Cheap Talk With Two-Sided Private Information

Job Market Paper

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Abstract

I investigate the strategic interaction between an informed expert and a decision maker when the latter has imperfect private information relevant to the decision. To analyse the effect of the decision maker's information, I extend the Crawford and Sobel (1982) canonical model of cheap talk by allowing the decision maker to access an unbiased and symmetric signal about the state of the world. I first show that, for symmetric preferences, partition equilibria exist in this more general environment. Second, for quadratic-loss preferences, I show that access to private information might reduce the informativeness of the partition used by the expert. Surprisingly, in a wide range of environments, the decision maker's private information cannot make up for the loss in communication implying that the welfare of both agents decreases.

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1 Introduction

Decision makers often seek advice from better informed experts before making a decision. Examples range from management consulting to political, financial and medical advice[1]. Frequently, the interests of the expert are not perfectly aligned with those of the decision maker and this creates an incentive for the expert to manipulate his information. Crawford and Sobel (1982)[2] (CS henceforth) studied the strategic information transmission between a biased expert (he) and an uninformed decision maker (she) when contracts or other commitment devices are not available. They showed that only coarse information can be transmitted in equilibrium, even though, when the divergence of preferences is small, the expert might prefer to truthfully reveal his information rather than to provide coarse information. The problem is that the expert cannot credibly submit more precise information, because if he were trusted, he would have an incentive to lie.

A natural reaction from the decision maker to this poor information transmission, would be to acquire some information by herself, in addition to consulting the expert. I argue that the decision maker should be cautious before taking such a move. In fact I show that, when the information structure satisfies certain conditions, the presence of an informative signal hampers communication between the agents and as a result, in a wide range of environments, the decision maker would be better off by not acquiring extra information.

To gain some intuition for the results, consider a decision maker who wants to choose an action (in $\mathbb{R}$) to match an unknown state of the world. For simplicity, suppose that the state of the world, θ, takes one of the these values $\{0, \frac{1}{2}, 1\}$ with equal probability. The decision maker consults an expert that perfectly knows the true state of the world, but who would like a higher action to be implemented. For instance, suppose that the expert always wants the decision maker to choose the action $y = \theta + \frac{1}{3}$, where $\frac{1}{3}$ represents the bias of the expert. Along the lines of CS, full revelation is not possible. In the most informative equilibrium[3]

[1]Cheap talk games have been applied to study communication in a wide variety of areas. See Morgan and Stocken (2003) for an application to finance, Gilligan and Krehbiel (1989); Austen-Smith (1993); Krishna and Morgan (2001b) and Morgan and Stocken (2008) for applications to political science and Galeotti et al. (2009) for an application to organisation design and sociology.

[2]Green and Stokey (2007), which circulated in 1981, also study the information transmission between two agents. They analysed the welfare implications of improving the information available to the expert.

[3]There is an issue of multiplicity of equilibria in cheap talk games. In particular there is always a babbling

the expert reveals the lowest state of the world and pools the two higher states. For the case of quadratic-loss preferences[4], the ex-ante expected utility of an uninformed decision maker is $EU^D = -\frac{1}{24}$. Suppose now that the decision maker has access to an informative signal, s, that takes values in $\{0, \frac{1}{2}, 1\}$ with the following probability matrix:

$$P = \begin{array}{c} s \setminus \theta \\ 0 \\ \frac{1}{2} \\ 1 \end{array} \begin{array}{c} \begin{array}{ccc} 0 & \frac{1}{2} & 1 \end{array} \\ \begin{pmatrix} 0.7 & 0.15 & 0 \\ 0.3 & 0.7 & 0.3 \\ 0 & 0.15 & 0.7 \end{pmatrix} \end{array}$$

where $p_{s\theta} = Prob(s|\theta)$. Given the signal structure, the expert can no longer credibly separate the lowest state from the other two. The reason is that when he observes $\theta = 0$, he knows that the decision maker will receive the signal $s = 0$ with high probability. If he lies and reports that $\theta \in \{\frac{1}{2}, 1\}$, with probability 0.7 the decision maker will choose $y = 0.5$ and with probability 0.3 she will choose $y = \frac{13}{20}$, leading to an expected utility to the expert of $-\frac{1783}{36000} \simeq -0.0495$, which is higher than the expected utility he would have if he truthfully revealed $\theta = 0$ (in that case the utility for the expert is $-\frac{1}{9} \simeq -0.1111$). Therefore, the introduction of the private information prevents the expert from revealing any information at all. Moreover, the ex-ante utility of the decision maker when she has access to the signal (and hence does not receive informative messages from the expert) is $EU^D = -\frac{6}{85}$ which is lower than what she had in the uninformed case.

This example shows that allowing the decision maker to have access to a private signal deteriorates the incentives of the expert to reveal information because he knows that the signal will shift the decision maker's action towards the true state of the world, making exaggeration more attractive. To generalise this intuition, I consider the CS model with a continuum of states and allow the decision maker to access a continuous signal distributed symmetrically around the state of the world prior to making her decision. The main contributions of the

equilibrium. CS showed that the most informative equilibrium is preferred by both agents to any other equilibrium and hence I focus on that one. For refinements of equilibria in cheap talk games see Matthews et al. (1991); Farrel (1993); Rabin (1990) and Chen et al. (2008) among others.

[4] Quadratic-loss preferences are given by $u(y, p) = -(y - p)^2$ where p represents the peak of the preferences which is θ for the decision maker and $\theta + b$ for the expert.

paper are as follows.

First, for general symmetric preferences, I show the existence of partition equilibria similar to those characterised by CS and extend the properties of the CS equilibria to this setup.

Second, for the quadratic-loss preferences case, I decompose the impact of private information on the expert's incentives to communicate into two opposing effects. On the one hand, there is an *information effect* that arises because more information allows the decision maker to choose better actions on average. The information effect is stronger in noisier messages because the decision maker weighs more her private signal if she has a less precise distribution over the states of the world. As a result, the shift in the expected action towards the true state of the world is larger if the message sent is a larger (and thus noisier) interval. This creates an incentive to exaggerate even more, leading to less communication transmission. On the other hand, there is a *risk effect* that occurs because the expert is no longer certain of how the decision maker will react to his messages. Since the expert is a risk averse agent, he prefers less noisy messages that lead to less noisy action lotteries, and this favours communication.

Third, I show that in some environments, the information effect dominates the risk effect, reducing the communication in equilibrium. I illustrate this result for two different models: the Uniform private information model and the Normal private information model, where I derive some comparative statics with respect to the accuracy of the signals: communication decreases with the accuracy of the signal.

Finally, I show through the uniform and normal models, that the acquisition of private information might lead to a welfare decrease for both agents, and hence in those cases the decision maker should abstain from acquiring information.

The rest of the paper is organised as follows. In Section 2, I discuss the related literature. In Section 3, I state the model and show the existence of the partition equilibria. In Section 4, I analyse the communication incentives and illustrate the welfare implications for the Uniform and Normal models. In Section 5, I relax some assumptions of the the model and discuss the implications on the results and finally in Section 6 I conclude.

2 Literature Review

Only a few papers have studied information transmission when the decision maker is privately informed. Two early references are Seidmann (1990) and Olszewski (2004). They show different ways in which private information might facilitate communication. In Seidmann (1990) the different types of experts share the same preferences over actions but differ in their preferences over lotteries. By introducing private information to the decision maker, the experts can be partially ranked whereas no information can ever be revealed in the uninformed case. Olszewski (2004) introduces private information alongside two kind of experts; sincere non-strategic experts and experts that are exclusively concerned with being perceived as honest. He shows that full revelation is the unique equilibrium because the decision maker can use her private information to cross-check the expert's statements.

My paper is most related to Chen (2009) and Lai (2010). Both of these papers introduce information to the decision maker within the standard framework of CS. Chen (2009) studies two-sided cheap talk and finds that the decision maker cannot elicit more information from the expert by communicating to him first. The present paper differs from hers in the question rather than in the structure. She compares the equilibria that arise if the decision maker's signal was made public prior to the communication stage, whereas I compare the equilibrium of the model with private information to the equilibrium of the uninformed case. Lai (2010) studies communication between an expert and an *amateur* who knows whether the state of the world is below or above a cutoff point that is her private information. As in the present paper, he finds that the expert in the *amateur* model is less willing to provide information. However, the decision maker always ex-ante benefits from having access to the extra information. The setup of my paper allows for more flexible signal structures. In particular I am able to explore the communication as the signals smoothly become more precise and I find that in some cases having access to information decreases the ex-ante welfare of the decision maker.[5]

Also related to this paper are models that introduce multiple experts because each expert represents a different source of information to the decision maker. Austen-Smith (1993)

[5] Another recent paper, Ishida and Shimizu (2010), analyses communication when both, the expert and the decision maker, have discrete imperfect signals about a binary state of the world. They show that when the two agents are equally informed, no information can be revealed in equilibrium for arbitrarily small biases.

analyses the case of an uninformed House that refers legislation to a two expert's committees (that are imperfectly informed) under open rule. He finds that any single committee is willing to provide more information under single referral than multiple referral. However, the information content of multiple referral is superior to single referral. In Krishna and Morgan (2001a) a decision maker can sequentially consult experts with different biases. They find that if the experts have likewise biases the decision maker cannot do better than ignoring the messages of the most biased expert. Galeotti et al. (2009) study communication across a network where all the agents are at the same time senders and receivers. They find that the willingness of a player to communicate with a neighbour decreases with the number of opponents that communicates to that neighbour. Although their setting is completely different from the setup of this paper, the force driving their *congestion effect* is the same force that drives my results, namely, the sensitivity of the decision maker to a signal decreases with the precision of her prior. In all these papers there is an equilibrium in which the decision maker ignores the report of all except one expert, and as a result, consulting multiple experts cannot be detrimental. By contrast, in the set up of this paper, it is never rational for the decision maker to ignore her signal, and hence the welfare implications can be negative.

Finally three papers study the role of uncertainty (to both agents[6]) on the incentives to communicate. Krishna and Morgan (2004) introduce a jointly controlled lottery together with multiple rounds of communication in the CS framework and show that the resulting equilibria Pareto dominates those of the original model. Blume et al. (2007) introduce error in the message transmission. They show that adding noise to the model almost always leads to a Pareto improvement. Goltsman et al. (2009) study optimal mediation in communication games. They find that mediators should optimally introduce noise in their reports because this eases the incentive compatibility constraints of the expert. In all these papers the uncertainty is independent of the state of the world. By contrast, I show that if instead of pure noise the decision maker receives an informative signal, the results are reversed.

[6] Another branch of the literature introduces uncertainty on the preferences of the expert. See for example Li and Madarasz (2007); Morgan and Stocken (2003); Wolinsky (2003) and Dimitrakas and Sarafidis (2005). They find that more information can be transmitted because the decision maker is less sensitive to the message of the expert.

3 The Model

3.1 Setup

There are two players, an expert (E or he) and a decision maker (D or she). The expert privately and perfectly observes the state of the world θ (also referred to as his type), whereas the decision maker only receives a signal $s \in \mathcal{S}$. The conditional distribution of the signal is common knowledge, but the realisation s is privately observed by the decision maker. After learning θ the expert sends a costless message $m \in \mathcal{M}$ to the decision maker who, taking into account her own private signal, chooses an action $y \in \mathbb{R}$ that affects both agents' payoffs.

The payoff functions of the players are defined by the following utility functions:

$u^D(y, \theta) = \tilde{u}^D(y - \theta)$

$u^E(y, \theta, b) = \tilde{u}^E(y - (\theta + b))$

where $\tilde{u}^D$ and $\tilde{u}^E$ are strictly concave, twice differentiable and symmetric functions around 0. Given this specification, the best action for the decision maker in state θ is to match the state of the world, whereas the expert always wants her to take a higher action, namely $\theta + b$.

The state of the world is uniformly distributed in $[0, 1]$[7], and given θ, the signal the decision maker receives is distributed symmetrically around θ with conditional distribution $F(s|\theta)$ and conditional density $f(s|\theta) = p(s - \theta)$, where $p(\cdot)$ is decreasing in absolute value and positive everywhere[8]. In particular I assume that θ and s are affiliated, meaning that higher realisations of s lead to higher posterior beliefs about θ in the first-order stochastic dominance (FOSD)[9].

Sometimes I will consider that the conditional distribution of the signal belongs to a parameterised family $\{F^\lambda(\cdot|\theta), \lambda \in [0, \infty)\}$, where λ represents the precision[10] of the signal, and such that $\lambda = 0$ corresponds to the uninformative signal, and in the limit as $\lambda \to \infty$, it corresponds to the degenerate distribution in θ.

[7] The assumption that the state of the world is uniformly distributed in $[0, 1]$ simplifies the argument and the presentation of the results, but the intuition behind the results can be translated to other prior distributions.

[8] The assumption that the signal has full support is not necessary but simplifies the argument and eliminates possible out of equilibrium beliefs.

[9] For unidimensional random variables, being affiliated is equivalent to saying that f is log-supermodular in (s, θ) or that f satisfies the *Monotone Likelihood Ratio Property* (MLRP): $f(s|\theta)f(s'|\theta') > f(s|\theta')f(s'|\theta)$ for all $s > s'$, $\theta > \theta'$.

[10] One signal is more precise than another if the latter is a mean preserving spread of the former.

I will refer to this model as the *private information* model and denote it by $F-PI$ where F refers to its signal structure.

3.2 Equilibrium

The equilibrium concept I consider is *Perfect Bayesian Equilibrium* (PBE). Given θ, a message strategy for the expert is a probability distribution over $\mathcal{M}$ denoted by $q(m|\theta)$. Upon receiving m and observing s, the decision maker updates her beliefs about the state of the world θ. Due to the concavity of $\tilde{u}^D$, the decision maker always has a unique preferred action that I denote by $y(m,s)$. The strategies $(q(\cdot), y(\cdot))$ constitute a PBE if:

1. for each θ, $\int_{\mathcal{M}} q(m|\theta)dm = 1$, and if $q(m^*|\theta) > 0$ then $m^* \in \arg\max_m \int_S \tilde{u}^E(y(m,s) - (\theta + b))p(s-\theta)ds$;
2. for each m and s, $y(m,s) \in \arg\max_y \int_0^1 \tilde{u}^D(y-\theta)g(\theta|m,s)d\theta$, where $g(\theta|m,s) = q(m|\theta)p(s-\theta)/\int_0^1 q(m|t)p(s-t)dt$

If the signal s were independent of θ, the setup would correspond to the canonical model of CS. However, when the signal is informative two main differences arise. First, the expert is no longer able to perfectly forecast the reaction of the decision maker to his message. Each message induces a lottery over actions and when the expert decides which message to send, he is in fact comparing lotteries and not actions. Second, since the signal depends on the state of the world, the distribution of the lotteries depends on the expert's type, and therefore two experts sending the same message face different lotteries. This implies that the set of experts that prefer one message over another does not need to form an interval as in CS[11]. This latter fact makes it difficult to provide a complete characterisation of the equilibria. However, as shown below, equilibria exist of a special kind. These special equilibria share the structure of the partition equilibria characterised by CS, and have the property that as the signal becomes less informative, they converge to the equilibria in (CS). In the remainder of the paper I focus exclusively on these equilibria.

[11] Chen (2009) in a similar setup provides an example of an equilibrium in which low and high types pool together whereas middle types send a different message. Krishna and Morgan (2004) also showed the existence of non-monotone equilibria (Example 2) when the expert faces the uncertainty of a jointly controlled lottery.

3.3 Partition Equilibria

In this section I show the existence of partition equilibria similar to those characterised in CS. An equilibrium is said to be a partition equilibrium if $[0,1]$ can be partitioned into intervals such that all the experts with types in a given interval use the same message strategy, which has disjoint support from the message strategies used in other intervals. Formally:

Definition An equilibrium $(q(\cdot), y(\cdot))$ is a *partition equilibrium of size* N, if there exists a partition $0 = a_0 < a_1 < ... < a_N = 1$ such that $q(m|\theta) = q(m|\theta')$ if $\theta, \theta' \in (a_i, a_{i+1})$[12], and if $q(m|\theta) > 0$ for $\theta \in (a_i, a_{i+1})$ then $q(m|\theta') = 0$ for all $\theta' \in (a_j, a_{j+1})$ with $j \neq i$.

Given a partition equilibrium, the only information that the decision maker learns upon receiving a message, is the interval in which the actual state of the world lies. As a result, I consider all the equilibria with the same partition as equivalent, and with some abuse of notation I will say that $m \equiv [\underline{a}, \overline{a}]$ if $[\underline{a}, \overline{a}] = \{\theta \in [0,1] \mid q(m|\theta) > 0\}$.

Before turning to the existence of partition equilibria I introduce two further pieces of notation that simplify the exposition of the argument. First I denote by $y(\underline{a}, \overline{a}, s; F)$ the best response of a decision maker with signal s upon receiving $[\underline{a}, \overline{a}]$:

$$y(\underline{a}, \overline{a}, s; F) = arg \max_{y} \int_{\underline{a}}^{\overline{a}} \tilde{u}^D(y - \theta) p(s - \theta) d\theta \tag{1}$$

Second, I denote by $U^E(\underline{a}, \overline{a}, \theta, b; F)$ the expected utility of an expert with type θ and bias b that sends message $m = [\underline{a}, \overline{a}]$:

$$U^E(\underline{a}, \overline{a}, \theta, b; F) = \int_{\mathcal{S}} \tilde{u}^E(y(\underline{a}, \overline{a}, s; F) - (\theta + b)) p(s - \theta) ds$$

The following proposition establishes that only a finite number of messages can be sent in a partition equilibrium. The intuition behind this result is that the intervals sent in equilibrium cannot be too small (except the first one). The reason is that if the size of an interval was smaller than $2b$, the expert on the lower bound of the interval would strictly

[12] The definition above does not determine the strategy of boundary types $\theta = a_i$. As we will see in the construction of the equilibria, those types are indifferent between the message strategies of adjacent intervals, and therefore there are many strategy specifications that lead to payoff equivalent equilibria.

prefer all the actions induced by this message to any possible action induced by a lower interval. By continuity an expert slightly below the lower bound of the interval would like to deviate and report that he belongs to the interval, violating the equilibrium conditions. Since a separating equilibrium is a partition equilibrium with an infinite number of messages, no separating equilibrium exists under the setup of this model.

Proposition 1 *The number of intervals sent in a partition equilibrium is finite. In particular, there is no separating equilibrium in the private information model.*[13]

Note that this proposition is true even if we drop the symmetry assumption of the payoff functions and the signal structure. This contrasts with the finding of Blume et al. (2007). In their setup there exist equilibria with an infinite (even uncountable) number of intervals. The reason is that in their model the decision maker cannot distinguish whether the message comes from an expert or has arrived by mistake and as a result the decision maker can rationally choose an action outside the interval induced by the message.

As in CS an equilibrium is determined by a partition $0 = a_0 < a_1 < ... < a_N = 1$ that satisfies the following arbitrage condition:

$$U^E(a_{i-1}, a_i, a_i, b; F) = U^E(a_i, a_{i+1}, a_i, b; F) \qquad (A^F)$$

Condition (A^F) means that the boundary type a_i is indifferent between sending message $m_i \equiv [a_{i-1}, a_i]$ and message $m_{i+1} \equiv [a_i, a_{i+1}]$. In CS this condition was necessary and sufficient to determine an equilibrium. When the decision maker has private information correlated with the state of the world, condition (A^F) alone might not be sufficient. The reason is that when an expert chooses between two messages, he is not choosing between two different actions but between two different lotteries over actions. If an expert with type a_i is indifferent between m_i and m_{i+1}, he must prefer the actions induced by m_i when the realisation of the signal is high, and the actions induced by m_{i+1} when the realisation of the signal is low. Since θ and s are affiliated, an expert with type $\theta > a_i$ allocates higher probability to high signals and as a result he might prefer m_i over m_{i+1}.

[13]All the proofs are relegated to the Appendix.

For general signal distributions and supermodular preferences, the change in the conditional density of the signal given a change in the state of the world needs to be bounded to prevent that a change in θ dramatically changes the probability allocated to actions, leading to a reversal of preferences. However, with symmetric preferences and symmetric signal structure, a shift in the conditional distribution corresponds to a shift in the induced actions that is less than one to one, and as a result there cannot be a reversal of preferences over messages.

Theorem 1 *If $b > 0$, there exists an integer $N(b, F)$ such that, for every $1 \leq N \leq N(b, F)$, there exists a partition equilibrium of size N. The equilibrium is characterised by a partition $0 = a_0 < a_1 < ... < a_N = 1$ satisfying (A^F).*

Note that as the signal becomes uninformative the conditional distribution becomes less sensitive to a change in θ making condition (A^F) sufficient as well as necessary for general preferences.

In the rest of this section I explore some properties of the equilibria which will be useful in the development of Section 4. In particular, I show that the intervals sent in a partition are increasing in length:

Proposition 2 *If $b > 0$, the intervals in a partition equilibrium are strictly increasing in length.*

To guarantee that there is a unique partition equilibrium of size N, for $1 \leq N \leq N(b, F)$, and hence to be able to compare equilibria and ultimately do welfare comparisons, I henceforth assume:

Assumption (S): *$U^E(a_i, a, a_i, b; F)$ is single-peaked in a for $a \geq a_i$.*

Assumption (S) says that when an expert of type a_i sends message $[a_i, a]$, his expected utility (which is increasing for $a_i < a < a_i + b$) might change its sign's derivative at most once. This assumption implies the following two propositions:

Proposition 3 *If $\hat{a}$ and $\tilde{a}$ are two partial partitions satisfying (A^F) with $\hat{a}_0 = \tilde{a}_0$ and $\hat{a}_1 > \tilde{a}_1$, then $\hat{a}_i - \hat{a}_{i-1} > \tilde{a}_i - \tilde{a}_{i-1}$ for all $i \geq 1$.*

Proposition 3 is a stronger version of the monotonicity condition (M) in CS. In particular it implies that there is a unique partition equilibrium of size N, for $1 \leq N \leq N(b, F)$.[14]

Proposition 4 allows me to Pareto rank the equilibria of the model[15]. It says that both agents ex-ante prefer equilibrium partitions with more intervals. In particular, the equilibrium with size $N(b, F)$ Pareto dominates all the others, and hence for the welfare analysis in Section 4 I will focus on the equilibrium partition with the highest number of steps.

Proposition 4 *For a fixed signal structure F and a fixed $b > 0$, both the decision maker and the expert prefer ex-ante equilibrium partitions with more intervals.*

In the next section I analyse how the introduction of the private information affects the informativeness of the equilibrium partitions comparing them to those of the CS model.

4 Communication and Welfare

In this section I analyse how access to private information affects the incentives of the expert to disclose information. Observe that, given the symmetric setup of the model, the decision maker in the private information model has the same preferences over partitions than the uninformed decision maker. Hence, it is meaningful to say that one partition is more communicative than another if ex-ante the uninformed decision maker prefers the former over the latter.

For $0 \leq a_{i-1} \leq a_i \leq a_{i+1} \leq 1$, denote by $V(a_{i-1}, a_i, a_{i+1}, b; F)$ the difference in expected utility to the expert with type a_i between sending $m_{i+1} = [a_i, a_{i+1}]$ and $m_i = [a_{i-1}, a_i]$:

$$V(a_{i-1}, a_i, a_{i+1}, b; F) = U^E(a_i, a_{i+1}, a_i, b; F) - U^E(a_{i-1}, a_i, a_i, b; F)$$

In particular, the arbitrage condition (A^F) can be written as $V(a_{i-1}, a_i, a_{i+1}, b; F) = 0$.

Proposition 5 states that to compare the communication under two different signal structures it is enough to look at how the arbitrage condition is modified when we change the

[14] Proposition 3 also implies what I showed in Proposition 2

[15] All the comparative statics with respect to the divergence of preferences b established in (CS) can also be transferred to the private information model. Since this is not the focus of the paper I don't state them here.

signal structure.

Given two signal structures F and F', consider the following condition:

Condition (C): For any $0 \leq a_{i-1} \leq a_i \leq a_{i+1} \leq 1$ such that $V(a_{i-1}, a_i, a_{i+1}, b, F) = 0$, then $V(a_{i-1}, a_i, a_{i+1}, b, F') > 0$.

Proposition 5 *Suppose that F and F' are two signal structures satisfying Condition (C), then there is less communication transmitted in the $F' - PI$ model than in the $F - PI$ model. Namely, if a and a' are two equilibrium partitions of size N of the $F - PI$ and the $F' - PI$ models respectively, then $a_i > a'_i$ for all $1 \leq i \leq N - 1$. Moreover, $N(b, F) \geq N(b, F')$.*

Intuitively, if $V(a_{i-1}, a_i, a_{i+1}, b, F) > 0$, the expert with type a_i strictly prefers message m_{i+1} over message m_i. As a result, the new indifferent type a such that $V(a_{i-1}, a, a_{i+1}, b, F) = 0$ would be to the left of a_i, but then the new partial partition $\{a_{i-1}, a, a_{i+1}\}$ provides less useful information to the decision maker. The reason is that m_{i+1} was larger than m_i, and hence a shift of a_i to the left makes the size of the intervals more uneven. Given the concavity of the decision maker's preferences, partition $\{a_{i-1}, a_i, a_{i+1}\}$ is preferred to partition $\{a_{i-1}, a, a_{i+1}\}$, because her ex-ante expected utility is higher under the former than under the latter.

Given Proposition 5, in order to analyse how the communication is affected by the acquisition of private information, I study how the preferences over messages change for the experts that were indifferent in the CS setup. In order to proceed I restrict attention to the case of quadratic-loss utilities[16] and derive some properties of these preferences that make the model more tractable. The quadratic-loss utility functions are given by:

$\tilde{u}^D(y - \theta) = -(y - \theta)^2$

$\tilde{u}^E(y - (\theta + b)) = -(y - (\theta + b))^2.$

Given these utilities, the decision maker's optimal action when she receives message m and signal s is to match her expectation about the state of the world: $y(m, s) = E[\theta|m, s]$. Moreover, the expected utility of an expert with type θ that sends message m can be written

[16] Section 5 provides a discussion of the results for other symmetric preferences.

as:

$$U^E(m,\theta) = -\hat{\sigma}^2(m,\theta) - (\hat{y}(m,\theta) - (\theta + b))^2 \tag{2}$$

where $\hat{y}(m,\theta)$ and $\hat{\sigma}^2(m,\theta)$ are the expectation and the variance of the actions chosen by the decision maker when the expert sends message m and has type θ[17]. Equation (2) states that the expert's expected utility only depends on the variance of the actions and the distance between his peak and the expected action of the decision maker.

Denote by $y_{CS}(m)$ the action chosen by an uninformed decision maker upon receiving message m. The change in the expert's expected utility due to the introduction of private information is:

$$U^E(m,\theta) - U^E_{CS}(m,\theta) = \underbrace{-\hat{\sigma}^2(m,\theta)}_{Risk\ \ Effect} + \underbrace{(y_{CS}(m) - (\theta + b))^2 - (\hat{y}(m,\theta) - (\theta + b))^2}_{Information\ \ Effect} \tag{3}$$

The introduction of private information has two effects on the expert's expected utility: an *information effect* and a *risk effect*. The information effect arises because the decision maker is able to make better decisions on average. This implies that her actions will in expectation be closer to θ than they were before. For a boundary expert, an action closer to the actual state of the world is also an action closer to his peak. Hence, fixing a message, the information effect has a positive impact on the expected utility of a boundary type. The risk effect occurs because the expert is no longer certain of how the decision maker will respond to his message. Since the expert is risk averse, he dislikes this uncertainty and fixing a message, the risk effect always has a negative impact in the expert's expected utility.

I now compare the information and risk effect across messages for an expert with type $\theta = a_i$ that is indifferent between sending messages $m_i = [a_{i-1}, a_i]$ and $m_{i+1} = [a_i, a_{i+1}]$ in the CS model. If there were no divergence of preferences between the agents ($b = 0$), the length of the two intervals would be the same and due to the symmetric setup, the signal would influence the decision maker in a symmetric way and the expert would still be indifferent between the two messages. However, the presence of a bias $b > 0$ implies that m_{i+1} is larger than m_i, and therefore the lotteries over actions induced by these two messages are qualitatively different.

[17] Namely, $\hat{y}(m,\theta) = \int_S y(m,s)p(s-\theta)ds$ and $\hat{\sigma}^2(m,\theta) = \int_S (y(m,s) - \hat{y}(m,\theta))^2 p(s-\theta)ds$.

Consider first the information effect. Observe that the message sent by the expert determines the prior of the decision maker before hearing her signal. Hence sending message m_{i+1} instead of m_i implies that the decision maker will have a less precise prior about the state of the world. A less precise prior implies that the decision maker will rely more on her signal when updating her posterior. In other words the actions of the decision maker are more sensitive to her private information the bigger the message sent is. From the point of view of the expert with type a_i, it means that the expected action of the decision maker will shift towards him by more when he sends m_{i+1} than when he sends m_i. Hence the expert with type a_i, strictly prefers $\hat{y}(m_{i+1}, a_i)$ to $\hat{y}(m_i, a_i)$. Abstracting from risk aversion, this result implies that the message m_{i+1} becomes more attractive to the expert than the message m_i. Hence, the information effect of the signal worsens the communication incentives of the expert.

Proposition 6 *The information effect hampers communication. Namely, if* $0 \leq a_{i-1} \leq a_i < a_{i+1}$ *are such that the expert with type* a_i *is indifferent between* $y_{CS}(m_i)$ *and* $y_{CS}(m_{i+1})$*, where* $m_i = [a_{i-1}, a_i]$ *and* $m_{i+1} = [a_i, a_{i+1}]$*, then the expert strictly prefers* $\hat{y}(m_{i+1}, a_i)$ *to* $\hat{y}(m_i, a_i)$*.*

Consider now the risk effect. Intuitively, sending a larger message spreads the decision maker's actions across the interval thereby increasing the variance of the lottery. Hence the risk effect is stronger in m_{i+1} than in m_i. Abstracting from the information effect, a risk averse agent prefers the lower message, easing the communication between the agents.

Proposition 7 *The risk effect eases communication. Namely, if* $0 \leq a_{i-1} \leq a_i < a_{i+1}$ *are such that the expert with type* a_i *is indifferent between* $y_{CS}(m_i)$ *and* $y_{CS}(m_{i+1})$*, where* $m_i = [a_{i-1}, a_i]$ *and* $m_{i+1} = [a_i, a_{i+1}]$*, then* $\hat{\sigma}^2(m_{i+1}, a_i) > \hat{\sigma}^2(m_i, a_i)$*.*

In the next proposition I determine some environments for which the change in the information effect dominates the change in the risk effect, leading to less communication in equilibrium.

Proposition 8 *For any information structure* F*, there exist* $\bar{b} < \frac{1}{4}$ *such that if* $b > \bar{b}$*, there is no communication in the* $F - PI$ *model.*

Moreover, for any b*, there exists a threshold of the precision of the signal, such that if the* *precision of the signal structure is above that threshold, then there is no communication in the private information model.*

The intuition behind the first statement of Proposition 8 is as follows. Suppose that in the CS model an expert was indifferent between perfectly revealing his type or pooling with some higher types. The introduction of private information does not affect the expected utility of the expert if he perfectly reveals his type, whereas the expected utility of the expert if he pools with the higher types strictly increases (namely, $V(0, 0, 4b, b) > 0$). By continuity this implies that for b sufficiently close to $\frac{1}{4}$, $V(0, 0, 1, b) > 0$ and no information can be transmitted in equilibrium. For the second statement, observe that for any b, there is a precision of the signal structure such that the lottery over actions induced by message $[0, 1]$ is preferred by an expert with type $\theta = 0$, over the constant action $y = 0$. In that case no information can ever be transmitted in equilibrium.

We have seen that in some environments the information effect dominates the communication effect and as a result there is less communication in equilibrium. Nevertheless, the signal itself might provide enough information to make up for the loss of communication. Clearly, if the divergence of preferences is such that there is no communication in the CS model ($b \geq \frac{1}{4}$), private information is always welfare improving[18]. Similarly, if the information is very precise, the decision maker is better off even if no information is transmitted from the expert.

However, I now analyse two different families of information structures and show that total welfare can be reduced by giving access to extra information.

4.1 Normal Private Information Model

Consider the case in which the signal is distributed normally around θ with variance σ^2. The parameter σ^2 is a measure of the dispersion of the signal[19].

To be more specific, suppose that the bias of the expert is $b = \frac{1}{20}$[20]. For this bias, the most

[18] See Persico (2000) and Athey and Levin (2001). They show that for decision problems where the signals are affiliated to the state of the world and the payoff of the decision maker satisfies the single crossing condition in (θ, y) (See Milgrom and Shannon (1994)), the ex-ante utility of the decision maker increases with the accuracy of the signals.

[19] Equivalently, $\frac{1}{\sigma^2}$ is a measure of the precision of the signal.

[20] This bias corresponds to the example illustrated in Crawford and Sobel (1982).

informative equilibrium in the standard CS model is determined by the following partition: $\{0, \frac{2}{15}, \frac{7}{15}, 1\}$. The expert reveals whether the state of the world lies in $[0, \frac{2}{15}]$, in $[\frac{2}{15}, \frac{7}{15}]$ or in $[\frac{7}{15}, 1]$, and the decision maker reacts by choosing the midpoint in each interval[21].

In the private information model with $\sigma = 0.3$, the most informative equilibrium is determined by the partition $\{0, 0.0863, 0.59, 1\}$. Figure 1 provides a graphical illustration of the two equilibria.

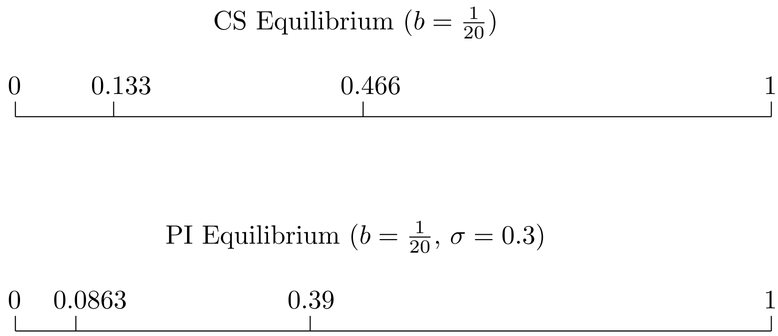


Figure 1

To compute the loss of communication due to the introduction of the signal I compute the ex-ante utility of an uninformed decision maker under both partitions. The loss of communication is $EU^D_{CS,\{0,\frac{2}{15},\frac{7}{15},1\}} - EU^D_{CS,\{0,0.0863,0.39,1\}} = (-0.0159) - (-0.0213) = 0.0054$.

Figure 2 shows the partition equilibria as a function of the variance of the signal. For every σ the partition can be read tracing the horizontal line at that level. The points of the partition correspond to the intersection with the solid lines. The case of $\sigma = 0.3$ is depicted as an example. The horizontal line cuts the solid lines at $a_1 = 0.0863$ and $a_2 = 0.39$, indicating that the partition equilibrium is $\{0, 0.0863, 0.39, 1\}$.

[21] It is easy to check that this in fact constitutes an equilibrium. For instance, when the expert observes $\theta = \frac{2}{15}$ he is indifferent between reporting the first interval or the second because they lead respectively to actions $a_1 = \frac{1}{15}$ and $a_2 = \frac{9}{30}$, that are equidistant to his preferred action $\frac{2}{15} + \frac{1}{20}$.

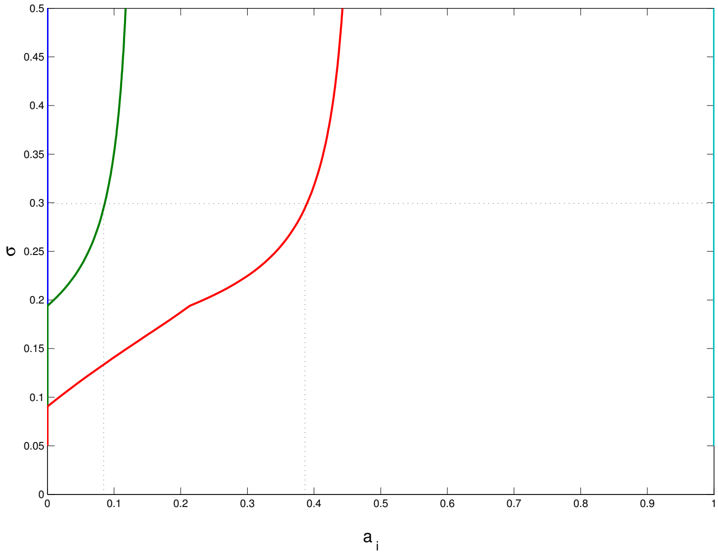


Figure 2. Partition equilibria for different variances of the signal. ($b = \frac{1}{20}$)

From Figure 2 we can see that for $\sigma < \sigma_0 \simeq 0.09$ no information is revealed in equilibrium. For $\sigma_0 < \sigma < \sigma_1 \simeq 0.193$ the partition equilibrium contains only two intervals and for $\sigma > \sigma_1$ the partition equilibrium is formed by three intervals. Finally, as σ increases, the equilibrium partition converges to the CS equilibrium.

Figure 2 suggests that the communication decreases with the precision of the signal. This comparative statics is proven in Theorem 2 for the case of the family of uniform signals.

The next figure shows the ex-ante expected utility of the decision maker for different variances of the signal. The horizontal dashed line corresponds to the ex-ante utility of the CS model.

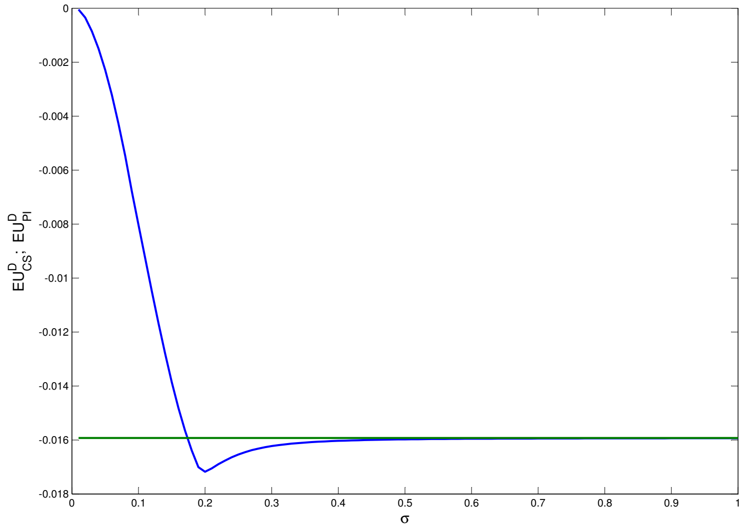


Figure 3. Ex-ante expected utility of the decision maker in the Normal Private Information model, for different variances of the signal. ($b = \frac{1}{20}$)

As can be seen from Figure 3, unless the precision of the signal is sufficiently high ($\sigma < 0.1735$), the decision maker is better off not seeking external information. Again, the minimum ex-ante utility is reached at $\sigma = 0.1930$ which corresponds to the case where the partition equilibrium the model passes from having size 3 to size 2.

To understand why the loss in communication might outweigh the gain in information, observe that the communication result is determined by the preferences of the indifferent experts in the partition, whereas the informational gain is computed as an average over all the types. Given an interval in equilibrium, the effect of information on the decision maker is stronger for boundary types. So the shift in the communication is based on those agents that are more sensitive to information. If the difference between the effect of information on these boundary types and the average effect is high enough then welfare decreases.

4.2 The Uniform Private Information Model

Suppose now that the signals are distributed uniformly on $[\theta - \delta, \theta + \delta]$. The parameter δ plays the same role as σ in the normal example. Upon receiving a signal s, and a message $m = [\underline{a}, \overline{a}]$ the decision maker's posterior distribution of θ is uniform on the interval $[\max\{\underline{a}, s-$

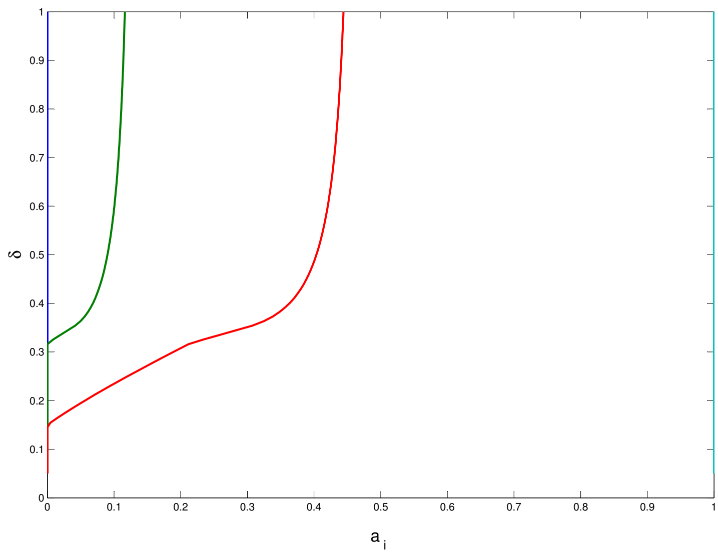
$\delta\}, \min\{\overline{a}, s+\delta\}]$[22]. Given those beliefs, the optimal action for the decision maker is[23]:

$$y(\underline{a}, \overline{a}, s, \delta) = \frac{\max\{\underline{a}, s-\delta\} + \min\{\overline{a}, s+\delta\}}{2}$$

In this more tractable case the comparative static results that we observed for the family of normal signals can be proven.

Theorem 2 *In the Uniform Private Information model, an increase in the precision of the signal (a decrease in δ) leads to less communication in equilibrium. Namely, if a^{δ} and $a^{\delta'}$ are two partition equilibria of size N satisfying ($A^{F^{\delta}}$) and ($A^{F^{\delta'}}$) respectively, with $\delta' < \delta$, then $a_i^{\delta'} < a_i^{\delta}$ for all $i = 1, ..., N-1$. Moreover $N(b, \delta') \leq N(b, \delta)$.*

In particular, Theorem 2 implies that for any δ (or in other words for any precision of the signal), and for any bias b, the acquisition of information hampers communication. Figure 4 illustrates the comparative static results for the family of uniform signals:



[22]Note that this signal structure does not satisfy the full rank assumption. As it is clear in the example, this assumption is not necessary for the existence of equilibrium and to establish the properties of the equilibria. However, the fact that the support of the signal varies with θ give rise to possible out of equilibrium beliefs. By threatening with extreme out of equilibrium actions, the decision maker could enforce more communication in equilibrium. Here I take a mild approach because I am interested in understanding how similar signals with full support (in which threatening with out of equilibrium actions is not possible) affect the incentives to communicate.

[23]All the functions previously defined will be indexed by δ to indicate the signal structure in consideration.

Figure 4. Partition equilibria for different values of δ. ($b = \frac{1}{20}$)

The intuition behind this result is that increasing the accuracy of the signal is equivalent to providing an extra signal on top of what the decision maker had before. Since by Remark 2 in the private information equilibria the intervals increase in size, the addition of the extra signal has more impact on larger messages and hence the previously indifferent expert, strictly prefers the upper interval when the precision of the signal increases leading to less communication.

The effect of the signal on communication becomes stronger when $\delta \simeq 0.35$ (See Figure 4). At this point the decision maker that receives the highest interval might, if the signal is sufficiently low, be able to reject completely the highest states of the world. Hence, as δ decreases the information effect in the upper interval becomes very strong. It is precisely for these values of δ, that welfare is more negatively impacted. Figure 5 shows the ex-ante expected utility of the decision maker for different precisions of the signal.

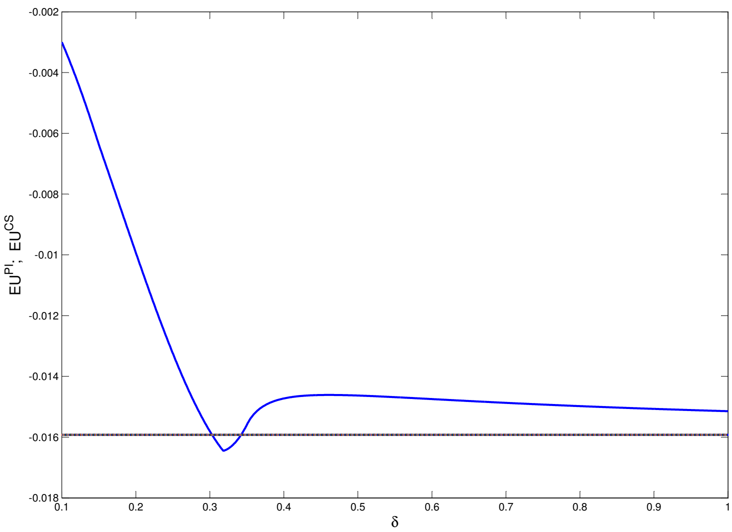


Figure 5. Ex-ante expected utility of the decision maker in the Uniform Private Information model for different dispersions of the signal. ($b = \frac{1}{20}$)

5 Discussion

In this section I discuss how the results would be affected if some of the assumptions of the model were relaxed.

5.1 Other Preferences

In the previous two sections I focused the analysis on the case of quadratic-loss utilities. Here I provide some intuition on how the results might change if we consider other functional forms for the preferences of the agents. In particular I consider the following families of utility functions:

$u^D(y, \theta) = -|y - \theta|^\rho$

$u^E(y, \theta, b) = -|y - (\theta + b)|^\varrho.$

where $\rho, \varrho \geq 1$. These families of utility functions were first introduced under this context by Krishna and Morgan (2004). One can interpret ρ and ϱ as a measure of risk-aversion since they measure the degree of concavity of $u^D(\cdot, \theta)$ and $u^E(\cdot, \theta, b)$ respectively. The case $\rho = \varrho = 2$ is equivalent to the quadratic-loss utilities studied before. The higher the ρ (ϱ) the more risk averse is the decision maker (expert).

In general, when $\varrho \neq 2$, the expected utility of the expert can no longer be written just in terms of the expectation and the variance of the decision maker's action. However it is useful to think of the information and risk effect to develop an intuition on these cases. Observe that the actions of the decision maker are completely independent of the preferences of the expert. Hence if we fix the preferences of the decision maker and we change the risk aversion of the expert, we are in fact comparing two fixed lotteries from the point of view of a risk averse agent. Intuitively, as ϱ decreases, the expert is more tolerant to risk and the risk effect diminishes. As a result larger intervals become more attractive leading to even less communication in equilibrium. In contrast, as the the expert's risk aversion increases, the risk effect becomes larger reducing the impact of the information effect. For high enough risk aversion it can be even the case that the risk effect outweighs the information effect leading to more communication in equilibrium. Consider for example an expert with preference's parameter $\varrho = 6$ and bias $b = \frac{1}{4}$ that faces a decision maker with quadratic preferences ($\rho = 2$). In the uninformed case no information can be transmitted in equilibrium. However, if the expert learns that the decision maker has access to a signal normally distributed around the state of the world and with standard deviation $\sigma = 0.5$, then his risk aversion allows him to reveal the following partition: $\{[0, 0.0119], [0.0119, 1]\}$. In this case obviously both agents are better off by the presence of the signal.

Alternatively, we could fix the preferences of the expert and change the preferences of the decision maker. In this case, however, a change in the preferences of the decision maker changes the lotteries over actions and hence indirectly the preferences of the expert. Intuitively a more risk averse decision maker is less sensitive to her private information because she dislikes the risk associated with the signal; in order for her to choose an action below (above) the middle of the interval, she needs to receive a lower (higher) signal compare to when she were less risk averse, so that she is more certain that the true state of the world is actually low (high). In fact, an increase in the risk aversion of the decision maker has a similar effect on her actions as a decrease in the accuracy of the signal structure. Hence, using the intuition of the comparative statics in Section 4, since the decision maker reacts less to her signal, the incentives to exaggerate are reduced and more communication arises in equilibrium.

For the case where $\rho = \varrho$, namely $u^E(y, \theta, 0) = u^D(y, \theta)$[24] the intuition is that although an increase in risk aversion smoothes the communication between the agents, the communication will still be worse compared to the canonical CS[25]. The reason is that, as discussed in Section 4, the value of information for the decision maker is bigger when her prior is less precise. A boundary expert with the same shape of preferences as the decision maker has nearly the same preferences as the decision maker when the state of the world is an extreme of the interval. Therefore the signal will make more attractive for the expert to report the higher interval leading to less communication in equilibrium than in the CS case.

To sum up, as we increase the risk aversion of both agents the communication between them improves, and as a result the welfare of the agents increases (as a function of their risk

[24] This assumption was made in CS to derive the comparative statics.

[25] Observe that in CS, the risk aversion of the agents does not play any role when the prior distribution is uniform in [0,1] and the preferences are symmetric as in this model. In fact, the equilibria in the CS are the same with any set of symmetric preferences (i.e. for any $\rho, \varrho \geq 1$).

aversion). This counterintuitive result was first highlighted by Krishna and Morgan (2004) although in their case only the risk aversion of the expert mattered because, absent of private information, the induced actions were independent of the risk aversion of the decision maker. This discussion extends their surprising conclusions to the risk aversion of the decision maker.

5.2 Other Information Structures

My analysis throughout the paper assumed that the signals were symmetrically distributed around the state of the world. The main reason for this assumption was that I did not want to bias my analysis by favouring certain states of the worlds over others. However, the same decomposition of the effect of private information on the incentives of the expert to reveal information can be used to study other signal structures. Moreover the information effect and the risk effect give us some intuition about which sort of signals will result in more information being transmitted and which signals will deteriorate the incentives to communicate. Suppose that an expert was indifferent between sending two messages in the CS setup. A signal structure that would have a higher information effect on the lower interval would make it more attractive to the expert and as a result the equilibrium partition would be more evenly spaced.

Example 2: Uniform $[0, \theta]$ ***signal structure.*** Suppose that the signal s is conditionally distributed uniformly on $[0, \theta]$.[26] An interpretation of this signal structure is that the decision maker is being told a lower bound for the state of the world.

Given this signal structure the decision maker values low intervals because they provide a (low) upper bound that complements her signal. More precisely, if the sender sends message $[\underline{a}, \overline{a}]$, the best reply form the decision maker is[27]:

$$y(\underline{a}, \overline{a}, s) = \begin{cases} \frac{\overline{a}-\underline{a}}{ln(\overline{a})-\ln(\underline{a})} & if \quad s < \underline{a} \\ \frac{\overline{a}-s}{ln(\overline{a})-\ln(s)} & if \quad \underline{a} < s < \overline{a} \end{cases} \tag{4}$$

[26] We could define a parameterised family of signals that includes this one. Define $F^{\eta}(s|\theta) = (\frac{s}{\theta})^{\eta}$ on $[0, \theta]$, with $\eta \in [0, \infty)$. The case $\eta = 1$ corresponds to the uniform signal in $[0, \theta]$ described in the example. An increase in η represents and increase in the accuracy of the signal (See Persico (1996)). As $\eta \to 0$ the signal becomes uninformative, and as $\eta \to \infty$ the signal becomes perfectly informative.

[27] For the illustration I ignore out of equilibrium actions and beliefs.

TO BE COMPLETED

6 Conclusion

I have examined a natural extension of the canonical paper Crawford and Sobel (1982), where the decision maker is allowed to access an informative signal about the state of the world. I have argued that in a symmetric setup, access to information hampers the communication between the agents, and as a result the welfare of both agents might decrease. I provide examples of these situations.

There are several extensions that might be interesting to examine. My assumption that the expert perfectly knows the state of the world is far from being realistic. The assumption was made to identify the strategic effects of the acquisition of information from the information aggregation issue that arises if the expert is not certain of the state of the world. A model that analyses the information transmission from an expert with imperfect information is left for further research.

Another feature that this model ignores is the possibility to organise communication in a different way. Krishna and Morgan (2004) have shown that adding a stage to the communication game, even if no more information is added in the process, helps to achieve better equilibria. How should communication be organised to extract the maximum amount of information from the expert? Can the decision maker use her private information in the process? What are the welfare implications of the acquisition of information once the optimal mechanisms are considered?

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Appendix A: Proofs

Proof of Proposition 1: The proposition follows as an immediate corollary of Lemma 1 and the fact that $[0, 1]$ is bounded. ■

Lemma 1 *If $b > 0$ and $m = [\underline{a}, \overline{a}]$ is a message sent in a partition equilibrium with $\underline{a} > 0$, then $\overline{a} - \underline{a} \geq 2b$.*

Proof of Lemma 1: Suppose by way of contradiction that we could find a partition equilibrium in which message $m = [\underline{a}, \overline{a}]$ with $\underline{a} > 0$ and $\overline{a} - \underline{a} < 2b$ was sent. Then in particular $|\overline{a} - (\underline{a} + b)| < b = (\underline{a} + b) - \underline{a}$ implying that an expert with type $\theta = \underline{a}$ strictly prefers the action $y = \overline{a}$ to action $y' = \underline{a}$. By continuity of preferences, there exists $\epsilon > 0$ such that $\underline{a} - \epsilon > 0$ and an expert with type $\theta' = \underline{a} - \epsilon$ strictly prefers $y = \overline{a}$ to $y' = \underline{a}$. Hence, by the concavity of the expert's preferences, all the actions $y(\underline{a}, \overline{a}, s)$, $s \in \mathcal{S}$ are preferred to $y' = \underline{a}$ which implies that type θ' strictly prefers message m to any interval message $m' \subset [0, \underline{a}]$, contradicting that m belongs to a partition equilibrium. ■

The following results will be used in the proof of Theorem 1. In Lemma 2 I derive some monotonicity properties of the decision maker's best action:

Lemma 2 *Given a message $m = [\underline{a}, \overline{a}]$, $y(\underline{a}, \overline{a}, s)$ is increasing in all its arguments and $\underline{a} \leq y(\underline{a}, \overline{a}, s) \leq \overline{a}$ for all $s \in \mathcal{S}$*

Proof of Lemma 2: $y(\underline{a}, \overline{a}, s)$ solves the first order condition[28]:

$$\int_{\underline{a}}^{\overline{a}} \tilde{u}_1^D(y(\underline{a}, \overline{a}, s) - \theta)p(s - \theta)d\theta = 0 \tag{5}$$

Since $\tilde{u}_{11}^D(\cdot) < 0$ and $p(\cdot) \geq 0$, there exists a $\bar{\theta} \in (\underline{a}, \overline{a})$ such that $\tilde{u}_1^D(y(\underline{a}, \overline{a}, s) - \bar{\theta}) = 0$ and therefore:

$$\tilde{u}_1^D(y(\underline{a}, \overline{a}, s) - \underline{a}) < 0 \quad \textit{and} \quad \tilde{u}_1^D(y(\underline{a}, \overline{a}, s) - \overline{a}) > 0 \tag{6}$$

[28] I denote partial derivatives with subscripts.

Differentiating (5) with respect to its first argument and rearranging:

$$y_1(\underline{a}, \overline{a}, s) = \frac{\tilde{u}_1^D(y(\underline{a}, \overline{a}, s) - \underline{a})p(s - \underline{a})}{\int_{\underline{a}}^{\overline{a}} \tilde{u}_{11}^D(y(\underline{a}, \overline{a}, s) - \theta)p(s - \theta)d\theta} > 0$$

where the inequality follows by (6) and $\tilde{u}_{11}^D(\cdot) < 0$. Analogously, differentiating (5) with respect to its second argument:

$$y_2(\underline{a}, \overline{a}, s) = -\frac{\tilde{u}_1^D(y(\underline{a}, \overline{a}, s) - \overline{a})p(s - \overline{a})}{\int_{\underline{a}}^{\overline{a}} \tilde{u}_{11}^D(y(\underline{a}, \overline{a}, s) - \theta)p(s - \theta)d\theta} > 0$$

To show that $y(\underline{a}, \overline{a}, s)$ is increasing in s it is sufficient to prove that $U(y, s) = \int_{\underline{a}}^{\overline{a}} \tilde{u}^D(y - \theta)p(s - \theta)d\theta$ is supermodular in (y, s). Given $y' > y$, $U(y', s) - U(y, s) = \int_{\underline{a}}^{\overline{a}} (\tilde{u}^D(y' - \theta) - \tilde{u}^D(y - \theta))p(s - \theta)d\theta$ which is increasing in s because $\tilde{u}^D(y' - \theta) - \tilde{u}^D(y - \theta)$ is increasing in θ by $\tilde{u}_{11}^D(\cdot) < 0$, and $p(s - \theta)$ is ordered in the FSO (Milgrom, 1981). Therefore $U(y, s)$ is supermodular in (y, s) and $y(\underline{a}, \overline{a}, s)$ is increasing in s.
Finally, (6) and $u_{11}^D(\cdot) > 0$ imply that $\underline{a} \leq y(\underline{a}, \overline{a}, s) \leq \overline{a}$ for all $s \in \mathcal{S}$. ■

In Lemma 3 I show that, given the symmetry of the setup, the decision maker's best response is completely determined by the length and the initial point of the interval sent and it is symmetric with respect to the mid point of the interval that the expert sends.

Lemma 3 *If $v^D(\cdot)$ and $p(\cdot)$ are symmetric:*

1. $Pr(\theta \in [a, a + h)|s) = Pr(\theta \in [0, h)|s - a)$.
2. $Pr(\theta \in [0, h)|\frac{h}{2} - s) = Pr(\theta \in [0, h)|\frac{h}{2} + s))$.
3. $g(\frac{h}{2} - \theta|0, h, \frac{h}{2} - s) = g(\frac{h}{2} + \theta|0, h, \frac{h}{2} + s)$.
4. $y(a, a + h, s) = a + y(0, h, s - a)$.
5. $y(0, h, \frac{h}{2} + s) - \frac{h}{2} = \frac{h}{2} - y(0, h, \frac{h}{2} - s)$. *In particular* $y(0, h, \frac{h}{2}) = \frac{h}{2}$.

Proof of Lemma 3: All the results are immediate implications of the symmetry of the functions and a change in variable.

1. $Pr(\theta \in [a, a+h)|s) = \int_a^{a+h} p(s-\theta)d\theta = \int_0^h p(s-a-\theta)d\theta = Pr(\theta \in [0,h)|s-a)$.

2. $Pr(\theta \in [0,h)|\frac{h}{2}-s) = \int_0^h p(\frac{h}{2}-s-\theta)d\theta = \int_0^h p(h-\theta-(\frac{h}{2}+s)) = \int_0^h p(\theta-(\frac{h}{2}+s))d\theta = Pr(\theta \in [0,h)|\frac{h}{2}+s))$.

3. $g(\frac{h}{2}-\theta|0,h,\frac{h}{2}-s) = \frac{p(s-\theta)}{Pr(\theta\in[0,h)|\frac{h}{2}-s)} = \frac{p(\theta-s)}{Pr(\theta\in[0,h)|\frac{h}{2}+s)} = g(\frac{h}{2}+\theta|0,h,\frac{h}{2}+s)$.

4. $0 = \int_a^{a+h} v_1^D(y(a,a+h,s)-\theta)p(\theta-s)d\theta = \int_0^h v_1^D(y(a,a+h,s)-a-\theta)p(\theta-(s-a))d\theta$ therefore $y(a,a+h,s)-a$ solves $\int_0^h v_1^D(y-\theta)p(\theta-(s-a))d\theta = 0$ which implies that $y(0,h,s-a) = y(a,a+h,s)-a$.

5. $0 = \int_0^h v_1^D(y(0,h,\frac{h}{2}+s)-\theta)p(\theta-(\frac{h}{2}+s))d\theta = \int_0^h -v_1^D(h-y(0,h,\frac{h}{2})-\theta)p(\frac{h}{2}-s-\theta)d\theta$ and therefore $y(0,h,\frac{h}{2}-s) = h-y(0,h,\frac{h}{2})$.

Finally using this equation for $s=0$, $y(0,h,\frac{h}{2}) = \frac{h}{2}$. ■

In Lemma 4 I show that, given the symmetric setup, there cannot be a reversal of preferences over messages:

Lemma 4 *If a_{i-1}, a_i and a_{i+1} satisfy (A^F), then:*

$$\begin{aligned} U^E(a_i,a_{i+1},\theta) - U^E(a_{i-1},a_i,\theta) > 0 \qquad if \quad \theta \in [a_i,a_{i+1}] \\ U^E(a_i,a_{i+1},\theta) - U^E(a_{i-1},a_i,\theta) < 0 \qquad if \quad \theta \in [a_{i-1},a_i]. \end{aligned}$$

Proof of Lemma 4: I prove it for $\theta \in [a_i,a_{i+1}]$, the case $\theta \in [a_{i-1},a_i]$ is symmetric. Denote by $\delta = \theta - a_i$, then:

$\int_{\mathcal{S}} \left[\tilde{u}^E(y(a_i,a_{i+1},s)-(\theta+b)) - \tilde{u}^E(y(a_{i-1},a_i,s)-(\theta+b))\right] p(s-\theta)ds =$

$\int_{\mathcal{S}}[\tilde{u}^E(y(a_i,a_{i+1},s+\delta)-\delta-(a_i+b)) - \tilde{u}^E(y(a_{i-1},a_i,s+\delta)-\delta-(a_i+b))]p(s-a_i)ds =$

$\int_{\mathcal{S}}[\tilde{u}^E(y(a_i-\delta,a_{i+1}-\delta,s)-(a_i+b)) - \tilde{u}^E(y(a_{i-1}-\delta,a_i-\delta,s)-(a_i+b))]p(s-a_i)ds >$

$\int_{\mathcal{S}} \left[\tilde{u}^E(y(a_i,a_{i+1},s)-(a_i+b)) - \tilde{u}^E(y(a_{i-1},a_i,s)-(a_i+b))\right] p(s-a_i)ds = 0$

Where the first equality follows by a change in variable, the second by Lemma 3.4, the inequality follows because of the concavity of $\tilde{u}^E(\cdot)$, and the last equality follows because by hypothesis a_{i-1}, a_i and a_{i+1} satisfy (A^F). ■

Proof of Theorem 1: The proof follows closely the proof of Theorem 1 of CS. I start by proving that there exists an integer $N(b,F)$, such that for every N, $1 \le N \le N(b,F)$, there exists a partition of size N satisfying the arbitrage condition (A^F).

First, note that, by the concavity of $\tilde{u}^E(\cdot)$ and lemma 2,

$$\frac{\partial}{\partial a}U^E(a, a_i, a_i) = \int_{\mathcal{S}} \tilde{u}_1^E(y(a, a_i, s) - (a_i + b))\frac{\partial y}{\partial a}(a, a_i, s)p(s - a_i)ds > 0 \quad (7)$$

so $U^E(a, a_i, a_i)$ is strictly increasing in a. Denote by $\hat{a}^i$ the strictly decreasing partial partition $\hat{a}_0 > \hat{a}_1 > ... > \hat{a}_i$ that satisfies (A^F). By the monotonicity of $U^E(a, \hat{a}_i, \hat{a}_i)$, there can be at most one value $\hat{a}_{i+1} < \hat{a}_i$ satisfying (A^F).[29]

Define $K(\hat{a}) \equiv \max\{i : \text{there exists } 0 \leq \hat{a}_i < .... < \hat{a}_2 < \hat{a} < 1 \text{ satisfying } (A^F)\}$. By Lemma 1, $K(\hat{a})$ is finite, well defined and uniformly bounded. Define $N(b, F) = sup_{\hat{a}\in[0,1)}K(\hat{a}) < \infty$. Note that $N(b, F)$ is achieved for certain $\bar{a} \in [0, 1)$ because $K(\hat{a}) \in \mathbb{N}$ and bounded. It remains to be proven that for each $1 \leq N \leq N(b, F)$ there is a partition a satisfying (A^F). Denote $a^{K(a)}$ the decreasing partial partition of size $K(a)$ satisfying (A^F) and such that $a_1^{K(a)} = a$. The partition changes continuously with a and therefore $K(a)$ is locally constant and can at most change by one at a discontinuity. Finally $K(0) = 1$, so $K(a)$ takes on all integer values between one and $N(b, F)$.

Now, I argue that the arbitrage condition (A^F) is also sufficient for the equilibrium. By $u_{11}^E(\cdot) < 0$, $U^E(m_i, \theta)$ is single-peaked in i and therefore condition (A^F) and Lemma 4 imply that $U^E(m_i, \theta) = \max_j U^E(m_j, \theta)$ for $\theta \in [a_i, a_{i+1}]$. ■

Proof of Proposition 2: Let a be a partition that supports an equilibrium, and let $h_i = a_i - a_{i-1}$ and $h_{i+1} = a_{i+1} - a_i$. Suppose that $h_{i+1} \leq h_i$, then for all $s \in \mathcal{S}$, $y(a_i, a_{i+1}, a_i + s) - a_i \leq y(a_i, a_i + h_i, a_i + s) - a_i = a_i - y(a_i - h_i, a_i, a_i - s) = a_i - y(a_{i-1}, a_i, a_i - s)$, where the inequality follows because $h_{i+1} \leq h_i$ and lemma 2 and the equality follows by lemma 3. But then if $b > 0$, an expert with type a_i strictly prefers $y(a_i, a_{i+1}, a_i + s)$ to $y(a_{i-1}, a_i, a_i - s)$ for all s, and since $p((a_i + s) - a_i) = p((a_i - s) + a_i)$ by the symmetry of $p(\cdot)$, $U^E(m_{i+1}, a_i) > U^E(m_i, a_i)$ contradicting the equilibrium condition. ■

Lemma 5 below will be used in the proof of Proposition 3 and Proposition 5. In Lemma

[29] In CS they use a symmetric argument with strictly increasing partial partitions. The reason I use decreasing partitions is that given that $b > 0$ the expected utility of an expert of type t_i when he sends message $m = [t, t_i]$ strictly decreases as t decreases. For increasing partitions, the monotonicity is harder to prove because there are actions on both sides of the expert's peak. (This is the role of Assumption (S), although it is not necessary for the proof of the Theorem).

5 I use Assumption (S) to derive some properties of the function $V(\cdot)$:

Lemma 5 *If* $0 \leq a_{i-1} < a_i < a_{i+1} \leq 1$ *and* $V(a_{i-1}, a_i, a_{i+1}, b) = 0$, *then* $U_i^E(a, a_i, a_i, b) > 0$ *and* $V_1(a, a_i, a_{i+1}, b) < 0$ *for all* $a \in [0, a_i)$, $U_2^E(a_i, a, a_i, b) < 0$ *and* $V_3(a_{i-1}, a_i, a, b) < 0$ *for all* $a \in [a_{i+1}, 1]$, *and* $V(a_{i-1}, a_i, a, b) > 0$ *for all* $a \in [a_i, a_{i+1})$.

Proof of Lemma 5: Equation (7) shows that $U^E(a, a_i, a_i)$ is increasing in a for all $a \leq a_i$ and hence $V_1(a, a_i, a_{i+1}, b) < 0$ for all $a \in [0, a_i]$. Condition (S) and the fact that $V(a_{i-1}, a_i, a_i) > 0$ (which follows by $U_1^E(a, a_i, a_i, b) < 0$ for all $a \in [0, a_i]$ and $V(a_{i-1}, a_i, a_{i+1}, b) = 0$) assures that $U_2^E(a_i, a, a_i, b) < 0$ and $V_3(a_{i-1}, a_i, a, b) < 0$ for all $a \in [a_{i+1}, 1]$, and $V(a_{i-1}, a_i, a, b) > 0$ for all $a \in [a_i, a_{i+1})$. ■

Proof of Proposition 3: Denote by $h_{i+1} = a_{i+1} - a_i$ and $h_i = a_i - a_{i-1}$. By Lemma 3 $U^E(a_i, a_{i+1}, a_i, b) = U^E(0, h_{i+1}, 0, b)$ and $U^E(a_{i-1}, a_i, a_i, b) = U^E(0, h_i, h_i, b)$. Hence $V(a_{i-1}, a_i, a_{i+1}, b)$ is a function only of the length of the intervals h_i and h_{i+1} and not of the location of the intervals. Denote this function as $\tilde{V}(h_i, h_{i+1})$. Namely, $\tilde{V}(h_i, h_{i+1}) = U^E(0, h_{i+1}, 0, b) - U^E(0, h_i, h_i, b)$. Given h, define $\phi(h)$ as the positive number, if it exists, that solves $\tilde{V}(h, \phi(h)) = 0$ (If this equation does not have a solution then I will consider that $\phi(h) = +\infty$). By Condition (S), there is at most one solution to this equation and therefore $\phi(h)$ is a well defined function of h. Proposition 3 is then reduced to prove that $\phi(h)$ is increasing in h, which is immediate from Lemma 5. ■

Proof of Proposition 4: The proof follows the proofs of Theorem 3 and 5 of CS. ■

Proof of Proposition 5: Suppose that for any $0 \leq a_{i-1} \leq a_i \leq a_{i+1} \leq 1$ such that $V(a_{i-1}, a_i, a_{i+1}, b, F) = 0$, we have that $V(a_{i-1}, a_i, a_{i+1}, b, F') > 0$[30]. First I prove that if $a(K)$ and $a'(K)$ are two partial partitions of size K satisfying (A^F) and $(A^{F'})$ respectively, with $a_0(K) = a'_0(K)$ and $a_K(K) = a'_K(K)$ then $a_i(K) > a'_i(K)$. The proof is done by induction on the size of the partition K. If $K = 1$ the statement is vacuous. Suppose $K > 1$ and the statement is true for all $K' = 1, .., K-1$. Suppose by way of contradiction that $a_j(K) \leq a'_j(K)$ for some $j = 1, ..., K-1$. Suppose further that j is the highest index for which this inequality

[30] The opposite case is symmetric.

is satisfied and hence $a_i(K) > a'_i(K)$ for all $j < i < K$. Define ${}^x a \equiv ({}^x a_0, {}^x a_1, ..., {}^x a_j)$ the partial partition that satisfies (A^F) such that ${}^x a_0 = 0$ and ${}^x a_1 = x$. By definition ${}^{a_1(K)} a_j = a_j(K) \leq a'_j(K)$. By the continuity of ${}^x a$ in x there exists an $\bar{x} \geq a_1(K)$ such that ${}^{\bar{x}} a_j = a'_j(K)$ and by Proposition 3, ${}^{\bar{x}} a_i \geq a_i(K)$ for all $0 < i < j$. Denote by $\bar{a} \equiv {}^{\bar{x}} a$. By Lemma 5, there exists a unique $\bar{a}_{j+1} > \bar{a}_j$ such that $V(\bar{a}_{j-1}, \bar{a}_j, \bar{a}_{j+1}, F) = 0$. By the condition of the Proposition, $V(\bar{a}_{j-1}, \bar{a}_j, \bar{a}_{j+1}, F') > 0$. By Proposition 3 $\bar{a}_{j+1} \geq a_{j+1}(K) > a'_{j+1}(K)$, and hence using the fact that $\bar{a}_j = a'_j(K)$ and Lemma 5:

$$V(\bar{a}_{j-1}, a'_j(K), a'_{j+1}(K), F') > 0 \tag{8}$$

On the other hand, applying the induction hypothesis to $(\bar{a}_0, ..., \bar{a}_j)$ and $(a'_0(K), ..., a'_j(K))$, $a'_i(K) < \bar{a}'_i$ for all $0 < i < j$. But then using Lemma 5,

$$V(\bar{a}_{j-1}, a'_j(K), a'_{j+1}(K), F') < V(a'_{j-1}(K), a'_j(K), a'_{j+1}(K), F') = 0$$

which contradicts (8) and establishes the result.

Finally, let $a'(N(b, F'))$ be the partition equilibrium of $F' - PI$ of size $N(b, F')$. Let $\bar{a}$ the partial partition satisfying (A^F) such that $\bar{a}_1 = a'_1(N(b, F'))$, then by Proposition 3 and the previous result, $\bar{a}_i < a'_i(N(b, F'))$. In particular, $\bar{a}$ is at least of length $N(b, F')$. Hence $N(b, F) \geq N(b, F')$ ■

The following results are used for the proof of Proposition 6. Lemma 6 transfer the symmetric properties of the best response established in Lemma 3 to the expected best response.

Lemma 6 *Given a message $m = [\underline{a}, \bar{a})$, and a type θ, the expected action of the decision maker $\hat{y}(\underline{a}, \bar{a}, \theta)$ satisfies the following properties:*

1. *$\hat{y}(\underline{a}, \bar{a}, \theta)$ is increasing in all its arguments and $\underline{a} < \hat{y}(\underline{a}, \bar{a}, \theta) < \bar{a}$*

2. *$\hat{y}(a, a + h, \theta) = a + \hat{y}(0, h, \theta - a)$.*

3. *$\hat{y}(0, h, \frac{h}{2} + \theta) - \frac{h}{2} = \frac{h}{2} - \hat{y}(0, h, \frac{h}{2} - \theta)$. In particular $\hat{y}(0, h, \frac{h}{2}) = \frac{h}{2}$.*

Proof of Lemma 6: All the results are immediate implications of Lemma 3, Lemma 2 and a change in variable.

1. It is a direct implication of Lemma 2 and the fact that s and θ are affiliated.

2. $\hat{y}(a, a+h, \theta) = \int_{\mathcal{S}} y(a, a+h, s)p(s-\theta)ds = \int_{\mathcal{S}} a + y(0, h, s-a)p(s-\theta)ds =$
$a + \int_{\mathcal{S}} y(0, h, s)p(s-(\theta-a))ds = a + \hat{y}(0, h, \theta - a)$.

3. $\hat{y}(0, h, \frac{h}{2}+\theta) - \frac{h}{2} = \int_{\mathcal{S}} y(0, h, s)p(s-(\frac{h}{2}+\theta))ds - \frac{h}{2} = \int_{\mathcal{S}} h - y(0, h, h-s)p(s-(\frac{h}{2}+\theta))ds - \frac{h}{2} =$
$\frac{h}{2} - \int_{\mathcal{S}} y(0, h, s)p(\frac{h}{2} - \theta - s)ds = \frac{h}{2} - \hat{y}(0, h, \frac{h}{2} - \theta)$.

Finally using this equation for $\theta = 0$, $\hat{y}(0, h, \frac{h}{2}) = \frac{h}{2}$. ■

Lemma 7 is the key result for Proposition 6. It states that as the length of the interval increases, the distance between the (CS) action and the expected action from the point of view of the boundary type increases.

Lemma 7 $\frac{\partial}{\partial h}(h/2 - \hat{y}(0, h, 0)) > 0$

Proof of Lemma 7: By Lemma 6.3, $\hat{y}(0, h, \theta) + \hat{y}(0, h, h-\theta) = h$. Totally differentiating this equation with respect to h:

$$\hat{y}_2(0, h, \theta) + \hat{y}_2(0, h, h-\theta) + \hat{y}_3(0, h, h-\theta) = 1$$

where all terms in the left hand side are positive by Lemma 6.1. It is therefore enough to show that if $\theta < h/2$ then $\hat{y}_2(0, h, \theta) \leq \hat{y}_2(0, h, h-\theta)$ since this would imply that $\hat{y}_2(0, h, \theta) < 1/2$ for all $\theta < h/2$, and in particular that $h/2 - \hat{y}(0, h, 0)$ is increasing in h.

First note that given quadratic loss utilities, $y(0, h, s) = \int_0^h \theta \frac{p(\theta-s)}{\int_0^h p(t-s)dt} d\theta$, and therefore:

$$y_2(0, h, s) = \int_0^h (h-\theta) \frac{p(h-s)p(\theta-s)}{(\int_0^h p(t-s)dt)^2} d\theta = \int_0^h (h-\theta) g(h|0, h, s) g(\theta|0, h, s) d\theta$$

and therefore if $s > 0$:

$$\begin{aligned} y_2(0,h,\tfrac{h}{2}+s) - y_2(0,h,\tfrac{h}{2}-s) &= \int_0^h (h-\theta)\left[g(h|0,h,\tfrac{h}{2}+s)g(\theta|0,h,\tfrac{h}{2}+s)\right. \\ &\quad \left. -g(h|0,h,\tfrac{h}{2}-s)g(\theta|0,h,\tfrac{h}{2}-s)\right] d\theta \\ &= \int_0^h (h-\theta)\left[g(h|0,h,\tfrac{h}{2}+s)g(h-\theta|0,h,\tfrac{h}{2}-s)\right. \\ &\quad \left. -g(h|0,h,\tfrac{h}{2}-s)g(h-\theta|0,h,\tfrac{h}{2}+s)\right] d\theta \\ &> 0 \end{aligned} \tag{9}$$

where the equality follows by Lemma 3.3 and the inequality follows because $g(\theta|\cdot,s)$ is log-supermodular in (θ,s) (recall that θ and s are affiliated).

Finally, if $\theta < \frac{h}{2}$,

$$\hat{y}_2(0,h,\theta) - \hat{y}_2(0,h,h-\theta) = \int_{\mathcal{S}} y_2(0,h,s)(p(s-\theta) - p(s-h+\theta))ds$$
$$= \int_{s>0} \left(y_2(0,h,\tfrac{h}{2}+s) - y_2(0,h,\tfrac{h}{2}-s)\right)\left(p(\tfrac{h}{2}+s-\theta) - p(\tfrac{h}{2}+s-(h-\theta))\right) ds \leq 0$$

where the second equality follows by dividing the signal space at $h/2$, and the inequality follows because the first term is always positive by (9) and the second is negative whenever $\theta < \frac{h}{2}$. ■

Proof of Proposition 6:

If $\hat{y}(m_{i+1},a_i) \leq a_i + b$, then by Lemma 6.1 $a_i < \hat{y}(m_{i+1},a_i) \leq a_i + b$ and $\hat{y}(m_i,a_i) < a_i$. So clearly $(\hat{y}(m_{i+1},a_i) - (a_i+b))^2 \leq b^2 < (\hat{y}(m_i,a_i) - (a_i+b))^2$. This together with the fact that $a_i + b$ is equidistant to $y_{CS}(a_{i-1},a_i)$ and $y_{CS}(a_i,a_{i+1})$, implies the information effect for message m_{i+1} is bigger than for message m_i.

Suppose now that $\hat{y}(m_{i+1},a_i) > a_i + b$. In this case comparing the distance between the expected actions and the expert's peak is equivalent to comparing the distance between the expected actions and the respective CS actions. The bigger the distance between the expected action and the CS action, the closer is the expected action to the expert's peak and hence the bigger is the information effect.

Using Lemma 6.2 and 6.3, the distance between the expected actions and the CS actions can be written as a function that depends only on the length of the intervals:

$$\begin{aligned}
y_{CS}(m_{i+1}) - \hat{y}(a_i, a_{i+1}, a_i) &= \tfrac{a_i + a_{i+1}}{2} - \hat{y}(a_i, a_{i+1}, a_i) = \tfrac{h_{i+1}}{2} - \hat{y}(0, \tfrac{h_{i+1}}{2}, 0) \\
\hat{y}(a_{i-1}, a_i, a_i) - y_{CS}(m_i) &= \hat{y}(a_{i-1}, a_i, a_i) - \tfrac{a_{i-1} + a_i}{2} = \hat{y}(0, h_i, h_i) - \tfrac{h_i}{2} = \tfrac{h_i}{2} - \hat{y}(0, \tfrac{h_i}{2}, 0)
\end{aligned} \tag{10}$$

where $h_{i+1} = a_{i+1} - a_1$ and $h_i = a_i - a_{i-1}$.

Since $h_{i+1} > h_i$, then to conclude that the information effect for message m_{i+1} is bigger than for message m_i it is enough to show that $\frac{h}{2} - \hat{y}(0, h, 0)$ increases with h, which is proved in Lemma 7. ■

Lemma 8 is used in the proof of Proposition 7. It establishes some useful symmetric properties to the variance of the decision maker actions:

Lemma 8 *Given a message $m = [\underline{a}, \overline{a})$, and a type θ, the variance of the actions of the decision maker $\hat{\sigma}(\underline{a}, \overline{a}, \theta)$ satisfies the following properties:*

1. $\hat{\sigma}(a, a + h, \theta) = \hat{\sigma}(0, h, \theta - a)$.

2. $\hat{\sigma}(0, h, \frac{h}{2} + \theta) = \hat{\sigma}(0, h, \frac{h}{2} - \theta)$.

Proof of Lemma 8: All the results are immediate implications of Lemma 3, Lemma 6 and a change in variable.

1. $\hat{\sigma}(a, a + h, \theta) = \int_{\mathcal{S}} (y(a, a + h, s) - \hat{y}(a, a + h, \theta))^2 p(s - \theta) ds =$
$\int_{\mathcal{S}} (y(0, h, s - a) - \hat{y}(0, h, \theta - a))^2 p(s - \theta) ds =$
$\int_{\mathcal{S}} (y(0, h, s) - \hat{y}(0, h, \theta))^2 p(s - (\theta - a)) ds = \hat{\sigma}(0, h, \theta - a)$.
2. $\hat{\sigma}(0, h, \frac{h}{2} + \theta) = \int_{\mathcal{S}} (y(0, h, s) - \hat{y}(0, h, \frac{h}{2} + \theta))^2 p(s - (\theta + \frac{h}{2})) ds =$
$\int_{\mathcal{S}} (y(0, h, h - s) - \hat{y}(0, h, \frac{h}{2} - \theta))^2 p(s - (\theta + \frac{h}{2})) ds =$
$\int_{\mathcal{S}} (y(0, h, s) - \hat{y}(0, h, \frac{h}{2} - \theta))^2 p(\frac{h}{2} - \theta - s) ds = \hat{\sigma}(0, h, \frac{h}{2} - \theta)$. ■

Proof of Proposition 7: Using Remark 8 the information effect for a boundary type can be written as a function of just the size of the interval sent: $\hat{\sigma}(a_i, a_{i+1}, a_i) = \hat{\sigma}(0, h_{i+1}, 0)$ and $\hat{\sigma}(a_{i-1}, a_i, a_i) = \hat{\sigma}(0, h_i, 0)$. Hence to compare the risk effect of sending m_i versus m_{i+1} it is enough to show that $\frac{\partial}{\partial h} \hat{\sigma}^2(0, h, 0) > 0$. But this follows because by (9) the distance between the decision maker's actions increases with h. ■

The following Lemma will be used in the proof of Proposition 8. It states that an expert with type $\theta = 0$, strictly prefers to send message $[0, 4b]$ to perfectly reveal himself.

Lemma 9 $V(0, 0, 4b, b) > 0$

Proof of Lemma 9: Recall that $V(0, 0, 4b, b) = U^E(0, 4b, 0, b) - U^E(0, 0, 0, b)$.

$$V(0,0,4b,b) = -\int_{\mathcal{S}}(y(0,4b,s)-b)^2 p(s)ds + b^2$$
$$= \int_{\mathcal{S}}(2b - y(0,4b,s))y(0,4b,s)p(s)ds$$
$$= \int_{s>0}(2b - y(0,4b,2b+s))y(0,4b,2b+s)p(2b+s) + (2b - y(0,4b,2b-s))y(0,4b,2b-s)p(2b-s)ds$$
$$= \int_{s>0}(2b - y(0,4b,2b-s))(y(0,4b,2b-s)p(2b-s) - y(0,4b,2b+s)p(2b+s))ds$$

where the third equality follows by dividing the signal space at $2b$, and the last equality follows by the symmetric properties of the functions (see Lemma 3). The first factor in the integral is always positive and using the fact that for quadratic-loss preferences $y(0, 4b, s) = \int_0^{4b} \theta g(\theta|0, 4b, s)d\theta$ the second factor can be written as:

$$y(0,4b,2b-s)p(2b-s) - y(0,4b,2b+s)p(2b+s) =$$
$$= \int_0^{4b} p(t-2b+s)dt \left[\int_0^{4b} \theta \left(g(\theta|0,4b,2b-s)g(4b|0,4b,2b+s) - g(\theta|0,4b,2b+s)g(4b|0,4b,2b-s)\right) d\theta\right]$$
$$> 0$$

where for the equality I am using the fact that $g(4b|0, 4b, 2b+s) = \frac{p(2b-s)}{\int_0^{4b} p(t-(2b+s))}$, $g(4b|0, 4b, 2b-s) = \frac{p(2b+s)}{\int_0^{4b} p(t-(2b-s))}$ and $\int_0^{4b} p(t-(2b+s)) = \int_0^{4b} p(t-(2b-s))$, and for the inequality I am using the affiliation of s and θ. ■

Proof of Proposition 8: To prove the first statement of the Proposition, observe that by Lemma 9, $V(0, 0, 1, \frac{1}{4}) > 0$. By continuity of $V(\cdot)$ in b, there exists a $\bar{b} < \frac{1}{4}$ such that $V(0, 0, 1, b) > 0$ for all $b \in (\bar{b}, \frac{1}{4}]$. By Lemma 5 $V(0, 0, a, b) > 0$ for all $a \in [0, 1]$, so there can no be information transmitted in equilibrium.

Finally the second statement follows by the fact that as $\lambda \to \infty$ the conditional distribution $G(\theta|s)$ converges to the degenerate distribution on s. And hence, there is a precision λ^b, such that the lottery induced by message $[0, 1]$ is preferred by the expert with type $\theta = 0$

and bias b to the constant action $y=0$. Namely, $V(0,0,1,b,F^{\lambda})>0$, and by Lemma 5, $V(0,0,a,b,F^{\lambda})>0$ for all $a\in[0,1]$, so there can no be information transmitted in equilibrium. ■

Appendix B: Uniform Private Information Model

Recall that the optimal action in this model is:

$$y(\underline{a},\overline{a},s,\delta)=\frac{\max\{\underline{a},s-\delta\}+\min\{\overline{a},s+\delta\}}{2}$$

If $\overline{a}-\underline{a}\le 2\delta$ the expectation and the second moment of the decision maker's actions from the point of view of the expert are given by:

$$\begin{aligned}\hat{y}(\underline{a},\overline{a},\theta,\delta) &= \tfrac{\underline{a}+\overline{a}}{2}+\tfrac{1}{8\delta}(\overline{a}-\underline{a})(2\theta-\overline{a}-\underline{a})\\ E(y^2|\underline{a},\overline{a},\theta,\delta) &= \tfrac{(\underline{a}+\overline{a})^2}{4}+\tfrac{1}{24\delta}[(\theta+\overline{a})^3-(\theta+\underline{a})^3-3(\underline{a}+\overline{a})^2(\overline{a}-\underline{a})]\end{aligned}$$

If $\overline{a}-\underline{a}>2\delta$, the expectation and second moment of the decision maker's actions are:

$$\hat{y}(\underline{a},\overline{a},\theta,\delta)=\begin{cases}\frac{\delta+\underline{a}+\theta}{2}+\frac{1}{8\delta}(\underline{a}-\theta)^2 & if\quad \theta<\min\{\underline{a}+2\delta,\overline{a}-2\delta\}\\ \frac{\underline{a}+\overline{a}}{2}+\frac{1}{8\delta}(\overline{a}-\underline{a})(2\theta-\overline{a}-\underline{a}) & if\quad \overline{a}-2\delta<\theta<\underline{a}+2\delta\\ \theta & if\quad \underline{a}+2\delta<\theta<\overline{a}-2\delta\\ \frac{\theta+\overline{a}-\delta}{2}+\frac{1}{8\delta}(\theta-\overline{a})^2 & if\quad \theta\ge\max\{\underline{a}+2\delta,\overline{a}-2\delta\}\end{cases}$$

$$E(y^2|\underline{a},\overline{a},\theta,\delta)=\begin{cases}\frac{1}{24\delta}[4(\underline{a}+\delta)^3+4(\theta+\delta)^3-(\underline{a}+\theta)^3] & if\quad \theta<\min\{\underline{a}+2\delta,\overline{a}-2\delta\}\\ \frac{1}{24\delta}[4(\underline{a}+\delta)^3-4(\overline{a}-\delta)^3+(\theta+\overline{a})^3-(\underline{a}+\theta)^3] & if\quad \overline{a}-2\delta<\theta<\underline{a}+2\delta\\ \theta^2+\frac{\delta^2}{3} & if\quad \underline{a}+2\delta<\theta<\overline{a}-2\delta\\ \frac{1}{24\delta}[(\overline{a}+\theta)^3-4(\overline{a}-\delta)^3-4(\theta-\delta)^3] & if\quad \theta\ge\max\{\underline{a}+2\delta,\overline{a}-2\delta\}\end{cases}$$

Given quadratic-loss utilities, $U^E(\underline{a},\overline{a},\theta,b,\delta)=-E(y^2|\underline{a},\overline{a},\theta,b)+2\hat{y}(\underline{a},\overline{a},\theta)-(\theta+b)^2$. In particular, denoting by $h_i=a_i-a_{i-1}$ and $h_{i+1}=a_{i+1}-a_i$, the expected utilities of an expert with type $\theta = a_i$ that sends message $[a_{i-1}, a_i]$ and $[a_i, a_{i+1}]$ are respectively:

$$\begin{aligned} U^E(a_{i-1}, a_i, a_i, b, \delta) &= \begin{cases} -(\frac{h_i}{2} + b)^2 + \frac{1}{12\delta}h_i^3 + \frac{b}{4\delta}h_i^2 & if \quad h_i \leq 2\delta \\ -\delta b - \frac{\delta^2}{3} - b^2 & if \quad h_i > 2\delta \end{cases} \\ U^E(a_i, a_{i+1}, a_i, b, \delta) &= \begin{cases} -(\frac{h_{i+1}}{2} - b)^2 + \frac{1}{12\delta}h_{i+1}^3 - \frac{b}{4\delta}h_{i+1}^2 & if \quad h_{i+1} \leq 2\delta \\ \delta b - \frac{\delta^2}{3} - b^2 & if \quad h_{i+1} > 2\delta \end{cases} \end{aligned} \tag{11}$$

Remark 1 *Condition (S) is satisfied in the Uniform private information model.*

Proof Remark 1: Taking the derivative of $U^E(0, h, 0, b, \delta)$ in equation (11) with respect to h: $\frac{\partial}{\partial h}U^E(0, h, 0, b, \delta) = \frac{1}{4\delta}(h - 2b)(h - 4\delta)$ if $h \leq 2\delta$, 0 otherwise. If $b > 2\delta$ no information can be sent in equilibrium and there is nothing to check[31]. If $b < 2\delta$, $U^E(0, h, 0, b, \delta)$ is increasing for $h < 2b$ and decreasing for $h > 2b$. ■

For the proof of Theorem 2 I use Proposition 9 and Proposition 10 below:

Proposition 9 *Suppose that $a_{i+1} - a_i < 2\delta$ and $V(a_{i-1}, a_i, a_{i+1}, \delta) = 0$, then $V(a_{i-1}, a_i, a_{i+1}, \delta') >$ 0 for all $\frac{a_{i+1} - a_1}{2} < \delta' < \delta$.*

Proof Proposition 9: By Lemma 2, if $V(a_{i-1}, a_i, a_{i+1}, \delta) = 0$, $h_{i+1} \equiv a_{i+1} - a_i > a_i - a_{i-1} \equiv h_i$, and hence, $h_{i+1} < 2\delta$ implies $h_i < 2\delta$. Since $V(a_{i-1}, a_i, a_{i+1}, \delta) = 0$ we have that:

$$\begin{aligned} V(a_{i-1}, a_i, a_{i+1}, \delta') &= V(a_{i-1}, a_i, a_{i+1}, \delta') - V(a_{i-1}, a_i, a_{i+1}, \delta) \\ &= (\frac{1}{12\delta'} - \frac{1}{12\delta})(h_{i+1}^2(h_{i+1} - 3b) - h_i^2(h_i - 3b)) \end{aligned} \tag{12}$$

which is positive for $\delta' < \delta$ as long as $h_{i+1} > 3b$. Note that as δ goes to infinity, the signal becomes uninformative resulting in the CS setup where $h_{i+1}^{CS} = h_i^{CS} + 4b \geq 4b$. Therefore, by (12), as the signal becomes more informative, the required h_{i+1} that makes $\theta = a_i$ indifferent between m_i and m_{i+1} becomes larger, implying that $h_{i+1} \geq 4b$ always holds, and thus $V(a_{i-1}, a_i, a_{i+1}, \delta') > 0$. ■

Consider now the case $a_{i+1} - a_1 > 2\delta$. Observe that it cannot be that $a_i - a_{i-1} > 2\delta$ as well, because in that case by (11) the expert with type a_i strictly prefers m_{i+1}. Since by

[31] See Remark 2

Lemma 2, intervals in equilibrium are increasing in size, the only interval that might be larger than 2δ is the last one. The following remark summarises this argument.

Remark 2 *For any equilibrium partition* $a = \{0 = a_0 < a_1 < ... < a_{N-1} < a_N\}$, $h_i = a_i - a_{i-1} < 2\delta$ *for* $1 \leq i \leq N-1$.

The following proposition shows that whenever $a_i - a_{i-1} < 2\delta < 1 - a_i$ and $V(a_{i-1}, a_i, 1, \delta) = 0$ then $V(a_{i-1}, a_i, 1, \delta') > 0$ for $\frac{a_i - a_{i-1}}{2} < \delta' < \delta$.

Proposition 10 *Suppose that* $a_i - a_{i-1} < 2\delta < 1 - a_i$ *and* $V(a_{i-1}, a_i, 1, \delta) = 0$ *then* $V(a_{i-1}, a_i, 1, \delta') > 0$ *for* $\frac{a_i - a_{i-1}}{2} < \delta' < \delta$.

Proof Proposition 10: By (11), $V(a_{i-1}, a_i, a_{i+1}, \delta) = \delta b - \frac{\delta^2}{3} - b^2 + (\frac{h_i}{2} + b)^2 - \frac{1}{12\delta} h_i^3 - \frac{b}{4\delta} h_i^2$. Taking the derivative with respect to δ:

$$\begin{aligned} \frac{\partial}{\partial \delta} V(a_{i-1}, a_i, a_{i+1}, \delta) &= b - \frac{2\delta}{3} + \frac{1}{12\delta^2} h_i^3 + \frac{b}{4\delta^2} h_i^2 \\ &< b - \frac{2\delta}{3} + \frac{1}{12\delta^2}(2\delta)^3 + \frac{b}{4\delta^2}(2\delta)^2 = 0 \end{aligned}$$

where the inequality follows because, by assumption $h_i < 2\delta$. $V(\cdot)$ decreasing in δ combined with $V(a_{i-1}, a_i, 1, \delta) = 0$ implies $V(a_{i-1}, a_i, 1, \delta') > 0$ for $\frac{a_i - a_{i-1}}{2} < \delta' < \delta$. ■

Proof Theorem 2: The theorem is a direct implication of Propositions 9, 10, Remark 2 and Proposition 5. ■