SEARCH WITH INCOMPLETE INFORMATION

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Abstract. We study a search model with incomplete information in which a decision maker has to select one agent from a sequence. The value of the agents for the principal is their private information, which can be reported to the principal. The first-order issue for the principal is the tradeoff between the quality of the selected agent and the ability to infer the agents' quality from their reports: selecting the agent with the highest reported quality destroys incentives for truthful reporting. The optimal policy balances these considerations by selecting an agent with a lower reported quality with positive probability and restricting the number of agents that are sampled. The model is tractable and we obtain an interesting non-stationary optimal search policy, which stands in stark contrast to the optimal cutoff policy under complete information.

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1. Introduction

We study a model of a decision maker (principal) who is searching for an agent. The agents arrive sequentially and their value for the principal is their private information. All agents would like to be chosen and they can make a report about their value. If the agent is selected, her value eventually becomes known and if it turns out to be different from the principal's perception, the principal can impose a penalty on the agent. The agent has limited liability and the penalty is bounded. This is a search model with recall, incomplete information, and deception costs.

There are multiple applications. The principal could be a politician who is searching for a bureaucrat to appoint to a post, a CEO of a company looking for a successor, or an owner of a small firm trying to find a partner to expand. The model can also apply to a venture capitalist selecting among business ideas of different entrepreneurs, a manager choosing among marketing strategies suggested by employees, or a municipal government open to alternative design proposals of a public project.

The first-order issue for the principal is the tradeoff between the quality of the selected agent and the ability to infer the agents' quality from their reports. The policy that selects the agent with the highest reported quality or the first agent with the reported policy that exceeds a certain threshold destroys incentives for truthful reporting: in equilibrium, each agent will find it optimal to report that she has the highest value for the principal since otherwise her chances of being selected are slim. As a result, these policies are not better than selecting the first available agent (Proposition 1). Consequently, the optimal policy provides incentives for informative reporting by rewarding low reports with positive probability of selection. The interim expected probability of being chosen by the principal is always positive, increases in the agent's report, but never reaches one.

How many agents should the principal consider? Sampling more agents is good as it improves the distribution of the first order statistics of the agent's quality. On the other hand, it also increases competition between the agents and weakens the incentives for the low quality agents to tell the truth since the chances of being selected are now smaller. In order to keep communication with the agents informative, the principal has to compensate the low quality agents by increasing the probability of choosing them. The optimal policy, thus, must balance these considerations and can potentially restrict the number of agents that are sampled. If the principal is infinitely patient and his costs of search are zero, a neutrality result obtains: if the principal samples at least two agents, his payoff is independent of their number. A principal who dislikes waiting, even if the waiting cost is arbitrarily small, strictly prefers to sample at most two agents.

The optimal search policy is non-stationary. We show that if the principal is sufficiently patient, the optimal policy takes the following form. When the principal meets the first agent, he chooses a deadline by which he will decide about whether to accept the agent. The deadline depends on the agent's report about her value to the principal; it is shorter if the reported quality is high. If the principal meets another agent prior to the deadline, he probabilistically chooses one of the two agents. Otherwise, he selects the first agent at the deadline.

Our model introduces incomplete information in the otherwise basic search model with exogenous arrival of potential matches. The model is tractable and we obtain an interesting non-stationary optimal policy, which stands in stark contrast to the optimal cutoff policy under complete information.[1] We hope that this way of modeling incomplete information will prove useful in other search environments.

Related literature. An important point of this paper is that competition might be harmful to the objectives of the principal and he can optimally choose to restrict the number of agents to be sampled. The observation that it might be desirable to check competition has been made in other contexts. In Manelli and Vincent (1995), a principal would like to procure a good from suppliers whose quality is uncertain. In their environment, a trading mechanism that selects the bidder with the lowest price might result in only low quality goods being offered for sale. In Chakravarty and Kaplan (2013) and Condorelli (2012), a benevolent principal would like to allocate an object to the agent with the highest valuation. The agents can compete by exerting socially wasteful effort and it might be optimal to select the winner randomly or only based on the publicly available information. In Bar and Gordon (2011), a firm would like to select a project(s) with the highest quality at the minimal cost, but the project's quality is not publicly known. The optimal mechanism might include randomization over projects whose reported quality exceeds a threshold. Randomization and inefficiency are the features of the optimal policy in our model; the effect that it is optimal to restrict participation of the agents is novel.

Our model is a basic search model with one principal and exogenously arriving agents. Guerrieri, Shimer and Wright (2010) analyze competitive search with incomplete information. In their model, uninformed principals compete in mechanisms that screen privately informed agents based on their reports. The substantive difference with respect to incomplete information is that in our model the mechanism (penalty) can condition *both* on the agent's report and on the outcome.[2] In addition, in Guerrieri, Shimer and Wright (2010), mechanism offers affect agents' incentives to apply to the principal, but, unlike in our model, cannot determine the match.

In a directed search model in Menzio (2007), agents makes statements about their privately known type. There is no penalty for lying; statements determine the principals' beliefs in the bargaining game about splitting the surplus from the match.

A number of papers focus on search environments in which the principal is privately informed. We refer the reader to Lauermann and Wolinsky (2011) and the references therein.

In Cremer, Spiegel and Zheng (2006, 2007), an auctioneer searches for bidders who are privately informed about their valuations. Cremer, Spiegel and Zheng (2007)

[1] See, e.g., the survey by Rogerson, Shimer and Wright (2005) or the references therein.

[2] See Mezzetti (2004) and Eraslan and Yimaz (2007) for mechanism design in environments in which the mechanisms can condition either on the agents' reports about the outcome or directly the outcome.

show that if the bidders types are independent, the seller's problem can be reduced to a search problem in which the surplus is measured in terms of virtual utilities. These models allow for monetary transfers and there is no possibility to learn the valuation of the winning bidder ex-post.

There is a literature on cheap talk and mechanism design in environments with either lying costs or limited ability of different types to imitate each other (see, e.g., Dziuda (2012), Severinov and Deneckere (2006), Deneckere and Severinov (2008), Kartik, Ottaviani and Squintani (2007), and Kartik (2009)).

Finally, Moldovanu and Shi (forthcoming) consider a model of search by a committee where each committee member has an ability to privately evaluate the value of an attribute of an alternative. In their model, the strategic issue is interaction between privately informed committee members who have conflicting preferences rather than between uninformed decision maker(s) and privately informed agents.

2. The Model

There is a *principal* who seeks to match with an agent. Agents arrive sequentially. We will consider two models: with deterministic arrival times $t = 1, 2, \ldots$, and with random arrival times on $[0, \infty)$, which are distributed according to the exponential distribution with mean arrival time $\lambda > 0$. The principal's undiscounted payoff from being matched with agent i is $x_i \in X \equiv [a, b]$. We depart from the standard assumption in search that x_i is observed by the principal and assume that:

Assumption 1. *The value of x_i is privately known by agent i.*

The values of x_i's are i.i.d. random draws, with continuously differentiable c.d.f. F whose support $X = [a, b]$ satisfies $0 < a < b$ and whose density f is positive almost everywhere on X. As long as Assumption 3 below holds, our results do not require the knowledge of the distribution function F by the principal: the optimal decision rule turns out to be independent of it.

Once an agent i has arrived, she makes a statement $y_i \in X$ about her value for the principal. It is irrelevant for the results whether or not agents are informed about their positions in the sequence, times of arrival, and the statements of the prior agents.

If the agent is not selected by the principal, she gets zero payoff. If the agent is chosen, she obtains utility from the match $v(x_i, y_i)$. We assume that the agent incurs a disutility cost of $c > 0$ if she is matched with the principal but her report is not truthful, $y_i \neq x_i$.

Assumption 2. *Lying about the value of x_i is costly for the agent:*

$$v(x_i, y_i) = \begin{cases} u(x_i), & \text{if } y_i = x_i, \\ u(x_i) - c, & \text{if } y_i \neq x_i, \end{cases} \tag{1}$$

where $u(x_i)$ is a strictly increasing function.

The cost c could represent the intrinsic disutility borne by the agent for being dishonest or the value of lost reputation. Another interpretation is that the principal's payoff from the match x_i becomes verifiable and, after the match, the principal can impose a penalty on the agent for a false statement. The agent has limited liability and the amount of penalty is bounded by c.

If the penalty for false reports c is large, the incentives to report truthfully are restored and the model reduces to the one with the commonly observed value of x_i. We assume that

Assumption 3. *The disutility of lying, c, is sufficiently small such that providing incentives for the agents to report their private information truthfully is nontrivial:*

$$\frac{1}{c} \geq \frac{1}{u(a)} + \int_a^b \frac{1}{u(x)} f(x)\mathrm{d}x \tag{2}$$

Since $\int_a^b \frac{1}{u(x)} f(x)\mathrm{d}x \leq \frac{1}{u(a)}$, a stronger sufficient condition is

$$c \leq u(a)/2. \tag{2'}$$

In particular, $c < u(a)$, so that even the agent with the lowest type has incentive to lie if by doing so she can guarantee to be selected with certainty.

Assumption 3 is stronger than $c < u(a)$ in order to guarantee that the optimal rule can be characterized in reduced form in terms of interim expected probabilities of match conditional on the agents' types and that there exists a feasible rule that is consistent with these probabilities.[3]

The payoffs are discounted: if a match with agent i occurs at time t, the principal's payoff is equal to $W(x_i, t) = x_i e^{-rt}$ and the selected agent's payoff is equal to

$$U(x_i, y_i, t) = \begin{cases} u(x_i)e^{-rt}, & \text{if } y_i = x_i, \\ (u(x_i) - c)e^{-rt}, & \text{if } y_i \neq x_i, \end{cases} \tag{3}$$

where $r > 0$ is a discount rate.

The search is with recall, i.e., the principal can choose any agent who has previously arrived and he commits to an assignment rule that prescribes when to stop and which agent to choose among those who have arrived conditional on their reports. The assignment rule is common knowledge among the agents. No results are affected if we impose a weaker assumption that the principal can recall only one agent, e.g., the agent who has arrived prior to the current one.

3. Simple Rules

If the value of x_i were commonly known, the optimal rule for the principal would be a threshold rule: the principal matches with the first agent whose value exceeds a certain cutoff. When the value is not observable, the rules must rely on the agents' reports. Thus, the appropriate definition of *threshold rule* is a rule under which the principal chooses the first agent who reports a value above some threshold $y^* \in X$.

The incentives of the agents, however, make this rule outcome equivalent to the naïve rule of choosing the first available agent: Since the penalty for lying is small and

[3] This issue is studied extensively in auction design. See, for example, Che, Kim and Mierendorff (2011) and Hart and Reny (2011).

$u(x)-c>0$ for all $x \in [a,b]$, it is optimal for the first agent to report $y_1 = \min\{x_1, y^*\}$ and be chosen with certainty even if she has to pay the penalty.

Instead of using the threshold rule, the principal can attempt to take advantage of the competition among the agents by choosing an agent with the highest report among, e.g., the first $n \geq 2$ agents, a kind of *n-agent auction.* In Appendix B, we show that in the unique symmetric equilibrium of n-agent auction every agent $i \in \{1,\dots,n\}$ reports the highest value $y_i = b$ irrespective of her type. Consequently, the agents' statements are uninformative in the auction and its outcome is equivalent to a lottery among the first n agents. Since choosing any agent other than the first involves a delay, the auction performs even worse than the threshold rule.

The existence argument is essentially identical to the argument for the threshold rule. If everyone is bidding the highest value, $y_j = b$, for all $j \neq i$, then agent i also finds it optimal to bid the highest value since this bid ensures $\frac{1}{n}(u(x_i)-c) > 0$ and every other bid has no chance of winning.

The uniqueness is shown by, first, observing that for low values of x bidding truthfully is dominated by paying penalty c and outbidding, possibly weakly, everyone else and, then, applying this argument inductively for other values of x.

We collect these observations in:

Proposition 1.

(I) *Choosing an agent who reports a value above a certain threshold is outcome equivalent to choosing the first agent.*

(II) *Choosing an agent with a highest report among n agents is payoff inferior to choosing the first agent.*

4. Two Agents are Enough

The previous section has established that requiring a minimal standard, in the form of a minimal reported value, or exploiting competition among agents' reports does not help the principal to improve over choosing the first available agent. We now show that the principal may do better by choosing between the first *two* agents, but it never pays to sample more.

We start by arguing that in searching for an optimal rule for the principal it is sufficient to consider only *truthful* assignment rules, that is, where it is a perfect Bayesian equilibrium for all agents to make truthful statements. This is not a standard revelation principle argument, because the reports are not cheap talk. Yet, the result holds because the principal's payoff and the penalty for lying are independent of the reports: Fix an equilibrium outcome of some rule and consider a corresponding direct rule. In truthful equilibrium of this new rule, the principal's payoff is unaffected, whereas the agents' payoffs from playing the equilibrium strategies are increased due to avoiding the cost of penalty, while the payoffs from deviations are unaffected or reduced by the cost of penalty.

A truthful assignment rule p is called *optimal* if it maximizes the ex-ante expected payoff of the principal among all rules.

Proposition 2. *In every optimal assignment rule, with probability one the choice is made between the first two agents. Furthermore, if discount rate r is sufficiently small, the ex-ante probability that the principal matches with the second agent is non-zero.*

The intuition for this result is simple and robust. Consider an infinitely patient principal, $r = 0$, and a symmetric assignment rule that chooses one of the first n agents and ignores the agents who arrive later. Incentives to report truthfully pin down the marginal probability of the match for the first n agents. The feasibility and symmetry require that the ex-ante expected probability of a match is equal to $\frac{1}{n}$. This implies that the interim probability that agent i with value x will be matched must have a functional form $p_i(x) = \frac{q(x)}{n}$. The expected payoff of the principal is then $W(p) = n\int_X p(x)f(x)\mathrm{d}x = \int_X q(x)f(x)\mathrm{d}x$ and is independent of n. This holds for any n and the argument straightforwardly extends to asymmetric assignment rules. Hence, the payoff of an infinitely patient principal is constant in all truthful assignment rules that sample more than one agent. It follows that if the principal is impatient and waiting for additional agents is costly, $r > 0$, it is optimal to sample at most two agents. Finally, if the costs of waiting are sufficiently small, it is optimal to sample more than one.

It is interesting to compare this result with the optimal rule in the environment in which the agents' values are observable to the principal. There, the principal's optimal strategy is a threshold rule. As the principal becomes more patient ($r \to 0$), the threshold $x^* = x^*(r)$ approaches the upper bound of X and the expected waiting time increases ad infinitum. Intuitively, if the values are observable, the marginal probability of the match is not determined by the incentives to report truthfully but instead is related to the order statistics; it is not independent of n. The interim probability has a different functional form $p_i(x) = q(x, n)$ and the principal's payoff for $r = 0$ is increasing in n.

In sharp contrast, when types are not observable and costs of lying are not high, the result stated in Proposition 2 holds for *all* $r > 0$, so the principal with arbitrarily high patience would still make his choice after observing statements of at most two agents, and the (expected) waiting time is bounded.

5. Optimal Assignment

In this section, we consider implementation of optimal assignment rules. For clarity and simplicity, we will focus on the limit case of infinitely patient principal, $r \to 0$. By continuity of the principal's payoff, these rules remain close to optimal when $r > 0$.

First, consider the model with exponential arrival of agents with mean arrival time λ. For this model, we would like to explore an implementation in which the waiting time is bounded and the rule has a flavor of betrothal. The principal (he) meets the first agent (she) and chooses a deadline, by which he promises to decide whether he accepts the agent. The deadline depends on the agent's statement and is decreasing in the declared value. If the agent reports the highest possible value, she is chosen immediately. If another agent does not arrive until the deadline, the agent is chosen and the search is over. Otherwise the choice is made probabilistically between the two agents. The probability that the principal switches to the second agent depends on that agent's announcement only and is increasing in it.

Proposition 3. *Let $r = 0$. Then there exist functions*[4] $\tau : X \to \mathbb{R}_+$ *and* $q_2 : X \to [0, 1]$ *such that the following assignment rule* p^I *is optimal:*

(a) when the first agent arrives and makes statement y_1, the principal waits $\tau(y_1)$ units of time;

(b) if no more agents arrive before the waiting time expires, then the first agent is chosen;

(c) otherwise, at the moment the second agent arrives and makes statement y_2, with probability $q_2(y_2)$ the principal chooses the second agent; with the complementary probability he chooses the first agent.

Moreover, the principal strictly prefers p^I to the rule of choosing the first agent.

In the model with deterministic arrival, we consider an implementation in which the principal chooses the first agent with some probability increasing the agent's report. Otherwise, the principal waits until the second period and chooses probabilistically between the first and the second agents.

Proposition 4. *Let $r = 0$. Then there exist functions*[5] $q_1 : X \to [0, 1]$ *and* $q_2 : X \to [0, 1]$ *such that following assignment rule* p^{II} *is optimal:*

(a) *when the first agent arrives and makes statement y_1, with probability $q_1(y_1)$ the principal chooses that agent immediately; with the complementary probability he waits for arrival of the second agent;*

(b) *when the second agent arrives and makes statement y_2, with probability $q_2(y_2)$ the principal chooses the second agent; with the complementary probability he chooses the first agent.*

Moreover, the principal strictly prefers p^{II} to the rule of choosing the first agent.

It can be verified that for $r > 0$ continuous modifications of assignment rules p^I and p^{II} (see footnotes 4 and 5) are incentive compatible, moreover, they are strictly superior to choosing the first agent for every discount rate r in a nonempty interval $[0, r_0)$ for some $r_0 > 0$.

Proposition 5. *For all $r > 0$, assignment rules p^I and p^{II} are incentive compatible.*

Appendix A. Simultaneous Arrival

Consider the model where there are $n \geq 2$ agents who arrive and make statements simultaneously and independently. In this model, the assignment rule p associates with every profile of statements $y = (y_1, \ldots, y_n)$ a probability distribution $p(y)$ over $\{1, 2, \ldots, n\}$, where for each $i \in \{1, \ldots, n\}$, $p_i(y)$ stands for the probability of choosing i. As in Section 4, we apply the revelation principle and restrict attention only to the assignment rules where truthtelling is a perfect Bayesian equilibrium.

[4] The closed form expressions are provided in the proof in Appendix B.

[5] See Footnote 4.

Denote by $\bar{F}$ the joint c.d.f. of all n agents and by $\bar{F}_{-i}$ the joint c.d.f. of all agents except i. Define

$$H = \int_X \left(1 - \frac{c}{u(z)}\right) \mathrm{d}F(z).$$

Proposition 6. *In the model with simultaneous arrival of $n \geq 2$ agents, assignment rule p is optimal if and only if it satisfies for each $i = 1, \ldots, n$ and for almost all $x_i \in X$*

$$\int_{x_{-i} \in X^{n-1}} p_i(x_i, x_{-i}) \mathrm{d}\bar{F}_{-i}(x_{-i}) = \frac{K_i}{H} \left(1 - \frac{c}{u(x_i)}\right), \tag{4}$$

for some nonnegative weights $(K_1, \ldots, K_n)$ such that $\sum_i K_i = 1$. Moreover, the principal's expected payoff is equal to

$$W(p) = \frac{1}{H} \int_{z \in X} \left(1 - \frac{c}{u(z)}\right) z f(z) \mathrm{d}z, \tag{5}$$

and it is strictly greater than the payoff from choosing the first agent, $\int_{z \in X} z f(z) \mathrm{d}z$.

Proof. Fix assignment rule p. Denote by $g_i(y_i)$ the expected that agent i is chosen after having reported y_i, assuming the rest of the agents report their types truthfully. Define $\bar{g}_i = \max_{y_i \in X} g_i(y_i)$. The incentive constraint of each agent i is therefore given by:

$$u(x_i) g_i(x_i) \geq (u(x_i) - c) \bar{g}_i \quad \text{for all } x_i \in X.$$

Hence g_i must satisfy

$$g_i(x_i) \geq \left(1 - \frac{c}{u(x_i)}\right) \bar{g}_i \quad \text{for all } x_i \in X. \tag{6}$$

Since $\sum_i p_i(x) = 1$ for all $x \in X^n$, the following feasibility constraint must hold:

$$\sum_{i=1}^{n} \int_{x_i \in X} g_i(x_i) f(x_i) \mathrm{d}x_i = \int_{x \in X^n} \left(\sum_{i=1}^{n} p_i(x)\right) \mathrm{d}\bar{F}(x) = 1. \tag{7}$$

The expected payoff of the principal is given by

$$\begin{aligned} W(p) &\equiv \int_{x \in X^n} \left(\sum_{i=1}^{n} x_i p_i(x)\right) \mathrm{d}\bar{F}(x) \\ &= \sum_{i=1}^{n} \int_{x_i \in X} x_i \left(\int_{x_{-i} \in X^{n-1}} p_i(x_i, x_{-i}) \mathrm{d}\bar{F}_{-i}(x_{-i})\right) f(x_i) \mathrm{d}x_i \\ &= \sum_{i=1}^{n} \int_{x_i \in X} x_i g_i(x_i) f(x_i) \mathrm{d}x_i. \end{aligned} \tag{8}$$

Let us now show that function[6] $p : X^n \to \Delta(\{1, \ldots, n\})$ maximizes $W(p)$ subject to (6) and (7) if and only if it satisfies the conditions of Proposition 6.

[6]We write $\Delta(A)$ for the set of probability distributions over a finite set A.

Indeed, in equation (8), expression $g_i(z)f(z)$ can be considered as a probability density, due to $g_i(z)f(z) \geq 0$ and (7). Consequently, if p maximizes $W(p)$ subject to the incentive constraint (6), then the latter must be a.e. binding,

$$g_i(x_i) = \left(1 - \frac{c}{u(x_i)}\right) \bar{g}_i \text{ for almost all } x_i \in X \text{ and all } i = 1, \ldots, n.$$

Otherwise, if (6) slacks on some interval for some i, one can reduce $g_i(x_i)$ on (a lower part of) that interval and increase it for greater values of x_i, so that the overall expected payoff goes up, a contradiction.

Next, let us define $K_i = \bar{g}_i H$. Since $\bar{g}_i \geq 0$, we obtain $K_i \geq 0$. By feasibility constraint (7) we obtain

$$\begin{aligned} 1 &= \sum_{i=1}^{n} \int_{x_i \in X} g_i(x_i) f(x_i) \mathrm{d}x_i = \sum_{i=1}^{n} \bar{g}_i \int_{x_i \in X} \left(1 - \frac{c}{u(x_i)}\right) f(x_i) \mathrm{d}x_i \\ &= \sum_{i=1}^{n} \bar{g}_i H = \sum_{i=1}^{n} K_i. \end{aligned}$$

Suppose that there exists p that satisfies the conditions of Proposition 6. By the above arguments it must be optimal, and the principal's expected payoff is given by

$$\begin{aligned} W(p) &= \sum_{i=1}^{n} \int_{x_i \in X} x_i \frac{K_i}{H} \left(1 - \frac{c}{u(x_i)}\right) f(x_i) \mathrm{d}x_i = \left(\sum_{i=1}^{n} \frac{K_i}{H}\right) \int_{z \in X} z \left(1 - \frac{c}{u(z)}\right) f(z) \mathrm{d}z \\ &= \frac{1}{H} \int_{z \in X} z \left(1 - \frac{c}{u(z)}\right) f(z) \mathrm{d}z, \end{aligned}$$

so we obtain (5). Moreover, if an assignment rule does not satisfy the conditions of Proposition 6, then it must slack the incentive constraint (6) on a positive measure of types (the feasibility constraint (7) is a part of the definition of an assignment rule), and hence yield a lower expected payoff, so it cannot be optimal.

It remains to show that an assignment rule that satisfies the conditions of Proposition 6 exists. This can be easily verified for the following rule, provided c is sufficiently small (specifically, Assumption 3 holds). Define $g^*(z) = \frac{1}{2H}\left(1 - \frac{c}{u(z)}\right)$ for all $z \in [a, b)$ and $g^*(b) = \frac{1}{2H}$. Fix a pair of agents i, i' in $\{1, \ldots, n\}$ and for every $x = (x_1, \ldots, x_n) \in X^n$ let

$$\begin{aligned} p_i(x_1, \ldots, x_n) &= \frac{1}{2} + g^*(x_i) - g^*(x_{i'}), \\ p_{i'}(x_1, \ldots, x_n) &= \frac{1}{2} + g^*(x_{i'}) - g^*(x_i), \quad \text{and} \\ p_j(x_1, \ldots, x_n) &= 0 \text{ for every } j \neq i, i'. \end{aligned}$$

This rule splits the unit weight equally between the chosen agents i and i', $K_i = K_{i'} = \frac{1}{2}$, and assign zero weights to the others. To verify the incentive constraints, observe that

$$\int_X g^*(z)f(z)\mathrm{d}z = \frac{1}{2H}\int_X \left(1-\frac{c}{u(z)}\right)f(z)\mathrm{d}z = \frac{1}{2H}H = \frac{1}{2}.$$

So

$$g_i(x_i) = \int_{x_{-i}\in X^{n-1}} p_i(x_i,x_{-i})\mathrm{d}\bar{F}_{-i}(x_{-i}) = \frac{1}{2}+g^*(x_i)-\frac{1}{2} = g^*(x_i)$$

satisfying the incentive constraint (6) as equality; and similarly for agent i'. For the rest of the agents (6) trivially holds.

To verify the feasibility, observe that $\sum_j p_j(x) = 1$ for all $x \in X^n$. It remains to check that $p_i(x) \geq 0$ and $p_{i'}(x) \geq 0$ for all $x \in X^n$. For agent i,

$$p_i(x) \geq \frac{1}{2}+g^*(a)-g^*(b) = \frac{1}{2H}\left(H+\left(1-\frac{c}{u(a)}\right)-1\right) \geq 0$$

if $H \geq \frac{c}{u(a)}$. Substituting H yields $1-\int_X \frac{c}{u(z)}\mathrm{d}F(z) \geq \frac{c}{u(a)}$, which holds for a small enough c (precisely, it is equivalent to condition (2)). The argument is the same for agent i'.

Finally, $W(p) = \frac{1}{H}\int_X z\left(1-\frac{c}{u(z)}\right)f(z)\mathrm{d}z$ is strictly greater than the payoff from choosing the first agent, $\int_X zf(z)\mathrm{d}z$, since $\frac{1}{H}\left(1-\frac{c}{u(z)}\right)$ is strictly increasing and $\int_X \frac{1}{H}\left(1-\frac{c}{u(z)}\right)f(z)\mathrm{d}z = 1$, so $\frac{1}{H}\left(1-\frac{c}{u(z)}\right)f(z)$ is the probability density that f.o.s.d. $f(z)$. ■

Appendix B. Proofs

Proof of Proposition 1. In Section 3 we already proved (I) and the existence part of (II). It remains to prove the uniqueness part of (II).

Consider the partition of $X = [a,b]$ to intervals with endpoints $(x^0, x^1, \ldots, x^S)$ with $a = x^0 < x^1 < \ldots < x^S = b$ that satisfy for every $s = 1,\ldots,S-1$

$$n(F(x^s)-F(x^{s-1}))^{n-1} = 1-\frac{c}{u(a)}.$$

Assumption 3 implies $c < u(a)$, and density f is assumed to be a.e. positive, so this partition is well defined and has a finite size.

Observe that in n-agent auction, since the cost of lying is fixed, each agent i either states the truth, $y_i = x_i$, or the value that maximizes the probability of winning.

We proceed by induction in $s = 0,1,\ldots,S-1$. Suppose that for types in $[a,x^s)$ stating the true value is not optimal (for $s=0$ this interval is empty). Then, if a type $x_i \geq x^s$ tells the truth, her payoff is almost surely at most $u(x_i)(F(x_i)-F(x^s))^{n-1}$. On the other hand, if i lies, she can ensure the payoff of at least $\frac{1}{n}(u(x_i)-c)$ by reporting $y_i = b$ whenever $x_i < b$. Hence, i strictly prefers to lie if $u(x_i)(F(x_i)-F(x^s))^{n-1} < \frac{1}{n}(u(x_i)-c)$, or, equivalently,

$$n(F(x_i)-F(x^s))^{n-1} < 1-\frac{c}{u(x_i)}.$$

Since $u(x_i) \geq u(a)$, the above inequality holds for all $x_i \in [x^s, x^{s+1})$. Consequently, types in $[a, x^{s+1})$ do not state the truth. Since everyone lies and pays the penalty that is independent of the report, the unique equilibrium outcome is for everyone to submit the highest possible report $y_i = b$. ■

Proof of Proposition 2. The proofs of this an the further results are based on Appendix A.

Consider all truthful assignment rules where the principal waits for at least $n \geq 2$ agents to arrive. By Proposition 6 any optimal assignment rule among n agents who arrive simultaneously yields the same payoff irrespective of $n \geq 2$. Consequently, when agents arrive sequentially and waiting is costly, every optimal rule must choose between the first two agents. ■

Proof of Proposition 3. Since we have assumed that $r = 0$, this is equivalent to a simultaneous arrival of two agents. Thus, we just need to verify that the described assignment rule p^I satisfies the conditions of Proposition 6 for some choice of functions τ and q_2.

Let $\tau(y_1) = \frac{1}{\lambda+r} \ln\left(1 - \frac{1}{S}\frac{c}{u(y_1)}\right)$ for $y_1 < b$ and $\tau(b) = 0$, where $S = 1 - \frac{\lambda}{\lambda+r}\int_X \frac{c}{u(x)}\mathrm{d}F(x)$, and let $q_2(y_2) = 1 - \frac{c}{u(y_2)}$ for $y_2 < b$ and $q_2(b) = 1$.

Define $H = \int_X \left(1 - \frac{c}{u(x)}\right)\mathrm{d}F(x)$. Observe that $S = H$ when $r = 0$, so $\tau(y_1) = \frac{1}{\lambda}\ln\left(1 - \frac{1}{H}\frac{c}{u(y_1)}\right)$

Denote by $q_1(y_1)$ the probability that agent 1 is chosen *before* agent 2 has arrived, in other words, $q_1(y_1)$ is the probability that agent 2 does not arrive within time interval $\tau(y_1)$:

$$q_1(y_1) = e^{-\lambda\tau(y_1)} = 1 - \frac{1}{H}\frac{c}{u(y_1)}.$$

Then, the probability that agent 1 is chosen, conditional on her statement y_1, is equal to

$$\begin{aligned}
&q_1(y_1) + (1 - q_1(y_1)) \int_{y_2 \in X} (1 - q_2(y_2))\mathrm{d}F(y_2) \\
&\quad = q_1(y_1) + (1 - q_1(y_1))\,(1 - H) = 1 - (1 - q_1(y_1))H \\
&\quad = 1 - \frac{c}{u(y_1)} = \frac{K_1}{H}\left(1 - \frac{c}{u(y_1)}\right).
\end{aligned}$$

Next, the probability that agent 2 is chosen, conditional on her statement y_2, is equal to

$$\begin{aligned}
&\int_{y_1 \in X} (1 - q_1(y_1))q_2(y_2)\mathrm{d}F(y_1) = \int_{y_1 \in X} \frac{1}{H}\frac{c}{u(y_1)}\mathrm{d}F(y_1)\left(1 - \frac{c}{u(y_2)}\right) \\
&\quad = \frac{1 - H}{H}\left(1 - \frac{c}{u(y_2)}\right) = \frac{K_2}{H}\left(1 - \frac{c}{u(y_2)}\right).
\end{aligned}$$

■

Proof of Proposition 4. Similarly to the proof of Proposition 3, we just need to verify that the described assignment rule p^{II} satisfies the conditions of Proposition 6 for some choice of functions q_1 and q_2.

Let $q_1(y_1) = 1 - \frac{1}{S}\frac{c}{u(y_1)}$ for $y_1 < b$ and $q_1(b) = 1$, where $S = 1 - e^{-r}\int_X \frac{c}{u(x)}\mathrm{d}F(x)$, and let $q_2(y_2) = 1 - \frac{c}{u(y_2)}$ for $y_2 < b$ and $q_2(b) = 1$.

Define $H = \int_X \left(1 - \frac{c}{u(x)}\right)\mathrm{d}F(x)$. Observe that $S = H$ when $r = 0$, so $q_1(y_1) = 1 - \frac{1}{H}\frac{c}{u(y_1)}$.

Let $K_1 = H$ and $K_2 = 1 - H$. The probability that agent 1 is chosen, conditional on her statement y_1, is equal to

$$q_1(y_1) + (1 - q_1(y_1))\int_{y_2 \in X}(1 - q_2(y_2))\mathrm{d}F(y_2).$$

The rest of the proof is the same as that of Proposition 3. ■

Proof of Proposition 5. In each rule, we need to compare only the payoff of each agent of each type $x \in X$ from the truthful statement and from the statement of the maximum value, b.

First consider rule p^{II}. If agent 1 with type $x_1 < b$ lies and reports $y_1 = b$, then she is chosen for sure, hence her payoff is $u(x_1) - c$. On the other hand, if agent 1 reports the truth, then her payoff is equal to

$$\begin{aligned}
\bar{U}(x_1) &= u(x_1)\left(q_1(x_1) + e^{-r}(1 - q_1(x_1))\int_{x_2 \in X}(1 - q_2(x_2))\mathrm{d}F(x_2)\right) \\
&= u(x_1)\left(q_1(x_1) + e^{-r}(1 - q_1(x_1))(1 - H)\right) \\
&= u(x_1)\left(1 - (1 - q_1(x_1))\left(1 - e^{-r}(1 - H)\right)\right) \\
&= u(x_1)\left(1 - \frac{c}{u(x_1)}\right) = u(x_1) - c,
\end{aligned}$$

so, in fact, the incentive constraint holds as equality.

Next, conditional on agent 2 having arrived, if agent 2 with type $x_2 < b$ lies and reports $y_2 = b$, then she is chosen for sure, hence her payoff is equal to $u(x_1) - c$. On the other hand, if agent 2 reports the truth, then her payoff is equal to

$$u(x_2)\left(1 - \frac{c}{u(x_2)}\right) = u(x_2) - c,$$

so the incentive constraint holds as equality as well.

Now consider rule p^I. Denote

$$q_1(x_1) = e^{-(\lambda + r)\tau(x_1)} = 1 - \frac{1}{S}\frac{c}{u(y_1)} = 1 - \frac{1}{1 - \frac{\lambda}{\lambda + r}(1 - H)}\frac{c}{u(y_1)}$$

If agent 1 with type $x_1 < b$ lies and reports $y_1 = b$, then $\tau(b) = 0$, so she is chosen immediately, hence her payoff is $u(x_1) - c$. On the other hand, if agent 1 reports the truth, then her payoff is equal to

$$\begin{aligned}\bar{U}(x_1) &= u(x_1)\left(q_1(x_1)+\frac{\lambda}{\lambda+r}(1-q_1(x_1))\int_{x_2\in X}(1-q_2(x_2))\mathrm{d}F(x_2)\right)\\ &= u(x_1)\left(q_1(x_1)+\frac{\lambda}{\lambda+r}(1-q_1(x_1))\,(1-H)\right)\\ &= u(x_1)\left(1-(1-q_1(x_1))\left(1-\frac{\lambda}{\lambda+r}(1-H)\right)\right)\\ &= u(x_1)\left(1-\frac{c}{u(x_1)}\right)=u(x_1)-c,\end{aligned}$$

so the incentive constraint holds as equality. The argument for the incentive constraint of agent 2 is the same as above for rule p^{II}. ■

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