

# Auction Design and Ex-Post Incentives in Procurement Contracts: Some Theory and Evidence

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## **Abstract**

In procurement, there is an important interplay between the mechanism which awards the contract and the incentive structure that constrains the ex-post behavior of the winning contractor. We examine this relationship in the context of highway procurement, where state agencies have built time based incentives into the bidding structure in recognition of the social costs of highway construction delays. Focusing on welfare-maximization, we show that if properly designed, these “A+B” contracts can yield efficient outcomes. Taking this theory to a large dataset of highway procurement auctions in Minnesota, we find that one particularly simple incentive structure may be optimal in practice. Next, we consider more standard highway contracts where there are no explicit time based incentives, but the state may assess penalties for late contract completion. We show that these standard contracts are a special case of our more general theory, and by appealing to this theory, argue that this incentive structure will generally yield inefficient outcomes. A second-best remedy is to increase the size of penalties, and so we empirically analyze the trade-off between shorter delays and higher bids induced by higher penalties.

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# 1 Introduction

It is estimated that 15 percent of world output is accounted for by the procurement of goods and services for the public sector from private sector firms. Establishing efficient mechanisms for procurement is therefore of considerable interest for guaranteeing the efficient allocation of goods and services. In the United States, much of the procurement system is modeled after the Federal Acquisition regulations, and therefore an auction is held and the lowest qualified bidder wins the contract.

Yet in many situations, the procurer cares not only about the price at which the good can be procured, but also about measures of quality, such as the speed with which the contract is completed. Some government agencies have started to experiment with auction designs in which bidders submit bids that include a commitment to a given level of quality. In this paper we examine one such type of design - a scoring auction - paying particularly careful attention to not only the auction itself, but also the ex-post incentive structure established by the contract form. As we will show, this interaction between auction design and ex-post incentives plays an important role in determining outcomes, though it has not yet received much attention in the academic literature.

We take as a case study the use of time incentives in highway procurement. This is an important topic in its own right, as road closures and delays due to highway construction can impose large daily time costs on road users. The size of this externality varies with the traffic flow on the road, but for important bridges can easily top \$100 000 a day. Our focus is on how a judicious choice of auction design and ex-post incentives can be used to achieve efficient outcomes in the face of these externalities. We examine this both theoretically and empirically, using a large dataset of contracts from the Minnesota State Department of Transportation (MNDOT). An attractive feature of our theoretical analysis is that we see two kinds of contracts in our data: the standard highway procurement contracts (with no time incentives) and the newer “A+B” contracts which include time incentives. Our theoretical analysis is able to capture both these contract forms within a single unified framework.

We find the A + B scoring auctions used by MNDOT and other agencies can be used to achieve efficient ex-ante allocations and ex-post outcomes, provided the incentives are correctly set. Taking this theory to the data, we find that while contractor behavior

generally conforms to the predictions of the theory, certain deviations suggest that a particularly simple incentive structure will be best in practice. Turning to the standard highway procurement contracts, we show that they are generally inefficient. Nonetheless, the theory suggests that social welfare could be improved by assessing harsher penalties for late completion. Again confronting the theory with the data, we find that the damages are weakly enforced, and so simply assessing penalties with greater frequency would be welfare improving.

The paper is in two parts. In the first part of the paper, we provide a characterization of equilibrium behavior by contractors participating in both A+B and standard procurement contracts, with a focus on how the time-based incentives affect contractor behavior both at the time of bidding and after winning the contract. We allow for both multidimensional private information and ex-ante uncertainty about costs after the auction completion. In the second part of the paper, we provide empirical analysis based on the MNDOT data, testing the predictions of the theory. We also assess the welfare-improving potential of assessing higher time penalties in standard contracts.

This paper is related to three main sets of literature. There is a growing literature on procurement when the procurer has preferences over dimensions of the contract other than price (Che (1993) and Asker and Cantillon (2008a)). There is also a long literature that can be traced back to Tirole (1986) on the importance of ex-post incentives and renegotiation on contract form, including the more recent paper by Bajari and Tadelis (2001). The empirical literature has lagged behind, but Bajari, Houghton, and Tadelis (2007) examine the role of adaptation costs on bidding in highway procurement.

Section 2 presents the theoretical analysis. Sections 3 and 4 provide an empirical analysis of A+B and standard highway procurement contracts, respectively. Section 5 concludes. All proofs are in the appendix.

## 2 Theory

### 2.1 Model

Our aim is to show that the A+B auction format used in highway procurement can achieve socially efficient outcomes, provided the incentive structure is well chosen. The model is a standard independent private values model of a scoring auction, augmented by two real-world features. The first is multidimensional costs, so that contractors can differ on both their materials costs and costs relating to completion time. The second is ex-ante uncertainty about cost realizations, so that at the time of bidding the contractor is uncertain as to how expensive it will be to complete this contract at a given time. These modeling features are incorporated below:

**AUCTION FORMAT:**  $n$  risk-neutral contractors bid on a highway procurement contract awarded by the government. A bid is a pair  $(b, d^b)$  indicating the base payment  $b$  that the winning contractor will receive (the “A-part”), and the number of days  $d^b \in [0, \bar{d}]$  that the contractor commits to complete the contract in (the “B-part”). The upper bound  $\bar{d}$  is the “maximum allowable days”, determined in advance by the project engineer. The bids are ranked according to the scoring rule  $s = s(b, d^b) = b + c_U d^b$ , and the contract is awarded to the contractor with the lowest score. The constant  $c_U$  in the scoring rule is known as the *user cost*. The contract also specifies ex-post incentives: a per day incentive payment  $c_I$  to be paid when the winning contractor completes the contract in advance of  $d^b$ , and per day disincentive  $c_D$  reducing the winning contractor’s base payment when the contractor completes the contract later than  $d^b$ . The three parameters  $(c_U, c_I, c_D)$  define the incentive structure specified by the A + B contract.

**SIGNALS AND PAYOFFS:** Losing bidders receive a payoff normalized to zero. The winning contractor can choose the actual completion time  $d^a$ . His payoff is given by:

$$\pi(b, d^b, d^a; \theta) = b + 1(d^b > d^a)(d^b - d^a)c_I - 1(d^b < d^a)(d^a - d^b)c_D - c(d^a; \theta) \quad (1)$$

where  $c(d^a; \theta)$  is his cost function. The cost function is parameterized by  $\theta \in \mathbb{R}^k$ , with  $k \geq 2$ . It is assumed to be twice continuously differentiable, decreasing and convex in  $d$  for all  $\theta$ , so that completing the contract more slowly lowers costs, but

at a decreasing rate. Overall, his payoff is just his bid, adjusted by the incentive or disincentive payments, less his private costs.

At the time of bidding, the contractor may not fully anticipate his private costs. To formalize this, we assume that each contractor  $i$  observes only a vector of signals  $x_i \in \mathbb{R}^k$  *affiliated* with  $\theta_i$  at the time of bidding. We refer to  $x_i$  as the contractor's type. We assume also that each contractor  $i$  draws the pair  $(x_i, \theta_i)$  *independently* from some distribution  $F_i$ , which has common support  $\mathbb{X} \times \Theta$  for all bidders. Thus the auction framework falls into the independent private values (IPV) framework, but with the added complications of a scoring rule, ex-post incentives and noisy ex-ante signals.

**EQUILIBRIUM:** A (Bayes-Nash) equilibrium of the game comprises a set of bidding strategies  $\beta_1(x) \cdots \beta_n(x)$  of the form  $\beta_i(x) = (b_i(x), d_i^b(x))$ , that are mutual best-responses; and an ex-post completion time strategy  $d_i^a(\theta; b, d^b)$  that may depend on the realized  $\theta$ .

**SOCIAL WELFARE AND EFFICIENCY:** Social welfare is given by  $W(b, d^a, \theta) = V - d^a c_T - c(d^a; \theta)$ . It reflects the total value of the highway project  $V$ , less a daily time delay cost  $c_T$  faced by motorists, less the private costs of the contractor. We will say that a contract is *ex-post efficient* if the incentive structure is such that the contractor chooses  $d^a$  to maximize welfare  $W$  for any realization of  $\theta$ . We will say an A+B contract is *ex-ante efficient* if the winning bidder is always the bidder who generates the highest expected social welfare  $E[W(b, d^a, \theta)|x]$  in equilibrium. These correspond intuitively to productive efficiency (any contractor to whom the job is given maximizes social welfare) and allocative efficiency (the contract is allocated to the contractor who maximizes social welfare in expectation).

**DISCUSSION:** At this point, it is worth discussing what is captured by the model, and what we abstract away from or ignore. The model highlights two aspects of the contracting process. First, there is the incentive structure, which specifies both how bids should be scored, and how subsequent behavior should be evaluated and rewarded or punished. This incentive structure corresponds exactly to that used in the MNDOT A + B auctions, except that the government sometimes additionally caps incentive payments at a maximum amount. The analysis doesn't change much if this feature is included, and so we omit it for simplicity.

Second, there are the contractor's private costs, which are multidimensional (since they are defined over time and various materials), and potentially uncertain at the time of bidding. This latter feature is especially important, since it means that the incentive structure must be flexible enough to accommodate contractors who sustain positive or negative cost shocks, and still attain an ex-post efficient outcome.

On the other hand, we omit a few features that play an important role in real-world contracting. We abstract away from the measurement and monitoring of the number of days taken by the contractor,  $d^a$ . In practice, the project engineer determines  $d^a$  from the number of work days charged, a measure that takes into account reasonable delays due to unforeseen weather circumstances, necessary work stoppages, and change orders. From talking to project engineers, we know that  $d^a$  is to some extent the outcome of negotiation between the contractor and the project engineer as to what is fair and reasonable.

Another simplification is that we assume the true social costs of time delay are fully captured in the constant flow cost  $c_T$ . In reality, traffic flows vary by day-of-week, and so even if we believe that the right measure of social costs is just the number of motorists delayed by the construction, the flow cost should vary across days. Extending the analysis to allow for this, or more complicated welfare functions is certainly possible and would lead to similar results, but with more messy analysis.

Finally, we take the incentive structure as given exogenously here, and analyze which incentive structures maximize social welfare. An alternative is to endow the government with a payoff function, and let them choose a mechanism to maximize their own payoff. As has been observed in other papers - most recently Asker and Cantillon (2008a) - this may lead to distortions away from efficiency as the government attempts to capture some of the contractors surplus. This line of analysis would take us away from our main goal, which is to understand the efficiency properties of these A + B auctions.

## 2.2 Analysis

Our analysis proceeds by backwards induction. First, we consider the optimal ex-post behavior of the winning contractor after he wins the contract and his true cost parameter  $\theta$  is realized. Second, we analyze how a given type with cost signals  $x$

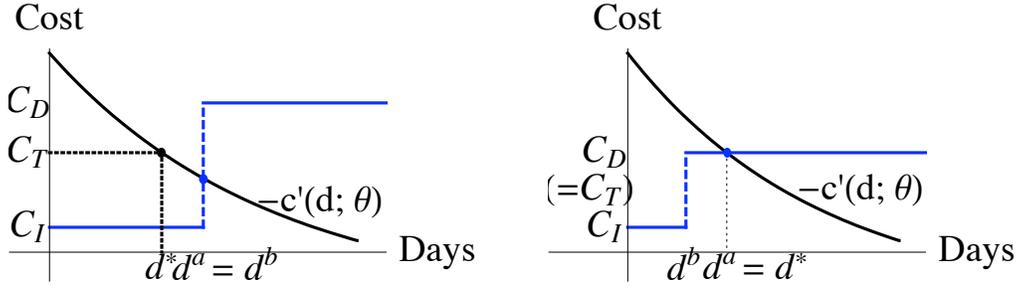


Figure 1: **Ex-Post Behavior and Incentives.** The figure depicts the marginal cost curve of the contractor and the incentives  $c_I < c_D$ , as well as the true social cost of delay  $c_T$ . In the left panel, the contractor has bid  $d^b$  and completes the contract in  $d^a = d^b$  since finishing early and finishing late are both unprofitable ( $c_I < c'(d^a; \theta) < c_D$ ). The ex-post efficient completion time is  $d^* < d^a$ . In the right panel, the contractor bids  $d^b$  but chooses to complete late at the ex-post efficient time  $d^a = d^*$ . He does so since his marginal gains to delay exceed the penalties ( $-c'(d^a; \theta) > c_D$ ).

should optimally structure his bid if he wants to bid a fixed score  $s$ . Finally, we look at the choice of this score  $s$ , and in so doing deduce equilibrium behavior in the auction.

In Figure 1 we depict the “moving parts” of the problem faced by the winning contractor once he has learned his true costs  $\theta$ . Since his costs are concave and decreasing in the number of days taken on the job, he faces a declining marginal benefit of delay<sup>1</sup>. This is depicted as the curve  $-c'(d^a; \theta)$  in the figure. He also faces a set of incentives, determined by his ex-ante commitment to complete in his days bid  $d^b$ , and the incentive rate  $c_I$  and disincentive rate  $c_D$ . At  $d^b$  the cost of delay discontinuously shifts from losing incentive payments at rate  $c_I$  to paying disincentives at rate  $c_D$ , as shown in the figure. Graphically, his optimal choice of completion time  $d^a$  is the point at which  $c'(d^a; \theta)$  cuts the step-function depicting his incentives.

Now consider what is required for ex-post efficiency. Taking a first order condition in  $d^a$  in the social welfare function  $W$  yields the efficient number of days  $d^*$  implicitly as the unique solution to  $c_T = -c'(d^*; \theta)$ . That is, the private benefit of delay must be equal to the external social costs imposed on motorists. A number of conditions may suffice to deliver ex-post efficient outcomes. One is that  $c_I = c_D = c_T$ , so that the private and social incentives exactly coincide. But weaker conditions may do as

<sup>1</sup>This is exactly analogous to the marginal cost of abatement curve you find in environmental economics

well. If  $d^b < d^*$ , then  $c_D = c_T$  will be enough, as is shown in the right panel of the figure. Analogously, if  $d^b > d^*$ , it will be enough to have  $c_I = c_T$ . But notice that the days bid  $d^b$  is endogenous, and so if one is to achieve ex-post efficiency under these weaker conditions, it is important to choose the user cost  $c_U$  carefully to promote the “right” bidding by contractors.

Notice that the equilibrium completion time strategies of all bidders must be symmetric with  $d_i^a(\theta; b_i, d^b) = d^a(\theta; d^b)$ . This is because after the auction is concluded this becomes a single-agent problem, and rationality requires that agents with the same payoffs function and state variables take the same actions. With that in mind, we turn now to examining how contractors choose how many days to bid, and how it may affect the eventual completion time. Define the optimal set of completion times:

$$d^a(\theta, d^b) = \{d^a : d^a \in \operatorname{argmin} c(d^a; \theta) + 1(d^a - d^b)(d^a - d^b)c_D - 1(d^b > d^a)(d^b - d^a)c_I\}$$

Then we have the following result:

**Lemma 1 (Bidding and Completion Time)** *If  $c_I \leq c_D$  then  $d^a(\theta, d^b)$  is a function, increasing in  $d^b$  if  $c_I < c_D$  and constant in  $d^b$  if  $c_I = c_D$ . If  $c_I > c_D$ , then  $d^a(\theta, d^b)$  is potentially multi-valued, decreasing in the strong set order in  $d^b$  and  $d^b \notin d^a(\theta, d^b)$ ,*

The intuition for this result is that when  $c_I < c_D$ , completion times  $d^a$  should be “sticky” around  $d^b$ , in the sense that for a range of realizations of  $\theta$  the contractor will complete the contract in the days bid. This is because the per-day benefits to finishing early,  $c_I$  are relatively smaller than the per-day costs of finishing late  $c_D$ , and so ex-post adjustments in either direction will be unprofitable. It follows that  $d^a$  is increasing  $d^b$ . But in the case where  $c_I = c_D$  incentives to adjust in either direction are equal, and so the number of days bid has no effect on ex-post incentives.

The opposite case where  $c_I > c_D$  creates perverse ex-post incentives. For any  $d^b$ , the contractor will either want to complete late (because the penalties are low), or early (because the rewards are high), but never on time. In fact, holding  $\theta$  fixed, as  $d^b$  increases it becomes relatively more attractive to complete early because the incentive payments will be higher (the contractor comes in more days early); hence the result.

For the rest of the analysis, we will maintain the assumption that  $c_I \leq c_D$ , which

holds in all of the data we examine. We will need one further assumption to simplify the analysis:

**Assumption 1 (Full Marginal Cost Support)** *For every  $d^a \in (0, \bar{d})$ ,  $\exists (\theta^L(d^a), \theta^H(d^a)) \in \Theta$  such that  $-c'(d^a; \theta^L(d^a)) < c_D$  and  $-c'(d^a; \theta^H(d^a)) > c_I$ .*

This ensures that there are always some realizations of  $\theta$  yielding marginal costs strictly below the disincentives (so it may be worth completing early), and others yielding marginal costs above the incentives (so it may be worth completing late). One way to think of the role of this assumption is as guaranteeing that there is genuine ex-ante uncertainty: contractors may need to make different decisions ex-post than they had anticipated at the time of bidding.<sup>2</sup>

Our next result examines concerns the choice of days bid:

**Lemma 2 (Incentives and Bidding)** *In equilibrium, for any type  $x$ :*

- (a) *If  $c_U \geq c_D > c_I$ ,  $d_i^b(x) = 0$  and  $d^a(\theta; d^b) = d^a(\theta)$  solves  $-c'(d^a(\theta); \theta) = c_D \forall \theta$*
- (b) *If  $c_U \leq c_I < c_D$ ,  $d_i^b(x) = \bar{d}$  and  $d^a(\theta; d^b) = d^a(\theta)$  solves  $-c'(d^a(\theta); \theta) = c_I \forall \theta$*
- (c) *If  $c_I < c_U < c_D$ , then  $d_i^b(x)$  is decreasing in  $c_U$ , and increasing in  $c_I$  and  $c_D$  for all  $x$ .*

To gain some intuition for the result, consider a bidder who is going to submit a total score  $s$ , and must decide how to weight the money (A-part) against the days bid (B-part). If the user cost is higher than the disincentives (case (a)), then it is optimal to bid  $b = s$  and  $d^b = 0$ , maximizing the money earned upfront, and then paying out penalties upon late completion at rate  $c_D$ . This is because bidding one additional day  $d^b$  means that you must reduce the bid to  $b = s - c_U$ , losing  $c_U$  - for which you gain only  $c_D$  in reduced penalties. The intuition for case (b) is similar. In the final case, it is immediately sub-optimal to be at a corner by the same marginal arguments, since now  $c_I < c_U < c_D$ . For the comparative statics, increasing  $c_U$  increases the

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<sup>2</sup>There are some incentive structures for which this assumption necessarily cannot hold (e.g. where  $c_D = 0$  so there are no penalties for being late), but these are generally uninteresting and never-used in practice.

“marginal cost” of days bid under the scoring rule, and so it is optimal to bid fewer days. But if either the positive or negative incentives increase, it is optimal to bid higher days so as to decrease expected penalties (if  $c_D$  increases) or further increase expected incentive payments (if  $c_I$  increases).

With this lemma in hand, we may get at the central topic of ex-post efficiency:

**Proposition 1 (Ex-Post Efficiency)** *In equilibrium:*

- (a) *If  $c_I \leq c_U = c_D$ , then the contract is ex-post efficient iff  $c_D = c_T$*
- (b) *If  $c_I = c_U \leq c_D$ , then the contract is ex-post efficient iff  $c_I = c_T$*
- (c) *If  $c_I < c_U < c_D$ , then the contract is ex-post inefficient.*

Cases (a) and (b) basically follow immediately from the lemma. For example, in case (a), the lemma tells us that contractors should optimally always bid 0 days; and so provided that the penalty  $c_D$  is set equal to the true social flow cost  $c_T$ , we will always get an ex-post efficient outcome. The inefficiency highlighted in case (c) is a consequence of the ex-ante uncertainty. Since bidders can't perfectly anticipate their ex-post costs, the days they bid  $d^b$  may not be equal to the socially efficient ex-post completion time  $d^*$ . But if  $c_I < c_U < c_D$ , there will be realizations of  $\theta$  for which the gap between  $c_I < c_D$  induces the winning contractor to “stick” to his original inefficient choice  $d^b$ , rather than adjusting.

We next consider the issue of ex-ante efficiency. Define the ex-post cost function

$$C(d^b; \theta) = c(d^a(\theta; d^b); \theta) + 1(d^a(\theta; d^b) - d^b)(d^a(\theta; d^b) - d^b)c_D - 1(d^b > d^a(\theta; d^b))(d^b - d^a(\theta; d^b))c_I$$

which gives the sum of costs and incentive payments for any days bid  $d^b$  and cost realization  $\theta$ , under the optimal completion time strategy. Using this, in turn define the bidder-specific ex-ante cost function  $C_i(d^b; x) = E_i[c_U d^b + C(d^b; \theta) | x]$ . Since the ex-post cost functions are globally concave in  $d^b$  for any  $\theta$ , the ex-ante function inherits this property and has a unique minimizer in  $d^b$ ,  $d_i^b(x)$ ; and a unique minimum  $P_i(x)$ . We will call the value  $P_i(x)$  the *pseudo-cost* of the contractor. In the following proposition, we use this to characterize equilibrium:

**Proposition 2 (Bidding and Ex-Ante Efficiency)** *There is a unique equilibrium, in which:*

- (a) *The strategies take the form  $\beta_i(x) = (s_i(x) - c_U d^b(x), d_i^b(x))$ , where  $d_i^b(x)$  is the ex-ante cost minimizer defined above and  $s_i(x)$  satisfies the first-order condition*

$$s_i(x) = P_i(x) + \frac{1}{\sum_{j \neq i} h_j(s)}$$

*where  $h_j(s)$  is the hazard function of the distribution of scores submitted by bidder  $j$ , and  $P_i(x)$  is bidder  $i$ 's pseudo-cost.*

- (b) *If the contract is ex-post efficient, it is also ex-ante efficient.*

The logic of the proof is as follows. Consider a bidder  $i$  with a signal  $x$ , who wants to submit a bid of  $s$ . From our earlier analysis, there is an optimal number of days to bid  $b_i(x)$ , regardless of what  $s$  is. Put another way, no matter how much the contractor's overall score is going to be, given his signal about his time costs, there's some number of days that balances the competing incentives to bid days down (given by  $c_U$ ) against the incentive to minimize ex-post costs (given by  $c_D$  and  $c_I$ ). But then we can compute the total expected costs of a contractor, including the implicit cost of days bid on his final score, regardless of the value of  $s$ . This is what  $P_i(x)$ , the pseudo-cost, is. After that, we are in the familiar case where contractors have a single-dimensional type, their pseudo-cost, and standard results apply. The first part of the proof is just iterative application of results in Asker and Cantillon (2008b) and Maskin and Riley (2003).

The second part is novel. From proposition 6, if the contract is ex-post efficient, then the incentive structure induces  $d_i^b(x) = 0$  or  $d_i^b(x) = \bar{d}$ . Simplifying the formula of the pseudo-cost in this special case, we find that social welfare is a decreasing affine function of pseudo-cost. Consequently, awarding the contract to the bidder with the lowest pseudo-cost achieves ex-ante efficiency. Since the equilibrium scores are monotone functions of the pseudo-costs, this will be achieved.

In practice, it may not be possible to achieve this first-best outcome. Budgetary constraints often prevent MNDOT and other agencies from setting the user cost equal to (their estimate of) the true daily cost. The following result characterizes

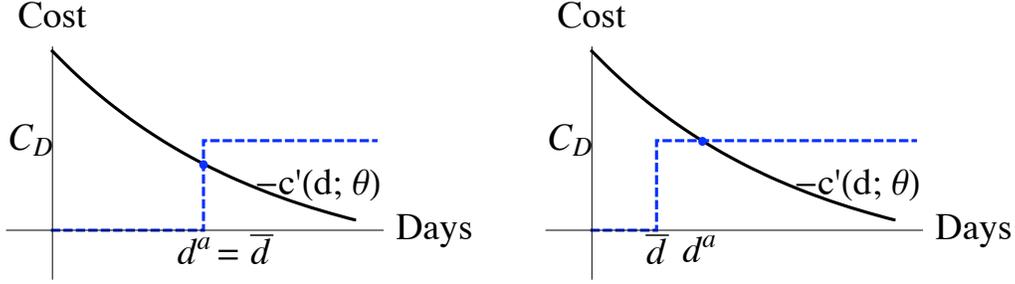


Figure 2: **A-only Contracts** The figure depicts the marginal cost curve of the contractor and the damages  $c_D$ . In the left panel, the contractor completes the contract on-time at the maximum days  $\bar{d}$  since his benefits of delay are less than the damages. In the right panel, the contractor completes the contract late since his benefits to delay exceed the costs.

how changes in the user cost can in general be used to manipulate outcomes, even when not using the efficient incentive structure.

**Proposition 3 (Completion Times and Contract Cost)** *If  $c_I \leq c_U \leq c_D$ , an increase in  $c_U$  causes the equilibrium distribution of  $d^a(\theta; d^b)$  to fall and the winning bid to increase (in both cases, in the sense of first order stochastic dominance). If  $c_U < c_I$  or  $c_U > c_D$  it has no effect on either.*

This documents the trade-off faced by the procurer. Raising the user cost results in the contract being completed more quickly in expectation, but at the cost of increasing the winning bid.

### 2.3 A-only Auctions

Most highway procurement auctions are A-only in that one submits only a bid on the materials cost, and not the time cost. On the other hand, damages are assessed for contracts that completed later than the maximum time  $\bar{d}$ . Thus one can view these auctions as a constrained case of the model above, where firms must choose to bid  $d = \bar{d}$ ,  $c_U$  is undefined (since all firms bid  $\bar{d}$  it is irrelevant),  $c_I = 0$  and  $c_D > 0$  is equal to the daily damages assessed.

Putting the model into this framework allows us to conclude a number of things, based on our earlier analysis. First, since there are no positive incentives and delay

has positive benefit, contractors will never complete contracts early. In fact, as can easily be seen from Figure 2 their completion time  $d^a(\theta)$  is equal to  $\bar{d}$  if  $-c'(\bar{d}; \theta) \leq c_D$  (left panel), and otherwise solves  $-c'(d^a; \theta) = c_D$  (right panel).

Second, these contracts will generally be ex-post inefficient. If there are *any* realizations of  $\theta$  under which the socially optimal solution requires that the contract be completed in  $d^* < \bar{d}$ , this will not occur since the contractor has no incentive to complete early. Moreover, even if the contract should always be completed late, to get the socially optimal completion time, we need  $c_T = c_D$ . But damage schedules are set state-wide across all contracts, and given that the social costs of delay vary with the population of the surrounding area and the disruption caused by the particular contract, it is almost certain that the damage schedule will not be equal to the true social costs for all contracts.

Third, the incentive structure here also generates a notion of “pseudo-cost” for the contractors. It is given by  $\tilde{P}_i(x) = E_i [c(d^a; \theta) + 1(d^a(\theta) > \bar{d})(d^a(\theta) - \bar{d})c_D|x]$ , which is just the sum of their expectations of private costs and damages. To obtain ex-ante efficiency we must have  $c_D = c_T$ , so that the pseudo-costs reflect the true social costs of delay. In this case the ex-ante efficient bidder will also have the lowest pseudo-cost, and will thus win the auction. Notice that even if  $c_D = c_T$ , outcomes will in general not be ex-post efficient (since no contracts are completed only). So this is a “second-best” result: within the constraints imposed by the “A-only” framework, the best option is to pick realistic damages so that the best bidder wins the contract, even if his ex-post behavior will sometimes not be socially optimal.

Finally, we note that  $c_D$  has a similar role to the user cost in  $A + B$  contracts, in that it determines how important punctual completion is. As  $c_D$  rises, the probability that a contract is completed late falls, while the bids increase in the sense of first order stochastic dominance. Summarizing:

**Proposition 4 (A-only auctions)** *There is a unique equilibrium of the A – only auction in which:*

1. *Bids obey the first order condition*

$$b_i(x) = \tilde{P}_i(x) + \frac{1}{\sum_{j \neq i} h_j(b)}$$

where  $h_j(b)$  is the hazard function of the distribution of bids submitted by bidder  $j$ , and  $\tilde{P}_i(x)$  is bidder  $i$ 's pseudo-cost.

2. The contract will be ex-post efficient iff  $c_D = c_T$  and  $-c'(\bar{d}; \theta) \geq c_T$  for all  $\theta \in \Theta$ ; and the contract will be ex-ante efficient if  $c_D = c_T$ .
3. The distribution of contract completion times is falling in  $c_D$  (in the sense of FOSD), while the distribution of winning bids is rising.

In the following sections we take these propositions to the data.

### 3 A+B Auctions

#### 3.1 Data

We have data from 29 A+B contracts awarded in the state of Minnesota between 2000 and 2007; and the 123 bids associated with these contracts. For each contract we observe all the bids, a number of contract characteristics including the engineer's cost estimate, and the incentives offered in the contract (the user cost  $c_U$ , disincentives  $c_D$  and incentives  $c_I$ ). In addition there are some other features not captured in the formal model, such as a minimum number of days to bid in some contracts; and capped positive incentives in others. For 23 of the A+B contracts we also observe the final completion time (the remaining contracts are still outstanding).

#### 3.2 Descriptive Evidence

Our approach in this section will be to see how various outcomes vary with changes in the incentive structure. There are four basic kinds of incentive structure that are observed in the data: (a) those with no positive incentives, and equal user cost and disincentive ( $0 = c_I < c_U = c_D$ ); (b) those with small positive incentives ( $0 < c_I < c_U = c_D$ ); (c) those with entirely equal incentives and disincentives ( $0 < c_I < c_U = c_D$ ); and (d) those with user cost higher than the other incentives ( $c_U > c_I = c_D > 0$ ). The breakdown of each type is shown in Table 1.

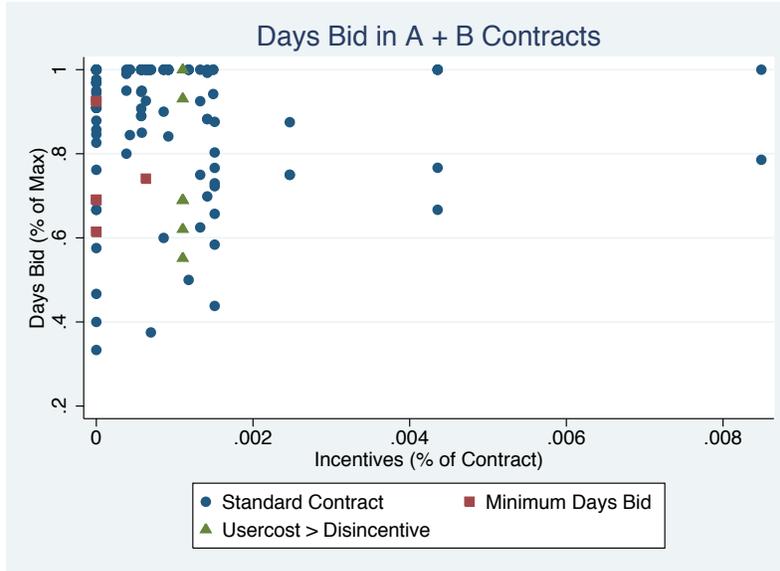


Figure 3: **Days Bid in A+B Contracts.** This figure shows how the number of days bid varies with the incentive structure.

Our analysis will be primarily descriptive, because we lack enough data to do too much more than this. We will treat the incentive structure as exogenous throughout the analysis.<sup>3</sup>

The first set of tests we run is on the bidding behavior. The theory predicts that bidders should bid fewer days when positive incentives are lower than disincentives, so as to “lock-in” the gains to finishing early; in fact it makes the strong prediction that with  $c_U = c_D$ , they should bid zero days (see Lemma 2). Likewise, when with high user cost  $c_U < c_D$ , the optimal bid is zero days. By contrast, in the case of equal incentives ( $c_U = c_I = c_D$ ) the theory makes no predictions on the optimal number of days to bid, as it has no equilibrium impact on payoffs. The observed days bid are shown in Figure 4.

Notice that there appears to be a pattern: the lower the incentives, the lower the days bid. In addition, there are four cases in which the minimum number of days was bid, and of those, 3 occur with no positive incentives. Lastly, in the one contract where the usercost was set above the disincentive, most of the days bids were well below the maximum bids. All of this goes towards supporting the theory. Yet the theory predicts something even stronger: bids of zero days. In practice, contractors

<sup>3</sup>We will control for some observable contract characteristics, so in practice we require only the weaker assumption of exogenous contract assignment conditional on those observables.

don't bid zero days though for fear that MNDOT will classify their bid as "irregular" and disqualify it from the auction. This explains why we don't see this theoretical prediction borne out in the data.

To confirm that there is a statistically significant difference in bidding behavior as the incentive structure changes, we run a tobit regression of days bid (as a percentage of the maximum allowed) on dummies for the different incentive structures. A tobit is used to account for the censoring of days bid both below (at the minimum days) and above (at the maximum days). The outcome is shown in the first column of Table 2. We find that relative to the case with equal incentives (the omitted group), the days bid is significantly and substantially lower with zero incentives or a higher usercost - the point estimates are at least 15% lower. The dummy for small positive incentives has no statistically significant effect.

We turn next to the actual dollar bid entered in these contracts. The theory predicts here that the dollar bid should be relatively higher in contracts with zero or small positive incentives, since the winning contractor will receive no bonus payment for finishing early. In fact, they should optimally bid zero days and build in all of their anticipated damages into their bid. The same is true for contracts with usercost exceeding disincentives.

In column 2 of Table 2 we show the outcome of an OLS regression of their dollar bid (as a percentage of the engineer's estimate) on the incentive structure. We add a dummy for whether there are more than two bidders as a control for the competitiveness of the auction; its coefficient is negative and significant, as one would expect. We find also that the bids are higher in contracts with no positive incentives and higher usercost, though only significantly so in the latter case. This is in line with the theory, although it is perhaps surprising that the case with no positive incentives is not generating much higher bids - more on this later.

Finally, we consider how the incentive structure impacts the final completion time. Figures 4 and 5 show what we observe in the data. Figure 4 plots the final completion time against the number of days bid. The theory predicts that when  $c_I < c_D$  the actual completion time should be "sticky" around  $d^b$ , since there is a wedge between the benefits to finishing earlier (which are low) and the costs of finishing late (which are high). By contract, the theory has nothing to say in the case with equal incentives;

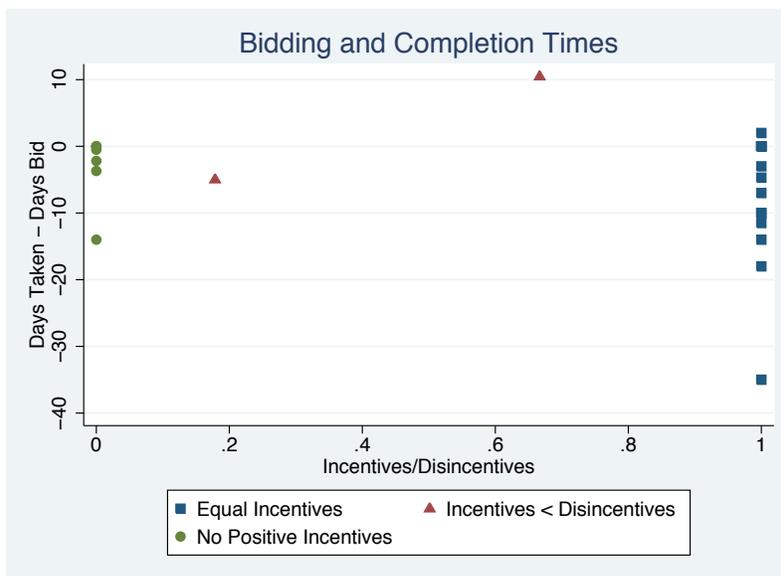


Figure 4: **Bidding and Completion Times.** This shows how the final days taken relates to the number of days bids, for varying incentive structures.

since the original number of days bid should have no impact on final completion time except insofar as there is some idiosyncratic correlation between how contractors pick the number of days they bid and when they finish their contracts (e.g. if they always bid the number of days they expect to take).

In Figure 4 we do observe the "stickiness" predicted by the theory. With one exception, whenever there are zero positive incentives, the contracts are completed either slightly early or just in time - in either case, very close to the days bid. There is a puzzle here though: the theory predicts that under zero positive incentives, contractors should systematically underbid the number of days they intend to take (to look in benefits upfront), and then finish late. We don't observe them finishing late in the data. This is worrying, since if they never finish late it suggests that outcomes may not be ex-post efficient, in the sense that they're committing to very achievable times and then have no incentive to adjust if handed a positive cost shock.

With equal incentives, they tend to finish the job well ahead of schedule, cashing in on the incentive payments. The theory makes no prediction here, so it is interesting to see that the preference of most contractors is to bid high days and then finish early, earning bonuses; versus the fully equivalent strategy of bidding low days, pushing up the materials bid accordingly, and then paying out damages. Look now at Figure 5,

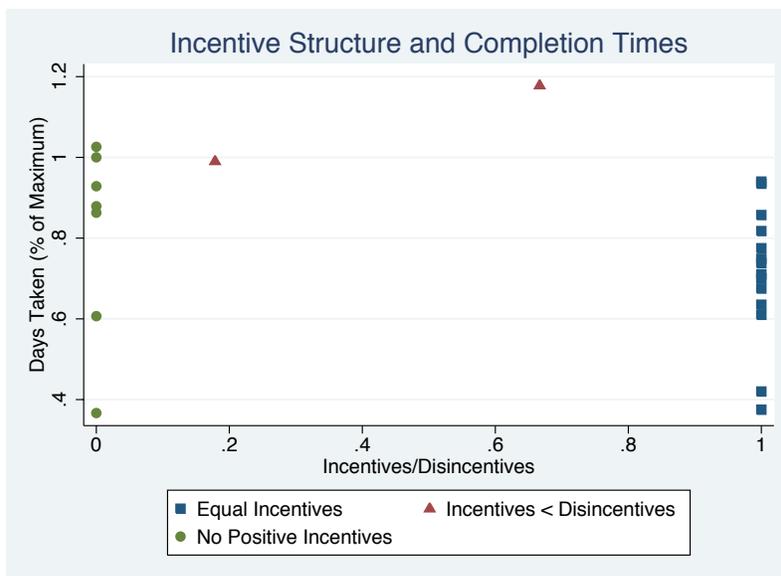


Figure 5: **Incentive Structure and Completion Times.** This shows how the completion time (as a percentage of the maximum time allowed) varies with the incentive structure.

which shows the relationship between incentive structure and final completion time (as a percentage of the maximum days). The theory predicts that all observed incentive structures should yield the same final completion time; none should systematically do better than the others (although obviously the magnitudes of the user cost etc may matter). In the figure, we see that in almost all contracts, the A+B bidding leads to an earlier completion time than the maximum allowed (and therefore earlier than the standard A-only contracts). It does seem that the contracts with positive incentives are more likely to lead to early completion times. But an OLS regression in the third column of Table 2 shows that the difference in days taken between the group with no positive incentives and that with positive incentives is not statistically significant, in line with the theory.

Overall, the predictions of the theory stand up well, at least in this primarily descriptive analysis. An important deviation from the theory is that bidders do not bid zero or even an unreasonably low number of days, when there are no positive incentives. This is possibly because they do not understand the game, or more likely because they are concerned that their bid will be rejected as irregular. This implies that the incentive structure with zero positive incentives will generally not deliver ex-post efficiency, even if the user cost is set equal to true social costs, because contractors

will not respond to positive cost shocks by completing early. On the other hand, with equal incentives, the mechanism is “strategy-proof” in an intuitive way: the number of days bid has no important implications for ex-post behavior. This suggests that choosing equal incentives in A+B contracts may be desirable from a social efficiency point of view in practice.

## 4 Standard Procurement Auctions

### 4.1 Data

Our data consists of a sample of 2311 contracts let by MNDOT during the period 1997-2007, and completed by the current date. From this data we construct two datasets. The first, used in both the bidding and ex-post analysis, consists of 3675 bids in the 940 contracts for which we have complete data, including all bids and bidder identities, as well as the engineer’s cost estimate, ex-post data on adjustments, extras and damages. For these contracts we also construct measures of competition, such as the capacity utilization of the contracts and the distances of the bidders to the contract site. These are constructed as follows....

[Discussion of variables used in bidding analysis to follow]

The second dataset is used to augment the first in the ex-post analysis. This dataset has data from an additional 1221 contracts where we are missing bidding data, but nonetheless observe the final contract amount and measures of the ex-post behavior of the winning contractor. To identify which contracts ran late, we have data on the liquidated damages that were assessed as a result of late completion. These damages are assessed according to a standard specification of damages that depends on the year in which the contract was let (see Table 3).

In reality, liquidated damages are not always assessed on late contracts. In one kind of contract, called a “calendar day contract” , in which a starting and ending date is specified explicitly in the contract, contractors are always penalized for late completion. But in “working day” contracts, it is the responsibility of the project engineer to count the number of “working days” used by the contractor, to assess whether he is late, and to decide whether or not to assess damages. So the project engineer

has tremendous discretion as to whether damages are charged, and the number of contracts in which damages are charged is an underestimate of the true number of late contracts. Fortunately, for a subsample of these working day contracts we have acquired the diary records of the recording engineer, which indicate both the working days allowed for the contract and the number of days charged by the engineer - which are days in which the contractor was, or should have been, at work. There are 236 of these contracts. For each of the ranges of contract size specified above in Table 3, we summarize the variables of interest in this subsample in Table 4. We compare this with the full sample in Table 5, breaking the full sample up into working day and calendar day contracts.

There are a number of features of interest. First, smaller contracts are more likely to finish early or on time than larger contracts. In fact, none of the contracts of size less than \$50 000 finish late in the diary subsample, and few are charged in the main sample. We will thus find it difficult to get at the effect of damages in small contracts, since there is too little variation in outcomes for identifying the effects of damages. Second, very few late working day contracts are actually assessed damages. Looking at the % late and % charged columns, we see that it is about 1/3rd of the late contracts (the exact percentage in the data is 24.3%). On smaller contracts though, it is generally more unusual to assess damages. Conditional on damages being assessed, they are a reasonable fraction of the contract value, ranging from around 2% in the smallest contracts, down to about 0.6% in the biggest. Finally, comparing working day and calendar day contracts, we see that calendar day contracts incur damages more often than working day contracts when the contract is small, but less often when large.

Also, the diary subsample looks somewhat different to the full sample. Contracts are charged more frequently in the diary subsample, suggesting that the engineer recorded the diary data with higher probability when he expected the contract to run late and he needed to document the contract for charging. This will not be a problem in the subsequent analysis, as all we require from this data is an estimate of the probability that the contract is charged conditional on being late, so the selection on the probability of being late will not matter. Importantly, the subsample is similar in other respects, conforming to the broad distribution of contract sizes we see for the full dataset.

## 4.2 Econometric Approach

### 4.2.1 Bidding

[Pat to insert econometrics here]

### 4.2.2 Ex-Post Behavior

In this section, we examine how the damages clauses of standard highway procurement contracts affect the completion time of these contracts. From the theory, we expect that higher penalties lead to more on-time completions. In principle, we can test this by looking at how the probability of a late completion co-varies with contract damages, controlling for all other observable contract characteristics. We face one major practical difficulty here: for most contracts (90% of the sample) we do not observe directly whether or not a contract was completed on time. Instead we observe whether MNDOT charged the contractor liquidated damages for being late. If MNDOT always charged damages on late contracts, there would be no problem, as we could identify late completions from the damage variable. But in fact we know that whether they do so depends on the type of contract, working day or calendar day.

To formalize these ideas and obtain an estimating equation, let  $y^*$  be a binary variable equal to 1 if the contract is completed late; let  $y$  be a binary variable equal to 1 if the contractor is charged liquidated damages; and let  $T$  be a binary variable taking on the value 1 if the contract is a working day contract. Now, the probability that the contract is late is given by:

$$\Pr(\text{Late}) = \Pr(y^* = 1) = \Pr(-c'(\bar{d}; \theta) > (1 - \alpha T)c_D) \quad (2)$$

This follows since a contract will be completed late whenever the marginal benefits to delay,  $-c'(\bar{d}; \theta)$  exceed the marginal costs. When  $T = 0$  as in a calendar day contract, the marginal costs are  $(1 - \alpha T)c_D = c_D$  since the damages will be assessed with certainty. But when  $T = 1$  as in a working day contract, we let marginal costs be  $(1 - \alpha T)c_D = (1 - \alpha)c_D < c_D$ , where  $1 - \alpha$  is the probability that damages will be assessed.

To obtain an estimating equation, let us log-linearly parameterize the marginal cost function:

$$-c'(\bar{d}; \theta) = \exp\{X\beta + \varepsilon\}$$

where  $X$  are contract-specific characteristics,  $\varepsilon$  is a mean-zero and normally distributed cost shock observed by the agent but not the econometrician, and  $\beta$  is a parameter vector to be estimated. Substituting this in, we obtain:

$$\Pr(y^* = 1) = \Pr(X\beta - \log c_D - \log(1 - \alpha T) + \varepsilon > 0) \quad (3)$$

If we observed  $y^*$ , given this specification we could simply run a probit to estimate the coefficients (up to scale). But since we don't observe late completions  $y^*$ , we translate this into a likelihood for being charged,  $y$ . Let  $\eta(X, T) = \Pr(y = 1 | y^* = 1, X, T)$ . We know that  $\Pr(y = 1 | y^* = 1, X, T = 0) = 1$ , because late calendar day contracts are always charged. We can estimate  $\eta(X, T)$  from the sub-sample of our data for which we observe both completion damages and liquidated damages.<sup>4</sup> Now,

$$\Pr(y = 1) = \eta(X, T) \Pr(X\beta - \log c_D - \log(1 - \alpha T) + \varepsilon > 0) \quad (4)$$

We estimate the model in two steps. First we estimate  $\eta$  conditional on contract characteristics and type, and then using this estimate we obtain the parameters of interest,  $(\alpha, \beta)$  by maximum likelihood. This second step is just a probit that is modified to take account of the fact that some working day contracts are delayed but not charged. Standard errors are obtained by bootstrapping.

There are two sources of identification of the model. On the one hand, the contract damage schedule given in Table 3 changes in 2005, giving us a plausibly exogenous source of variation. Since there is only one change in this schedule, though, this is fairly weak. The other source of identification is the assignment of contracts by MNDOT to either calendar day or working day status. Calendar day contracts have stronger incentives for completion, since the project engineer has less discretion in whether or not to assess damages. From the perspective of risk neutral contractors, this amounts to higher expected costs to delay, and so we should expect to see that

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<sup>4</sup>One might be concerned that this raises a selection problem, in that this sub-sample is not a random selection of our data. We do not believe this is important, as the two samples are similar in their observable characteristics  $X$ .

calendar day contracts finish late less often than working day contracts, and thus are charged less often.

## 4.3 Results

### 4.3.1 Bidding

[Pat to insert results here]

### 4.3.2 Completion Times

We present the results on completion time in Table 6. The results show that bigger contracts have a significantly higher absolute probability of delay, about 6% more for each doubling of contract size. Since the average probability of delay in the data is estimated to be 20%, expressed as a relative change this is very large, about 30% higher. On the other hand, while increased damages per day are associated with lower probabilities of delay, about 1% in absolute terms or 5% in relative terms for a doubling of damages, this effect is not significant. This appears due to the limited variation in damages per day in the data; it is very hard to separate the effect of contract size on completion time from the effect of damages, since these are linked together by the standard schedule of damages.

We do find a big effect of changing incentives through the choice of contract type, however. Using a working day contract instead of a calendar day contract increases the probability of delay by 20% in absolute terms, doubling that probability in relative terms. This may seem like an unreasonably large effect, except that the data tells us that on average late working day contracts are charged only around 20% of the time, so the damages contractors expect to face in working day contracts are around 5 times lower on average. Given this, the large effect of the switch is incentive structure is not all that surprising. Adding additional controls, as in columns (2) and (3) does not significantly change the results.

## 5 Conclusion

Building time incentives into highway procurement contracts is important from the perspective of social welfare. In this paper, we have shown that the A + B scoring auctions used by MNDOT and other agencies can be used to achieve efficient ex-ante allocations and ex-post outcomes, provided the incentives are correctly set. We have provided a characterization of the optimal incentive structure. Taking this theory to a small dataset, we have found evidence that suggests that setting equal incentive and disincentive payments ex-post may achieve the best outcomes in practice.

We have also shown that the standard highway procurement contracts are generally ex-post inefficient, but that a second-best ex-ante efficiency result can be achieved by setting damages equal to the social costs of time delay. We have investigated this proposal empirically, finding contractors build their expected damage payments into their bids at a high rate. We also find that a far more important determinant of the probability of delay is the assignment of these contracts to either working day or contract day status. The results suggests that by assigning the more time sensitive projects to be calendar day contracts may improve social welfare.

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## 6 Appendix

### 6.1 Proof of Lemma 1

$d^a(\theta; d^b)$  is the set of minimizers of  $\phi(d^a) = c(d^a; \theta) + 1(d^b > d^a)(d^b - d)c_I - 1(d^a - d^b)(d^a - d^b)c_D$ . For  $c_I \leq c_D$ ,  $\phi(d^a)$  is globally concave and hence has unique solution, so  $d^a(\theta; d^b)$  is a function. Moreover, if  $c_I = c_D$  then  $d^a(\theta, d^b)$  is defined implicitly by  $-c'(d^a; \theta) = c_D$ , independent of  $d^b$ . If  $c_I < c_D$ , then for any  $\theta$ ,  $d^a(\theta; d^b) = d^b$  if  $c_I < -c'(d^b; \theta) < c_D$ , and is constant in  $d^b$  otherwise. Thus  $d^a(\theta; d^b)$  is increasing in  $d^b$  for any  $\theta$ . Finally, if  $c_I > c_D$ ,  $\phi(d^a)$  is not globally concave, and in fact always attains maximal value at one (or both) of the points  $d^1 < d^2$  where  $-c'(d^1; \theta) = c_I$  or  $-c'(d^2; \theta) = c_D$ . As  $d^b$  increases, the solution proceeds through the sets  $\{d^2\}, \{d^1, d^2\}, \{d^1\}$ , so  $d^a(\theta; d^b)$  is decreasing in the SSO in  $d^b$ .

### 6.2 Proof of Lemma 2

For part (a), it will be sufficient to show that  $d_i^b(x) = 0$  is (weakly) optimal for all realizations of  $\theta$ , and strongly optimal on a set of positive measure. So fix  $\theta$  and compare the payoffs to  $d_i^b(x) = 0$  versus any deviation  $d'$ . There are two possibilities. Either  $d' \leq d^a(d'; \theta)$  in which case the contractor will not complete early both on-path and in the deviation, with payoff difference  $(c_D - c_U)d' \leq 0$ ; so bidding 0 days is weakly optimal. Or  $d' > d^a(d'; \theta)$  in which case the contractor completes early in the deviation and late on-path, with payoff difference  $c_I(d' - d^a(d'; \theta)) + c_D d^a(0; \theta) - c_U d' < 0$ ; so bidding 0 days is strongly optimal. Moreover, assumption 1 guarantees that there exists some  $\theta^L$  with positive density with  $-c'(d'; \theta^L) < c_D$ , so that bidding and completing at  $d'' < d'$  where  $-c'(d'; \theta^L) = c_D$  would yield strictly higher payoff; and this payoff is by the above logic  $\leq$  the payoff to bidding  $d^b = 0$  and completing at  $d''$ . This gives the result. Part (b) is similar.

For part (c), note that  $d^b(x)$  is implicitly defined as the solution to the first order condition given by:

$$-E \left[ c'(d^a; \theta) \frac{\partial d^a}{\partial d^b} | x \right] - \frac{\partial}{\partial d^b} P(d^a(d^b; \theta) > d^b | x) c_D + \frac{\partial}{\partial d^b} P(d^a(d^b; \theta) > d^b | x) c_I = c_U$$

The LHS is increasing in  $d^b$ , and the *RHS* is increasing in  $c_U$  so by the IFT, we have  $d^b(x)$  increasing in  $c_U$ . Similarly since the LHS is increasing in  $c_D$  and  $c_I$ ,  $d^b(x)$  must be decreasing in  $c_I$  and  $c_D$ .

### 6.3 Proof of Proposition 1

In case (a), part (a) of lemma 2 shows that for  $d^a(\theta; d^b) = d^a(\theta)$  solves  $-c'(d^a(\theta); \theta) = c_D \forall \theta$ . From earlier analysis, ex-post efficiency requires  $d^a(\theta)$  to solve  $-c'(d^a; \theta) = c_T$ . These solutions coincide iff  $c_D = c_T$ . Case (b) follows similarly from part(b) of lemma 2. Finally, in case (c), assumption 1 guarantees that for some realizations of  $\theta$  the contractor will want to complete late; while for others he will want to complete early (regardless of  $d^b$ ). Moreover, by part (c) of lemma 2  $d^b \in (0, \bar{d})$ , so this is feasible. But since the ex-post efficient completion time requires  $-c'(d^a; \theta) = c_T$  and  $c_U < c_D$  cannot both be equal to  $c_T$ , there are some  $\theta$  for which the completion time is inefficient; and thus the contract as a whole is ex-post inefficient.

### 6.4 Proof of Proposition 2

The objective function faced by a bidder with signal  $x_i$  is to maximize  $E [\pi(b, d^b, d^a; \theta) | x_i] \prod_{j \neq i} H_j(s) = (s - C_i(d^b; x)) \prod_{j \neq i} H_j(s)$  by choice of  $s$  and  $d^b$ , where  $H_j(s)$  is the distribution of scores submitted by bidder  $j$ . From this objective function, it is clear that the optimizer in  $d^b$  does not depend on  $s$ ; and moreover must minimize  $C_i(d^b; x)$ . From earlier analysis, this is  $d_i^b(x)$  - so this must be part of the equilibrium bid. But then we may replace  $C_i(d^b; x)$  with its minimized value  $P_i(x)$ , and obtain the standard objective function in an FPA  $(s - P_i(x)) W_i(s)$ . Asker and Cantillon (2008b) show the set of equilibria in this modified auction are the same as in the original auction, and Maskin and Riley (2003) show the uniqueness of equilibrium for this asymmetric FPA. The FOC follows directly from taking a derivative in  $s$  in the objective function.

For the second part, by Proposition 6, if the contract is ex-post efficient, either  $d^b = 0$  and  $c_I \leq c_U = C_D = C_T$  or  $d^b = \bar{d}$  and  $c_I = c_U = c_T \leq C_D$ . Consider the first case (the second is similar). Then the pseudo-cost simplifies to  $P_i(x) = E_i [c(d^*; \theta) + c_T d^* | x]$  where  $d^*$  is the socially-efficient completion time. The contract is ex-ante efficient if it awards the contract to the bidder who maximizes expected

social welfare given by  $E[V - c(d^a; \theta) - d^a c_T | x]$ . Clearly the bidder with the lowest pseudo-cost maximizes this function (since all bidders choose the optimal  $d^*$ ), and since the winner of the auction has the lowest pseudo-cost, the contract is ex-ante efficient.

## 6.5 Proof of Proposition 3

For the first part, from Lemma 2 we have  $d_i^b(x)$  decreasing in  $c_U$ , and from Lemma 1  $d^a(d^b; \theta)$  is increasing in  $d^b$  for each  $\theta$ . Putting these together it follows that  $d^a(\theta, d^b)$  is increasing (in the sense of FOSD) in  $c_U$ . To see that the winning bid increases, notice that at the old equilibrium bids the payoff conditional on winning has fallen - since they complete quicker - while the probability of winning remains the same. It is then optimal for every bidder to deviate by increasing their score and bidding more; resulting in an equilibrium with a higher winning bid (in the sense of FOSD). For the second part, consider the case  $c_U < c_I$  (the other case is similar). Then the bidder bids  $d^b = \bar{d}$  even if  $c_U$  increases marginally, and his completion time is independent of  $c_U$ . To see that the winning bid doesn't change, notice that if all bidders submit the same bids as before, their payoffs have not changed (they did not depend on  $c_U$ ) and their probability of winning does not change; so this remains the unique equilibrium.

## 6.6 Proof of Proposition 4

[to be completed]

## 6.7 Tables

Table 1: Observed Incentive Structures

	# Auctions	# Bids
No positive incentives	10	38
Small positive incentives	3	12
Equal incentives	15	67
Higher usercost	1	6
Total	29	123

**Summary of the observed incentive structures.** “No positive incentives” means  $0 = c_I < c_U = c_D$ , “Small positive incentives” means  $0 < c_I < c_U = c_D$ , “Equal Incentives” means  $c_I = c_U = c_D$ , and “Higher usercost” means  $c_I = c_D < c_U$ .

Table 2: A+B Regressions

	Days Bid (% of Max)	\$ Bid (% of Est)	Days Taken (% of Max)
No positive incentives	-0.1523 (0.0745)	0.1157 (0.0752)	0.1024 (0.1049)
Small positive incentives	0.0930 (0.0615)	-0.0240 (0.0385)	–
Higher usercost	-0.1963 (0.0516)	0.0634 (0.0328)	–
More than two bidders		-0.1062 (0.0661)	
N	123	123	23

**Regression results.** The excluded group is equal incentives. Robust standard errors are in parentheses, clustered by contract in the first two columns. In the first column, a modified tobit is used to account for the censoring of days bid at the minimum and the maximum days bid. In the second and third columns, the regression technique is OLS. The last two groups are omitted in the final regression because there are too few observations.

Table 3: Damage Specifications

Contract Value (\$)	Damages per Day (\$)	
	1995–2004	2005–
Below 25K	75	150
25K-50K	125	300
50K-100K	250	300
100K-500K	500	600
500K-1M	750	1000
1M-2M	1250	1500
2M-5M	1750	2000
5M-10M	2500	3000
Over 10M	3000	3500

**MNDOT schedule of liquidated damages.** These are part of the standard contract specifications.

Table 4: Summary Statistics on Diary Subsample (Working Day Contracts)

Contract Value (\$)	# Obs	Avg. Days Late	% Late	% Charged	Damages (% of value)
Below 25K	8	-6.0	0	0	-
25K-50K	6	-2.5	0	0	-
50K-100K	28	-2.7	17.9	7.1	1.2
100K-500K	90	-2.8	20.0	3.3	0.2
500K-1M	33	3.9	54.5	18.1	0.4
1M-2M	39	0.5	43.6	17.9	0.2
2M-5M	24	3.7	45.8	12.5	0.04
5M-10M	8	4.3	62.5	0	-

**Summary Statistics for diary subsample.** The over 10M category is excluded since no contracts of that size are observed in the subsample. % late indicates the percentage of contracts that were completed late; % charged indicates the percentage of contracts that were actually charged damages. The damages column indicates what percentage of the total contract value was eventually assessed in damages, conditional on damages being assessed.

Table 5: Summary Statistics on Full Sample

Contract Value (\$)	# WD	% WD Charged	# CD	% CD Charged	Damages (% of Value)
Below 25K	42	2.4	33	3.0	4.7
25K-50K	70	0	26	3.8	2.1
50K-100K	184	2.7	42	4.8	3.1
100K-500K	649	4.3	169	1.8	1.6
500K-1M	253	10.7	80	3.8	1.1
1M-2M	222	12.1	69	10.1	1.0
2M-5M	160	11.9	79	7.6	0.6
5M-10M	34	8.8	24	4.2	0.8
Above 10M	5	0	20	0	-

**Summary Statistics for the full sample.** % charged indicates the percentage of contracts that were actually charged damages. The damages column indicates what percentage of the total contract value was eventually assessed in damages, conditional on damages being assessed.

Table 6: Probability of Late Completion

	(1)	(2)	(3)
Log Engineers Estimate	0.06403 (0.02006)	0.06361 (0.02132)	0.06440 (0.01938)
Log Damages per Day	-0.01098 (0.02285)	-0.00956 (0.02246)	-0.00667 (0.020194)
Working Day Contract	0.22538 (0.06770)	0.21470 (0.07730)	0.17673 (0.06012)
Contract Type FE	No	Yes	Yes
District FE	No	No	Yes

**Completion Times.** The table presents maximum likelihood estimates of the marginal effects of the covariates on the probability of a late completion time. The marginal effect here is the effect of a 1 unit change in the covariate on the change in probability of late completion for a contract of the average characteristics. Standard errors are in parentheses, and were obtained by bootstrapping the two step procedure 500 times. Specifications (2) and (3) include contract type and district fixed effects, respectively.