# Property Rights*

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PRELIMINARY AND INCOMPLETE

November 3, 2008

## 1 Introduction

Every organization, whether a firm, a nonprofit organization, or a society, must confront two basic problems. The first is the creation of incentives for efficient behavior among its members. The second is the efficient allocation among those members of the resources available to and produced by the organization. The achievement of these two goals are closely related, because the rule for allocating resources generally affects individuals' incentives. As a result, the two goals often come into conflict, necessitating trade-offs between them.

Organizations have many ways of trying to achieve these goals. Sometimes they use contracts. These may explicitly specify behavior or may attempt to indirectly encourage desirable behavior through incentive pay. Other times they may allocate decision rights to parties and leave the parties considerable discretion. In this chapter, we focus on one particular instrument: the allocation of property rights over assets.

The basic concept of a property right is relatively simple: A property right gives the owner of an asset the right to the use and benefits of the asset, and the right to exclude others from them. It also, typically, gives the owner the freedom to transfer these rights to others. Roman law referred to these elements as *usus* (the right to use), *abusus* (the right to encumber or transfer), and *fructus* (the right to the fruits). The American jurist Oliver Wendell Holmes put it this way,

> But what are the rights of ownership? They are substantially the same as those incident to possession. Within the limits prescribed by policy, the owner is allowed to exercise his natural powers over the subject matter uninterfered with, and is more or less protected in excluding other people from such interference. The owner is allowed to exclude all, and is accountable to no one but him.[1]

*Forthcoming in R. Gibbons and J. Roberts, eds., *Handbook of Organizational Economics*, Princeton University Press.

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[1] **CITE** [1881], quoted in Grossman and Hart [1986].

In practice, however, complications can arise. First, these rights of ownership are not necessarily bundled together. For example, some stockholders of a firm may own a share of its profits, but may not have the right to vote on the use of the firm's assets. Similarly, an individual may possess the right to use an asset, such as a community garden or lake, but may not have the right to exclude others from doing so. Lastly, in some cases an owner may not possess the right to transfer his ownership rights to others, as with the prohibition against slavery.

In addition, property rights are in practice often held collectively. For example, no single shareholder in a firm may be able to use the firm's assets as he sees fit. Yet, a majority of the shareholders, should they reach an agreement, can do so.

The specification of property rights is, in fact, both a contract and an example of specifying decision rights *(*also called *entitlements* in the legal literature*),* which may provide rights to take certain actions ("rights of inclusion") or to prevent others from taking certain actions ("rights of exclusion"). As such, although our focus is on property rights, we will note that many of the basic principles we derive apply as well to the design of other types of contracts and to the specification of other types of decision rights..

Our discussion focuses on implications of the allocation of property rights, with a particular emphasis on determining the surplus-maximizing allocation of those rights. This, of course, is also the subject of the celebrated *Coase Theorem (Coase [1960]).* Two versions of this "Theorem" are worth distinguishing. What one might call the "first-best" Coase Theorem says that if bargaining takes place under conditions of complete information and the parties can fully specify behavior in a contract (or, as we shall see, at least eliminate negative externalities), then the outcome will be first-best efficient regardless of the allocation of property rights. In such settings, property rights are simply irrelevant. We will be concerned instead with situations in which such contracts are not available. The "second-best" Coase Theorem makes a less ambitious claim: if property rights are tradable and bargaining takes place under conditions of complete information, we may expect individuals to trade to a surplus-maximizing allocation of property rights, which will be independent of their initial allocation.[2] Unlike the first-best Coase Theorem, however, the final outcome need not achieve the first-best. When this second-best Coase Theorem applies, the initial allocation of property rights has no efficiency implications, but the final property rights allocation that is achieved through bargaining matters. In that case, efficiency requires that the property rights end up at the surplus-maximizing property rights allocation we identify. In some cases, however, bargaining over the allocation of property rights may be imperfect, or even impossible. It could be plagued by asymmetric information (see Section 4) or generate externalities on third parties (see Section 5), or the affected parties may not even meet until after important economic decisions have been made.

[2] This statement presumes a setting with quasi-linear preferences. Absent quasi-linearity, the final allocation of property rights will be Pareto efficient, but may depend on the initial allocation of property rights due to wealth effects.

In such cases, the initial allocation of property rights matters, and efficiency will be served by initially specifying them in the surplus-maximizing manner we identify.

The chapter is organized as follows. We begin in Section 2 by identifying two fundamental, but potentially conflicitng, forces affecting the efficient allocation of property rights: while all else equal an asset should be owned by the party who values it most, ownership may also affect individuals' incentives for taking noncontractible actions. Efficiency often involves a trade-off between these two concerns. We first illustrate this trade-off in a classic model of property rights, the Tragedy of the Commons, and then derive some general results about the effects of property rights and their surplus-maximizing allocation in a static model.

In Section 3 we explore the role of property rights in dynamic settings in which parties may make ex ante investments, and may also bargain ex post. We show how the lessons from the static model of Section 2 can be applied in this setting to derive several well-known results, including those of Edlin and Reichelstein [1995] and Hart and Moore [1990]. We also discuss the sensitivity of Hart and Moore's conclusions to various of their assumptions.

In Sections 4 and 5 we investigate the role of the property rights allocation in generating efficiency when bargaining is possible, but imperfect. Section 4 examines the case in which there is asymmetric information among the parties, while Section 5 focuses on settings in which multiple parties bargain in a bilateral fashion.

In Section 6 we expand our perspective by considering the possibility of more complicated property rights allocations, such as options to own. We ask what optimal property rights mechanisms look like. Our discussion also connects to the legal literature on property rules versus liability rules.

## 2 Property Rights, Externalities, and Efficiency

Property rights can affect economic efficiency through both direct and indirect channels. The direct effect is simple: other things equal, efficiency calls for the individuals who most value access to an asset to hold the property rights to it. (This value may include benefits from preventing others' access to the asset and may also be uncertain.) The indirect effects, however, are more subtle. By determining who has access to an asset, the allocation of property rights can alter individual behavior.

The importance of property rights for incentives has long been recognized. Consider the following passage from an early 19th century text:[3]

> Suppose that the earth yielded spontaneously all that is now produced by cultivation; still without the institution of property it

[3] A much earlier example can be found in Aristotle's *Politics*: "[T]hat which is common to the greatest number has the least care bestowed upon it. Everyone thinks chiefly of his own, hardly at all of the common interest; and only when he is himself concerned as an individual." (*Politics*, 1261b34) **CHECK CITE AND QUOTE**

> could not be enjoyed; the fruit would be gathered before it was ripe, animals killed before they came to maturity; for who would protect what was not his own...?
>
> Mrs. J.H. Marcet
>
> *Conversations in Political Economy, 3rd. Edition* [1819, 60-1][4]

When behavior generates externalities, as in the quote above, efficiency may call for allocating an asset to an individual or individuals who do not derive the greatest benefit from it, to induce desirable behavior.

In this section, we explore this trade-off in a simple static setting. We first illustrate these effects in a classic model of property rights, the Tragedy of the Commons, and then derive some more general results about the efficiency effects of property rights. These results will turn out to apply as well to a variety of dynamic settings, as we'll see in Section 3.

## 2.1 The Tragedy of the Commons

Consider the classic Tragedy of the Commons, simplified to the case in which there are two individuals, denoted $i = 1, 2$. There is a single asset, a lake. Each individual $i$ can exert some effort fishing, denoted by $a_i \in [0, \overline{a}_i] \subset \mathbb{R}$. Access to the lake, which is determined by the specification of property rights, is captured by the variables $(x_1, x_2) \in \{0,1\}^2$. Individual $i$ has access to the lake if $x_i = 1$, and does not if $x_i = 0$. Private ownership of the lake gives the owner access to the lake and the ability to exclude all others from access to it, and given the negative externalities we assume below, the owner always exercises this right of exclusion.[5] So private ownership by individual 1 results in $(x_1, x_2) = (1, 0)$; private ownership by individual 2 leads to $(x_1, x_2) = (0, 1)$. In contrast, with common (public) ownership, both individuals have the right to use the lake and neither has a right to exclude the other, so $(x_1, x_2) = (1, 1)$. (Note that this is an instance in which the right to use the asset and the right to exclude others from its use are not always bundled together.)

Given access $(x_1, x_2)$ and efforts $(a_1, a_2)$, individual $i$'s benefit from being on the lake, including both enjoyment of the lake and fish caught is given by the function $b_i(x_i, x_j, a_i, a_j)$. His cost of fishing effort is $\phi_i(a_i)$. The function $b_i(\cdot)$ is nonincreasing in $x_j$, since when individual $j$ has access to the lake, he exerts a (weakly) negative externality on individual $i$. It is nonincreasing in $a_j$ because greater effort by individual $j$ weakly reduces individual $i$'s catch and/or may reduce his enjoyment of the lake. In addition, $b_i(0, 1, a_i, a_j) = 0$ for all $(a_i, a_j)$: individual $i$ experiences no benefit from $a_i$ nor any externality from $a_j$ when he is denied access to the lake, and we normalize his benefit in that case to zero. The quantity $b_i(1, 1, 0, a_j) \geq 0$ represents either individual $i$'s inherent enjoyment of the lake when he has access to it, or his catch with zero effort ($a_i = 0$ is the level of effort he would exert just for pleasure; this benefit

[4] Quoted in Baumol and Oates [1988, 28n].

[5] We will return to the issue of whether rights of exclusion will in fact be exercised in Section ???.

is nonnegative since individual $i$ has the option of whether to use the lake when he has access.) The cost of zero effort is normalized to zero: $\phi_i(0) = 0$.

The first-best assignment of property rights and effort levels are

$$(x_1^*, x_2^*, a_1^*, a_2^*) \in \Sigma^* \equiv \arg \max_{(x_1,x_2,a_1,a_2)} b_1(x_1, x_2, a_1, a_2) + b_2(x_2, x_1, a_2, a_1) - \phi_1(a_1) - \phi_2(a_2). \tag{1}$$

Provided that only others' fishing generates negative externalities, not their access per se, so that $b_i(1, x_j, a_i, 0)$ does not depend on $x_j$ for $i = 1, 2$ and $j \neq i$, there is a first-best outcome in which both individuals have access to the lake, since any outcome in which some individual is denied access generates weakly less aggregate surplus than an outcome in which that individual has access but does fish. Efficient effort intensities in that case satisfy

$$(a_1^*, a_2^*) \equiv \arg \max_{(a_1,a_2)} b_1(1, 1, a_1, a_2) + b_2(1, 1, a_2, a_1) - \phi_1(a_1) - \phi_2(a_2). \tag{2}$$

Suppose, however, that neither the precise level of fishing effort nor an individual's catch can be monitored. Then, if both individuals have access to the lake due to common ownership, they will each choose their fishing effort to maximize their individual payoff. An equilibrium $(a_1^{\circ\circ}, a_2^{\circ\circ})$ then satisfies

$$a_i^{\circ\circ} \in \arg \max_{a_i} b_i(1, 1, a_i, a_j^{\circ\circ}) - \phi_i(a_i) \text{ for } i = 1, 2. \tag{3}$$

When the first-best involves access for both individuals but negative externalities create inefficiencies, assigning ownership to one individual may be preferable. The owner will deny access to the non-owner (who will choose to exert no effort), and will choose his fishing effort to equal

$$a_i^{\circ} = \arg \max_{a_i} b_i(1, 0, a_i, 0) - \phi_i(a_i). \tag{4}$$

Since $b_j(1, 1, 0, a_i^{\circ}) \geq 0$ for $j \neq i$, aggregate surplus would be at least as large allowing individual $j$ access to the lake, provided his fishing effort remained zero (and only fishing, not access, generates externalities). Unfortunately, denial of access may be the only way to keep him from fishing.

In general, either private or common ownership may generate a larger aggregate surplus. The effort level $a_i^{\circ}$ given in (4) is efficient given the fact that the non-owner $j$ has no access to the lake. But since denial of access to the non-owner results in the loss of his net benefit $b_j(a_j^{\circ\circ}, a_i^{\circ\circ}, 1, 1) - \phi_j(a_j^{\circ\circ})$, this assignment of property rights may not result in a greater surplus than common ownership. As an illustration, consider the following linear-quadratic example:

**Example 1** ***The Linear-Quadratic Tragedy of the Commons*** *Let* $b_i(x_i, x_j, a_i, a_j) = x_i \max\{0, \beta_i + v_i a_i - \gamma_j a_j x_j\}$ *and* $\phi_i(a_i) = \mu_i a_i + (c_i/2)(a_i)^2$*, where* $(v_i, \mu_i, c_i) >> 0$*,* $(\beta_i, \gamma_i) \geq 0$*, and* $v_i \geq \mu_i + \gamma_i$*. Assume also that individual i's largest possible effort level,* $\overline{a}_i$*, satisfies* $\overline{a}_i > (v_i - \mu_i)/c_i \geq 0$ *for* $i = 1, 2$ *and* $j \neq i$*. The parameter* $\beta_i$ *is individual i's enjoyment from having access to the lake,* $v_i$ *is his value of fish, and* $(\mu_i, c_i)$ *affect his cost of effort. The parameters* $\gamma_1$ *and*

*$\gamma_2$ capture the extent of the negative externality: externalties are absent when $\gamma_1 = \gamma_2 = 0$. In contrast, if $\gamma_i = v_i - \mu_i$, then externalities eliminate the social value of individual i's fishing when individual j also uses the lake.*

*Since an individual generates an externality only if he fishes, there is always an efficient outcome in which both individuals have access to the lake. Indeed, if $\beta_i > \gamma_j \overline{a}_j$ for $i = 1, 2$ and $j \neq i$, using the lake without fishing generates a strictly positive payoff for an individual regardless of the other individual's level of fishing effort, which implies that all first-best outcomes involve giving both individuals access to the lake. The efficient effort levels, which satisfy (2), are then $a_i^* = (v_i - \mu_i - \gamma_i)/c_i$ for $i = 1, 2$. In contrast, equilibrium effort levels with common ownership, which satisfy (3), are $a_i^{\circ\circ} = (v_i - \mu_i)/c_i$ for $i = 1, 2$. When $\gamma_i = \gamma_j = 0$ so that externalities are absent, the equilibrium is efficient. This might be the case if the lake is large, or if fish populations are localized (such as shellfish) and access can be restricted to particular sections of the lake. If instead $(\gamma_1, \gamma_2) >> 0$, then equilibrium effort levels under common ownership are inefficiently high.*

*In some cases, there is also a first-best outcome in which only one individual has access to the lake. For example, suppose that $v_2 = \mu_2$ and $\beta_2 = 0$. Then, individual 2's fishing does not generate a positive social surplus, nor does individual 2 have any direct benefit from access. In that case, there is a first-best outcome in which only individual 1 has access to the lake: $(x_1^*, x_2^*, a_1^*, a_2^*) = (1, 0, (v_1 - \mu_1)/c_1, 0)$. With private ownership by individual 1, there are no externalities from fishing and equilibrium effort levels equal their first-best levels.*

*Which property rights allocation maximizes aggregate surplus when the first-best is not achievable? As an illustration, suppose that the individuals are symmetric, so that $\overline{a}_1 = \overline{a}_2 \equiv \overline{a}$, $\beta_1 = \beta_2 \equiv \beta$, $v_1 = v_2 \equiv v$, $\mu_1 = \mu_2 \equiv \mu$, $c_1 = c_2 \equiv c$, $\gamma_1 = \gamma_2 \equiv \gamma$. Suppose also that $\beta > \gamma\overline{a}$, so the first-best outcome gives both individuals access. Then aggregate surplus with common ownership is $S_{CO} = 2\beta + \frac{(v-\mu)^2}{c} - 2\gamma\frac{(v-\mu)}{c}$ and with private ownership is $S_{PO} = \beta + (1/2)\frac{(v-\mu)^2}{c}$. So common ownership is optimal when $2\gamma\frac{(v-\mu)}{c} \leq \beta + \frac{1}{2}\frac{(v-\mu)^2}{c}$. Since $\beta \geq 0$, common ownership is necessarily better when $\gamma \leq (1/4)(v - \mu)$, so that externalities are small. When externalities are large, common ownership remains optimal if the direct benefit to the second individual from granting him access, $\beta + \frac{1}{2}\frac{(v-\mu)^2}{c} - \gamma\frac{(v-\mu)}{c}$, outweighs the negative externalities it generates on the first individual, $-\gamma\frac{(v-\mu)}{c}$. Note that when private ownership is optimal, incentive effects outweigh the fact that, fixing an individual's fishing effort at zero, granting him access maximizes surplus.*

## 2.2 A General Static Model

Consider now what can be said more generally about the optimal allocation of property rights in a static setting. Suppose there are $N$ agents, each of whom chooses private action $a_i \in A_i \subseteq \mathbb{R}^{M_i}$; the profile of actions is $a = (a_1, ..., a_N)$. We shall assume the set $A_i$ is a compact lattice. The feasible contractible property rights are $x \in X = \prod_{k=1}^{K} X_k$. (For example, $x$ may be a vector of zeros and ones indicating, for each asset and individual, whether the individual has access to the asset.) Agent $i$ has quasi-linear utility of the form $U_i(x, a) + t_i$. The aggregate surplus given property rights $x$ and action profile $a$ is therefore $S(x, a) = \sum_i U_i(x, a)$, and the first-best outcomes are denoted

$$\Sigma^* = \arg \max_{x \in X, a \in A} S(x, a).$$

It is also useful to define the efficient actions given some fixed property rights $x \in X$ as $A^*(x) = \arg\max_{a \in A} S(x, a)$ and the efficient allocation of property rights given some fixed actions $a \in A$ as $X^*(a) = \arg\max_{x \in X} S(x, a)$. If actions were fixed at $a$ so that incentive effects were not an issue, the optimal property rights would be an element of $X^*(a)$, reflecting the principle that the individual or individuals who most value access to an asset should have it.

We will also at times be interested in examining the optimal allocation of a particular dimension of $x$, such as the choice of the ownership of a single asset $x_k$. Writing $x = (x_k, x_{-k})$, we define the efficient choices for $x_k$ given actions $a$ and the allocation of other property rights $x_{-k}$ by $X_k^*(x_{-k}, a) \equiv \{x_k : (x_k, x_{-k}) \in X^*(a)\}$.

Given an allocation of property rights $x$, the agents choose their actions noncooperatively to maximize their individual payoffs. The resulting Nash equilibrium set is

$$A^{\circ}(x) = \left\{ a \in A : a_i \in \arg \max_{a_i' \in A_i} U_i(x, a_i', a_{-i}) \text{ for all } i \right\},$$

where $A = A_1 \times ... \times A_N$. The optimal (2nd-best) assignment of property rights given this behavior results in the outcomes

$$\widehat{\Sigma} = \arg\max_{x \in X, a \in A^{\circ}(x)} S(x, a).$$

We also let $\widehat{X} = \{\widehat{x} \in X : \text{there exists } \widehat{a} \in A \text{ such that } (\widehat{x}, \widehat{a}) \in \widehat{\Sigma}\}$ denote the set of second-best property rights allocations.

**Remark 2** *The model incorporates the possibility that the ability to take some actions may depend on the ownership allocation. For example, an action that is taken by one party $i$ when $x = 0$ and another party $j$ when $x = 1$ can be captured by letting both parties formally take the action, but having only party $i$'s choice affect payoffs when $x = 0$ and only party $j$'s choice affect payoffs when $x = 1$.*

## 2.3 Achieving Efficiency

We begin with a result identifying conditions under which a first-best outcome can be achieved. The basic idea is that efficiency can be sustained in equilibrium if there is an efficient outcome $(x^*, a^*)$ at which harmful externalities are absent.

**Proposition 3** *Suppose that there exists* $(x^*, a^*) \in \Sigma^*$ *at which there are no harmful externalities:*

$$U_i(x^*, a_i^*, a_{-i}) \geq U_i(x^*, a^*) \text{ for all } i, a_{-i}. \tag{5}$$

*Then* $a^* \in A^\circ(x^*) \subseteq A^*(x^*)$*; that is, given property rights* $x^*$*, action profile* $a^*$ *is sustained in a Nash equilibrium, and every Nash equilibrium outcome given property rights* $x^*$ *is efficient. Moreover, every Nash equilibrium given property rights* $x^*$ *results in the same payoff for each player.*[6]

**Proof.** For all $j$, $a_j \in A_j$,

$$\begin{aligned} U_j(x^*, a^*) &= S(x^*, a^*) - \sum_{i \neq j} U_i(x^*, a^*) \\ &\geq S\left(x^*, a_j, a_{-j}^*\right) - \sum_{i \neq j} U_i\left(x^*, a_j, a_{-j}^*\right) \\ &= U_j\left(x^*, a_j, a_{-j}^*\right). \end{aligned}$$

where the inequality holds because $(x^*, a^*) \in \Sigma^*$ and any externalities are nonharmful. Hence, $a^* \in A^\circ(x^*)$.

Next, for any $a^\circ \in A^\circ(x^*)$,

$$U_i(x^*, a^\circ) \geq U_i\left(x^*, a_i^*, a_{-i}^\circ\right) \geq U_i(x^*, a^*), \tag{6}$$

where the first inequality follows from Nash equilibrium and the second inequality from nonharmful externalities at $(x^*, a^*)$. Summing this expression, yields

$$\sum_i U_i(x^*, a^\circ) \geq \sum_i U_i\left(x^*, a_i^*, a_{-i}^\circ\right) \geq \sum_i U_i(x^*, a^*) = S(x^*, a^*).$$

Since $S(x^*, a^*) \geq \sum_i U_i(x^*, a)$ for all $a$, this implies that $\sum_i U_i(x^*, a^\circ) = S(x^*, a^*)$; i.e., that $a^\circ \in A^*(x^*)$. Equation (6) then implies that $U_i(x^*, a^\circ) = U_i(x^*, a^*)$ for all $i$. ■

Consistent with this result, recall that in the Linear-Quadratic Tragedy of the Commons of Example 1 common ownership is efficient if, because the lake is large, externalities are absent. Similarly, whenever there is an efficient outcome involving private ownership (so that only one individual has access to the lake), private ownership leads to a first-best outcome.

Note also that, according to Proposition 3, in both of these cases efficiency would be preserved if, in addition, the individuals could take actions (make

[6] It is clear from the proof that for the first result only, the nonharmful externalities assumption could be weakened to requiring that for each $j$ and $a_j$, $\sum_{i \neq j} u_i\left(x^*, a_j, a_{-j}^*\right) \geq \sum_{i \neq j} u_i(x^*, a^*)$ (i.e., that *unilateral* deviations have no harmful externalities on the *aggregate* payoff of the other agents).

"investments") that enhanced their own value from using the lake or reduced their cost of fishing since such actions have no external effects.

Externalities are entirely absent in both of these examples. An example with externalities that are nonharmful would arise if the maximal feasible amount of fishing for each individual $i$, $\overline{a}_i$, were low enough so that the first-best outcome involved common ownership and fishing levels $(\overline{a}_1, \overline{a}_2)$. We will see in Section 3 that externalities that are nonharmful also arise quite naturally in dynamic settings.

**Remark 4** *While we have emphasized the interpretation of the variable $x$ as capturing the assignment of property rights, the results also apply more generally to any contract terms or decision rights $x$. For example, if a contract specifies that each fisherman will confine his fishing to his own section of the lake, and if fish species are localized so that harmful externalities are eliminated, then Proposition 3 implies that the first best will be realized. Likewise, in all of the other results in this chapter, the variable $x$ can be interpreted more generally as applying to any enforceable contractual terms or decision rights including, but not limited to, property rights allocations.*

## 2.4 Externalities and Distortions

When harmful externalities are instead present at all first-best outcomes, efficiency typically cannot be achieved. As intuition suggests, equilibrium actions will be distorted in directions that generate harmful externalities. To formalize this point, we begin with a definition:

**Definition 5** *Actions generate negative (positive) externalities at $x$ if, for all $i$, all $j \neq i$, and all $a_{-i} \in A_{-i}$, $U_j(x, a)$ is nonincreasing (nondecreasing) in $a_i$.*

First we show that if a single agent has a one-dimensional action choice that generates negative (positive) externalities, in equilibrium he will choose an action that is too high (low) relative to the efficient level:

**Proposition 6** *Suppose that only agent $i$ takes actions, he has a one-dimensional action choice $a \in \mathbb{R}$, and his action choice generates negative (positive) externalities at $x$. Then $A^{\circ}(x) \geq (\leq) A^*(x)$ in the strong set order.*[7]

**Proof.** Consider the case of positive externalities. Define the function

$$\Psi_i(x, a, \lambda) \equiv U_i(x, a) + \lambda_i \sum_{j \neq i} U_j(x, a) = (1 - \lambda_i) U_i(x, a) + \lambda_i S(x, a), \qquad (7)$$

[7] The strong set order comparison $A^{\circ}(x) \geq (\leq) A^*(x)$ means that for all $a \in A^*(x) \backslash A^{\circ}(x)$, $b \in A^*(x) \cap A^{\circ}(x)$, and $c \in A^{\circ}(x) \backslash A^*(x)$, we have $a \leq b \leq c$ $(a \geq b \geq c)$. If we know that the action has *strictly* negative (positive) externalities at $x$, in the sense that $U_j(a, x)$ is *strictly* decreasing (increasing) in $a_i$ for all $j \neq i$ and all $a_{-i} \in A_{-i}$, then we can derive the stronger conclusion that $a^{\circ} \geq (\leq) a^*$ for all $a^{\circ} \in A^{\circ}(x)$ and $a^* \in A^*(x)$. This follows from the Monotone Selection Theorem (see Milgrom and Shannon [1994]).

where $\lambda_i \in [0,1]$. Observe that $A^*(x) = \arg\max_{a\in A} \Psi_i(x,a,1)$ and $A^\circ(x) = \arg\max_{a\in A} \Psi_i(x,a,0)$. Positive externalities imply that the function $\Psi_i(\cdot)$ has increasing differences in $(a, \lambda_i)$. The result follows by Topkis's Monotonicity Theorem (see Milgrom and Roberts [1990]). The result for negative externalities follows similarly. ■

The result also implies that when many agents choose actions and/or actions are multidimensional, the equilibrium level of any single dimension that generates negative (positive) externalities will be too high (low) holding fixed all of the other dimensions. To get a result comparing the entire equilibrium action profile with the first-best action profile, we need to make assumptions on the interactions among different dimensions. The following proposition provides such a result:

**Proposition 7** *Suppose that for a given property rights allocation $x \in X$, $U_i(x, a_i, a_{-i})$ is supermodular in $a_i$ and has increasing differences in $(a_i, a_{-i})$ for all $i$, and that $S(x,a) = \sum_i U_i(x,a)$ is supermodular in $a$. Then there exist smallest and largest elements of the sets $A^\circ(x)$ and $A^*(x)$, denoted respectively by $(\underline{a}^\circ, \overline{a}^\circ)$ and $(\underline{a}^*, \overline{a}^*)$. If, in addition, actions generate negative (positive) externalities at $x$, then $\overline{a}^\circ \geq \overline{a}^*$ $(\underline{a}^* \geq \underline{a}^\circ)$.*

**Proof.** Consider the case of positive externalities. For all $i$, consider again function (7), but now where $a = (a_i, a_{-i})$. Observe first that the fact that $S(x,a)$ is supermodular in $a$ implies, by Tarski's Fixed Point Theorem (see Milgrom and Roberts [1990]), that there exist smallest and largest elements of the set $A^*(x)$, denoted $(\underline{a}^*, \overline{a}^*)$. Next, observe that $A^\circ(x)$ is the set of Nash equilibria of the game in which each individual $i$'s payoff function is $\Psi_i(x, a_i, a_{-i}, 0)$, while $A^*(x)$ is a subset of the set of Nash equilibria of the game in which each individual $i$'s payoff function is $\Psi_i(x, a_i, a_{-i}, 1)$. The functions $\Psi_i(\cdot)$ are supermodular in $a_i$ and, due to positive externalities, have increasing differences in $a_i$ and $\lambda_i$, as well as in $a_i$ and $a_{-i}$. The existence of smallest and largest elements of the set $A^\circ(x)$, denoted $(\underline{a}^\circ, \overline{a}^\circ)$, follows from Theorem 5 in Milgrom and Roberts [1990]. The Corollary of Theorem 6 in Milgrom and Roberts [1990] implies that the smallest Nash equilibrium action profile when $\lambda_i = 1$ for all $i$ is larger than the smallest Nash equilibrium action profile when $\lambda_i = 0$ for all $i$ $(\underline{a}^\circ)$. Since the set of maxima of $S(a,x)$ are a subset of the set of Nash equilibria of the game when $\lambda_i = 1$ for all $i$, it must be that $\underline{a}^* \geq \underline{a}^\circ$. The result for negative externalities follows similarly. ■

Propositions 6 and 7 do not rule out the possibility that an equilibrium action profile could be efficient. However, if payoff functions are differentiable and the marginal externalities are strict, there is necessarily inefficiency:

**Proposition 8** *Suppose that for a given property rights allocation $x \in X$, some action $a_{ik} \in \mathbb{R}$ generates negative (positive) externalities at $x$, player $i$'s payoff $U_i(x,a)$ is differentiable in $a_{ik}$, and $\partial U_j(x,a)/\partial a_{ik} \neq 0$ for some $j \neq i$. If $a^\circ \in A^\circ(x)$ and $a^\circ$ is interior, then $a^\circ \notin A^*(x)$.*

**Proof.** If $a^\circ \in A^\circ(x)$, then $\partial \Psi_i(x, a_{ik}^\circ, a_{-ik}^\circ, 0)/\partial a_{ik} = 0$, while if $a^\circ \in A^*(x)$, then $\partial \Psi_i(x, a_{ik}^\circ, a_{-ik}^\circ, 1)/\partial a_{ik} = 0$ ($a_{-ik}$ represents all actions other than $a_{ik}$). However, under the assumptions of the Proposition $\partial \Psi_i(x, a_{ik}^\circ, a_{-ik}^\circ, \lambda)/\partial a_{ik}$ is strictly increasing (decreasing) in $\lambda$, which establishes the result. ■

## 2.5 Second-best Property Rights Allocations

When there are unavoidable externalities and therefore inefficiency, what can be said about the welfare effect of the property rights allocation, and about optimal (second-best) property rights allocations?

We first observe that since actions are too high (low) with negative (positive) externalities, a change in property rights that is directly beneficial and discourages (encourages) those actions is welfare-enhancing:

**Lemma 9** *Consider two property rights allocations $x$ and $x'$ and a pair of Nash equilibria resulting from these property rights allocations, $a \in A^\circ(x)$ and $a' \in A^\circ(x')$. If actions generate negative (positive) externalities at $x'$, $S(x', a) \geq S(x, a)$, and $a' \leq (\geq) a$, then $S(x', a') \geq S(x, a)$.*

**Proof.** Note that

$$\begin{aligned} S(x', a') - S(x, a) &= \sum_j [U_j(x', a') - U_j(x', a)] + [S(x', a) - S(x, a)] \\ &\geq \sum_j \left[U_j\left(x', a_j, a'_{-j}\right) - U_j(x', a)\right] + [S(x', a) - S(x, a)] \\ &\geq 0, \end{aligned}$$

where the first inequality follows from the revealed preference of agent $j$, and the second inequality follows from negative (positive) externalities at $x'$, $a' \leq (\geq)\ a$, and $S(x', a) \geq S(x, a)$. ■

As an illustration, consider a change in property rights in the Linear-Quadratic Tragedy of the Commons from private ownership by individual 1 [$x = (1, 0)$] to common ownership [$x' = (1, 1)$] when $\beta_2 > \gamma_1 \overline{a}_1$. Holding fishing efforts fixed at $a = (a_1^\circ, 0) \in A^\circ(x)$, this raises surplus by giving individual 2 the benefit of using the lake for non-fishing purposes. If, however, externalities from individual 2's fishing efforts are absent (i.e., if $\gamma_2 = 0$), then there are (weakly) positive externalities arising from the behavioral change to $a' = (a_1^{\circ\circ}, a_2^{\circ\circ}) \in A^\circ(x')$. So aggregate surplus increases from this change to common ownership.

An interesting special case arises when, given the actions chosen by the agents, the allocation of property rights has no direct efficiency consequences, so that $X^*(a) = X$ for all $a$. In that case, any change in property rights that increases all actions when actions generate positive externalities, or reduces all actions when actions generate negative externalities, is welfare-enhancing. We'll see in the next section that in dynamic settings, the condition that $X^*(a) = X$ for all $a$ arises as a natural consequence of efficient renegotiation.

Lemma 9 considered property rights changes that both increase the ex post surplus holding actions fixed, and also lead to surplus-enhancing behavioral changes. Such changes involve no trade-offs. In general, the optimal (second-best) property rights allocation will trade off these two effects. As a result, when we look at an optimal (second-best) property rights allocation, in each dimension it will be distorted away from what would be efficient ex post given the actions individuals take and the other dimensions of the property rights allocation, and this distortion will be in the direction that reduces (increases) incentives for actions with negative (positive) externalities:

**Proposition 10** *Suppose $X \subset \mathbb{R}^m$, let $x = (x_k, x_{-k})$ where $x_k \in \mathbb{R}$, and fix $x_{-k}$. Suppose that, for all i, $U_i(x, a_i, a_{-i})$ is supermodular in $a_i$, has increasing differences in $(a_i, a_{-i})$, and has increasing differences in $(x_k, a_i)$. Take any second-best outcome $(\widehat{x}, \widehat{a}) \in \Sigma$, and any ex post optimal property right in dimension k, $x'_k \in X^*_k(\widehat{x}_{-k}, \widehat{a})$. If actions generate negative (positive) externalities at $(x'_k, \widehat{x}_{-k})$, then either $(x'_k, \widehat{x}_{-k}) \in \widehat{X}$ or $\widehat{x}_k < (>)\ x'_k$.*

**Proof.** Suppose that actions generate positive externalities and $\widehat{x}_k \leq x'_k$. Under the hypotheses of the proposition, Theorems 5 and 6 in Milgrom and Roberts [1990] imply that given the property rights allocation $x' = (x'_k, \widehat{x}_{-k})$ there is a largest Nash equilibrium action profile $a' = \max A^{\circ}(x')$ and, in addition, $a' \geq \widehat{a}$. Since $x'_k \in X^*_k(\widehat{x}_{-k}, \widehat{a})$ implies that $S(x', \widehat{a}) \geq S(\widehat{x}, \widehat{a})$, we can apply Lemma 9 to see that $S(x', a') \geq S(x, a)$, which implies that $x' \in \widehat{X}$. The proof for the case of negative externalities is similar. ■

The statement of Proposition 10 is complicated somewhat by the possibility that there are multiple second-best outcomes. In the case in which $\Sigma$ is a singleton so that $\Sigma = \{(\widehat{x}, \widehat{a})\}$, and we have negative (positive) externalities at all $x$, Proposition 10 implies that in every dimension $k$ we have $\widehat{x}_k \leq \inf X^*_k(\widehat{x}_{-k}, \widehat{a})$ ($\widehat{x}_k \geq \sup X^*_k(\widehat{x}_{-k}, \widehat{a})$), i.e., at the second-best optimal outcome the property rights allocation in dimension $k$ is distorted weakly downward (upward) from any ex post efficient allocation given the actions chosen and the other dimensions of the property rights allocation.

An interesting special case of the proposition arises when $X^*(a) = X$ for all $a$. Below we will see that this arises in the Property Rights Theory of Hart-Moore (1990), in which the allocation of property rights is always renegotiated efficiently after actions are chosen, so the property rights only affect surplus through their incentive effect on actions, but not directly. Proposition 10 implies that in this case if $\Sigma \neq \varnothing$ and actions generate positive externalities then $x_k$ can optimally be set at its maximal level $\max X_k$. In fact, if there is a collection of property rights $(x_1, ...x_K) \in \mathbb{R}^K$ such that $U_i(x, a_i, a_{-i})$ has increasing differences in $(x_k, a_i)$ for every $k = 1, ..., L$ in this collection, then applying Proposition 10 iteratively implies that there is an optimal property rights allocation in which the property rights over dimension $1, ..., K$ are set at at their largest possible levels:

**Corollary 11** *Suppose that $X^*(a) = X$ for all $a$ and $\widehat{\Sigma} \neq 0$, actions generate positive externalities, and there is a collection of property rights $(x_1, ...x_K) \in \mathbb{R}^L$*

*such that the assumptions of Proposition 10 hold for each $x_k$ in this collection. Then there is an optimal property rights allocation in which each element of this collection of property rights is set at its largest possible level; i.e.,* $(x_1, ...x_K) = (\max X_1, ..., \max X_K)$.

As an illustration, suppose that there are two agents, 1 and 2, and that only agent 1 takes actions. Restrict attention to private property rights allocations, so that either agent 1 or agent 2 has access to each asset, but not both. Suppose that only agent 1 takes actions and that agent 1's actions increase the more assets he has access to. Measure $x$ such that larger values represent greater access for agent 1 (and less access for agent 2). Specifically, let $x_k = 1$ if agent 1 owns asset $k$ (and hence can exclude agent 2) and $x_k = 0$ if agent 2 owns asset $k$. Then, Corollary 11 implies that if agent 1's actions generate positive externalities, he should own all of the assets.

# 3 Dynamic Models

Most economic relationships – especially within organizations – are not one-shot affairs. Rather the parties interact over time. A body of work has emerged in recent years exploring the role of property rights in such settings. In this so-called "hold-up" literature, parties make ex ante investments and then engage in some productive activity ex post. The initial allocation of property rights affects agents' investment incentives.

An important feature of dynamic settings is that the property rights allocation may be renegotiated over time. This renegotiation can enhance efficiency. This is true for two reasons. The first is straightforward: surplus can be enhanced by renegotiating what would otherwise be inefficient allocations of assets. The second reason is more subtle. Renegotiation of property rights over time can allow the direct and indirect effects of property rights to be separated: property rights can initially be set to encourage efficient investments, and can then be renegotiated once those investments have been made. At the same time, the presence of renegotiation leads to a new source of externalities: if the parties renegotiate their initial contract in states where it would lead to inefficiency, past actions may affect the payoffs that parties achieve in renegotiation. The following example illustrates these issues:

**Example 12** ***The Dynamic Linear-Quadratic Tragedy of the Commons*** *Consider again the Tragedy of the Commons model from Example 1. Suppose now, however, that the setting is dynamic. Specifically, we shall interpret the effort $a_i$ as an investment that individual $i$ makes in learning to fish better. This effort affects individual $i$'s catch should he have access to the lake and choose to fish, and also affects the level of externalities his fishing generates (fishing is now a binary choice of whether to fish or not). This investment is observable to all of the individuals, but not verifiable, and hence cannot be specified in a contract. Once these investments are made, the ownership of the lake can be renegotiated prior to the actual act of fishing. Individual* *i's investment is* $a_i \in [0, \overline{a}_i]$ *and, as in Example 1, the payoff to individual* $i$ *is* $b_i(x_i, x_j, a_i, a_j) = x_i \max\{0, \beta_i + v_i a_i - \gamma_j a_j x_j\}$ *less his investment cost* $\phi_i(a_i) = \mu_i a_i + (c_i/2)(a_i)^2$*, where* $(v_i, \mu_i, c_i) >> 0$*,* $(\beta_i, \gamma_i) \geq 0$*, and* $v_i \geq \mu_i + \gamma_i$*.*

*Suppose that* $\beta_i > (\gamma_1 \overline{a}_1 + \gamma_2 \overline{a}_2)$ *for* $i = 1, 2$*, so that following any pair of investments* $(a_1, a_2)$ *the ex post efficient property rights allocation involves common ownership. Suppose as well that* $v_2 = \mu_2$*, so that individual 2 optimally exerts no effort in the first-best outcome* ($a_2^* = 0$)*; that is, only individual 1 should invest. The first-best investment for individual 1 is* $a_i^* = (v_1 - \mu_1 - \gamma_1)/c_1$*.*

*We've seen that the first best is not achievable in a static setting (with no renegotiation of property rights) when* $\gamma_1 > 0$ *because individual 1 will over-invest under common ownership, instead investing* $a_1^{\circ\circ} = (v_1 - \mu_1)/c_1$*. The outcome is the same in the dynamic setting if the initial property rights allocation involves common ownership since, in that case, no renegotiation will occur ex post. But consider what happens in the dynamic setting if the initial property rights allocation instead allocates ownership of the lake to individual 1. Specifically, suppose that individual 1 receives a share* $\lambda_1 \in [0, 1]$ *of the renegotiation surplus, which equals* $R(a_1, a_2) \equiv (\beta_2 + v_2 a_2 - \gamma_1 a_1 - \gamma_2 a_2)$ *when the initial private ownership by individual 1 is renegotiated to common ownership following investments* $(a_1, a_2)$*. Then individual 1's investment solves*

$$\max_{a_1} [v_1 a_1 - \mu_1 a_1 - (c_1/2)(a_1)^2] + \lambda_1 R(a_1, a_2),$$

*resulting in* $a_1 = (v_1 - \mu_1 - \lambda_1 \gamma_1)/c_1$*. (Individual 2 will invest nothing.*[8]*) When individual 1 has all the bargaining power, so that* $\lambda_1 = 1$*, the first best is achieved. In this case, individual 2's post-renegotiation payoff always equals 0, his payoff when he has no access to the lake, so there are no externalities arising from individual 1's investment choice. If, however,* $\lambda_1 \in (0, 1)$*, the outcome is more efficient than if instead common ownership were specified at the start, but is still inefficient. The inefficiency stems from the presence of a harmful bargaining externality: when individual 1 reduces his investment, he reduces the payoff that individual 2 gets through bargaining. Notice that this externality is present despite that fact that at the initial property rights allocation (private ownership of the lake by individual 1), no direct externalities are present.*

**Remark 13** *The investment decision by individual 1 in Example 12 can be viewed as an investment that alters the value of a product (access to the lake) that individual 1 can sell to individual 2. Anticipating bargaining over the sale of this product, individual 1's incentives when he possess all of the bargaining power are efficient.*

In this section, we formulate a general dynamic model with two stages, referred to as "ex ante" and "ex post." In the "ex ante" stage, the parties specify property rights $x$, and then choose noncontractible but observable actions $a$. Next, between the ex ante and ex post stages, some uncertainty $\varepsilon$ may be

[8] Specifically, individual 2's investment solves the problem $\max_{a_2 \geq 0} (1 - \lambda_1) R(a_1, a_2) - [\mu_2 a_2 + (c_2/2)(a_2)^2]$, whose solution is $a_2 = 0$.

realized, part of which may be verifiable ($\varepsilon_v$) and part not ($\varepsilon_u$). Then, in the "ex post" stage, the parties can renegotiate the allocation of the property rights $x$, and may also bargain over some decisions $q$ that are contractible ex post but not ex ante.

Paralleling our discussion of the static model, we begin by considering when the parties can achieve the first best. We then consider settings in which harmful externalities cannot be eliminated, so that a first-best outcome cannot be achieved. We focus there on the "Property Rights Theory of the Firm" of Grossman and Hart [1986], Hart and Moore [1990], and Hart [1995].

## 3.1 A Model of Hold-up

We will consider the following hold-up model involving $N$ agents, $i = 1, ..., N$, in which the timing of their interaction unfolds as follows (see also Figure 1 TO BE ADDED):

1. The agents contract on the property rights allocation $x$ and side payments;
2. Each agent $i$ chooses investments $a_i \in A_i$. The investments are observable to all of the agents but are not verifiable, and thus are noncontractible;
3. Uncertainty $\varepsilon = (\varepsilon_v, \varepsilon_u)$ is realized; $\varepsilon_v$ is verifiable and can be directly contracted upon, while $\varepsilon_u$ is not verifiable;
4. Each agent $i$ takes actions $q_i \in Q_i$ that, while noncontractible ex ante, are contractible ex post. Prior to these actions being chosen, the agents may be able to renegotiate the property rights allocation $x$ and also specify both the ex post decisions $q = (q_1, ..., q_N) \in Q \equiv \prod_i Q_i$ and side payments.

   Agent $i$'s payoff given $(x, a, \varepsilon, q)$ and transfer $t_i$ is $v_i(x, a, \varepsilon, q) + t_i$. If bargaining fails, $x$ is unchanged and the parties noncooperatively choose the $q_i$'s. For convenience, we assume that noncooperative choice of the $q_i$'s given $(x, a, \varepsilon)$ results in a unique value of $v_i(\cdot)$ for each party $i$, we denote this "disagreement payoff" by $v_i^\circ(x, a, \varepsilon)$.[9,10]

Given $(a, \varepsilon)$, an ex post surplus-maximizing outcome $(x^*(a, \varepsilon), q^*(a, \varepsilon))$ satisfies

$$(x^*(a, \varepsilon), q^*(a, \varepsilon)) \in \arg \max_{x \in X, q \in Q} \sum_i v_i(x, a, \varepsilon, q).$$

We denote the resulting maximal ex post surplus given $(a, \varepsilon)$ by

$$S(a, \varepsilon) = \max_{x \in X, q \in Q} \sum_i v_i(x, a, \varepsilon, q).$$

[9] For example, if there is a unique Nash equilibrium $q^\circ(x, a, \varepsilon)$ then $v_i^\circ(x, a, \varepsilon) = v_i(x, a, \varepsilon, q^\circ(x, a, \varepsilon))$ for all $i$.

[10] We could also allow for some actions $z_i$ that are never contractible by assuming that the noncooperative choice of the $z_i$'s given $(x, a, \varepsilon, q)$ leads to unique Nash equilibrium payoffs, given by our functions $v_i(x, a, \varepsilon, q)$. We would also need to alter the definition of $S(a, \varepsilon)$ to reflect the fact that the $z_i$'s are always chosen noncooperatively.

A first-best investment profile $a^*$ satisfies $a^* \in \arg\max_{a \in A} S(a, \varepsilon)$. A (state-contingent) first-best outcome is a collection $(x^*(a^*, \cdot), a^*, q^*(a^*, \cdot))$.

**Remark 14** *In Grossman and Hart [1986] the ability to take the decisions $q$ depends on the ownership allocation. Just as in Section 2, the model can incorporate the possibility that an action $q_k$ is taken by one party $i$ when $x_k = 0$ and another party $j$ when $x_k = 1$ by letting both parties formally take the action, but having only party $i$'s choice affect payoffs when $x_k = 0$ and only party $j$'s choice affect payoffs when $x_k = 1$.*

**Remark 15** *Observe that when there are no decisions $q$ that are ex ante contractible but ex post noncontractible, maximizing surplus ex post involves only the optimal reassignment of the property rights $x$, so $S(a, \varepsilon) = \max_{x \in X} \sum_i v_i^\circ(x, a, \varepsilon)$. In contrast, when there are such decisions $q$, then typically $S(a, \varepsilon) > \max_{x \in X} \sum_i v_i^\circ(x, a, \varepsilon)$.*

To formulate results that apply to a variety of settings, we will sometimes avoid a specific model of bargaining and instead make only two weak assumptions on the "post-renegotiation" utilities that the parties obtain by bargaining, denoting by $u_i(x, a, \varepsilon)$ party $i$'s post-renegotiation payoff starting from ownership allocation $x$ following investments $a$, and uncertainty realization $\varepsilon$.[11] The first is *feasibility*:

$$\sum_i u_i(x, a, \varepsilon) \leq S(a, \varepsilon) \text{ for all } x, a, \varepsilon. \tag{8}$$

Note in particular that if bargaining is fully efficient then we have equality in (8) for all $(x, a, \varepsilon)$. The second is *individual rationality*:

$$u_i(x, a, \varepsilon) \geq v_i^\circ(x, a, \varepsilon) \text{ for all } i, x, a, \varepsilon. \tag{9}$$

The two assumptions allow for a range of bargaining processes. At one extreme we could have no renegotiation at all (bargaining always fails), in which case as noted above there is noncooperative (Nash equilibrium) choice of the $q_i$'s. (Note that if there are no $q_i$'s then with no renegotiation we are back in the static model of the last section, albeit with some uncertainty in payoffs due to $\varepsilon$.) At the other extreme, we could have efficient bargaining, which could in principle take various forms.

At other times, we will be more specific, considering situations in which the Nash bargaining solution applies. In that case,

$$u_i(x, a, \varepsilon) = v_i^\circ(x, a_i, \varepsilon) + \lambda_i R(x, a, \varepsilon), \tag{10}$$

[11] Note that this reduced-form utility is assumed independent of the transfer $t$, since we think of $t$ as a transfer made ex ante before bargaining. For some bargaining solutions (e.g., the Nash solution) an ex post transfer is equivalent to an ex ante transfer, so is irrelevant for bargaining. For other solutions – such as outside option bargaining considered in Section ?? below – an ex post transfer differs from ex ante transfer and affects bargaining. In this case, the ex post transfer would need to be viewed as part of $x$.

where $\lambda_i \geq 0$, $\sum_i \lambda_i = 1$, and $R(x, a, \varepsilon) = S(a, \varepsilon) - \sum_j v_j^\circ(x, a, \varepsilon)$ is the renegotiation surplus.

We will say that there is "no renegotiation" at $(x, a, \varepsilon)$ if (9) holds with equality for each $i$, that is, if each party receives a payoff exactly equal to the continuation payoff he would get under the original property rights allocation $x$. (Note that, strictly speaking, the parties might renegotiate their original contract in such a case, but this renegotiation would yield none of them any gain; their payoffs would be the same as if they did not renegotiate.)

## 3.2 Achieving Efficiency

We begin by considering circumstances in which a first-best outcome can be achieved. In this section, we allow for the property rights allocation to be "indexed" on the verifiable uncertainty $\varepsilon_v$ according to some function $\widehat{x}(\varepsilon_v)$. The simplest such arrangement is of course an unconditional specification of property rights, $\widehat{x}$. Indexing, however, allows the ownership allocation to depend on verifiable states of the nature.[12]

Given an indexed ownership allocation $\widehat{x}(\cdot)$, the timing of the model implies that in any subgame perfect Nash equilibrium of the dynamic game the parties' investments $a$ will be a Nash equilibrium in the static game induced by the utility functions $U_i(\widehat{x}(\cdot), a) \equiv E_\varepsilon[u_i(\widehat{x}(\varepsilon_v), a, \varepsilon)]$.

We first observe that when it is possible to specify a first-best property rights allocation at which there are no harmful externalities absent renegotiation, the first best can be achieved. This works because renegotiation is eliminated on the equilibrium path, which implies that any bargaining externalities are nonharmful as well. The following example illustrates this idea:

**Example 16** *Consider again the Dynamic Linear-Quadratic Tragedy of the Commons model discussed in Example 12 but now assume that $\beta_1 = \beta_2 = 0$, $v_1 = v_2 \equiv v$, $c_1 = c_2 \equiv c$, and $\gamma_1 = \gamma_2 \equiv \gamma$. Assume also that $\mu_1 < \mu_2 = v$. In this case, as noted in Example 1, there is a first-best outcome that involves private ownership by individual 1. Individual 1's investment is $a_1^* = (v - \mu_1)/c$, while individual 2 does not invest ($a_2^* = 0$). Under this first-best ownership structure, externalities are absent if renegotiation is impossible, as in the static setting of Section 2, so the equilibrium outcome is first best.*

*In contrast, in the dynamic setting, the possibility of renegotiation leads to externalities. In particular, observe that once the agents have chosen their investments $(a_1, a_2)$, private ownership by individual 1 is efficient if and only if*

$$va_1 \geq \max\{va_2, (v - \gamma)(a_1 + a_2)\}. \tag{11}$$

*If efficient investments are chosen, no renegotiation occurs. Suppose, though, that individual 2 deviates so that (11) fails to hold. Then the two individuals will*

[12] We will consider the possibility of more complicated property rights arrangements, such as option contracts, in Section 6.

*renegotiate to an efficient ownership structure, increasing individual 1's payoff when $\lambda_1 > 0$. However, observe that this bargaining externality is nonharmful. Since the deviation raises individual 1's payoff and lowers aggregate surplus, individual 2 will not have an incentive to deviate in this fashion. A similar conclusion applies to deviations by individual 1 that lead to renegotiation. Since deviations that do not lead to renegotiation do not generate externalities, we can conclude that the first best will result.*

The following proposition extends this observation to our more general hold-up model, which also allows for decisions that are contractible ex post but not ex ante, as well as uncertainty $\varepsilon$. The result holds regardless of the details of the bargaining process, including efficient renegotiation and no renegotiation as special cases.

**Proposition 17** *Suppose that $(x^*(a^*,\cdot), a^*, q^*(a^*,\cdot))$ is a (state-contingent) first-best outcome at which*

**(i)** *$x^*(a^*,\cdot)$ does not depend on unverifiable uncertainty $\varepsilon_u$,*

**(ii)** *there are no harmful externalities:*

$$\mathbb{E}_\varepsilon\left[v_i\left(x^*\left(\varepsilon_v\right), a^*, \varepsilon, q^*\left(\varepsilon\right)\right)\right] \leq \mathbb{E}_\varepsilon\left[v_i\left(x^*\left(\varepsilon_v\right), a_i^*, a_{-i}, \varepsilon, q_i^*\left(\varepsilon\right), q_{-i}\left(\varepsilon\right)\right)\right] \text{ for all } i, a_{-i}, q_{-i}(\cdot)\,.$$

*Then under the indexed property rights allocation $\widehat{x}(\cdot)$ in which $\widehat{x}(\varepsilon_v) = x^*(a^*, \varepsilon_v)$ for all $\varepsilon_v$, there is an equilibrium without renegotiation that leads to the first-best outcome $(x^*(a^*,\cdot), a^*, q^*(a^*,\cdot))$ and, moreover, all equilibria under this indexed property rights allocation lead to identical (first-best) payoffs for every agent.*

**Proof.** Let $U_i(x(\cdot), a) \equiv \mathbb{E}_\varepsilon[u_i(x(\varepsilon), a, \varepsilon)]$. By individually rational bargaining, the Nash best-response property, and nonharmful externalities at the first-best outcome $(x^*(a^*,\cdot), a^*, q^*(a^*,\cdot))$, for any $i$ and $a_{-i}$ we have

$$\begin{aligned} U_i\left(\widehat{x}(\cdot), a_i^*, a_{-i}\right) &\geq \mathbb{E}_\varepsilon\left[v_i^\circ(x^*(a^*, \varepsilon_v), a_i^*, a_{-i}, \varepsilon)\right] \\ &\geq \mathbb{E}_\varepsilon\left[\min_{q_{-i}} v_i(x^*(a^*, \varepsilon_v), a_i^*, a_{-i}, \varepsilon, q_i^*\left(a^*, \varepsilon\right), q_{-i})\right] \\ &\geq \mathbb{E}_\varepsilon\left[v_i\left(x^*\left(a^*, \varepsilon_v\right), a^*, \varepsilon, q^*\left(a^*, \varepsilon\right)\right)\right]. \qquad (12) \end{aligned}$$

Writing these inequalities with $a_{-i} = a_{-i}^*$ for each $i$ and adding them up yields:

$$\sum_i U_i\left(\widehat{x}(\cdot), a^*\right) \geq \sum_i \mathbb{E}_\varepsilon\left[v_i\left(x^*\left(a^*, \varepsilon_v\right), a^*, \varepsilon, q^*\left(a^*, \varepsilon\right)\right)\right]. \qquad (13)$$

On the other hand, the bargaining feasibility condition implies that

$$\sum_i U_i(\widehat{x}(\cdot), a^*) \leq \mathbb{E}_\varepsilon\left[S\left(a^*, \varepsilon\right)\right] = \sum_i \mathbb{E}_\varepsilon\left[v_i\left(x^*\left(a^*, \varepsilon_v\right), a^*, \varepsilon, q^*\left(a^*, \varepsilon\right)\right)\right],$$

so

$$\sum_i U_i\left(\widehat{x}(\cdot), a^*\right) = \sum_i \mathbb{E}_\varepsilon\left[v_i\left(x^*\left(a^*, \varepsilon_v\right), a^*, \varepsilon, q^*\left(a^*, \varepsilon\right)\right)\right]. \qquad (14)$$

Combining (12) and (14) we see that for each $i$,

$$U_i\left(\widehat{x}(\cdot), a^*\right) = \mathbb{E}_\varepsilon\left[v_i\left(x^*\left(a^*, \varepsilon_v\right), a^*, \varepsilon, q^*\left(a^*, \varepsilon\right)\right)\right]. \qquad (15)$$

Conditions (12) and (15) imply that when considering the static investment game induced by later renegotiation, $(\widehat{x}(\cdot), a^*)$ is a first-best outcome at which there are no harmful externalities. Hence, Proposition 3 applied to this induced static game implies that $\widehat{x}(\cdot)$ supports $a^*$ as equilibrium choices, and that every equilibrium profile of investment choices leads to the same payoff for every agent. Finally, observe that once $a^*$ is chosen, and taking $\widehat{x}(\varepsilon_v) = x^*(a^*, \varepsilon_v)$ as given, in each state $\varepsilon$ the action profile $q^*(a^*, \varepsilon)$ is an efficient outcome at which there are no harmful externalities. Therefore, under the indexed ownership allocation $\widehat{x}(\cdot)$, once $a^*$ is chosen, $q^*(a^*, \varepsilon)$ is a Nash equilibrium action profile in the continuation game. ■

The intuition behind Proposition 17 is that by specifying an efficient plan we can eliminate bargaining in equilibrium. Any party's investment deviation therefore creates bargaining surplus and so has beneficial bargaining externalities on the others. If, in addition, investments don't generate harmful externalites absent renegotiation (either directly or by inducing a change in the ex post actions $q$), then nobody will deviate.

The efficiency conclusion in Example 16 is an application of this result. The following examples describe two other applications of this result in the literature.

**Example 18** *Grossman and Hart [1986] consider a setting with two agents in which there are ex post contractible but ex ante noncontractible actions $q_1$ and $q_2$, in addition to ex ante investments that are noncontractible. There is no uncertainty $\varepsilon$. The possible ownership allocations are agent 1 integration [$(x_1, x_2) = (1, 1)$], agent 2 integration [$(x_1, x_2) = (0, 0)$], and nonintegration [$(x_1, x_2) = (1, 0)$], where $x_i = 1$ ($x_i = 0$) indicates that agent 1 (agent 2) chooses $q_i$. Under agent i integration, agent i chooses both $q_1$ and $q_2$ absent renegotiation; under nonintegration, each agent i chooses $q_i$. Following Grossman and Hart, we let the ex post actions be $(q_{11}, q_{12}, q_{21,} q_{22})$ and the payoff functions $v_i(x, a, \varepsilon, q)$ take the form $v_i(a_i, \phi_i(x_1 q_1^1 + (1 - x_1) q_1^2, x_2 q_2^1 + (1 - x_2) q_2^2))$, where $\phi_i(\cdot)$ is a real-valued function that is increasing in both $a_i$ and $\phi_i$. Then, the ex post noncooperative (Nash equilibrium) choices of q are independent of a and there are no direct externalities from investments (they are "self-investments"). Proposition 17 implies that, in some cases, the first best can be achieved. For example, if $\phi_{-i}(\cdot)$ is a constant function, then agent i integration achieves the first best. Likewise, if $\phi_1(\cdot) = \phi_1(x_1 q_1^1 + (1 - x_1) q_1^2)$ and $\phi_2(\cdot) = \phi_2(x_2 q_2^1 + (1 - x_2) q_2^2)$, then nonintegration is optimal.*

**Example 19** *Edlin and Reichelstein [1996] study a hold-up model of trade in which a seller (agent 1) and a buyer (agent 2) each make ex ante noncontractible* *investments that affect their own value from trade with each other. The level of trade is a real-valued number that is contractible ex ante and may be renegotiated ex post. Their model fits into our property rights setting if the "good" being traded is an asset. Investments raise each agent's value of owning the asset. Formally, $x = 0$ if the seller owns the asset and $x = 1$ if the buyer does. Suppose that there is no uncertainty $\varepsilon$ so that the payoffs are $v_i(x, a_i)$ for $i = 1, 2$. In this setting, the first-best is achievable with a noncontingent property rights allocation. For example, suppose that agent 2 ownership is efficient given the first-best investments $(a_1^*, a_2^*)$. Then a contract that specifies $\widehat{x} = 1$ implements the first best.*

If part or all of $\varepsilon$ is not verifiable, there may be no first-best outcome in which the property rights depend only on $\varepsilon_v$, the verifiable part of $\varepsilon$, as imagined in Proposition 17. Nevertheless, in some situations we can still implement the first-best with a *noncontingent* property rights allocation $\widehat{x}$, as shown by Edlin and Reichelstein [1996]. Specifically, they assume:

**No Ex Ante Noncontractibles:**: Payoffs $v_i(x, a, q, \varepsilon)$ do not depend on $q$.

**Self-Investments**: Each agent $i$'s Nash equilibrium payoff $v_i^\circ(x, a, \varepsilon)$ does not depent on $a_{-i}$.

The Self-Investments assumption implies that there would be no externalities in the absence of renegotiation; the entire source of externalities is bargaining. Together, the two assumptions allow us to write $v_i^\circ(x, a, \varepsilon) = v_i(x, a_i, \varepsilon)$.[13] Edlin and Reichelstein also introduce a separability assumption:

**Separability**: $X$ is a convex subset of $\mathbb{R}^K$, and for each $i$,

$$v_i(x, a_i, \varepsilon) = \overline{v}_i(a_i) \cdot x + \widehat{v}_i(x, \varepsilon) + \widetilde{v}_i(a_i, \varepsilon).$$

With these assumptions, we get the following result:[14]

**Proposition 20 (Edlin-Reichelstein)** *Suppose that the Self-Investments, No Ex Ante Noncontractibles, and Separability assumptions hold, and bargaining is given by the Nash bargaining solution. Then the noncontingent property rights allocation specifying $\widehat{x} = \mathbb{E}_\varepsilon[x^*(a^*, \varepsilon)]$ – i.e., the* expected *efficient decision given the first-best investments $a^*$– sustains $a = a^*$ as an equilibrium, and every equilibrium given $\widehat{x}$ results in the same payoff for each agent.*

**Proof.** Let

$$U_i(x, a) = \mathbb{E}_\varepsilon[u_i(x, a, \varepsilon)] = \mathbb{E}_\varepsilon[v_i(x, a_i, \varepsilon)] + \lambda_i \mathbb{E}_\varepsilon[R(x, a, \varepsilon)].$$

[13] Note, however, that the Self-Investments assumption is formulated to allow cases with ex ante noncontractible decisions - e.g., it is satisfied whenever the Nash equilibrium $q^\circ(a, \varepsilon)$ does not depend on $a$ and $v_i(x, a, q, \varepsilon)$ does not depend on $a_{-i}$, as in Grossman-Hart [1986].

[14] Edlin and Reichelstein assume that $x \in \mathbb{R}$ and $a_i \in \mathbb{R}$, assumptions that we dispense with here.

Note that $(\widehat{x}, a^*)$ results in the first-best (state-contingent) outcome $(x^*(a^*,\cdot), a^*)$ for any value of $\widehat{x}$ due to efficient renegotiation (specifically, $\sum_i U_i(\widehat{x}, a) = \mathbb{E}_\varepsilon[S(a,\varepsilon)]$ for any $\widehat{x}$). We will show that there are no harmful externalities at $(\hat{x}, a^*)$, which yields the result by Proposition 3.

Given Self-Investments, the only externalities are in the expected renegotiation surplus $\mathbb{E}_\varepsilon[R(\hat{x}, a, \varepsilon)]$, so it suffices to show that this expectation is minimized at $a = a^*$. To see this, observe that for any $a$,

$$\begin{aligned}\mathbb{E}_\varepsilon[R(\hat{x},a,\varepsilon)] &= \mathbb{E}_\varepsilon \max_x \left[\sum_i [\overline{v}_i(a_i)\cdot(x-\hat{x}) + \widehat{v}_i(x,\varepsilon) - \widehat{v}_i(\hat{x},\varepsilon)]\right] \\ &\geq \mathbb{E}_\varepsilon\left[\sum_i [\overline{v}_i(a_i)\cdot(x^*(a^*,\varepsilon)-\hat{x}) + \widehat{v}_i(x^*(a^*,\varepsilon),\varepsilon) - \widehat{v}_i(\hat{x},\varepsilon)]\right] \\ &= \sum_i \mathbb{E}_\varepsilon[\widehat{v}_i(x^*(a^*,\varepsilon),\varepsilon) - \widehat{v}_i(\hat{x},\varepsilon)]\end{aligned}$$

Since the inequality holds with equality at $a = a^*$, the result follows. ■

Finally, recall that efficiency also resulted in Example 12 when $\lambda_1 = 1$. In contrast to the cases considered in Propositions 17 and 20 (and Example 16), harmful externalities were present at the first-best property rights allocation. In Example 12, even though the ex post efficient allocation of property rights could be specified ex ante, a first-best outcome was achieved by initially specifying a property rights allocation that was not ex post efficient and was later renegotiated. Our third efficiency result generalizes the idea behind Example 12:

**Proposition 21** *Suppose that only agent $i$ invests, that agent $i$ has all of the bargaining power in any renegotiation (in the sense that he captures all of the renegotiation surplus), that renegotiation is efficient, and that there is a property rights allocation $\widehat{x}$ at which there are no direct externalities from agent $i$'s investments. Then the initial property rights allocation $\widehat{x}$ results in a first-best outcome.*

**Proof.** A deviation by agent $i$ has no effect on the ultimate (post-renegotiation) payoff of any agent $j \neq i$. Hence, agent $i$ will invest efficiently. Renegotiation will also ensure that the final property rights allocation is efficient. ■

## 3.3 Externalities and Distortions

We now shift attention to settings in which the first-best cannot be achieved with a property rights allocation. We focus here on cases studied in the Property Rights Theory of Hart and Moore [1990] (and Hart [1995]). This theory examines situations, like those considered by Edlin and Reichelstein [1995], in which parties make self-investments (such as investments in their own human assets) so that there are no externalities in the absence of renegotiation, but the investments can still generate harmful bargaining externalities. Unlike in Edlin and Reichelstein [1995], however, harmful bargaining externalities cannot be eliminated as in Proposition 17 because "incomplete contracting" makes the parties unable to specify the ex post efficient allocation in an ex ante contract.

**Example 22** *Return to the setting of the dynamic Tragedy of the Commons Example 12, with the following modifications. First, assume that* $\gamma_1 = \gamma_2 = 0$*, i.e., there are no direct externalities. Second, add an ex post "cooperation" decision* $q_i \in \{0, 1\}$ *for each party* $i$*. Modify the payoffs to be* $b_i(x_i, x_j, a_i, a_j) = x_i q_j \max\{0, \beta_i + v_i a_i\} - \tau q_i$*, where* $\tau > 0$ *is small. Thus, each party can prevent the other from deriving a benefit from the lake by choosing* $q_i = 0$*, thereby recieving a small benefit itself. The efficient ex post outcome is to have common access and cooperation:* $(q_1^*, q_2^*) = (x_1^*, x_2^*) = (1, 1)$*, but the Nash equilibrium has no cooperation:* $(q_1^\circ, q_2^\circ) = (0, 0)$*.*

*In this model, the initial allocation of property rights has no role (this model can be reinterpreted as stemming from a legal system where long-term property rights allocation is impossible and the default is no access to anybody). Ex post the parties renegotiate to common access and cooperation. Assuming Nash bargaining, each party* $i$ *receives* $\lambda_i \max\{\beta_i + v_i a_i\} - \phi_i(a_i)$*, and will choose ex ante investment* $a_i$ *to maximize this expression. This entails underinvestment relative to the first-best investment which maximizes* $\max\{\beta_i + v_i a_i\} - \phi_i(a_i)$*. Note that the underinvestment is due to the parties' inability to specify cooperation* $(q_1^*, q_2^*) = (1, 1)$ *in the ex ante contract: indeed, if they were able to do it, then by Proposition 17 (incorporating* $q$ *into* $x$*) they would be able to implement the first-best.*

We now consider a two-agent Property Rights Theory model (we discuss the extension to an arbitrary number of agents in Section 3.5). As noted above, Hart and Moore [1990] assume Self-Investments, so that we can write each agent $i$'s disagreement payoff as $v_i^\circ(x, a_i, \varepsilon)$.[15] We also assume Nash bargaining, as given in (10). Note that with Self-Investments, any externalities from investments come through the renegotiation surplus $R(x, a, \varepsilon)$. Thus, we have positive externalities when the renegotiation surplus is increasing in investments, i.e., when the incremental benefit of investment on the total surplus $S(a, \varepsilon)$ is greater than on the parties' disagreement payoffs, as was true in Example 22. This condition is called *relationship specificity*:

**Definition 23** *We have relationship specificity of agent* $i$*'s investment at* $x$ *if* $S(a, \varepsilon) - v_i^\circ(x, a, \varepsilon)$ *is nondecreasing in* $a_i$ *for all* $i, x, a_{-i}$*, and* $\varepsilon$*.*

One set of more primitive sufficient conditions ensuring relationship specificity is given in Lemma 24 below. The lemma shows that we get relationship specificity if the returns to increasing investments are largest when the property rights $x$ and ex post contractible decisions $q$ are set at their ex post efficient levels and the Nash equilibrium decisions are independent of investments.

[15] Hart and Moore [1990] do not explicitly allow for any uncertainty $\varepsilon$, but one can view their payoff functions as expectations given some uncertainty $\varepsilon$.

**Proposition 24** *Suppose that* $q^{\circ}(a,\varepsilon)$ *does not depend on* $a$*, and that for all* $i, a, a_i', x,$ *and* $\varepsilon$ *such that* $a_i' > a_i$*,*

$$\begin{aligned} & v_i\left(x^*(a,\varepsilon), q^*(a,\varepsilon), a_i', \varepsilon\right) - v_i\left(x^*(a,\varepsilon), q^*(a,\varepsilon), a_i, \varepsilon\right) \\ \geq\ & v_i\left(x, q^{\circ}(\varepsilon), a_i', \varepsilon\right) - v_i\left(x, q^{\circ}(\varepsilon), a_i, \varepsilon\right). \end{aligned}$$

*Then we have relationship specificity.*

**Proof.** We then obtain, for $a_i' > a_i$

$$\begin{aligned} & S\left(a_i', a_{-i}, \varepsilon\right) - S(a,\varepsilon) \\ \geq\ & \sum_i v_i\left(x^*(a,\varepsilon), q^*(a,\varepsilon), a_i', \varepsilon\right) - \sum_i v_i\left(x^*(a,\varepsilon), q^*(a,\varepsilon), a_i, \varepsilon\right) \\ \geq\ & \sum_i [v_i\left(x, q^{\circ}(\varepsilon), a_i', \varepsilon\right) - v_i\left(x, q^{\circ}(\varepsilon), a_i, \varepsilon\right)] \end{aligned}$$

which implies relationship specificity. ■

Under relationship-specificity, we have positive externalities, and we know by Proposition 6 that positive externalities imply underinvestment when one agent makes a one-dimensional investment. To get the same result with multi-agent and/or multidimensional investments, we can apply Proposition 7, provided that the investment game is supermodular:

**Proposition 25** *Suppose that there is Nash bargaining, self investments, relationship-specificity,* $S(a,\varepsilon)$ *is supermodular in* $a$*, and* $v_i^{\circ}(x, a_i, \varepsilon)$ *is supermodular in* $a_i$ *for* $i = 1, 2$*. Then there exist smallest and largest elements of the sets* $A^{\circ}(x)$ *and* $A^*(x)$*, denoted respectively by* $(\underline{a}^{\circ}, \overline{a}^{\circ})$ *and* $(\underline{a}^*, \overline{a}^*)$*, and* $\underline{a}^* \geq \underline{a}^{\circ}$*.*

**Proof.** Under the assumptions, the induced investment game payoffs $U_i(x,a) = E_{\varepsilon}[u_i(x,a,\varepsilon)]$ are supermodular in $a_i$ and have increasing differences in $(a_i, a_{-i})$, and investments generate positive externalities. Applying Proposition 7 yields the result. ■

Provided that there is a unique investment equilibrium, Proposition 25 implies that the equilibrium investments are (weakly) less than in any first-best outcome.

## 3.4 Second-best Property Rights Allocations

In the models of Hart and Moore [1990] (and Hart [1995]), the ex ante contractible decision $x \in X$ is a "control structure" over a set $K$ of assets. In contrast to our treatment of the Tragedy of the Commons above, Hart and Moore rule out control structures in which more than one agent can independently access an asset. Thus, with two agents, letting $x_{ik} \in \{0,1\}$ denote whether agent $i$ has access to asset $k$, we must have $x_{1k} + x_{2k} \leq 1$. The set of feasible control structures over $K$ assets is then

$$X = \left\{(x_1, x_2) \in \{0,1\}^K \times \{0,1\}^K : x_{1k} + x_{2k} \leq 1 \text{ for all } k\right\}.$$

Hart and Moore refer to the case in which neither agent has control of an asset $k$, so that $x_{1k} = x_{2k} = 0$, as "joint ownership" of the asset. (This is the opposite of common ownership in the Tragedy of the Commons, in the sense that each agent can exclude the other without himself having the right to access the asset.) Hart and Moore also assume that there are no externalities from access in the following sense:

**Definition 26** *There are no externalities from access to assets if $v_i^\circ(x, a, \varepsilon)$ does not depend on $x_{-i}$.*

When there are no externalities from access, an agent's (Nash equilibrium) disagreement payoff depends on the assets he has access to, but not on the set of assets that can be accessed by other agents. In that case, agent $i$'s disagreement payoff can be written as $v_i^\circ(x_i, a, \varepsilon)$. Note that this assumption may be violated if agent $-i$'s choice of actions $q_{-i}$ depends on $x_{-i}$ and generates externalities. Also, the assumption is not natural if common ownership (where both agents can have access to the same asset without reaching an agreement) is possible.

The important aspect of this assumption is that, with Self-Investments and Nash bargaining, it implies that each agent's post-renegotiation utility $u_i(x, a, \varepsilon)$ has a form that is separable between $a_i$ and $x_{-i}$:

$$u_i(x, a, \varepsilon) = \lambda_i S\left(a, \varepsilon\right) + (1 - \lambda_i)v_i^\circ(x_i, a_i, \varepsilon) - \lambda_i v_{-i}^\circ(x_{-i}, a_{-i}, \varepsilon).$$

Hence, only agent $i$'s asset ownership affects agent $i$'s investment incentives.

To make statements about optimal second-best control structures, Hart and Moore also assume "asset specificity," i.e., that access to assets increases agents' marginal incentives to invest:

**Definition 27** *There is asset specificity if $v_i^\circ\left(x_i, a, \varepsilon\right)$ has increasing differences in $(x_i, a_i)$.*

This property implies that an agent's investment is increased by shifting assets to him. Since under relationship specificity we have underinvestment, this implies that when only one agent makes a one-dimensional investment, he should optimally own all of the assets:

**Proposition 28** ***(Hart and Moore's Proposition 2)*** *If only agent $i$ invests (i.e., $A_{-i}$ is a singleton), his investment is a one-dimensional asset-specific relationship-specific self-investment, there are no externalities from access to assets, and $\widehat{X} \neq \varnothing$, then it is optimal for agent $i$ to own all of the assets, i.e., $(\widehat{x}_i, \widehat{x}_{-i}) = \left(\left(1, ..., 1\right), \left(0, ..., 0\right)\right) \in \widehat{X}$.*

**Proof.** To begin, consider the relaxed problem in which we ignore the restriction that $x_{1k}+x_{2k} \leq 1$, allowing the feasible property rights to be $(x_1, x_2) \in \{0,1\}^K \times \{0,1\}^K$. Under the assumptions of the proposition, $U_i\left(x, a_i\right) = E_\varepsilon[u_i(x, a_i, \varepsilon)]$ has increasing differences in $(x_i, a_i)$ and (weakly) in $(-x_{-i}, a_i)$. Relationship specificity means that the investment $a_i$ generates positive externalities. Moreover, since ex post bargaining is efficient, the aggregate surplus $\sum_i U_i(x, a_i)$ is independent of $x$, or in terms of the notation of Section 2, $X^*(a_i) = X$. Corollary 11 then implies that $(\widehat{x}_i, \widehat{x}_{-i}) = ((1, ..., 1), (0, ..., 0))$ is an optimal property rights allocation in the relaxed problem. Since this allocation also satisfies the constraint that $x_{1k} + x_{2k} \leq 1$ it is in fact optimal. ■

If more than one agent invests then in general we have tradeoffs, since with asset specificity moving an asset from agent $i$ to agent $j$ has the direct effect of raising $i$'s investment incentives (i.e., his best-response curve) and lowering $j$'s. In some special cases, however, agent $j$'s incentives may not be affected. When a shift in ownership can raise one party's incentives without lowering the other's, we can derive unambiguous welfare conclusions, provided that $S(a, \varepsilon)$ is supermodular in $a$.[16] In such cases, we can conclude that all equilibrium investments increase whenever all agents' investment incentives do. As Hart and Moore note, a number of conclusions follow immediately. Here we present a few of these results as an illustration. To do so, we first need to state two definitions:

**Definition 29** *Agent $i$ is indispensable to asset $k \in K$ if $v^\circ_{-i}(x', a, \varepsilon) - v^\circ_{-i}(x, a, \varepsilon)$ does not depend on $a_{-i}$ whenever $x'_{il} = x_{il}$ for all $l \neq k$.*

Intuitively, indispensibility of agent $i$ to asset $k$ means that ownership of asset $k$ does not affect the marginal product of agent $-i$'s investments in the absence of agent $i$.

**Definition 30** *Assets $l, m \in K$ are perfectly complementary if for each $i$, $v^\circ_i(x', a, \varepsilon) - v^\circ_i(x, a, \varepsilon)$ does not depend on $a_i$ whenever $x'_{ik} = x_{ik}$ for $k \notin \{l, m\}$, $x_{il} + x_{im} \leq 1$, and $x'_{il} + x'_{im} \leq 1$.*

Intuitively, when two assets are perfectly complementary, the marginal product of investment for agent $i$ when he has only one of the two assets equals the marginal product when he has neither.

**Proposition 31** ***(Hart and Moore's Propositions 4, 6, and 8)*** *If investments are asset-specific relationship-specific self-investments, there are no externalities from access to assets, $S(a, \varepsilon)$ is supermodular in $a$, and $\widehat{\Sigma} \neq \varnothing$, then*

*1.* **(i)** *joint ownership is unnecessary in an optimal control structure; i.e., there exists a second-best ownership structure $\widehat{x} \in \widehat{X}$ such that $\widehat{x}_{1k} + \widehat{x}_{2k} = 1$ for all $k$;*

**(ii)** *if agent $i$ is indispensable to asset $k$, then agent $i$ optimally owns asset $k$, i.e., there exists $\widehat{x} \in \widehat{X}$ such that $\widehat{x}_{ik} = 1$.*

**(iii)** *it is optimal for perfectly complementary assets to be owned together, i.e., there exists $\widehat{x} \in \widehat{X}$ such that $\widehat{x}_{il} = \widehat{x}_{im} = 1$ for some $i$.*

[16] When $S(a, \varepsilon)$ is supermodular in $a$, Proposition 28 can also be extended to multidimensional investments by agent $i$.

**Proof.** Result (i) follows from Corollary 11 by noting that, for each agent $i$, $U_i(x, a_i) = E_\varepsilon[u_i(x, a_i, \varepsilon)]$ has increasing differences in $(x_i, a_i)$ and $(-x_{-i}, a_i)$. Next, note that given result (i), when considering results (ii) and (iii) we can describe the assets that each agent has access to by means of $x_i$, the assets agent $i$ controls. For result (ii), note that when agent $i$ is indispensable to asset $k$, $v_j^\circ(x_i, a_j)$ has increasing differences in $(x_{ik}, a_j)$ for $j = 1, 2$. Using also the other assumptions, the result follows by Corollary 11. For result (iii), suppose without loss of generality that $x_{1l} = 1$ and $x_{1m} = 0$. Then, fixing all dimensions of $x_1$ other than $m$, $v_j^\circ(x_1, a_i)$ has increasing differences in $(x_{1m}, a_j)$ for $j = 1, 2$. The result follows from Corollary 11. ■

#### 3.4.1 Violation of Self-Investments: "Cooperative" Investment

We now examine how the conclusions of the Hart and Moore's Property Rights Theory are modified when the assumptions about investments are changed. We begin by changing the assumption of Self-Investments. While Self-Investments was an appropriate assumption to describe an agent's investment in his own human capital (self-training), it is not appropriate when instead an agent invests in other agents' human capital (training others). An investment may also have both self- and cooperative components, and this may also depend on asset ownership. For example, investment in the value of a physical asset is self-investment when the asset is owned by the investing person, but is purely cooperative when the asset is owned by someone else. Investments that have cooperative components have been examined by Che and Hausch [1999], Demski and Sappington [19??], Edlin and Hermalin [2000], and Segal and Whinston [2002].

When an investment has a cooperative component, it can generate externalities in two ways: (i) bargaining externalities through renegotiation surplus as before, and (ii) a direct externality. For simplicity, we shall assume (ii) is positive:

**Cooperative Investments:** $v_{-i}^\circ(x, a, \varepsilon)$ is nondecreasing in $a_i$.

Cooperative investments introduce another way in which increases in an agent's investments generate positive externalities. As a result, the one-sided underinvestment result generalizes:

**Proposition 32** *If only agent $i$ invests, his investment is a one-dimensional cooperative relationship-specific investment, and we have Nash bargaining, then there is under-investment for all ownership structures: $A^\circ(x) \leq A^*(x)$ in the strong set order for all $x \in X$.*

**Proof.** The expected payoff of agent $j \neq i$ is

$$\mathbb{E}_\varepsilon\left[(1-\lambda_j)\, v_j^\circ(x, a_i, \varepsilon) + \lambda_j\left(S(a_i, \varepsilon) - v_i^\circ(x, a_i, \varepsilon)\right)\right]$$

and it is nondecreasing in $a_i$. Apply Proposition 6. ■

With bilateral investment, supermodularity of the game is harder to ensure when there is a cooperative component to investments, because investments then interact in $v_i(\cdot)$ directly. When supermodularity holds, the underinvestment result extends to this case as well. For example, this is the case if investments are purely cooperative, so that $v_j^\circ(x, a, \varepsilon)$ depends only on $a_j$ for $j \neq i$, provided that $S(a, \varepsilon)$ is supermodular in $a$.

Since under-investment holds, Proposition 28 continues to hold with cooperative investments: If only agent $i$ invests (i.e., $A_{-i}$ is a singleton), his investment is a one-dimensional asset-specific relationship-specific cooperative-investment, there are no externalities from access to assets, and $\widehat{\Sigma} \neq \varnothing$, then it is optimal for agent $i$ to own all of the assets. However, the conclusions of Proposition 31 no longer hold, as the following example illustrates for the case where an agent is indispensible to an asset [Proposition 31(ii)]:

**Example 33** *Consider a modification of the linear-quadratic Tragedy of the Commons model where only agent 1 invests. Suppose now that the investment $a_1$ of agent 1 is training agent 2 to utilize the asset (fish in the lake) more productively. (This can be equivalently interpreted as agent 1's investment in the physical asset that only has value when the asset is used by agent 2, e.g., the asset could be "produced" by agent 1 for use by agent 2, as in Che and Hausch [1999].) The agents' payoffs are*

$$\begin{aligned} v_1(x, a_1, \varepsilon) - \phi_1(a_1) &= V - a_1 \\ v_2(x, a_1, \varepsilon) &= B_2(a_1)\, x_2, \end{aligned}$$

*where $B_2(\cdot)$ is increasing in $a_1$. We focus on private ownership. The parties engage in Nash bargaining ex post where agent 1's bargaining share is $\lambda_1 \in (0,1)$. Suppose $B_2(a_1) > 0$ for all $a_1$. Then it is always ex post efficient to give the asset to agent 2: $x_2^*(a_1) \equiv 1$ for all $a_1$. However, specifying this ownership allocation ex ante will induce agent 1 to make no investment in training agent 1 (i.e., he will choose $a_1 = \min A_1$). On the other hand, if ownership is initially given to agent 1, he will maximize $\lambda_1 B_2(a_1) - a_1$, so his investment is (weakly) higher. Since in both cases there is underinvestment relative to the first-best, giving agent 1 ownership is optimal, even though agent 2 is indispensible. (If $\lambda_1 = 1$, however, the first-best is achieved in accord with Proposition 21.)*

**Remark 34** *In Example 33 there are no ex ante noncontractible decisions. (This is true generally of the models in Che and Hausch [1999], Demski and Sappington [19??], and Edlin and Hermalin [2000]). Thus, it is possible to eliminate equilibrium renegotiation by specifying the ex post efficient ownership in the ex ante contract (namely, specifying $x_2 = 1$). However, unlike with Self-Investments, this does not sustain efficient investment, because of the direct externality that is not eliminated, and gives rise to underinvestment. It is instead optimal to stimulate the seller's investment by specifying an ex post inefficient ownership that induces renegotiation in equilibrium. This creates a renegotiation externality but reduces the direct externality. As the renegotiation externality proves to be smaller than the direct externality, it turns out that the seller's incentive is optimized by eliminating the direct externality completely* *by giving the seller the asset ex ante, which he always sells ex post, and (provided $\lambda_1 < 1$) we still have underinvestment due to the renegotiation externality. More generally, for an examination of the otpimal contract when investments have cooperative and self-components see Segal and Whinston [2002], Subsection 3.2.*

**Remark 35** *Even if there is no "physical asset," a "control right" could be created for agent 1 over agent 2 to induce him to invest in agent 2's human capital. For example, if agent 2 requires some outside market to realize his benefit $B_2$, then an "exclusive dealing" contract preventing agent 2 from realizing $B_2$ without agent 1's permission would raise agent 1's investment. Marvel [19??] offers this motive as a justification for exclusive dealing contracts that manufacturers have with retailers.*

As Example 33 illustrates, in contrast to Proposition 31(ii), it may be strictly suboptimal for an agent that is indispensible to an asset to own it when investments are cooperative. The reason is that even if one agent $i$ is indispensable to an asset $k$, agent $j$'s investment incentives may be affected by the assignment of the property right to the asset. Although it does not affect $j$'s disagreement point, it does affect agent $i$'s disagreement point, which is now sensitive to agent $j$'s investment. As a result, agent $j$'s incentives could still be improved by giving him the asset.

For similar reasons, with cooperative investments it may be optimal for perfectly complementary assets to be owned separately or to have joint ownership of an asset [Proposition 31(i) and (iii)]. For the latter, consider the following example from Hart [1995]:

**Example 36 (Hart 1995, p. 68)** *There is one asset whose value is $B(a_1, a_2)$ in either party's hands, and which is increasing in both parties' investments $(a_1, a_2)$. The payoffs are*

$$v_i(x, a_1, a_2) - \phi_i(a_i) = x_i B(a_1, a_2) - a_i.$$

*Thus $S(a) = B(a_1, a_2) - a_1 - a_2$. Note that ex post efficiency dictates that the asset be owned privately by either agent: $X^* = \{(1,0), (0,1)\}$. However, if ex ante we give private ownership to agent $i$ then agent $-i$ will have no incentive to invest, and will choose $a_{-i} = \min A_{-i}$. It may thus be better to specify joint ownership of the asset: $x_1 = (0,0)$. It will be renegotiated ex post, and assuming that each agent $i$ will capture 50% of the ex post surplus, his payoff will be*

$$u_i(a, x) = \frac{1}{2} B(a_i, a_{-i}) - a_i.$$

*So both agents will invest, and the total surplus may be higher with joint ownership than with private ownership.*

### 3.4.2 Violation of Relationship-Specificity: "External" Investment and Overinvestment

Relationship-specificity asserted that an agent $i$'s investment increased the efficient surplus $S(a, \varepsilon)$ by more than it increased the agent's disagreement payoff $v_i^{\circ}(x, a, \varepsilon)$. This condition implied that bargaining externalities were posiitve, which in turn implied under-investment. In some cases, however, an agent's investment may instead increase his disagreement payoff by more than it increases the efficient surplus, leadign to over-investment. As the following example shows this may make it optimal to take asset's away from an agent, even when that agent is the only investing agent:

**Example 37** *Consider again Example 22 but now assume that only agent 1 invests. The payoffs, excluding investment costs, are now* $b_1(x_1, x_2, a_1) = x_1 q_2 \max\{0, \beta_1\} + x_1(1-q_2)a_1 - \tau q_1$, *and* $b_2(x_1, x_2, a_1) = x_2 q_1 \max\{0, \beta_2\} - \tau q_2$ *where* $\beta_1 > \overline{a}_1$. *As in Example 22, the efficient ex post outcome is to have common access and cooperation:* $(q_1^*, q_2^*) = (x_1^*, x_2^*) = (1, 1)$, *and the Nash equilibrium has no cooperation:* $(q_1^{\circ}, q_2^{\circ}) = (0, 0)$. *Now, however, agent* 1*'s disagreement payoff is* $v_1^{\circ}(x_1, a_1) = x_1(1 - q_2)a_1$, *while the efficient surplus is* $S(a_1) = \beta_1 + \beta_2$, *so relationship specificity does not hold. The first-best investment is* $a_1^* = 0$. *When agent 1 owns the asset, however, his investment is strictly positive. When instead, agent 1 does not own the asset, he chosoes not to invest and the first-best is achieved.*

As other examples of this point, Holmstrom and Tirole [19??] consider "negotiated transfer pricing" and show that it may be optimal to ban trade outside the firm to reduce rent-seeking investment. This motivation can also lead to exclusive dealing contracts being adopted to induce retailers not to allocate effort/investment towards other manufacturers, see e.g., Areeda and Kaplow [19??] and Segal and Whinston [2000].

### 3.4.3 Violation of Asset-Specificity: Ownership Reduces Investment Incentives

In contrast to the examples considered previously, we can also have examples in which the marginal benefit of investment is higher when an agent does not own an asset than when he does. For example, investment may only be valuable when the agent does not have access to the asset. Equivalently increasing the investment reduces the (opportunity) cost of transferring the asset to another agent (e.g., when the asset is "produced" by the seller and investment reduces the production cost). In this case, investment is increased by taking the asset away from the agent. For example, when the agent is a seller who invests in reducing the cost of producing the asset, it may be optimal to specify that he must deliver the asset to the buyer, which increases his incentive to invest in cost reduction.

### 3.5 More Than Two Agents

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## 4 Bargaining Inefficiencies: Asymmetric Information

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## 5 Bargaining Inefficiencies: Bilateral Contracting with Externalities

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## 6 Property Rights Mechanisms

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## 7 Conclusion

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