

Some Unpleasant Bargaining Arithmetics?

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- ▶ The agreement rule (e.g., unanimity, majority, super-majority) is a key component of multilateral bargaining environments.
- ▶ Large literature on comparing the performance of different voting rules in a variety of bargaining models.
- ▶ Emphasis has been primarily on the efficiency of equilibrium outcomes.

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- ▶ We focus on comparing the distributional consequences (equity properties) of alternative voting rules in a simple bargaining environment.
- ▶ It is commonly believed that, contrary to majority rule, unanimity rule protects minorities from the possibility of expropriation and is therefore more equitable.
- ▶ We show that this is not necessarily the case in bargaining: unanimity rule can induce equilibrium outcomes that are more unequal (or less equitable) than equilibrium outcomes under majority rule.

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- ▶ The players have to collectively decide which project to implement (if any), and how to distribute the available surplus.

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 - ▶ If a proposal is submitted, all players then vote (sequentially) on whether or not to approve it.
 - ▶ If q players vote in favor, then the proposal is implemented and the game ends. Otherwise, a new player is selected and the process repeats itself (possibly *ad infinitum*).

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- ▶ $i \in \{1, \dots, n\}$ specifies each player's ranking in the endowment distribution (with 1 denoting the least productive and n the most productive player).

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- ▶ Merlo and Wilson (1995, 1998) and Eraslan and Merlo (2002) with perfect correlation between the “cake” and the “proposer” processes.
- ▶ Deliberately “egalitarian” protocol, and no additional dimensions of heterogeneity other than in the players’ technology endowments.

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- ▶ Let G^U and G^M denote the Gini coefficients for the equilibrium payoff distribution under unanimity and majority rule.

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- ▶ For every $\epsilon > 0$, if $\delta > \frac{(1-\epsilon)n}{(1-\epsilon)n+\epsilon}$, in the unique SSP equilibrium of the unanimity game

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 - ▶ In equilibrium, we must have $\mu_i \geq 0$ and $\mu_i \leq 1 - \frac{1}{n}$ for all i ; $y_1 - (q-1)\delta v^M \geq \delta v^M$; and $\sum_{i=1}^n \mu_i = q-1$.

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 - ▶ Since $\epsilon < \frac{\delta}{2 + \delta \frac{n-1}{n}}$, we can find μ_1, \dots, μ_n satisfying these.

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- ▶ In this environment

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- ▶ This “egalitarian” force generates regression toward the mean that equalizes expected equilibrium payoffs.

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- ▶ Payoffs are monotone: $v_i^q \leq v_{i+1}^q$.
- ▶ In the q -game, there is always agreement when player q proposes.

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$$v_i^U = \max\left\{0, \frac{1}{n(1-\delta)}\left(y_i - \frac{\delta}{n - \delta(\kappa^U - 1)} \sum_{j=\kappa^U}^n y_j\right)\right\}.$$

where

$$\kappa^U = \min\left\{i : y_i - \delta \sum_j v_j^U \geq 0\right\}.$$

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- ▶ Player 1 needs the approval of either player 2 or 3 but not both: $\mu_2 = \mu_3 = 1/6$ if $0.4 \geq \delta v_j^M$
- ▶ If there is no agreement then players 2 and 3 both receive their continuation payoffs with probability $1/3$: $\mu_2 = \mu_3 = 1/3$ if $0.4 < \delta v_j^M$

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- ▶ G^q is not monotonic in q ($G^1 = G$).

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- ▶ There exists a $\bar{q} < n$ such that for any $q < \bar{q}$ and $q' > \bar{q}$, and for any equilibria of q -game and q' -game, we have $G^q \leq G \leq G^{q'}$.

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