

Bargaining with Optimism: A Structural Analysis of Medical Malpractice Litigation

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Abstract

We study identification and estimation of a structural model of bargaining with optimism where players have heterogeneous beliefs about the final resolution of a dispute if they fail to reach an agreement. We show the distribution of the players' beliefs and the stochastic bargaining surplus are nonparametrically identified from the probability of reaching an agreement and the distribution of transfers in the final resolution of the dispute. We use a Maximum Simulated Likelihood approach to estimate the beliefs of doctors and patients in medical malpractice disputes in Florida during the 1980s and 1990s. We find strong evidence that beliefs for both parties vary with the severity of the injury and the qualification of the doctor named in the lawsuit, even though these characteristics are statistically insignificant in explaining whether the court rules in favor of the plaintiff or of the defendant. We also quantify the reduction in settlement amounts that would result from the introduction of a (counterfactual) policy that imposes caps on the total compensation for plaintiffs.

Key words: Bargaining, optimism, nonparametric identification, medical malpractice litigation

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1 Introduction

Excessive optimism is often invoked as a possible explanation for why parties involved in a negotiation sometimes fail to reach an agreement even though a compromise could be mutually beneficial. For example, consider a medical malpractice dispute where a patient (the plaintiff) suffered a damage allegedly caused by a doctor's (the defendant) negligence or wrongdoing. If the plaintiff and the defendant are both overly optimistic about their chances of getting a favorable jury verdict, there may not be any settlement that can satisfy both parties' exaggerated expectations. The general argument dates back to Hicks (1932), and was later developed by Shavell (1982), among others, in the context of legal disputes. A recent theoretical literature originated by the work of Yildiz (2003, 2004), extends this insight and studies a general class of bargaining models with optimism (see Yildiz (2011) for a survey). These models have also been used in a variety of empirical applications that range from pretrial negotiations in medical malpractice lawsuits (Watanabe (2006)), to negotiations about market conditions (Thanassoulis (2010)), and cross-license agreements (Galasso (2012)).

Despite the recent surge of interest in the theory and application of bargaining with optimism, none of the existing contributions formally addresses the issue of identification in this class of models. That is, under what conditions can the structural elements of the model be unambiguously recovered from the history of bargaining outcomes reported in the data? This is the main question we address in this paper. We introduce an empirical framework for structural estimation of bargaining models with optimism, and show that the model elements are nonparametrically identified.

We consider a bilateral bargaining environment where players are optimistic about the probability that a stochastic outcome favors them if they fail to reach an agreement. The players have a one-time opportunity for reaching an agreement at an exogenously scheduled date during the bargaining process, and make decisions about whether or not to settle and, if so, the amount of the settlement based on their beliefs and time discount factors. We show that all structural elements in the model are identified nonparametrically from the probability of reaching an agreement and the distribution of transfers in the final resolution of the dispute. The identification strategy does not rely on any parametrization of the structural primitives such as players' beliefs or the bargaining surplus distribution. We then propose a Maximum Simulated Likelihood (MSL) estimator based on flexible parametrization of the joint beliefs of the players, and estimate the model using data on medical malpractice disputes in the State of Florida during the period 1984-1999.²

²Sieg (2000) and Watanabe (2006) also use the same source of data for their empirical analyses of medical malpractice litigation. Sieg (2000) estimates a bargaining model with one-sided incomplete information; Watanabe (2006) a bargaining model with optimism and learning. Neither study addresses the issue of identification, which is the main focus of our paper.

The bargaining environment we consider is simpler than the one studied by Yildiz (2004). Rather than allowing for multiple rounds of offers and counteroffers, in our model there is a single settlement opportunity for the players to reach an agreement. Hence, in our bargaining environment there are no dynamic “learning” considerations in the players’ decisions, and the dates of the final resolution of the bargaining episodes are solely determined by the players’ optimism, their patience, and their perception of the surplus available for sharing.³

There are both theoretical and empirical reasons that motivated our choice of the bargaining environment. Data limitations would prevent us from deriving robust (parametrization-free) arguments for the identification of structural elements in general models of bargaining with optimism that admit multiple rounds of offers and counteroffers. For instance, none of the data sets which are used in empirical applications of bargaining models with optimism contains information on the sequence of proposers in a negotiation or the timing and size of rejected offers. By abstracting from the dynamic learning aspects that would be introduced into the theoretical analysis if we were to consider a more general bargaining environment with multiple rounds of (unobserved) offers and counteroffers, we take a pragmatic approach and specify a model that is identifiable under realistic data requirements and mild econometric assumptions. At the same time, despite this simplification, our model captures the key insight of bargaining with optimism in that the timing of agreement is determined by the players’ optimism and their patience. Thus, our work represents a first important step toward addressing the issue of nonparametric identification in more general models of bargaining with optimism.⁴ Our modeling choice is also motivated by the specific empirical application we consider here, which analyzes medical malpractice disputes in Florida. The law of the State of Florida (Florida Statutes, Title XLV, Chapter 766, Section 108), requires that a one-time, mandatory settlement conference between the plaintiff and the defendant be held “at least three weeks before the date set for trial”. The settlement conference is scheduled by the county court, is held before the court, and is mediated by court-designated legal professionals.

Our identification strategy builds on the following insights. First, if the length of time between the scheduled settlement conference and the trial (henceforth, the “wait-time”) were reported in the data, we would be able to recover the distribution of optimism by observing how the conditional settlement probability varies with the wait-time. Second, the

³Sieg (2000) also considers a bargaining environment where there is a one-time opportunity for the players to settle out of court. Rather than studying a model of bargaining with optimism, Sieg (2000) assumes that the defendant has an informational advantage over the plaintiff. In other words, his analysis of medical malpractice disputes is based on a bargaining model with one-sided incomplete information, where the defendant knows the actual probability of a verdict in his/her favor, while the plaintiff does not.

⁴Watanabe (2006) studies medical malpractice disputes in the context of a dynamic model of bargaining with optimism and learning. As we already mentioned, however, his analysis is fully parametric and does not address the issue of identification.

distribution of the potential surplus to be divided between players can be recovered from the distribution of total compensation awarded to the plaintiff by the court decision, provided the surplus distribution is orthogonal to the beliefs of the plaintiff and the defendant and the court decision. Third, because the accepted settlement offer reflects a plaintiff's time-discounted expectation of his or her share of the total surplus, we can identify the conditional distribution of the plaintiff's belief given that there is a settlement using the distribution of accepted settlement offers given the length of the wait-time. This is done through a deconvolution argument using the distribution of surplus recovered above. Finally, since optimism is defined as the sum of both parties' beliefs minus one, the objects identified from the preceding steps can be used to back out the joint distribution of the beliefs through a standard Jacobian transformation.

A key challenge for implementing this identification strategy in our empirical application is that the wait-time is not reported in the data. In order to solve the issue of unobserved wait-time, we tap into a recent literature that uses eigenvalue decomposition to identify finite mixture models or structural models with unobserved heterogeneity (see, for example, Hall and Zhou (2003), Hu (2008), Hu and Schennach (2008), Kasahara and Shimotsu (2009), An, Hu and Shum (2010) and Hu, McAdams and Shum (2013)). In particular, we first exploit the institutional details of our environment to group lawsuits into smaller clusters defined by the county and the month in which a lawsuit is filed. We argue that the lawsuits within each cluster can be plausibly assumed to share the same, albeit unobserved wait-time between the mandatory settlement conference and the trial. We then use the cases in the same cluster as instruments for each other and apply eigenvalue decomposition to the joint distribution of settlement decisions and accepted offers within the cluster. This allows us to recover the settlement probability and the distribution of accepted offers conditional on the unobserved wait-time. Then, the arguments in the previous paragraph apply to identify the joint distribution of beliefs.

Using data from medical malpractice lawsuits in Florida in the 1980s and 1990s, we find clear evidence in our estimates that the beliefs of the plaintiffs and the defendants vary with observed characteristics of the lawsuits such as the severity of the injury arising out of medical malpractice and the professional qualification of the doctor named in the lawsuit. On the other hand, we find that the observed case characteristics are statistically insignificant in explaining the court and jury decisions.

We use our estimated model to assess the effects of a counterfactual tort reform which limits the liability of defendants by imposing caps on the total compensation received by plaintiffs. For each level of severity of the injury arising out of malpractice, we set a cap equal to the 75th percentile of the total compensation paid by the defendant following a jury verdict observed in the data. Our calculations show these caps can induce sizeable reductions in the average settlement amounts. Specifically, the reductions are 33%, 45%

and 24% for low, medium and high severity cases, respectively. There is also clear evidence that the impact of these caps varies with the qualification of the defendant: doctors who are board certified would benefit from a significantly more sizable reduction in the average settlement amounts they would have to pay to the plaintiffs than their non-board-certified counterparts in medium- and high-severity cases.

The rest of the paper is organized as follows. Section 2 introduces the model of bilateral bargaining with optimism. Section 3 establishes identification of the structural elements in the model. Section 4 presents the Maximum Simulated Likelihood (MSL) estimator. Section 5 describes the data and the institutional details of the empirical application which focuses on medical malpractice lawsuits in Florida. Sections 6 and 7 present and discuss our estimation results and policy analysis, respectively. We conclude with Section 8. All the proofs and a monte carlo study are contained in the appendices.

2 The Model

Consider a lawsuit following an alleged instance of medical malpractice involving a plaintiff (the patient) and a defendant (the doctor). The total amount of potential compensation related to the injury arising out of the malpractice, C , is assumed to be common knowledge among the plaintiff and the defendant. This amount can be interpreted as a sunk cost for the defendant, which the defendant may or may not be able to recover, in part or in total, depending on the final outcome of the legal dispute. After the filing of the lawsuit, the plaintiff and the defendant are notified of a date for a *one-time* settlement conference, which is mandatory by state law, according to Title XLV, Chapter 766, Section 108 of the Florida Statutes. The conference, which is held before the court, requires attendance by both parties (and their attorneys), as well as legal professionals designated by the county court where the lawsuit is filed. This conference must take place at least three weeks before the date set for trial.⁵

During the settlement conference, the defendant has the opportunity to make a settlement offer of S to the plaintiff. If the plaintiff accepts it, then the case is settled out of court with the plaintiff receiving S and the defendant reclaiming $C - S$. Otherwise, the case is taken to court and decided by a jury. Both the defendant and the plaintiff are aware that the trial must take place at least three weeks after the settlement conference and are informed of the exact date of the trial, which is determined by the court schedule and the backlogs of the county court judges.⁶ Let T denote the length of time between the settlement conference

⁵The current Florida Statutes pertaining to medical malpractice and related matters are available online at <http://www.flsenate.gov/Laws/Statutes/2014/Chapter766>

⁶Cases are assigned randomly among all the county court judges in the county court where the suits are filed depending on their availability.

and the trial (henceforth, the “wait-time”).

Let $A \equiv 1$ denote the event that a settlement is reached at the conference, and $A \equiv 0$ that the case goes to trial. In the latter case, at the end of the trial, the jury makes a binary decision, D , and rules either in favor of the plaintiff, $D = 1$, in which case the plaintiff is awarded the compensation C , or in favor of the defendant, $D = 0$, in which case the defendant is not awarded any compensation.⁷

The plaintiff and the defendant have heterogeneous beliefs about the probability that the jury would rule in their favor in the event that the case goes to trial. We let $\mu_p, \mu_d \in [0, 1]$, denote the subjective probability of winning the trial believed by the plaintiff and the defendant, respectively. Excessive optimism arises from the assumption that the joint support of (μ_p, μ_d) is $\Omega_\mu \equiv \{(\mu, \mu') \in (0, 1]^2 : 1 < \mu + \mu' \leq 2\}$. The realized value of (μ_p, μ_d) is common knowledge between both parties participating in the settlement conference. We also maintain the following assumption throughout the paper.

Assumption 1 (i) (μ_p, μ_d) and C are independent from the wait-time T ; and the distributions of (μ_p, μ_d) is continuous with positive density over Ω_μ . (ii) Conditional on $A = 0$, the jury decision D is orthogonal to C and T .

Assumption 1 allows the beliefs of the plaintiff and the defendant to be correlated and be asymmetric with different marginal distributions. This is empirically relevant because the marginal distribution of beliefs may very well differ between patients and doctors due to factors such as informational asymmetries (e.g., doctors may be better informed about the cause and severity of the damage arising out of medical malpractice) or unobserved individual heterogeneities.

The beliefs of the plaintiff and the defendant are also likely to be correlated through unobserved heterogeneity at the case level. For example, the doctor and the patient may both know aspects related to the cause and severity of the damage arising out of medical malpractice that are not recorded in data. Such aspects lead to correlations between patients’ and plaintiffs’ beliefs from the perspective of the econometrician. Assumption 1 also accommodates correlation between (μ_p, μ_d) and C .

The independence between the wait time T and the beliefs of the parties in the lawsuit is plausible because the wait-time T is mostly determined by the availability of judges and juries in the county court where the suit is filed. This, in turn, depends on the court schedule and the backlogs of the county court judges, which are idiosyncratic and orthogonal to the parties’ beliefs (μ_p, μ_d) .

The orthogonality of C from D given T and $A = 0$ in condition (ii) is also plausible. On the one hand, C is a monetary measure of the magnitude of the damage suffered by the plaintiff regardless of its cause; on the other hand, D captures the jury’s judgement about

⁷We abstract here from the legal costs associated with the lawsuit.

the cause of the damage based on the evidence presented at trial. It is likely that the jury decision is correlated with specific features of the lawsuit that are reported in the data and that may also affect the beliefs of both parties. Nevertheless, once we condition on such observable features, jury decisions are likely to be orthogonal to the measure of damage captured by C . At the end of this section, we discuss how to extend our model to account for heterogeneities across lawsuits reported in the data.

We now characterize the Nash equilibrium at the settlement conference. The plaintiff accepts an offer if and only if $S \geq \delta^T \mu_p C$, where δ is a constant time discount factor fixed throughout the data-generating process. The defendant offers the plaintiff $S = \delta^T \mu_p C$ if and only if $C - S \geq \delta^T \mu_d C$. Hence, in equilibrium a settlement occurs during the conference if and only if:

$$C - \delta^T \mu_p C \geq \delta^T \mu_d C$$

which is equivalent to $\mu_d + \mu_p \leq \delta^{-T}$. It then follows that the distribution of settlement amounts, conditional on the wait-time between the settlement conference and the trial being $T = t$, is:

$$\Pr(S \leq s \mid A = 1, T = t) = \Pr(\mu_p C \leq \delta^{-t} s \mid \mu_d + \mu_p \leq \delta^{-t}), \quad (1)$$

where the lower case letters denote realized values for random variables, and the equality follows from part (i) in Assumption 1. Besides the distribution of the potential compensation, conditional on there not being a settlement in a conference $T = t$ periods ahead of the trial and conditional on the jury ruling in favor of the plaintiff, is:

$$\Pr(C \leq c \mid A = 0, D = 1, T = t) = \Pr(C \leq c \mid \mu_d + \mu_p > \delta^{-t}), \quad (2)$$

where the equality follows from both conditions in Assumption 1.

The data we use in this paper reports characteristics of plaintiffs and defendants, such as the professional qualification (board certification) of the defendant and the age of the plaintiff. It also reports the severity of the injury arising out of medical malpractice. These variables, denoted by a vector X , are correlated with the potential compensation C and the beliefs (μ_p, μ_d) .

The model described above can incorporate such observed case-heterogeneity by letting the primitives (i.e., the distribution of (μ_p, μ_d) , compensations C , jury decisions D , and the wait-time T) depend on X . If both restrictions in Assumption 1 hold conditional on X , then the structural links between the data and the model elements are characterized in the same way as above, except that all distributions need to be conditioned on X . Since these characteristics that vary across lawsuits are reported in the data, our identification argument in Section 3 should be interpreted as conditional on X . We suppress dependence of the structural elements on X only for the sake of notational simplicity. We only make the dependence explicit when needed.

3 Identification

This section shows how to recover the distribution of both parties’ beliefs from the probability of reaching settlements and the distribution of accepted settlement offers. We consider an empirical environment where for each lawsuit the data reports whether a settlement occurs during the mandatory conference (A). For each case settled at the conference, the data reports the amount paid by the defendant to the plaintiff (S). For each of the other cases that were taken to the court, the data reports the jury decision (D) and, if the court rules in favor of the plaintiff, the amount of total compensation paid by the defendant (C). However, the exact dates of settlement conferences and the scheduled court hearings (if necessary) are never reported in the data.⁸ Thus, the wait-time T between settlement conferences and scheduled court hearings, which is known to both parties at the time of the conference, is not available in data.

To address this issue with unreported wait-time, we propose sequential arguments that exploit an implicit panel structure of the data. In particular, we note that lawsuits filed with the same county court during the same period (week) practically share the same wait-time T . The reason for such a pattern is as follows: First, the dates for settlement conferences are mostly determined by availability of authorized legal professionals affiliated with the county court, and are assigned on a “first come, first served” basis. Thus, the settlement conferences for the cases filed with the same county court at the same time are practically scheduled for the same period. Besides, the dates for potential court hearing are determined by the schedule and backlog of judges at the county court. Hence, the cases filed with the same county court simultaneously can be expected to be handled in court in the same period in the future. This allows us to effectively group lawsuits into clusters with the same wait-time, despite unobservability of T in data. We formalize this implicit panel structure as follows.

Assumption 2 *The data is partitioned into known clusters, each of which consists of multiple (potentially more than three) cases that share the same wait-time T . Across the cases within a cluster, (μ_p, μ_d, C) and D (if necessary) are independent draws from the same distribution.*

This implicit panel structure in our data allows us to use accepted settlement offers

⁸For example, the data we use in Section 5 reports the “date of final disposition” for each case. However, for a case settled outside the court, this date is defined not as the exact date of the settlement conference, but as the day when official administrative paperwork is finished and the claim is declared closed by the insurer. There is a substantial length of time between the two. For instance, for a large proportion of cases that are categorized as “settled within 90 days of the filing of lawsuits”, the reported “dates of final disposition” are actually more than 150 days after the initial filing. Similar issues also exist for cases that were taken to the court in that the reported “dates of final disposition” do not correspond to the actual dates of court hearings.

in the lawsuits within the same cluster as instruments for each other, and apply eigen-decomposition-based arguments in Hu (2008) and Hu and Schennach (2008) to recover the joint settlement probability and the distribution of accepted settlement offers conditional on the unobserved T . We then use these quantities to back out the joint distribution of beliefs using variations in T .

For the rest of this section, we first present arguments for the case where T is discrete (i.e. $|\mathcal{T}| < \infty$). At the end of this section, we explain how to generalize them for identification when T is continuously distributed. We maintain that there is positive probability that a cluster contains at least three cases.

3.1 Conditional distribution of settlement offers

We first recover the settlement probability and the distribution of settlement offers given the wait-time T before court hearings. Let \mathcal{S}, \mathcal{T} denote the unconditional support of S, T respectively.

Assumption 3 (i) *The support of T is finite ($|\mathcal{T}| < \infty$) with a known cardinality and $\inf\{\delta^t : t \in \mathcal{T}\} \geq 1/2$. (ii) *Given any (μ_p, μ_d) , the potential compensation C is continuously distributed with positive density over $[0, \bar{c}]$.**

That the support \mathcal{T} is known is empirically relevant in our setting. Without loss of generality, denote the elements in \mathcal{T} by $\{1, 2, \dots, |\mathcal{T}|\}$. Condition (i) also rules out unlikely cases where a court hearing is scheduled so far in the future or the one-period discount factor is so low that the compounded discount factor is less than one half. Condition (i), together with the non-increasingness of $\mathbb{E}[A_i | T = t]$ over $t \in \mathcal{T}$ under Assumption 1, pin down the index for eigenvalues and eigenvectors in the aforementioned decomposition. Condition (ii) is a mild restriction on the support of potential compensation. It is implied if C is orthogonal from (μ_p, μ_d) with a bounded support.⁹ The role of (ii) will become clear as we discuss the identification result below.

Lemma 1 *Under Assumptions 1, 2 and 3, $\mathbb{E}(A | T = t)$ and $f_S(s | A = 1, T = t)$ are jointly identified for all $t \in \mathcal{T}$ and $s \in \mathcal{S}$.*

This intermediate result uses arguments similar to that in Hu, McAdams and Shum (2013) for identifying first-price sealed-bid auctions with non-separable auction heterogeneities. It exploits the conditional independence of beliefs across lawsuits within a cluster in Assumption 2. These conditions allow us to break down the joint distribution of the incidence of

⁹It is worth noting that our identification argument remains valid even with \bar{c} being unbounded, as long as the full-rank condition in Lemma B1 in Appendix B holds for some partition of \mathcal{S} .

settlement and the size of accepted offers across multiple lawsuits within one cluster into the composition of three linear operators.

More specifically, let $f_{R_1}(r_1, R_2 = r_2 | \cdot)$ be shorthand for $\frac{\partial}{\partial \tilde{r}} \Pr\{R_1 \leq \tilde{r}, R_2 = r_2 | \cdot\}|_{\tilde{r}=r_1}$ for any discrete random vector R_2 and continuous random vector R_1 . For any three lawsuits i, j, k sharing the same wait-time T , let $A_{i,k} = 1$ be a shorthand for “ $A_i = A_k = 1$ ”. By construction,

$$\begin{aligned} & f_{S_i, S_k}(s, s', A_j = 1 | A_{i,k} = 1) \\ &= \sum_{t \in \mathcal{T}} f_{S_i}(s | S_k = s', A_j = 1, T = t, A_{i,k} = 1) \mathbb{E}[A_j | S_k = s', T = t, A_{i,k} = 1] f_{T, S_k}(t, s' | A_{i,k} = 1) \\ &= \sum_{t \in \mathcal{T}} f_{S_i}(s | A_i = 1, T = t) \mathbb{E}[A_j | T = t] f_{T, S_k}(t, s' | A_{i,k} = 1). \end{aligned} \quad (3)$$

The second equality follows from Assumption 1, the fact that “ $S = \delta^T \mu_p C$ whenever $A = 1$ ” and “ $A = 1$ if and only if $\mu_p + \mu_d \leq \delta^{-T}$ ” and that (μ_p, μ_d, C) are independent draws across the lawsuits i, j, k under Assumption 2.

To illustrate the identification argument, it is useful to adopt matrix notations. Let \mathcal{D}_M denote a partition of the *unconditional* support of accepted settlement offers \mathcal{S} into M non-degenerate intervals, each of which is denoted by d_m .¹⁰ For a given partition \mathcal{D}_M , let L_{S_i, S_k} be a M -by- M matrix whose (m, m') -th entry is the probability that $S_i \in d_m$ and $S_k \in d_{m'}$ conditional on $A_{i,k} = 1$ (settlements are reached in the two cases i, k); and let Λ_{S_i, S_k} be a M -by- M matrix with its (m, m') -th entry being $f(S_i \in d_m, A_j = 1, S_k \in d_{m'} | A_{i,k} = 1)$. Note that both Λ_{S_i, S_k} and L_{S_i, S_k} are directly identifiable from data. Thus a discretized version of (3) is:

$$\Lambda_{S_i, S_k} = L_{S_i | T} \Delta_j L_{T, S_k} \quad (4)$$

where $L_{S_i | T}$ be a M -by- $|\mathcal{T}|$ matrix with (m, t) -th entry being $\Pr(S_i \in d_m | A_i = 1, T = t)$; Δ_j be a $|\mathcal{T}|$ -by- $|\mathcal{T}|$ diagonal matrix with diagonal entries being $[\mathbb{E}(A_j | T = t)]_{t=1, \dots, |\mathcal{T}|}$; and L_{T, S_k} be a $|\mathcal{T}|$ -by- M matrices with its (t, m) -th entry being $\Pr(T = t, S_k \in d_m | A_{i,k} = 1)$. Besides,

$$L_{S_i, S_k} = L_{S_i | T} L_{T, S_k} \quad (5)$$

due to conditional independence in Assumption 2.

Part (ii) in Assumption 3 implies the supreme of the conditional support of accepted offers given $T = t$ is $\delta^t \bar{c}$ and hence decreases in t . This, in turn, guarantees there exists a partition $\mathcal{D}_{|\mathcal{T}|}$ such that $L_{S_i | T}$ as well as L_{S_i, S_k} are non-singular (see Lemma B1 in Appendix B for details). Then (4) and (5) imply

$$\Lambda_{S_i, S_k} (L_{S_i, S_k})^{-1} = L_{S_i | T} \Delta_j (L_{S_i | T})^{-1} \quad (6)$$

¹⁰That is, $d_m \equiv [s_m, s_{m+1}]$ for $1 \leq m \leq M$, with $(s_m : 2 \leq m \leq M)$ being a vector of ordered endpoints on \mathcal{S} such that $s_1 < s_2 < \dots < s_M < s_{M+1}$ and $s_1 \equiv \inf \mathcal{S}$, $s_{M+1} \equiv \sup \mathcal{S}$.

where the left-hand side consists of directly identifiable quantities. The right-hand side of (6) takes the form of an eigen-decomposition of a square matrix, which is unique up to a scale normalization and unknown indexing of the columns in $L_{S_i|T}$ and diagonal entries in Δ_j (i.e. it remains to find out the specific value of $t \in \mathcal{T}$ that corresponds to each diagonal entry in Δ_j).

The scale in the eigen-decomposition is implicitly fixed because the eigenvectors in $L_{S_i|T}$ are conditional distributions and needs to sum up to one. The question of unknown indices is solved because in our model $\mathbb{E}[A_j | T = t]$ is monotonically decreasing in t over \mathcal{T} provided the parties follow rational strategies described in Section 2. This is again due to the independence between timing and the beliefs in Assumption 1 and the moderate compounded discounting in Assumption 3. This establishes the identification of Δ_j and $L_{S_i|T}$, which are used for recovering L_{T,S_k} and then the conditional density of accepted settlement offers over its full support \mathcal{S} (see proof of Lemma 1 in Appendix B).

3.2 The joint belief distribution

We now explain how to identify the joint distribution of beliefs (μ_p, μ_d) from the quantities recovered from Lemma 1 under the following orthogonality condition.

Assumption 4 *The joint distribution of (μ_p, μ_d) is independent from C .*

This condition requires the magnitude of potential compensation to be independent from plaintiff and defendants' beliefs. This condition is plausible because C is meant to capture an objective monetary measure of the severity of damage inflicted upon the patient. On the other hand, the beliefs (μ_p, μ_d) should depend on the evidence available as to whether the defendant's neglect is the main cause of such damage. It then follows from (2) that the distribution of C is directly identified as:

$$\Pr(C \leq c) = \Pr(C \leq c | A = 0, D = 1). \quad (7)$$

Let $\mathcal{S}_t \equiv [0, \bar{c}\delta^t]$ denote the conditional support of accepted settlement offers $S = \delta^T \mu_p C$ given $A = 1$ and $T = t$; and let $\varphi_t(s)$ denote the probability that a settlement is reached when the wait-time between the settlement conference and the trial is t and that the accepted settlement offer is no greater than s .¹¹ That is, for all (s, t) ,

$$\varphi_t(s) \equiv \Pr(S \leq s, A = 1 | T = t) = \Pr(\mu_p C \leq s/\delta^t, \mu_d + \mu_p \leq 1/\delta^t) \quad (8)$$

where the equality is due to Assumption 1. The non-negativity of C and (μ_p, μ_d) and an application of the law of total probability on the right-hand side of (8) implies:

$$\varphi_t(s) = \int_0^{\bar{c}} \Pr\left(\frac{1}{\mu_p} \geq \frac{c}{s}\delta^t, \frac{1}{\mu_d + \mu_p} \geq \delta^t\right) f_C(c) dc = \int_0^{\bar{c}} h_t(c/s) f_C(c) dc \quad (9)$$

¹¹In general, we could also allow \mathcal{S} , \mathcal{T} and \mathcal{S}_t to depend on observed heterogeneity of lawsuits. Throughout this section, we refrain from such generalization in order to simplify exposition.

where $f_C(c)$ is the density of C and $h_t(v) \equiv \Pr\{\mu_p^{-1} \geq v\delta^t, (\mu_d + \mu_p)^{-1} \geq \delta^t\}$; and the first equality is due to orthogonality between C and (μ_p, μ_d) .

Changing variables between C and $V \equiv C/S$ for any fixed t and s , we can write (9) as:

$$\varphi_t(s) = \int_0^\infty h_t(v)\kappa(v, s)dv \quad (10)$$

where $\kappa(v, s) \equiv sf_C(vs)$. With the distribution (and hence density) of C recovered from (7), the kernel function $\kappa(v, s)$ is considered known for all (v, s) hereinafter for identification purposes. Also note for any $s > 0$, $\kappa(\cdot, s)$ is a well-defined conditional density with support $[0, \bar{c}/s]$.¹² Let $F_{V|A=1, T=t}$ denote the distribution of V given $T = t$ and $A = 1$, whose support is denoted as \mathcal{V}_t .

Assumption 5 *For any t and $g(\cdot)$ such that $\mathbb{E}[g(V) | A = 1, T = t] < \infty$, the statement “ $\int_0^\infty g(v)\kappa(v, s) = 0$ for all $s \in \mathcal{S}_t$ ” implies the statement “ $g(v) = 0$ a.e. $F_{V|A=1, T=t}$ ”.*

This condition, known as the “completeness” of kernels in integral operators, was introduced in Lehmann (1986) and used in Newey and Powell (2003) for identification of nonparametric regressions with instrumental variables. Andrews (2011) and Hu and Shiu (2012) derived sufficient conditions for various versions of such completeness conditions when $g(\cdot)$ is restricted to belong to difference classes. This condition is analogous to a “full-rank” condition on κ if the conditional supports of S and V were finite.¹³

Proposition 1 *Under Assumptions 1-5, $\Pr(\mu_p \leq \mu, \mu_p + \mu_d \leq \delta^{-t})$ is identified for all $\mu \in (0, 1)$ and $t \in \mathcal{T}$.*

For the rest of this section, we discuss how to generalize results above when \mathcal{T} is infinite (T is continuously distributed over a known interval). First off, the key idea of using eigen-decompositions in Section 3.1 remains applicable, except that $L_{S_i|T}$ and L_{T, S_k} become linear integral operators, and their invertibility needs to be stated as an assumption as opposed to being derived from restrictions on model primitives and implications of rational strategies (as is the case when T is discrete).

Under the support condition that $\inf\{\delta^t : t \in \mathcal{T}\} \geq 1/2$, the eigenvalues in the decomposition $\mathbb{E}[A_j | T = t]$ remains strictly monotonic over the interval support \mathcal{T} when

¹²This is because $\kappa(v, s) > 0$ for any $v \geq 0, s > 0$, and that $\int_0^\infty \kappa(v, s)dv = \int_0^{\bar{c}/s} sf_C(vs)dv = 1$ for any s .

¹³If the support of potential compensation is unbounded, there are plenty of examples of parametric families of densities that satisfy the completeness conditions. For example, suppose potential compensations follow a Gamma distribution with parameters $\alpha, \beta > 0$. That is, $f_C(t) = \frac{\beta^\alpha}{\Gamma(\alpha)} t^{\alpha-1} \exp\{-t\beta\}$. Then, with $s > 0$, the kernel $\kappa(v, s) \equiv sf_C(vs) = \frac{[s\beta]^\alpha}{\Gamma(\alpha)} v^{\alpha-1} \exp\{-v(s\beta)\}$ is a density of a Gamma distribution with a shape parameter $\alpha > 0$ and a scale parameter $s\beta > 0$. That is, $\kappa(v, s)$ remains a conditional density within the exponential family, and satisfies the sufficient conditions for the completeness condition in Theorem 2.2 in Newey and Powell (2003).

T is continuously distributed. On the other hand, the argument that uses monotonicity of the eigenvalues over a finite support \mathcal{T} to index them is no longer applicable when T is continuously distributed. However, in our model the supremum of the support of accepted settlement offers given $T = t$ must be $\delta^t \bar{c}$. With the supremum of the support of compensations \bar{c} identified and known, this means t can be expressed through a known functional of the eigenvectors $f_{S_i}(\cdot | A_i = 1, T = t)$ in the eigen-decomposition identified in the first step. Thus the issue with indexing eigenvalues is also solved.

The remaining step of identifying the joint distribution of $1/\mu_p$ and $1/(\mu_p + \mu_d)$ from $f_S(\cdot | A = 1, T = t)$ and $\mathbb{E}[A | T = t]$ follow from the same argument as in the discrete case. An additional step based on Jacobian transformation leads to identification of the joint distribution of (μ_p, μ_d) when T is continuously distributed.

4 Maximum Simulated Likelihood Estimation

A nonparametric estimator based on the identification result in Section 3 would require a large data set, and the ‘‘curse of dimensionality’’ aggravates if the data also report case-level variables that may affect both parties’ beliefs (such as the severity of the injury arising out of medical malpractice and the qualification of the defendant) and therefore should be conditioned on in estimation. To deal with case-heterogeneity in moderate-sized data, we propose in this section a Maximum Simulated Likelihood (MSL) estimator based on a flexible parametrization of the joint belief distribution.

Consider a data containing N clusters. A cluster is indexed by n and consists of $m_n \geq 1$ cases, each of which is indexed by $i = 1, \dots, m_n$. For each case i in cluster n , let $A_{n,i} = 1$ when there is an agreement for settlement outside the court and $A_{n,i} = 0$ otherwise. Define $Z_{n,i} \equiv S_{n,i}$ if $A_{n,i} = 1$; $Z_{n,i} \equiv C_{n,i}$ if $A_{n,i} = 0$ and $D_{n,i} = 1$; and $Z_{n,i} \equiv 0$ otherwise. Let T_n denote the wait-time between the settlement conference and the scheduled date for court decisions, which is shared by all cases in cluster n . We propose an MSL estimator for the joint distribution of (μ_p, μ_d) that also use variation in the heterogeneity of lawsuits reported in the data. Throughout this section, we assume the identifying conditions hold once conditional on such observed heterogeneity of the lawsuits.

Let $x_{n,i}$ denote the vector of case-level variables reported in the data that affects the distribution of C . (We allow $x_{n,i}$ to contain a constant in the estimation below.) The total potential compensation C in a lawsuit with observed features $x_{n,i}$ is drawn from an exponential distribution with the rate parameter given by:

$$\lambda(x_{n,i}; \beta) \equiv \exp\{x_{n,i}\beta\}$$

for some unknown constant vector of parameters β . In the first step, we pool all observations

where the jury is observed to rule in favor of the plaintiff to estimate β :

$$\hat{\beta} \equiv \arg \max_{\beta} \sum_{n,i} d_{n,i} (1 - a_{n,i}) [x_{n,i} \beta - \exp\{x_{n,i} \beta\} c_{n,i}].$$

Next, let $w_{n,i}$ denote the vector of case-level variables in the data that affects the joint belief distribution. (The two vectors $x_{n,i}$ and $w_{n,i}$ are allowed to have overlapping elements.) We suppress the subscripts n, i for simplicity when there is no ambiguity. In the second step, we estimate the belief distribution conditional on such a vector of case-level variables W using $\hat{\beta}$ above as an input in the likelihood. To do so, we adopt a flexible parametrization of the joint distribution of (μ_p, μ_d) conditional on W as follows. For each realized w , let $(Y_1, Y_2, 1 - Y_1 - Y_2)$ be drawn from a Dirichlet distribution with concentration parameters $\alpha_j \equiv \exp\{w \rho_j\}$ for $j = 1, 2, 3$ for some constant vector $\rho \equiv (\rho_1, \rho_2, \rho_3)$. In what follows, we suppress the dependence of α_j on w to simplify the notation.

Let $\mu_p = 1 - Y_1$ and $\mu_d = Y_1 + Y_2$. The support of (μ_p, μ_d) is $\{(\mu, \mu') \in [0, 1]^2 : 1 \leq \mu + \mu' \leq 2\}$, which is consistent with our model with optimism. (Table C1 in Appendix C shows how flexible such a specification of the joint distribution of (μ_p, μ_d) is in terms of the range of moments and the location of the model it allows.) Also note $Y_2 = \mu_p + \mu_d - 1$ by construction, so it is a measure of optimism. Under this specification, the marginal distribution of Y_1 conditional on $W = w$ is $Beta(\alpha_1, \alpha_2 + \alpha_3)$, where of course α_j 's are functions of w . The conditional distribution $Y_2 \mid Y_1 = \tau, W = w$ is the same as the distribution of $(1 - \tau)Beta(\alpha_2, \alpha_3)$. For any y and $\tau \in (0, 1)$, we can write:

$$\Pr\{Y_2 \leq y \mid Y_1 = \tau, W = w\} = \Pr\left\{\frac{Y_2}{1 - \tau} \leq \frac{y}{1 - \tau} \mid Y_1 = \tau, W = w\right\}$$

where the right-hand side is the c.d.f. of a $Beta(\alpha_2, \alpha_3)$ evaluated at $y/(1 - \tau)$.

Let $q_{n,i} \equiv \Pr(D_{n,i} = 1 \mid A_{n,i} = 0, W_{n,i} = w_{n,i})$. Recall that we maintain D is orthogonal to (T, C) conditional on $A = 0$ and W . Hence $q_{n,i}$ does not depend on $c_{n,i}$. This conditional probability is directly identifiable from the data. Let $h_n(\cdot; \theta)$ denote density of the wait-time T_n in cluster n . This density in general depends on cluster-level variables reported in the data, and is specified up to an unknown vector of parameters θ .

The log-likelihood of our model is:

$$L_N(\rho, \beta, \theta) \equiv \sum_{n=1}^N \ln \left[\sum_{t \in \mathcal{T}} h_n(t; \theta) \prod_{i=1}^{m_n} f_{n,i}(t; \rho, \beta) \right]$$

where $f_{n,i}(t; \rho, \beta)$ is shorthand for the conditional density of $Z_{n,i}, A_{n,i}, D_{n,i}$ given $T_n = t, W_{n,i} = w_{n,i}$ and with parameter ρ , evaluated at $(z_{n,i}, a_{n,i}, d_{n,i})$. Specifically,

$$\begin{aligned} f_{n,i}(t; \rho, \beta) &\equiv [g_{1,n,i}(t; \beta, \rho)]^{a_{n,i}} \times \{g_{0,n,i}(t; \beta) [1 - p_{n,i}(t; \rho)] q_{n,i}\}^{(1-a_{n,i})d_{n,i}} \\ &\times \{[1 - p_{n,i}(t; \rho)] (1 - q_{n,i})\}^{(1-a_{n,i})(1-d_{n,i})} \end{aligned}$$

where

$$\begin{aligned} p_{n,i}(t; \rho) &\equiv \Pr(A_{n,i} = 1 \mid T_n = t, W_{n,i} = w_{n,i}; \rho) = \Pr(\mu_{p,n,i} + \mu_{d,n,i} \leq \delta^{-t} \mid w_{n,i}; \rho) \\ &= \Pr(Y_2 \leq (1 - \delta^t) / \delta^t \mid w_{n,i}; \rho); \end{aligned}$$

$$\begin{aligned} g_{0,n,i}(t; \beta) &\equiv g_0(z_{n,i}, x_{n,i}, t; \beta) \\ &\equiv \left. \frac{\partial \Pr(C_{n,i} \leq Z \mid A_{n,i} = 0, T_n = t, X_{n,i} = x_{n,i}; \beta)}{\partial Z} \right|_{Z=z_{n,i}} = f_C(z_{n,i} \mid x_{n,i}; \beta); \end{aligned} \quad (11)$$

with $f_C(\cdot \mid x_{n,i}; \beta)$ being the conditional density of the potential compensation given $X_{n,i} = x_{n,i}$; and

$$\begin{aligned} g_{1,n,i}(t; \beta, \rho) &\equiv g_1(z_{n,i}, w_{n,i}, x_{n,i}, t; \beta, \rho) \equiv \left. \frac{\partial \Pr(S_{n,i} \leq Z, A_{n,i} = 1 \mid T_n = t, w_{n,i}, x_{n,i}; \beta, \rho)}{\partial Z} \right|_{Z=z_{n,i}} \\ &= \left. \frac{\partial}{\partial Z} \int_0^\infty \Pr\left(Y_1 \geq 1 - Z/(c\delta^t), Y_2 \leq \frac{1-\delta^t}{\delta^t} \mid w_{n,i}; \rho\right) f_C(c \mid x_{n,i}; \beta) dc \right|_{Z=z_{n,i}} \end{aligned} \quad (12)$$

In the derivation above, we have used the conditional independence between $C_{n,i}$ and $D_{n,i}$, T_n , $(\mu_{p,n,i}, \mu_{d,n,i})$ conditional on $W_{n,i}, X_{n,i}$. Under regularity conditions that allow for the change of the order of integration and differentiation in (12), $g_{1,n,i}(t; \beta, \rho)$ equals:

$$\int_{z_{n,i}\delta^{-t}}^\infty \Pr\left\{Y_2 \leq \frac{1-\delta^t}{\delta^t} \mid Y_1 = 1 - z_{n,i}\delta^{-t}/c, w_{n,i}; \rho\right\} f_{Y_1}(1 - z_{n,i}\delta^{-t}/c \mid w_{n,i}; \rho) \frac{f_C(c \mid x_{n,i}; \beta)}{c\delta^t} dc$$

where the lower limit is $z_{n,i}\delta^{-t}$ because the integrand is nonzero only when $1 - z_{n,i}\delta^{-t}/c \in (0, 1) \Leftrightarrow c \in (\frac{z_{n,i}}{\delta^t}, +\infty)$. Changing variables between c and $\tau \equiv 1 - z_{n,i}\delta^{-t}/c$ for any i, n and fixed t , we can write $g_{1,n,i}(t; \beta, \rho)$ as:

$$\int_0^1 \Pr\left\{\frac{Y_2}{1-\tau} \leq \frac{1-\delta^t}{\delta^t(1-\tau)} \mid Y_1 = \tau, w_{n,i}; \rho\right\} \frac{f_{Y_1}(\tau \mid w_{n,i}; \rho) f_C\left(\frac{z_{n,i}}{\delta^t(1-\tau)} \mid x_{n,i}; \beta\right)}{\delta^t(1-\tau)} d\tau$$

where the first conditional probability in the integrand is a Beta c.d.f. evaluated at $\frac{1-\delta^t}{\delta^t(1-\tau)}$ and parameters $(\alpha_2(w_{n,i}; \rho_2), \alpha_3(w_{n,i}; \rho_3))$ and the second term $f_{Y_1}(\tau \mid w_{n,i}; \rho)$ is the Beta p.d.f. with parameters $(\alpha_1(w_{n,i}; \rho_1), \alpha_2(w_{n,i}; \rho_2) + \alpha_3(w_{n,i}; \rho_3))$. For each n, i, t and a fixed vector of parameters (λ, ρ) , let $\hat{g}_{1,n,i}(t; \lambda, \rho)$ be an estimator for $g_{1,n,i}(t; \lambda, \rho)$ using $S > N$ simulated draws of τ . (We experiment with various forms of density for simulated draws.) It follows from the Law of Large Numbers that $\hat{g}_{1,n,i}(t; \lambda, \rho)$ is an unbiased estimator for each n, i and (λ, ρ) .

Our Maximum Simulated Likelihood estimator for the belief parameter ρ in the second step is

$$(\hat{\rho}, \hat{\theta}) \equiv \arg \max_{\rho, \theta} \hat{L}_N(\rho, \theta, \hat{\beta}). \quad (13)$$

where $\hat{L}_N(\rho, \theta, \beta)$ is an estimator for $L_N(\rho, \theta, \beta)$ by replacing $g_{1,n,i}(t; \beta, \rho)$ with $\hat{g}_{1,n,i}(t; \beta, \rho)$ and replacing $q_{n,i}$ with a parametric (logit or probit) estimate $\hat{q}_{n,i}$; and $\hat{\beta}$ is the estimates for the parameters in the distribution of potential compensation in the first step.

Under appropriate regularity conditions, $(\hat{\rho}, \hat{\theta})$ converge at a \sqrt{N} -rate to a zero-mean multivariate normal distribution with some finite covariance as long as $N \rightarrow \infty$, $S \rightarrow \infty$ and $\sqrt{N}/S \rightarrow 0$. The covariance matrix can be consistently estimated using the analog principle, which involves the use of simulated observations. (See equation (12.21) in Cameron and Trivedi (2005) for a detailed formula.)

5 Data Description

Since 1975 the State of Florida has required all medical malpractice insurers to file reports on their resolved claims to the Florida Department of Financial Services. Using this source, we construct a sample that consists of 13,351 medical malpractice lawsuits filed in Florida between 1984 and 1999.¹⁴ Our sample includes those cases that are either resolved through the mandatory settlement conference or by a jury decision following a trial. For each lawsuit, the data reports the date when the suit is filed (*Suit_Date*) and the county court with which it is filed (*County_Code*), the date of the final disposition (*Year_of_Disposition*) corresponding to the date when the claim is closed with the insurer, and whether the case is resolved by a settlement at the settlement conference or by a jury decision in court ($A = 1$ or $A = 0$). The data also reports the size of the transfer from the defendant to the plaintiff upon the resolution of the lawsuit. This transfer is equal to the settlement amount accepted by the plaintiff (S), if the case is settled out of court, or the total compensation awarded to the plaintiff according to the court decision (C), otherwise. In addition, we also observe case-level variables that may be relevant to the joint distribution of beliefs and/or to the distribution of potential compensations. These variables include the severity of the injury arising out of medical malpractice (*Severity*), the age (*Age*) of the patient who suffered the injury and whether the doctor named in the lawsuit is board-certified (*Board_Code*), where *Board_Code* = 1 denotes that the doctor is reported to be certified by at least one professional board and 0 otherwise.

The dates of the settlement conference and of the scheduled jury trial for each lawsuit are not reported in the data, regardless of whether the case is settled out of court or decided by a jury. In fact, the recorded date of the final disposition of a case only reports when the claim is closed with the insurer, which typically occurs later than the actual date when an agreement is reached in the settlement conference or when a decision is made by the jury in court, and includes administrative delays which may vary across cases and are not

¹⁴Sieg (2000) and Watanabe (2009) also use the same source of data for their empirical analyses of medical malpractice lawsuits.

directly measurable. Therefore, the wait-time between the settlement conferences and the trials cannot be recovered from data.

Despite these data limitations, we define clusters within which the cases could be reasonably assumed to share the same length of wait-time. It is, in fact, plausible that the lawsuits filed with the same county court in the same month would be scheduled for court proceedings in the same month, since the schedule for hearings in a county court is mostly determined by the backlog of unresolved cases filed with that court, and by the availability of judges and other legal professionals from that court. By the same token, the schedule for settlement conferences, which require the presence of court officials who have the authority to facilitate a settlement, are also mostly determined by the backlog of cases as well as the availability of attorneys representing both parties. Based on these considerations, we maintain that the lengths of wait-time between settlement conferences and court hearings for all lawsuits filed with the same county court in the same month are the same. As explained in Section 3, the distribution of settlement decisions and accepted offers in lawsuits from these clusters are sufficient for recovering the joint beliefs of plaintiffs and defendants.

The data consists of 3,545 clusters defined by month-county pairs. In total there are 1,344 clusters which report at least three medical malpractice lawsuits. About half of these clusters (661 clusters) contain at least six cases. Moreover, among these 1,344 clusters, 1,294 have at least two lawsuits that were settled out of court during the mandatory settlement conference. These features of the data confirm that we can apply our identification strategy from Section 3 to recover the joint distribution of patients' and doctors' beliefs. It is worth mentioning that in our MSL estimation, the likelihood includes all 3,545 clusters to improve the efficiency of the estimator, even though in theory identification only requires the joint distribution of settlement decisions and accepted offers from the subset of clusters that have at least two settlements out of three or more cases.

Table 1(a): Settlement probability and accepted offers

Board Cert'n	Severity	# obs	\hat{p}_{settle}	$s.e.(\hat{p}_{settle})$	$\hat{\mu}_{S A=1}$ (\$1k)	$s.e.(\hat{\mu}_{S A=1})$ (\$1k)
certified	low	1,129	0.717	0.013	42.553	2.313
	medium	2,996	0.805	0.007	158.521	4.234
	high	2,623	0.831	0.007	263.089	7.213
uncertified	low	1,783	0.812	0.009	41.300	2.256
	medium	2,192	0.863	0.007	128.796	4.748
	high	2,628	0.880	0.006	333.161	12.357

Next, we report some evidence from the data that the beliefs of the plaintiffs and the defendants are affected by certain observed characteristics that vary across lawsuits. Table

1(a) summarizes the settlement probability and the average size of accepted settlement offers in the sample after controlling for the doctors’ qualification and the level of severity of the injury arising out of medical malpractice. There is evidence that both the settlement probability and the size of accepted settlement offers differ systematically across the sub-groups. Table 1(b) reports the p-values of two-sided t-tests (using the unequal variance formula) for the equality of settlement probabilities in sub-groups. We let (u,c) and (l,m,h) be shorthand for the realized values of (*uncertified*, *certified*) in *Board_code* and (*low*, *medium*, *high*) in *Severity* respectively. With the exception of three pair-wise tests, the nulls in the other tests are all rejected at the 2% significance level. Among the three exceptions, the null for equal settlement probability between (u,m) and (u,h) is also rejected at the 10% level. The only two cases where the null can not be rejected even at the 10% significance level are “(u,l) versus (c,m)” and “(u,l) versus (c,h)”. This is consistent with the intuition that a plaintiff may be relatively more optimistic that the jury would rule in his or her favor when the injury suffered is relatively more severe, or when the doctor’s qualification is not supported by board certification. Our estimates in the next section are also consistent with this intuition.

The failure to reject the null of equal settlement probability between the two subgroups (u,l) and (c,h), for example, may be due to the fact that the impact of severity and of board certification on the plaintiff’s belief offset each other. Pairwise t-tests for the equality of average accepted settlement offers between the sub-groups defined by severity and doctor qualification also demonstrate similar patterns. Specifically, the null of equal average settlement offers is almost always rejected at the 1% significance level for all pair-wise t-tests using unequal variances, with the only exception being the test comparing (u,l) versus (c,l).

Table 1(b): *p*-values for t-tests: settlement probability

	u,l	u,m	u,h	c,l	c,m	c,h
u,l	–	< 0.001	< 0.001	< 0.001	0.601	0.105
u,m		–	0.072	< 0.001	< 0.001	0.002
u,h			–	< 0.001	< 0.001	< 0.001
c,l				–	< 0.001	< 0.001
c,m					–	0.014
c,h						–

The data also contains some evidence that the distribution of total compensation may be partly determined by the age of the plaintiff and the severity of the injury. Out of the total 2,298 lawsuits which were not resolved through settlement, 359 were ruled in favor of the plaintiff by the court. The observations of the realized total compensation in these cases

are useful for inference on the distribution of C . Figure 1(a) and 1(b) in Appendix A report histograms of the accepted offers (S) from the cases settled outside the court and the total compensation (C) from the cases where the court ruled in favor of the plaintiff, conditioning on the information about the plaintiffs. The variable Age is discretized into three categories: young ($Age < 33$), older ($Age > 54$) and middle, with the two cutoffs being the 33rd and the 66th percentiles in the data. Figure 1(a) suggests the younger plaintiffs tend to receive higher transfers either through accepted offers in settlement or through the total compensation paid by the defendant when the court rules in favor of the plaintiff. Figure 1(b) shows the cases with more severe injuries in general are associated with higher transfers. Both patterns are intuitive, and consistent with our estimates presented in the next section below.

To further compare the distribution of accepted settlement offers with that of total compensations awarded by the court, we compare the percentiles of both variables conditional on Age and $Severity$. We find that the 10th, 25th, 50th, 75th and 90th conditional percentiles of the accepted settlement offers are consistently lower than those of the total compensations awarded by the court. This is consistent with the notion that an accepted settlement offer is equal to the discounted expectation of the total compensation that could be awarded by the court.

The qualification of the doctors does not seem to have any noticeable effect on the distribution of the total compensation. Figure 1(c) reports the histogram of the total compensation for the cases where the court ruled in favor of the plaintiff, conditioning on the board certification status of the doctors. A t-test for the equality of the average compensation for the two subgroups with and without board certification reports an asymptotic p-value of 0.5036 (assuming unequal population variance). Furthermore, a one-sided Komolgorov-Simirnov test against the alternative that the distribution of C is stochastically lower when the defendant is board-certified yields a test statistic of 0.0705 and an asymptotic p-value of 0.4078. Thus, in either test the null can not be rejected even at the 15% significance level.

On the other hand, it is reasonable to postulate that the total potential compensation in a malpractice lawsuit is positively correlated with the contemporary income level in the county where the lawsuit is filed. In order to control for such an income effect, we collect data on household income in all counties in Florida between 1981 and 1999. We collect data on the median household income in each Florida county in 1989, '93, '95, '97, '98 and '99 from the Small Area Income and Poverty Estimates (*SAIPE*) produced by the U.S. Census Bureau.¹⁵ We also collect a time series of state-wide median household income in Florida each year between 1984 and 1999 from the U.S. Census Bureau's Current Population Survey. We combine this latter state-wide information with the county-level information from *SAIPE* to extrapolate the median household income in each Florida county in the years 1984-89,

¹⁵See <http://www.census.gov/did/www/saibe/data/statecounty/data/index.html>

'92, '94 and '96.¹⁶ We then incorporate this yearly data on household income in each county while estimating the distribution of total compensation next year.

6 Estimation Results

As the first step in estimation, we use a logit regression to fit the court decisions in those lawsuits that are resolved through court hearings. The goal is to provide some evidence about whether the jury decisions were affected by case characteristics reported in the data. Moreover, the predicted probability for $D = 1$ (the jury ruled in favor of the plaintiff) from the logit regression will be used in the MSL estimation of the joint beliefs of doctors and patients.

Table 2. Logit Estimates for Court Decisions

(Response Variable: D . Total # of observations: 2,289 cases.)

	(1)	(2)	(3)
<i>Board_Code</i>	-0.0701 (0.120)	-0.207 (0.282)	-0.233 (0.288)
<i>Severity</i>	0.045** (0.023)	0.032 (0.033)	0.083 (0.056)
<i>Age</i>	0.003 (0.003)	0.003 (0.003)	0.021* (0.012)
<i>Severity</i> × <i>Board_Code</i>		0.025 (0.046)	0.029 (0.046)
<i>Age</i> ²			-0.014 (0.011)
<i>Severity</i> × <i>Age</i>			-0.012 (0.011)
<i>Constant</i>	-2.042*** (0.189)	-1.954*** (0.228)	-2.393*** (0.391)
Log likelihood	-953.5592	-953.4136	-952.2051
Pseudo-R ²	0.0026	0.0028	0.0041
p-value for L.R.T.	0.1676	0.2533	0.2557

Notes: Standard errors are reported in parentheses. *** significant at 1%; ** significant at 5%; * significant at 10%. Age² is reported in units of “100 yr²”.

Table 2 reports the logit regression estimates under different specifications, using 2,289 lawsuits from the data that were not settled outside the court and thus had to be resolved

¹⁶The extrapolation is done based on a mild assumption that a county’s growth rate relative to the state-wide growth rate remains steady in adjacent years. For example, if the ratio between the growth rate in County A between 1993 and 1995 and the contemporary state-wide growth rate is α , then we maintain the yearly growth rates in County A in 1993-94 (and 1994-95) are both equal to $\sqrt{\alpha}$ times the state-wide growth rates in 1993-94 (and 1994-95 respectively). With the yearly growth rate in County A between 1993-1995 calculated, we then extrapolate the median household income in County A in 1994 using the data from the *SAIPE* source.

through scheduled court hearings. The case heterogeneity used in the logit regressions include *Board_Code*, *Severity* and the age of the patients *Age*. In all three logit regressions, the constant term is highly statistically significant at the 1% level. The severity is statistically insignificant in the latter two specifications. Besides, the age of the patient is only significant at 10% level in the third specification. The board certification of doctors and the interaction terms in the logit regressions are all insignificant.

The pseudo R-squares are low for all three specifications. This suggests that the patient and case characteristics considered are rather insignificant in explaining the court decisions. Furthermore the p-values for the likelihood ratio tests of the joint significance of all slope coefficients are 0.1676, 0.2533 and 0.2557 in the three specifications respectively. Therefore we conclude from Table 2 that the doctor’s board certification, the severity of the malpractice and the age of the plaintiff do not have significant impact on jury decisions in the court.

Next, we estimate the distribution of total potential compensation using a subset of the observations of lawsuits above where the court ruled in favor of the plaintiffs ($A = 0$ and $D = 1$). The descriptive statistics in Section 5 show that the severity of the injury and the age of the plaintiffs have a noticeable impact on the size of the total potential compensation, while the doctors’ board certification does not. In one of the specifications, we include the county-level income in the same year when the lawsuit is filed in order to allow for the effect of income level. We adopt an exponential specification where the density of the compensation given case characteristics $x_{n,i}$ is $\lambda_{n,i} \exp\{-\lambda_{n,i}c\}$, where $\lambda_{n,i} \equiv \exp\{x_{n,i}\beta\}$.

Table 3(a). Coefficient Estimates in the Distribution of Total Compensation

(Total # of observations: 351 cases.)		
	(1)	(2)
<i>M.Sev.</i>	-1.198*** (0.157)	-1.014*** (0.392)
<i>H.Sev.</i>	-1.855*** (0.159)	-0.707** (0.348)
<i>M.Sev. × Age</i>		-0.003 (0.009)
<i>H.Sev. × Age</i>		-0.023*** (0.008)
<i>Age</i>	0.004 (0.003)	0.016** (0.007)
<i>Income (in \$1k)</i>	-0.026*(0.014)	-0.031** (0.014)
<i>Constant</i>	-3.810*** (0.506)	-4.288*** (0.554)
Log likelihood	-1179.27	-1167.48
Pseudo-R ²	0.513	0.541
p-value for LRT	<0.001	<0.001

Notes: *** significant at 1%; ** significant at 5%; * significant at 10%.

Table 3(a) presents the maximum likelihood estimates for β . We calculate the standard errors using a standard robust form that consists of estimates for the Hessian and cross-products of the Jacobian of the likelihood. In both specifications, likelihood ratio tests for the joint significance of all slope coefficients yield p-values that are lower than 0.001, thus rejecting the null of joint insignificance even at the 1% level. In both specifications, signs of the estimates show that more severe injuries lead to higher total potential compensation on average; and the expectation of such compensation is higher when the local median household income in the county is higher. (Note that under our specification the conditional mean of the compensation is $1/\lambda_{n,i} = \exp\{-x_{n,i}\beta\}$, so a negative coefficient implies a positive effect on the conditional mean.)

The patient’s age also has a significant impact on the total compensation awarded in the second (richer) specification. A likelihood ratio test for the joint significance of all three age-related coefficients yields a p-value of 0.002, thus offering strong evidence that age matters in determining the size of compensation. Furthermore, it is worth mentioning that the sign of the estimated age effects vary across malpractices with different severity. For cases with low and medium severity, a patient’s age is estimated to have a negative impact on the conditional mean of compensation (because $0.016 > 0$ and $0.016 - 0.003 > 0$); for case with high severity, the estimated effect is positive ($0.016 - 0.023 < 0$). This pattern may be due to the nature of the interaction between the patient’s age and the damage inflicted by malpractice. When the damage is moderate (severity=“low” or “medium”), the compensation that is required to keep the patients’ life-quality is likely to be proportional to the patient’s remaining life span. On the other hand, the seniority in age may aggravate severe damage (severity=“high”) much more than it does moderate damage. Thus costs for maintaining the life-quality (or even sustaining the life) of the patient when the severity is high could increase drastically with the patient’s age.

To better understand the magnitude of age, severity and income effects in terms of monetary units, we report estimates for the average marginal effects (AME) of severity, income and age on the total potential compensation in Table 3(b). The standard errors are calculated using a bootstrap resampling method. While reporting the AME of *Income* and *Age*, we condition on the severity of the malpractice because the latter is shown in Table 3(a) to be statistically significant in determining compensation.

Table 3(b). Average Marginal Effects on Potential Compensation
(units: one thousand US \$)

	Estimates	90% C.I.
<i>Severity</i>		
low to medium	237.108	[39.149, 342.710]
medium to high	228.637	[177.517, 626.792]
<i>Income</i>		
low sev.	3.046	[1.203, 14.433]
medium sev.	10.780	[3.640, 57.358]
high sev.	20.240	[7.331, 254.106]

The average marginal effect on the total potential compensation is estimated to be \$237,108 when the severity increases from low to medium, and \$228,637 when from medium to high. In both cases, the 90% confidence interval does not include zero, which suggests such increases in mean compensation are statistically significant. A \$1k-increment in the local income raises the average compensation by \$3,046 when the severity of malpractice is low; and raises it by \$10,780 (and \$20,240) if the severity is medium (and high). That such an increase is greater for more severe damage may be ascribed to the fact that more severe damage requires more resources for care, which cost more in counties with higher household income.

Our last step is to estimate the joint distribution of plaintiff and defendant beliefs. To do so, we plug in the logit estimates from specification (3) in Table 2 and the MLE estimates from the second specification in Table 3 (a) in our MSL estimator defined in (13). As explained in Section 4, we maintain the specification that $\mu_p = 1 - Y_1$ and $\mu_d = Y_1 + Y_2$, where Y_1, Y_2 are the first two components in the draw from a Dirichlet distribution with concentration parameters $(\alpha_1, \alpha_2, \alpha_3)$. We allow α_j 's to be different parameters across the classification of cases based on the severity and the board certification of the doctors. We define a period in the model as a quarter in the calendar year, and adopt a binomial specification for the distribution of the wait-time T . Specifically, we let $T \sim binomial(\bar{T}, p)$ where $\bar{T} \equiv 4$. We use a quarterly discount factor of 0.99 (which is consistent with a 4% annual inflation rate). While implementing MSL, we use $S = 2,000$ simulated draws from the standard uniform distribution to evaluate the integral in the likelihood for each observation.

Table 4. Estimates for Parameters in the Belief Distribution

		Board Certified: Yes		Board Certified: No	
		μ_p	μ_d	μ_p	μ_d
Sev. = low	mean	0.402	0.692	0.407	0.691
		[0.366, 0.480]	[0.607, 0.785]	[0.396, 0.507]	[0.666, 0.703]
	std.dev.	0.173	0.163	0.168	0.158
		[0.167, 0.249]	[0.144, 0.244]	[0.167, 0.181]	[0.157, 0.166]
skewness		0.251	-0.523	0.233	-0.509
		[0.059, 0.345]	[-0.867, -0.321]	[-0.017, 0.262]	[-0.564, -0.439]
correlation		-0.613		-0.607	
		[-0.642, -0.411]		[-0.646, -0.447]	
Sev. = med.	mean	0.431	0.671	0.542	0.549
		[0.427, 0.464]	[0.625, 0.677]	[0.512, 0.602]	[0.543, 0.600]
	std.dev.	0.172	0.163	0.183	0.183
		[0.169, 0.172]	[0.162, 0.165]	[0.179, 0.183]	[0.175, 0.185]
skewness		0.173	-0.452	-0.108	-0.128
		[0.088, 0.183]	[-0.468, -0.314]	[-0.274, -0.006]	[-0.266, -0.112]
correlation		-0.804		-0.833	
		[-0.846, -0.701]		[-0.845, -0.671]	
Sev. = high	mean	0.510	0.585	0.733	0.338
		[0.499, 0.522]	[0.571, 0.596]	[0.728, 0.746]	[0.334, 0.366]
	std.dev.	0.176	0.173	0.163	0.175
		[0.173, 0.177]	[0.171, 0.175]	[0.162, 0.166]	[0.173, 0.181]
skewness		-0.026	-0.216	-0.686	0.445
		[-0.056, 0.002]	[-0.246, -0.176]	[-0.746, -0.669]	[0.361, 0.457]
correlation		-0.825		-0.844	
		[-0.845, -0.714]		[-0.864, -0.783]	

For each severity level and certification status of doctors, Table 4 reports the point estimates and 90% confidence intervals for the mean, standard deviation, skewness and correlation of the beliefs of the doctor and the patients, which are calculated using the MSL estimates for $\hat{\alpha}_j$. The closed form for mappings from the concentration parameters α_j to the mean, standard deviation, skewness and the correlation of (μ_p, μ_d) is presented in the appendix. The confidence intervals reported in Table 4 are constructed using the empirical distribution of the respective estimates from $B = 200$ bootstrap samples. Figure 2 and Figure 3 in the appendix plots the estimated marginal and joint distribution of (μ_p, μ_d) .

The estimates in Table 4 demonstrate several informative patterns about how the joint distribution of plaintiff and defendant beliefs vary with case characteristics. First, the plaintiffs tend to be more optimistic about their chance of winning the lawsuit when the injuries

are more severe, regardless of the doctor’s certification status. This is evident from the pattern that the estimated mean of μ_p is greater for high severity cases, and that the estimated skewness of μ_p decreases (shifting more probability masses toward 1) as severity increases. Comparing confidence intervals for the mean and skewness of μ_p across severity levels, we can see that such differences in the patient’s belief are statistically significant at 10% or 15% level for most cases. The only exception is that this difference may be statistically insignificant as severity increase from low to medium when the doctor is board certified.

Second, defendants are less optimistic about the court decision as severity increases. Our estimates show the defendant belief μ_d becomes more positively skewed (shifting probability mass toward 0) with a lower mean when the injuries are more severe. Again, the differences in the mean and skewness of μ_d across classifications are mostly statistically significant, except for when the severity increases from low to medium and the doctor is board certified.

One explanation for the two patterns above could be the “sympathy factor” in the jury decision perceived by plaintiffs and defendants. That is, both parties might believe the jury is inclined to rule in favor of the patient out of sympathy if the severity of damage is high. Such an explanation is consistent with the earlier observation that the differences in beliefs are more pronounced when the severity reaches a high level as opposed to a medium level.

Third, for cases with medium or high severity, the plaintiffs tend to be significantly more optimistic (with beliefs μ_p more skewed to the left with a greater mean) when the doctors are not reported as board certified. However, for cases with low severity, the doctor’s certification does not seem to have any significant impact on the skewness the patient’s belief. In comparison, the distribution of defendant beliefs move in an opposite direction, with the doctors with board certification being more optimistic. This is probably because board certification serves as a professional endorsement of a doctor’s qualification and capabilities. Thus, both parties of the lawsuit may perceive board certification as a potential influence on the jury’s decision that is favorable to the defendant.

Last but not the least, the joint distribution of (μ_p, μ_d) is estimated to be significantly negatively correlated, regardless of the severity and the doctor’s qualification. The point estimates for the correlation is smaller for cases with lower severity, which conforms with the patterns mentioned above.

7 Policy Experiment

We use our estimates to predict the impact of a hypothetical tort reform that limits the liability of defendants. In particular, we consider a policy which imposes a binding cap on the maximum compensation payable by the defendant if the court rules in favor of the plaintiff. We allow these caps to vary with the reported severity of the injury arising out of medical malpractice. Specifically, they are set to the 75-th percentiles of the total compensation

reported for each severity level in the data. Using our estimates from Table 4, we calculate the counterfactual mean of accepted settlement offers (that is, the mean of $S \mid A = 1$) under the caps, and compare them with the reported empirical mean in the data. Table 5 reports the point estimates as well as 90% confidence intervals calculated using bootstrap resampling.

Table 5: Impact of compensation caps on mean accepted settlement offers
(units: one thousand US \$)

	Board Certified: Yes			Board Certified: No	
Low sev.	emp.	42.159	[38.166, 46.794]	41.203	[37.239, 45.749]
	c.f.	28.250	[21.118, 41.724]	27.522	[19.901, 34.190]
	diff.	13.909	[-1.280, 21.841]	13.680	[5.618, 22.195]
Med sev.	emp.	159.001	[151.180, 165.098]	129.145	[120.558, 137.454]
	c.f.	88.326	[73.371, 99.440]	117.121	[92.572, 132.746]
	diff.	70.675	[56.109, 88.235]	12.024	[-4.538, 36.286]
High sev.	emp.	263.059	[251.389, 277.277]	332.938	[312.392, 356.387]
	c.f.	199.944	[174.914, 226.242]	314.611	[260.384, 350.431]
	diff.	63.115	[32.691, 91.445]	18.327	[-28.745, 73.341]

Notes: “emp.” denotes the observed mean in the data; “c.f.” the counterfactual mean under the caps; and “diff.” equals “emp.” – “c.f.”

Table 5 shows that on average the binding caps could induce sizable reductions in the accepted settlement offers. For example, the point estimates suggest that when the severity level is low, imposing a cap on total potential compensation that equals the 75% empirical quantile of court-ruled compensation would lead to over thirty-percent reduction in the mean of accepted settlement offer (32.99% when the doctor is reported to be board-certified; and 33.20% when there is no reported board certification). On the other hand, there is also evidence that the impact of compensation caps (conditional on medium and high severity) interacts with the reported qualification of the doctors. For cases with medium severity, the reduction in mean accepted offer is 44.45% when the doctors is board certified, compared with 9.31% when there is not certification reported in the data. Likewise, when severity is high, the reduction is as high as 23.99% with board certification but only 5.5% when no certification is reported in data. The bootstrap confidence intervals suggest that the impact of the means are statistically insignificant for cases with medium and high severity without board certification information.

The heterogeneity in the estimated impact of compensation caps across severity and doctor qualification can be explained by the difference in the joint beliefs and the distribution

of compensation across these categories. Recall that the caps we consider condition on the severity levels but not the doctor’s qualification. If we ignore the difference in the joint belief of plaintiffs and defendants across categories, the impact of caps on the mean of accepted settlement should be greater when the distribution of total compensation has a thinner (far-stretched) tail beyond the binding cap (imposed at 75% empirical quantile). For instance, when there is no certification information, the reduction in the counterfactual mean is greater with medium severity than with low severity. This is partly attributed to the fact that the range of realized compensations in the data that are censored by the caps (which are imposed at the 75% quantile) is larger in the former category, as seen the right panels in Figure 1(b).

Moreover the difference in the joint beliefs also interacts with that in the compensation distribution to affect the proportion of reduction. To see this, note that as severity becomes high, the range of censored compensation is even greater than that under a medium severity category. Nevertheless, as our estimates in Table 4 suggest, the plaintiffs subject to high-severity damage are significantly more optimistic. This could in part explain why the reduction in the mean of accepted settlement offers is not as high as in the case with medium severity.

To further investigate the impact of caps on the distribution of S conditional on $A = 1$, we report in Table 6 the estimated quantiles of accepted settlement offers between two scenarios: one with a binding cap on total compensation; and the other with practically no binding cap (that is, as if the cap were set to maximum reported in the data).

Table 6: Impact of compensation caps on quantiles of settlement offers
(units: one thousand US \$)

		Board Certified: Yes		Board Certified: No	
Low sev.	25%.	0.138	[0.039, 0.456]	0.038	[-0.013, 0.330]
	50%	2.154	[1.072, 4.957]	1.650	[0.718, 4.010]
	75%	9.066	[-18.000, 13.867]	8.299	[-12.149, 11.319]
Med sev.	25%.	0.729	[0.353, 1.263]	0.863	[0.533, 1.562]
	50%	9.938	[7.750, 15.158]	13.565	[9.841, 20.102]
	75%	92.947	[-97.099, 44.544]	60.188	[48.003, 79.916]
High sev.	25%.	1.410	[0.525, 2.364]	2.135	[0.518, 3.318]
	50%	15.676	[11.699, 26.511]	22.250	[14.389, 33.895]
	75%	76.946	[54.004, 98.928]	108.094	[75.332, 154.836]

The estimates suggest the conditional quantiles of accepted settlement offers are lower when a binding cap is imposed. Thus these estimates conforms to the theoretical implication that under a binding cap the distribution of accepted settlement offers is stochastically lower

once a binding cap is imposed. Moreover, the difference in quantiles is statistically more significant for cases with greater severity. This is consistent with the pattern shown in Figure 1(b) that the range of compensation values censored by the caps is greater when severity is higher.

8 Concluding Remarks

A fundamental step in the empirical analyses of bargaining outcomes is to show how assumptions of a model warrant that its structural elements can be unambiguously recovered from the data. Merlo and Tang (2012) discuss the identification of stochastic sequential bargaining models under various scenarios of data availability. In the current paper, we have addressed the same identification question in a prototypical model of bargaining with optimism. We have shown that all structural elements of the model are identified nonparametrically under realistic data requirements.

Based on our identification results, we have proposed a feasible estimation procedure using maximum simulated likelihood, and applied it to a data set on medical malpractice lawsuits in Florida during the 1980s and 1990s. We have found that doctors' and patients' beliefs vary with case characteristics, such as the severity of the injury and the qualification of the doctor, even though these characteristics are statistically insignificant in explaining which party the court rules in favor of. Using our structural estimates, we have also quantified the reduction in settlement amounts that would result from a counterfactual cap on the total compensation for plaintiffs.

In our analysis, we have abstracted from considering the role played by legal costs in dispute resolution due to data limitations. In particular, our data only reports litigation costs for a relatively small fraction of defendants and does not contain any information on the litigation costs paid by plaintiffs.¹⁷ We plan to address the question of identification in a richer bargaining framework which explicitly incorporates legal costs under different data scenarios in future work.

¹⁷Sieg (2000) incorporates legal costs into his empirical analysis of medical malpractice litigation which uses the same data. He deals with the data limitation issue by adopting a multivariate normal parameterization and by matching implied and sample moments under an assumption of free entry of plaintiff lawyers. The argument is that with free entry of plaintiff lawyers, there is a long-run equilibrium in which the average cost for the plaintiff lawyers equals the expected revenue. We cannot adopt this approach in our analysis because our model maintains a different informational assumption than Sieg's. Specifically, we do not assume the defendant has complete knowledge about the degree of his/her liability. That is, unlike Sieg (2000), we do not assume the defendant knows the actual probability that the jury rules in his/her favor. Hence the "zero profit" argument based on rational expectations cannot be applied in our setting.

Appendix A: Figures

Figure 1(a): Distribution of transfers by plaintiff age

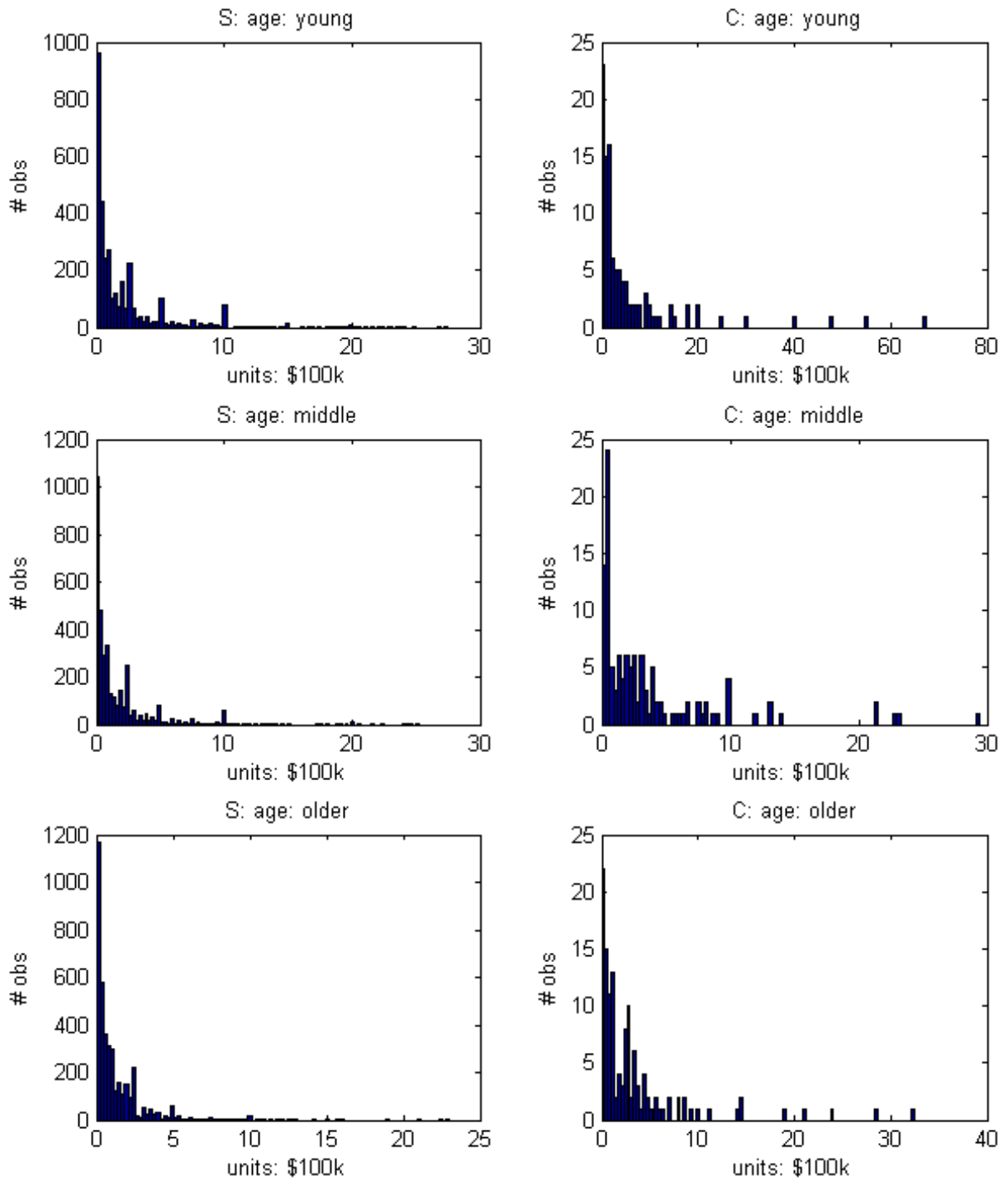


Figure 1(b): Distribution of transfers by severity

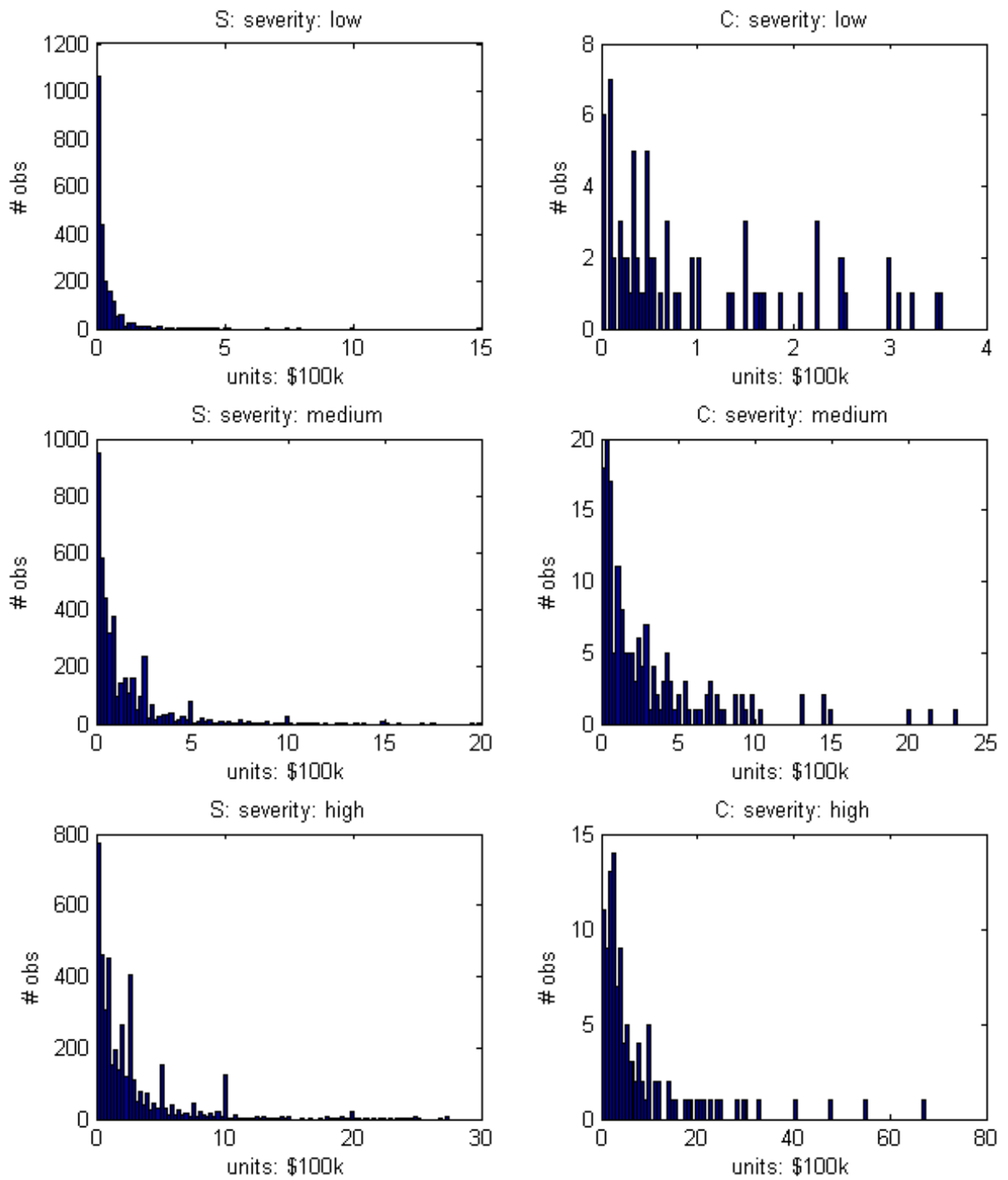
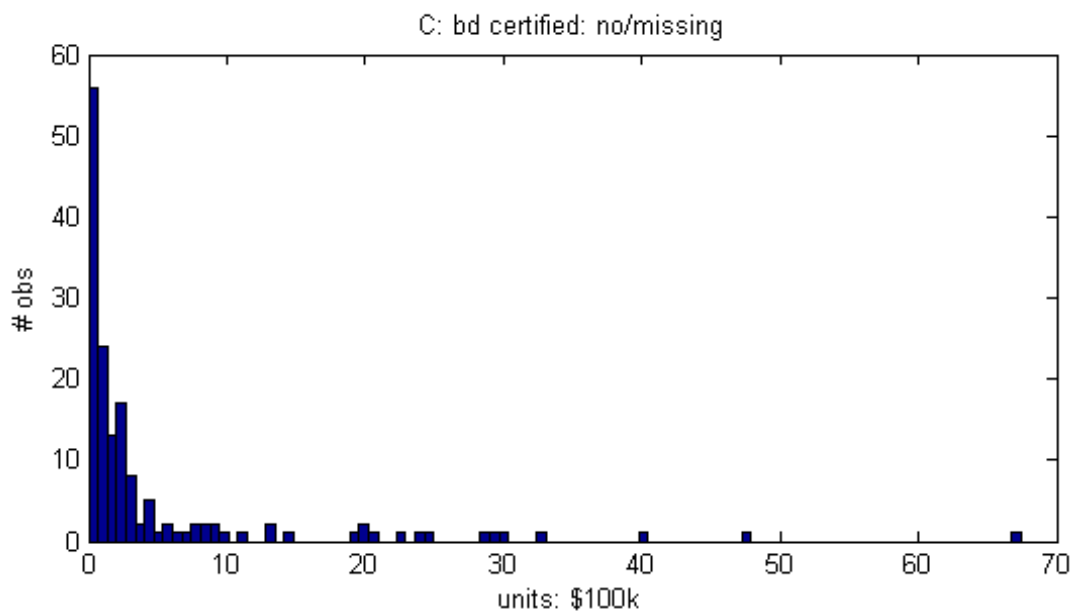
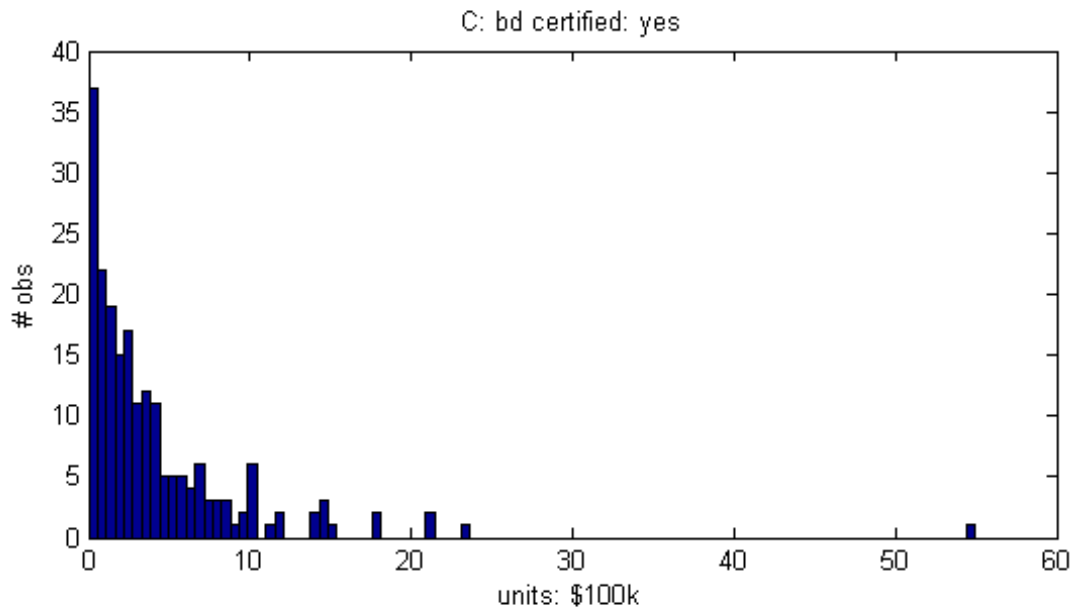
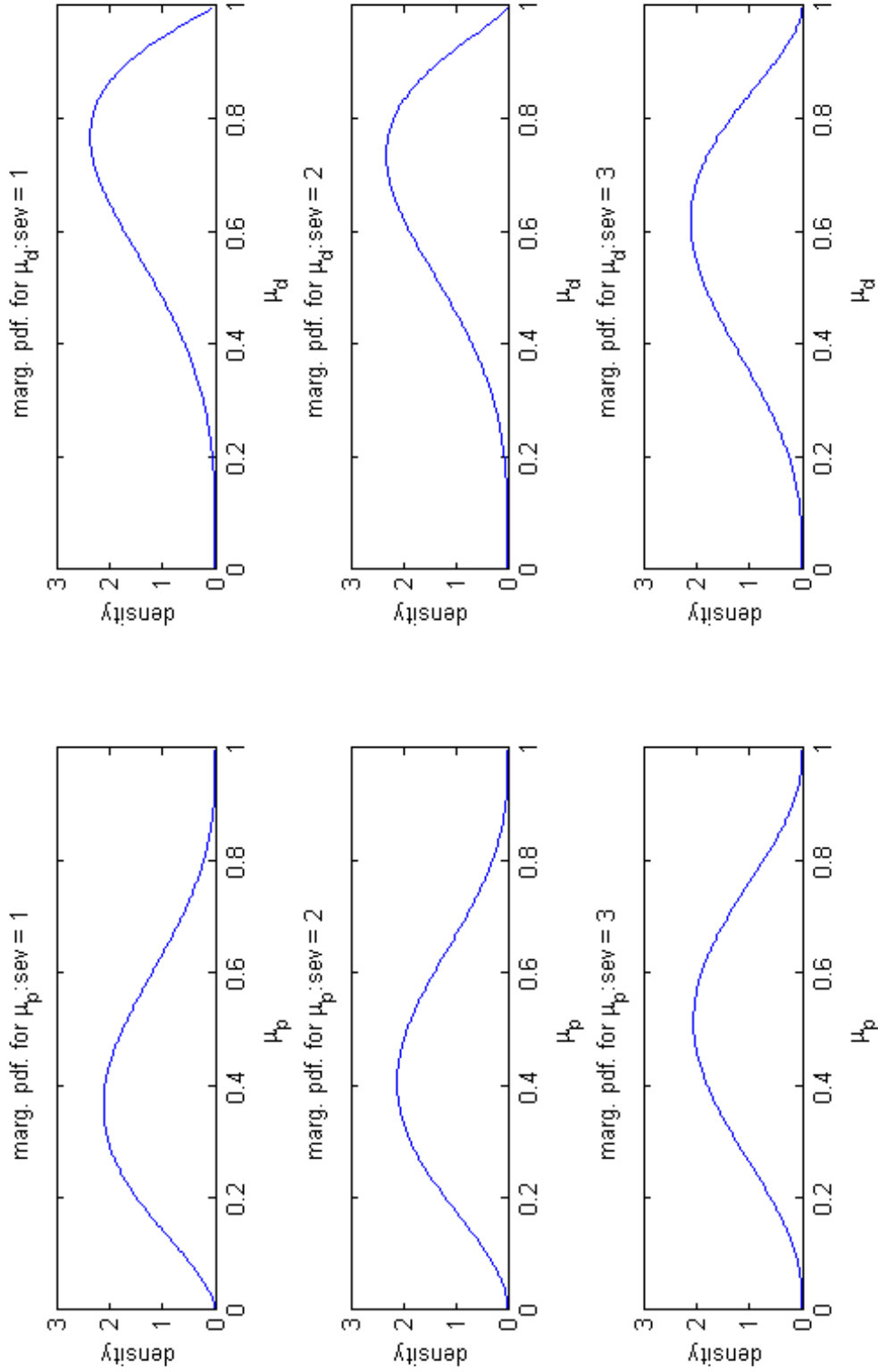


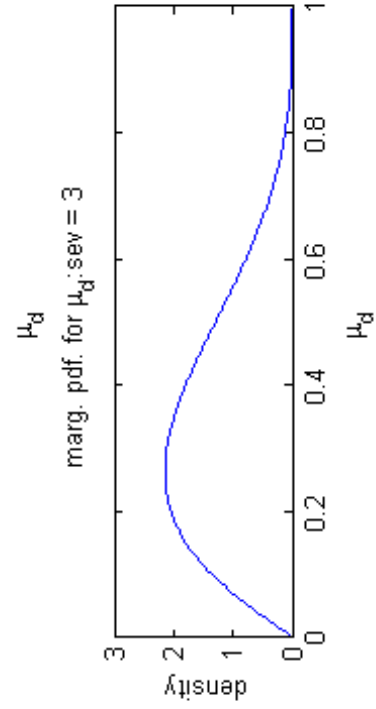
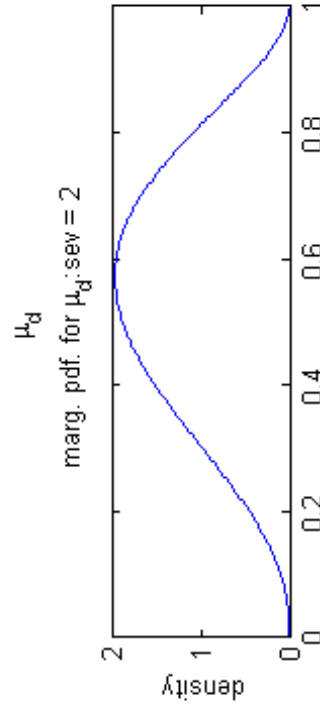
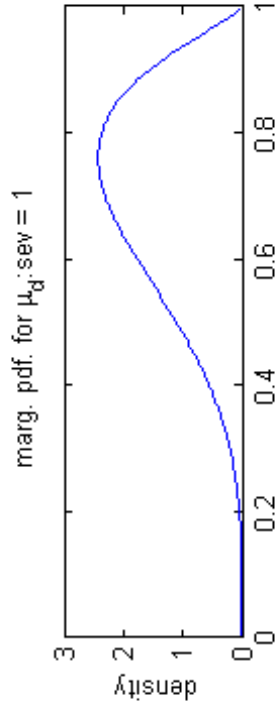
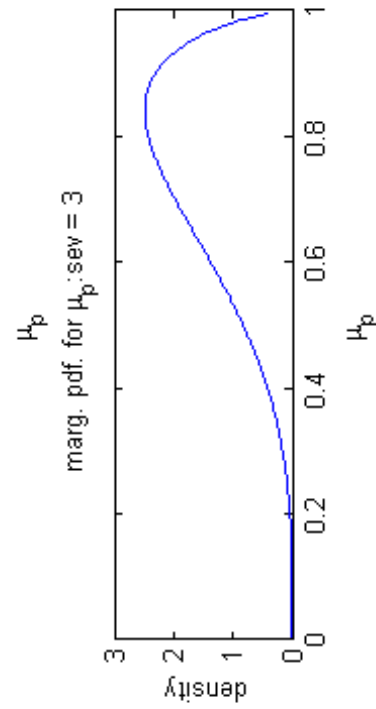
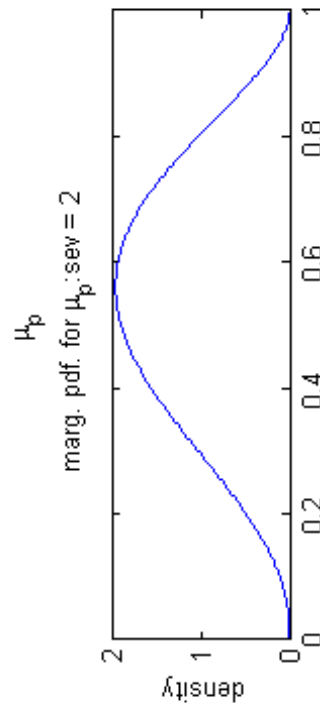
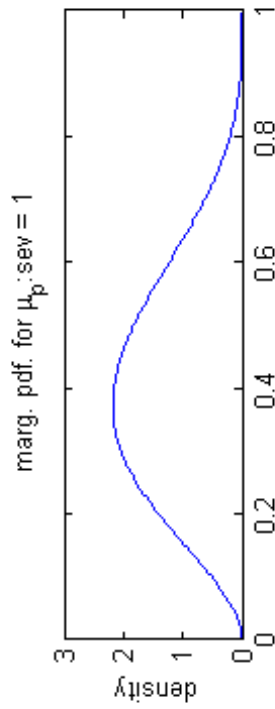
Figure 1(c): Distribution of transfers by certification



Estimated Marginal Belief Distributions : Certification = yes



Estimated Marginal Belief Distributions : Certification = no/missing



Appendix B: Proofs

Lemma B1 *Under Assumption 1, 2 and 3, there exists a partition $\mathcal{D}_{|\mathcal{T}|}$ such that L_{S_i, S_k} has full-rank $|\mathcal{T}|$.*

Proof of Lemma B1. By construction, the supremum and the infimum of the support of μ_p given $A = 1$ ($\mu_p + \mu_d \leq \delta^{-t}$) and $T = t$ are 1 and 0 respectively. It then follows from (ii) in Assumption 3 implies the supremum and the infimum of the support of $S = \delta^T \mu_p C$ conditional on “ $A = 1$ and $T = t$ ” are $\delta^t \times 1 \times \bar{c} = \delta^t \bar{c}$ and $\delta^t \times 0 \times 0 = 0$ respectively. For any $t \in \mathcal{T} \equiv \{1, 2, \dots, |\mathcal{T}|\}$, let $\bar{s}(t) \equiv \bar{c} \delta^t$ denote the supreme of the support of S conditional on “ $A = 1$ and $T = t$ ”. Let $\mathcal{D}_{|\mathcal{T}|}$ be a partition of the unconditional support of settlement offers \mathcal{S} into $|\mathcal{T}|$ intervals, which are characterized by the sequence of endpoints $\bar{s}(1) > \bar{s}(2) > \bar{s}(3) > \dots > \bar{s}(|\mathcal{T}|) > \bar{s}(|\mathcal{T}| + 1) \equiv 0$. (That is, the t -th smallest interval in $\mathcal{D}_{|\mathcal{T}|}$ is $[\bar{s}(|\mathcal{T}| - t + 2), \bar{s}(|\mathcal{T}| - t + 1)]$ for $t = 1, 2, \dots, |\mathcal{T}|$.) Because conditional on $A = 1$ and $T = t$ the settlement offer S is continuously distributed over $[0, \delta^t \bar{c}]$ with positive densities, the square matrix $L_{S_i|T}$ based on the partition $\mathcal{D}_{|\mathcal{T}|}$ must be triangular with full rank $|\mathcal{T}|$. Next, note that $L_{S_i, S_k} = L_{S_i|T} D_T (L_{S_k|T})'$ where $(L_{S_k|T})'$ is the transpose of $L_{S_k|T}$ and D_T is a diagonal matrix with diagonal entries being $[\Pr(T = t)]_{t=1, \dots, |\mathcal{T}|}$. Since $L_{S_k|T}$ has full rank by symmetric arguments and D_T is non-singular by construction, it then follows that L_{S_i, S_k} has full rank. \square

Proof of Lemma 1. Lemma B1 below shows that, under Assumptions 1, 2 and 3, there exists a partition $\mathcal{D}_{|\mathcal{T}|}$ such that L_{S_i, S_k} has full rank $|\mathcal{T}|$. This necessarily means $L_{S_i|T}$ must also be invertible (see proof of Lemma B1). Hence

$$L_{T, S_k} = (L_{S_i|T})^{-1} L_{S_i, S_k}. \quad (14)$$

Substituting (14) into (4) leads to

$$\begin{aligned} \Lambda_{S_i, S_k} &= L_{S_i|T} \Delta_j (L_{S_i|T})^{-1} L_{S_i, S_k} \\ \Leftrightarrow \Lambda_{S_i, S_k} (L_{S_i, S_k})^{-1} &= L_{S_i|T} \Delta_j (L_{S_i|T})^{-1}. \end{aligned} \quad (15)$$

Note the two matrices on the left-hand side of (15), L_{S_i, S_k} and Λ_{S_i, S_k} , are directly identifiable from data. The equation (15) suggests $\Lambda_{S_i, S_k} (L_{S_i, S_k})^{-1}$ admits an eigenvalue-eigenvector decomposition, with each eigenvalue being $\mathbb{E}(A_j | T = t)$ and corresponding eigenvector being $[\Pr(S_i \in d_m | A_i = 1, T = t)]_{m=1, \dots, |\mathcal{T}|}$ for any $t \in \mathcal{T}$. Because each eigenvector is a conditional probability mass function with entries summing up to 1, the scale in the decomposition is fixed implicitly. Furthermore, Assumption 1-(i) and Assumption 3-(i) imply that $\mathbb{E}[A_j | T = t]$ must be strictly decreasing in t over the support of wait-time between settlement conferences and court hearings \mathcal{T} . This rules out the possibility of duplicate eigenvalues in the decomposition, and also uniquely links the eigenvalues and eigenvectors

to each specific elements in \mathcal{T} . Thus $L_{S_i|T}$ and Δ_j are identified using the partition $D_{|\mathcal{T}|}$. It then follows that L_{T,S_k} is also identified from (14) once $L_{S_i|T}$ is identified.

It remains to show that $f_{S_i}(\cdot | A_i = 1, T = t)$ is identified over its full support. (Note $L_{S_i|T}$ identified above is only a discretized version of this conditional density.) For any $s \in \mathcal{S}$, define a $|\mathcal{T}|$ -vector l_{s,S_k} whose m -th coordinate is given by $f_{S_i}(s, S_k \in d_m | A_{i,k} = 1)$, where d_m is the m -th interval in the partition $D_{|\mathcal{T}|}$ used for identification of $L_{S_i|T}$ and Δ_j above. By construction,

$$l_{s,S_k} = (L_{T,S_k})' \lambda_s \quad (16)$$

where $(L_{T,S_k})'$ is the transpose of L_{T,S_k} ; and λ_s is a $|\mathcal{T}|$ -vector with the t -th coordinate being $f_{S_i}(s | A_i = 1, T = t)$.¹⁸ The coefficient matrix $(L_{T,S_k})'$ does not depend on the realization of $S_i = s$ while vectors λ_s and l_{s,S_k} both do. With L_{T,S_k} invertible and identified above and with l_{s,S_k} directly identifiable, λ_s is recovered as the unique solution of the linear system in (16) for any $s \in \mathcal{S}$. \square

Proof of Proposition 1. By definition, the function $\varphi_t(s)$ on the L.H.S. of (10) is directly identifiable for all s, t . For any given $t \in \mathcal{T}$, suppose there exists $\tilde{h}_t \neq h_t$ such that $\varphi_t(s) = \int_0^\infty \tilde{h}_t(v) \kappa(v, s) dv$ for all $s \in \mathcal{S}_t$. Then (10) implies:

$$\int_0^\infty [\tilde{h}_t(v) - h_t(v)] \kappa(v, s) dv = 0$$

for all $s \in \mathcal{S}_t$. It then follows from Assumption 5 that $\tilde{h}_t(v) = h_t(v)$ almost everywhere $F_{V|t}$ for such t . This establishes the identification of $h_t(v)$ for any $t \in \mathcal{T}$ and $v \in \mathcal{V}_t$. Thus $\Pr(\frac{1}{\mu_p} \geq b, \frac{1}{\mu_p + \mu_d} \geq \delta^t)$ is identified for all $t \in \mathcal{T}$ and $b \in [1, +\infty)$, i.e. the support of μ_p^{-1} given $\mu_p + \mu_d \leq \delta^{-t}$. (To see this, note that for any t , the support of $V\delta^t$ given $\mu_p + \mu_d \leq \delta^{-t}$ is by construction identical to that of μ_p^{-1} given $(\mu_p + \mu_d)^{-1} \geq \delta^t$.) It then follows that $\Pr(\mu_p \leq \mu, \mu_p + \mu_d \leq \delta^{-t})$ is identified for all $t \in \mathcal{T}$ and $\mu \in (0, 1)$ using Jacobian transformation. \square

¹⁸To see this, note for any s and d_m , the Law of Total Probability implies $f_{S_i}(s, S_k \in d_m | A_{i,k} = 1)$ can be written as:

$$\begin{aligned} & \sum_{t \in \mathcal{T}} f_{S_i}(s | T = t, S_k \in d_m, A_{i,k} = 1) \Pr(T = t, S_k \in d_m | A_{i,k} = 1) \\ &= \sum_{t \in \mathcal{T}} f_{S_i}(s | A_i = 1, T = t) \Pr(T = t, S_k \in d_m | A_{i,k} = 1) \end{aligned}$$

where the equality follows from Assumption 1.

Appendix C: Monte Carlo Study

In this section we present some evidence of the finite-sample performance of the Maximum Simulated Likelihood estimator proposed in Section 4. For the sake of simplicity, we focus on a simple design where cases are homogenous. Let the data-generating process be defined as follows. Let $R \in \{r \in [0, 1]^3 : r_1 + r_2 + r_3 = 1\}$ follow a Dirichlet distribution with concentration parameters $(\alpha_1, \alpha_2, \alpha_3)$. Let $\mu_p = 1 - R_1$ and $\mu_d = R_2 - (1 - R_1) + 1 = R_1 + R_2$. By construction, the support of (μ_p, μ_d) is $\{(\mu, \mu') \in [0, 1]^2 : 1 \leq \mu + \mu' \leq 2\}$, which is consistent with our model of bargaining with optimism. The marginal distribution of μ_p is $Beta(\alpha_2 + \alpha_3, \alpha_1)$ (because the marginal distribution of R_1 is $Beta(\alpha_1, \alpha_2 + \alpha_3)$); and the marginal distribution of μ_d is $Beta(\alpha_1 + \alpha_2, \alpha_3)$.¹⁹ Let $\alpha_0 \equiv \alpha_1 + \alpha_2 + \alpha_3$. Table C1 below summarizes the relation how the concentration parameters determine the key features of the distribution of (μ_p, μ_d) :

Table C1: Summary of the Joint Distribution of (μ_p, μ_d)

	μ_p	μ_d
Marg. distr.	$Beta(\alpha_2 + \alpha_3, \alpha_1)$	$Beta(\alpha_1 + \alpha_2, \alpha_3)$
Mean	$\frac{\alpha_2 + \alpha_3}{\alpha_0}$	$\frac{\alpha_1 + \alpha_2}{\alpha_0}$
Variance	$\frac{\alpha_1(\alpha_2 + \alpha_3)}{\alpha_0^2(\alpha_0 + 1)}$	$\frac{\alpha_3(\alpha_1 + \alpha_2)}{\alpha_0^2(\alpha_0 + 1)}$
Skewness	$\frac{2(\alpha_1 - \alpha_2 - \alpha_3)\sqrt{\alpha_0 + 1}}{(\alpha_0 + 2)\sqrt{\alpha_1(\alpha_2 + \alpha_3)}}$	$\frac{2(\alpha_3 - \alpha_1 - \alpha_2)\sqrt{\alpha_0 + 1}}{(\alpha_0 + 2)\sqrt{\alpha_3(\alpha_1 + \alpha_2)}}$
Mode (marginal)	$\frac{\alpha_2 + \alpha_3 - 1}{\alpha_0 - 2}$	$\frac{\alpha_1 + \alpha_2 - 1}{\alpha_0 - 2}$
Correlation	$-\frac{\sqrt{\alpha_1 \alpha_3}}{\sqrt{(\alpha_2 + \alpha_3)(\alpha_1 + \alpha_2)}}$	

We also use the simple design where the distribution of the cake: $gamma(1, 1)$. $\Pr(D = 1) = 0.05$ and that the distribution of wait-time T is Binomial with parameters 5 and 0.4. The results are reported in the following tables:

¹⁹The covariance between μ_p and μ_d is given by: $Cov(\mu_p, \mu_d) = Cov(1 - R_1, R_1 + R_2) = Var(R_1) - Cov(R_1, R_2)$, which is used to calculate the expression reported in Table C1.

Table C2: Results for DGP # 1: $\alpha \equiv [1.25, 1.50, 2.70]$

DGP1		mean	std. dev	l.q.	med.	h.q.	r.m.s.e.	m.a.e.
$N = 200$	α_1	1.580	0.864	0.945	1.456	1.996	0.923	0.467
	α_2	1.635	0.860	1.155	1.401	1.801	0.868	0.328
	α_3	3.090	1.334	2.157	2.841	3.734	1.387	0.723
	λ	0.385	0.213	0.208	0.343	0.539	0.213	0.172
$N = 400$	α_1	1.424	0.599	0.998	1.350	1.705	0.623	0.350
	α_2	1.482	0.489	1.131	1.403	1.711	0.489	0.312
	α_3	2.757	0.960	2.149	2.645	3.224	0.960	0.541
	λ	0.378	0.175	0.232	0.362	0.517	0.176	0.149
$N = 800$	α_1	1.291	0.431	0.999	1.210	1.495	0.431	0.246
	α_2	1.330	0.318	1.126	1.275	1.494	0.359	0.266
	α_3	2.489	0.574	2.127	2.439	2.920	0.609	0.436
	λ	0.355	0.135	0.256	0.324	0.453	0.142	0.111

Table C3: Results for DGP # 2: $\alpha = [3.60, 2.00, 1.40]$

DGP1		mean	std. dev	l.q.	med.	h.q.	r.m.s.e.	m.a.e.
$N = 200$	α_1	5.014	2.406	3.503	4.465	5.798	2.785	1.187
	α_2	1.958	0.655	1.515	1.832	2.274	0.655	0.443
	α_3	2.056	0.916	1.460	1.828	2.290	1.124	0.432
	λ	0.263	0.161	0.142	0.220	0.351	0.211	0.199
$N = 400$	α_1	4.562	1.496	3.428	4.248	5.400	1.775	0.788
	α_2	1.727	0.363	1.481	1.672	1.921	0.454	0.374
	α_3	1.871	0.563	1.474	1.730	2.154	0.733	0.342
	λ	0.230	0.105	0.165	0.204	0.277	0.200	0.197
$N = 800$	α_1	4.383	0.964	3.644	4.273	4.964	1.239	0.743
	α_2	1.695	0.259	1.533	1.696	1.798	0.400	0.333
	α_3	1.828	0.378	1.559	1.759	2.087	0.570	0.359
	λ	0.231	0.089	0.168	0.216	0.273	0.191	0.184

Appendix D: Calculating Counterfactuals

In this appendix we provide further details for calculating the counterfactual mean settlement offer under compensation caps considered in Section 7. First, we calculate the counterfactual density of $S \mid A = 1$ under the proposed caps on C . Let x denote the variables that affect the distribution of C (that is, age, income and severity, which is suppressed in the notation below). Let $f(\cdot \mid x)$ denote the conditional density of C ; h denote the probability mass for wait-time T (which is orthogonal to μ, C). By construction,

$$\frac{\partial}{\partial s} \Pr\{S \leq s \mid A = 1, X = x\} = \frac{\frac{\partial}{\partial s} \Pr\{S \leq s, A=1 \mid X=x\}}{\Pr\{A=1 \mid X=x\}} \quad (17)$$

where the denominator is

$$\sum_t \Pr\left\{\mu_p + \mu_d - 1 \leq \frac{1-\delta^t}{\delta^t}\right\} h(t)$$

due to independence between T and (μ, C) ; and the numerator is:

$$\begin{aligned} & \frac{\partial}{\partial s} \sum_t \left[\int_0^{\hat{c}} \Pr\left(\mu_p \leq \frac{s}{c\delta^t}, \mu_p + \mu_d \leq \frac{1}{\delta^t}\right) f(c \mid x) dc \right] h(t) \\ &= \sum_t \left[\frac{\partial}{\partial s} \int_0^{\hat{c}} \Pr\left(Y_1 \geq 1 - \frac{s}{c\delta^t}, Y_2 \leq \frac{1-\delta^t}{\delta^t}\right) f(c \mid x) dc \right] h(t) \end{aligned}$$

where $Y_1 \equiv 1 - \mu_p$ and $Y_2 = \mu_p + \mu_d - 1$. Assuming the order of integration and differentiation can be changed and using the fact that “ $1 - s\delta^{-t}/c \in (0, 1) \Leftrightarrow c \in (\frac{s}{\delta^t}, +\infty)$ ”, we can write the term in the square brackets on the right-hand side for each t as

$$\begin{aligned} & \int_{s\delta^{-t}}^{\infty} \Pr\left\{Y_2 \leq \frac{1-\delta^t}{\delta^t} \mid Y_1 = 1 - s\delta^{-t}/c\right\} f_{Y_1}(1 - s\delta^{-t}/c) \frac{f_C(c \mid x)}{c\delta^t} dc \\ &= \int_0^1 \Pr\left\{\frac{Y_2}{1-\tau} \leq \frac{1-\delta^t}{\delta^t(1-\tau)} \mid Y_1 = \tau\right\} \frac{f_{Y_1}(\tau) f_C\left(\frac{s}{\delta^t(1-\tau)} \mid x\right)}{\delta^t(1-\tau)} d\tau \end{aligned}$$

where the equality follows from changing variables between c and $\tau \equiv 1 - \frac{s}{c\delta^t}$.

Suppose we put a cap \hat{c} on total compensation, then the expression in the numerator on the right-hand side of (17) becomes

$$\frac{\partial}{\partial s} \sum_t \left[\frac{\int_0^{\hat{c}} \Pr\left(\mu_p \leq \frac{s}{c\delta^t}, \mu_p + \mu_d \leq \frac{1}{\delta^t}\right) f(c \mid x) dc + \Pr\{C \geq \hat{c} \mid x\} \Pr\left(\mu_p \leq \frac{s}{\hat{c}\delta^t}, \mu_p + \mu_d \leq \frac{1}{\delta^t}\right)}{\Pr\{A=1 \mid X=x\}} \right] h(t).$$

By construction, $\frac{\partial}{\partial s} \Pr\left(\mu_p \leq \frac{s}{\hat{c}\delta^t}, \mu_p + \mu_d \leq \frac{1}{\delta^t}\right)$ is

$$\frac{\partial}{\partial s} \Pr\left(Y_1 \geq 1 - \frac{s}{\hat{c}\delta^t}, Y_2 \leq \frac{1-\delta^t}{\delta^t}\right) = \Pr\left(Y_2 \leq \frac{1-\delta^t}{\delta^t} \mid Y_1 = 1 - \frac{s}{\hat{c}\delta^t}\right) \frac{f_{Y_1}(1 - s\delta^{-t}/\hat{c})}{\hat{c}\delta^t}.$$

A similar expression exists with \hat{c} replaced by c . Given our estimates for the distribution of C and (Y_1, Y_2) from Section 4, we calculate the counterfactual mean of settlement offers using these formulas and the simulation-based integration. The estimated counterfactual means and their differences with empirical means in the data are reported in Table 5.

Next, we explain how to calculate the distribution of $S \mid A = 1$ under counterfactual caps on C . Recall by construction

$$\Pr\{S \leq s \mid A = 1, X = x\} = \frac{\Pr\{S \leq s, A=1 \mid X=x\}}{\Pr\{A=1 \mid X=x\}}$$

where the denominator is calculated as before. The numerator under a cap \hat{c} is

$$\sum_t \left[\int_0^{\hat{c}} \Pr(\mu_p \leq \frac{s}{c\delta^t}, \mu_p + \mu_d \leq \frac{1}{\delta^t}) f(c | x) dc + \Pr(C \geq \hat{c} | x) \Pr(\mu_p \leq \frac{s}{\hat{c}\delta^t}, \mu_p + \mu_d \leq \frac{1}{\delta^t}) \right] h(t). \quad (18)$$

To calculate the term in the square brackets, we need to consider two cases.

Case 1: $\hat{c} \leq s\delta^{-t}$. Then $\frac{s}{\hat{c}\delta^t} \geq 1$ and

$$\Pr(\mu_p \leq \frac{s}{\hat{c}\delta^t}, \mu_p + \mu_d \leq \frac{1}{\delta^t}) = \Pr\left(Y_2 \leq \frac{1-\delta^t}{\delta^t}\right).$$

Besides,

$$\begin{aligned} \int_0^{\hat{c}} \Pr(\mu_p \leq \frac{s}{c\delta^t}, \mu_p + \mu_d \leq \frac{1}{\delta^t}) f(c | x) dc &= \int_0^{\hat{c}} \Pr\left(Y_1 \geq 1 - \frac{s}{c\delta^t}, Y_2 \leq \frac{1-\delta^t}{\delta^t}\right) f(c | x) dc \\ &= \Pr\left(Y_2 \leq \frac{1-\delta^t}{\delta^t}\right) \int_0^{\hat{c}} f(c | x) dc = \Pr\left(Y_2 \leq \frac{1-\delta^t}{\delta^t}\right) \Pr(C \leq \hat{c} | x) \end{aligned}$$

where the second equality holds because $\hat{c} \leq s\delta^{-t}$ implies $1 - \frac{s}{c\delta^t} < 0$ for all $c \leq \hat{c}$. Therefore the square bracket in (18) is

$$\begin{aligned} &\Pr\left(Y_2 \leq \frac{1-\delta^t}{\delta^t}\right) \Pr(C \leq \hat{c} | x) + \Pr(C \geq \hat{c} | x) \Pr\left(Y_2 \leq \frac{1-\delta^t}{\delta^t}\right) \\ &= \Pr\left(Y_2 \leq \frac{1-\delta^t}{\delta^t}\right) \end{aligned}$$

Case 2: $\hat{c} > s\delta^{-t}$. Then $\frac{s}{\hat{c}\delta^t} < 1$ and

$$\begin{aligned} &\int_0^{\hat{c}} \Pr(\mu_p \leq \frac{s}{c\delta^t}, \mu_p + \mu_d \leq \frac{1}{\delta^t}) f_C(c | x) dc \\ &= \int_{s\delta^{-t}}^{\hat{c}} \Pr\left(Y_1 \geq 1 - \frac{s}{c\delta^t}, Y_2 \leq \frac{1-\delta^t}{\delta^t}\right) f_C(c | x) dc + \Pr\left(Y_2 \leq \frac{1-\delta^t}{\delta^t}\right) \Pr(C \leq s\delta^{-t} | x) \\ &= \int_0^{1-\frac{s}{\hat{c}\delta^t}} \Pr\left(Y_1 \geq \tau, Y_2 \leq \frac{1-\delta^t}{\delta^t}\right) f_C\left(\frac{s}{\delta^t(1-\tau)} \middle| x\right) \frac{s}{(1-\tau)^2\delta^t} d\tau + \Pr\left(Y_2 \leq \frac{1-\delta^t}{\delta^t}\right) \Pr(C \leq s\delta^{-t} | x) \end{aligned}$$

where the first equality uses the fact that $1 - \frac{s}{c\delta^t} < 0$ if $c > s\delta^{-t}$ and the second uses the change of variables between c and $\tau \equiv 1 - \frac{s}{c\delta^t}$. Furthermore

$$\Pr\left(Y_1 \geq \tau, Y_2 \leq \frac{1-\delta^t}{\delta^t}\right) = \int_{\tau}^1 \Pr\left(Y_2 \leq \frac{1-\delta^t}{\delta^t} \middle| Y_1 = \zeta\right) f_{Y_1}(\zeta) d\zeta.$$

Hence the square bracket in (18) is

$$\begin{aligned} &\int_0^{1-\frac{s}{\hat{c}\delta^t}} \Pr\left(Y_1 \geq \tau, Y_2 \leq \frac{1-\delta^t}{\delta^t}\right) f_C\left(\frac{s}{\delta^t(1-\tau)} \middle| x\right) \frac{s}{(1-\tau)^2\delta^t} d\tau \\ &+ \Pr\left(Y_2 \leq \frac{1-\delta^t}{\delta^t}\right) \Pr(C \leq s\delta^{-t} | x) + \Pr(C \geq \hat{c} | x) \Pr\left(Y_1 \geq 1 - \frac{s}{\hat{c}\delta^t}, Y_2 \leq \frac{1-\delta^t}{\delta^t}\right). \end{aligned}$$

Again given our estimates for the distribution of C and (Y_1, Y_2) from Section 4, we can calculate the counterfactual distribution of settlement offers using these formulas and simulation-based integration. These estimated counterfactual distributions of $S | A = 1$ are then inverted to estimate counterfactual quantiles, whose differences with the empirical quantiles are reported in Table 6.

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