Production Function Estimation with Measurement Error in Inputs*

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Abstract

Production functions are a central component in a variety of economic analysis. However, these production functions often first need to be estimated using data on individual production units. More than any other input in the production process, there is reason to believe that there are severe errors in the recording of a producer's capital stock. Thus, when estimating production functions, we need to account for the ubiquity of measurement error in capital stock. This paper shows that commonly used estimation techniques in the productivity literature fail in presence of plausible amounts of measurement error in capital. We propose an estimator that addresses this measurement error, while controlling for unobserved productivity shocks. Our main insight is that investment expenditures are informative about a producer's capital stock, and we propose a hybrid IV-Control function approach that instruments capital with (lagged) investment, while relying on standard intermediate input demand equations to offset the simultaneity bias. We rely on a series of Monte-Carlo simulations and find that standard approaches yield downward biased capital coefficients. We apply our estimator to two standard datasets, the census of manufacturing firms in India and Slovenia, and find capital coefficients that are, on average, twice as large. We discuss the implications for productivity analyses.

Key words: *Production function estimation; measurement error*

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1 Introduction

> The measurement of capital is one of the nastiest jobs that economists have set to statisticians. (Hicks (1981) p. 204)

Production functions are a central component in a variety of economic analysis. However, these production functions often first need to be estimated using data on individual production units. Measurement of capital assets poses a problem for estimation of production functions. More than any other input in the production process, there is reason to believe that there are severe errors in the recording of a producer's capital stock. These errors are likely to be large, and are extremely difficult to reduce through improved collection efforts since firms themselves have difficulty evaluating their capital stock. Thus, when estimating production functions, we need to account for the ubiquity of measurement error in capital stock. This paper shows that commonly used estimation techniques in the productivity literature fail in presence of plausible amounts of measurement error in capital. We show that using *both* investment and the book value of capital can correct the presence of measurement error in the capital stock. This idea follows the standard insight of relying on two measures of the same underlying (true) variable of interest, and using one of these measures as an instrument for the other.

The presence of substantial, or at least the potential of, measurement error in capital is reflected in a well-documented fact that when estimating production functions with firm fixed-effects, capital coefficients are extremely low, and sometimes even negative. Griliches and Mairesse (1998) state ''*In empirical practice, the application of panel methods to micro-data produced rather unsatisfactory results: low and often insignificant capital coefficients and unreasonably low estimates of returns to scale.*''. One obvious other interpretation is that capital is a fixed factor of production, and therefore the variation left in the time series is essentially noise. However, this also implies that changes in capital, which is by definition equal to investment minus depreciation, is heavily contaminated by measurement error. Indeed, Becker and Haltiwanger (2006) do an in depth study of measurement issues related to capital, and find that different ways of measuring capital that ought to be equivalent, such as using perpetual inventory methods or inferring capital investment from the capital producing sectors, lead to different results for a variety of outcomes, such as parameter estimates of the production function, and the investment and capital patterns.

The recent literature on the estimation of production functions (Olley and Pakes, 1996; Levinsohn and Petrin, 2003; Ackerberg, Frazer, and Caves, 2015), has exploited control functions to solve the problem of endogenous inputs. However, these control function approaches are difficult to reconcile with the presence of measurement error in inputs. We propose an estimator that deals with the measurement error in capital, while controlling for unobserved productitvity shocks. In particular, we leverage the log-linearity from the Cobb-Douglas production function and productivity process, to construct an estimator that jointly addresses these sources of bias, is simple to use, and requires no additional data.

Related Literature

There has a been a long literature on the estimation and identification of production functions when a researcher has access to a panel data set of producers over time (of a given) industry with information on output and inputs. Olley and Pakes (1996) (henceforth OP) and Levinsohn and Petrin (2003) (henceforth LP) have renewed interest in addressing the simultaneity bias, due to the unobserved productivity term ω_{it}, when estimating the relationship between output and input. More recently Ackerberg, Frazer, and Caves (2015) (henceforth ACF) refined the precise conditions under which these production functions are identified, and provided an alternative estimator. In a related literature, the focus has moved away from the classic simultaneity problem to the one of unobserved prices, for both output and inputs (De Loecker and Warzynski, 2012; De Loecker, Goldberg, Khandelwal, and Pavcnik, 2016).[1]

Van Biesebroeck (2007) evaluates the performance of various production function estimators, including the so-called control function approaches, in the presence of measurement error, although not with a specific focus on measurement error in capital. He compares various methods in the presence of log additive mean-zero independent and standard normally distributed errors to all inputs, measurement error in output and input prices. While the focus of Van Biesebroeck (2007)

[1]In most settings, we observe firms charging different prices for their output, and paying different prices for inputs, which leads to an additional complication since researchers typically only have access to (deflated) revenues and expenditures on inputs. We believe this to be a very important concern, but we abstract away from this issue in this paper. In other words we start our analysis by having correctly converted the revenue and expenditure data to the comparable units in a physical sense. This is precisely the setup of Ackerberg, Frazer, and Caves (2015). Of course, to the extent that input prices vary across firms, and are not correlated with the actual levels of input choices, our approach is relevant.

is on the bias in the estimated coefficients, we provide an estimator that is robust to the presence of such measurement error, in the context of endogenous input choices.

Kim, Petrin, and Song (2016) also study the identification of production function with measurement error in inputs, and suggest an estimator that leverage recent research in the econometric literature on non-linear measurement error models. Their paper relies on different condition than ours for identification, essentially leveraging the fact that the classic models of investment and input choice used for production functions, famously outlined in (Olley and Pakes, 1996), is a first-order Markov process. Thus further temporal dependance in observable transitions can be used to structure the measurement error in capital (similarly to Hu and Shum (2013)). This estimator is more complex, which explains why, to our knowledge, their estimator has not yet been used. In contrast, our estimator is simple to program and use with standard statistical software packages.

2 Sources of Measurement Error in Capital

In this section we discuss the potential sources of measurement error in capital, and how we incorporate measurement error into estimation of the production function. This discussion leads to, what we believe, to conclude that investment is a natural instrument for recorded capital.

2.1 Construction of Capital Stock: Book Value and Perpetual Inventory Methods

Capital stock is typically measured in two ways, using either book value, or the so-called perpetual inventory method (PIM, hereafter). The book value of capital is measured using direct information on the value of capital, as recorded in a firm's balance sheet. The PIM requires data on investment, and recorded depreciations to construct capital stock. Of course, both of these approaches are related, since the book value of capital is typically the outcome of firms themselves applying the PIM in their internal accounting. PIM is the most common approach to construct capital stock series, see Becker and Haltiwanger (2006) for an excellent overview. In essence this approach measures the capital stock of a particular asset K_a using:

$$K_{at} = \sum_{j=0}^{\infty} \theta_{ajt} I_{at-j}, \tag{1}$$

where θ_{ajt} is the weight at time t of asset a of vintage j, and thus captures the depreciation profile, and I_{at-j} is the real gross investment of vintage j. Literally applying equation (1) is virtually impossible, even when we rely on the highest quality dataset, such as for example the U.S. Census of Manufacturers. Instead, applied work typically relies on a more familiar law of motion for capital:

$$K_{it}^{e} = (1 - \delta_{st})K_{it-1}^{e} + I_{it-1}, \tag{2}$$

where we now calculate current capital stock for a more aggregated asset e, such as equipment and buildings, and rely on an industry-wide depreciation rate for assets δ_{st}, where s indicates the industry. Finally, real gross investment expenditure is ideally corrected for sales and the retirement of capital assets.

This immediately raises a few important potential measurement issues. First, this approach requires an initial stock of capital, K_{e0}, which is not just the first year of recorded data, but the date at which production started. Second, investment price deflators are rarely available at the producer level — typically these are computed at the industry level. This is a problem, since asset mix can be differ considerably across producers within the same industry. Third, depreciation rates are assumed to not vary across producers and vintage of the capital stock, which again creates measurement error in capital. Indeed, depreciation is very hard to reckon. Moreover, depreciation does not simply follow a fixed factor, and this all is further compounded by reported depreciation being governed by tax treatment of depreciation, rather than economic depreciation.[2]

In contrast, investment is more precisely recorded through the purchases of various capital goods and services in a given year. This is in stark contrast to capital stock, which is accumulated over time, and this further exacerbates the problem. While to some extent every input of the production function, including labor and intermediate inputs, is subject to measurement error, capital is distinct in this dimension.

The use of book value as recorded in a producer's balance sheet is also subject to measurement error. In principle, one can rely on both measures, the book value and the constructed capital stock using PIM, and see how they line up. In the U.S. Census data on manufacturing, such as the Annual Survey of Manufacturing and

[2] For example, when regulators set electricity rates (see for instance Progress Energy – Carolinas (2010)), they often have hundreds of pages of asset specific depreciations depending on the lifespan of a boiler, car, truck, or building, and these depreciation rates typically have fairly intricate time series patterns.

the Census of Manufacturers, these perpetual inventory and direct assets measures differ by 15 to 20 percent (see Becker and Haltiwanger (2006)). This suggests a reasonable amount of measurement error in capital that is likely to be persistent over time. Given that we see measurement error even in the highest quality data sources such as the U.S. Census of Manufacturers, capital measurement error may be more prevalent for datasets covering developing countries. In the latter, we are often precisely interested in identifying factors driving productivity growth, and the (mis) allocation of resource, and therefore accurately measuring the marginal products, and capital growth is of first order importance.

2.2 Investment as an Instrument

When we turn to the actual solution and implementation of our estimator, we rely on the commonly assumed errors-in-variable structure, where the observed log of the capital stock (k^*) is the sum of the log true capital stock (k) and the measurement error (ϵ^k):

$$k^*_{it} = k_{it} + \epsilon^k_{it}, \tag{3}$$

where i indexes the producer, and t is time. We will use the $*$ notation to denote variables measured with error, and the unstarred notation to denote the true value — the one that is typically observed by the firm. We refer to this representation, loosely, as the *reduced form* for the various measurement error sources we have described.[3] We assume that ϵ^k_{it} is classical measurement error – i.e. it is uncorrelated with true capital stock k_{it}. More precisely, in all that follows, we assume that $\mathbb{E}[\epsilon^k_{it}] = 0$. We do, however, allow for ϵ^k_{it} to be serially correlated over time (within a producer). Since capital is constructed using historical information on the cost of assets, it is incredible that there is no serial dependence in measurement error of the value of assets.

Our main premise is that investment (at $t-1$) is informative about the capital stock at time t, conditional on lagged capital, but is not correlated with the measurement error in capital ϵ^k_{it}. Formally, this first means that $\mathbb{E}(k_{it} i_{it-1}) \neq 0$ – i.e., lagged investment is informative about current capital. This is of course a testable assumption. Second, this means that $\mathbb{E}(\epsilon^k_{it} i_{it}) = 0$. Since current capital is just

[3] In Appendix D we discuss a different process for measurement error where $K_{it} = (1-\delta_{it})K_{it-1} + I_{it}$ and $\delta_{it} = (\delta + \epsilon^d_{it})$; i.e., there is measurement error in depreciation rates. We perform similar Monte Carlo simulations. For this process for measurement error, the estimator we propose still performs fairly well for reasonable amounts of measurement error in capital.

the addition of past investment choices, our approach leverages the idea that the source of measurement error is the accumulated errors in depreciation, rather than the new addition to capital.

3 Estimation in the presence of errors-in-variables

3.1 Setup

We are interested in estimating a standard Cobb-Douglas production function given in logs by:

$$y_{it} = \beta_l l_{it} + \beta_k k_{it} + \omega_{it}, \tag{4}$$

where y_{it}, l_{it}, k_{it} denote (log) output, labor and capital respectively. [4]

We focus on measurement error in capital, rather than other inputs such as labor or materials, since we believe this is inherently the most difficult input to measure. Of course, this does not mean that other inputs do not share some of the same difficulties, simply that these errors are likely to be considerably smaller. Finally, the literature has explicitly allowed for measurement error in output, and we follow this tradition by relying on measured output y_{it} where $y_{it} = y^*_{it} + \epsilon^y_{it}$, where again ϵ^y_{it} is classical measurement error that is potentially serially correlated. Throughout we denote the true variable with a superscript *.

The question we address is whether we can correctly estimate the coefficients of the production ($\boldsymbol{\beta} = \{\beta_l, \beta_k\}$), and also therefore recover productivity (ω_{it}) when we have data on $< y_{it}, k_{it}, l_{it}, d_{it} >$, where l_{it} and d_{it} capture labor and intermediate inputs, which assume are measured without error, $d_{it} = d^*_{it}$ and $l_{it} = l^*_{it}$.[5]

Even in the absence of the standard simultaneity problem we therefore cannot obtain consistent estimates of $\boldsymbol{\beta}$ using OLS estimates of the following:

$$y_{it} = \beta_l l_{it} + \beta_k k_{it} + \omega_{it} + \epsilon^y_{it} - \beta_k \epsilon^k_{it}, \tag{5}$$

due to the error-in-variables problem.

Note that estimates of the production function are predominantly used for two reasons. In many applications, say when looking at misallocation of factors, or

[4] The restriction to Cobb-Douglas production functions is more substantial than in most papers on the estimation of production functions, since we require log-linearity of the estimating equation.

[5] We can allow for measurement error in labor, but we focus instead on error in capital. We do, however, do not consider measurement error in intermediate inputs, since we rely on it to control for unobserved productivity shocks, as is standard in the literature. The treatment of an imperfect control variable lies beyond the scope of this paper.

computing structural models of investment, the marginal products of inputs are of direct interest, and thus bias in β's leads to biased marginal products. As well, production function coefficients are also used as an intermediate input in the construction of productivity, and biases in β, such as underestimating the capital coefficient, will lead to capital intensive firms appearing more productive than they really are.

3.2 Solution

We propose a simple IV-strategy to deal with the measurement error in the capital stock. In particular we rely on a separate but related measure of the capital stock: investment. The main advantage is that investment is usually already observed, and thus, does not constitutes an extra burden on the researcher in terms of data collection.

Investment has been proposed by Olley and Pakes (1996) (hereafter, OP) to offset the simultaneity problem, and we will argue that some of the *appealing* features of the OP approach, leading to substantially higher capital coefficients for example, are related to our insights of relying on investment as an instrument rather than a control variable. We do in fact not rely on a dynamic control, such as investment, but rather exploit the Levinsohn and Petrin (2003) insight of using a static control (d_{it}), to exploit the (log) linearity of the production function and the associated first order conditions.

In particular throughout the paper we rely on an intermediate input demand equation to control for unobserved productivity: $\omega_{it} = h(d_{it}, k_{it}, \boldsymbol{z}_{it})$, where $\boldsymbol{z}$ is a vector of variables capturing departures from the standard setup considered in ACF.[6] The choice of the specific variable input, d_{it}, to use depends not only on data availability but more so on which production technology is assumed. We follow the literature and consider a value-added production function, but our approach can equally accommodate a gross output production function.

Our approach leverages the log linear structure of the Cobb-Douglas production function, and the associated (variable) input demand equations. Albeit restrictive, it is the predominant functional form used in applied work. Under a Cobb-Douglas production function, we obtain a log-linear intermediate input demand

[6] See De Loecker and Warzynski (2012) for a detailed treatment

equation, which after inverting for productivity gives us:

$$\omega_{it} = (1 - \beta_d)d_{it} - \beta_l l_{it} - \beta_k k_{it} - \ln(\beta_d) + \boldsymbol{z}_{it}, \tag{6}$$

where lower cases denote logs and we collect the output and input price terms in $\boldsymbol{z}_{it} = p^d_{it} - p_{it} + \mu_{it}$.[7] In the standard setup considered in the literature, this last term $\boldsymbol{z}_{it}$ drops out due to perfect output and input markets. Our proposed estimator does not rely on this restriction, but throughout the Monte Carlo analysis we do not consider departures from this standard setup.

The second component that preserves the linear structure is the linear productivity processes – i.e. we consider an AR(1) process for productivity:

$$\omega_{it} = \rho\omega_{it-1} + \xi_{it}. \tag{7}$$

This is a departure from the literature that typically assumes a first-order markov processes, but in theory can allow for a non-linear process of the form $g(\omega_{it-1})$. However, in practice the AR(1) process is often used, and even the properties of new estimators, such as ACF, are evaluated on data generating processes with exactly this AR(1) process.[8]

The insight to rely on lagged investment to instrument the capital stock suggests an IV approach, given the linearity we have assumed in the productivity process and the production function, and therefore in the material demand equation. The actual implementation now depends on whether we consider a so-called one-step estimator, as suggested by Wooldridge (2009), or a two-step estimator, as suggested by Ackerberg, Frazer, and Caves (2015). While from a theoretical point of view both approaches are very similar, in practice they are expected to perform differently depending on the degree of persistence of the capital stock over time. We start with the one-step approach since this gives rise to a standard GMM estimator with the well-known properties, and analytically provided standard errors, and then present our approach works using the two-step approach of Ackerberg, Frazer, and Caves (2015).

[7] To obtain this expression, we take the first-order condition for the intermediate input d_{it} with the profit function $\Pi_{it} = P_{it}Q_{it} - (P^d D_{it} + P^l L_{it} + P^k K_{it})$, and taking logs and invert for productivity. Formally, we can deal with an linear control function $\omega_{it} = \theta_d d_{it} + \theta_l l_{it} + \theta_k k_{it}$, but we wish to provide a theoretical grounding for a linear control.

[8] While we consider an AR(1) process, the approach goes through for higher order AR processes of the form $\sum_{p=1}^{P} \rho_p \omega_{t-p}$. If we considered dynamic controls, such as investment, having a higher order Markov process would cause considerable problem since the control variable would typically be a function of many unobserved productivity terms $\omega_{it}, \omega_{it-1}, \cdots$. It thus remains an empirical matter how to evaluate the trade off between allowing for non linearities and higher order AR terms.

3.3 One-step approach

The first step of our approach follows precisely the insight of Wooldridge (2009) and substitutes the productivity shock by the empirical counterpart of its law of motion (7), and relies on the intermediate input equation (6):

$$
\begin{aligned}
y_{it} &= \beta_l l_{it} + \beta_k k_{it} + \omega_{it} + \epsilon^y_{it} - \beta_k \epsilon^k_{it} \\
&= \beta_l l_{it} + \beta_k k_{it} + \rho\omega_{it-1} + \xi_{it} + \epsilon^y_{it} - \beta_k \epsilon^k_{it} \\
&= \beta_l l_{it} + \beta_k k_{it} + \theta_k k_{it-1} + \theta_l l_{it-1} + \theta_d d_{it-1} + \boldsymbol{z}'_{it-1}\gamma + \xi_{it} + \epsilon_{it}
\end{aligned} \tag{8}
$$

with $\epsilon_{it} \equiv \epsilon^y_{it} - \tilde{\epsilon}^k_{it}$ and $\tilde{\epsilon}^k_{it}$ collects all the relevant terms related to the measurement error in capital – i.e. $\tilde{\epsilon}^k_{it} \equiv \beta_k \epsilon^k_{it} + \theta_1 \epsilon^k_{it-1}$. We combine the persistence and production parameters in θ, e.g. $\theta_d \equiv \rho(1 - \beta_d)$.

In the absence of the measurement error in capital ϵ^k, we can rely on standard techniques to obtain consistent estimates of the production function. The specific estimator will of course depend on a host of assumptions about the environment in which firms produce, and the degree of variability of the inputs. However, our focus is specifically on the bias induced by the presence of the measurement error such that $\mathbb{E}(k_{it}\epsilon_{it}) \neq 0$, regardless of the specific environment under consideration. The details of our approach do, however, vary with the exact assumptions regarding the variability of the labor input and the degree of competition in output markets, and we consider these cases separately below.

We start in section 3.3.1 with the simplest case of perfectly competitive output markets and static labor choices. This is the predominant set of assumptions in the literature. We then consider the case with labor adjustment costs (section 3.3.2), and finally the case with imperfect competition in output markets in section 3.3.3.

3.3.1 Standard case: perfect competition and static labor choices

The classic setup relies on perfectly competitive output markets and labor being a static input choice. This implies that we can immediately obtain an estimate of the labor coefficient by exploiting the first-order condition for the choice of labor, which yields $\beta_l = \frac{WL_{it}}{PY_{it}}$, where P is the price for output, which is the same across producers due to the assumption of perfect competition, and W is the wage rate for labor which is also a constant due to perfect competition in the labor market.

The presence of measurement error in output then calls for a simple estimator for the labor coefficient:

$$\hat{\beta}_l = Median\left[\frac{WL_{it}}{PY_{it}}\right], \tag{9}$$

the median of the sales to labor cost ratio, where the median is used instead of the mean, due to the error in the denominator of this expression.[9] This estimator is a consistent estimator of β_l if we assume that the output measurement error satisfies $Med[\epsilon^y_{it} = 0]$. To see this consider the first-order condition of profits with respect to labor, and rearrange terms to obtain:

$$Median\left[\frac{WL_{it}}{PY_{it}}\frac{1}{\exp(\epsilon^y_{it})}\right] = \frac{WL_{it}}{PY_{it}}\frac{1}{\exp(Median[\epsilon^y_{it}])}, \tag{10}$$

where we use the property of the exponential being a monotone function. Thus, our estimator proposed in equation (9) is consistent estimator of β_l.

Using this estimator $\hat{\beta}_l$ and that the output market is perfectly competitive simplifies equation (8) to:

$$\tilde{y}_{it} = \beta_k k_{it} + \theta_k k_{it-1} - \rho\tilde{l}_{it-1} + \theta_d d_{it-1} + \xi_{it} + \epsilon_{it}, \tag{11}$$

where $\tilde{y}_{it} = y^*_{it} - \hat{\beta}_l l_{it}$ – i.e. output net of labor variation, and $\tilde{l}_{it-1} = \hat{\beta}_l l_{it-1}$.

In the absence of the measurement error in capital we could run the equation (11) above with OLS and obtain a consistent estimate of the capital coefficient given the timing assumptions: capital at t (and at $t-1$) are orthogonal to the productivity shock ξ_{it}, and so is the intermediate input choice at $t-1$. We call this the One-Step Control Function estimator. It is precisely because of errors in the capital stock that we require an instrument.

Our strategy is to use lagged investment as an instrument for current capital, and an analogous strategy for the lagged capital term, which controls (partially) the persistent part of productivity. Specifically we use following moment conditions:

$$\mathbb{E}\left[(\epsilon_{it} + \xi_{it})\begin{pmatrix} i_{it-1} \\ i_{it-2} \\ \tilde{l}_{it-1} \\ d_{it-1} \end{pmatrix}\right] = 0, \tag{12}$$

and the resulting estimator is called the IV One-Step Control Function estimator.

To sum up we rely on a simple linear IV to obtain consistent estimates of the production function in the presence of serially correlated unobserved productivity *and* measurement error in capital.

[9]Strictly speaking, we could use a mean estimator for equation (9). The point of the median is to allow for measurement error in labor that is also median zero. Of course, this assumption is difficult to square with the rest of the setup of the paper, which is why we do not discuss it.

3.3.2 Perfect competition and labor adjustment costs

In the case where labor faces adjustment cost, the labor input now constitutes a state variable and labor choices will not entirely react to productivity innovation shocks ξ_{it}. In addition, labor choices are no longer described by a simple first order condition as described by equation (9) and therefore we can no longer net out the labor variation. The equation of interest thus becomes:

$$y = \beta_l l_{it} + \beta_k k_{it} + \theta_k k_{it-1} + \theta_l l_{it-1} + \theta_d d_{it-1} + \xi_{it} + \epsilon_{it}, \tag{13}$$

and depending on the specific source of invariability of the labor input we can specify the relevant instruments. In the case of one-period hiring, labor at time t and $t-1$ are exogenous variables and do not require instruments. The moments conditions to obtain the estimates for $(\beta_l, \beta_k, \theta_k, \theta_l, \theta_d)$ are given by:

$$\mathbb{E}\left[(\epsilon_{it} + \xi_{it})\begin{pmatrix} l_{it} \\ i_{it-1} \\ i_{it-2} \\ l_{it-1} \\ d_{it-1} \end{pmatrix}\right] = 0. \tag{14}$$

The source of identification for the labor coefficient is then precisely that adjustment cost vary across firms, to the extent that these vary with the labor stock (see Bond and Söderbom (2005) for a discussion).

3.3.3 Imperfect competition and static labor

The case of imperfect competition is similar to the previous subsection in that the static first order condition cannot be used to recover the labor coefficient. However, there are two extra complications. First, we need to address the simultaneity of labor, and this requires additional assumptions on labor markets, and the wage rate in particular. Second, the control function (equation (6)) now consists of the extra term $\boldsymbol{z}_{it}$, and we need to include the relevant variables in the control function to avoid an omitted variable bias. The relevant estimating equation is thus precisely equation (8).

A data generating process discussed in De Loecker and Warzynski (2012) considers l_{it-1} and l_{it-2}, to instrument for l_{it} and l_{it-1}. This requires of course that labor choices are linked over time, and this is the case when wages do not only vary across firms, but also are serially correlated over time. Note that in this case we require to observe wages, and in fact they become the d_{it-1} variable in equation (13). In this case the relevant moment conditions are now:

$$\mathbb{E}\left[(\epsilon_{it}+\xi_{it})\begin{pmatrix} l_{it-1} \\ i_{it-1} \\ i_{it-2} \\ l_{it-2} \\ d_{it-1} \\ z_{it-1} \end{pmatrix}\right]=0, \tag{15}$$

where the last moment, $\mathbb{E}[(\epsilon_{it}+\xi_{it})z_{it-1}]=0$, highlights that firms produce in an imperfect output market and face different input prices, here wages.

3.4 Two-step approach

The two-step approach relies on the same assumptions as the one-step approach, but instead of replacing the unobserved productivity shock by its productivity process, which is directly a function of observables, in the production function, it replaces the productivity shock by the inverted intermediate input demand equation. This yields a so-called first stage regression:

$$y_{it}=\theta_k k_{it}+\theta_l l_{it}+\theta_d d_{it}+\boldsymbol{z}_{it}'\gamma+\tilde{\epsilon}_{it}, \tag{16}$$

where we highlight that the error term in this regression, $\tilde{\epsilon}_{it}$, is different from the one-step approach. Unlike the first stage in ACF, an OLS regression of equation (16) gives biased estimates of the first stage parameters due to the errors in variables problem. Therefore, our first stage requires an instrumental variables regression where i_{it-1} is used as an instrument for k_{it}, to obtain unbiased estimates of the coefficients.This provides us with a consistent estimate of predicted output, ϕ_{it} in the notation of ACF, and now we obtain an expression for productivity given the parameters (β_l,β_k), using this first stage regression:

$$\omega_{it}=\phi_{it}-\beta_l l_{it}-\beta_k k_{it}, \tag{17}$$

with the only wrinkle that we only observe capital with measurement error and therefore we have to incorporate the measurement error in capital. This, however, does not affect the remainder of the procedure.

The remainder of the ACF approach relies on the specific law of motion of productivity to generate moment conditions directly on the productivity shock, ξ_{it}, where the latter is obtained, for a given value of β_l and β_k, by projecting ω_{it} on its lag, given the linear productivity process assumed throughout. Formally, this gives moment conditions on ξ_{it} and they are very similar to the one-step approach and given by:

$$\mathbb{E}[\xi_{it}(\boldsymbol{\beta})i_{it-1}] = 0. \tag{18}$$

This moment compares to the standard moment condition in ACF, $\mathbb{E}(\xi_{it}k_{it}) = 0$, which highlights our IV strategy, whereby we rely on lagged investment to instrument for the potentially mis-measured capital stock.

3.5 Discussion

3.5.1 Comparison to Olley-Pakes

The moment condition of the two-step ACF approach, $\mathbb{E}(\xi_{it}i_{it-1}) = 0$, is different from the Olley-Pakes estimator, where both current and lagged (measured) capital are used. However, it is useful to contrast this to a special case of the Olley-Pakes estimator where investment variation is implicitly used to identify the capital coefficient. To see this let us consider a simple Martingale process for productivity, $\omega_{it} = \omega_{it-1} + \xi_{it}$, and consider the final stage of the Olley-Pakes procedure. This final stage is very similar to equation (11), where the labor variation is subtracted, and the productivity process has been substituted. In this case the Olley-Pakes regression is given by:

$$\Delta\tilde{y}_{it} = \beta_k \Delta k_{it} + \xi_{it} + \Delta\epsilon^y_{it}, \tag{19}$$

where Δ denotes the first difference operator – i.e. $\Delta x_{it} = x_{it} - x_{it-1}$, and capital is assumed to be observed without error. Given the law of motion on capital (equation (2)) this implies that the capital coefficient is identified from the moment condition $\mathbb{E}[(\xi_{it} + \epsilon^y_{it})\tilde{i}_{it-1}] = 0$. However, in practice the difference in the measured capital stock, Δk_{it}, is used.[10] This is precisely the focus of this paper, and we propose to use the actual recorded investment at $t-1$, which will eliminate the measurement error present in both current and lagged capital. The comparison to the (final stage) Olley-Pakes specification only holds under the case where we can directly compute the labor coefficient; and therefore immediately nets out labor variation.[11] It is, however, well-known that the Olley-Pakes estimator leads

[10]We use the notation $\tilde{i}_{it-1}$ to reflect that $\Delta k_{it} \neq i_{it}$, but $\Delta k_{it} = k_{it-1} - \ln[(1-\delta)K_{it-1} + I_{it-1}]$. However, the variation in Δk_{it} comes from the lagged investment component.

[11]In the absence of this first order approach to measuring β_l, the measurement error in capital no longer enters linearly: it is now part of the unspecified nonparametric function $\phi(k + \epsilon^k, i)$. Instead we focus on a simple IV strategy while still encompassing a relatively rich environment.

to substantial higher capital coefficients, and our approach suggests that this could partly reflect the presence of measurement error in the capital stock.

3.5.2 Departures from linearity

The choice between the various data generating processes, such as is labor a static variable input or not, depends of course on the specific application and the institutional details of the industry. While our approach works under a wide variety of economic environments, the main restrictions we have imposed is to specify a Cobb-Douglas production technology, and an associated linear control function, and a linear process for productivity. While this captures a large majority of the empirical applications considered by previous studies, this is still a restriction.

The main issue with allowing for either a) an non Cobb-Douglas Production Function, such as a translog, b) a non-linear productivity control function of the form $\omega_{it} = h(d_{it}, k_{it}, \boldsymbol{z}_{it})$, or c) a non-linear Markov process for productivity $\omega_{it} = g(\omega_{it}) + \xi_{it}$, has to do with addressing an errors-in-variables problem for non-linear specifications. To our knowledge, the non-linear error in variables literature, such as Schennach (2004), deals with instrumental variables with a specific so-called *double measurement* form $i_{it} = k_{it} + \epsilon^{i}_{it}$, or as in Hausman, Newey, Ichimura, and Powell (1991), looks at specific functional forms such as $\tilde{g}(\cdot) = \sum_{k=0}^{K} \alpha_k x^k$, which are not met by our framework.

3.5.3 Validity of instrument

Our approach has a very transparent first stage regression to check the validity of the instrument, although that this is met by construction when we rely on the capital stock computed using PIM law of motion – i.e. $K_{it} = (1-\delta)K_{it-1} + I_{it-1}$. When we rely on the reported book value of capital, this is not the case. However, in both instances it is a useful check whether lagged investment has sufficiently explanatory power to predict current capital stock. Obviously, we rule out that the measurement in capital is correlated with investment, or if investment itself is measured with error, we rule out that there is a correlation between both measurement errors. As discussed in section 2.1, we belief the main source of measurement error to come from the difficulty to appropriately measure depreciations over a long period of time across heterogeneous assets and production processes. We do, however, require that this measurement error is unrelated to the measurement of gross investment.

We demonstrate our estimator in a controlled setting using a Monte Carlo analysis, in the next section, based on the framework introduced by ACF, where we introduce measurement error in capital to evaluate the performance of our estimator. We then turn to two datasets covering standard production and input data for plants active in manufacturing in India and Slovenia.

4 Monte Carlo Analysis

We evaluate our estimator in a series of Monte Carlo analysis where the main interest lies in comparing the capital coefficient across methods as we ramp up measurement error in capital. We follow Ackerberg, Frazer, and Caves (2015) closely and start from their data generating process, and add measurement error to capital. We do we depart from ACF by adding time-varying investment costs in the investment policy function.

We refer the reader to Appendix C for the details of the underlying model of investment, but the main features of our setup are as follows. We rely on a constant returns to scale production function with a quadratic adjustment cost for investment. This model yields closed form solutions for both labor and capital, where labor is set using a static first-order condition given firm-specific wages.[12] Productivity and wages follow an AR(1) process, and we consider a quadratic adjustment cost for investment: $\phi_{it} I_{it}^2$, where ϕ_{it} — which should be considered as the price of capital — itself follows a first-order Markov process. We solve the model in closed form, following the work in Syverson (2001) and discussed in Appendix C.2, and this generates our perfectly measured monte-carlo dataset on output, inputs, investment and productivity.

We then overlay measurement error on this dataset composed of AR(1) processes with normally distributed shocks:

$$\begin{aligned} \epsilon_{it}^y &= \rho^y \epsilon_{it-1}^y + u_{it}^y \\ \epsilon_{it}^k &= \rho^k \epsilon_{it-1}^k + u_{it}^k, \end{aligned} \tag{20}$$

where $u^y \sim \mathcal{N}(0, \sigma_y^2)$, and $u^k \sim \mathcal{N}(0, \sigma_k^2)$. In other words we allow for serially correlated measurement error in output and capital.

[12] ACF also deal with two other data generating processes, other than the approach we described (called DGP1 in their paper). In appendix C.4, we also consider optimization error in labor (DGP2) and an interim productivity shock between labor and materials as in ACF (DGP3) along with optimization error in labor.

Within this setup we focus mainly on the impact of increasing ϵ^k which is governed by the variance σ_k^2. We distinguish between the role of measurement error *within* a given Monte Carlo, and the overall distribution of estimated coefficients *across* 1000 Monte Carlo runs.

Table 1 shows the parameters used in our Monte Carlo. We pick the same parameters for the size of the dataset, production function, and processes for productivity and wages as in ACF. For the process for the price of capital, denoted ϕ_{it}, we pick parameters that match the the cross-sectional dispersion of capital ($std.[k_{it}] = 1.6$), and the time-series variation in capital ($Corr.[k_{it} - k_{it-1}] = 0.93$) in the Annual Survey of Industries in India (discussed in section 5), choosing an autocorrelation term for the process of ϕ of 0.9, and a shock variance of 0.3. Indeed, it is this last moment that the ACF monte carlo has difficulty replicating: it predicts a serial correlation coefficient of capital of 0.997. Clearly this will make the one-stage approach problematic, as both current and lagged capital are highly collinear, much more so than in any producer level dataset we are aware of.

Finally, we pick parameters for the measurement error in inputs and outputs. We choose a measurement error for output with a standard deviation of 30 percent, and a low autocorrelation of 0.2. For capital, we choose a serial correlation coefficient of 0.7, so a fairly high persistence, and a standard deviation of 0.2. Importantly, this assumption on the time series process for capital measurement error yields a difference between k and k^* which has a standard deviation of 30 percent. However, we precisely verify the sensitivity to the value of the capital measurement error's standard deviation.

4.1 The impact of measurement error

We will compare the performance of estimators that use investment to instrument for mismeasured capital and those that do not, that have been discussed in section 3.3 and 3.4. We name these estimators one-step and two-step control function estimators, some of which use first-order conditions (FOC) to estimate the labor coefficient. Estimators that use investment as an instrument have the IV prefix in front of them. Appendix B provides precise details on these estimators.

In figure 1 we plot the average estimate of β_k over 100 replications against the variance of capital measurement error σ_k. Panel (a) shows the two-step control function estimator, panel (b) shows the two-step control function FOC estimator, and panel (c) shows the one-step control function estimator FOC.[13]

The first main result of our Monte Carlo simulations is that we find that standard estimators, both the one-step FOC, two-step, and two-step FOC, become progressively more biased as the measurement error in capital increases. It is of course difficult to guess the relevant range of this variance, but the main takeaway is that our IV-based estimator is insulated from this problem. The simulations do suggest that standard methods deliver an estimate of half the magnitude for a standard deviation in the capital measurement error σ_k of about 0.2, which corresponds to a standard deviation between k and k^* of 0.28 in the stationary distribution.

It is important to note that our estimators are robust to capital measurement error, while still undoing the simultaneity bias that typically plagues the production function estimation. Therefore, applying our estimator when capital stock is accurately measured provides consistent estimates of the production function coefficients.

4.2 Distribution of estimated coefficients

So far we have inspected the sensitivity of the estimated coefficients to the degree of the measurement error in capital. Now we consider the distribution of our estimators, given $\sigma_k = 0.2$, over 1,000 monte carlo replications. In figure 2 we plot the distribution over the estimated capital coefficient, comparing the same IV and non-IV estimators as in the previous figure.

Again, the takeaway from this analysis is simple but very stark: our IV estimators are centered around the true parameter (0.4), while the other estimators generate, albeit tight, distributions that are biased substantially towards zero, and which often do not include the true value in its support. Interestingly enough, the two-step estimator generates an implied distribution that is much closer to one obtained with the IV approach, but it is centered around about 0.3 and still generates severely biased capital coefficients. As a side point, the IV one-step estimator generates similar mean estimates of β_k as the IV two-step estimators, but has far more variance.

[13]We do not show the one-step control function estimator since it is subject to the ACF critique of material control functions: we would need a monte carlo with adjustment costs for labor in order to properly evaluate this estimator.

4.3 Alternative sources of measurement error

As discussed before we have so far considered, what we refer to as, a reduced form for the measurement error in capital. I.e. we consider the standard representation of an errors-in-variable, whereby the measurement error is (log) additive – here $k + \epsilon^k$. In Appendix D we discuss an alternative source of measurement error in capital, derived structurally from the measurement error in depreciation rates: $K_{it} = (1 - \delta_{it})K_{it-1} + I_{it}$ and $\delta_{it} = (\delta + \epsilon^d_{it})$; i.e., there is measurement error in depreciation rates.

This form of measurement error does not map into a log-additive structure, and we therefore evaluate our estimator in the presence of this alternative setup. The main takeaway from figure D.1 in the appendix is that our estimator outperforms the other approaches (both in a one-step and two-step setting), but given the formal violation of the moment conditions, leads to a small bias of the capital coefficient for large values of the variance of the capital measurement error.

The evidence from the Monte Carlo unequivocally favors our estimator in the presence of measurement error in capital, and moreover suggests that the bias can be quite severe for moderate measurement error in capital. To verify how large this problem is in real data we now apply our estimator, exactly as performed in our Monte Carlo analysis, to two different datasets of manufacturing plants, in India and Slovenia.

5 Applications to plant-level data

We show our methodology in a couple of separate applications using plant-level microdata. The first data that we use is the Annual Survey of Industries from India. This a plant-level survey for over 600,000 plants on a 20 year period. This dataset has been previously used and described in Allcott, Collard-Wexler, and O'Connell (2016). The second data set is the Slovenian Database, as used in De Loecker and Warzynski (2012) and De Loecker (2007) and covers all establishments in the Slovenian manufacturing sector for the period 1994-2000. All variables are deflated using industry-specific price deflators. Appendix A describes each dataset briefly, and presents basic summary statistics.

These two data sets have been used extensively to study productivity dynamics, but at the same time have distinct features related to the measurement of capital. The data on Slovenian establishments reports the book value of plants and investment, while the Indian census data reports both the book value and the (constructed) capital stock using the perpetual inventory method. In addition, the economic environments are different in an important ways. There is substantial investment during the process of economic transition in Slovenia, while in India, labor represents about 20 percent of value added, which is far below the cost share of labor in most other countries. We expect these differences to materialize in the estimated coefficient, and the role and importance of measurement error in the capital stock.

Throughout we will compare the production function coefficients obtained by simple OLS, IV (without the simultaneity control), One-Step Control (i.e. the LP approach), Two-Step Control (i.e. the ACF approach) and our approach, either IV One-Step or IV Two-Step. We estimate a separate capital and labor coefficient for each industry in both Slovenia and India.

We start by reporting the average labor and capital coefficients across the various estimators in Table 2 below. We confirm a well known result in the literature that using fixed effects lowers the capital coefficient substantially, from an average of 0.39 to 0.20 in India, and from 0.24 to 0.18 in Slovenia. By itself, this does not conclusively show that there is measurement error in capital. However, if capital is fixed over a long period of time we can simply not identify its marginal product using the time series variation within producers. Our next specification, IV (investment), considers a two-stage least squares regression of output on capital and labor, where we instrument for capital with investment. Thus, the IV estimator also ignores the simultaneity bias. We find substantially higher capital coefficients compared to OLS, of 0.59 versus 0.39 in India, and 0.35 versus 0.24 in Slovenia. This reinforces our prior that instrumenting for capital with lagged investment may lead to a higher capital coefficient. In fact, the first-stage of this IV regression — a univariate regression of capital on lagged investment — has a R^2 of 0.79 and 0.64 in Slovenia and India. However, investment and unobserved productivity are very likely to be positively correlated, so the increase in the capital coefficient in the IV regression might be due to endogeneity problem as well.

The second panel lists the standard control function estimators used in the literature; i.e. those that do not use investment as an instrument for capital, One-Step and Two-Step Control, and also considers the FOC approach to estimating labor (denoted by FOC). They produce reasonable parameter estimates that are line with the literature – i.e. capital coefficients of around 0.25.

The third panel lists the estimators based on our IV strategy, again for the one-step and two-step approach, and interacted with the FOC approach. We obtain much higher capital coefficients across both datasets and various specifications. For instance, the IV One-Step Control produces an average capital coefficient of 0.41 and 0.41 for India and Slovenia, respectively, compared to 0.23 and 0.19 when we do not instrument the capital stock with lagged investment. Likewise, instrumenting with investment raises the Two-Step control estimate of capital from 0.31 to 0.46 in India, and from 0.26 to 0.32 in Slovenia. More generally, the IV estimators produce higher capital coefficient for all but one country-estimator pair.

These differences are not only statistically significant (at any level of significance), but most of all, are economically meaningful. The implied marginal product of capital and associated objects of interest, such as productivity dynamics are widely different.

Our estimators also controls for the simultaneity of inputs, and this is reflected in the labor coefficients: we find lower coefficients than obtained using OLS – again a standard finding in the literature. An attentive reader will also notice that the estimators that use first-order conditions give very different labor coefficients (with a smaller effect on capital coefficients). In particular, in India the labor coefficient falls from 0.63 in the IV Two-Step Control, to 0.22 for the IV Two-Step Control FOC. Note that the labor coefficient in all the FOC methods is the same, since it is derived from the input cost share of labor: in India, labor accounts for 22 percent of value added, versus 54 percent in Slovenia. Thus, the implausibly low labor coefficients in India are not a result of our particular techniques. Instead, they suggest that static labor choices are a particularly bad assumption in the Indian context.[14]

Finally, we plot the industry-specific capital coefficients by country in Figure 3, for our one-step and two-step estimators. For each panel, the left subpanel shows the results for India, and the right subpanel shows the results for Slovenia. The vertical axis shows the capital coefficient from our IV estimator, while the horizontal axis shows the capital coefficient that does not instrument with investment. Most observations are above the 45 degree line (in red), indicating that our IV estimates are higher than the non-IV estimates for capital. On average we obtain capital coefficients that are about two times larger, and this holds across all industries in all our data sets. If we go back to our Monte Carlo results, say in figure 1, dropping

[14]This is to be expected given the evidence on the prevalence of substantial labor adjustment cost in the Indian labor market, which would invalidate the use of the FOC approach. See e.g. De Loecker, Goldberg, Khandelwal, and Pavcnik (2016) for a discussion.

the capital coefficient by half would indicate that the variance of the measurement error σ_k is around 0.2.

This is very much in line with the results in Van Biesebroeck (2007), in particular section V (ii) and Table II panel (4c) is relevant for our purpose: The reported bias in the capital coefficient, between the estimated and the true value, is around -0.2 for the Olley and Pakes method, the most similar to our approach, and given the selected value for the capital coefficient of 0.4. Taking these results at face value would suggest that measurement error in capital could lead to obtain capital coefficients that are significantly lower – i.e. half the magnitude. This has important consequences for any subsequent productivity analysis, and we discuss this in the next section.

6 Implications for productivity analysis

The results from the Monte Carlo and analysis of various census datasets points to a rather large bias in the capital coefficient. This biases our estimates of marginal products of capital, and propagate to productivity analysis as productivity estimates, such as measures of TFP, rely on estimates of the production function.

In what follows, we abstract away from the bias in the labor coefficient, and therefore we can write the implied bias in the measured productivity residual ω^m as follows:

$$\omega_{it}^m = \omega_{it} + b \cdot k_{it} - \hat{\beta}_k \epsilon_{it}^k, \tag{21}$$

where b measures the bias: $b = (\beta_k - \hat{\beta}_k)$.[15] Our monte carlo exercises in section 4 have shown that, without correcting for measurement error in capital, we should expect $\hat{\beta}_k < \beta_k$, and thus $b > 0$. This will lead to a spurious positive relationship between productivity and size, if we measure size by the capital stock, of magnitude b. Note that many of the outcomes that researchers have studied, such as the relationship between productivity and other important firm characteristics such as R&D activities or import and export behavior, may also suffer from bias in β_k, to the extent that these characteristics are correlated with capital stock — which they are. This is precisely what we find both datasets.

[15]The last term $\hat{\beta}_k \epsilon_{it}^k$ will not affect most of the analysis we are interested in here, as $\mathbb{E}(\epsilon_{it}^k) = 0$.

Productivity dispersion

The dispersion of productivity has recently received tremendous amount of attention, starting from earlier work by Syverson (2004), among others, and brought to an cross-country context in the work of Hsieh and Klenow (2009). The latter identified the mere presence of productivity dispersion as a potential diagnostic of misallocation of resources, and consequently an indication as to where to look for drivers of difference in income across countries.

It is easy to show that the dispersion of productivity, which is typically measured using the standard deviation of log productivity ($Std.(\omega)$), is again a function of the bias in our estimate of β_k. Indeed, the difference between the standard deviation of productivity, with and without measurement . term and therefore will also depend on the variance of the capital stock and the covariance of capital and output. The difference between the variance with and without the bias in β_k is given by: $b \cdot var(k) + 2b \cdot cov(y, k)$, and therefore we can expect to find a larger dispersion of TFPR, which would then suggest for example a (potential) larger degree of misallocation.

To highlight the possibility of such a finding, in Figure 4 we plot the distribution of the corrected and uncorrected (ACF based) productivity for the entire Indian manufacturing dataset. We confirm that the mean of productivity is higher, and that the standard deviation of productivity is also higher.

7 Conclusion

This paper revisits the estimation of production functions, using standard data on output and inputs, in the presence of measurement error in inputs, and in capital in particular. Our starting point is that appropriately measuring capital is one of the most difficult tasks that go into estimating a production function. There is, however, rather surprisingly little work that directly deals with the potential presence of measurement error in capital, or any input for that matter.

We introduce an estimator that relies on a hybrid IV-control function approach, and we build on what have now become standard techniques to address the simultaneity bias, and add an IV strategy to correct for the measurement error of capital. We propose a simple strategy that relies on investment to inform us about the marginal product of capital; and specifically we use investment as an instrument for the capital stock while still controlling for the standard simultaneity bias.

Investment is an instrument and therefore an excluded variable from the control function.

Monte Carlo simulations show that our estimator performs well even in cases of rather large measurement error. We also apply our estimator to Indian and Slovenian census data. We find capital coefficients that are about twofold of those obtained with standard techniques. These results suggest that correcting for measurement error in capital can be a first order concern, and it has immediate implications for the literature that studies productivity dynamics, firm growth, investment, and the covariates of productivity growth, to name but a few.

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Tables and Figures

Table 1: Monte Carlo Parameters

Data Size		
Number of Firms	N= 1,000	Taken from ACF.
Time Periods	T= 10	
Production Function Parameters		
Capital Coefficient	β_k = 0.4	
Labor Coefficient	β_l = 0.6	
Depreciation Rate	δ = 0.2	
Productivity Process	ρ_ω=0.7, $\sigma_\omega = 0.3$	
Wage Process	ρ_w=0.3, $\sigma_w = 0.1$	
Cost Capital ϕ	ρ_ϕ=0.9, $\sigma_\phi = 0.3$	Dispersion of Log Capital of 1.6 Autocorrelation of Capital of 0.93
Measurement Error Parameters		
Error in Capital	ρ^k = 0.7, σ_k = 0.2	High Persistence, and 30 percent measurement error in stationary distribution
Error in Output	ρ^y = 0.2, σ_y = 0.3	Low Persistence, and 30 percent measurement error in stationary distribution

Table 2: Mean Industry-Level Coefficients

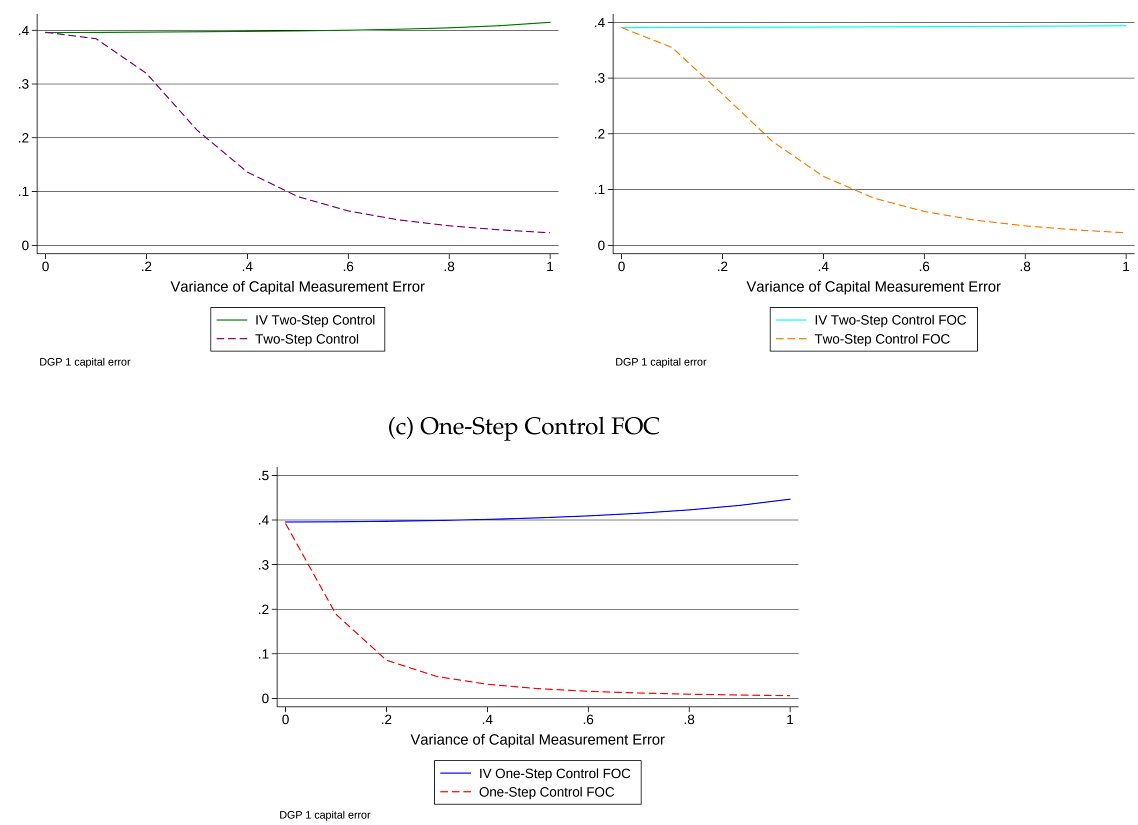
	India (Nr Ind. =19)		Slovenia (Nr Ind. =18)	
	Capital	Labor	Capital	Labor
OLS	0.39	0.78	0.24	0.83
FE	0.20	0.59	0.18	0.77
IV (investment)	0.59	0.51	0.35	0.70
One-Step Control	0.23	0.41	0.19	0.65
Two-Step Control	0.31	0.91	0.26	0.47
Two-Step Control (Adj)	0.36	0.71	0.21	0.85
One-Step Control, Labor FOC	0.25	0.22	0.20	0.54
Two-Step Control, Labor FOC	0.57	0.22	0.42	0.54
IV One-Step Control	0.41	0.37	0.41	0.61
IV Two-Step Control	0.46	0.63	0.32	0.67
IV Two-Step Control (Adj)	0.56	0.53	0.32	0.74
IV One-Step Control, Labor FOC	0.47	0.22	0.44	0.54
IV Two-Step Control, Labor FOC	0.68	0.22	0.40	0.54

Notes: We report the average capital across all industries for each dataset. We consider value-added based Cobb-Douglas production functions with material demand as a control for productivity, and an AR(1) process for productivity. *FOC* refers to case where the labor coefficient is obtained using the FOC approach – i.e. we compute the median of the wage bill to sales ratio for each industry separately. All estimators labeled with *FOC* thus have the same estimate for labor – i.e. the median of the wage bill over sales, by industry. (Adj) refers to the specification with both labor and capital adjustment costs – i.e. current labor is used as the instrument.

Figure 1: Impact of measurement error on capital coefficient in a Monte Carlo ($\beta_k = 0.4$):

(a) Two-Step Control

(b) Two-Step Control FOC

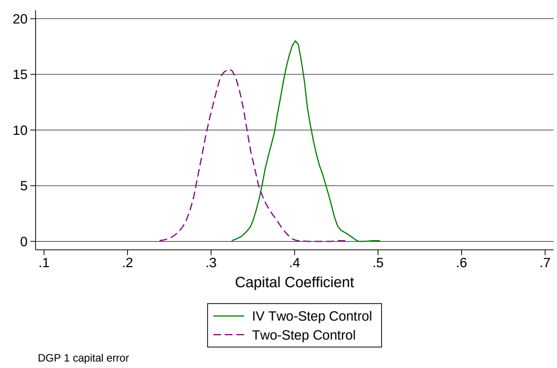


(c) One-Step Control FOC

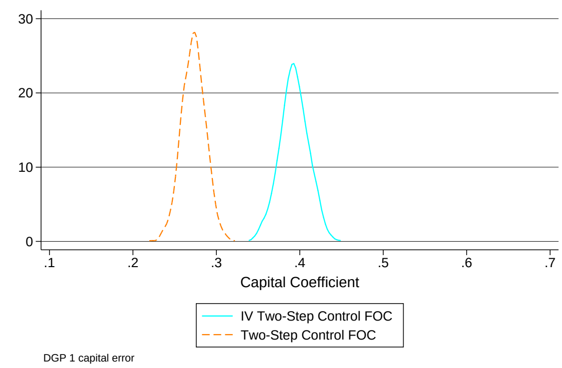
Note: We plot the estimated capital coefficient as a function of the variance in the capital measurement error (σ_k^2). Average of 100 monte-carlo replications per value of σ_k.

Figure 2: The distribution of the estimated capital coefficient in a Monte Carlo ($\beta_k = 0.4$)

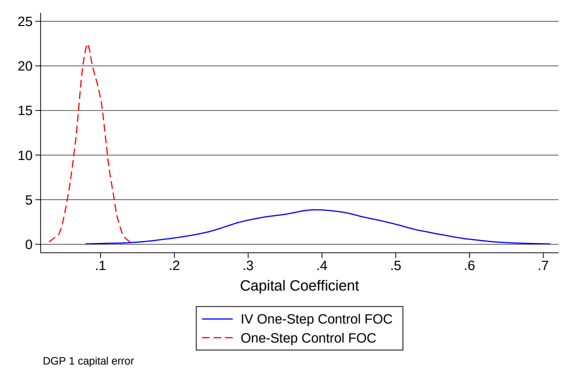
(a) Two-Step Control



(b) Two-Step Control FOC



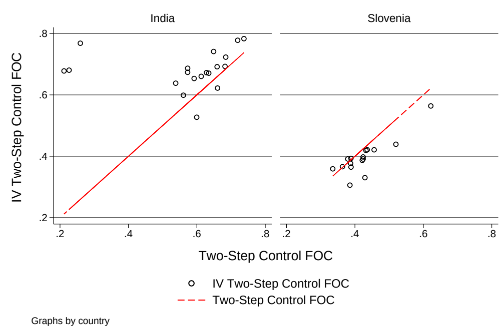
(c) One-Step Control FOC



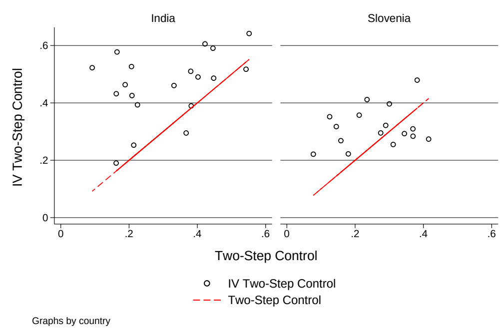
Note: We plot the distribution of the estimated capital coefficient across 1000 Monte Carlo replications, with $\beta_k = 0.4$, and $\sigma_k^2 = 0.2$.

Figure 3: Industry-specific Coefficients

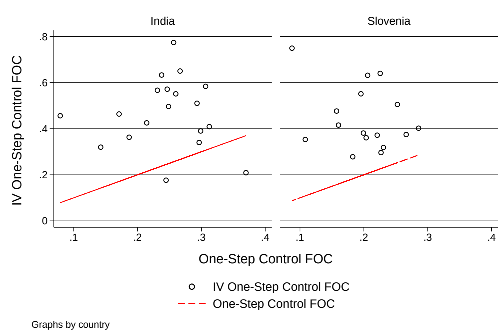
(a) Two-Step Control FOC



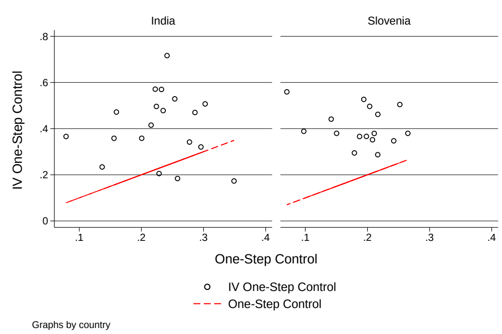
(b) Two-Step Control



(c) One-Step Control FOC



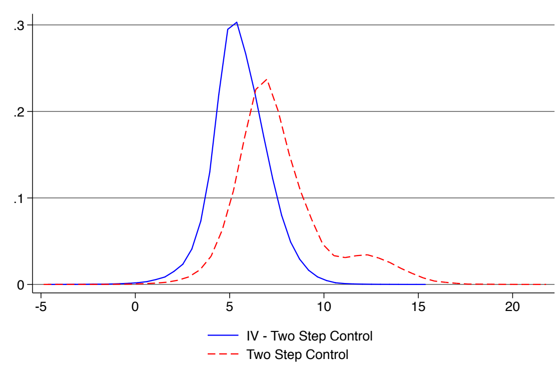
(d) One-Step Control



Notes: Each observation is a two digit industry (as classified by the respective national industry classification), and we plot the capital coefficient obtained from our procedure (i.e. IV) against the alternative, either the one-step GMM estimator (WLP) or the two-step GMM estimator (ACF). The red line is the 45 degree line. All specifications consider value-added based Cobb-Douglas production functions, with material demand to control for productivity and an AR(1) process for productivity. Both the labor and the capital coefficients are estimated, and therefore do not impose perfect competition and laber being a variable input of production.

Figure 4: Productivity Distribution and Dispersion

(a) Distribution



Note: In panel (a) we plot the histogram of TFPR for all plant-year observations (543,365) in Indian manufacturing. The blue histogram is based on the corrected productivity estimates (using IV Two-Step Control) and the red one is obtained using productivity estimates from the standard ACF (Two-Step Control) routine.

A Data Appendix

We apply our estimator to two datasets, covering manufacturing plants in India and Slovenia. There have been numerous productivity studies using these data, and therefore are completely standard in which variables are reported, and how they are constructed.

A.1 Slovenian manufacturing

We refer the reader to De Loecker (2007) for a detailed discussion of the data. For this setting it is important to note that the data contain standard information on establishment-level production and that similar data have been used throughout the literature. See, for example Olley and Pakes (1996) and Levinsohn and Petrin (2003).

In particular, and as mentioned in the paper, the data represent the population of producers of manufacturing products over the period 1994-2000. The estimation of the production function requires information on plant-level output (revenues deflated with detailed producer price indices), (deflated) value added, and input use: labor as measured by full-time equivalent production workers, raw materials and a measure of the capital stock. The latter is constructed from the balance sheet information on total fixed assets broken down into 1) machinery and equipment, 2) land and buildings and 3) furniture and vehicles. Appropriate depreciation rates (based on actual depreciation rates) are used to construct a firm-level capital stock series using standard techniques. See, for example, the data appendix in Olley and Pakes (1996).

In addition, the data report investment and provide detailed information on ownership, firm entry and exit. Finally, the export status and export revenues – at every point in time – provide information whether a firm is a domestic producer, an export entrant or a continuing exporter. This gives rise to an unbalanced panel of about six thousand producers, covering the period 1994-2000.

A.2 Indian manufacturing

We use India's Annual Survey of Industries (ASI) for establishment-level microdata, and this dataset is described in more detail in Allcott, Collard-Wexler, and O'Connell (2016). Registered factories with over 100 workers (the "census scheme") are surveyed every year, while smaller establishments (the "sample scheme") are typically surveyed every three to five years. The publicly available ASI includes establishment identifiers that are consistent across years beginning in 1998, but we have plant identifiers going back to 1992. We have a plant-level panel for the entire 1992-2010 sample.

The ASI is comparable to manufacturing surveys in the United States and other countries. Variables include revenues, value of fixed capital stock, total workers employed, total costs of labor, and materials. Industries are grouped using India's NIC (National Industrial Classification) codes, which are closely related to SIC (Standard Industrial Classification) codes.

There are 615,721 plant-by-year observations at 224,684 unique plants. 107,032 plants will be immediately dropped from our estimators because they are observed only once. For plants observed multiple times, 60 percent of intervals between observations are one year, while 91 percent are five years or less.

The mean (median) plant employs 79 (34) people and has gross revenues of 139 million (20 million) Rupees, or in U.S. dollars approximately $3 million ($400,000).

B Estimators: Details

In this section we describe the estimators proposed in this paper in great detail, enough so that these estimator can easily be coded up by other researchers, and code for these estimators is also available in STATA. In what follows, we use materials as the static control function decision d^*.

B.1 One-Step Estimators, Labor FOC

1. Estimate β_l

$$\hat{\beta}_l = Median\left(\frac{WL_{it}}{PY_{it}}\right).$$

2. Produce output $\tilde{y}_{it}$ netted out from labor contribution.

$$\tilde{y}_{it} = y_{it} - \hat{\beta}_l l_{it}$$

3. Estimate

$$\tilde{y}_{it} = \beta_k k_{it} + \theta_k k_{it-1} + \theta_l l_{it-1} + \theta_d d_{it-1} + \epsilon_{it}$$

using instruments $\boldsymbol{x}_{it} = [i_{it-1}, i_{it-2}, l_{it-1}, d_{it-1}]$

Notice that we refer to the non-IV version of this estimator as the estimator that estimates $\tilde{y}_{it} = \beta_k k_{it} + \theta_k k_{it-1} + \theta_l l_{it-1} + \theta_d d_{it-1}$ by OLS.

B.2 One-Step Estimators, Labor Adjustment Costs

Estimate:

$$y_{it} = \beta_l l_{it} + \beta_k k_{it} + \theta_k k_{it-1} + \theta_l l_{it-1} + \theta_d d_{it-1}$$

by two-stage least-squares using instruments $\boldsymbol{x}_{it} = [l_{it}, i_{it-1}, i_{it-2}, l_{it-1}, d_{it-1}]$. Notice that we refer to the non-IV version of this estimator as the estimator that estimates this previous equation by OLS.

B.3 Two-Step Estimators, Labor FOC

1. Estimate β_l

$$\hat{\beta}_l = Median\left(\frac{WL_{it}}{PY_{it}}\right).$$

2. Produce output $\tilde{y}_{it}$ netted out from labor contribution.

$$\tilde{y}_{it} = y_{it} - \hat{\beta}_l l_{it}$$

3. Estimate

$$\tilde{y}_{it} = \theta_d d_{it}$$

by OLS, and obtain $\hat{y}_{it} = \hat{\theta}_d d_{it}$.

4. For a parameter β_k, minimize the criterion $\mathcal{Q}(\beta_k)$ using:

   (a) Compute $\omega_{it} = \hat{y}_{it} - \beta_k k_{it}$

   (b) Estimate the AR(1) process for productivity, $\omega_{it} = \rho\omega_{it-1}$, by OLS, obtain $\hat{\rho}$. Recover productivity shock $\xi_{it} = \omega_{it} - \hat{\rho}\omega_{it-1}$.

   (c) Compute $\mathcal{Q}(\beta_k)$ as the empirical analogue of the moment condition $\mathbb{E}[\xi_{it} i_{it-1}] = 0$.

   $$\mathcal{Q}(\beta_k) = (\boldsymbol{\xi}\boldsymbol{z})'(\boldsymbol{z}'\boldsymbol{z})^{-1}(\boldsymbol{\xi}\boldsymbol{z})$$

   where $\boldsymbol{\xi}$ denotes the stacked vector of ξ_{it}, and $\boldsymbol{z}$ denotes the stacked vector of i_{it-1}.

   (d) Find $\hat{\beta}_k$ as the minimizer of $\mathcal{Q}(\beta_k)$.

B.4 Two-Step Estimators, Labor Adjustment Costs

1. Estimate the regression:

   $$y_{it} = \theta_d d_{it},$$

   by OLS.

   Obtain $\hat{y}_{it} = \hat{\theta}_d d_{it}$.

2. For a parameter $\boldsymbol{\beta} = [\beta_k, \beta_l]$, minimize the criterion $\mathcal{Q}(\boldsymbol{\beta})$ using:

   (a) Compute $\omega_{it} = \hat{y}_{it} - \beta_k k_{it} - \beta_l l_{it}$

   (b) Estimate the AR(1) process for productivity, $\omega_{it} = \rho\omega_{it-1}$, by OLS, obtain $\hat{\rho}$. Recover productivity shock $\xi_{it} = \omega_{it} - \hat{\rho}\omega_{it-1}$.

   (c) Compute $\mathcal{Q}(\boldsymbol{\beta})$ as the empirical analogue of the moment condition $\mathbb{E}[\xi_{it} \begin{pmatrix} l_{it} \\ i_{it-1} \end{pmatrix}] = 0$ given by:

   $$\mathcal{Q}(\boldsymbol{\beta}) = (\boldsymbol{\xi}\boldsymbol{Z})'(\boldsymbol{Z}'\boldsymbol{Z})^{-1}(\boldsymbol{\xi}\boldsymbol{Z}) \tag{B.1}$$

   where $\boldsymbol{\xi}$ denotes the stacked vector of ξ_{it}, and $\boldsymbol{Z}$ denotes the stacked matrix of $\boldsymbol{x}_{it} = [l_{it}, i_{it-1}]$.

   (d) Compute β_k as the minimizer of $\mathcal{Q}(\beta_k)$.

C Monte Carlo

In this section, we describe details of the Monte-Carlo that we will use to evaluate the performance of our estimator. We will need to specify laws of motion for each of the variables in the data generating process.

C.1 Timing

First, we specify the timing assumptions in our model. Investment is chosen with one period time to build. Materials are chosen statically, i.e., after the firm knows it's productivity Ω_{it}. Labor is chosen statistically in for DGP 2 and DGP 3, and in an interim period for DGP 1, i.e. part of the productivity shock is revealed before the firm makes its labor choice.

Second, there are three exogenous state variables, productivity A_{it}, wages W_{it}, output prices P_{it}, and the price of capital ϕ_{it}, which all have log AR(1) processes. The only endogenous state variable is capital.

Logged productivity A has a first-order Markov evolution:

$$a_{it} = \rho^a a_{it-1} + u^a_{it}, \tag{C.1}$$

where $u^a \sim \mathcal{N}(0, \sigma_a^2)$.

As well, log wages have a first-order markov process:

$$w_{it} = \rho^w w_{it-1} + u^w_{it}, \tag{C.2}$$

and likewise for the logged price for output (P):

$$p_{it} = \rho^p p_{it-1} + u^p_{it}. \tag{C.3}$$

where $u^w \sim \mathcal{N}(0, \sigma_w^2)$ and $u^p \sim \mathcal{N}(0, \sigma_p^2)$. For the purposes of the Monte-Carlo, we will normalize $p_{it} \equiv 1$, the case of perfect competition.

C.2 Derivation of Investment Policy as in Syverson (2001)

In this section, we derive a closed form for the investment function in Syverson (2001), to show that we can allow a time varying cost of capital ϕ_{it}. This derivation is very close to Syverson (2001), so our goal is merely to show that this model admits a time-varying cost of capital ϕ_{it}.

Firms have flow profits given by:

$$\Pi_{it} = P_{it} A_{it} L^{\alpha}_{it} K^{1-\alpha}_{it} - W_{it} L_{it} - \frac{\phi_{it}}{2} I^2_{it}, \tag{C.4}$$

where P is the price of output, A is physical productivity, W refers to firm specific wages, and I is investment.

The firm's value function V is given by:

$$\begin{aligned} V(P_{it}, A_{it}, K_{it}, W_{it}, \phi_{it}) = \underset{L_{it}, K_{it}}{max}\, & P_{it} A_{it} L^{\alpha}_{it} K^{1-\alpha}_{it} - W_{it} L_{it} \\ & + \beta \mathbb{E}_{it} V(P_{it+1}, A_{it+1}, K_{it+1}, W_{it+1}, \phi_{it+1}) \\ \text{such that } & K_{it+1} = (1-\delta) K_{it} + I_{it} \end{aligned} \tag{C.5}$$

where δ is the depreciation rate of capital.

Labor is chosen using the usual first-order condition $\frac{\partial \Pi_{it}}{\partial L_{it}} = 0$:

$$\begin{aligned} P_{it} A_{it} \alpha L_{it}^{\alpha-1} K_{it}^{1-\alpha} &= W_{it} \\ \rightarrow L_{it} &= \left[\frac{\alpha P_{it} A_{it}}{W_{it}}\right]^{\frac{1}{1-\alpha}} K_{it} \end{aligned} \tag{C.6}$$

And likewise, investment solves the Euler Equation, $\frac{\partial V}{\partial I} = 0$ giving,

$$\phi_{it} I_{it} = \beta \mathbb{E}_{it} V_K(P_{it+1}, A_{it+1}, K_{it+1}, W_{it+1}, \phi_{it+1}). \tag{C.7}$$

The envelope condition yields:

$$\begin{aligned} V_K(P_{it}, A_{it}, K_{it}, W_{it}, \phi_{it}) =& (1-\alpha) P_{it} A_{it} L_{it}^{\alpha} K_{it}^{-\alpha} \\ &+ (1-\delta) \mathbb{E}_{it} V_K(P_{it+1}, A_{it+1}, K_{it+1}, W_{it+1}, \phi_{it+1}) \end{aligned} \tag{C.8}$$

Substituting into the first-order conditions, the envelope condition becomes

$$\phi_{it} I_{it} = \beta \mathbb{E}_{it} \left[(1-\alpha) \alpha^{\frac{\alpha}{1-\alpha}} W_{it+1}^{-\frac{\alpha}{1-\alpha}} P_{it+1}^{\frac{1}{1-\alpha}} A_{it+1}^{\frac{1}{1-\alpha}}\right] + \beta(1-\delta) \mathbb{E}_{it} \phi_{it+1} I_{it+1} \tag{C.9}$$

And then iterating this equation forward; i.e., replacing $\mathbb{E}_{it} \phi_{it} I_{it}$ with the right hand side in equation (C.9), yields:

$$\begin{aligned} \phi_{it} I_{it} =& \beta \mathbb{E}_{it} \left[(1-\alpha) \alpha^{\frac{\alpha}{1-\alpha}} W_{it+1}^{-\frac{\alpha}{1-\alpha}} P_{it+1}^{\frac{1}{1-\alpha}} A_{it+1}^{\frac{1}{1-\alpha}}\right] \\ &+ \beta(1-\delta) \mathbb{E}_{it} \beta \left[(1-\alpha) \alpha^{\frac{\alpha}{1-\alpha}} W_{it+2}^{-\frac{\alpha}{1-\alpha}} P_{it+2}^{\frac{1}{1-\alpha}} A_{it+2}^{\frac{1}{1-\alpha}}\right] \\ &+ [\beta(1-\delta)]^2 \mathbb{E}_{t+1} \phi_{it+2} I_{it+2} \\ \phi_{it} I_{it} =& \beta \mathbb{E}_{it} \left[(1-\alpha) \alpha^{\frac{\alpha}{1-\alpha}} W_{it+1}^{-\frac{\alpha}{1-\alpha}} P_{it+1}^{\frac{1}{1-\alpha}} A_{it+1}^{\frac{1}{1-\alpha}}\right] \\ &+ \beta(1-\delta) \mathbb{E}_{it} \beta \left[(1-\alpha) \alpha^{\frac{\alpha}{1-\alpha}} W_{it+2}^{-\frac{\alpha}{1-\alpha}} P_{it+2}^{\frac{1}{1-\alpha}} A_{it+2}^{\frac{1}{1-\alpha}}\right] \\ &+ [\beta(1-\delta)]^2 \beta \left[(1-\alpha) \alpha^{\frac{\alpha}{1-\alpha}} W_{it+3}^{-\frac{\alpha}{1-\alpha}} P_{it+3}^{\frac{1}{1-\alpha}} A_{it+3}^{\frac{1}{1-\alpha}}\right] \\ &+ [\beta(1-\delta)]^3 \mathbb{E}_{t+2} \phi_{it+3} I_{it+3}. \end{aligned} \tag{C.10}$$

Writing in the form of geometric series

$$I_{it} = \frac{\beta(1-\alpha)}{\phi_{it}} \alpha^{\frac{\alpha}{1-\alpha}} \mathbb{E}_{it} \sum_{j=0}^{\infty} \left\{[\beta(1-\delta)]^j W_{it+1+j}^{-\frac{\alpha}{1-\alpha}} P_{it+1+j}^{\frac{1}{1-\alpha}} A_{it+1+j}^{\frac{1}{1-\alpha}}\right\} \tag{C.11}$$

Given that we have assume that P_{it}, A_{it} and W_{it} follow the log-linear AR(1) process with normal error terms:

then the investment function becomes

$$
\begin{aligned}
I_{it} = & \frac{\beta(1-\alpha)}{\phi_{it}} \alpha^{\frac{\alpha}{1-\alpha}} \mathbb{E}_{it} \sum_{j=0}^{\infty} \{[\beta(1-\delta)]^j W_{it}^{-\frac{\alpha\phi_w^{i+1}}{1-\alpha}} \prod_{s=0}^{j} (u_{t+1+i-s}^w)^{-\frac{\alpha\phi_w^s}{1-\alpha}} P_{it}^{\frac{\phi_p^{i+1}}{1-\alpha}} \\
& \cdot \prod_{s=0}^{j} (u_{t+1+i-s}^p)^{\frac{\phi_p^s}{1-\alpha}} A_{it}^{\frac{\phi_a^{i+1}}{1-\alpha}} \prod_{s=0}^{j} (u_{t+1+i-s}^a)^{\frac{\phi_a^s}{1-\alpha}} \} \\
= & \frac{\beta(1-\alpha)}{\phi_{it}} \alpha^{\frac{\alpha}{1-\alpha}} \prod_{s=0}^{j} \{[\beta(1-\delta)]^j W_{it}^{-\frac{\alpha\phi_w^{i+1}}{1-\alpha}} \prod_{s=0}^{j} \mathbb{E}_{it} (u_{t+1+i-s}^w)^{-\frac{\alpha\phi_w^s}{1-\alpha}} P_{it}^{\frac{\phi_p^{i+1}}{1-\alpha}} \\
& \cdot \prod_{s=0}^{j} \mathbb{E}_{it} [(u_{t+1+i-s}^p)^{\frac{\phi_p^s}{1-\alpha}}] A_{it}^{\frac{\phi_a^{i+1}}{1-\alpha}} \prod_{s=0}^{j} \mathbb{E}_{it} [(u_{t+1+i-s}^a)^{\frac{\phi_a^s}{1-\alpha}}] \}
\end{aligned}
$$

Since for $\epsilon \sim \mathcal{N}(0, \sigma^2)$, we have $\mathbb{E}(u^{\frac{\phi^s}{1-\alpha}}) = \exp(\frac{\sigma^2 \phi^{2s}}{2(1-\alpha)^2})$, then the investment function can be further simplified as:

$$
\begin{aligned}
I_{it} = & \frac{\beta(1-\alpha)}{\phi_{it}} \alpha^{\frac{\alpha}{1-\alpha}} \mathbb{E}_{it} \sum_{i=0}^{\infty} \{[\beta(1-\delta)]^j W_{it}^{-\frac{\alpha\phi_w^{i+1}}{1-\alpha}} \prod_{s=0}^{j} \exp\left(\frac{\alpha^2 \sigma_w^2 \phi_w^{2s}}{2(1-\alpha)^2}\right) P_{it}^{\frac{\phi_p^{i+1}}{1-\alpha}} \\
& \cdot \prod_{s=0}^{j} \exp\left(\frac{\sigma_p^2 \phi_p^{2s}}{2(1-\alpha)^2}\right) A_{it}^{\frac{\phi_a^{i+1}}{1-\alpha}} \prod_{s=0}^{j} \exp\left(\frac{\sigma_a^2 \phi_a^{2s}}{2(1-\alpha)^2}\right) \}.
\end{aligned} \tag{C.12}
$$

C.3 Process for the Price of Capital

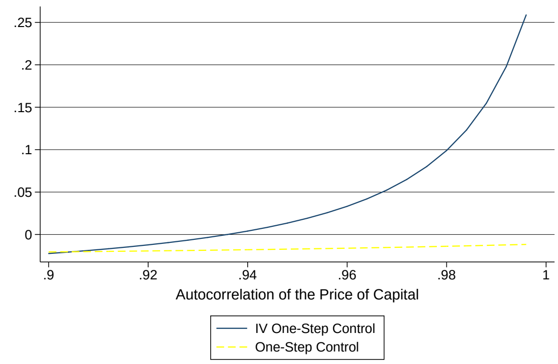
In the original Monte-Carlo proposed by Ackerberg, Frazer, and Caves (2015), the authors extend the model proposed by Syverson (2001) by allowing for the price for capital ϕ to differ between firms, i.e. to allow the price for capital to be a firm specific ϕ_i. This is important, in the context of their Monte-Carlo, since it allows a higher cross-sectional dispersion of capital between firms than what is generated by reasonable processes of productivity, given the patterns in standard producer-level data.

In this paper, we also need capital to move around more than what would be generated by the process considered by Ackerberg, Frazer, and Caves (2015), which generates a serial correlation coefficient for capital of 0.99. Instead, we have the following AR(1) process:

$$\phi_{it} = \rho^{\phi} \phi_{it-1} + \sigma^{\phi} u_{it}^{\phi}, \tag{C.13}$$

where $u_{it}^{\phi} \sim \mathcal{N}(0, 1)$.

Figure C.1 shows the relationship between the autocorrelation of the price of capital and estimates of the capital coefficient using both the one and two step estimators proposed in the paper. To make these estimates more comparable, when we change ρ^{ϕ}, we adjust σ^{ϕ} so that the stationary distribution of ϕ, given by the usual formula for an AR(1) process with normal errors $\frac{\sigma}{\sqrt{1-\rho^2}}$, is unchanged. In panel a, showing one-step estimators, for a wide range range of ρ^{ϕ} parameters below one, our estimator performs very well. However, at very high levels of persistence of ϕ, our IV one-step estimate drop to 0.2. In contrast, the estimates for panel b showing two-step estimators, do not change much as we vary the persistence of the price of capital ϕ.



Notes: Autocorrelation of price of capital refers to the parameter ρ^{ϕ}, where the process for ϕ is $\phi_{it} = \rho^{\phi}\phi_{it-1} + u_{it}$. The variance term σ_k for measurement error is $\sigma_k = 0$. Average estimated coefficient over 100 monte carlo replications.

Figure C.1: Persistence of the price of capital ϕ; i.e., ρ^{ϕ}, and estimate of the capital coefficient

C.4 Alternative Data Generating Processes

We evaluate the performance of our estimator in two alternative data generating processes (DGPs), as considered in ACF in their Monte-Carlos.

- DGP 1:

  DGP 1 is the case considered in the main Monte-Carlo's in the paper. Importantly, labor is chosen a half-period before materials are picked. More precisely, labor is chosen at time $t - 0.5$, and materials are chosen at time t, where the productivity process is adjusted so that the stochastic process for $\omega_{it-0.5}$ is given by:

  $$\omega_{it} = \rho^{0.5}\omega_{it-0.5} + \xi^{b}_{it}$$

  where ξ^{b}_{it} is an appropriately adjusted normally distributed shock.

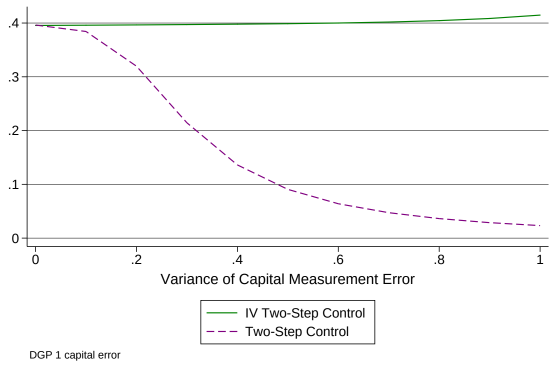
- DGP 2:

  DGP2 refers to the case of optimization error in labor. The variance of the wage distribution is shut down, $\sigma_w = 0$, but instead, firms face an optimization error in labor. Thus, $l_{it} = l^{*}_{it} + \epsilon^{l}_{it}$ where $\epsilon^{l}_{it} \sim \mathcal{N}(0, 0.37)$.

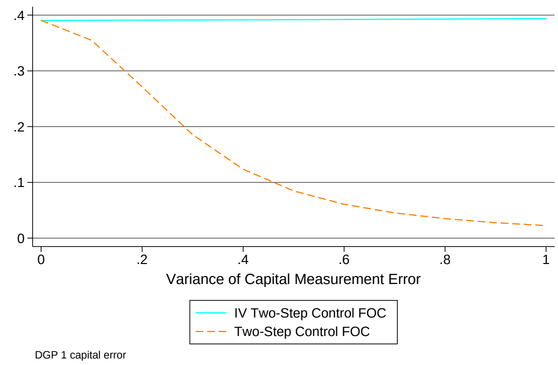
- DGP 3:

  DGP 3, has the same process as DGP 1, but adds in optimization error in labor,$l_{it} = l^{*}_{it} + \epsilon^{l}_{it}$ where $\epsilon^{l}_{it} \sim \mathcal{N}(0, 0.37)$, as in DGP 2.

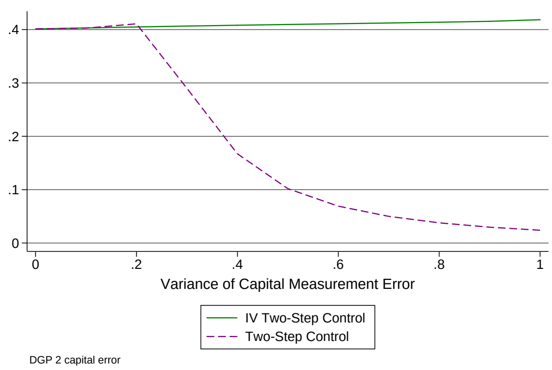
Figure C.2, below replicates figure 1 for DGP 1, 2 and 3, and shows the sensitivity of our IV and non-IV two-step estimators to the measurement error in capital. Notice that the pattern that we document in the monte-carlos for DGP 1 in the main paper is the same as what we find for these alternative DGPs: our estimator performs well with varying degrees of capital measurement error, while the standard control function approaches are biased for reasonable amounts of measurement error.



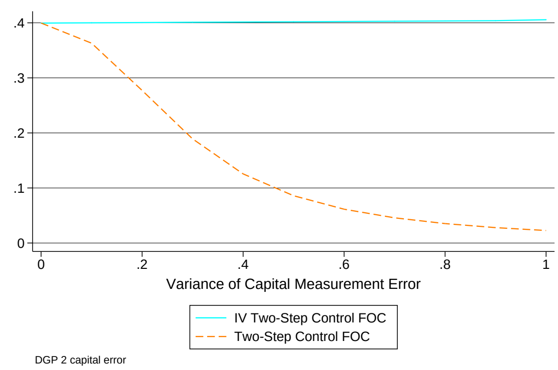
(a) Two-Step Control DGP 1



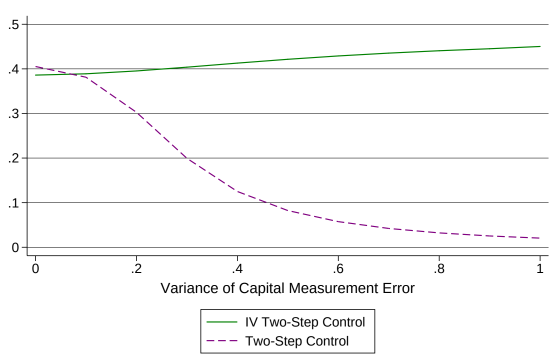
(b) Two-Step Control FOC DGP 1



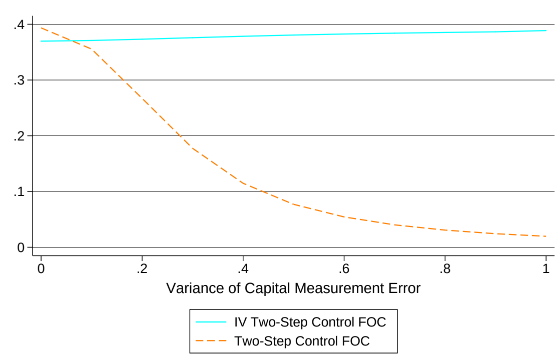
(c) Two-Step Control DGP 2



(d) Two-Step Control FOC DGP 2



(e) Two-Step Control DGP 3



(f) Two-Step Control FOC DGP 3

Note: We plot the estimated capital coefficient as a function of the variance in the capital measurement error (σ_k^2). Average of 100 monte-carlo replications per value of σ_k. The true value of $\beta_k = 0.4$.

Figure C.2: Relationship between β_k and Measurement error σ_k in Capital for different DGPs

D Different Processes for Measurement Error in Capital

In our main specifications in this paper, we rely on the process for measurement error having the form:

$$k_{it} = k^*_{it} + \epsilon^k_{it} \tag{D.1}$$

where ϵ^k_{it} is a mean zero measurement error, which may be serially correlated. While this is the standard formulation for an *errors-in-variables* structure, we contrast this approach to, what we refer to as, a structural derived measurement error for capital. After we introduce this setup, we perform an analogous Monte Carlo analysis to evaluate our estimator, and we make sure that both approaches are directly comparable through their implied variance in the measurement error of capital.

As discussed in section 2 we consider the main source of the capital measurement error to stem from errors in depreciation $D_{it} = \delta_{it} K_{i-1t}$, where the correct measure is given by $d_{it} = \delta^*_{it} K^*_{it-1}$. Applying the same law of motion for capital as before it is easy to show that the measured capital stock under the error in depreciation rates is given by:

$$K_{it} = \sum_{\tau=0}^{t} I^*_{i\tau} - \sum_{\tau=0}^{t} D_{i\tau-1} \tag{D.2}$$

where we kept the assumption that we observe investment without error, and for simplicity also the initial capital stock.[16] The source of measurement error is thus from the cummulative depreciation errors, and we capture as follows: $\delta_{it} = \delta^* + \epsilon^d_{it}$. Both the reduced form and the structural approach generate a wedge between the measured and true capital stock, in levels K and K^*, respectively. After some algebra, we have a direct mapping between the structural measurement error, ϵ^d_{it}, and the reduced form measurement ϵ^k_{it}:

$$\epsilon^d_{it} = \frac{K^*_{it}}{K^*_0} \exp\left(\epsilon^k_{it}\right). \tag{D.3}$$

This relationship is important to compare the performance of our estimator across both Monte Carlo's: a much smaller variance in the depreciation error, ϵ^d_{it}, is needed to generate a certain variance of the classical measurement error, ϵ^k_{it}.[17]

D.1 Monte Carlo Analysis

All parameters of the monte carlo are the same as before, except that we parameterize $\epsilon^d_{it} \sim \mathcal{N}(0, \sigma_d)$. True depreciation is given by $D^*_{it} = \delta K^*_{i-1t}$. However, measured depreciation is given by $D_{it} = (\delta + \epsilon^d_{it}) K_{it}$.

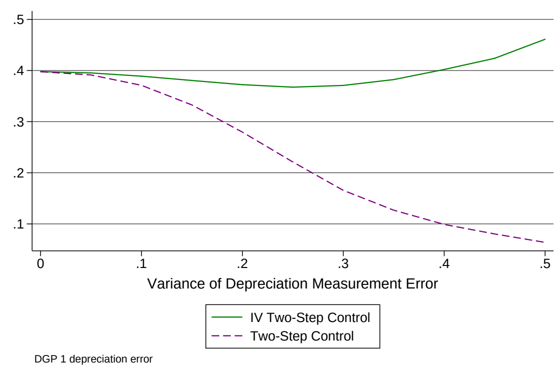
Figure D.1 presents the results of this monte carlo for both the two-step IV control method we propose and a two-step control method that does not use investment as an instrument. Plot the relationship between the mean estimate of β_k over 1,000 replications, as we vary method and a two-step , across the three data generating process considered by ACF.

[16] This is without loss of generality for the purpose of this Appendix.

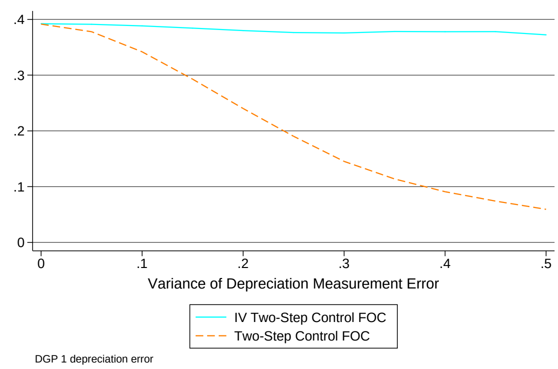
[17] We have traced this out in our Monte Carlo analysis, and we obtain about a 20-1 ratio – i.e., we obtain similar implied variances for the measurement error using either $\sigma^k = 0.2$ or $\sigma^d = 0.01$.

Figure D.1 shows that under small amount of measurement error in depreciation, say on the order of a 0.01 variance shock to a depreciation rate of 0.1, the IV control function method we have proposed does fairly well, with mean estimates around the true value of 0.4. In contrast, the mean estimates that do not instrument with investment, show a drop of β_k to 0.3, with a 0.01 variance shock to the depreciation rate, in line with the previous results we showed illustrating that measurement error in capital stock leads to downward bias on the capital coefficient.

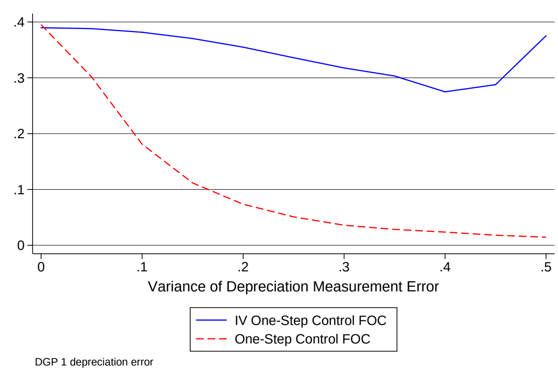
Notice that figure D.1 does not show that our two-step IV control function estimator is consistent for any value of measurement error in depreciation. Indeed, the process for measurement error in depreciation does not lead to the log additive error structure on capital that we need for estimation. Instead, our goal is merely to point out that our estimator might perform well for alternative measurement error structures, at least for some small deviations from our structure.



(a) Two-Step Control



(b) Two-Step Control FOC



(c) One-Step Control FOC

Note: We plot the estimated capital coefficient as a function of the variance in the capital measurement error (σ_k^2). Average of 100 monte-carlo replications per value of σ_d. The true value of $\beta_k = 0.4$.

Figure D.1: Relationship between β_k and Measurement error σ_d in Depreciation