In equation (4), $W_3(\tau, T_{-i}, \theta_i)$ is theater i 's value when it exits at τ , given that theaters j and k follow some symmetric strategy $T_3(\cdot)$. Again, note that the value of playing the two-player subgame that a theater obtains in equilibrium, $V_2(t, \tilde{\theta}; \theta)$, is embedded into the value of the three-player game.

Remember that $\tilde{\theta}$ represents the highest possible value of θ of surviving opponents. For argument's sake, suppose for the time being that any equilibrium strategy $T_3(\theta)$ has a differentiable inverse function.⁷ Denote the inverse function as $\Phi_3(t)$. That is, $\Phi_3(t)$ is the value of exit for a player which in equilibrium exits at t . Then, $\tilde{\theta}$ in equation (4) is given by

$$\tilde{\theta} = \Phi_3(t_{jk}^*(\theta_j, \theta_k)). \quad (5)$$

To see this more clearly, assume that $\theta_k > \theta_l$. Since I am focusing on a symmetric equilibrium, this implies that $t_{jk}^*(\theta_j, \theta_k) = T_3(\theta_k)$.⁸ Substituting this into (5) gives $\tilde{\theta} = \theta_k$. That is, if player

⁷See Lemma 4.

⁸A symmetric equilibrium with a strictly monotonic strategy $T_3(\theta)$ implies that a player with a higher exit value exits the market earlier.

k exits first, the maximum possible value of exit for survivors is simply θ_k . This completes a full characterization of equation (4).

Definition 2 $(\widehat{T}_3(\theta_i), \widehat{T}_3(\theta_j), \widehat{T}_3(\theta_k))$ is a symmetric Bayesian equilibrium of the three-player subgame if for all $l \in \{i, j, k\}$, $\theta_l \in [\underline{\theta}, \bar{\theta}]$, and $\tau \geq 0$,

$$W_3(\widehat{T}_3(\theta_l), T_{-l}; \theta_l) \geq W_3(\tau, T_{-l}; \theta_l).$$

Putting all these subgames together, I define the equilibrium of the entire game.

Definition 3 A set of symmetric strategies $(\widehat{T}_1(\theta, t_{\mathcal{M}}), \widehat{T}_2(\theta, t_{\mathcal{D}}, \tilde{\theta}), \widehat{T}_3(\theta))$ with posterior beliefs $g(\theta|\theta \leq \tilde{\theta})$ is a symmetric perfect Bayesian equilibrium if

1. $\widehat{T}_1(\theta, t_{\mathcal{M}})$ is given by equation (2) for all $\theta \in [\underline{\theta}, \bar{\theta}]$ and $t_{\mathcal{M}} \in [0, \infty]$.
2. $\widehat{T}_2(\theta, t_{\mathcal{D}}, \tilde{\theta})$ constitutes a symmetric Bayesian equilibrium of this two-player subgame as given by Definition 1.
3. $\widehat{T}_3(\theta)$ satisfies the condition given by Definition 2.
4. For any opponent j , $g(\theta_j|\theta_j \leq \tilde{\theta})$ is given by

$$g(\cdot|\theta_j \leq \tilde{\theta}) = \begin{cases} \frac{g(\theta_j)}{\Pr(\theta_j \in \Theta)}, & \theta_j \in \Theta \\ 0, & \theta_j \notin \Theta \end{cases}$$

where Θ is the set of types θ that can survive on-equilibrium path.

3.2 Properties of the Symmetric Equilibrium

Since $T_1(\theta_i, t_{\mathcal{M}})$ is simply a solution to a single-agent optimal stopping problem, I start from $\widehat{T}_2(\theta_i, t_{\mathcal{D}}, \tilde{\theta})$. Fudenberg and Tirole (1986) model a two-player game with $t_{\mathcal{D}} = 0$ and $\tilde{\theta} = \bar{\theta}$. They show that any equilibrium strategy $\widehat{T}_2(\theta_i, 0, \bar{\theta})$ is strictly decreasing in θ_i and it has a differentiable inverse function. Denote the inverse function as $\Phi_2(t) \equiv T_2^{-1}(t, 0, \bar{\theta})$, where $\Phi_2(t)$ is the exit value of a player which in equilibrium exits at time t .

Given these properties, the first order condition of $W_2(\tau, T_2(\cdot, 0, \bar{\theta}), 0, \bar{\theta}; \theta_i)$ gives

$$\Phi_2'(t) = -\frac{G(\Phi_2(t))}{g(\Phi_2(t))} \left[\frac{\Phi_2(t) - \Pi_2(t)}{V_1(\Phi_2(t), t) - \Phi_2(t)/r} \right]. \quad (6)$$

Fudenberg and Tirole (1986) demonstrate that the differential equation (6) with the boundary conditions

$$\Phi_2(0) = \Pi_1(0) \quad (7)$$

$$\lim_{t \rightarrow \infty} \Phi_2(t) = \lim_{t \rightarrow \infty} \Pi_2(t) \quad (8)$$

has a unique solution. They also show that for all $t > 0$ and all i , $0 < \Pi_2(t) < \Phi_2(t) < \Pi_1(t)$. A typical form of $\Phi_2(t)$ is shown in Figure 3. These propositions in Fudenberg and Tirole (1986) still hold when $t_D > 0$ and $\tilde{\theta} \neq \bar{\theta}$. Before showing this, I construct the equilibrium in the three-player game. All proofs are presented in Online Appendix B.

The first order condition of equation (4) is given by

$$\Phi_3'(t) = -\frac{G(\Phi_3(t))}{2g(\Phi_3(t))} \left[\frac{\Phi_3(t) - \Pi_3(t)}{V_2(t, \Phi_3(t); \Phi_3(t)) - \Phi_3(t)/r} \right]. \quad (9)$$

This serves as a policy function that I use in estimation.

Lemma 4 *Given $\{T_3(\theta_i), T_3(\theta_j), T_3(\theta_k)\}$ is the equilibrium strategy, (i) For all $\theta \in [\Pi_2(0), \bar{\theta}]$, $T_3(\theta) = c > 0$ cannot be true. (ii) $T_3(\theta)$ is continuous and strictly decreasing on $(\underline{\theta}, \Pi_2(0))$. (iii) $\Phi_3(t)$ is differentiable on $(0, \infty)$. Its derivative is given by (9).*

Next, I construct the boundary conditions. For this purpose, I need the following assumption.

Assumption 2 *If more than one firm has the same optimal exit time, the firm with the highest value of θ exits first. Then, every other firm reappraises its optimal stopping time, taking into account the fact that there is now one less firm.*

Since $T_3(\theta)$ is strictly decreasing on $(\underline{\theta}, \Pi_2(0))$, the event where more than one firm has the same optimal exit time has measure zero when t is strictly positive. However, it is possible that exiting immediately is optimal for more than one firm. In this case, the symmetric equilibrium does not exist. Assumption 2 avoids this problem. See Haigh and Cannings (1989) and Argenziano and Schmidt-Dengler (2008) for discussion.

Lemma 5 *Suppose Assumption 2 holds. For one of the solutions to equation (9) to be an equilibrium, the boundary conditions should be given by*

$$\Phi_3(0) = \Pi_2(0) \quad (10)$$

$$\lim_{t \rightarrow \infty} \Phi_3(t) = \lim_{t \rightarrow \infty} \Pi_3(t). \quad (11)$$

The following proposition guarantees that one set of parameters in the model generates a unique solution.⁹

Proposition 6 *The symmetric equilibrium, if it exists, is unique.*

⁹I do not have a formal proof for existence. In estimation, however, for any set of parameters, I numerically find the $\Phi_3(t)$ that satisfies equations (9), (10), and (11). Moreover, as Proposition 6 shows, if I find a symmetric equilibrium, it is the unique symmetric equilibrium.

The next lemma establishes the shape of $\Phi_3(t)$.

Lemma 7 $0 < \Pi_3(t) < \Phi_3(t) < \Pi_2(t)$.

Now I discuss how players move from the three-player game to the two-player game when one player exits at time $t_{\mathcal{D}}$. The case with $t_{\mathcal{D}} = 0$ was already given by equations (6)-(8).

Lemma 8 *Suppose $\max\{\theta_i, \theta_j, \theta_k\} < \Pi_2(t)$. Assume player k is the first to exit and $T_3(\theta_k) > 0$. Players i and j do not exit at any $t \in [T_3(\theta_k), \Pi_2^{-1}(\theta_k)]$. That is, $\Phi_2(t) = \theta_k$ for $t \in [T_3(\theta_k), \Pi_2^{-1}(\theta_k)]$. For $t \in [\Pi_2^{-1}(\theta_k), \infty]$, the policy function $\Phi_2(t)$ follows (6) with the boundary conditions*

$$\Phi_2(\Pi_2^{-1}(\theta_k)) = \theta_k \tag{12}$$

$$\lim_{t \rightarrow \infty} \Phi_2(t) = \lim_{t \rightarrow \infty} \Pi_2(t). \tag{13}$$

After one player exits at some positive time, there will be no selection until the duopoly payoff becomes low enough. Once the selection restarts, the game becomes the two-player game with $t_{\mathcal{D}} = \Pi_2^{-1}(\theta_k)$ and $\tilde{\theta} = \theta_k$. Finally, I summarize the results.

Theorem 9 *Equations (2) and (6)-(11) constitute a symmetric Bayesian equilibrium of the entire game.*

Based on all the claims I have made so far, a typical path of $\Phi_3(t)$ is shown in Figure 4. Suppose that none of the players exit immediately and that player k is the first to exit, i.e., $\max\{\theta_i, \theta_j, \theta_k\} = \theta_k < \Pi_2(0)$. A typical shape of $\Phi_2(t)$ in this case is shown in Figure 4.

3.3 Computing the Equilibrium of the Model

For a given payoff function and exit values of theaters, I can calculate the equilibrium exit times. First, the policy function $\Phi_3(t)$ is obtained by solving the differential equation given by equations (9)-(11). Assuming $\theta_k = \max\{\theta_i, \theta_j, \theta_k\}$, theater k 's exit time, t_k , is such that $\theta_k = \Phi_3(t_k)$. Then, I set $\Phi_2(t) = \theta_k$ for $t \in [t_k, \Pi_2^{-1}(\theta_k)]$. That is, $\Phi_2(t)$ stays constant until it equals $\Pi_2(t)$. From this time on, $\Phi_2(t)$ is given by (6) with the boundary conditions (12) and (13). Assuming $\theta_j > \theta$, theater j 's exit time, t_j , is given by $\theta_j = \Phi_2(t_j)$. Finally, I find $T_1(\theta_i, t_j)$ from (2). Note that this procedure works in the same way for an N -player game.

The key to the tractability is that the evaluation of $V_2(t, \Phi_3(t); \Phi_3(t))$ in (9) does not involve opponent's strategy, as I showed in (3). Thus, computing an equilibrium repeatedly is feasible in my framework.¹⁰

¹⁰Other models such as Ericson and Pakes (1995) are hard to compute, and hence are difficult to use for a full solution approach.

4 Data

4.1 Data Source and Selection Criteria

The main data for this study comes from *The Film Daily Yearbook of Motion Pictures* (1949, 1950, 1951, 1952, 1954, and 1955)¹¹, which contains information on every theater that has existed in the U.S. The dataset includes the name, location, number of seats, and type of the theater (indoor, drive-in, etc.) of each theater.¹² Data does not show the exact date of exit, so I constructed the exit year in the following way. If a theater was observed in year t but not in year $t + 1$, I assume that the theater exited sometime between years t and $t + 1$.

I assume that wars of attrition started in 1949, when demand started to shrink rapidly in an exogenous way. I define all the non-drive-in theaters that were open in 1949 as players of the exit game. Theaters that entered after 1949 are treated as exogenous demand shifters. While the focus of this analysis is on single screen theaters, theaters that entered after 1949 were brand-new, and sometimes equipped with concession stands and nicer seats. There was certainly competition between classic single screen theaters and these new theaters. It is not unreasonable, however, to assume that the game I developed was played among old theaters, and the entry/exit of new theaters was exogenous from the viewpoint of the old theaters.

Movie theaters compete in local markets (Davis, 2005). In this paper, I define a market as a county. One big advantage of doing this is that data on demand shifters, such as TV penetration and demographics, are at the county-level. One drawback of this market definition is that a geographical area of each market may be too large, because customers would not drive for long time to go to a movie theater. Another problem is that some counties extend over many cities and contain hundreds of theaters (e.g., San Francisco county). To partially alleviate these problems, I focus on markets (counties) with less than or equal to 10 theaters in 1949. Because of this selection, 326 markets out of 3002 markets were dropped.

I assume that the diffusion of televisions was the main driving force behind the decline in demand for classic movie theaters. Gentzkow and Shapiro (2008a) provide TV penetration rates by county and year.¹³ The TV penetration rate is defined as the share of households which have at least one TV set. I discard markets that experienced a decrease in TV penetration, since the theoretical model presented in Section 3 accounts only for the case of monotonic demand changes over time. I specify the theater's profit as a decreasing function

¹¹The yearbook is not available for 1953.

¹²For location variables, the exact address is often missing. However, we know the name of the city where the theater is/was located. The number of seats is often missing, too.

¹³The dataset covers only 1950, 1953, 1954, and 1955 during the period of my study. Thus, I assume that every county had a TV penetration of 0% in 1949. Then, for all $t \in [1949, 1955]$, I use the linear interpolation.

of TV penetration, so I need to focus on markets with monotonically increasing TV penetration. Fortunately, the share of such markets is only about 3%. Thus, due to this selection, another 84 markets were dropped.¹⁴

Other data sources also provide across-market variations that will help to identify the theaters' payoff functions. Basic demographic/market variables, including population size, the median age, the median income, the share of population living in urban areas, the employment share of population, and land size, are obtained for every county from the U.S. Census. I also discard markets with missing covariates. Because of this, 78 markets were dropped from the sample.

Thus, for estimation, I am left with 2,513 markets, which have a total of 9,367 theaters in 1949.

4.2 Data Description

Table 1 shows the frequency of markets by the initial number of competitors. First note that there are a lot of monopoly markets, which under my assumptions on exogenous entry, helps identify the theater's payoff function, as decisions in such markets are a single-agent optimal stopping problem. The majority of markets has a few competitors in 1949; there are many duopoly and triopoly markets, and almost 80% of all markets have five competitors or less. In markets with more than 4 initial competitors, for example, around 30% of the competitors exited in the 6 years of the sample period. Meanwhile, only 15% of the competitors exited in monopoly markets.

Table 2 shows summary statistics of the market level variables. Population determines the potential market size of a county. I assume that the median age, family income, urban share, and employment share also shift demand. Counties are substantially different in terms of geographic sizes, which may affect the profitability of theaters. To account for this, I will also include land area in the theater's payoff function. As I discuss in the next section, I assume that these variables determine the base demand for theaters, which is market-specific and constant over time.

The diffusion process of televisions varies across markets. I calculated the 5th, 25th, 75th, and 95th percentiles of the TV penetration rate across markets for each year. In 1950, in the

¹⁴To see the effect of this selection, I calculate the summary statistic for markets with monotonic and non-monotonic TV penetration, respectively. The obvious difference is that the average size of markets with non-monotonic TV penetration is bigger, as they have an average of approximately 36,000 people compared to 22,500 people for markets with monotonic TV rates. The average number of initial competitors for markets with non-monotonic and monotonic TV rates is 4.52 and 3.68, respectively. I believe this selection does not produce/distort my estimation results.

90% of the markets, TV penetration rates were lower than 7%. In 1955, however, the 90% interval ranged from 9% to 92%, indicating a wide variation in the diffusion process across counties. This rich cross-section and time-series variation of the TV penetration rate is the main source of identification of theaters' payoff function.¹⁵ In estimation, I specify the demand decay as a flexible function of the TV penetration rates.

I also assume that the change in population during the sample period affects the decline in demand. The 5th, 25th, 75th, and 95th percentiles of the population change are -23.7%, -11.3%, 9.5%, and 37.8%, respectively. Because of these large population changes, it is important to control for population growth when measuring demand decline.

Finally, I address the entry of theaters. In 1,543 markets (61.4%), there was no entry in any year studied. In 634 markets (25.2%), the average number (yearly average) of theaters operating that entered after 1949 was one or smaller. That is, if this number in a market is 0.5, there was one new entrant and it operated for three years (out of six years in the sample) in the market. In the remaining 336 markets (13.4%), the corresponding average was bigger than one.

5 Estimation Strategy

I adopt a full solution approach instead of a two-step estimation strategy (e.g., Aguirregabiria and Mira (2007), Bajari, Benkard, and Levin (2007), Pakes, Ostrovsky, and Berry (2007), and Pesendorfer and Schmidt-Dengler (2008)). The full solution approach is preferred for the following reasons. First of all, the first stage estimation of the policy functions is most likely imprecise. In a two-step method, a policy function should be calculated for every possible state. In a declining environment, however, a state with low demand and a large number of competitors is rarely observed. Therefore, in this case, the small sample bias of two-step methods is a significant problem. Second, unobservable variables (theaters' learning and market level heterogeneity) play an important role in my analysis. However, it is difficult to introduce unobservable variables in a two-step method in a general way (see Aguirregabiria and Mira (2007) for example). For these reasons, I use a full solution approach.

This section is organized as follows. I begin by specifying the payoff function. Then in Section 5.2, I discuss the initial condition problem and propose a simulation method to deal with it. Finally, I present an indirect inference estimator and explain the estimation procedure in Section 5.3.

¹⁵I assume the diffusion of televisions across households is exogenous. Alternatively, Gentzkow and Shapiro (2008b) use the year in which each geographical market began receiving television broadcasts as an instrument for the TV diffusion across households.

5.1 Specification

To numerically solve the model, I parameterize the payoff function. From this section on, I use a market subscript $m \in \{1, \dots, M\}$ for all variables that differ across markets. Let $\Pi_n(t, m)$ be a theater's instantaneous profit in market m at time t , when there are n surviving theaters. I assume that

$$\Pi_n(t, m) = \pi_n(m) \cdot d(t, m),$$

where $\pi_n(m)$ is the base demand and $d(t, m)$ is a decay function. I assume that $d(t, m)$ decreases over time to satisfy Assumption 1 in Section 3. I assume that the decay function is given by

$$d(t, m) = \{1 - \lambda_0 TV_{tm} - \lambda_1 TV_{tm}^2\}^{\exp(\lambda_2 + \lambda_3 \Delta POP_m + \lambda_4 NEW_m)},$$

where TV_{tm} is the TV penetration rate in market m at time t , ΔPOP_m is the growth rate of population in market m throughout the sample period, and NEW_m is the total number of new entrants in market m during the sample period. I restrict $\lambda_0 \geq 0$, $\lambda_1 \geq 0$, and $1 - \lambda_0 TV_{tm} - \lambda_1 TV_{tm}^2 \geq 0$. A sufficient condition for the the last condition, for example, is $\lambda_0 + \lambda_1 \leq 1$. Introducing λ_1 can capture the accelerating/decelerating effect of the TV penetration on decay. Note that $d(t, m)$ is between zero and one, and decreases over time as long as TV_{tm} is increasing in time. This specification is flexible and potentially captures various types of decay.

The base demand is specified as additively separable:

$$\pi_n(m) = \alpha_m + X_m \beta + \delta \log(n_m + 1), \quad (14)$$

where α_m is unobservable (to the econometrician) market level heterogeneity, X_m is a vector of observable variables, and δ is a parameter that captures the effect of competition. I assume δ is negative to guarantee that $\Pi_n(t, m)$ is decreasing in n for all t . The set of observable variables X_m includes a constant, population size, the median age of population, the median income, the share of population living in urban areas, the employment share of population, and land size. I also assume that α , θ , and X are all ex-ante independent of one another.

In Online Appendix D, I discuss several assumptions of my econometric model.

5.2 The Initial Condition Problem

As mentioned earlier, if unobservable market heterogeneity (α_m in equation (14)) affected theaters' profit before 1949, the number of theaters in 1949 and market heterogeneity would be correlated through selection. In fact, there is wide variation in the initial number of theaters among similarly sized markets. Table 3 shows the average, standard deviation, maximum, and

minimum of numbers of theaters in 1949 by population size in 1950.¹⁶ Furthermore, even when I controlled for other observable covariates, there still was a variation in the initial number of theaters. Thus, it is important to account for unobservable market-level heterogeneity.

Firm level unobservables, if they are serially correlated, introduce an additional source of endogeneity.¹⁷ That is, the initial competitors (incumbent firms in 1949) are a selected sample so the distribution of exit values of incumbents is different from the population distribution. In addition, market heterogeneity and firms' exit values are correlated, even though they are assumed to be ex-ante independent.

Formally, letting $f_Y(y|z)$ be the joint density of $Y \in \mathbb{R}^K$, conditional on $Z = z$, the above argument suggests that

$$f_{\theta,\alpha}(\boldsymbol{\theta}, \alpha|n, X) \neq f_{\theta,\alpha}(\boldsymbol{\theta}, \alpha|X),$$

where n is the observed number of theaters in 1949 and X is a set of observable market covariates. In addition, by assumption, we have

$$f_{\theta,\alpha}(\boldsymbol{\theta}, \alpha|X) = f_{\boldsymbol{\theta}}(\boldsymbol{\theta}|X) f_{\alpha}(\alpha|X)$$

but

$$f_{\theta,\alpha}(\boldsymbol{\theta}, \alpha|n, X) \neq f_{\boldsymbol{\theta}}(\boldsymbol{\theta}|n, X) f_{\alpha}(\alpha|n, X).$$

Thus, I need to obtain $f_{\theta,\alpha}(\boldsymbol{\theta}, \alpha|n, X)$, which is endogenously generated from the game before 1949 in order to calculate the likelihood function or to simulate moments of the dynamic game of exit. One way to address this issue is to simulate the model starting from the time when the industry was born, and infer the distributions of $\boldsymbol{\theta}$ and α that are consistent with the industry structure in 1949. The movie theater industry, however, had a non-stationary structure in the sense that it experienced a boom in the 1920's and 1930's, and afterwards faced declining demand in the 1950's. Therefore, simulating the entire history of the industry requires one to model the life cycle of the industry, which is beyond the scope of the paper.

Instead, I approximate $f_{\theta,\alpha}(\boldsymbol{\theta}, \alpha|n, X)$ by simulation as follows. I assume that \bar{N} potential players play an entry game at time zero (in 1949 in my model), and entrants play the exit game afterwards. Specifically, each potential entrant draws its value of exit, and by comparing it with the expected value of entry (including market level heterogeneity), it chooses whether or not to enter the market. For a given value of market heterogeneity and set of exit values of all potential entrants, the entry game predicts the number of entrants in equilibrium. Since the initial number of theaters is observed, I use the entry game to restrict the support of market heterogeneity and exit values so that, in equilibrium, the entry game predicts the

¹⁶I use demographic data of 1950 because census data are available only every 10 years.

¹⁷Unobservable time-invariant exit values in this paper are an extreme example of this with perfect serial correlation.

same number of entrants as is observed. Since it is hard to analytically characterize such a restriction, I use a simulation to approximate the joint distribution of market heterogeneity and exit values, conditional on the observed number of entrants. This simulated distribution can then be used as an input to simulate and solve the dynamic stage of the game.

My approach can be regarded as a reduced form of a full simulation of the entire history of the industry. Heckman (1981) proposes to introduce an additional reduced form equation to account for the potential correlation between the initial state of a sample and unobservable heterogeneity, and jointly estimates the model with the additional equation. Thus, the current approach can also be regarded as the extended version of Heckman (1981)'s model in the context of games with serially correlated private information. Agguiregabiria and Mira (2007) also accounts for the initial condition problem associated with market level unobservables, but in their context private information are iid over time and thus players are not selected samples in that aspect.

To begin, I make the following assumption:

Assumption 3 *All potential players make an entry decision simultaneously at time zero without knowing that a war of attrition will immediately follow the entry game. That is, every player assumes that, upon entry, it earns the same per-period payoff forever.*

I first solve an entry game for each market repeatedly to simulate the joint distribution of (θ, α) given n . Then I randomly draw (θ, α) from the simulated distribution to solve the dynamic exit game (i.e., the war of attrition).

5.2.1 Entry Model

The model is similar to Seim (2006). \bar{N} potential players simultaneously decide whether or not to enter the market in 1949. Before making a decision, each player draws its own value of exit, θ , which is private information. If n_m players enter the market as a result of their decisions, the per-period payoff for each entrant is given by

$$\Pi_n(0, m) = \alpha_m + X_m\beta + \delta_e \log(n_m + 1), \quad (15)$$

where (β, δ_e) is the set of parameters to be estimated.¹⁸ Although I use the same form of $\Pi_n(0, m)$ as in (14), I allow the effect of competition, δ_e , to be different from δ in the base

¹⁸Rigorously speaking, $\Pi_n(0, m)$ should be given by

$$\Pi_n(0, m) = (\alpha_m + X_m\beta + \delta_e \log(n_m + 1)) d(0, m).$$

However, $d(0, m)$ is constant so I omit it from this argument for notational simplicity.

demand of the exit stage to capture the effect of the Paramount Decree on competition.¹⁹ On the other hand, if player i does not enter, it earns θ_{mi} . By Assumption 3, each player simply compares $\Pi_n(0, m)$ with its own value of exit when making an entry decision.

I assume α_m and θ_{mi} follow the distributions $N(\mu_\alpha, \sigma_\alpha^2)$ and $TN(\mu_\theta, \sigma_\theta^2; l_\theta, h_\theta)$, respectively, where $TN(\mu, \sigma^2; l, h)$ represents the truncated normal distribution with mean μ , variance σ^2 , and the lower and higher truncation points l and h . Note that these unobservables are carried over to the dynamic stage of the game.

Player i 's choice $D_i = \{0, 1\}$ is then given by

$$\begin{aligned} D_i &= \mathbf{1}\{\alpha_m + X_m\beta + \delta_e E[\log(n_m + 1)] \geq \theta_i\} \\ &= \mathbf{1}\left\{\alpha_m + X_m\beta + \delta_e \sum_{k=1}^{\bar{N}} \Pr(n_m = k) \log(k + 1) \geq \theta_i\right\}, \end{aligned}$$

where \bar{N} is the number of potential entrants. Let $P(X_m, \alpha_m)$ denote a player's subjective probability that a competitor enters the market. The symmetric equilibrium belief $P^*(X_m, \alpha_m)$ is thus given by a fixed point of the following equation:

$$P = G\left(\alpha_m + X_m\beta + \delta_e \sum_{k=1}^{\bar{N}} \binom{\bar{N}}{k} P^k (1 - P)^{\bar{N}-k} \log(k + 1)\right), \quad (16)$$

where G is the CDF of θ .²⁰ In the actual estimation, I assume $\alpha \sim N(0, \sigma_\alpha^2)$ and $\theta \sim TN(0, \sigma_\theta^2; 0, 15)$, where $(\sigma_\alpha^2, \sigma_\theta^2)$ are parameters to be estimated.²¹

Define $K(P) = \sum_{k=1}^{\bar{N}} \binom{\bar{N}}{k} P^k (1 - P)^{\bar{N}-k} \log(k + 1)$. The number of entrants predicted by this entry model equals the number of θ_i s in $\{\theta_1, \dots, \theta_{\bar{N}}\}$ that satisfy

$$\alpha_m + X_m\beta + \delta_e K(P^*(X_m, \alpha_m)) - \theta_i \geq 0. \quad (17)$$

In estimation, I set $\bar{N} = 11$.²²

5.2.2 Simulating the Joint Distribution of α and θ s

Let $F_{\theta, \alpha}(\boldsymbol{\theta}, \alpha | n, X)$ be the CDF of the density $f_{\theta, \alpha}(\boldsymbol{\theta}, \alpha | n, X)$. Based on the entry model, I simulate the joint distribution of θ and α conditional on n_m in the following way:

Step 1: Draw α^0 from $N(0, \sigma_\alpha^2)$.

¹⁹Since $\Pi_n(0, m)$ is the reduced-form profit, the Paramount Decree is expected to have affected β as well as δ . To reduce the number of parameters to be estimated, however, I assume the effect of the decree is captured by the difference between δ and δ_e .

²⁰Since $\delta_e < 0$, the symmetric equilibrium belief is unique.

²¹I arbitrarily choose $h_\theta = 15$. However, my estimation results are not sensitive to this choice. I re-estimate the model with $h_\theta = 12$ for example, and obtained very similar results.

²²I arbitrarily set \bar{N} at 11, since the maximum number of actual entrants is 10 in my sample.

Step 2: Calculate $P^*(X_m, \alpha^0)$ using (16). Then, define θ^* as

$$\theta^* = \alpha^0 + X_m\beta + \delta_e K(P^*(X_m, \alpha^0)).$$

That is, θ^* is the threshold of exit values below which a theater finds it profitable to enter the game.

Step 3: Draw a value of exit n_m times from $TN(0, \sigma_\theta^2; 0, \theta^*)$ and $\bar{N} - n_m$ times from $TN(0, \sigma_\theta^2; \theta^*, 15)$.

Sort these values in an ascending order. Call them $\theta^0 = (\theta_1^0, \dots, \theta_{n_m}^0, \theta_{n_m+1}^0, \dots, \theta_{\bar{N}}^0)$.

Step 4: Define α_l and α_h such that

$$\begin{aligned}\theta_{n_m}^0 &= \alpha_l + X_m\beta + \delta_e K(P^*(X_m, \alpha_l)) \\ \theta_{n_m+1}^0 &= \alpha_h + X_m\beta + \delta_e K(P^*(X_m, \alpha_h)).\end{aligned}$$

Step 5: Draw α^1 from $TN(0, \sigma_\alpha^2; \alpha_l, \alpha_h)$.

That is, α^1 is large enough to support n_m entries but not enough to support $n_m + 1$ entries.

Step 6: Return to Step 2 and repeat these steps J times to get $(\theta^j, \alpha^j)_{j=1}^J$.²³ Call this $\hat{F}_{\theta, \alpha|n_m}$.

Note that this procedure is done for each market. Any random draw (θ, α) from $\hat{F}_{\theta, \alpha|n_m}$ supports exactly n_m entrants in a symmetric equilibrium of the entry game.

5.3 An Indirect Inference Estimator

I use indirect inference for estimation. I choose several moments of the data which seemingly capture the relevant features of the data. I simulate moments from the model and minimize the distance between the simulated moments and data moments. The major advantage of this approach over likelihood is its ease of implementation. Above all, the fact that exact dates of exit are not available even though the model is constructed in continuous time makes calculating the likelihood a daunting task, and hence indirect inference is preferable.²⁴ I jointly use moments from the dynamic game and from the entry game.

²³I discard the first 100 sets of (θ^j, α^j) before storing.

²⁴For a discussion on several estimation issues in continuous-time models, see Arcidiacono, Bayer, Blevins, and Ellickson (2010). See also Doraszelski and Judd (2010) for a discussion on tractability of continuous time models.

To begin, I use p to denote the five time periods:

$$p = \begin{cases} 1 & \text{if } t \in [1949, 1950) \\ 2 & \text{if } t \in [1950, 1951) \\ 3 & \text{if } t \in [1951, 1952) \\ 4 & \text{if } t \in [1952, 1954) \\ 5 & \text{if } t \in [1954, 1955]. \end{cases}$$

Let γ be a set of structural parameters and ρ be a set of auxiliary parameters that summarize certain features of the data. For any arbitrary moment x , I use x and \hat{x} to denote the empirical and computed (from the model) moments, respectively. Thus, $\hat{\rho}(\gamma)$ denotes the set of auxiliary parameters estimated from the simulated data. Note that I keep the dependence of $\hat{\rho}$ on γ explicit. Let e_{mp} denote the rate of theaters' exit during period p in market m . Let n_{mp} be the number of theaters at the beginning of period p in market m . The elements in ρ are

$$M^{-1} \sum_{m=1}^M e_{mp} \quad \text{for } p = 1, 2, 3, 4, 5 \quad (18)$$

$$M^{-1} \sum_{m=1}^M e_{mp} n_{mp} \quad \text{for } p = 1, 2, 3, 4, 5 \quad (19)$$

and

$$M^{-1} \sum_{m=1}^M e_{m49-55} \Delta POP_{m49-55} \quad (20)$$

$$M^{-1} \sum_{m=1}^M e_{m49-55} NEW_{m49-55} \quad (21)$$

$$M^{-1} \sum_{m=1}^M e_{m49-52} \Delta TV_{m49-52} \quad (22)$$

$$M^{-1} \sum_{m=1}^M e_{m52-55} \Delta TV_{m52-55}, \quad (23)$$

where $e_{mt-t'}$ denotes the rate of theaters' exit between years t and t' , $\Delta POP_{mt-t'}$ ($\Delta TV_{mt-t'}$) is the change in population (TV penetration) from t to t' , and $NEW_{mt-t'}$ is the average number of theaters that entered the market after 1949 (the average is taken over years from t to t').

I also add moments from the entry model:

$$Cov(n_m, X_{mq}) \quad \text{for } q = 1, \dots, 6 \quad (24)$$

and

$$E(n_m), Var(n_m), \quad (25)$$

where X includes population size, the median age of population, the median income, the share of population living in urban areas, the employment share of population, and land size. Thus, there are 22 moments.

The elements in $\hat{\boldsymbol{\rho}}(\boldsymbol{\gamma})$ are the same as (18)-(25) with e_{mp} , n_{mp} , and n_m being replaced by their simulated counterparts, \hat{e}_{mp} , \hat{n}_{mp} , and \hat{n}_m , respectively. I use the entry game not only to approximate $F_{\theta, \alpha | n_m}$ but also to form the additional moments in equations (24) and (25).

The indirect inference estimator $\hat{\boldsymbol{\gamma}}$ is given by

$$\hat{\boldsymbol{\gamma}} = \arg \min_{\boldsymbol{\gamma}} (\boldsymbol{\rho} - \hat{\boldsymbol{\rho}}(\boldsymbol{\gamma}))' \Omega (\boldsymbol{\rho} - \hat{\boldsymbol{\rho}}(\boldsymbol{\gamma})), \quad (26)$$

where Ω is a positive definite weighting matrix.

Using $\hat{F}_{\theta, \alpha | n}$ obtained as described in Section 5.2.2, I can simulate the moments from the dynamic game of exit. The procedure to calculate the value of the objective function is described below.

First, I simulate \hat{n}_m . The following two steps describe this process.

Step 1: Choose the set of structural parameters $\boldsymbol{\gamma}$.

Step 2: Draw $\{\boldsymbol{\theta}^{ns}\}_{ns=1}^{NS}$ and $\{\alpha^{ns}\}_{ns=1}^{NS}$ independently from their ex ante distributions. Use (16) and (17) to solve the entry game to calculate \hat{n}_m^{ns} for $ns = 1, \dots, NS$ and form $\hat{n}_m = \frac{1}{NS} \sum_{ns=1}^{NS} \hat{n}_m^{ns}$.

Next I simulate \hat{e}_{mp} and \hat{n}_{mp} , using steps 3 to 5.

Step 3: For X_m and n_m , simulate $\hat{F}_{\theta, \alpha | n_m}$, following the process of Section 5.2.2.

Step 4: Draw $(\boldsymbol{\theta}^{ns}, \alpha^{ns})_{ns=1}^{NS}$ randomly from $\hat{F}_{\theta, \alpha | n_m}$. For each simulation draw, calculate the equilibrium of the dynamic game of exit: $\{(t_1^{ns}, \dots, t_{n_m}^{ns})\}_{ns=1}^{NS}$.

Step 5: Calculate the rate of theaters' exit for each time period \hat{e}_{mp}^{ns} and form $\hat{e}_{mp} = \frac{1}{NS} \sum_{ns=1}^{NS} \hat{e}_{mp}^{ns}$. In addition, calculate the number of theaters at the beginning of each time period \hat{n}_{mp}^{ns} and form $\hat{n}_{mp} = \frac{1}{NS} \sum_{ns=1}^{NS} \hat{n}_{mp}^{ns}$. Note that these steps are done for each market.

Finally, the objective function is evaluated in step 6.

Step 6: Calculate moments based on (18) to (25) and obtain the value of the criterion function

$$J(\boldsymbol{\gamma}) = (\boldsymbol{\rho} - \hat{\boldsymbol{\rho}}(\boldsymbol{\gamma}))' \Omega (\boldsymbol{\rho} - \hat{\boldsymbol{\rho}}(\boldsymbol{\gamma})).$$

Then, repeat Steps 1-6 to minimize $J(\boldsymbol{\gamma})$.

The vector of parameters to be estimated is

$$\boldsymbol{\gamma} = (\delta, \delta_e, \beta_1, \beta_2, \beta_3, \beta_4, \beta_5, \beta_6, \beta_7, \lambda_0, \lambda_1, \lambda_2, \lambda_3, \lambda_4, \sigma_\alpha, \sigma_\theta).$$

The estimator $\hat{\gamma}$ is consistent and the asymptotic distribution is

$$\sqrt{M}(\hat{\gamma} - \gamma) \xrightarrow{d} \mathcal{N}(0, W), \quad (27)$$

where W is given by

$$W = \left(1 + \frac{1}{NS}\right) [H'\Omega H]^{-1} H'\Omega (E\rho\rho') \Omega H [H'\Omega H]^{-1},$$

with $H = \partial\hat{\rho}(\gamma)/\partial\gamma'$. An optimal weight matrix $\Omega = (E\rho\rho')^{-1}$ is used so I have $W = \left(1 + \frac{1}{NS}\right) [H'(E\rho\rho')^{-1}H]^{-1}$. For implementation, I bootstrap the data 1000 times to get $\{\rho_b\}_{b=1}^{1000}$, and then calculate its variance-covariance matrix Σ_M . Then I replace $(E\rho\rho')^{-1}$ and H with Σ_M^{-1} and H_M , respectively.

6 Estimation Results

This section first presents parameter estimates. Using these estimated parameters, I then perform several counter-factual analyses. The model fit is presented in Online Appendix D.

6.1 Parameter Estimates

6.1.1 Base Demand

Table 4 presents estimates of the structural parameters. The coefficient on population (β_2) is positive and statistically significant. This implies that theaters earn higher profits in bigger markets. The coefficients on median age (β_3), income (β_4), urban share (β_5), and land area (β_7) are all positive and significant. The coefficients on employment share (β_6) is negative. One possible interpretation for this is that once I control for other observable and unobservable (to the econometrician) market characteristics, employed people have less time to watch movies. The parameter that captures competition (δ) is negative and significantly different from zero, implying that a theater's profit is eroded by competition. To see the relative sizes of these estimates, I calculate the value of base demand (14) at the sample mean of X . Then, $\bar{X}\hat{\beta} = 1.69$. The monopoly profit is given by $\bar{X}\hat{\beta} + \hat{\delta} \ln 2 = 1.51$. Duopoly and triopoly profits are 1.41 and 1.33, which are about 7% and 12% lower than monopoly profits, respectively. In the entry stage, using the same mean $\bar{X}\hat{\beta}$ and a different competition effect $\hat{\delta}_e$, duopoly and triopoly profits are 8% and 14% lower than monopoly profits, respectively.

6.1.2 Decay Function

Table 4 reports the parameters in the decay function. The fact that the quadratic term for TV rate (λ_1) is significantly different from zero means that the effect of TV diffusion is dampened

in the decay function, since TV_{mt} is less than one, and the values of λ_0 and λ_1 are restricted by $1 - \lambda_0 TV_{tm} - \lambda_1 TV_{tm}^2 \geq 0$. The coefficient on population growth (λ_3) is negative. Since $1 - \lambda_0 TV_{tm} - \lambda_1 TV_{tm}^2$ lies between zero and one, the negative estimate of λ_3 means that, if population grows faster, then the demand decay is less severe. That is, in a county with outflow of people, decay in demand is faster. The coefficient on the number of theaters that entered after 1949 (λ_4) is positive. Although this is insignificant (the p -value is 0.12), the sign is consistent with our intuition that entry of a new competitor erodes incumbents' profit.

6.1.3 Estimates of Standard Deviations

Estimates of σ_θ and σ_α are reported in Table 4. The standard deviation of exit values is 2.953 and is statistically significant. This implies that the mean of exit values is 2.356, and 95% of theaters have an exit value below 5.747.²⁵ Meanwhile, the standard deviation of market level heterogeneity is 0.340, which means that 95% of the value of heterogeneity is between -0.680 and 0.680. Compared with the value of base demand (14) evaluated at the sample mean of X and the estimated parameters (i.e., $\bar{X}\hat{\beta} = 1.69$), this variation explains a substantial proportion of the variation in initial numbers of competitors among similarly sized markets.

The variance of exit values can be interpreted as the extent of asymmetric information. If the variance is small, for example, a theater's assessment about its competitors' exit values is more precise. Hence, if a theater's value of exit is significantly higher than the mean when the variance is small, the theater would give up and exit relatively earlier. As the previous paragraph suggests, the value of exit ranges widely, implying that theaters should stay in the market in hopes of outlasting their competitors.

6.1.4 Role of Initial Conditions

If a market has a high unobservable demand shifter, we would observe more firms in the market than otherwise. If one ignores the unobservables in such a case, he would underestimate the negative effect of competition, since many firms appear to be able to operate profitably. On

²⁵Remember that μ_θ is normalized to zero. The mean value is calculated by

$$\mathbb{E}(\theta | 0 < \theta < 15) = \mu_\theta + \frac{\phi\left(\frac{a-\mu_\theta}{\sigma_\theta}\right) - \phi\left(\frac{b-\mu_\theta}{\sigma_\theta}\right)}{\Phi\left(\frac{b-\mu_\theta}{\sigma_\theta}\right) - \Phi\left(\frac{a-\mu_\theta}{\sigma_\theta}\right)}\sigma_\theta = 2.356$$

and the value of 5.747 is calculated by

$$0.95 = \frac{\Phi\left(\frac{5.747-\mu_\theta}{\sigma_\theta}\right) - \Phi\left(\frac{a-\mu_\theta}{\sigma_\theta}\right)}{\Phi\left(\frac{b-\mu_\theta}{\sigma_\theta}\right) - \Phi\left(\frac{a-\mu_\theta}{\sigma_\theta}\right)}.$$

the other hand, if the unobservable demand shifter is low, the number of competitors would be small. In such a case, one would infer that the effect of competition is strong. Furthermore, ignoring the initial condition problem affects estimates of other parameters as well.

To see the role of the initial condition problem, I estimate the model using the ex-ante distributions of market level heterogeneity and exit values; i.e., I use $F_{\theta, \alpha | n_m}$ instead of $\hat{F}_{\theta, \alpha | n_m}$. The second column of Table 4 shows the parameter estimates. The effect of competition in the dynamic game, δ , is estimated to be smaller in absolute value than the full model, while that in the entry game, δ_e is larger in absolute value. Thus, ignoring the initial condition problem significantly changes estimates of competition effects.

It is difficult, however, to evaluate the importance of initial conditions, since parameters in the payoff function are reduced-form parameters.²⁶ To better understand the role of initial conditions, I first simulate the model starting from 1949 until 1955, using the ex-ante distribution of unobservables. To fix observable variables, I only use Steuben county in Indiana for the simulation, as the TV penetration rate in 1955 and the total population in 1950 are close to the median values in the sample.²⁷ Then, I assume 1955 is the “initial” period of my hypothetical sample. Put differently, I assume that the industry was born in 1949 and the data is available from 1955. Panel (a) in Figure 5 plots the distribution of market level heterogeneity (α) conditional on the number of surviving theaters in 1955. Using the notation of section 5.2, this result clearly shows that $f_\alpha(\alpha | n, X) \neq f_\alpha(\alpha | X)$. Next, I simulate $\hat{F}_{\theta, \alpha | n_m}$ using the simulation method I proposed in section 5.2, and integrate over θ to calculate $\hat{f}_\alpha(\alpha | n, X)$ for $n = 4$. Panel (b) in Figure 5 plots this and compares it with the true distribution of α conditional on $n = 4$, which I already showed in panel (a), along with the ex-ante distribution of α . The simulated distribution well approximates the “true” distribution of α .

Since unobservable exit values are time invariant, selection based on exit values may be substantial too. In panel (c) in Figure 5, I plot the distribution of exit values when the industry was born in this exercise (i.e., ex-ante distribution) and the distribution of exit values of surviving theaters in 1955. As expected, these two distributions are significantly different

²⁶Another reason is that I use the entry model not only for addressing the initial condition problem but also for identifying parameters in the base demand. One alternative would be to identify all the parameters from “dynamic” moments only (e.g. use $Cov(e_{mp}, X_{mq})$ and $Var(e_{mp})$ instead of moments in (24) and (25)), and to use the entry model only for addressing the initial condition problem. Then, I would be able to compare the estimation results with and without the entry model. I did not use this strategy because the correlations between exit rates and X_m are weak in the data, and thus I wouldn’t well identify β in the base demand.

²⁷The TV penetration rate in 1955 and the total population in 1950 of Steuben county are 0.54 and 17,087, respectively. The median value for each variable is 0.54 and 17,031, respectively. The number of theaters in 1949 in this county was 3 in the data, but I start from 4 theaters to have more variations in the number of theaters in 1955.

from one another. In particular, the distribution of survivors' exit values has a high density at the low end compared to the unconditional distribution. I also calculated $\hat{f}_\theta(\theta|n=4, X)$ for surviving theaters in 1955, using the proposed simulation method. Panel (c) plots this distribution. Again, the simulated distribution very well approximates the true distribution of exit values of surviving theaters.

Thus, the proposed method shows a satisfactory performance. One caveat is that in this exercise the game played by the theaters before the "initial" period (i.e., 1955) is the exit game where the payoff function is very similar to that of the entry game. As I discussed in section 5.2, in the current application the type of game which was played before 1949 may have been different from the exit game. However, if the game before the initial period was different, the parameter estimates in the entry game would change accordingly, and hence be able to capture the correlation between n , θ , and α .

6.2 Simulation Analysis

6.2.1 Delay and Cost of Asymmetric Information

First, I quantify delays in exit generated by the war of attrition. As discussed in Section 3, theaters expect a higher profit if they outlast their competitors and thus stay until the expected benefit of waiting becomes lower than the expected cost of waiting. As time goes by, theaters become discouraged and exit if their competitors remain in the market. Notice that this dynamic selection may occur even if demand is not declining.²⁸ In order to quantify delays in exit generated by the war of attrition, I fix the TV penetration rate at its initial level in each market so that the decay function is constant over time. There are three types of theaters in equilibrium. The first set of theaters does not exit. Since demand is constant, their instantaneous profits are higher than their values of exit forever. The second set of theaters exits as soon as a war of attrition starts. They chose to enter the market in the static entry game. However, playing the exit game is not profitable for them, so they exit immediately. The third set of theaters stays in the market for a while, in hopes that they will outlast their competitors. Notice that this is due to asymmetric information. With complete information, they would exit the market immediately.

Holding demand constant, 146 theaters exited in the sample period. Therefore, 9,221 theaters remained in the market (the first set of theaters described above). Out of the 146 exits, 63 exits occurred right after the game started (the second set of theaters). Thus, the remaining 83 theaters stay in the market initially, hoping that they will outlast their

²⁸For example, the original game in Fudenberg and Tirole (1986) is mainly for the case of a growing industry. The case of constant demand may be simply thought of as a special case of either a declining or growing market.

competitors. Figure 6 plots the distribution of the months before they exit. Most of the delays are less than 2 years. However, there are a small number of theaters that wait more than 5 years because of outlasting motives. This can possibly accumulate a big negative profit.

These delays come from asymmetric information. If exit values are common knowledge, in equilibrium, any theater exits the game exactly when its profit becomes lower than its exit value. Thus, no theater incurs a loss.²⁹ I call this the complete information case. The difference in cumulative industry profits under a war of attrition and complete information can be regarded as the cost of asymmetric information. Let n_s and n_s^* represent the number of theaters at time s in the war of attrition and in complete information, respectively.

Define Q_m and Q_m^* as

$$Q_m = \int_{\underline{s}}^{1955} \sum_{k=1}^{n_s} [\Pi_{n_s}(s, m) - \theta_k] e^{-rs} ds \quad (28)$$

$$Q_m^* = \int_{\underline{s}}^{1955} \sum_{k=1}^{n_s^*} [\Pi_{n_s^*}(s, m) - \theta_k] e^{-rs} ds, \quad (29)$$

where \underline{s} denotes the moment when at least one theater started to earn a negative profit in the market (when the war of attrition started). That is, Q_m measures the cumulative profits that all theaters in market m have earned since the firm with the highest exit value started to earn a lower profit than its exit value. Then, I calculate the percentage difference:

$$\frac{\sum_{m=1}^{2513} Q_m^* - \sum_{m=1}^{2513} Q_m}{\sum_{m=1}^{2513} Q_m} = 0.768\%.$$

Thus, the industry profit will increase by 0.768% under complete information. Define

$$R_m = \int_{\underline{s}}^{1955} \sum_{k=1}^{n_s} \mathbf{1}_{\{\Pi_{n_s}(s, m) - \theta_k < 0\}} [\Pi_{n_s}(s, m) - \theta_k] e^{-rs} ds.$$

This is similar to (28), but R_m sums up only the negative part of the theaters' profit. Then, I calculate the relative magnitude of negative profits as follows:

$$\frac{\sum_{m=1}^{2513} R_m}{\sum_{m=1}^{2513} Q_m} = -0.062\%.$$

This is significantly smaller than $(\sum_{m=1}^{2513} Q_m^* - \sum_{m=1}^{2513} Q_m) / \sum_{m=1}^{2513} Q_m$. Although the negative profit is not negligible, most of the differences in the industry profit between the war of attrition and complete information come from lowered profits of surviving theaters due to business stealing. To better understand the source of the cost of asymmetric information, Table

²⁹A loss or a negative profit in this context is in terms of economic profit. That is, if the profit of a theater is lower than its exit value (the value of outside option), I call this "incur a loss" or "make a negative profit."

5 summarizes these statistics by the number of initial competitors. The cost of asymmetric information differs greatly across markets. The difference in a duopoly (12.7%) is substantially larger than any other market structure. For a given player, the probability of winning the war of attrition, i.e., the probability of being a monopolist, is highest in a duopoly, and therefore theaters have the greatest incentive to wait. Moreover, the increment of profit when one competitor exits is highest in duopoly, which also partly explains the big difference in the industry profit between the two cases. As n_m gets large, competition becomes closer to perfect competition, and hence outlasting motives become smaller.

Finally, I calculate the incidence of delayed exits. The differences calculated above are conditional on theaters' exits being delayed. To see the significance of delayed exits in the industry, I calculate the number of months delayed, splitting the sample by the number of initial competitors. This is reported in the last column of Table 5. In duopoly markets, on average, some theater makes a negative profit for a month in the sample period. Thus, the above result means that, during the one month, the total profit for duopoly markets decreases by 12.7% compared to the complete information benchmark. The number of months delayed is smaller for markets with a larger number of initial competitors.

6.2.2 Source of Variation in Exit Process

I also quantify sources of across-market variation in theaters' exits. I focus on variables in the decay function, TV penetration rates, population growth (ΔPOP_m), the number of theaters that entered after 1949 (NEW_m). Since these three variables enter the decay function in a non-linear way, it is difficult to fully decompose their effects. For every county, I set the value of each variable at certain common levels to see how the equilibrium exit process will change. For each variable, I use the 25th and 75th percentiles. The 25th and 75th percentiles of population growth are -11.3% and 9.5%, respectively. For TV penetration, I use the TV penetration rate in 1955 to calculate the 25th and 75th percentiles (34.4% in Cook county in Minnesota for the 25th percentile and 72.9% in Nolan county in Texas for the 75th percentile). In terms of new entrants, the majority of counties had no entrant. Thus, I fix NEW_m at one and two, and calculate the equilibrium path in each of these scenarios.

Table 6 summarizes the results. When I use the TV penetration rates of Cook county for every county, the exit process is significantly delayed. The number of surviving theaters in 1955 increases by 21.1% as compared to the equilibrium outcome. When I use the TV penetration rates of Nolan county instead, the number of surviving theaters in 1955 decreases by 10.4%. Meanwhile, population growth explains a small portion of the variation of the exit process. The demand decline is severe in a county with a large population outflow (a county of at the 25th percentile). In particular, the number of surviving theaters in 1955 is only 5.7% less

than in equilibrium. In terms of NEW_m , the number of surviving theaters in 1955 decreases only by 5.2% even in a scenario where NEW_m is set at two. Note that the estimate for the parameter on the number of theaters that entered after 1949 (λ_4) is statistically insignificant, so this result should not be taken as conclusive. Overall, most of variations in the exit process across markets are explained by the difference in TV penetration.

Finally, it is worth mentioning the effect of a delayed/hastened diffusion process of television on the theaters' exits. If I use the 25th percentile of TV penetration rates, the survival rate of theaters would increase by more than 20%. On the other hand, if the diffusion process is hastened (i.e., using the 75th percentile of TV penetration rates), the survival rate would increase by 10%. This is potentially policy-relevant because delaying the diffusion of a new technology is an important option when the government wants to protect (at least in the short run) the affected sector. Similarly, the government may want to use the diffusion of a new technology to facilitate the replacement of the old technology. We would like to know the effectiveness of such a policy.

7 Conclusion

This paper estimates the effect of competition among theaters and their beliefs about competitors separately from the effect of exogenous demand decline on the devolution of the industry. I modify Fudenberg and Tirole (1986)'s model of exit in duopoly with incomplete information to work in an oligopoly. I use data on the U.S. movie theater industry and rich cross-section and time-series variations of TV penetration rates to estimate theaters' payoff functions and the distribution of exit values. By imposing the equilibrium condition, the model predicts the distribution of theaters' exit times for a given set of parameters and unobservables. I use indirect inference and estimate the model parameters by matching the predicted distribution with the observed distribution of exit times.

I control for the endogeneity generated by the correlation between the initial number of theaters and the unobservable variables in the model. To achieve this goal, I set up a model of entry at the beginning of my sample period and use a simulation to approximate the joint distribution of unobservable variables, consistent with the observed number of entrants. This simulated distribution can be used to simulate the theoretical distribution of exit times. The simulation analysis shows that the proposed method can well approximate the distribution of market level heterogeneity and exit values of surviving theaters conditional on the observed number of initial competitors.

Using the estimated model, I show that many theaters delay their exits, even when making negative profits, in hopes that their competitors will exit first. The difference between the

total net profit in the war of attrition and under complete information can be regarded as the cost of asymmetric information. I find that the cost of asymmetric information is relatively large in markets with a small number of competitors.

A war of attrition is costly for all theaters. If they are allowed, theaters may want to negotiate with their competitors on exit times. For example, the theater that exits early could be compensated by another theater that stays in the market. Such an arrangement can potentially improve all theaters' payoffs. However, this is illegal under anti-trust laws. Policymakers could consider an exception to the anti-trust laws in this case because sometimes a war of attrition is very costly. For example, think of the following mechanism. Each theater would "bid" the amount of compensation with which it is willing to exit immediately. Based on each theater's "bid," the mechanism assigns an outcome so that every theater is weakly better off.

The framework in this paper can be applied to analyze other industries where exogenous demand decline creates a nonstationary environment in an oligopoly. Think of the example of video rental stores. The government may potentially want to delay the diffusion process of online movie services to mitigate the severity of demand decline for video rental stores. Policies targeted toward import competing sectors are also relevant. Consider an unskilled-labor intensive industry that will inevitably shrink in the long run due to competition from low-wage countries. The government wants to use trade policies to maximize the national welfare. If the market is an oligopoly, competition among firms and their expectations about competitors that I emphasize in this paper may play an important role. I believe that the framework proposed here can be used to discuss several alternative policies in such environments.

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Tables and Figures

Table 1: Number of competitors

Number of Theaters in 1949	Frequency	Percent
1	433	17.2
2	506	20.1
3	434	17.3
4	355	14.1
5	242	9.6
6	199	7.9
7	137	5.5
8	80	3.2
9	73	2.9
10	54	2.2
Total	2,513	100

Table 2 : Summary statistics of demographic variables in 1950

Variable	Obs	Mean	Std. Dev.	Min	Max
Population	2,513	22,460	19,070	1,870	194,182
Median age	2,513	4.09	1.03	1.00	7.00
Median family income	2,513	4.88	1.56	0.00	9.00
Urban share	2,513	0.24	0.23	0.00	0.95
Employment share	2,513	0.96	0.02	0.81	1.00
Land area (square miles)	2,513	965	1,288	46	18,573

Note: Median age and median family income are categorical variables.

Source: Hanes, Michael R., and the Inter-university Consortium for Political and Social Research.

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Table 3: Summary statistics of number of theaters in 1949 by population size

Population of counties	Observations	Mean	Std. Dev.	Minimum	Maximum
0-10,000	619	2.0	1.1	1	7
10,000-20,000	851	3.0	1.6	1	9
20,000-30,000	457	4.2	1.9	1	10
30,000-40,000	250	5.4	2.2	1	10
40,000-50,000	155	6.2	2.1	1	10
50,000-75,000	116	7.2	1.8	2	10
75,000-100,000	43	7.9	1.8	5	10
100,000-150,000	20	8.5	1.2	6	10
150,000+	2	8.0	1.4	7	9

Source: Author's calculation based on the population data used in Table 2 and theater's data from The Film Daily Yearbook of Motion Pictures.

Table 4: Estimates of structural parameters

Variables	Full Model	Model without initial condition adjustment
$\bar{\delta}$ (competition in dynamic stage)	-0.260 *** (0.0234)	-0.094 * (0.0519)
$\bar{\delta}_e$ (competition in entry stage)	-0.308 *** (0.0083)	-1.073 *** (0.0616)
β_1 (constant)	1.891 *** (0.0655)	1.577 *** (0.2452)
β_2 (population)	2.404 *** (0.1118)	3.850 *** (0.2797)
β_3 (median age)	1.090 *** (0.0447)	1.482 *** (0.1526)
β_4 (median income)	1.193 *** (0.0952)	1.094 *** (0.1589)
β_5 (urban share)	0.732 *** (0.0148)	0.506 *** (0.1313)
β_6 (employment share)	-2.038 *** (0.0971)	-1.323 *** (0.2326)
β_7 (land area)	0.033 *** (0.0028)	0.166 *** (0.0513)
λ_0 (TV rate)	0.0001 (0.0090)	0.076 *** (0.0256)
λ_1 (TV rate squared)	0.943 *** (0.1567)	0.382 *** (0.0648)
λ_2 (constant in power function)	-0.487 *** (0.1525)	1.167 *** (0.1186)
λ_3 (change in population in power function)	-1.306 *** (0.2853)	-1.062 *** (0.2823)
λ_4 (new entrants in power function)	0.159 (0.1033)	0.285 *** (0.0353)
σ_θ (std. of exit value)	2.953 *** (0.0733)	2.167 *** (0.1481)
σ_α (std. of demand shifter)	0.340 *** (0.0107)	0.614 *** (0.0395)
J Statistic	452.287	274.052
(p-value)	(0.000)	(0.000)
No. of Obs.	2,513	2,513

Note:

Standard errors in parentheses.

*** p<0.01, ** p<0.05, * p<0.1

Low p values in the J statistic mean that I cannot reject the over-identifying restrictions.

Table 5: Cost of asymmetric information, by number of initial competitors

# of initial competitors	# of markets	Q_m	Q_m^*	R_m	$(Q_m^* - Q_m)/Q_m$	R_m/Q_m	Average # of months
1	433	0.0	0.0	0.0	-	-	0.00
2	506	637.1	717.9	-12.3	12.677%	-1.924%	1.15
3	434	2,370.4	2,454.7	-7.2	3.558%	-0.304%	1.10
4	355	4,656.5	4,717.4	-3.6	1.308%	-0.076%	0.88
5	242	5,378.1	5,411.4	-1.4	0.619%	-0.026%	0.67
6	199	7,047.4	7,073.8	-0.9	0.375%	-0.013%	0.62
7	137	6,392.9	6,410.8	-0.5	0.280%	-0.007%	0.58
8	80	4,901.3	4,908.8	-0.2	0.154%	-0.004%	0.43
9	73	5,480.5	5,488.4	-0.2	0.143%	-0.003%	0.47
10	54	5,392.5	5,398.1	-0.1	0.103%	-0.002%	0.47
Total	2,513	42,256.8	42,581.4	-26.3	0.768%	-0.062%	

Note: Let Q_m and Q_m^* be the total cumulative profit (less value of exit) earned by all theaters in market m in a war of attrition and in the case of complete information, respectively. Let R_m be the total negative profit earned by all theaters in market m . Then, I calculate $(Q_m^* - Q_m) / Q_m$ and define this as a cost of asymmetric information. I also calculate R_m/Q_m . The table above summarizes this statistic by the number of initial competitors.

Table 6: Sources of cross-market variation in exits

	Number of theaters					
	1949	1950	1951	1952	1954	1955
Data	9,367.0	9,140.0	9,112.0	8,996.0	8,138.0	6,950.0
In equilibrium	9,367.0	9,229.5	9,194.8	9,087.8	8,321.9	7,376.7
Different Scenarios						
<u>TV diffusion rate fixed at:</u>						
25th percentile	9,367.0 (0.00%)	9,230.1 (0.01%)	9,214.4 (0.21%)	9,187.0 (1.09%)	9,038.4 (8.61%)	8,933.7 (21.11%)
75th percentile	9,367.0 (0.00%)	9,218.1 (-0.12%)	9,182.9 (-0.13%)	9,045.3 (-0.47%)	8,114.9 (-2.49%)	6,606.3 (-10.44%)
<u>Population change fixed at:</u>						
25th percentile	9,367.0 (0.00%)	9,226.6 (-0.03%)	9,175.9 (-0.21%)	9,021.7 (-0.73%)	8,057.1 (-3.18%)	6,959.8 (-5.65%)
75th percentile	9,367.0 (0.00%)	9,229.9 (0.00%)	9,192.6 (-0.02%)	9,080.6 (-0.08%)	8,330.6 (0.10%)	7,445.6 (0.93%)
<u># of theaters that entered after 1949 fixed at:</u>						
One	9,367.0 (0.00%)	9,229.3 (0.00%)	9,193.4 (-0.02%)	9,077.6 (-0.11%)	8,271.7 (-0.60%)	7,291.1 (-1.16%)
Two	9,367.0 (0.00%)	9,227.7 (-0.02%)	9,184.9 (-0.11%)	9,045.3 (-0.47%)	8,109.0 (-2.56%)	6,991.1 (-5.23%)

Note: To identify the source of variations in exit times across markets, I fix the TV diffusion rate of each county at the 25th and 75th percentiles of the sample, and calculate the equilibrium path. Similarly, I fix the growth rate of population of each county at the 25th and 75th percentile of the sample, and calculate the equilibrium path. I also calculate the equilibrium exit rate for two additional scenarios, when one new theater enters the market after 1949 and when two new theaters enter the market after 1949. The numbers in parentheses are deviations (in percentage) from the original equilibrium.

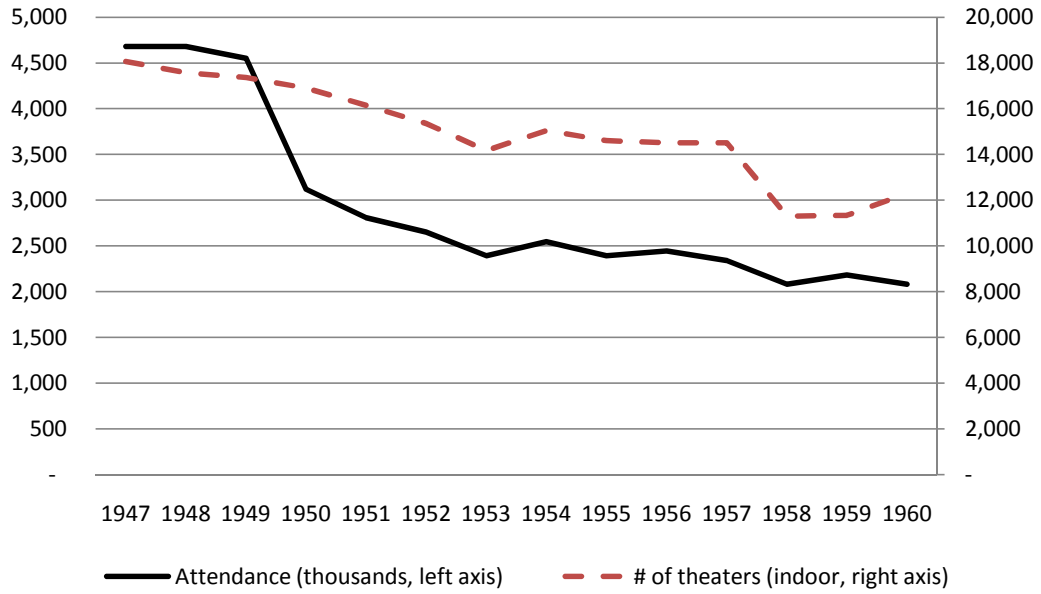


Figure 1: Movie attendance and number of movie theaters

Note: The movie theater yearly attendance and the total number of indoor movie theaters.
 Source: The Film Daily Yearbook of Motion Pictures.

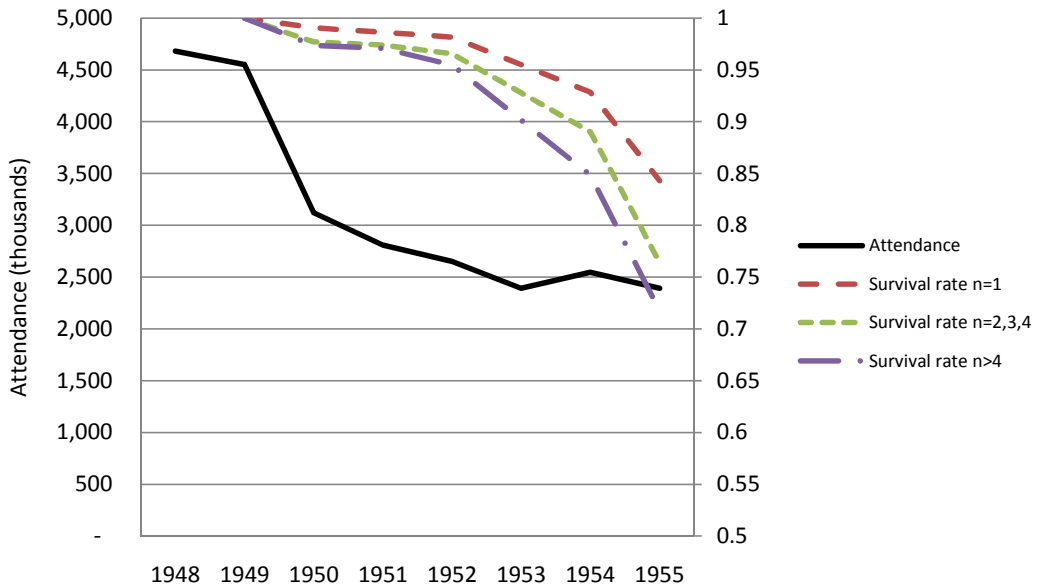


Figure 2: Movie attendance and survival of movie theaters

Note: The movie theater yearly attendance (left axis) and the survival rates by the initial number of theaters (right axis).
 Source: The Film Daily Yearbook of Motion Pictures and author's calculation.

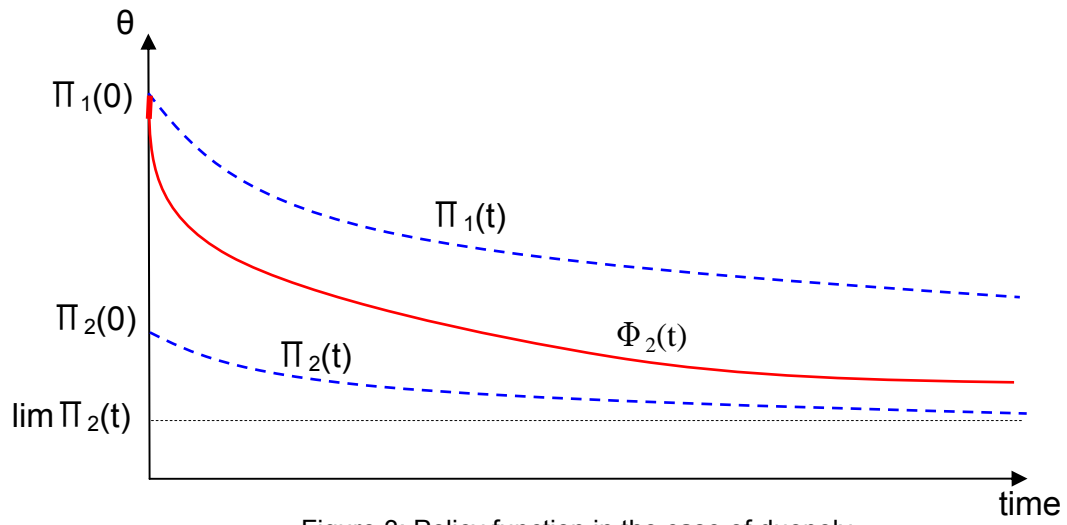


Figure 3: Policy function in the case of duopoly

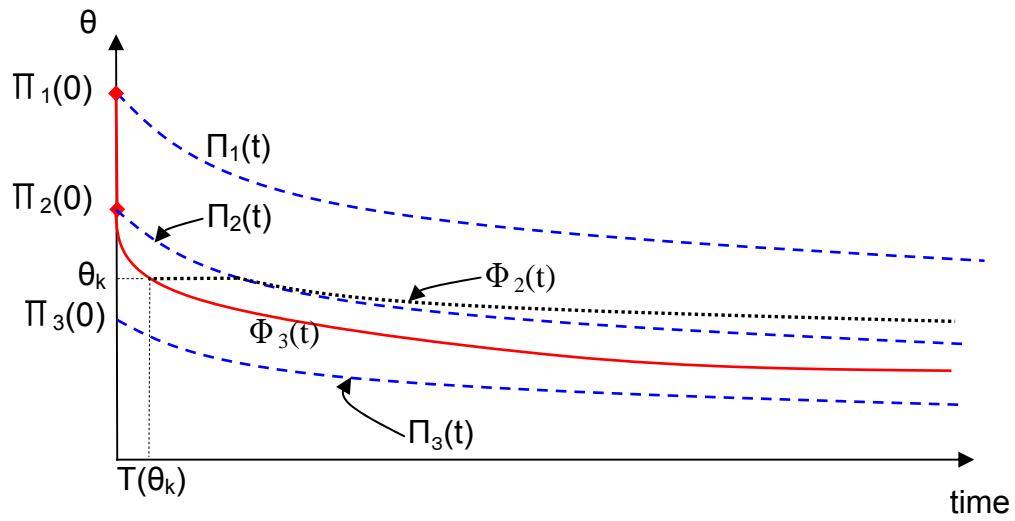


Figure 4: Policy function in the case of oligopoly

Player k exits at $T(\theta_k)$ and surviving players form a new policy function, $\Phi_2(t)$.

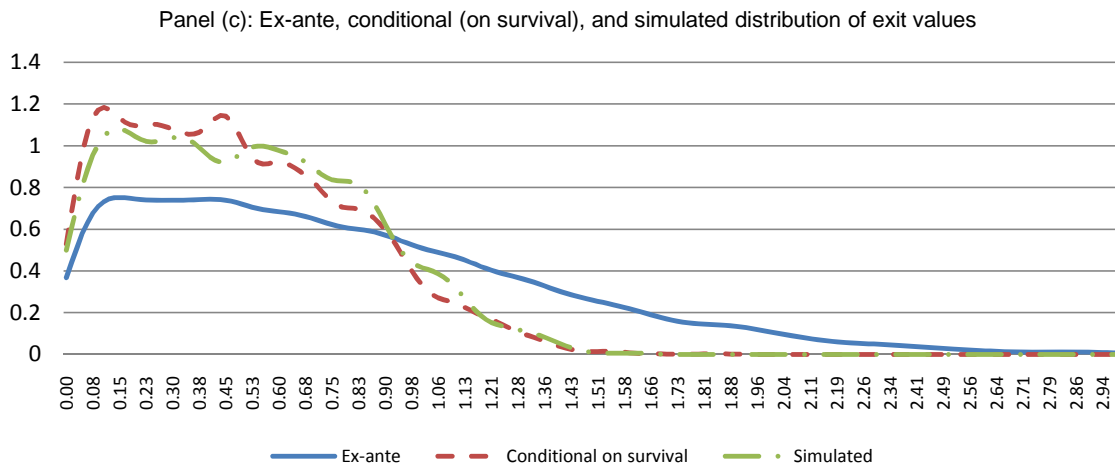
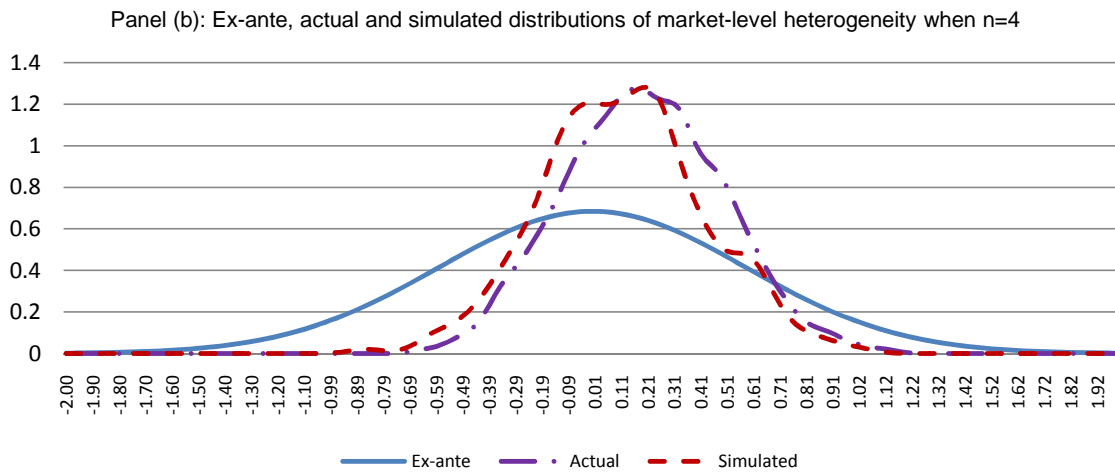
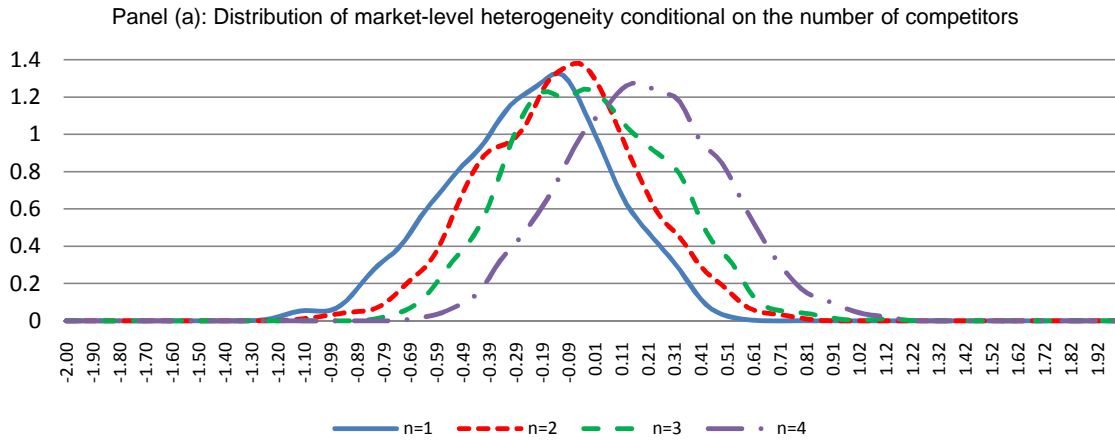


Figure 5 Importance of initial condition problem

Note: I simulate the model many times starting from 1949 until 1955, using the ex-ante distributions of unobservables. I fix observable covariates at the level of the median county. Panel (a) plots the distribution of market level heterogeneity, splitting the simulated outcomes by the number of surviving theaters in 1955. Then, assuming that 1955 is the initial period of my hypothetical sample, I simulate the distribution of market level heterogeneity for $n=4$, using the method I described in section 5. Panel (b) plots the resulting distribution and the distribution conditional on $n=4$ from panel (a), along with its ex-ante distribution. Finally, panel (c) plots the ex-ante, conditional (on survival in 1955), and simulated distributions of exit values, which I obtained from the above simulation. The difference between the unconditional and conditional distribution shows the significance of the selection process. The similarity between the conditional and simulated distributions shows that the simulation method I proposed approximates the conditional distribution very well.

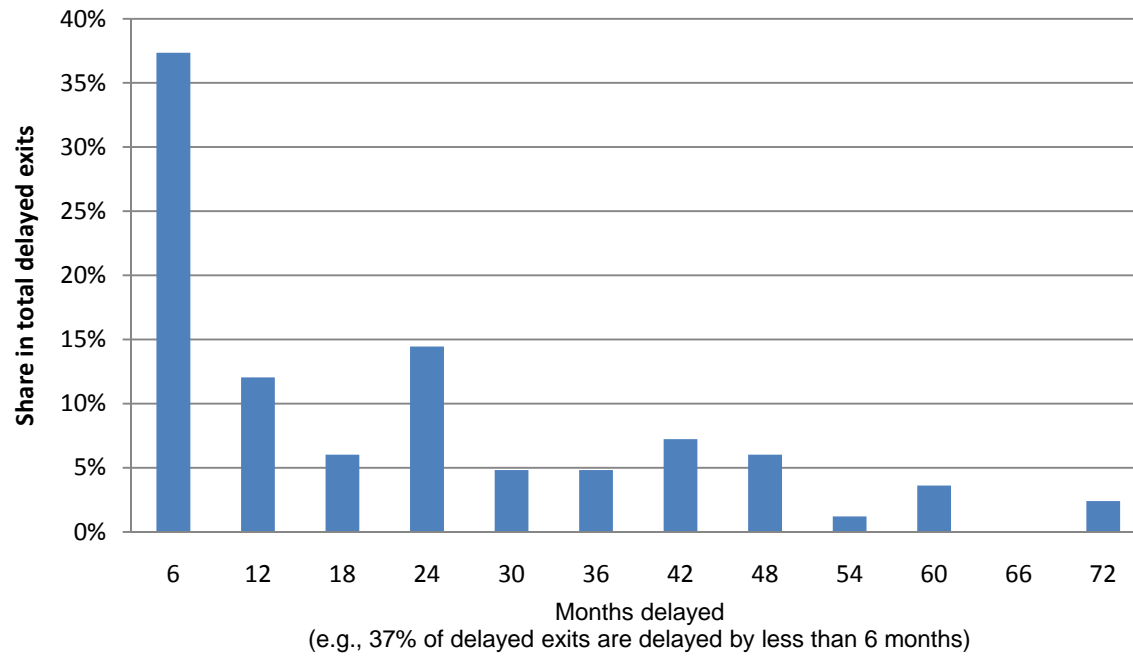


Figure 6: Distribution of months delayed
 Out of 146 total exits in a constant demand environment, 83 exits are delayed.

1 Appendix A (Online Appendix, Not for Publication)

Claim Consider a two player game. Pick θ arbitrarily such that $\theta > \lim_{t \rightarrow \infty} \Pi_2(t)$. Let $T_2(\cdot, 0, \bar{\theta})$ be the equilibrium strategy of the game if the two player subgame starts at $t = 0$. Let $\Phi_2(t, 0, \bar{\theta})$ be its inverse. For any t_0 such that $0 < t_0 < T_2(\theta, 0, \bar{\theta})$, let $T_2(\theta, t_0, \tilde{\theta})$ denote the equilibrium strategy of the game starting from t_0 . Let also $\Phi_2(t, t_0, \tilde{\theta})$ be its inverse. Then, $\tilde{\theta} = \Phi_2(t_0, 0, \bar{\theta})$ and $T_2(\theta, 0, \bar{\theta}) = T_2(\theta, t_0, \tilde{\theta})$.

In words, the optimal exit time that is planned at time zero equals the one that is planned at some later time, conditional on nobody having exited until then.

Proof of Claim Since both players follow the same strategy, player i knows that θ_j is equal to or smaller than $\Phi_2(t_0, 0, \bar{\theta})$. If not, player j would be better off by exiting at $t_0 - \varepsilon$, which contradicts the construction of $\Phi_2(t_0, 0, \bar{\theta})$. Thus, $\tilde{\theta} = \Phi_2(t_0, 0, \bar{\theta})$. At time t_0 , player i knows that θ_j is equal to or smaller than $\Phi_2(t_0, 0, \bar{\theta})$. If player i has exit value θ and chooses stopping time $\tau \geq t_0$, the present discounted value of its expected profit over time at t_0 is

$$\begin{aligned} & W_2(\tau, T_2(\cdot, t_0, \tilde{\theta}), t_0, \tilde{\theta}; \theta_i) \\ &= \Pr(T_2(\theta_j, t_0, \tilde{\theta}) \geq \tau | \theta_j \leq \tilde{\theta}) \left[\int_{t_0}^{\tau} \Pi_2(s) e^{-r(s-t_0)} ds + \frac{\theta}{r} e^{-r(\tau-t_0)} \right] \\ &+ \int_{\{\theta_j | T_2(\theta_j, t_0, \tilde{\theta}) < \tau\}} \left(\int_{t_0}^{T_2(\theta_j, t_0, \tilde{\theta})} \Pi_2(s) e^{-rs} ds \right. \\ &\left. + e^{-r(T_2(\theta_j, t_0, \tilde{\theta})-t_0)} V_1(\theta_i, T_2(\theta_j, t_0, \tilde{\theta})) \right) g(\theta_j | \theta_j \leq \tilde{\theta}) d\theta_j. \end{aligned}$$

Taking the first-order condition and rearranging gives

$$\begin{aligned} \Phi_2'(t, t_0, \tilde{\theta}) &= - \frac{G(\Phi_2(t, t_0, \tilde{\theta}) | \theta_j \leq \tilde{\theta})}{g(\Phi_2(t, t_0, \tilde{\theta}) | \theta_j \leq \tilde{\theta})} \left[\frac{\Phi_2(t, t_0, \tilde{\theta}) - \Pi_2(t)}{V_1(\Phi_2(t, t_0, \tilde{\theta}), t) - \Phi_2(t, t_0, \tilde{\theta})/r} \right] \\ &= - \frac{G(\Phi_2(t, t_0, \tilde{\theta}))}{g(\Phi_2(t, t_0, \tilde{\theta}))} \left[\frac{\Phi_2(t, t_0, \tilde{\theta}) - \Pi_2(t)}{V_1(\Phi_2(t, t_0, \tilde{\theta}), t) - \Phi_2(t, t_0, \tilde{\theta})/r} \right]. \end{aligned} \quad (\text{A1})$$

where the second equality follows from $\frac{G(\Phi_2(t, t_0, \tilde{\theta}) | \theta_j \leq \tilde{\theta})}{g(\Phi_2(t, t_0, \tilde{\theta}) | \theta_j \leq \tilde{\theta})} = \frac{G(\Phi_2(t, t_0, \tilde{\theta}))}{g(\Phi_2(t, t_0, \tilde{\theta}))}$. The boundary condition is

$$\Phi_2(t_0, t_0, \tilde{\theta}) = \Phi_2(t_0, 0, \bar{\theta}). \quad (\text{A2})$$

Since (6) and (A1) are the same, (A2) implies

$$\Phi_2(t, t_0, \tilde{\theta}) = \Phi_2(t, 0, \bar{\theta}) \quad \forall t \geq t_0.$$

Equivalently, $T_2(\theta, 0, \bar{\theta}) = T_2(\theta, t_0, \tilde{\theta})$. ■

2 Appendix B (Online Appendix, Not for Publication)

Proof of Lemma 4 (i) Arbitrarily pick $\theta \in [\Pi_2(0), \bar{\theta}]$. Suppose $T_3(\theta) > 0$. Then, θ is the marginal type at $t = T_3(\theta)$ that is indifferent between exiting at t and exiting at $(t + dt)$. The cost of waiting until $(t + dt)$ is $\{(\theta - \Pi_3(t)) dt\} > 0$ because $\theta \geq \Pi_2(0) > \Pi_3(0)$. The value of waiting until $(t + dt)$ is the probability that either player j or player k drops out in $[t, t + dt]$ conditional on it having survived until t , times the value of entering the two-player subgame. If player j drops out in the interval, the value of entering the subgame depends on θ_k . Since θ is the marginal type at $T_3(\theta)$, it follows that $\Pr(\theta < \theta_k) = 0$. Otherwise, player θ_k should have dropped out earlier. Thus, player θ drops out immediately after the two-player subgame starts. Therefore, the value of staying should be zero, and player θ cannot be the marginal type. This is a contradiction.

(ii), (iii) By the same argument as Lemma 1 of Fudenberg and Tirole (1986). ■

Proof of Lemma 5 In the previous lemma, I proved that any strategy $T_3(\cdot)$ such that $T_3(\theta) = c > 0$ cannot be an equilibrium. Without Assumption 2, any strategy $T_3(\cdot)$ such that $T_3(\theta) = 0$ for $\theta \in [\Pi_2(0), \bar{\theta}]$ cannot be an equilibrium either. To show this, assume $T_3(\theta) = 0$ for $\theta \in [\Pi_2(0), \bar{\theta}]$. Suppose $\theta_i \in [\Pi_2(0), \bar{\theta}]$. Then there exists $\varepsilon > 0$ such that

$$\Pr(T_3(\theta_j) = T_3(\theta_k) = 0) V_1(\theta_i, 0) > [\theta_i - \Pi_2(0)] \varepsilon.$$

Thus, player i becomes better off by waiting for ε . This contradicts $T_3(\theta) = 0$. Therefore, Assumption 2 is necessary. With this assumption, the first boundary condition is given by (10). The logic behind the second boundary condition is the same as that of (8) ■

Proof of Proposition 6 In the symmetric case, a slight modification of the proof for Lemma 3 in Fudenberg and Tirole (1986) suffices. In particular, letting Φ_3 and $\tilde{\Phi}_3$ be distinct solutions of (9), (10), and (11), I can obtain a contradiction.

Proof of Lemma 7 Firm $\Phi_3(t)$ does not drop out if its triopoly profit is strictly larger than its exit value. If $\Pi_3(t) = \Phi_3(t)$, the firm does not drop out either, because there is a positive probability that some of its competitors exits in the next instant; i.e., $P(\Phi_3(t) < \theta_j < \Phi_3(t + s)) > 0$ for all t and s . Since $\Pr(\theta_j < \lim_{t \rightarrow \infty} \Pi_2(t)) > 0$, there is always a positive probability that firm j stays in. Therefore, $\Phi_3(t) < \Pi_2(t)$. ■

Proof of Lemma 8 When $t \in [T_3(\theta_k), \Pi_2^{-1}(\theta_k))$, θ_i is strictly lower than $\Pi_2(t)$. Thus, regardless of what firm j does, firm i does not exit the game. When $t = \Pi_2^{-1}(\theta_k)$, only type $\theta_i = \theta_k$ is indifferent between staying and exiting. But the event has measure zero. The same logic applies to firm j 's strategy. For $t \in [\Pi_2^{-1}(\theta_k), \infty]$, the policy

function follows (6). Since no type has exited since $T_3(\theta_k)$, the maximum possible value of opponent's exit value is still θ_k . Thus (12) obtains. The second boundary condition is the same as (8).■

3 Appendix C: Discussion of Assumptions in the Model (Online Appendix, Not for Publication)

I assume that theaters are different from one another only in terms of unobservable and privately known exit values. In reality, theaters are different in observable ways as well. First of all, the capacity differs across theaters, which should affect the per-period payoff function. Second, theaters have different locations. One street is busy and many people may come to watch a movie, while another street is empty. This is also observable from the competitors' viewpoint.

However, I assume theaters are homogeneous in terms of observable variables for the following reasons. First, the capacity variable (the number of seats) is often missing in my dataset. In addition, the name of the street where a theater is located is frequently missing. Second, the differential equation (9) that I use for estimation will be very complicated, once I abandon the symmetry assumption. This would make computation highly demanding. Thus, rather than a firm-level analysis, this study may be considered as a market level analysis; e.g., how the initial market structure and the exit process in the market are related.

Next, the assumption of deterministic demand is worth mentioning. Demand uncertainty and demand learning often play important roles in the dynamics of an industry. A relevant example is that theaters may delay their exit, making a negative profit, simply because they want to learn about the stochastic demand process. This motive for delaying exit may also rationalize the data of the current study. In this paper, however, demand uncertainty is assumed away for the sake of tractability. Therefore, uncertainty from the players' viewpoint comes only from competitors' exit times.

4 Appendix D: Model Fit (Online Appendix, Not for Publication)

To investigate the model fit, I randomly drew a set of structural parameters γ from the estimated asymptotic distribution $\mathcal{N}(0, W)$ given in (27) 300 times, and simulated the survival rate of theaters for each draw. Then, I calculated the 98% confidence interval (top 1% and

bottom 1% of exit rates) for the exit rate for different market structures.¹ Figure D1 shows four graphs. The model fits the data well, except for the survival rate in 1955 for markets with two or three theaters in 1949. For most of other years for other markets, the model shows a good fit.

¹For some region of parameters, the model cannot be calculated. I restrict $\lambda_0 \geq 0$, $\lambda_1 \geq 0$, and $1 - \lambda_0 TV_{tm} - \lambda_1 TV_{tm}^2 \geq 0$. Hence, if a draw of parameters violates one of these conditions, I discarded the draw. This would narrow, but not widen, the 98% interval. Therefore, the point I made in this subsection is still valid; most of data points are within the interval.

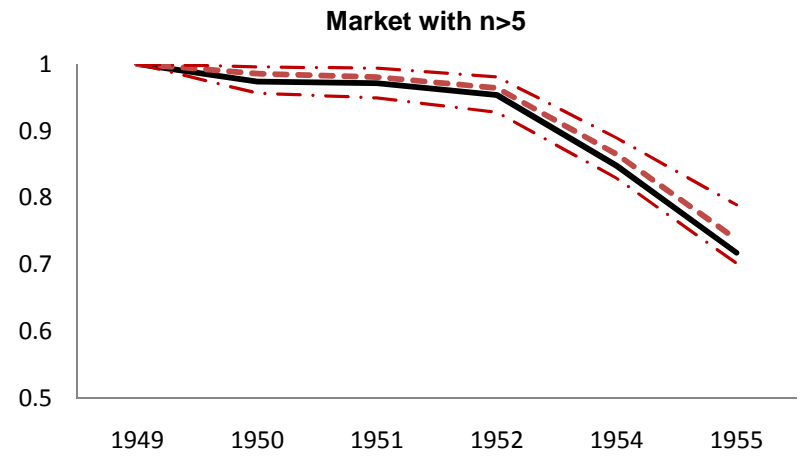
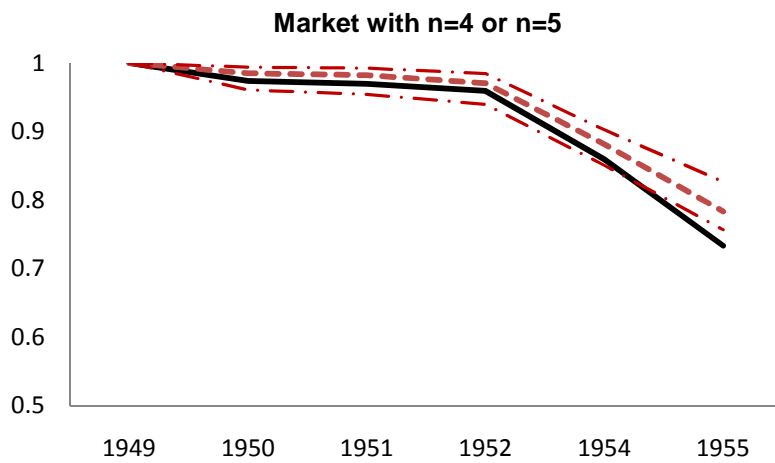
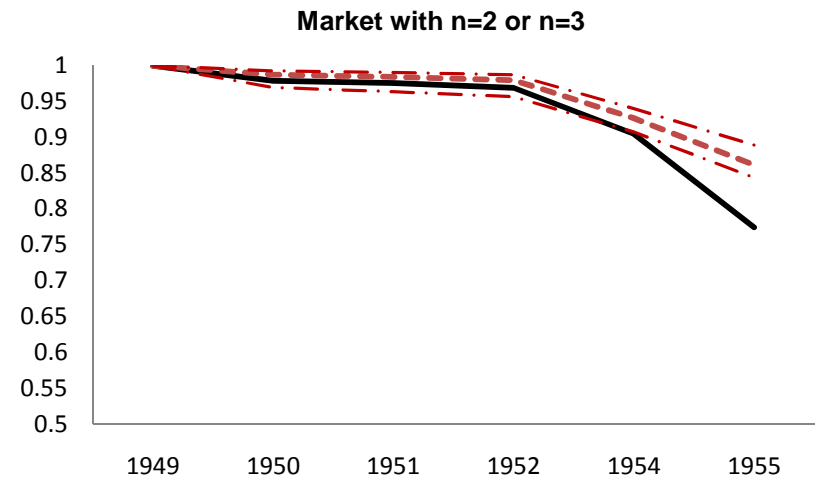
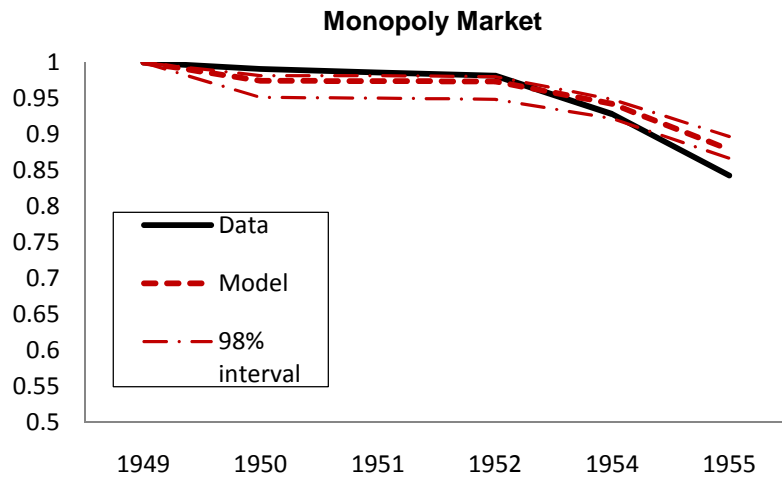


Figure D1 Model Fit

Note: I randomly drew a set of structural parameters γ from the estimated asymptotic distribution $N(0,W)$ 300 times, and simulated the survival rate of theaters for each draw. Then, I calculated the 98% confidence interval (top 1% and bottom 1% of exit rates) for the exit rate for different market structures.