Inefficient durable-goods monopolies

João Montez*

Abstract

We identify inefficient equilibria in a standard durable-goods monopoly model with a finite number of buyers. They are the result of high valuation buyers randomizing their purchase date, while deciding whether to buy at a high initial price or wait for a lower price which is offered only once a critical mass has purchased. This buyer war of attrition, together with the monopolist's pricing decision, can significantly delay market clearing and rationalizes unscheduled price cuts in monopolized durable-goods markets.

Keywords: durable-goods monopoly, real-time delay, stochastic price cuts, buyer attrition.

1 Introduction

Durable-goods theory challenges the classic association of monopoly with inefficiency by proposing a captivating idea: that a monopolist, unable to commit to future prices, will clear the market in a twinkle of an eye. In this paper we show that the model supporting this view also has inefficient equilibria where the market can take a long time to clear.

In these equilibria the timing of price cuts is stochastic, a common feature of many durable-goods markets which the existing theory—supporting deterministic price paths—has left unexplained. Indeed, while buyers facing a high initial price may rightly expect a price cut in the future, these are typically unscheduled and buyers are unable to forecast their timing. "This is life in the technology lane" according to Steve Jobs, "there is always someone who bought a product before a particular cutoff date and misses the new price" he wrote following a $200 price cut on Apple's i-Phone in September 2007.[1]

In the equilibria we study here buyers randomize their purchase date, deciding whether to buy at a high initial price or wait to buy at the lower future price which is offered once a critical mass of sales is reached. For this reason the precise date at which each buyer will purchase is unknown. Eventually, as if succumbing to the temptation, some end up purchasing at the high price, a critical mass is reached, and the monopolist finally brings

*London Business School, Sussex Place Regent's Park, London NW1 4SA UK. E-mail: jmontez@london.edu. I am grateful to Luis Cabral, Dezsö Szalay, Thomas von Ungern-Sternberg and Lucy White. I would also like to thank Patrick Bolton, Yeon-Koo Che, Alessandro Lizzeri, Eloy Perez, Rohan Pitchford, and participants at several conferences and seminars. Financial support from the Portuguese and the Swiss National Science Foundation is gratefully acknowledged.

[1] Another publicized unscheduled price cut was Motorola's Razr phone. It was introduced at $499 in early 2005 and priced at $199 six months later. These price cuts cannot be explained by competition or cost alone.

the price down for the remaining buyers. The market then clears, but this can take some time.

In an influential paper Coase (1972) argued that durability severely limits monopoly power. His idea is that, once high-valuation buyers have purchased and left the market, a monopolist selling a durable-good will want to lower the price and sell to the remaining buyers as well. Buyers anticipating this price-cutting behavior will be reluctant to accept high prices and, to avoid delaying sales, the monopolist will choose to offer a low price at the outset of the game. The idea has been formalized by studying subgame perfect equilibria in an infinite-horizon model of complete information: a monopolist, producing at a constant known marginal cost, posts a price in each period and buyers with unit demand simultaneously decide to either accept or reject the price.

With a *continuum* of buyers this model has a unique equilibrium path in stationary strategies. As Coase conjectured, when the time between two consecutive offers converges to zero the opening price converges to the lowest buyer valuation (above cost) and the competitive quantity is sold in a twinkle of an eye (Gul, Sonnenschein, and Wilson, 1986). With a *finite number* of buyers this result changes dramatically. In this case each sale has a nonnegligible effect on profits and the monopolist can therefore credibly condition price reductions on single purchases. If prices can be revised frequently there exists a stationary subgame perfect equilibrium where the monopolist sells in sequence to each buyer at their own valuation—as if *eating* down the demand curve—and achieves virtually the profits of a perfectly discriminating monopolist (Bagnoli, Salant, and Swierzbinski, 1989 and von der Fehr and Kuhn, 1995).[2] While the effect of durability on market power is ambiguous, in both cases equilibrium price paths are deterministic and, since *all* gains from trade are realized almost instantaneously, market outcomes are efficient.

Here we use this same standard setting to identify a new source of monopoly inefficiency: *buyer attrition*. We show that in a market with a finite number of buyers there also exist mixed-strategy equilibria where the market clearing date remains bounded away from zero, even as the interval between offers shrinks to zero. We do this by constructing (both stationary and non-stationary) equilibria that can be described as follows: The monopolist initially chooses a high price and waits for a critical mass of buyers to purchase before lowering the price to the remaining buyers. While buyers' payoff is decreasing in time, those who purchase first pay a higher price and get a lower payoff. As early buyers create a positive externality for the remaining ones, from the high-valuation buyers' perspective the game resembles a war of attrition and they delay purchases in the hope others purchase first—in equilibrium buyers randomize over (conditional) purchasing dates. This attrition behavior, together with the monopolist's pricing decision, creates inefficient real-time delay.

In the equilibria we study here, the monopolist would like to make a *one period* price cut to increase the probability some high-valuation buyers purchase in that period and therefore reduce the interest which would otherwhise be lost while delaying the remaining sales. Similar to the effect described by Coase, this potentially creates an unravelling of prices which limits the monopolist's profit. When strategies depend not only on current demand but also directly on past prices, i.e. are non-stationary, the threat that if the monopolist cuts the price then all players coordinate in a low profit equilibrium thereafter can be enough to make the monopolist adhere to a path where buyers engage in a war of

[2] For this reason the outcome is known as Pacman, after a classic computer game.

attrition. The harsh consequence of loosing his reputation disciplines the monopolist and makes him set a high initial price.

Such trigger strategies are ruled out when strategies depend only on current demand, i.e. are stationary. In this case a similar punishment effect can replace reputation. We find that, when buyers hold heterogeneous postures, a small price cut can induce early purchases by *soft* buyers, leaving only *tough* buyers and a low continuation profit.[3] The monopolist avoids reaching that state by adhering to a high price which keeps the soft buyers in the market—until some tough buyers purchase first—and willingly enduring a buyer war of attrition and losses through delay.

Given the significant implications of the standard model to regulation and antitrust, identifying and understanding sources of inefficiency in monopolized durable-goods markets has been an important research topic. Authors have studied the role of reputation (Ausubel and Deneckere, 1989), the effect of time varying demand (Sobel, 1991) and challenged technology and informational assumptions—for example accounting for imperfect durability (Deneckere and Liang, forthcoming), limited capacity (Bulow, 1982 and McAfee and Wiseman, forthcoming) and private information on cost (Ausubel and Deneckere, 1992).

The present article adds to this literature and complements the work of Ausubel and Deneckere (1989) on the relationship between reputation and inefficiency. They showed in the standard model with a *continuum* of buyers that market outcomes can be inefficient if strategies are non-stationary. In the reputational equilibria they constructed the monopolist retains market power, the price path is deterministic and sales necessarily occur over infinite time—so the market can never clear. Importantly these equilibria exist only if the lowest buyer valuation does not exceed the monopolist's cost—the *no-gap* case. Here we show instead that in the same standard setting with a *finite number* of buyers reputational concerns are not necessary for inefficiency since the monopolist may let buyers engage in a war of attrition even when strategies are stationary—in addition no-gap is irrelevant. Importantly, in this setting the market eventually clears and price paths can be stochastic.

More generally, the mechanism we describe here is relevant to the study of bargaining—the durable-goods monopoly model is a special bargaining game where one player makes non-discriminating offers to the remaining ones. Gomes (2005) showed, by means of an example, that in bargaining games with *exogenous* positive externalities there can be real-time delay in stationary strategies when the grand coalition is inefficient or unfeasible since attrition behavior can be sustained by the belief of each player that a coalition not involving themselves will form. In our setting externalities arise *endogenously* from equilibrium play—here attrition behavior is sustained by the belief that some other player may accept the current offer and trigger better future offers (such as a price reduction). This belief can be consistent when a player makes non-discriminating offers to the remaining ones, which they then simultaneously accept or reject. Non-discrimination can therefore lead to real-time delay even in the absence of exogenous externalities, and when the grand coalition is, as here, efficient.

This mechanism has other interesting applications. For example, in takeovers and acquisitions minority shareholder protection typically rules out discrimination.[4] A large literature has studied whether a raider, who can improve the management, has the incentive to incur the bidding cost anticipating that current shareholders may fail to tender their

[3] These are rational postures, unlike the irrational types often used in reputation models.
[4] The European Directive on Takeover Bids and the Williams Act.

shares while trying to free-ride on the expected appreciation (e.g. Grossman and Hart, 1980 and Holmstrom and Nalebuff, 1992). Such exogenous externalities can lead to inefficiencies. In many cases there will be no externalities since an acquisition may not increase the target's value. For example there may be only synergies between the acquiring company and the target, a situation which is formally similar to the durable-goods game when current shareholders disagree on the target's value. Our analysis suggests that also in those cases successive tender offers can generate endogenous externalities and therefore takeovers may take an inefficiently long time to be completed.

The remainder of the article is organized as follows. In section 2 we present the model and in section 3 we briefly look at pure strategies. The core of the paper is section 4, in which we focus on mixed strategies to characterize and explain buyer-attrition. We conclude in section 5. (The main proofs are instructive and were included in the text; minor proofs were relegated to an appendix.)

2 The model

We consider the standard durable-goods monopoly model with a finite number of buyers. A monopolist seller, indexed by m, can produce any amount of a durable good at a constant marginal cost in any period $t = 0, 1, 2, ...$ There is a set $N = \{1, .., n\}$ of buyers and each buyer has a valuation $v(i) > 0$ for a single unit of the good. $H = \max\{v(i)|\, i \in N\}$, $L = \min\{v(i)|\, i \in N\}$, and z is the number of buyers with valuation L. We set the marginal cost to zero to interpret prices and valuations as net of the cost.

The monopolist is precluded from setting prices based on buyers' valuations, so in each period t the monopolist posts a common price $p_t \in R$. Buyers then simultaneously accept the current price and leave the game or reject it and continue to the next period—action $a_t^i = 0$ denotes a rejection in period t by buyer i and $a_t^i = 1$ an acceptance; each buyer i may choose $a_t^i = 1$ at most once. There is complete information, so all valuations, prices, and purchases are observed.

The discount factor is $\delta \equiv e^{-\rho\triangle} \in (0, 1)$, where $\triangle > 0$ denotes the real time between two successive offers and $\rho > 0$ the common discount rate. The payoff of buyer i is

$$u^i = \begin{cases} \delta^t \left[v(i) - p_t\right] \text{ if } a_t^i = 1 \text{ for some } t \\ 0 \text{ if } a_t^i = 0 \text{ for all } t \end{cases},$$

and the monopolist's payoff is the discounted revenue

$$u^m = \sum_{t=0}^{\infty} \delta^t \left(p_t \sum_{i \in N} a_t^i \right).$$

A t-period history $\eta(t)$ is a list of prices and purchases from period 0 to $t-1$. A pure strategy is a function specifying a player's action plan at each period for each history prior to that period. Denote the vector of strategies by players other than j by $\mathbf{s}^{-j}$ and the vector of all strategies by $\mathbf{s} = (s^j, \mathbf{s}^{-j})$. For a given $\mathbf{s}$, player j's expected payoff is

$$\mu^j(\mathbf{s}) \equiv E\left[u^j \,|\mathbf{s}\right].$$

Let $\mu^j(\mathbf{s}|\, \eta(t))$ denote player j's expected payoff if after history $\eta(t)$ players behave according to $\mathbf{s}$. A strategy is Stationary Markov, or simply a *stationary strategy*, if it only depends on the payoff relevant history, which in this game is the set $I(t) \subseteq N$ of buyers remaining in the market at t. A Stationary Markov Equilibrium, or simply a *stationary equilibrium* (SE), is a strategy vector $\mathbf{s}^*$ such that for all $I(t)$ and for any alternative strategy s^j satisfies

$$\mu^j(s^{j*}, \mathbf{s}^{-j*} | I(t)) \geq \mu^j(s^j, \mathbf{s}^{-j*} | I(t)) \text{ for all } j \in N \cup m.$$

In line with the literature we focus on the case where the time between offers $\triangle$ is close to zero. Most of the work should be understood in this light and we will typically drop the qualifier "for δ close to 1" in the remainder of the paper. We want to know whether all outcomes are real-time efficient:

Definition 1. *Real-time efficiency*: As $\delta \to 1$ all gains from trade are realized, i.e.

$$\lim_{\delta \to 1} \left[\sum_{i \in N} \mu^i + \mu^m \right] = \sum_{i \in N} v(i).$$

The game is formally infinite but it is practically over when all buyers have purchased, i.e. the market has cleared. We now present a first and important lemma (the proof is simple and included in the appendix):

Lemma 1. Before the market clears, the relevant price space is $p_t \in [L, \infty)$ and the market clears at t if and only if $p_t = L$, i.e. $p_t = L \Leftrightarrow I(t+1) = \emptyset$.

So either all L-buyers are in the market or the game is over. Moreover the monopolist never sets the price below L since that would lower the revenue without generating additional sales. Although L-buyers use a simple cut-off strategy—buying whenever the price does not exceed L—and get no surplus, they affect the monopolist's cost of waiting which is an important determinant of equilibrium behavior.

3 Pure strategies and efficiency

In this section we briefly look at pure strategies and introduce two outcomes the literature has focused on: the Pacman and the Coasian outcomes. Bagnoli, Salant, and Swierzbinski (1989) showed that the *Pacman* strategy always forms a SE: the monopolist posts a price equal to the highest valuation still in the market and all buyers with that valuation purchase immediately, i.e. for all $I(t) \neq \emptyset$

$$p_t = \max \{v(i) : i \in I(t)\} \text{ and } a_t^i = \begin{cases} 1 \text{ if } p_t \leq v(i) \\ 0 \text{ otherwise} \end{cases}$$

The outcome is real-time efficient since the market clears in at most n periods and as $\delta \to 1$ the monopolist's payoff becomes that of a perfectly discriminating monopolist, i.e.

$$\lim_{\delta \to 1} \mu^m = \sum_{i \in N} v(i) \text{ and } \mu^i = 0 \text{ for all } i \in N.$$

The Pacman equilibrium is unique only if every buyer has a valuation which is "large relative to the sum of valuations of all buyers with a lower willingness to pay" (von der Fehr and Kuhn, 1995 pp. 791). Otherwise additional SE exist and some may be Coasian.

In a *Coasian* equilibrium prices are never significantly higher than L, i.e. $\lim_{\delta\to 1} p_t^* = L$ for all $I(t) \neq \emptyset$, so any benefit from price discrimination vanishes as the time between offers becomes arbitrarily close to zero, i.e.

$$\lim_{\delta\to 1} \mu^m = nL \text{ and } \lim_{\delta\to 1} \mu^i = v(i) - L \text{ for all } i \in N.$$

This outcome is also real-time efficient. If differences in valuations are not too large there always exists a Coasian equilibrium with an opening price $p_0^* = L$, which all buyers immediately accept (see Claim 1 in the appendix).

Any SE with low profits can support additional history-dependent equilibria. For example, equilibria where the monopolist always posts a price larger than H before some date t and then players coordinate on the Pacman outcome after that date can be sustained by the threat to revert to a Coasian equilibrium if the monopolist posts a price lower than H before t—provided that t is not too large. These reputational equilibria have a zero probability of trade for some time and do not seem to capture any relevant real-world phenomena. This contrasts with similar reputational equilibria identified by Ausubel and Deneckere (1989) in the model with a continuum of buyers. In both cases price paths are deterministic but the latter equilibria have an arbitrarily-slow (but positive) rate of sales. Stationary rules out reputational equilibria: all SE in pure strategies are real-time efficient (see Claim 2 in the appendix).

4 Mixed strategies, attrition and delay

In the remainder of the paper we focus on mixed strategies. We present buyer attrition—a new source of monopoly inefficiency—and explain why even in such a standard setting reputation is unnecessary for real-time delay.

First we note that a mixed strategy equilibrium is not *per se* inefficient. An outcome with a positive rate of sales is real-time inefficient if and only if, when the interval between offers is close to zero, the per period probability of acceptance is arbitrarily close to zero for all buyers.[5] In the equilibria we construct below we have real-time delay since the probability of making a sale before a certain date converges to an exponential distribution.

We will focus on *symmetric* equilibria—buyers with the same valuation use the same strategy—and we let consumers' valuations take three values: low, medium and high, i.e. $v(i) \in \{L, M, H\}$ with $0 < L < M < H$. In this case the payoff relevant history of the game $I(t)$ can be summarized by

$$h(t) = (n_t^H, n_t^M, n_t^L),$$

where $n_t^v \in \mathbb{N}$ is the number of buyers with valuation v in the market at time t. To present our results we also find it convenient to focus most of the time on the simplest case with outcomes where buyers engage in a war of attrition in both non-stationary and stationary strategies:

$$h(0) = (2, 1, z).$$

[5]At least in some state reached with positive probability. Suppose not, then the game transitions from state to state in the twinkle of an eye until the market clears.

To simplify notation we henceforth make reference to each state $h(t)$ in the text and, since stationary play depends on h alone, we denote equilibrium actions (not strategies) by p_h^* and a_h^{v*}.

We proceed by studying the subgames, looking at buyer postures (subsection 4.1) and the Coase temptation (subsection 4.2). In these steps we review the forces behind the Pacman and Coasian outcomes and use them to trace delay. We then identify buyer attrition in both non-stationary (subsection 4.3) and stationary equilibria (subsection 4.4).

4.1 Soft and tough buyers

We can restrict our attention to subgames where $n_t^L = z$ and we move backwards by looking at the subgames with one additional buyer.[6] We show that, following Lemma 2, this additional buyer can hold one of two postures: *soft* or *tough*.

Let $x \equiv (H-L)/L$ and $y \equiv (M-L)/L$, then:

Lemma 2. The game with one H-buyer and z L-buyers has two SE if $z \geq x$: the Pacman and a Coasian with $p_h^* = L$. The Pacman outcome is unique otherwise. (Similarly for the game with a single M-buyer if $z \geq y$.)

Proof. In the case of $h(t) = (1, 0, z)$ we look in step 1 at $p_h > L$ and in step 2 at $p_h = L$ (the proof for the state $h(t) = (0, 1, z)$ is analogous).

Step 1: Suppose that in state $h(t) = (1, 0, z)$ some price $p > L$ is part of a (mixed) equilibrium price strategy and that $E[p_h]$ is the expected price of this strategy profile. L-buyers always refuse p and, since strategies are stationary, the H-buyer only accepts this price with positive probability if the payoff of accepting p is higher than the payoff of waiting an additional period, i.e.

$$H - p \geq \delta(H - E[p_h]).$$

Since for all $\delta \in (0, 1)$ the H-buyer accepts with probability 1 any price $p \leq E[p_h]$, the monopolist will only choose prices $p_t \geq E[p_h]$. So the monopolist uses a pure strategy: charges one price p_h which the H-buyer always accepts.

Also $p_h \notin (L, H)$ since there always exists some higher price p' which the H-buyer will accept and it also generates a higher profit, i.e.

$$\forall p \in (L, H)\ \exists p' \in (p, H] : H - p' > \delta(H - p)\ \wedge\ p' + \delta z L > p + \delta z L.$$

The price $p_h = H$ however is an equilibrium price—the Pacman outcome—since no price p' exists and for δ sufficiently close to 1 it generates a profit larger than the non-discriminating profit

$$H + \delta z L > (z + 1)L.$$

Step 2: Looking at (1) below, the price $p_h = L$ is also an equilibrium price if the left-hand term—the premium the H-buyer is willing to pay to avoid a one period delay in

[6] Since, from Lemma 1, in games with only L-buyers the offer $p_h^* = L$ is immediatly accepted by all buyers.

consumption when he expects the next period price to be L—is lower than the right-hand term—the monopolist's lost interest from delaying the sales to the L-buyers—, i.e. if

$$[[(1-\delta)H + \delta L] - L] \leq (1-\delta)Lz, \tag{1}$$

since then the monopolist prefers to sell immediately to all buyers (profit on the left-hand below) rather than price discriminate against the H-buyer (profit on the right-hand below):

$$(z+1)L \geq [(1-\delta)H + \delta L] + \delta z L \ \Leftrightarrow z \geq x.$$

If however the premium on the H-buyer is significant—the last condition does not hold—then $p = L$ cannot be an equilibrium price and it follows from step 1 and Lemma 1 that the Pacman equilibrium is unique. *Q.E.D.*

So, when the differences in valuations is not too large, buyers may hold different postures and these are associated with distinct outcomes:

Definition 2. *Soft and tough buyers:* Buyer i is *soft* if in the subgame with only the L-buyers and buyer i the equilibrium outcome is the Pacman, i.e. $p_h^* = v(i)$, and the buyer is *tough* if the outcome Coasian, i.e. $p_h^* = L$.

These are genuine strategic postures, unlike the irrational postures often used in reputation models where each "type" is identified by a fixed acceptance or rejection rule (see e.g. Abreu and Gul, 2000).

4.2 A Coase conjecture with a finite number of buyers

We now study the subgame $h(t) = (2, 0, z)$ when the H-buyers are *tough*, so once a single H-buyer purchases the monopolist reduces the price to L. In this setting the Coase conjecture is verified: in the unique symmetric equilibrium the opening price is virtually L. In fact this result holds more generally in any game where buyers' valuations take two values.

The intuition is the following. For any stationary price larger than L the H-buyers' payoff structure is similar to a stationary war of attrition: their payoff is decreasing in time but those who purchase first get a lower payoff. In the unique symmetric equilibrium H-buyers randomize their acceptance and are indifferent between buying today or waiting, see if some buyer purchases first, and buy tomorrow. This would delay the first sale and make the monopolist loose the interest on subsequent sales.

When buyers use a stationary strategy a one period price cut at t has then two effects: it reduces the profit on those sales made today and, by increasing H-buyers' acceptance rate at t, the monopolist gains the interest in the sales to the remaining buyers which are then expected to be done earlier.

The small loss associated with the former is outweighed by the gain on the latter since the probability of having someone buying at t will then be strictly positive—without a price reduction this probability is instead arbitrarily close to zero. Since this is true for any high price, the monopolist cannot sustain such prices in any equilibrium. Interestingly, the latter point of this explanation echoes the intuition for the Coase conjecture proposed by Gul, Sonnenschein, and Wilson (1986) in the same setting with a continuum of buyers.

With $W \equiv (1-\delta)H + \delta L$ we have:

Lemma 3. When H-buyers are tough, the play of the unique symmetric SE in state $h(t) = (2, 0, z)$ is Coasian. It has the following equilibrium actions:

$$\begin{cases} p_h^* = L \text{ and } a_h^{H*} = a_h^{L*} = 1 \text{ if } z > 2x. \\ p_h^* = W,\ a_h^{H*} = 1 \text{ and } a_h^{L*} = 0 \text{ otherwise.} \end{cases}$$

Proof. In step 1, 2 and 3 we show by contradiction that there can be no equilibrium with $p_h^* \in (W, H]$. In step 4 we look at $p_h^* \in [L, W]$.

Step 1: Suppose that in state $h(t) = (2, 0, z)$ there exists some $p_h^* \in (W, H]$ when $p_h^* = L$ in state $h(t) = (1, 0, z)$. Then the first buyer creates a positive externality for the remaining one in the form of a low price: if H-buyer i accepts the offer p_t^* his payoff is $\mu^i = H - p_t^*$, while H-buyer's j payoff is $\mu^j = \delta(H - L)$ if he waits one period as $p_{t+1} = L$, the p_h^* in state $h(t + 1) = (1, 0, z)$.

Thus, from the H-buyers' perspective, the game resembles a war of attrition: the returns to buying decrease with time but, at any time, each buyer is better off if the other buys first. For this reason, for a given p_h^* there are several SE but a unique symmetric equilibrium and it involves mixed strategies.

If the monopolist is expected to offer an equilibrium stationary price $p_h^* \in (W, H]$ in every period after t, we can let $q^i(p_t)$ denote the probability of a typical H-buyer i accepting an offer p_t at t conditional on reaching this period t by

$$q^i(p_t) = \Pr(a_t^i(p_t) = 1 \big| \, p_h^*) \text{ if } v(i) = H.$$

This conditional probability amounts to a behavioral strategy (the strategy is a probability distribution over all dates). In a mixed-strategy equilibrium each buyer must be indifferent between purchasing today or waiting, see if the other buyer purchases first, and purchase tomorrow. So H-buyer i's best response function is:

$$\left.\begin{array}{r} q^i(p_t) = 1 \text{ if } H - p_t > \\ q^i(p_t) \in [0, 1] \text{ if } H - p_t = \\ q^i(p_t) = 0 \text{ if } H - p_t < \end{array}\right\} \delta\left[q^j(p_t)(H - L) + (1 - q^j(p_t))(H - p_h^*)\right].$$

In the symmetric equilibrium, for a given p_t and $p_h^* \in (W, H]$, we have

$$q^i(p_t) = q^j(p_t) = q(p_t) \begin{cases} = 1 \text{ if } p_t < W \\ = \frac{(H - p_t) - \delta(H - p_h^*)}{\delta(p_h^* - L)} \in (0, 1) \text{ if } p_t \in [W, (1 - \delta)H + \delta p_h^*] \\ = 0 \text{ if } p_t > (1 - \delta)H + \delta p_h^* \end{cases} \tag{2}$$

As expected the value of the equilibrium randomization depends on δ.

Step 2: We now fix a price $p_h^* \in (W, H]$ in state $\widetilde{h}(t) = (2, 0, z)$. For the H-buyers the game becomes a standard stationary war of attrition with the conditional acceptance probability given by

$$q(p_h^*) = \frac{(1 - \delta)(H - p_h^*)}{\delta(p_h^* - L)}.$$

Not only is q decreasing in δ, but for δ arbitrarily close to 1 each H-buyer will accept the current offer with a probability arbitrarily close to zero. Formally, we divide the numerator and denominator of $q(p_h^*)$ by δ and, since $\delta \equiv e^{-\rho\triangle}$, we have

$$\lim_{\triangle\to 0}\frac{q(p_h^*)}{\triangle} = \lim_{\triangle\to 0}\frac{(H-p_h^*)}{(p_h^*-L)}(\frac{1-e^{-\rho\triangle}}{\triangle}) = \rho(H-p_h^*)/(p_h^*-L).$$

That is, in the limit each H-buyer's mixed strategy over purchasing dates is characterized by an exponential distribution with parameter $\rho(H-p_h^*)/(p_h^*-L)$.

The monopolist's expected payoff in this subgame if he charges the price p_h^*, $\mu_h^m(p_h^*)$, is the solution to

$$\mu_h^m(p_h^*) = q(p_h^*)^2\left[2p_h^* + \delta zL\right] + q(p_h^*)(1-q(p_h^*))\left[p_h^* + \delta(z+1)L\right] + (1-q(p_h^*))^2\delta\mu^m(p_h^*)$$

or

$$\mu_h^m(p_h^*) = \frac{1}{q(p_h^*)(2-q(p_h^*))}\left[q(p_h^*)^2\left[2p_h^* + \delta zL\right] + q(p_h^*)(1-q(p_h^*))\left[p_h^* + \delta(z+1)L\right]\right]. \quad (3)$$

Letting the time between offers converge to zero we have that

$$\lim_{\delta\to 1}\mu_h^m(p_h^*) = \alpha(p_h^*)\cdot\left[p_h^* + (z+1)L\right] \text{ where } \alpha(p_h^*) = \frac{2(H-p_h^*)}{2H-p_h^*-L}.$$

So the monopolist's expected profit is only a share $\alpha(p_h^*) \in (0,1)$ of the profit he would get in case one of the H-buyers accepts p_h^* immediately.

Step 3: Still supposing that $p_h^* \in (W,H]$ is a stationary equilibrium price, when buyers' strategies are stationary the monopolist can increase the acceptance rate at t by offering a slightly lower price at t and this will not affect the future. He then i) loses profits on buyers who accept p_t but ii) gains the additional interest on the sales to all remaining buyers that are the expected to be made earlier.

We now show that, for δ arbitrarily close to 1, i) is smaller than ii) since a small price-cut has a negligible effect on the revenue from current sales but generates a non-negligible increase in the acceptance probability of current period. Therefore it is always optimal to undercut any price $p_h^* \in (W,H]$, which in turn implies that no such price can be part of a SE.

Formally, the monopolist's payoff when he offers a price $p_t \in (W,(1-\delta)H + \delta p_h^*)$ is

$$\mu_h^m(p_t) \equiv q(p_t)^2(2p_t + \delta zL) + 2q(p_t)(1-q(p_t))(p_t + \delta(z+1)L) + (1-q(p_t))^2\delta\mu^m(p_h^*),$$

and with (2) we have that for all $p_h^* \in (W,H]$

$$\begin{aligned}\lim_{\delta\to 1}\left.\frac{d\mu_h^m(p_t)}{dp_t}\right|_{p_t=p_h^*} &= \lim_{\delta\to 1}\left[\frac{\partial\mu_h^m(p_t)}{\partial p_t} + \frac{\partial\mu_h^m(p_t)}{\partial q(p_t)}\frac{\partial q(p_t)}{\partial p_t}\right]_{p_t=p_h^*} \\ &= -\frac{2}{p_h^*-L}(1-\alpha(p_h^*))\left[p_h^* + (z+1)L\right] < 0.\end{aligned}$$

Step 4: Now we focus on $p_h \in [L,W]$ which, by Lemma 1, both H-buyers accept with probability 1 (L-buyers only accept offers which do not exceed L). Depending on the number of L-buyers z the price which maximizes the seller's payoff in that range is either L or W. *Q.E.D.*

As mentioned above, this result and its intuition extends to any game with two valuations:

Proposition 1. When H-buyers are tough, the unique symmetric SE of any game $h(0) = (n-z, 0, z)$ is Coasian.

Proof. By induction on n. Suppose H-buyers are tough and that when $h(t) = (n-z-1, 0, z)$ the outcome is Coasian, i.e. $\lim_{\delta\to 1} p_h^* = L$ (from Lemma 3 this is verified for $n-z=2$). We will show by contradiction that when $h(t) = (n-z, 0, z)$ there cannot be a symmetric SE where $\lim_{\delta\to 1} p_h^* > L$. For the interest of space, in the remainder we consider only the limiting case where δ is arbitrarily close to 1.

Step 1: If there was a non-Coasian SE the H-buyers payoff structure is similar to a continuous time war of attrition and in the unique symmetric profile each H-buyer's mixed strategy is represented by an exponential distribution with parameter $\frac{\rho}{n-z-1}(H-p_h^*)/(p_h^*-L)$ and

$$\lim_{\delta\to 1} \mu_h^m(p_h^*) = \beta(p_h^*)\cdot[p_h^* + (n-1)L] \text{ where } \beta(p_h^*) = \frac{(n-z)(H-p_h^*)}{(n-z)H - p_h^* - (n-z-1)L}.$$

Step 2: We now consider how buyers would respond to a one period price cut at t. If all H-buyers except i accept the current price with probability q, then the payoff of buyer i if he refuses the current offer will be approximately

$$(1-q)^{n-z-1}\cdot(H-p_h^*) + (1-(1-q)^{n-z-1})\cdot(H-L)$$

In a symmetric strategy profile all H-buyers accept with the same probability q and in the unique symmetric solution we have:

$$q(p_t)\begin{cases} = 1 \text{ if } p_t \le L \\ = -e^{\frac{\ln\frac{p_t-L}{p_h^*-L}}{n-z-1}} + 1 \text{ if } p_t \in (L, p_h^*] \\ = 0 \text{ if } p_t > p_h^* \end{cases}$$

That is, the cumulative distribution representing each H-buyer equilibrium strategy now contains a mass point $q(p_t)$ at t (and is an exponential distribution in the remainder of the support).

For every $p_t, p_h^* \in (L, H]$ the monopolist's profit is

$$\mu_h^m(p_t) = (1-q(p_t))^{n-z}\cdot\mu_h^m(p_h^*) + \sum_{s=0}^{n-z-1}[(n-z-s)p_t + (z+s)L]\cdot q(p_t)^{n-z-s}\cdot(1-q(p_t))^s.$$

Step 3: For every $p_t \in (L, p_h^*)$ we have (with equality for $p_t = p_h^*$)

$$\mu_h^m(p_t) > (1-q(p_t))^{n-z}\mu^m(p_h^*) + (1-(1-q(p_t))^{n-z})(p_t + (n-1)L).$$

We Differentiate the expression with respect to p_t and we have that its limit, as $p_t \to p_h^*$, is

$$\frac{\partial}{\partial p_t}(1-q(p_t))^{n-z} \cdot (\beta(p_h^*)-1)\,[p_h^* + (n-1)L]\,,$$

that is, the product of the change in the probability of no one buying at t by the difference in the profit when buyers engage in a war of attrition and when one buyer purchases immediately. It is negative since that probability is increasing in the current price p_t and $\beta(p_h^*) < 1$ for every $p_h^* \in (L, H]$. So undercutting the proposed equilibrium price p_h^* offers always a profitable deviation for the monopolist when buyers' response is symmetric. Since this holds for every $p_h^* > L$, when H-buyers are tough there can be no symmetric SE which is not Coasian. *Q.E.D.*

If it is reasonable to expect that strategies should depend only on current market conditions—and without reason to treat similar buyers differently—,in a market with only a high and a low valuation buyers, a monopolist is able to price discriminate only if he is not expected to reduce its price until each and every high-valuation buyer has purchased—the Pacman outcome. To the extent that a small number of low-valuation buyers can be enough to induce a monopolist to cut its price before all high-valuation buyers have purchased, it is reasonable to expect Coasian outcomes even in markets with many high valuation buyers and few low-valuation buyers.

4.3 Buyer attrition with reputation

We have shown that the subgame $(2, 0, z)$ has no symmetric SE with delay. We will now construct non-stationary symmetric equilibria of this same subgame with high prices which rely on reputation.

In these equilibria the monopolist posts the same price in every period and patiently wait for H-buyers to select in a war of attrition who purchases first and only then he reduces the price to clear the market. Moreover if the monopolist deviates from this price path all players coordinate in a new equilibrium with low continuation profits—the Coasian outcome. That is, by charging a lower price the monopolist looses his reputation of holding to high prices and so even a small price cut will have a large negative effect on profits. This can render a price cut unprofitable and allows the monopolist to sustain high prices.

Let $k \equiv 2(H - 2L)/L$, then for any $\widehat{p} \in (L, H - (\frac{z+2}{2})L)$:

Proposition 2. When $z \in (x, k)$ the following actions form a subgame perfect equilibrium of the game $h(0) = (2, 0, z)$:

$$\begin{cases} p_t = \widehat{p},\ a_t^H = \frac{(1-\delta)(H-\widehat{p})}{\delta(\widehat{p}-L)} \text{ and } a_t^L = 0 \text{ if } h(t) = (2,0,z) \text{ and } p_{t-1} = \widehat{p} \\ \\ \text{the Coasian outcome of Lemma 3 otherwise.} \end{cases}$$

The outcome is real-time inefficient. In the limit, as $\delta \to 1$, each H-buyer acceptance follows a Poisson process with parameter $\rho(H - \widehat{p})/(\widehat{p} - L)$ and the monopolist's expected payoff is

$$\alpha(\widehat{p}) \cdot [\widehat{p} + (z+1)L] \ \text{ where } \alpha(\widehat{p}) = \frac{2(H-\widehat{p})}{2H - \widehat{p} - L}.$$

Proof. Step 1: When $z > x$ and δ is close to 1, the Coasian outcome described in Lemma 3 is a stationary equilibrium of the subgame $h(t) = (2, 0, z)$ which gives the monopolist a sure profit arbitrarily close to $(z + 2)L$. This outcome can be used as a punishment if the monopolist ever charges a price $p_t \neq \widehat{p}$ when $h(t) = (2, 0, z)$. The monopolist adheres to the price path described above, which gives the outcome described in step 2 of Lemma 3, if

$$(3)|_{p_h^* = \widehat{p}} > (z + 2)L.$$

This condition will be satisfied if $\widehat{p} \in (L, H - \frac{z+2}{2}L)$. This set is nonempty if $z < k$.

Step 2: The outcome is real-time inefficient since

$$\lim_{\delta \to 1} \left[\sum_{i \in N} \mu^i + \mu^m \right] = 2(H - \widehat{p}) + \alpha(\widehat{p})(\widehat{p} + (z + 1)L) < \sum_{i \in N} v(i)$$

because $\alpha(\widehat{p}) \in (0, 1)$ for every $\widehat{p} \in (L, H - (\frac{z+2}{2})L)$. *Q.E.D.*

Here H-buyers engage in a war of attrition. As the time between offers converges to zero the cumulative probability that a H-buyer accepts an offer before date t, conditional on the other buyer not accepting before, converges to an exponential distribution with parameter $\rho(H - M)/(H - L)$. Thus a monopoly distortion remains here in the form of real-time delay. This dissipates part of the H-buyers' potential gains and the monopolist can expect to get only a share α of the profit he would get if one H-buyer accepted the opening price immediately.

As in the previous subsection, similar results can be extend to histories with multiple H-buyers. For example, if the price sequence as buyers leave the market is strictly decreasing there will be some delay between each sale.[7] Like the reputational equilibria studied by Ausubel and Deneckere (1989), these reputational equilibria are inefficient—but do not rely on no-gap—, they entail delay and can have an arbitrarily-slow (but positive) real-time rate of sales. They have however different properties: the market eventually clears—even if perhaps after a long-time—and price cuts are stochastic.

These same properties can be obtained even without relaxing the stationarity assumption and for this reason reputation concerns turn out to be unnecessary for inefficiency in the standard model with a finite number of buyers. We turn to this in the next subsection.

4.4 Buyer attrition without reputation

Before looking at the overall game $h(0) = (2, 1, z)$ we still need to consider the subgame with a *tough* H-buyer and a *soft* M-buyer. There is a SE where the M-buyer accepts an offer before the H-buyer.

Lemma 4. For $h(t) = (1, 1, z)$, with a tough H-buyer and a soft M-buyer, the actions $p_h^* = M$, $a_h^{M*} = 1$ and $a_h^{H*} = a_h^{L*} = 0$ form the equilibrium play of a SE.

Proof. Suppose that, for $z \geq x$, the outcome of the game $h(t) = (1, 0, z)$ is Coasian and the outcome of the subgame $h(t) = (0, 1, z)$ is the Pacman. We use the one-stage-deviation

[7] H-buyers engage in a sequence of multiplayer wars of attrition where rewards are monotone in the order in which they buy.

principle: The M-buyer has no advantage in refusing the price M since he would still get the same price in the future while the H-buyer should refuse M since he expects a payoff $\delta(H - L) > H - M$ (and for the same reason he should refuse any price $p_t > (1 - \delta)H + \delta L$). The monopolist can deviate to any $p_t \in [L, (1 - \delta)H + \delta L]$ and sell to the H-buyer immediately, in addition to the M-buyer. However for δ close to 1 such a deviation would give him a payoff lower than his equilibrium payoff $M + \delta(z+1)L$. *Q.E.D.*

In this outcome there is no *skimming*—which states that higher valuation buyers purchase no later than lower valuation buyers. In a model with a continuum of buyers there is always skimming since the equilibrium price path is independent of single buyer decisions and, since delaying a purchase is more costly to high valuation buyers, these always buy earlier. In a model with a finite number of buyers the decision of a single buyer can have a significant effect on the price path and a high valuation buyer can therefore benefit from waiting for some other buyer to purchase first. Skimming fails here because buyers hold heterogeneous postures and their individual actions have a nonnegligible effect on the equilibrium price path.

We have shown that when the difference in valuations is not too high, i.e. $z \geq x$, the actions below form the equilibrium play of a symmetric SE in each proper subgame when the M-buyer is *soft* while H-buyers are *tough* (with $x \equiv (H-L)/L$ and $W \equiv (1-\delta)H+\delta L$):

$h(t)$	p_h^*	a_h^{H*}	a_h^{M*}	a_h^{L*}
$(0, 0, z)$	L	–	–	1
$(0, 1, z)$	M	–	1	0
$(1, 0, z)$	L	1	–	1
$(1, 1, z)$	M	0	1	0
$(2, 0, z)$	L if $z > 2x$	1	–	1
	W if $z \in [x, 2x]$	1	–	0

Now we present an attrition equilibrium for the overall game. In this equilibrium the monopolist first sets the price M and H-buyers delay purchasing for a (random) length of time; then one purchases. The M-buyer purchases next and then the market clears as the price drops to L—the equilibrium play of subgame $h(t) = (1, 1, z)$. So H-buyers play a war of attrition where the looser is the first to buy at price M and the winner is the third to buy at price L (the M-buyer purchases second at a price equal to his valuation).

Let $F \equiv (2H - (z + 2)L)/3$, then:

Proposition 3. For $M \in (W, F)$ and $z > x$ there exists a SE of the game $h(0) = (2, 1, z)$ such that in state $h(t) = (2, 1, n)$ we have $p_h^* = M$ which the M-buyer refuses and each H-buyer accepts with probability

$$\frac{(H - M)(1 - \delta)}{\delta\left[(1 - \delta)H + M - \delta L\right]}. \tag{4}$$

It is real-time inefficient since in the limit, as $\delta \to 1$, each H-buyer's acceptance follows a Poisson process with parameter $\rho(H - M)/(M - L)$ and the monopolist's expected payoff is

$$\alpha(M) \cdot (2M + (z + 1)L). \tag{5}$$

Proof. An argument similar to the proof of Lemma 3 shows that a $p_t \in (M, H]$ cannot be part of a symmetric SE of the game we are studying.

Consider now $p_t \in (W, M)$. The M-buyer accepts such price with probability 1 since no lower price is offered in any other state he is present. H-buyers should therefore reject p_t since $p_{t+1} = p_h^* \leq W$ in $h(t+1) = (2, 0, z)$ (unless p_t is arbitrarily close to L, in which case they should accept). So any $p_t \in (W, M)$ induces a subset of buyers to accept immediately and, for δ arbitrarily close to 1, this gives the monopolist an assured profit of approximately

$$p_t + (z + 2)L \tag{6}$$

which has the supremum for $p_t = M$ (which the M-buyer is indifferent between accepting or rejecting).

Suppose the M-buyer always rejects $p_t = M$ when $h(t) = (2, 1, z)$. Why is it that the monopolist does not charge a price slightly lower than M (which the M-buyer always accepts) and virtually get $M + (z + 2)L$ in the twinkle of an eye?

To answer this question we need to compute the profit when the price is M. If $p_h^* = M$ in state $h(t) = (2, 1, z)$, from the H-buyers' perspective the game resembles once again a stationary war of attrition (see step 2 from the proof of Lemma 3). In the only symmetric equilibrium H-buyers accept the monopolist's offer with probability (4) and the monopolist gets, in the limit, an expected profit of (5). When δ is arbitrarily close to 1, the monopolist sets the price $p_h^* = M$ in state $h(t) = (2, 1, z)$ if this profit is larger than the supremum of the sure profit (6), i.e.

$$\alpha(M) \cdot (2M + (z + 1)L) > M + (z + 2)L. \tag{7}$$

The soft M-buyer essentially makes the profit function discrete at $p_t = M$. Under condition (7) a price cut is unprofitable and the monopolist can credibly wait out for additional sales before lowering the price. This discreteness breaks the Coase temptation.

For δ arbitrarily close to 1 the proposed strategies form an equilibrium if $M \in (W, F)$ and $z > x$: the upper bound on M ensures that (7) is verified and $z > x$ ensures the actions in table 1 are part of a SE. Finally we have that (W, F) is not empty (plotting the values of L as a function of z for which both conditions are satisfied for a given value of H—for each point (z, L) in that region the range of admissible values of M is (W, F)).

Since the market takes potentially a long time to clear the outcome is real-time inefficient (see also step 2 of proposition 2). *Q.E.D.*

So there is a SE of the overall game where buyers engage in a war of attrition if H-buyers' acceptance rate can be made high, i.e. $(H - M)/(M - L)$ is high, but the profit made with the sales to low valuation buyers is not too high. In that situation the benefits of setting the higher price M and let a war of attrition select the first buyer will outweigh the cost of delaying sales to the remaining ones.

When the ratio $(H - M)/(M - L)$ is low the H-buyers' hazard rate is also low and it can therefore take a very long time to make the first sale. Waiting for the H-buyers to select in a war of attrition who makes the first purchase will substantially delay the remaining sales. The monopolist will therefore choose to precipitate market clearing by slightly cutting the price, selling immediately to the soft M-buyer, and selling in the next period to all remaining buyers.

In a nutshell, the reason a *soft* M-buyer enables the monopolist to sustain a high price is the following: In the initial state of the game any price lower than M is accepted by the M-buyer, so it moves the game to a state where the monopolist can't resist the Coase temptation and therefore he makes a small profit. So the monopolist will not price below M to keep the *soft* buyer in the market until at least one of the *tough* buyers has purchased.

In some sense the soft buyer replaces the role of reputation. With non-stationary strategies a small price cut changes the non-payoff relevant history of the game and buyers delay purchases anticipating a punishment path with an even lower price (and profit). In that case trigger strategies discipline the monopolist's pricing behavior. In the stationary case a small price cut leads to a purchase, which triggers a change in the payoff relevant history, but this also moves the game to a state with a low continuation profit. While no trigger strategies are used in the latter case, a similar punishment effect renders a price cut unprofitable.

5 Conclusion

We studied a standard durable-goods monopoly model with a finite number of buyers. We first saw that all stationary pure-strategy equilibria are real-time efficient. Then, in a simple tractable setting, we showed that inefficient delay and unscheduled price cuts can arise in both stationary and non-stationary mixed-strategy equilibria where buyers randomize their purchasing date. These properties are distinct from those found in the previous literature, including the inefficient reputational equilibria models with a continuum of buyers. For this reason the present work also offers a complementary view—and predictions—on the role and effect of reputation.

Most of all, buyer attrition can help us understand common price patterns in durable-goods markets, where the price of many hit goods seem to start high and after some time, once some buyers have purchased, the price finally drops and the good becomes available to many buyers. We showed that these are equilibrium features even of the standard complete information model with a finite number of buyers.

But the same qualitative features can be found as well in incomplete information settings. For example, the standard model assumes common knowledge of the payoff relevant history. We could have instead restricted buyers to observe only prices and the monopolist to additionally know the number of buyers who purchased in each period. There are perfect-Bayesian equilibria of this game where the monopolist initially charges a high price and high valuation buyers engage in a (unobservable actions) war of attrition. Purchases still follow a Poisson process, so over time some buyers leave the market and purchases become less frequent. When a certain number of units has been sold the monopolist finds it optimal to finally cut its price and sell to all remaining buyers. In turn, the anticipation of such price cut—although at an unknown date—rationalizes the high valuation buyers' randomization.

We find similar features even when buyers have private valuations. Suppose for instance that there are two types of buyers—high and low—and that each high valuation is independently drawn from a common distribution.[8] There are perfect-Bayesian equilibria of this game in which the monopolist posts a high price until a certain number of units is sold and decreases the price thereafter to clear the market. The equilibrium features of this related model are consistent with the frenzies surrounding the launch of many hit goods, which are followed by a market slowdown before a price cut. The reason is that, from the high-valuation buyer's perspective, the game resembles a generalized war of attrition as studied by Bulow and Klemperer (1999).[9] There are many other interesting ways of introducing incomplete information in durable-goods markets and it would be interesting to explore further how buyer attrition can determine equilibrium behavior under alternative assumptions.

In this context selling can be less efficient than leasing since losses from inefficient delay can outweight the standard static deadweight loss associated with leasing. This contrasts with the more conventional view that requiring a durable-goods monopolist to sell its products rather than leasing is a good policy when it does not induce the monopolist to make inefficient durability or investment decisions.

In our view, once we leave behind the focus on two particular outcomes—Coasian and the Pacman—, the standard setting can still significantly enhance our understanding of durable-goods markets, providing new, alternative, and empirically relevant predictions.

Appendix

Proof of Lemma 1. We prove the first statement in step 1 and the second in step 2.

Step 1: Denote by $\overline{p}$ the highest price any buyer would accept with probability 1 in all subgames, i.e. $\overline{p} = \sup\{p : a_t^i = 1\ \forall i \in I(t) \subseteq N\}$. The monopolist's equilibrium profit has to be non-negative, so a profit maximizing monopolist will always offer $p_t \geq \overline{p} \geq 0$. For any $\mathbf{s}^*$ we have $\mu^i \leq v(i) - \overline{p}$ for all $i \in N$. Buyer i will also accept with probability 1 any other price $\widetilde{p}$ such that

$$v(i) - \widetilde{p} \geq \delta(v(i) - \overline{p}) \Leftrightarrow \widetilde{p} \leq (1-\delta)v(i) + \delta\overline{p}.$$

Since buyer i refuses prices larger than $v(i)$, $\overline{p} = L$ by the definition of $\overline{p}$. So without loss of generality we can restrict the monopolist's action space to $p_t \geq L$.

Step 2: By step 1, all buyers accept with probability 1 a price $p_t = L$ when it is offered, i.e. $a_t^i = 1$ for all i since

$$v(i) - L \geq \delta(v(i) - p_t) \text{ for all } v(i) \in [L, H] \text{ and } p_t \geq L.$$

Buyers with $v(i) = L$ refuse all prices $p_t > L$. *Q.E.D.*

Proof of Claim 1. Suppose that $p_t^* = L$ for every payoff relevant history $I(t)$. Then, from Lemma 1, if the monopolist offers a price $p_t > L$

$$a_t^i = 1 \text{ for all } i : v(i) - p_t \geq \delta(v(i) - L) \Leftrightarrow p_t \leq v(i)(1-\delta) + \delta L. \quad (8)$$

[8] To be more specific, a distribution $F(v)$ with $F(H-\varepsilon) = 0$, $F(H) = 1$, a strictly positive finite derivative everywhere and $\varepsilon < H - L$. This contrasts with a setting in which *all* buyers can have a low valuation, i.e. $F(L) = 0$. In the latter case we expect the Coase conjecture to be verified.

[9] If the monopolist is expected to drop the price to the lowest market valuation after k units have been sold, then $k-1$ buyers purchase immidiatly and *one too many* high-valuation buyers are left competing to buy at a lower price.

By the one period deviation principle, the proposed strategy forms an equilibrium if its profit is higher than the profit of selling at a premium to those l buyers satisfying (8), i.e. if

$$nL \geq l\left[v(i)(1-\delta)+\delta L\right]+\delta\left[(n-l)L\right] \Leftrightarrow nL \geq lv(i) \tag{9}$$

Since $l \leq n-z$ and $v(i) \leq H$, (9) is always satisfied if

$$(n-z)H \leq nL \Leftrightarrow \frac{H-L}{L} \leq \frac{z}{n-z},$$

i.e. if the difference between the highest and the lowest valuation is not too high. *Q.E.D.*

Proof of Claim 2. If we restrict attention to pure strategies which are stationary, a price which all buyers refuse cannot be part of a stationary equilibrium since the expected payoff of all players would be zero from that time on. So in each state a nonempty subset of the remaining buyers has to accept every equilibrium price with probability one. In equilibrium the market always clears in at most n periods and it is therefore real-time efficient. *Q.E.D.*

References

Abreu, D. and Gul, F. "Bargaining and Reputation." Econometrica, Vol. 68 (2000), pp. 85-117.

Ausubel, L.M. and Deneckere , R.J. "Reputation in Bargaining and Durable Goods Monopoly." Econometrica, Vol. 57 (1989), pp. 511–531.

Ausubel, L.M. and Deneckere , R.J. "Durable Goods Monopoly with Incomplete Information." Review of Economic Studies, Vol. 59 (1992), pp. 795–812.

Bagnoli, M., Salant, S.W., and Swierzbinski, J.E. "Durable-Goods Monopoly with Discrete Demand." Journal of Political Economy, Vol. 97 (1989), pp. 1459–1478.

Bulow, J. "Durable Goods Monopolists." Journal of Political Economy, Vol 90 (1982), pp. 314-332.

Bulow, J. , Klemperer, P. "The Generalized War of Attrition." American Economic Review, Vol. 89 (1999), pp. 175-189

Coase, R. "Durability and Monopoly." Journal of Law and Economics, Vol. 15 (1972), pp. 143-149.

Deneckere, R. and Liang, M. "Imperfect Durability and the Coase Conjecture." Rand Journal of Economics (forthcoming).

von der Fehr, N.M. and Kuhn, K.U. "Coase versus Pacman: Who Eats Whom in the Durable-Goods Monopoly?" Journal of Political Economy, Vol. 103 (1995), pp. 785-812.

Gomes, A. "Multilateral Contracting with Externalities." Econometrica, Vol. 73 (2005), pp. 1329-50.

Grossman, S and Hart, O. "Takeover Bids, The Free-Rider Problem, and the Theory of the Corporation " Bell Journal of Economics, Vol. 11 (1980) pp. 42-64.

Gul, F., Sonnenschein, H., and Wilson, R. "Foundations of Dynamic Monopoly and the Coase Conjecture." Journal of Economic Theory, Vol. 39 (1986), pp. 155–190.

Holmstrom, B and Nalebuff, B. "To The Raider Goes The Surplus? A Reexaminationof the Free-Rider Problem" Journal of Economics & Management Strategy, Vol 1 (1992), pp. 37-62.

McAfee, P and Wiseman, T. "Capacity Choice Counters the Coase Conjecture." Review of Economic Studies, Vol. 75 (2008), pp. 317-332.

Sobel, J. "Durable Goods Monopoly with Entry of New Consumers." Econometrica, Vol. 59 (1991) pp. 1455-85.