Manipulated Electorates and Information Aggregation*

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Abstract

We study information aggregation with a biased election organizer who recruits voters at some costs. Voters are symmetric ex-ante and prefer policy a in state A and policy b in state B, but the organizer prefers policy a regardless of the state. Each recruited voter observes a private signal that is imperfectly informative about the unknown state, but does not learn the size of the electorate. In contrast to existing results for large elections, there are equilibria in which information aggregation fails: As the voter recruitment cost disappears, a perfectly informed organizer can ensure that policy a is implemented independent of the state by appropriately choosing the number of recruited voters in each state.

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1 Introduction

Voting is considered to be an effective mechanism to aggregate information that is dispersed among voters about which of the available policies or candidates is better for the society. Indeed, Feddersen & Pesendorfer (1997) showed that in large elections, the majority decision will often be *as if* there was no uncertainty about which policy is the best one for the society. Hence, simple majority rules not only allow voters to express their preferences, but also allow the society to better aggregate dispersed information, provided that the electorate is sufficiently large.

Elections, however, take place in larger contexts in which interested parties try to influence election outcomes. For instance, one often sees interested parties spend resources to affect voter turnout. Examples of such activities include bussing in elections or in referenda, activities of a CEO directed at increasing participation in shareholder voting, or prodding of colleagues by a department chair. Hence, it is natural to ask the extent to which the ability of an election organizer to manipulate voter participation rate might benefit him to influence the outcomes of elections, when the voters have dispersed information about the correct policy choice.

To this end, we analyze a model in which a number of voters have to decide between two policies, policy a and policy b. Voters prefer policy a in state A and policy b in state B, i.e., voters prefer that the implemented policy matches the state of the world. However, no individual voter knows the state, and hence voters are uncertain about the correct policy. Although uncertain about the state, each voter has a small piece of information in the form of a noisy signal.

The key element in our model is that the number of voters is chosen by an *election organizer* who privately observes the realization of the state of the world, and recruits a number of voters after observing the state. Recruitment is a costly activity, and the total recruitment cost is increasing in the number of recruited voters. Each voter, then has an equal chance of being selected to participate in the election. The election organizer is biased -has a conflict of interest with respect to the voters- in the sense that he always prefers that policy a is implemented independent of the state. We assume that the number of recruited voters is not observed by the voters. However, the voters will make Bayesian inferences about the state from being recruited, since the organizer may choose different participation rates in different states, making the event that a voter is selected to be partially informative about the state.

To fix ideas, consider a referenda to build a bridge in a town. The cost of the bridge is unknown, and the voters prefer that the bridge is built only if the cost of building the bridge is low. The governor observes the cost, and chooses the level of advertisement activity that determines the degree of awareness about the referenda among the voters, which in turn affects the voter turnout. The governor prefers the bridge to be built no matter what the cost is, since either it will increase his popularity, and hence re-election chances in the next elections, or because he has private benefits from doing business with the construction company in charge of building the bridge. Our paper then explores how much the ability to influence the voter turnout can translate into the ability to influence the policy outcome.

We show in our main result that the ability to manipulate turnout significantly affects the performance of elections. Strikingly, if the number of voters in an election can be manipulated by an interested party, then elections may fail to aggregate any information at all, and lead to outcomes that are not preferred by the voters, and coincide perfectly with the organizer's preferences. In particular, there are equilibria in which the majority chooses policy a almost always, independent of the state, when the recruitment cost is almost zero, and when the potential number of voters is arbitrarily large. We view our results as having implications for the use of elections as a means to control a common agent by a dispersed group of principals (e.g., shareholders controlling a manager or faculty controlling a chair). In particular, our main result suggests that elections may be an ineffective form of control if the agent can manipulate turnout.

Our result also has implications for the ability of the electorates to make the correct choices. Specifically, we are interested in the probability that the majority votes for the correct policy, that is, for policy a in state A and for policy b in state B. Our main result implies that the presence of the biased organizer reduces information aggregation. When the recruitment costs are small and the electorate is large there are symmetric equilibria in which the selected outcome is independent of the state of the world; thus information aggregation completely fails. Moreover, in such equilibria the organizer's favorite policy is implemented with a probability that approaches one in the limit, hence the organizer ensures that information aggregation fails in the most drastic way that is favorable for him. This result is in stark contrast to earlier papers on voting and information aggregation (see, Feddersen and Pesendorfer (1997)) who showed that in a large electorate without an organizer, the outcome of the election coincides with the correct policy choice for the voters.

Driving this negative result is a recruitment effect. The intensity of the recruitment activity by the organizer depends on his private information. This introduces an endogenous relationship between the number of voters and the state. In particular, the number of voter participation is state dependent. There are two distinct, but related ways in which the recruitment effect manifests itself in our main result. First, being recruited contains information about the state. Specifically, the information that a voter is recruited is evidence for the state being the one in which the organizer recruits more aggressively. Second, and more subtly, the recruitment activity affects the number of voters participating in each state, and hence affects the chances that a voter is pivotal in each state. All else equal, a voter is more likely to be pivotal in the state where participation is lower, i.e., where recruitment is less aggressive. Because voters vote as if their vote is pivotal, the asymmetry in the numbers of voters across states makes a vote more likely to be pivotal in the state in which there is fewer voter participation. Hence, the organizer manages to manipulate the outcome of the election, by recruiting more voters in state B and less voters in state A, which together with a voting behavior that is aggressively in support of policy a induces beliefs for the pivotal vote that the state is much more likely to be A.

In our second set of results (Theorems 2 and 3), we characterize the limit equilibrium behavior across *all equilibria* when the recruitment costs are small and when the population size is large. First and foremost, there is no equilibrium that aggregates information. Moreover, apart from equilibria in which the organizer recruits no voter—which exists for some parameters—there is only one additional limit equilibrium outcome. In this type of equilibrium, the expected vote shares in state A for each policy approaches a half, and hence there is a close race in state A. The election outcome in state A is deterministic and policy a is implemented if the number of voters grows without bound, and is not deterministic otherwise. In state B, the organizer recruits no voter, and hence the implemented policy is not deterministic in state B.

In the final part of our analysis, we tackle an *election design* question. In particular, Theorem 4 presents our results when there is a participation requirement, i.e., when there is a requirement that the number of voters that are participating be above a certain threshold. If the required threshold increases without bound, then there is always an equilibrium sequence that aggregates information. Moreover, if the threshold increases sufficiently fast relative to the rate at which the recruitment cost disappears, then all symmetric equilibria aggregate information. However, if the threshold increases at a slower rate than the recruitment cost disappears, then a limit outcome in which the organizer ensures that policy a is implemented exists.

Another election design tool that we examine is the use of unanimity rule for policy a to be implemented. If the minimum number of voters that participate grows without bound, then the symmetric equilibrium outcomes do not fully aggregate information. However, the outcome is very different than the outcome of manipulated equilibria, and the amount of inefficiency that comes from the failure of information aggregation is smaller if the highest signal is more precise. Therefore, when the information content of the highest signal is sufficiently precise, then the worst equilibrium outcome of elections with unanimity rule may be better for the voter welfare than the worst equilibrium outcomes of elections with majority rules. This result stands in contrast to Feddersen & Pesendorfer (1998) who showed that unanimity rules are the only voting rules among all supermajority rules that fail to aggregate information in large elections.

2 Model

There is a finite number N of potential voters, and one of the two available policies $\{a, b\}$ will be implemented. Voters are unsure about which policy is the better one for themselves. In particular, there are two possible states of the world, $\Omega = \{A, B\}$, and a generic element is ω. Voters are homogenous, and have the following utility function:

$$\begin{aligned} u(a, A) &= u(b, B) = 1, \\ u(a, B) &= u(b, A) = -1, \end{aligned}$$

where $u(x, \omega)$ denotes the utility if policy x is chosen in state ω. In other words, voters prefer that the implemented policy matches the true state of the world, but they do not know what the state of the world is.[1]

Information Structure: There is a common prior belief $\pi \in (0, 1)$ that the state is A. Each voter receives a private signal, $s \in S := [0, 1]$. The signals are distributed across the population conditionally independently and according to an identical distribution function $F(s|\omega)$. The distribution F admits a continuous density function, denoted by $f(s|\omega)$.[2] We assume that a strict version of the Monotone Likelihood Ratio Property condition (MLRP) holds:[3]

Assumption 1.

$$\frac{f(s|A)}{f(s|B)} \qquad \textit{is strictly decreasing in } s.$$

Assumption 1 implies that voters who receive higher signals attach a strictly larger probability to the state of the world being state A. Another implication of Assumption 1 is that signals carry some information about the state of the world, i.e., $f(s|A)$ is not identical to $f(s|B)$ for all $s \in S$. Our second assumption which we present below puts a bound on the informativeness of the signals.

Assumption 2. *There exists a number $\eta > 0$ such that,*

$$\eta < f(s|\omega) < \frac{1}{\eta} \qquad \textit{for } \omega \in \Omega \textit{ and for } s \in S.$$

[1] Note that, here also we make the simplifying assumption that $u(a, A) - u(b, A) = u(b, B) - u(a, B)$, but none of our results depends on this specification.

[2] The assumption that the densities $f(\cdot|\omega)$ are continuous is for expositional simplicity. All our results go through without making this assumption.

[3] We make the strict version of the MLRP condition only for expositional simplicity. All of our results go through if we assume a weak MLRP condition, together with a condition which states that "$f(s|A)$ is not everywhere identical to $f(s|B)$."

An implication of Assumption 2 is that there is no single voter type who has arbitrarily precise information about the state of the world.

Organizer's Actions and Preferences: There is a single election organizer who observes the realization of the state of the world ω, and recruits (or solicits) a number of voters that will participate in the election. He prefers that policy a is implemented irrespective of the state of the world. Recruitment is costly and in particular recruiting each additional pair of voters costs a strictly positive amount, c to the organizer. In particular, if the organizer recruits n pairs of voters, then the number of participants in the electorate is equal to

$$2n + 1 \in \{1, 3, 5..., N\}.$$

If he recruits no one, a randomly chosen voter becomes the unique voter.

In particular, his payoff function is:

$$\begin{aligned} u_O(a, n) &= 1 - cn, \\ u_O(b, n) &= -cn, \end{aligned}$$

where the first argument is the policy that the majority of the electorate chooses to implement, and the second argument is the number of pairs of voters that the organizer recruits.

Timing of the voting game:

- The organizer learns the state.
- Organizer chooses n.
- Nature chooses (recruits) $2n + 1$ voters, each equally likely, from the population.
- Each recruited voter observes his private signal, but does not observe the number of voters that are recruited. In other words, voters do not observe n.
- Only the recruited voters participate in the election and each recruited voter votes for policy a or policy b.
- The policy that receives more votes is implemented.

Strategies and Equilibrium:

A strategy for the organizer is a pair of distributions over integers,

$$\tilde{n} \equiv (\tilde{n}_A, \tilde{n}_B) \in \Delta\left(\left\{0, 1, ..., \frac{N-1}{2}\right\}\right)^2,$$

which denotes the recruitment choice of the organizer in states A and B respectively.

A pure strategy[4] for voter i is a mapping

$$d : S \to \{a, b\},$$

that prescribes how the voter will vote as a function of his signal, conditional on being recruited. When a voter is not recruited, he does not have a ballot to cast.

A symmetric Nash equilibrium is a tuple $(\tilde{n}, d)$ in which the organizer's strategy $\tilde{n}$ is a best response to a voter strategy profile in which each voter uses the same strategy d, and the strategy d is a best response to the strategy profile in which the organizer's strategy is $\tilde{n}$ and all other voters are using the strategy d. From hereon, equilibrium refers to symmetric Nash equilibrium.

For any given symmetric voter strategy d, let the expected vote share for policy a in state ω be:

$$q_\omega(d) := \Pr\left(d\left(s\right) = a|\omega\right) = \int_{s\in[0,1]} \mathbf{1}_{d(s)=a} f\left(s|\omega\right) ds$$

Inference of voters and cutoff strategies

In our model voters are consequential, i.e., they only care about the implemented policy, and not directly about how they vote. In other words, a single vote will make an impact on the outcome of the election only when the number of votes cast for either alternative without that single vote are exactly equal, i.e., when that vote is *pivotal.* More precisely, a voter votes as if his vote is pivotal, as is typical in voting models with incomplete information.

In our model, there is an additional source of information that the voters use in their inference about the state of the world, because there is some

[4] As will be clear in the rest of the analysis, voters' best replies will have a cut off structure, and therefore focusing on pure strategies for the voters will be without loss of generality.

information carried in the event that a voter is being recruited. This is because, the number of recruited voters depends on the state of the world, and hence a voter learns some information about the state of the world from being recruited. Certainly, the inference made from the recruitment event will depend on the organizer's strategy only, and the inference from conditioning on being pivotal will depend both on the voters' and the organizer's strategy, because both the *expected vote shares* and *the number of participants* affect the probability of an equal split in the vote counts.

The posterior likelihood ratio that the state is A, conditional on being recruited and conditional on being pivotal for a voter who received signal s, when all other voters are using the strategy d, and the organizer is using a pure strategy $\tilde{n} = (n_A, n_B)$ is calculated as below:[5]

$$\beta(s, piv, rec; \tilde{n}, d) := \underbrace{\frac{\pi}{1-\pi}}_{prior} \underbrace{\frac{f(s|A)}{f(s|B)}}_{signal} \underbrace{\frac{\frac{2n_A+1}{N}}{\frac{2n_B+1}{N}}}_{recruited} \underbrace{\frac{\binom{2n_A}{n_A}(q_A)^{n_A}(1-q_A)^{n_A}}{\binom{2n_B}{n_B}(q_B)^{n_B}(1-q_B)^{n_B}}}_{pivotal}, \tag{1}$$

where we omit the dependence of q_ω on the voter strategy d for ease of reading.

This likelihood ratio, that we refer to as *critical likelihood ratio* guides a voter's voting decision. In particular, a voter with a signal s votes for policy a if his critical likelihood ratio is above 1, and votes for policy b otherwise. Therefore, we make a preliminary observation that in all symmetric equilibria, voters use cutoff strategies. This is a very standard result that is also shared in many other voting models, and follows from the MLRP condition assumed in Assumption 1.

Lemma 1. *Any equilibrium voting strategy has a cutoff structure. There is a signal $\hat{s}$ such that a recruited voter casts a vote for policy* b *if $s > \hat{s}$ and for policy* a *if $s < \hat{s}$.*

From hereon we will use $\hat{s} \in S$ to denote a generic cutoff strategy, and $q_\omega(\hat{s})$ to denote the expected vote share for policy a in state ω when voters use

[5] Note that the extension of the expression to the case in which the organizer is using a mixed strategy is rather straightforward, and we skip it in order to highlight the recruitment effect and the pivotal effect in the displayed expression. For completeness, we write the critical likelihood ratio when $\tilde{n}$ is a mixed strategy in Equation 6, which is in the Appendix.

a cutoff strategy $\hat{s}$. Lemma 1 allows us to conveniently express the expected vote share for policy a as:

$$q_\omega(\hat{s}) = F(\hat{s}|\omega).$$

Remark 1. *The critical likelihood ratio is found by using the information in the voter's prior belief, in his signal, conditioning on the events that he is recruited and that his vote is pivotal. Holding everything else constant, if the number of recruited voters in state ω increases, than the recruitment effect will push the critical likelihood ratio of the voter towards state ω. However, if the outcome is not deterministic, then the probability of an equal split in state ω, i.e., the pivotal probability decreases if the number of recruited voters in state ω increases. In an equilibrium where the cut off signal is an interior signal, these forces will be relatively similar and will be offsetting each other.*

In an equilibrium with an interior cutoff—$0 < s^* < 1$—the cutoff type is indifferent between voting for either option, so

$$\beta(s^*, piv, rec; \tilde{n}, s^*) = 1.$$

Organizer's best reply

The organizer chooses the size of the participation in order to maximize the total probability with which policy a is implemented, less the recruitment cost. Therefore, the best reply of the organizer to a given symmetric cutoff strategy $\hat{s}$ of the voters in state ω is found by the following equation:

$$\tilde{n}_\omega(\hat{s}, c) = \underset{n \in \left\{0,1,\ldots,\frac{1}{2}(N-1)\right\}}{\arg\max} \sum_{i=n+1}^{2n+1} \binom{2n+1}{i} (q_\omega(\hat{s}))^i (1 - q_\omega(\hat{s}))^{2n+1-i} - nc \quad (2)$$

The first term in the objective function of the organizer represents the total probability that policy a is implemented when the probability that a randomly selected voter votes for policy a is $q_\omega(\hat{s})$, and when the size of participation is $2n+1$. The second term is the cost of choosing participation size of $2n+1$.

Observe that the number of potential voters, N, appears in the recruitment effect in Equation 1, and in the organizer's best reply in Equation 2. However, the term N cancels out in the recruitment effect, and hence has an impact on equilibrium behavior only if it is sufficiently small that it becomes a binding constraint in the organizer's best reply in equation 2. We make the following assumption on the size of the potential voters, N, which ensures that the size of the population is never a binding constraint for the organizer.[6]

Assumption 3.

$$N \geq \left\lfloor \frac{2}{c} \right\rfloor$$

Note that Assumption 3 is a lower bound on the size of the population. All our analysis would go through if we assumed that there is an infinitely many and countable number of potential voters in the population, in which case we could dispense this assumption.

To get more insight into the best reply of the organizer we calculate the incremental gain he gets from recruiting an additional pair of voters:

$$\begin{aligned}\Delta\left(n-1,\omega,\hat{s}\right) \quad := \quad & \sum_{i=n+1}^{2n+1}\binom{2n+1}{i}\left(q_{\omega}\left(\hat{s}\right)\right)^{i}\left(1-q_{\omega}\left(\hat{s}\right)\right)^{2n+1-i} \\ & -\sum_{i=n}^{2n-1}\binom{2n-1}{i}\left(q_{\omega}\left(\hat{s}\right)\right)^{i}\left(1-q_{\omega}\left(\hat{s}\right)\right)^{2n-1-i}\end{aligned}$$

A counting argument allows us to simplify the above expression to the following one:

$$\Delta\left(n-1,\omega,\hat{s}\right)=\binom{2n}{n}\left(q_{\omega}\right)^{n}\left(1-q_{\omega}\right)^{n}\left(2q_{\omega}-1\right)$$

In particular, the increase in the probability that policy a is implemented when the size of the recruited voters increases from $2n-1$ to $2n+1$ is equal to the probability of a tie in the vote counts for policies a and b, multiplied by $2q-1$.

Notice that, if $q_{\omega}(\hat{s}) \leq 1/2$, then $\Delta\left(n-1,\omega,\hat{s}\right) \leq 0$ for every n, so, the organizer recruits no one since recruitment is costly. Indeed, when the odds are against him, the organizer recruits as few people as possible in order to

[6]The term $\lfloor x \rfloor$ refers to the smallest integer not greater than x.

maximize the variance of the outcome of the election, and also to save on recruitment costs.

On the other side, if $q_\omega(\hat{s}) > 1/2$, then $\Delta(n-1,\omega,\hat{s}) > 0$ and $\Delta(n-1,\omega,\hat{s}) > \Delta(n,\omega,\hat{s})$. The implication of this is that if $q_\omega(\hat{s}) > 1/2$, then there is a unique n such that $\Delta(n-1,\omega,\hat{s}) \geq c$ and $\Delta(n,\omega,\hat{s}) < c$. Notice that, when $q > 1/2$, the odds are with the organizer, so he wants to minimize the variance of the election outcome by recruiting many people. For instance, if the organizer recruited an infinite number of voters, then by the law of large numbers, policy a would be implemented. However, there is a cost of recruiting voters, so the organizer recruits as many voters as possible until he reaches a point in which the marginal benefit of recruiting an additional pair of voters is not more than the marginal cost.

Therefore, the organizer's best reply will either be unique (meaning one integer for each state), or it will be a mixed strategy in which the support consists of two adjacent integers, for one or both of the states.

3 Manipulated Electorates

We are interested in the election outcomes when c is small. When c is small, the organizer may recruit many voters and we compare our result to those for exogenously large elections. To this end, we fix the common prior π and information structure F which satisfies Assumptions 1 and 2. Let $\{G(c)\}_{c>0}$ be a collection of voting games in which for each game $G(c)$, the prior belief is π, information structure is F, the recruitment cost to the organizer is c and the number of potential voters, $N(c)$, is some integer that satisfies Assumption 3.

Theorem 1. *Let $\{c_k\}_{k=1,2,\ldots}$ be a sequence of positive numbers that converge to zero. Then, there is a sequence of symmetric strategy profiles $\{\Sigma_k\}_{k=1,2,\ldots}$ where each Σ_k is a symmetric Nash equilibrium of $G(c_k)$ such that in both states:*

1. *The probability that policy* a *is implemented converges to one,*

2. *The number of recruited voters grows without bound,*

3. The organizer's payoffs converges to one.

Theorem 1 states that, as the recruitment cost disappears and the number of potential voters becomes large, there are equilibria in which policy a is elected with a probability that is arbitrarily close to one in *both* states. Therefore information aggregation fails in the most drastic way that is beneficial to the organizer. In other words, in such equilibria the organizer's favorite outcome is implemented regardless of the state. Moreover, in both states the number of recruited voters becomes large and the expected payoff of the organizer becomes one. Hence, an endogenously large electorate may lead to the failure of information aggregation, and in the limit, the organizer incurs no cost from the recruitment efforts despite recruiting an unbounded number of voters.

In such *manipulated equilibria*, a randomly selected voter votes for policy a with a probability strictly larger than $1/2$ in *both states of the world*. Therefore, when the recruitment cost is small, the organizer can ensure that the majority selects policy a by recruiting many voters.

The aggressive voter behavior in favor of policy a is a consequence of the *asymmetry* in the number of voters that are recruited in states A and state B. In such equilibria, the organizer recruits more voters in state B than in state A. Such asymmetry in the numbers of voters in different states affects the voter behavior in two ways. First, because there are more voters in state B than in state A, a voter is more likely to be recruited in state B, and his posterior belief that the state is B goes up when he is recruited. This is the *recruitment effect*. The other effect that works in the opposite direction is the *pivotality effect*. Because there are more voters in state B than in state A, the pivotal probability in state A is larger than the pivotal probability in state B. Among the two effects, the pivotality effect is the more dominant one, and the overall net effect leads to the voting behavior in favor of policy a.

We now turn to the organizer's incentives. To provide some insights, we want to argue that whenever voters are using a cutoff s^* such that $q_A(s^*) > q_B(s^*) > 1/2$, then it will be the case that the organizer recruits more voters in state B than in state A, when the recruitment cost is small. To see why, recall that the marginal benefit of recruiting one more voter is:

$$\Delta(n-1,\omega,s^*) = \underbrace{\binom{2n}{n} q_\omega(s^*)^n(1-q_\omega(s^*))^n}_{\text{pivot probability}}(2q_\omega(s^*)-1).$$

The first term reflects how likely it is that an additional voter change the outcome. The second term reflects how likely it is that an additional voter swings the election in the organizer's favor (rather than against). Comparing the relative magnitude of the two terms in the two states shows that they go in opposite directions,

$$q_A(s^*)(1-q_A(s^*)) < q_B(s^*)(1-q_B(s^*)), \text{ whereas}$$
$$2q_A(s^*)-1 > 2q_B(s^*)-1.$$

In general, the relative marginal benefits of additional voters in the two states will depend on both terms. However, for sufficiently large n, the first term unambiguously dominates the second. Because the organizer recruits a growing number of voters when the recruitment cost is small,

$$\Delta(n^*-1,A,s^*) < \Delta(n^*-1,B,s^*),$$

for every integer n^* pair of voters he recruits in state A. Because the marginal benefit $\Delta(n,\omega,s)$ is decreasing in n, it follows that the organizer recruits strictly more voters in state B than in state A. Figure 1 depicts how the probability that the majority selects policy a changes with n for q_A and q_B, and that the curve for q_B is steeper than that of q_A when n is large.

In order to prove the first part of the theorem, we show that there is an equilibrium sequence in which the probability that a randomly selected voter votes for policy a stays bounded above from a half in each state of the world. Denoting s_B to be the signal that satisfies $q_B(s_B) = 1/2$, the main step is to prove that equilibrium cut points converge to a signal $s^* > s_B$. To see why this suffices to prove the theorem, first observe that because $s^* > s_B$, $q_\omega(s^*) > 1/2$ for each $\omega \in \{A, B\}$. This is because, the expected vote share for policy a is increasing in the voters' cut point, and because the expected vote share in state B is weakly higher than that in state A. If the limit cut point $s^* = 1$,

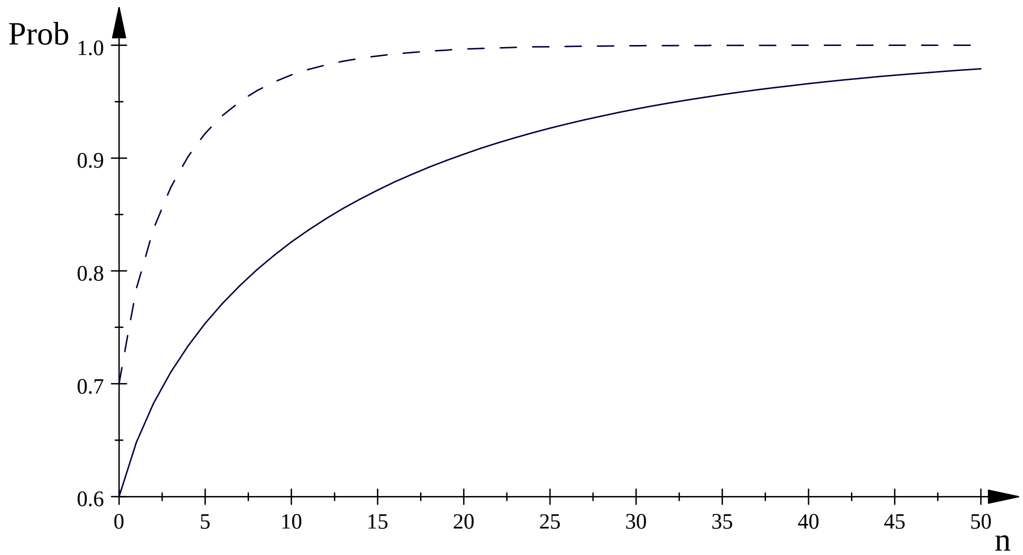

Figure 1: The probability that policy a receives a majority of votes given the number of recruited voters n for $q = 0.6$ (straight) and for $q = 0.7$ (dashed).

then the voters ignore their signals and vote for policy a almost independent of their signals, resulting in policy a be implemented for sure. If $s^* < 1$, then as c disappears, the organizer recruits an arbitrarily large number of voters in both states.

There may be multiple equilibrium sequences, with different limit cutoffs s^*, and with different organizer behavior. However, provided that the limit cutoff $s^* > s_B$, the policy that is implemented becomes deterministic, and is independent of the state. Moreover, there is at least one equilibrium sequence along which the number of recruited voters grows without bound in both states. As we show in Lemma 7, located in the Appendix, and in the subsequent remark, the ratio of the number of recruited voters in states A and B stays bounded away from zero and infinity.

Contrast with voting models with state independent number of voters.

The existence of equilibria with a limit cut point $s^* > s_B$ stands in sharp contrast to Feddersen and Pesendorfer, who showed that in a voting model where the number of voters is state independent, all symmetric equilibrium cut points have limit points $s^* \in (s_A, s_B)$. This then implies that all equilibria aggregate information. What drives Feddersen and Pesendorfer's result is that otherwise, for any s^* that is not in the interval (s_A, s_B), the ratio of pivot probabilities is degenerate in the limit. Note that the ratio of pivot probabilities is

$$\lim_{n\to\infty} \frac{\binom{2n}{n}(q_A(s^*))^n(1-q_A(s^*))^n}{\binom{2n}{n}(q_B(s^*))^n(1-q_B(s^*))^n}. \tag{3}$$

This expression goes to 0 if $s^* > s_B$ and to ∞ if $s^* < s_A$. Therefore, critical likelihood ratio cannot be one and there cannot be an equilibrium with a limit cut point $s^* \notin (s_A, s_B)$.

In our model, the limit effect of pivot considerations are shaped by both the expected vote shares *and* the relative ratio of the number of participants in each state. This is because an equal split is less likely in the state with larger number of participants. Indeed, the organizer's decision of how many voters to recruit is linked to the expected vote shares in a way that keeps the inference made by being pivotal relatively moderate, compared to the case in which the number of voters is exogenous. Hence, the existence of the organizer opens up the possibility that the majority votes for policy a.

The key to the result that $s^* > s_B$ can be a limit cut point is through the organizer's recruitment strategy, and its interaction with the pivotal probabilities in different states. For a given probability $q > 1/2$, that represents the probability that a randomly selected voter voters for right policy, the organizer chooses the number of recruited voters, $2n+1$, such that:

$$\binom{2n}{n} q^n(1-q)^n(2q-1) \approx c$$

The approximation in the above statement represents the error that comes from ignoring the integer constraints. Therefore, if the voters' cut points are any $s > s_B$, then the ratio of pivot probabilities in each state of the world stays bounded away from 0 and infinity, and is approximated as:

$$\frac{\binom{2n_A}{n_A}(q_A)^{n_A}(1-q_A)^{n_A}}{\binom{2n_B}{n_B}(q_B)^{n_B}(1-q_B)^{n_B}} \approx \frac{2q_B-1}{2q_A-1}. \tag{4}$$

Note that the right hand side is independent of c. This is because, the organizer's choice of the size of the electorate keeps the pivot probabilities in each state relatively at the same order, and when c disappears, the relative pivot probabilities stay bounded away from 0 and infinity. This is unlike the case in which the number of voters is state independent, as is depicted by equation 3.

4 All Limit Equilibria

In this section we characterize systematically the limiting equilibrium outcomes that can be generated by any equilibrium sequence, as the recruitment cost vanishes. The recruitment activity limits information aggregation in all symmetric equilibria. Let s_ω denote the median signal in state ω, i.e.,

$$F(s_A|A) = F(s_B|B) = 1/2.$$

Trivial Equilibrium.

An equilibrium is a *trivial equilibrium* if: *i)* the organizer recruits no voter in either state, so that there is only one voter casting a ballot in both states, and *ii)* selected voter votes for policy a with a probability not more than 1/2 in both states. In a trivial equilibrium, the organizer is passive and information is not aggregated because only one voter makes the decision on the implemented policy.

A voting game G admits a trivial equilibrium for all recruitment costs c if and only if the distribution of signals, F satisfies the following inequality:

$$\frac{\pi}{1-\pi}\frac{f(s_A|A)}{f(s_A|B)} \leq 1. \tag{5}$$

The above inequality is satisfied if the prior belief that the state is A is lower than a threshold belief $\bar{\pi} \in (0,1)$.

It is clear that when c is large, then a trivial equilibrium exists even if inequality 5 fails. However, the essence of the claim is that when c is small and inequality 5 fails, a trivial equilibrium does not exist.

Notice that if inequality 5 holds, then in a trivial equilibrium, the voter who is selected to cast his vote votes for policy a with a probability not more than 1/2 in both states of the worlds. This in turn justifies the organizer's behavior in which he recruits no one. Conversely, if inequality 5 fails, then a trivial equilibrium does not exist when the recruitment cost c is sufficiently small. This is because, in a putative trivial equilibrium, the probability that the voter voting for policy a is strictly larger than $1/2$ in state A, and hence if the recruitment cost is small, then the organizer's best reply is to recruit some voters.

All Non-trivial Equilibria.

In Theorem 2 below, we argue that, fixing all parameters of the environment other than the recruitment cost, any limit point of a non-trivial equilibrium cutoff sequence, as the recruitment cost disappears, has to be either equal to s_A or strictly larger than s_B.

Theorem 2. *Let $\{c_k\}_{k=1,2,\ldots}$ be a sequence of positive numbers converging to zero.*

1. *For any s^* which is a limit point of the non-trivial equilibrium cutoffs of the sequence of voting games $\{G(c_k)\}_{k=1,2,\ldots}$, either $s^* = s_A$ or $s^* > s_B$.*
2. *$\{G(c_k)\}_{k=1,2,\ldots}$ has a sequence of non-trivial equilibria with limit cutoff $s^* = s_A$, and another sequence with limit cutoff $s^* > s_B$.*

The theorem states that, there are only 2 types of limit points of non-trivial equilibrium cutoffs, as the recruitment cost disappears. None of these equilibria aggregate information fully, so that information aggregation failure is inevitable in any equilibrium.

One type of limit equilibrium outcome is when $s^* > s_B$. These types of equilibria are identical to the equilibrium outcomes of equilibria that we presented in Theorem 1. In such equilibria the majority selects policy a, i.e., there is *full manipulation* and a drastic failure of information aggregation.

The second type of equilibrium sequences are those where there is a close race between the two policies in state A. This is because, $s^* = s_A$, and hence, the probability that a randomly selected voter votes for policy a converges to $1/2$. On the other side, in state B, the organizer recruits no one, and policy b is implemented with a probability $1-F(s_A|B)$ in state B. In the next theorem, we identify the properties of the limit outcomes of such equilibrium sequences.

Theorem 3.

1. *If inequality 5 is not satisfied, then along all equilibrium sequences with limit cutoff* $s^* = s_A$*:*

    - *The number of voters recruited in state* A *grows without bound.*
    - *Policy a is implemented in state A.*

2. *If inequality 5 is satisfied, then for each of the following outcomes, there is a corresponding equilibrium sequence with limit cutoff* $s^* = s_A$ *such that:*

    - *The number of voters recruited in state* A *grows without bound, and policy* a *is implemented in state* A.
    - *The number of voters recruited in state* A *stays bounded, and policy* a *is implemented with a probability between 0 and 1 in state* A.

In this type of equilibrium sequences with limit points $s^* = s_A$, whether policy a is implemented in state A for sure or not depends on the number of voters that are recruited. If inequality 5 fails, then all such equilibrium outcomes have the organizer recruiting a number of voters that grows without bound, and hence even if there is a close race between the policies, policy a prevails as the winner in state A.

Remark 2. *One may be tempted to think that equilibrium cut points that converge to* s_A *cannot be sustained, since in state* B *there is only one voter, and hence conditional on being pivotal, the voters should believe that the state is* B *and hence vote for* B. *The main force that sustains this type of equilibrium is the recruitment effect. Because in state* A *the electorate gets very large, recruitment effect pushes the belief towards state* A, *and the pivotality effect pushes it towards the opposite direction. When the expected vote share is* $1/2$, *the pivotal probability decreases to zero at the rate at which* $\frac{1}{\sqrt{n}}$ *decreases to 0, where n is the number of voters recruited. Therefore, the recruitment effect, which is at the order of n, becomes stronger than the pivotal effect in the close neighborhood of the vote fraction* $1/2$.

Finally, we illustrate the ordering of the equilibrium cutoffs with Figure 2.

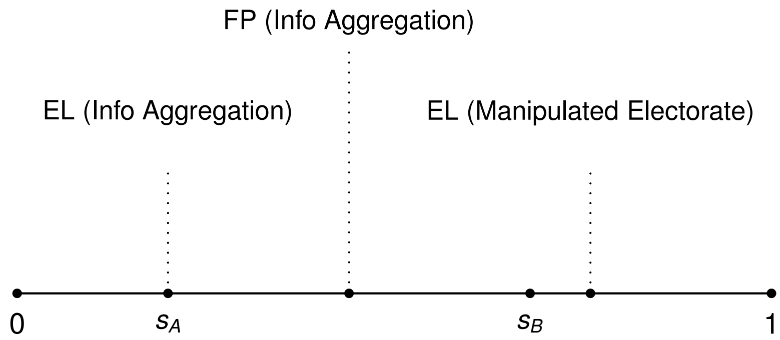

Figure 2: $F(s_A|A) = F(s_B|B) = 1/2$

5 Robust Election Design

In this section we explore whether electorate design can be a remedy for an organizer's ability to manipulate election outcomes. To this end, we analyze 2 elections design tools that provide protections against manipulation, namely, quorum requirement and unanimity rule.

5.1 Participation Requirement

We start by relaxing the assumption that the minimum number of voters participating in the election when the organizer is passive is 1, and instead introduce the minimum number of voters that are participating as an election design tool, denoted by the parameter $2m + 1$. Our main result in this setup is that if the number of participants that are present already without any recruitment activity grows large, then there exists an equilibrium sequence in which the majority votes for the correct policy; thus, information aggregates.

Specifically, suppose the number of voters is $2(m + n) + 1$ for some integer m, where n is the number of pairs of voters recruited by the organizer and m is the number of voters who are already there by default. We consider games parameterized by c (a real number) and m, denoted $G(c, m)$. The number of potential voters is $N = \left\lfloor 2\left(m + \frac{1}{c}\right) + 1 \right\rfloor$. The following result shows that information can be aggregated in the limit and, if m grows sufficiently quickly, information will be aggregated.

Theorem 4. *Let* $\{c_k, m_k\}_{k=1,2..}$ *be a sequence of positive numbers* c_k *converging to zero and integers* m_k *diverging to infinity. We are interested in the symmetric equilibrium sequences of the voting games* $\{G(c_k, m_k)\}_{k=1,2,...}$.

1. *There exists a sequence of equilibria such that the probability with which the correct policy is implemented converges to one (that is, policy* a *in state* A *and policy* b *in state* B*).*

2. *If* m_k *diverges sufficiently fast relative to the rate at which* c_k *converges to 0, in all sequences of equilibria the probability with which the correct policy is implemented converges to one.*

3. *If* m_k *diverges sufficiently slowly relative to the rate at which* c_k *converges to 0, then there is a sequence of equilibria in which policy* a *is implemented with probability converging to one in both states (i.e., information aggregation fails and the outcome is manipulated).*

The first part of the result is related to Theorem 2. If m_k increases sufficiently slowly compared to the rate at which c_k disappears, then there is an equilibrium sequence along which the voter cutoffs converge to s_A. In the limit, there is a close race in state A, and the vote shares for the available policies are equal. However, despite the close race, in state A, policy a wins. In state B, the vote share for policy B is strictly above 1/2 and because m_k diverges, policy b wins.

The second part of the result is an immediate consequence of the result by Feddersen and Pesendorfer (1997). When m_k increases very fast, relative to the speed by which c_k disappears, the organizer doesn't recruit anyone in either state, and hence we get an election in which the number of participants is independent of the state of the world.

The third result is an immediate consequence of Theorem 1. Recall that there exists a sequence of equilibria along which the number of voters diverges in both states. Hence, if m_k diverges sufficiently slowly so as not to exceed that number, the originally identified equilibria stays an equilibrium.

One interpretation for the parameter m is that there is a "quorum" requirement, i.e., the minimal number of voters that the organizer must recruit is m.

Another interpretation is a mandatory voting requirement that increases the participation rate even in the absence of the organizer's recruitment efforts. In the equilibria that aggregate information stated in the theorem, this constraint on the lower bound on the number of participating voters will bind in state B and, if m is sufficiently large, it will also bind in state A. Thus, the theorem suggest that when the number of voters can be manipulated, quorums are one instrument to improve the performance of elections.

Another such instruments one may consider is a unanimity rule, where policy a is only elected if all voters vote for a, which we analyze next.

5.2 Unanimity Rule

In this subsection, we analyze what happens with unanimity rule, i.e., all of the participants' votes are required for policy a to be implemented. It is clear that the organizer will recruit no additional voter, because recruitment is costly, and the probability that policy a is selected weakly decreases with the electorate size. Because the organizer recruits no new person regardless of his cost, we will denote the minimum number of voters participating in the election with m, i.e., we will drop the subscript c.

Because the organizer recruits no new voter, voters' participation rates will not depend on the state of the world. Therefore, our model corresponds to the model analyzed by Feddersen & Pesendorfer (1997), with unanimity rule. It is well known by now that unanimity rule is the only supermajority rule that fails to aggregate information in large electorates. However, the extent of how much information aggregation fails depends on the informativeness of the extreme signals. In our environment, it will only depend on

$$\frac{f(1|A)}{f(1|B)}.$$

Let s_m be an interior equilibrium cutoff used by the voters in the game with m voters.[7] Surely there may be multiple equilibria, and hence multiple cutoffs for any given m, but we pick a convergent subsequence of a sequence of cutoffs indexed by m. A voter is pivotal only if all other $m-1$ voters vote for

[7]Existence of an interior equilibrium cutoff is guaranteed when the number of participating voters is large.

a. Hence, a voter is pivotal in state ω with probability $(m-1)F(s_m|\omega)^{m-1}$. The indifference condition of the cutoff signal delivers that,

$$\frac{\pi}{1-\pi}\frac{f(s_m|A)}{f(s_m|B)}\left(\frac{F(s_m|A)}{F(s_m|B)}\right)^{m-1}=1.$$

Lemma 2. $\lim_{m\to\infty} s_m = 1$.

Proof. On the way to a contradiction, suppose that $s_m \to x \in (0,1)$. Then $F(x|A) > F(x|B)$ by the MLRP condition. However, then the indifference condition cannot be satisfied at the limit, which is a contradiction.

Suppose now that $s_m \to 0$. Then, $\lim_{s\to 0}\frac{F(s|A)}{F(s|B)} = \frac{f(0|A)}{f(0|B)} > 1$. But then the indifference condition cannot be satisfied at the limit, which is a contradiction. □

In the following theorem, we show that the limit equilibrium probabilities that the correct policy is implemented depends only on the informativeness of the highest signal. This observation was made by Feddersen & Pesendorfer (1998) in a related voting model, and the authors concluded that unanimity voting rule fails to aggregate information. Therefore, unanimity rule is an inferior voting rule among other supermajority rules. In our model, however, simple majority rule gives rise to effective manipulation by a conflicted organizer, while unanimity rule mitigates this type of manipulation. Hence, when the highest signal is sufficiently informative, the inefficiency induced by the unanimity rule may be less than the inefficiency inflicted by the organizer's manipulation.

Theorem 5. *Consider the unanimity rule with a quorum of m voters. Suppose $m \to \infty$. The probability that policy* a *is selected in state* A *converges to*

$$\left(\frac{1-\pi}{\pi}\frac{f(1|B)}{f(1|A)}\right)^{\frac{f(1|A)}{f(1|B)-f(1|A)}},$$

and the probability that policy a *is selected in state* B *converges to*

$$\left(\frac{1-\pi}{\pi}\frac{f(1|B)}{f(1|A)}\right)^{\frac{f(1|B)}{f(1|B)-f(1|A)}}.$$

As $\frac{f(1|A)}{f(1|B)} \to 0$*, these probabilities converge to* 1 *and* 0*, respectively. Hence, information is arbitrarily close to being aggregated when signal 1 is very informative.*

6 Robustness and Discussion of Assumptions

In this section, we analyze various extensions of the model to highlight the robustness of our model to variations on our assumptions.

6.1 Multiple States

We now extend the model to incorporate a continuum of states. To this end, suppose that $\Omega := [0, 1]$, with $g(\omega)$ being the prior p.d.f. over the states of the world. Assume that $g(\omega)$ has a strictly positive lower bound on its support. We assume that the set of signals is $S := [0, 1]$, and that MLRP condition is satisfied, i.e., for every pair of states $\omega_1 < \omega_2$, $\frac{f(s|\omega_1)}{f(s|\omega_2)}$ is strictly decreasing in s. Also, we assume that $f(s|\omega)$ has a uniform strictly positive lower and upper bounds for all $s \in S$ and $\omega \in \Omega$. Voters have identical preferences, where $v(\omega) := u(b, \omega) - u(a, \omega)$ is a strictly increasing function, and that $v(0) < 0$ and $v(1) > 0$.

As is before, the organizer observes the true state ω and picks a number of pairs of voters to participate in the election. In this scenario, our main result, i.e., an appropriate version of the statements in Theorem 1 continues to hold. In particular, as c disappears, there is a sequence of equilibria in which the probability that policy a is implemented converges to 1 in *all* states of the world, the number of voters grows without bound, and the organizer's payoff converges to 1 in all states.

6.2 Heterogenous Preferences

Now we extend the model to accommodate heterogeneity in voter preferences. We still maintain the assumption that there are two states, $\Omega = \{A, B\}$, and that there is a finite number of voter types, $T := \{t_1, .., t_R\}$, where each $t_i \in (0, 1)$, and t_i is increasing in i. The type t_i represents the probability that a voter of type t_i has to attach to state of the world being state A in order to be indifferent between the two outcomes a and b. Voters with higher preference types are more difficult to be convinced to vote for a than voters with higher preference types. However, it is feasible to persuade each voter to vote in favor of policy j by providing a sufficiently informative evidence in favor of state j. Each voter's preference type is drawn according to a p.d.f. $h(\cdot)$ with full support, and independent of the state, and independent of the signal distribution.

In this environment, all of our results stated in Theorems 1-3 go through with minor but straightforward modifications. In particular, the manipulated equilibria are sustained in this setup. Also, the structure of all limit equilibria as identified in Theorem 2, and information aggregation result stated in Theorem 4 continues to hold.

6.3 Competitive Organizers

In the paper we assume that there is a single organizer who does all of the recruitment choices. Suppose that there is a second organizer, that we refer to as O_1 who prefers that policy b is implemented regardless of the state, and incurs the same marginal recruitment cost as the organizer who prefers that policy a is implemented independent of the state, that we refer to as O_0.

In this scenario, there is always a sequence of manipulated equilibria in which policy a is implemented with probability that converges to one in both states, and O_0 makes all recruitment activity and O_1 is passive. Similarly, there is also a sequence of manipulated equilibria in which policy b is implemented with probabilities that converge to one in both states, and O_1 is active and O_0 is passive. There is, however, one more sequence of equilibria, in which O_0 chooses to recruit many voters in state A, and O_1 chooses to recruit many voters in state B, and information gets aggregated, i.e., correct policy is implemented with probability that converges to one. Therefore, competition among organizers opens up the possibility for information aggregation.

6.4 Role of Recruitment Cost

Strictly positive recruitment cost is essential for our main result in Theorem 1. This is because, the organizer's ability to manipulate the election outcome comes from his ability to pick different voter participation rates in different states. In the manipulated equilibria, the organizer recruits more voters in state B than in state A, because the probability that a randomly selected voter votes for policy a is strictly higher in state A then in state B, and also because marginal recruitment cost is strictly positive. If, contrary to what we assume, recruitment is costless, the manipulated equilibria would not be sustained. In particular, when faced with an aggressive voter behavior in which voters vote for policy a with a probability strictly more than $1/2$, as is the case in manipulated equilibria, the organizer's best reply would be to recruit all potential voters in both states, which would take away his ability to recruit different numbers of voters in different states. Intuitively, strictly positive recruitment cost makes it optimal for the organizer to exhibit a recruitment strategy that sways the voter behavior aggressively in favor of policy b.

6.5 Role of Organizer's Private Information

Organizer's ability to manipulate the electorate relies on his private information about the state of the world. If we had assumed that he does not hold any further information than the common prior belief, then his recruitment strategy would be independent of the state of the world, and hence the voters would not be able to infer any new information about the state of the world from being recruited. As we argued before, the asymmetry in the number of voters that are recruited in different states is essential for manipulation, and such an asymmetry is not possible if the organizer has no private information. For instance, if the minimum number of people that the organizer can choose to recruit is very large, then a result similar to Feddersen & Pesendorfer (1997) would hold, and the voters would be selection the correct policy with arbitrarily high accuracy.

6.6 Abstention, Costly Voting, and Subsidies

Abstention.

Feddersen & Pesendorfer (1996) observed that in an election in which voters have common interests, some voters who are not well informed may have strict incentives to abstain and their abstention has significant effects on the election outcome. In the equilibrium of our model, however, there is never a strict incentive to abstain: For all signals above the equilibrium cutoff, a voter strictly prefers to vote for policy b (rather than to abstain or vote differently) and she strictly prefers to vote for a for signals below that cutoff. With a signal exactly equal to the cutoff, a voter is just indifferent between each vote and abstaining.

The fact that there is no incentive to abstain in this equilibrium relies on the fact that the number of participating voters is odd so that there are never any ties. However, once voters can abstain, there may be additional equilibria in which each voter does abstain with strictly positive probability, implying a positive probability of an even number of voters and hence a tie. Thus, our original equilibrium remains if voters can choose to abstain but additional equilibria may arise.

Costly Voting and Subsidies.

Suppose that, in contrast to our model, all citizen can vote but voting is costly. Here, recruitment may correspond to a subsidy by the organizer. Concretely, suppose that there are N citizens and each citizen can vote at a cost r. This cost may correspond to the cost of collecting information or to the physical act of going to the voting booth. The organizer can reduce the cost of voting to zero by paying c, for example, by bussing voters to the voting booth. If the voting costs r are sufficiently high, only those citizens who receive a subsidy will actually vote.[8]

Note that the cost r doesn't actually have to be very large. To see this point, suppose that voters receive their signal only after making the participation decision (as in Krishna & Morgan (2012)). Now note that in the original "manipulated equilibrium" of our model, we have $n_B > n_A$, that is, not being recruited is evidence in favor of state A. Since in that equilibrium in that state the election outcome is (correctly) a with high probability, this suggests that participation incentives in that equilibrium are small. Moreover, note that when a citizen contemplates participation, she compares her participation cost r to her private benefit from the correct decision *weighted* by the probability

[8] In this context, one can interpret the "participation requirement" m as the number of voters who have zero voting costs while the cost of voting is r for the $N - m$ remaining voters.

that she is pivotal. As we observed before, organizer chooses recruitment such that the recruitment cost c (the cost of the subsidy) equals his private benefit from policy zero, weighted by the probability that the election is tied. Thus, the probability of being pivotal may be small—and the participation incentives may be weak—whenever the organizer's private benefit is large relative to the individual voter's benefits and if the subsidy c is of similar magnitude as r. Thus, there may be many scenarios where one may expect that many citizens who are not recruited will optimally choose to not participate even for intermediate voting costs.

Further analysis of costly voting with subsidies may be an interesting extension of the current model and such analysis may yield a better understanding of exactly what such scenarios may be and when to expect voter subsidies to have substantial effects on voting behavior.

7 Literature Review

Information aggregation in elections with strategic voters has been studied by Austen-Smith & Banks (1996), Feddersen & Pesendorfer (1997, 1996, 1999a, 1998, 1999b), McLennan (1998), Myerson (1998a,b), among others.[9] These papers study equilibrium outcomes with an exogenously large number of voters.

In particular, Feddersen & Pesendorfer (1997) show that in a model with multiple states—and both private and common values—under all supermajority rules except unanimity rule large electorates aggregate information. Similar to us, they provide a complete characterization of all equilibria. The main difference from their model is that here the number of participating voters is selected by a conflicted organizer, and hence the number of voters participating in the election is endogenously state dependent.

Myerson (1998b) introduces a Poisson model with population uncertainty

[9] Bhattacharya (2013) observes the necessity of preference monotonicity for information aggregation. Bouton & Castanheira (2012) find that voters' imperfect information may help solve certain coordination problems. Mandler (2012) shows that uncertainty about the informativeness of the signals can also lead to failure of information aggregation. A recent generalization is Barelli & Bhattacharya (2013). Gul & Pesendorfer (2009) shows that information aggregation fails when there is policy uncertainty.

in which the expected number of voters may be state dependent. He shows that large electorates aggregate information along some sequence of equilibria. In his model, the ratio of the expected numbers of voters across different states is exogenously fixed along the sequence as the expected numbers of voters grow. In our model, similar to Myerson's model, the number of voters that are participating is state dependent. However, the ratio of the number of voters is endogenously determined via the choice of an organizer who incurs a cost for increasing the size of the participating voters. A second difference is that we characterize the limiting outcomes of all symmetric equilibria. We show that there is no equilibrium in which information fully aggregates when the number of voters is endogenous and there also exist equilibria in which the organizer's favorite outcome is implemented regardless of the state.

Methodologically, information aggregation in elections is related to work on large auctions. Among others, this has been studied by Wilson (1977), Milgrom (1979), Pesendorfer & Swinkels (1997); Feddersen & Pesendorfer (1996), and Atakan & Ekmekci (2014). The papers study auctions in which the number of bidders becomes large exogenously. Lauermann & Wolinsky (2012) introduce an auction model in which the number of bidders is random and endogenously state dependent.

Related studies of voter (non-)participation in election include especially Feddersen & Pesendorfer (1996)—who identify the swing voters' curse when voters can abstain—and the vast literature on costly voting, especially Palfrey & Rosenthal (1985) and Krishna & Morgan (2011). Those models emphasize the voters' side whereas our model emphasizes the ability of the organizer to affect participation.

A related paper that endogenizes the issues that are voted for by a strategic proposer is Bond & Eraslan (2010). Similar to us, they show that unanimity rule may be superior to other supermajority voting rules, since the voting behavior under different rules have different implications for the proposals put on the table, in a way that disciplines the proposer to make offers preferred by the voters. In our model, unlike in theirs, the alternatives are fixed, but the participation rate is endogenously determined by a strategic organizer. Similar to their paper, in our model also unanimity rule restricts the organizer's ability to create the asymmetry of participation rates across the states.

Finally, a large literature analyzes the ability of a conflicted agent to manipulate one or more decision makers to act in favor of the agent's interests, either through informational tools or by taking actions that directly affect the decision makers' incentives. An important instance are models of cheap-talk, emanating from Crawford & Sobel (1982) who analyzed the ability of a sender to transmit information, and hence induce behavior partially beneficial to the sender. Our model shares with these models the feature that the organizer has superior information, biased preferences, and takes actions that affect the beliefs of the decision makers. Importantly, the organizer lacks the power to commit to a plan of actions ex-ante. The lack of commitment power separates our model from the current literature on persuasion, initiated by Kamenica & Gentzkow (2011), and applied to a voting context by Wang (2012).

8 Conclusion

Understanding the performance of voting mechanisms to pick the best alternatives for the society has always received attention, dating all the way back to Athenian leader Cleisthenes, and later to Condorcet. In this paper, we studied the ability of voting mechanisms to aggregate dispersed information among voters when the election is taking place in the presence of an organizer who has tools to change the turnout rate, and who has interests that are not aligned with those of the voters. Our main result is that such an organizer can manage to influence the election outcomes drastically, in his favor, thereby preventing information aggregation completely. This result suggests that although voting mechanisms may be very effective in aggregating information, they may be quite susceptible, and hence not robust, to manipulation activities by outsiders. An interesting feature of our model is that, small electorates in which the organizer is not allowed to intervene may perform much better than large electorates in which the organizer can influence the turnout rate.

The ability of the organizer to get his desired outcome relies on him being able to recruit many voters, and does not rely on abilities of cherry picking voters who have information supporting his favorite policy, or voters who are *a priori* more inclined to vote towards his favorite policy. In practice, however, many of the manipulation schemes involve the use of tools, such as timing of elections, or targeted subsidies. Because in our model the organizer can only pick the turnout rate, and cannot distinguish voters with different characteristics, our sharp result provides a *lower bound* on the ability of an organizer who has more tools than only to pick the turnout rate of the election.

A Appendix

A.1 Miscellaneous Results

In this part, we explore some properties of organizer's best reply correspondence, and the critical likelihood ratio that will be used in proving the theorems.

Organizer's Best Reply:

Let $\tilde{n} := (\tilde{n}_A, \tilde{n}_B)$ be a generic mixed strategy of the organizer, and let the set of all mixed strategies for the organizer in the voting game with recruitment cost c be $\tilde{N}(c)$. The term $\tilde{n}_\omega(i)$ denotes the probability that the strategy $\tilde{n}$ assigns to the integer i in state ω. The organizer's best reply correspondence to a voter cutoff s when the recruitment cost is c is $\eta(s,c) := (\eta_A(s,c), \eta_B(s,c)) \subset \tilde{N}(c)$. Moreover, $\tilde{n}^* \in \eta(s,c)$ if and only if each positive integer that is in the support of $\tilde{n}^*_\omega$ solves

$$\max_{n \in \left\{0,1,\ldots,\frac{1}{2}(N-1)\right\}} \sum_{i=n+1}^{2n+1} \binom{2n+1}{i} \left(q_\omega\left(s\right)\right)^i \left(1-q_\omega\left(s\right)\right)^{2n+1-i} - nc$$

<u>Properties of $\eta(s,c)$:</u>

This part has some repetition from the main text, but we include this to help the reader in the rest of the Appendix. Let

$$\begin{aligned}\Delta\left(n-1,\omega,\hat{s}\right) \quad := \quad & \sum_{i=n+1}^{2n+1} \binom{2n+1}{i} \left(q_\omega\left(\hat{s}\right)\right)^i \left(1-q_\omega\left(\hat{s}\right)\right)^{2n+1-i} \\ & -\sum_{i=n}^{2n-1} \binom{2n-1}{i} \left(q_\omega\left(\hat{s}\right)\right)^i \left(1-q_\omega\left(\hat{s}\right)\right)^{2n-1-i}\end{aligned}$$

A simplification of the above expression delivers the following identity.

$$\Delta\left(n-1,\omega,\hat{s}\right) = \binom{2n}{n} \left(q_\omega\right)^n \left(1-q_\omega\right)^n \left(2q_\omega - 1\right)$$

If $q_\omega(\hat{s}) \leq 1/2$, then $\Delta\left(n-1,\omega,\hat{s}\right) \leq 0$ for every n, so, the organizer recruits no one since recruitment is costly.

If $q_\omega(\hat{s}) > 1/2$, then $\Delta(n-1,\omega,\hat{s}) > 0$ and $\Delta(n-1,\omega,\hat{s}) > \Delta(n,\omega,\hat{s})$. The implication of this is that if $q_\omega(\hat{s}) > 1/2$, then there is a unique n such that $\Delta(n-1,\omega,\hat{s}) \geq c$ and $\Delta(n,\omega,\hat{s}) < c$.

Hence, the support of η_ω contains at most two integers, and if it includes two integers, they have to be adjacent integers.

Critical likelihood ratio, when the organizer uses a mixed strategy

$$\beta(\hat{s}, piv, rec; s, \tilde{n}) = \frac{\pi}{1-\pi} \frac{f(\hat{s}|A)}{f(\hat{s}|B)} \frac{\sum_{i\geq 0} \tilde{n}_A(i)(\frac{2i+1}{N})\binom{2i}{i} q_A(s)^i (1-q_A(s))^i}{\sum_{i\geq 0} \tilde{n}_B(i)(\frac{2i+1}{N})\binom{2i}{i} q_B(s)^i (1-q_B(s))^i} \quad (6)$$

Lemma 3. *Fix $s \in [0,1]$. For every $\hat{s} \in [0,1]$,*

$$\max_{\tilde{n}\in\eta(s,c)} \beta(\hat{s}, piv, rec; s, \tilde{n})$$

exists, and is attained by some pure strategy $n \in \eta(s,c)$. Moreover, the set of maximizers for each $\hat{s}$ is the same. Similarly,

$$\min_{\tilde{n}\in\eta(s,c)} \beta(\hat{s}, piv, rec; s, \tilde{n})$$

exists, and is attained by some pure strategy $n \in \eta(s,c)$. Moreover, the set of minimizers for each $\hat{s}$ is the same.

Proof. The function β is continuous in $\tilde{n}$, and the the maximum of a continuous function over a compact domain exists. Independence of the maximizers from $\hat{s}$ is seen by inspection of the function β. □

Operator $\tilde{\beta}$:

Definition 1. *Let*

$$\tilde{\beta} : [0,1] \times [0,\infty) \rightrightarrows \mathbb{R}_+$$

be a correspondence that takes a signal and cost of recruitment as arguments, and returns a positive number, that denotes a likelihood ratio. In particular, $x \in \tilde{\beta}(\hat{s}, c)$ if there is a strategy $\tilde{n} \in \eta(\hat{s}, c)$ such that $\beta(\hat{s}, piv, rec; \hat{s}, \tilde{n}) = x$.

The mapping $\tilde{\beta}$ takes the signal $\hat{s}$ as the cutoff strategy of the voters, then calculates the best reply correspondence of the organizer to the cutoff strategy

$\hat{s}$, and then returns the number that is equal to the critical likelihood ratio of type $\hat{s}$ when all other voters follow the cutoff strategy $\hat{s}$ and when the organizer is following a strategy that belongs to the set of best replies to the cutoff strategy with cutoff $\hat{s}$.

Lemma 4. *The correspondence $\tilde{\beta}(\hat{s}, c)$ is convex valued and is upper-hemicontinous in its first argument, $\hat{s}$.*

Proof. The best reply correspondence, $\eta(\hat{s}, c)$ is upper-hemicontinous in $\hat{s}$, and is convex valued. The function $\beta(\hat{s}, piv, rec; \hat{s}, \tilde{n})$ is continuous in $\tilde{n}$. Moreover, because the densities $f(\cdot|\omega)$ are continuous for each $\omega \in \{0, 1\}$, the claim that $\tilde{\beta}$ is upper-hemicontinous follows from Berge's maximum theorem. Convex-valuedness follows from the continuity of β in $\tilde{n}$, and convexity of the best reply correspondence of the organizer. □

Lemma 5. *A signal $s \in (0, 1)$ is an equilibrium cutoff signal of $G(c)$ if and only if $1 \in \tilde{\beta}(s, c)$.*

Proof. By construction, if $1 \in \tilde{\beta}(s, c)$, then there is a strategy $\tilde{n}$ which is a best reply for the organizer to the voter cutoff strategy s such that $\beta(s, rec, piv; s, \tilde{n}) = 1$. Now, observe that $(s, \tilde{n})$ is an equilibrium, because $\tilde{n}$ is a best response of the organizer to voter behavior s, and using s as a cutoff is a best response for any given voter, since he is indifferent between the two alternatives in the events that he has signal s, he is recruited and he is pivotal, when other voters are following the same strategy and when the organizer follows the strategy $\tilde{n}$. The other direction is straightforward, so we skip it. □

A.2 Proof of Theorem 1

Proof:

Definition 2. *Let $s_\omega \in (0, 1)$ be the signal that satisfies*

$$q_\omega = F(s_\omega|\omega) = 1/2.$$

In other words, s_ω is the median signal in state ω.

Our proof strategy is that we will first show that for all small c, there is some $s(c) > s_B + \epsilon$ such that $1 \in \tilde{\beta}(s(c), c)$. This means, there are equilibria in which the voters vote for policy a with probability more than 1/2 in both states. The second part of the proof will show that in such equilibria, as c disappears, a get selected with probability approaching 1, that the number of voters grows without bound, and that the organizer's payoff approaches 1.

We will start by showing the existence of equilibria with a large cutoff. There are two steps we will show in the following development:

Claim 1:

$$\exists \epsilon > 0 \text{ such that } \lim_{c \to 0} \max \tilde{\beta}(s_B + \epsilon, c) < 1,$$

and

Claim 2:

$$\exists \epsilon_c > 0, \text{ with } \lim_{c \to 0} \epsilon_c \to 0, \text{ such that } \lim_{c \to 0} \max \tilde{\beta}(1 - \epsilon_c, c) = \infty.$$

These two findings together with the upper-hemicontinuity and convex valuedness of $\tilde{\beta}$ (Lemma 4) imply, via intermediate value theorem, that for all c smaller than a cutoff $\bar{c} > 0$, there is a $s(c) \in (s_B + \epsilon, 1 - \epsilon_c)$ such that $1 \in \tilde{\beta}(s(c), c)$, delivering the desired result.

Claim 1:

$$\exists \epsilon > 0 \text{ such that } \lim_{c \to 0} \max \tilde{\beta}(s_B + \epsilon, c) < 1.$$

Proof of Claim 1:

Remember that $q_\omega(x)$ denotes the probability that a randomly selected voter votes for policy a in state ω, and let $n_\omega(x, c)$ be a pure best reply of the organizer if the voters are using the cutoff strategy x. We will drop the arguments in these objects occasionally to save on notation and for ease of reading.

Using the derivations we have for the organizer's best reply conditions before, i.e., that $\Delta(n_\omega - 1, \omega, x) \geq c$ and $\Delta(n_\omega, \omega, x) < c$, we now posit and prove the following:

Lemma 6.

$$\frac{2c}{2q_\omega - 1} \leq \binom{2n_\omega}{n_\omega} (q_\omega)^{n_\omega} (1 - q_\omega)^{n_\omega} \leq \frac{c}{q_\omega (1 - q_\omega)(2q_\omega - 1)} \frac{(n_\omega + 1)}{(2n_\omega + 1)}.$$

Proof: Rewriting the hypothesis,

$$\begin{aligned} \Delta(q_\omega, n_\omega) &\leq c \implies \\ \binom{2n_\omega + 1}{n_\omega} (q_\omega)^{n_\omega + 1} (1 - q_\omega)^{n_\omega + 1} (2q_\omega - 1) &\leq c \implies \\ (2n_\omega + 1) \binom{2n_\omega}{n_\omega} (q_\omega)^{n_\omega} (1 - q_\omega)^{n_\omega} &\leq \frac{c(n_\omega + 1)}{q_\omega (1 - q_\omega)(2q_\omega - 1)} \end{aligned}$$

and

$$\begin{aligned} \Delta(q_\omega, n_\omega - 1) &\geq c \implies \\ \binom{2(n_\omega - 1) + 1}{n_\omega - 1} (q_\omega)^{n_\omega} (1 - q_\omega)^{n_\omega} (2q_\omega - 1) &\geq c \implies \\ \binom{2n_\omega - 1}{n_\omega - 1} (q_\omega)^{n_\omega} (1 - q_\omega)^{n_\omega} (2q_\omega - 1) &\geq c \implies \\ \frac{n_\omega}{2n_\omega} \binom{2n_\omega}{n_\omega} (q_\omega)^{n_\omega} (1 - q_\omega)^{n_\omega} (2q_\omega - 1) &\geq c \implies \\ \binom{2n_\omega}{n_\omega} (q_\omega)^{n_\omega} (1 - q_\omega)^{n_\omega} &\geq \frac{2c}{2q_\omega - 1} \end{aligned}$$

Taken together, the claim follows.

□

For the rest of the Appendix, we will occasionally use Stirling's approximation, which is:

$$\lim_{n \to \infty} \frac{n!}{\sqrt{2\pi n} \left(\frac{n}{e}\right)^n} = 1.$$

Lemma 7. *If $q_\omega(x) \in (0.5, 1)$, then for any selection of pure strategy best replies by the organizer, $\{n_A(x, c), n_B(x, c)\}_{c>0}$*

$$\lim_{c \to 0} \frac{n_B(x, c)}{n_A(x, c)} = \frac{\ln(4(q_A)(1 - q_A))}{\ln(4(q_B)(1 - q_B))}.$$

Proof: If $q_\omega(x) \in (0.5, 1)$, then $\lim_{c \to 0} n_\omega(x, c) = \infty$.

Rewriting the approximation for state A,

$$\begin{aligned} & \binom{2n_A}{n_A}(q_A)^{n_A}(1-q_A)^{n_A} \\ = & \frac{(2n_A)!}{(n_A!)^2}(q_A)^{n_A}(1-q_A)^{n_A} \\ \cong & \frac{\sqrt{2\pi 2n_A}\left(\frac{2n_A}{e}\right)^{2n_A}}{\left(\sqrt{2\pi n_A}\left(\frac{n_A}{e}\right)^{n_A}\right)^2}(q_A)^{n_A}(1-q_A)^{n_A} \\ = & \frac{(4q_A(1-q_A))^{n_A}}{\sqrt{\pi}\sqrt{n_A}} \end{aligned}$$

The approximation for state B is similar, and hence,

$$\begin{aligned} & \lim_{c\to 0}\frac{\binom{2n_A}{n_A}(q_A)^{n_A}(1-q_A)^{n_A}}{\binom{2n_B}{n_B}(q_B)^{n_B}(1-q_B)^{n_B}} \\ = & \lim_{c\to 0}\sqrt{\frac{n_B}{n_A}}\left(\frac{(4q_A(1-q_A))}{(4q_B(1-q_B))^{\frac{n_B}{n_A}}}\right)^{n_A} \end{aligned} \tag{7}$$

where the equality is from Stirling's approximation and the previous rewriting.

From Lemma 6 and $q_\omega(x) \in (0.5, 1)$, the ratio (7) must be bounded away from zero and infinity, which requires that

$$\lim_{c\to 0}\frac{n_B(x,c)}{n_A(x,c)} = K \in (0,\infty)$$

(if $\lim_{c\to 0}\frac{n_B}{n_A} = 0$, then the ratio vanishes, if $\lim_{c\to 0}\frac{n_B}{n_A} = \infty$ it explodes). Using again that the ratio is bounded, this requires

$$\lim_{c\to 0}\frac{(4q_A(1-q_A))}{(4q_B(1-q_B))^{\frac{n_B}{n_A}}} = 1$$

from which the claim of this step follows.

□

Now, from Lemma 6, it follows that,

$$\begin{aligned}
\max \tilde{\beta}(s,c) &= \max_{(n_A,n_B)\in\eta(s,c)} \frac{f(s|A)}{f(s|B)} \frac{(2n_A+1)\binom{2n_A}{n_A}(F(s|A)(1-F(s|A)))^{n_A}}{(2n_B+1)\binom{2n_B}{n_B}(F(s|B)(1-F(s|B)))^{n_B}} \\
&\leq \max_{(n_A,n_B)\in\eta(s,c)} \frac{f(s|A)}{f(s|B)} \frac{(2n_A+1)\frac{c}{q_A(1-q_A)(2q_A-1)}\frac{(n_A+1)}{(2n_A+1)}}{(2n_B+1)\frac{2c}{2q_B-1}} \\
&= \max_{(n_A,n_B)\in\eta(s,c)} \frac{f(s|A)}{f(s|B)} \frac{2n_A+1}{2n_B+1} \frac{2q_B-1}{2q_A(1-q_A)(2q_A-1)} \frac{n_A+1}{2n_A+1}
\end{aligned}$$

Note that $\max \tilde{\beta}(s,c)$ denotes the biggest element of the correspondence $\tilde{\beta}$. The term (n_A, n_B) denotes a pure strategy that puts probability 1 to integers n_A and n_B in states A and B respectively. Applying Lemma 7, we obtain that, for any fixed s such that $q_B(s) \in (0.5, 1)$,

$$\lim_{c\to 0} \max \tilde{\beta}(s,c) \leq \frac{f(s|A)}{f(s|B)} \frac{2q_B-1}{4q_A(1-q_A)(2q_A-1)} \frac{\ln(4(q_B)(1-q_B))}{\ln(4(q_A)(1-q_A))}.$$

Note that the right side vanishes for $s \to s_B$ from above, because $q_B(s) \to 1/2$ (while $q_A(s)$ stays strictly larger than $1/2$) there exists some ε and c^* such that for all $c \leq c^*$,

$$\max \tilde{\beta}(s_B+\varepsilon, c) < 1.$$

Note also that the number 1 on the right hand side of the inequality is arbitrary, and we could have proven this inequality for any positive number.

Remark 3. *A slightly more complicated argument shows that the same result is true if* $\lim x(c) = 1$*, i.e., if* $\lim n_A(x(c),c)/n_B(x(c),c) \to \infty$*, and if* $\lim x(c) = 1$*, then* $\beta(s, piv, rec; x(c), n(c)) = 0$ *for every* $s \in [0,1]$.

Claim 2:

$$\exists \epsilon_c > 0, \text{ with } \lim_{c\to 0} \epsilon_c \to 0, \text{ such that } \lim_{c\to 0} \sup \tilde{\beta}(1-\epsilon_c, c) = \infty.$$

Proof of Claim 2:

Let $f(x) := x(1-x)(2x-1)$, and let $\epsilon_c > 0$ be a number that is close to zero that satisfies $f(q_A(1-\epsilon_c)) = c$. Existence of such a $\epsilon_c > 0$ is guaranteed when c

is small, and $\lim_{c\to 0}\epsilon_c = 0$. We are going to show that $\lim_{c\to 0}\max\tilde{\beta}(1-\epsilon_c, c) = \infty$.

The definition of $1-\epsilon_c$ is that in state A, the organizer with a marginal cost c is indifferent between recruiting no additional voter and recruiting one pair of voters.

Note that,

$$f'(q) = \frac{\partial}{\partial q}\left(f(q)\right) = 12q\left(1-q\right) - 1$$

and $f'(q) < 0$ for q sufficiently close to one. Hence, for x close to 1, $q_B(x) < q_A(x)$ implies whenever $f(q_A(x)) = 2q_A(x)(1-q_A(x))(2q_A(x)-1) = c$, $f(q_B(x)) = 2q_B(x)(1-q_B(x))(2q_B(x)-1) > c$. Hence, the best reply of the organizer to the cutoff strategy $1-\epsilon_c$ is that in state B, he recruits at least 1 pair, and actually as $1-\epsilon_c \to 1$, exactly one pair. This is because, $\delta(1,1) = \binom{4}{2}q_B(x)^2(1-q_B(x))^2(2q_B(x)-1) < f(q_A(x)) = c$ for all x sufficiently close to one. Writing down the pivotal probability in state B, we get $2q_B(x)(1-q_B(x))$, and if in state A the organizer recruits no one, then the pivotal probability in state A is 1. Because $\lim_{x\to 1}\frac{q_B(x)}{q_A(x)} = 1$, and $\lim_{x\to 1}\frac{1-q_B(x)}{1-q_A(x)} = \frac{f(1|B)}{f(1|A)}$, and that $\frac{f(1|A)}{f(1|B)} > 0$, we have that

$$\begin{aligned}\lim_{c\to 0}\max\tilde{\beta}(1-\epsilon_c, c) &= \lim_{c\to 0}\frac{f(1-\epsilon_c|A)}{f(1-\epsilon_c|B)}\frac{1}{3}\frac{1}{2q_B(1-\epsilon_c)(1-q_B(1-\epsilon_c))} \\ &= \infty.\end{aligned}$$

<u>Combining Claims 1 and 2 and Lemma 4:</u>

Because $\tilde{\beta}(s,c)$ is upper-hemicontinous and convex valued (Lemma 4), and combining this with Claims 1 and 2, it follows via intermediate value theorem that there is a $\bar{c} > 0$ and $\epsilon > 0$ such that for every $c < \bar{c}$, there is an $s(c) > s_B + \epsilon$ such that $1 \in \tilde{\beta}(s(c), c)$. Hence, there is an equilibrium in which the voters vote for policy a with a probability more than 1/2.

<u>Modifying Proof of Claim 2 to ensure large participation:</u>

In this part, we will modify the second part of the above proof (i.e., the proof of claim 2) to show that ϵ_c can be chosen in such a way that the organizer, when faced with voters using a cutoff $1-\epsilon_c$, is indifferent between $m(c)$ and $m(c)-1$ pairs of voters in state A and recruits $m(c)$ pairs of voters in state B, and that $\lim_{c\to 0} m(c) = \infty$.

The alternative mapping that we consider is $x_m(c)$, defined analogously as the solution to

$$\binom{2m}{m}(q_A(x))^m(1-q_A(x))^m(2q_A(x)-1)=c.$$

As before, for given c sufficiently small,

$$\binom{2m}{m}(q_B(x))^m(1-q_B(x))^m(2q_B(x)-1)>c>\binom{2m+2}{m+1}(q_B(x))^m(1-q_B(x))^m(2q_B(x)-1).$$

Thus, we can pick some $\hat{x}(c)$ just below $x_n(c)$ such that in state A, the organizer recruits $m-1$ pairs of voters and in state B recruits m pairs of voters. As $c\to 0$, it must be that $\hat{x}(c)\to 1$ and similar to before,

$$\lim_{c\to 0}\max\tilde{\beta}(\hat{x}(c),c)=\infty.$$

Now consider a sequence of equilibria whose existence has been shown to exist with cutoffs bounded away (above) s_B, i.e., $\lim_{c\to 0}s(c)=s^*>s_B$.

If $s^*<1$, then $\lim_{c\to 0}n_\omega(s(c),c)\to\infty$. This follows directly from the properties of the best reply correspondence of the organizer.

If $s^*=1$, then note the following: Consider the function $g(m,x)=x^m(1-x)^m(2x-1)$. There is some $\epsilon>0$ such that $g(m,x)$ is decreasing in x for every positive integer m, and for every $x>1-\epsilon$. Therefore, $s(c)<x_m(c)$, and that $\lim_{c\to 0}s(c)=1$ implies that $n_\omega(s(c),c)\geq m(c)$. We can always choose the sequence $m(c)$ in a way that it grows unboundedly, hence participation grows without bound.

Showing that policy a gets selected:

Let $s(c)$ denote the equilibrium cutoff sequence that we showed the existence of in the previous parts of this Proof. By construction, $1>s(c)>s_B+\varepsilon$ for all c smaller than $\bar{c}>0$. We show that the probability of the majority voting for policy a approaches one as $c\to 0$. Without loss of generality, suppose $s(c)$ converges. The claim is obvious if

$$\lim_{c\to 0}s(c)=1.$$

If not, then

$$1 > \lim_{c \to 0} s(c) \geq s_B + \varepsilon$$

implies

$$\lim_{c \to 0} q_A(s(c)) > \lim_{c \to 0} q_B(s(c)) > 0.5$$

and $\lim_{c \to 0} n_\omega(c) \to \infty$. But this implies the claim also for the second case, because a law of large numbers applies. Thus, we have proven that across the sequence of equilibria we have shown the existence of, the probability that policy a gets implemented approaches 1, as c disappears.

Showing that organizer's payoff is 1 in both states:

Let $U_O^c(s(c), \tilde{n}(c))$ denote the equilibrium payoff of the organizer in the election in which the marginal recruitment cost is c and the equilibrium strategy profile $s(c), \tilde{n}(c)$ is the one identified in the previous parts. Consider the following alternative strategy $\bar{n}(c) := (\lfloor \frac{1}{\sqrt{c}} \rfloor, \lfloor \frac{1}{\sqrt{c}} \rfloor)$, i.e., $\bar{n}(c)$ is the strategy in which the organizer invites $\lfloor \frac{1}{\sqrt{c}} \rfloor$ pairs of voters in both states. Note that, as $c \to 0$, recruitment cost incurred by the strategy $\bar{n}$ disappears. Moreover, because $c \to 0$, the number of recruited voters go to ∞, and because $s(c) \to s^* > s_B + \epsilon$, by a weak version of Law of Large Numbers, the probability that the majority votes for policy a approaches 1 when the organizer employs strategy $\bar{n}(c)$. Hence, $\lim_{c \to 0} U_O^c(s(c), \bar{n}(c)) \to 1$. Because $\tilde{n}(c)$ is a best reply to voter cutoff strategy $s(c)$, it has to be that $U_O^c(s(c), \tilde{n}(c)) \geq U_O^c(s(c), \bar{n}(c))$, for every $c > 0$. Therefore, $\lim_{c \to 0} U_O^c(s(c), \tilde{n}(c)) = 1$, as well. Since, in each state, the organizer's payoff is bounded above by 1, and since each state occurs with positive probability, the organizer's payoff conditional on each state converges to 1, as well.

■

A.3 Proof of Theorem 2

This theorem characterizes all limit points of non-trivial equilibrium cutoffs:

PART 1: Sequences with limit points $s^* > s_B$: Manipulated Equilibria

These are the manipulated equilibria identified in our main theorem. They always exist for small c. Remaining is to show that all sequences for which $s^* > s_B$ yield the same outcome as identified in the first two bullet points of

Theorem 1. This is very similar to the proof of Theorem 1, and hence we skip it here.

PART 2: Sequences with limit points $s^* = s_A$

Step 1: To show s_A is the only possible limit point of non-trivial equilibria, not more than s_B.

Lemma 8. *Let $\{s(c), n_A(c), n_B(c)\}_{c>0}$ be a selection of cutoffs $s(c)$ for the voters, and a pair of integers $(n_A(c), n_B(c))$ that are in the support of the organizer's best reply to voter strategy $s(c)$ with recruitment cost c. If $\lim_{c\to 0} s(c) > s_\omega$, then (we drop the dependence of n_ω on c here)*

$$\lim_{c\to 0}(2n_\omega + 1)\binom{2n_\omega}{n_\omega} q_\omega(s(c))^{n_\omega}(1 - q_\omega(s(c)))^{n_\omega} = 0$$

Proof. First, if $\lim s(c) > s_\omega$, then $\lim q_\omega(s(c)) = q^* > 1/2$. Therefore, $\lim n_\omega(c) = \infty$. By Stirling's approximation, we get that

$$\lim_{c\to 0} \frac{\binom{2n_\omega}{n_\omega} q_\omega(s(c))^{n_\omega}(1 - q_\omega(s(c)))^{n_\omega}}{(4q^*(1 - q^*))^{n_\omega}} = 1$$

Because $q^* > 1/2$, $4q^*(1 - q^*) < 1$, and hence

$$\lim_{c\to 0}(2n_\omega + 1)(4q^*(1 - q^*))^{n_\omega} = 0.$$

Combining this with the above equality delivers the result. □

Clearly, if $s^* < s_A$, then the probability that a randomly selected voter votes for policy a is strictly less than 1/2 in both states, and the organizer recruits no one. But then this is a trivial equilibrium. So if there is any non-trivial equilibrium sequence, its limit point, $s^* \geq s_A$. If $s^* > s_B$, then these are manipulated equilibria, and we showed their existence and analyzed its properties in Theorem 1. So first suppose that $s_A < s^* < s_B$. In that case, when k is sufficiently large, the organizer recruits no one in state B, and many voters in state A. In fact, because $s^* > s_A$, $q_A(s(c)) \to q^*_A > 1/2$, and therefore, in any sequence of equilibria, for any selection of integers $\{n(A, c)\}$ that are in the support of the equilibrium recruitment strategy of the organizer,

$n(A,c) \to \infty$. Therefore, by Lemma 8 above,

$$(2\tilde{n}(A,c)+1)\binom{2\tilde{n}(A,c)}{\tilde{n}(A,c)} q_A(s(c))^{\tilde{n}(A,c)}(1-q_A(s(c)))^{\tilde{n}(A,c)} \to 0.$$

Therefore, $\max \tilde{\beta}(s(c),c) \to 0$, which is a contradiction to $s(c)$ being an equilibrium cutoff.

On the way to a contradiction, suppose that $s^* = s_B$.

We will now argue that this cannot be the case since

$$\lim \max \tilde{\beta}(s(c),c) = 0 \text{ if } s(c) \to s_B.$$

First, note that

$$\lim_{c \to 0} q_A(s(c)) = q_A(s_B) > 1/2,$$

and this implies

$$\lim_{c \to 0}(2n_A+1)\binom{2n_A}{n_A} q_A^{n_A}(1-q_A)^{n_A} = 0.$$

for every sequence of integers $n_A(c)$ in the support of the organizer's best reply, via Lemma 8. So, there cannot be any infinite subsequence of the sequence equilibrium cutoffs in which $q_B(s(c)) \le 1/2$. This is because, otherwise, along such a subsequence, $n_B(c) = 0$, and hence $\lim \max \tilde{\beta}(s(c),c) = 0$. So, we now look at the infinite subsequence along which $q_B > 1/2$.

Also remember that $\max \tilde{\beta}(s(c),c)$ is attained by some pure strategy that is in $\eta(s(c),c)$, and we will denote such a pure strategy with a pair of integers, n_A and n_B which correspond to the integers in the support of the strategy in states 0 and 1, respectively. Surely these integers depend on c, but for ease of reading, when it does not cause confusion, we will drop the dependence of these integers on c.

In what follows, we will bound $\lim \max \tilde{\beta}(s(c),c)$ from above, by either putting a lower bound on the multiplication of two terms on the denominator, which is

$$(2n_B+1)\binom{2n_B}{n_B} q_B^n(1-q_B)^{n_B},$$

or by directly arguing that

$$\frac{(2n_A+1)\binom{2n_A}{n_A}q_A^{n_A}(1-q_A)^{n_A}}{(2n_B+1)\binom{2n_B}{n_B}q_B^{n_B}(1-q_B)^{n_B}} \to 0.$$

For any given $q > 1/2$, the function $f(q,n) := (2n+1)\binom{2n}{n}q^n(1-q)^n$ can have at most one peak, when viewed as a function of n. This is because,

$$\frac{f(q,n+1)}{f(q,n)} = \frac{2n+3}{2n+1}\frac{(2n+2)(2n+1)}{(n+1)^2}q(1-q) = \frac{4n+6}{n+1}q(1-q).$$

A simple calculation shows that the expression for $\frac{f(q,n+1)}{f(q,n)}$ is a strictly decreasing function of n. When q is sufficiently close to 1/2, $f(q,n)$ is strictly increasing in n at $n = 0$. Therefore, for every nonnegative integer N^*, the minimum of $f(q,n)$ in the domain $n \in \{0,1,...,N^*\}$ is attained at one of the extreme points, i.e., either at $n = 0$ or $n = N^*$.

Pick any infinite subsequence along which $n_A(c) \geq n_B(c)$. From the above argument when c is small,

$$f(q_B(s(c)), n_B(c) \geq \min\{f(q_B(s(c)), n_A(c); f(q_B(s(c)), 0)\}.$$

Along a subsequence at which the above minimum is attained at $n = 0$, $f(q_B(s(c)), 0) = 1$, and hence our claim follows. This is because,

$$(2n_B+1)\binom{2n_B}{n_B}q_B^n(1-q_B)^{n_B} \geq \min_{n\in\{0,1,...,n_B\}} f(q_B,n) \geq \min_{n\in\{0,1,...,n_A\}} f(q_B,n) = 1$$

Where the first inequality follows from the definition of $f(q,n)$, and the second one follows from the property of the subsequence that $n_A \geq n_B$, and hence the min is taken over a larger set. This together with

$$\lim(2n_A+1)\binom{2n_A}{n_A}q_A^{n_A}(1-q_A)^{n_A} = 0$$

delivers that $\lim\max\tilde{\beta}(s(c),c) = 0$ along such a sequence.

Along the remaining sequence along which the minimum of the above expression is attained at $f(q_B(s(c)), n_A(c)$,

$$\frac{(2n_A+1)\binom{2n_A}{n_A}q_A^{n_A}(1-q_A)^{n_A}}{(2n_B+1)\binom{2n_B}{n_B}q_B^{n_B}(1-q_B)^{n_B}} \leq \frac{(2n_A+1)\binom{2n_A}{n_A}q_A^{n_A}(1-q_A)^{n_A}}{(2n_A+1)\binom{2n_A}{n_A}q_B^{n_A}(1-q_B)^{n_A}} = \frac{q_A^{n_A}(1-q_A)^{n_A}}{q_B^{n_A}(1-q_B)^{n_A}} \to 0.$$

The last line follows from the fact that $q_A(1-q_A) < q_B(1-q_B)$, and that $n_A \to \infty$. $q_A(1-q_A) < q_B(1-q_B)$ because $q_A > q_B \geq 1/2$.

Now the only remaining subsequence is the one along which $n_A(c) < n_B(c)$. For such a subsequence, notice that the optimality of the organizer's best reply delivers:

$$\binom{2n_B}{n_B}q_B^{n_B}(1-q_B)^{n_B} \geq \frac{c}{2q_B-1},$$

and

$$\binom{2n_A}{n_A}q_A^{n_A}(1-q_A)_A^n \leq \frac{c}{2q_A(1-q_A)(2q_A-1)}\frac{n+1}{2n+1} \leq \frac{c}{2q_A(1-q_A)(2q_A-1)}$$

Notice that, $2q_B - 1 \to 0$, and combining this with $n_A(s(c)) < n_B(s(c))$ delivers:

$$\lim_{c\to 0}\frac{(2n_A+1)\binom{2n_A}{n_A}q_A^{n_A}(1-q_A)_A^n}{(2n_B+1)\binom{2n_B}{n_B}q_B^{n_B}(1-q_B)_B^n} = 0$$

which then implies that $\lim\max\tilde{\beta}(s(c),c) = 0$ along such a sequence as well.

<u>Step 2:</u> To show s_A is an attainable limit point.

The proof strategy here will be similar to the proof for the existence of manipulative equilibria in Theorem 1.

Using Lemma 7 (above), it is relatively straightforward by now that there is some small $\bar{\varepsilon} > 0$ such that for every $\epsilon < \bar{\varepsilon}$, $\lim\max\tilde{\beta}(s_A+\varepsilon,c) = 0$. We will show that there is an $\epsilon(c) > 0$ with $\lim\epsilon(c) = 0$, such that $\lim\max\tilde{\beta}(s_A+\epsilon(c),c) = \infty$. Clearly, then, via the intermediate value theorem, again an equilibrium with a cut point $s(c) \in (s_A+\epsilon(c), s_A+\varepsilon)$ exists, for every small c. By the previous step, the limit point of $s(c)$ have to be s_A. So all we have to show is that there is a mapping $\epsilon(c) > 0$ with $\lim\epsilon(c) = 0$ such that

$\lim \max \tilde{\beta}(s_A + \epsilon(c)) = \infty$.

Note that in state B, organizer recruits no one, for $\epsilon(c)$ sufficiently small. So our task is to show that $(2n_A+1)\binom{2n_A}{n_A} q_A^{n_A}(1-q_A)^{n_A}$ can be made arbitrarily large, for small c, with a cutoff $s_A + \epsilon(c)$.

Pick an arbitrary large number, a. Let $s(a,c) > s_A$ be such that the organizer is indifferent between recruiting a and $a+1$ pairs of voters when the voters are using a cutoff strategy with a cutoff $s(a,c)$. In particular, $(2q_A(s(a,c)) - 1)\binom{2a}{a} q_A(s(a,c))^a (1 - q_A(s(a,c)))^a = c$. It's evident by now that we can do the selection of $s(a,c)$ such that $\lim_{c\to 0} s_A(a,c) \to s_A$ for every integer a. (Note that such a selection could be made by picking cutoffs that converge to 1 as well)

Let $s(a) > s_A$ be equal to $\min\{\tilde{s}(a), s_A + \bar{\varepsilon}\}$, where $\tilde{s}(a)$ is the signal that has the property that, for every $q \in [1/2, q_A(\tilde{s}(a))]$

$$2 - \frac{a(2q-1)^2}{q(1-q)} > 0$$

For every a, such a $\tilde{s}(a) > s_A$ exists, by inspection of the inequality. Moreover, for a sufficiently large, $s(a) = \tilde{s}(a) < \bar{\varepsilon}$.

Note that, $\lim \max \tilde{\beta}(s(a), c) = 0$. Moreover, $\lim \max \tilde{\beta}(s(a,c), c) = O(\sqrt{a})$, from Stirling's approximation. In particular (and more formally),

$$\lim_{a\to\infty} \lim_{c\to 0} \max \tilde{\beta}(s(a,c), c) = \infty$$

Therefore, via intermediate value theorem, for each a sufficiently large, there is a $\bar{c}$ such that for all $c < \bar{c}$, there is $s^*(a,c) \in (s(a,c), s(a))$ such that $1 \in \tilde{\beta}(s^*(a,c), c)$.

Step 3: To show that an equilibrium sequence whose limit point is s_A and in state A, majority selects policy a.

Note that if $s^* = s_A$, then in state B, no one is recruited, and hence there is only one voter for every c. Therefore, as $c \to 0$, The term $(2n_A + 1)\binom{2n_A}{n_A} q_A^{n_A}(1 - q_A)^{n_A}$ converges to a number $k \in (0, \infty)$. Now consider the cutoff $s^*(a,c)$ defined in step 2, above. Note that the organizer's best reply to this cutoff in state A is to recruit at least a voters, for c small enough. This is because, $s^*(a,c) < \tilde{s}(a)$, where for every $q \in [1/2, q_A(\tilde{s}(a))]$,

$$2 - \frac{a(2q-1)^2}{q(1-q)} > 0.$$

The marginal benefit of the organizer,

$$\frac{\partial \binom{2a}{a}(q(1-q))^a(2q-1)}{\partial q} = \binom{2a}{a}(q(1-q))^a[2 - \frac{a(2q-1)^2}{q(1-q)}] > 0$$

for every $q \in [1/2, q_A(\tilde{s}(a))]$, and hence the support of the organizer's best replies at state A are at least a pairs of voters, whenever the voters are using a cutoff between $s(a,c)$ and $\tilde{s}(a)$. Because the equilibrium cutoff $s^*(a,c)$ that we identified is in that interval, the organizer indeed recruits at least a pairs of voters in state A.

Since a is arbitrary, we can construct a sequence of equilibria along which $s(c) \to s_A$, and the number of voters recruited in state A grows without bound.

Now we will show that if $s(c) \to s_A$ and if the number of voters in state A grows without bound, then the majority selects policy a with a probability that converges to 1. As we stated in the previous paragraph:

$$\lim_{c \to 0}(2n_A + 1)\binom{2n_A}{n_A} q_A^{n_A}(1-q_A)^{n_A} = k \in (0, \infty)$$

Where again, n_A, q_A clearly depend on $s(c)$ and c, but we dropped the dependence for ease of reading. Because $n_A \to \infty$, $s(c) > s_A$. The probability that the majority selects policy a in state A is:

$$\sum_{i=n_A+1}^{2n_A+1} \binom{2n_A+1}{i} q_A^i (1-q_A)^{2n_A+1-i}.$$

To show that this probability converges to 1, we use the following lemma.

Lemma 9. *Let $\{q(c)\}_{c>0}$ be a selection of probabilities with $\lim q(c) \to q*$, and $\{n(c)\}_{c>0}$ be a selection of integers such that $\lim n(c) \to \infty$. If*

$$\lim_{c \to 0}(2n(c) + 1)\binom{2n(c)}{n(c)} q(c)^{n(c)}(1-q(c))^{n(c)} = k \in (0, \infty)$$

then,

$$\lim_{c\to 0}\sum_{i=0}^{n(c)}\binom{2n(c)+1}{i}q(c)^i(1-q(c))^{2n(c)+1-i}=0$$

Proof. Pick any pair q, n. Let

$$t(i,n) := \frac{\binom{2n+1}{n+1}q^{n+1}(1-q)^n}{\binom{2n+1}{i}q^i(1-q)^{2n+1-i}} = (\frac{q}{1-q})^{n+1-i}\frac{(2n+1-i)(2n-i)...(n+2)}{n(n-1)...(i+1)}.$$

Note that $t(i,n) > 1$ for $i \leq n$ because $q > 1/2$. Moreover, $t(i,n)$ is decreasing in i.

Pick an arbitrary $\epsilon > 0$. Let $1+\kappa(\epsilon)$ be a lower bound strictly larger than 1 for the term

$$\frac{2n+1-(n(1-\epsilon))}{n(1-\epsilon)+1}.$$

For $i \leq (1-2\epsilon)n$, we have that $t(i,n) \geq (1+\kappa(\epsilon))^{\epsilon n}$. Therefore,

$$\sum_{i=0}^{n}\binom{2n+1}{i}q(n)^i(1-q(n))^{2n+1-i} \leq ((n(1-2\epsilon))(1+\kappa(\epsilon))^{-\epsilon n}+2\epsilon n)\binom{2n+1}{n}q^{n+1}(1-q)^n.$$

Taking $n \to \infty$, and then using the fact that ϵ was arbitrary, and the fact that

$$(2n(c)+1)\binom{2n(c)}{n(c)}q(c)^{n(c)}(1-q(c))^{n(c)} \to k \in (0,\infty)$$

delivers the result.

□

Step 4: Proof that if inequality 5 holds, there is an equilibrium with limit cutoff s_A and with bounded number of voters.

Note that $\tilde{\beta}(s_A, c)$ is single valued for every $c > 0$, and that value is equal to $\frac{\pi}{1-\pi}\frac{f(s_A|A)}{f(s_A|B)}$. This is because, $\eta(s_A, c)$ has a single element for every $c > 0$, and this single element is a pure strategy that recruits no one in both states.

Hence, if inequality 5 holds, then $\max\tilde{\beta}(s_A, c) \leq 1$, for every $c > 0$. By the argument in step 2, there is some large a and $\bar{c} > 0$ such that for every $c < \bar{c}$, $\max\tilde{\beta}(s(a,c), c) > 1$. Therefore, by the intermediate value theorem, there is some $s(c) \in [s_A, s(a,c)]$ such that $1 \in \tilde{\beta}(s(c), c)$. Because $s(c) < s(a,c)$, and because for all sufficiently small c, $s(a,c) < \tilde{s}$, and because $s(a,c)$ is the cutoff signal to which the organizer best reply is to recruit at most $a+1$ pairs of voters in state A, the organizer recruits not more than $a+1$ pairs of voters in state A when the voters are using the cutoff $s(c)$. Because this is true for every $c < \bar{c}$, and because $\lim_{c\to 0} s(a,c) = s_A$, we can construct a sequence of equilibrium cutoffs that converge to s_A and along such equilibria, the organizer recruits a bounded number of voters in state A (and no one in state B).

Step 5: Proof that if inequality 5 is not satisfied, all equilibria with limit cutoff s_A has growing number of voters.

On the way to a contrary, suppose that there is an equilibrium sequence with limit cutoff s_A, and which has a bounded number of voters in state A, say less than k. Notice that,

$$\liminf_{c\to 0} \sum_{i>0} \tilde{n}(c)(i) \times (2i+1)\binom{2i}{i}(q_A(s(c))(1-q_A(s(c))))^i \geq 1,$$

where $\tilde{n}(c)$ is the equilibrium strategy of the organizer. This is because, $q_A(s(c)) \to 1/2$, $(2i+1)\binom{2i}{i}(1/4)^i$ is strictly increasing in i, and because $\tilde{n}(c)(i) = 0$ for every $i > k$. Moreover, $\frac{f(s|A)}{f(s|B)}$ is strictly increasing in s, and hence, for every $s > s_A$,

$$\frac{\pi}{1-\pi}\frac{f(s|A)}{f(s|B)} > 1.$$

But this contradicts the equilibrium requirement that $1 = \beta(s(c), piv, rec; s(c), \tilde{n}(c))$.

A.4 Proof of Theorem 4

As we argued in the main text after the statement of the Theorem, items 2 and 3 follow from a slight modification of Feddersen and Pesendorfer (1997). In the rest of the proof, we will show that there is a sequence of equilibria that aggregates information.

For every k, let s_k be an equilibrium cutoff of the voting game in which there are $2m_k + 1$ voters in both states of the worlds. This is the setup of Feddersen and Pesendorfer, and $s_k \to s^* \in (s_A, s_B)$ since $m_k \to \infty$.

We will now construct a sequence of equilibrium cutoffs, $\tilde{s}_k$, such that information aggregates along such equilibria. Consider the mapping $\tilde{\beta}^k(s,c)$ which is a modification of the mapping $\tilde{\beta}(s,c)$ by incorporating that the minimum number of voters is $2m_k + 1$. Clearly for large enough k, $\tilde{n}_B(s_k, c_k) = m_k$, i.e., the organizer recruits no one in state B. If $\tilde{n}_A(s_k, c_k) = m_k$, i.e., if in state A also the organizer recruits no one, then it is an equilibrium that the organizer recruits no one, and the voters use a cutoff strategy with a cutoff s_k, and let $\tilde{s}_k = s_k$ in this case.

If $\tilde{n}_A(s_k, c_k) > m_k$, then because $(2m+1)\binom{2m}{m} q_A(s_k)^m (1 - q_A(s_k))^m)$ is decreasing in m when m is large and s_k is sufficiently close to s^*, $\max \tilde{\beta}^k(s_k, c_k) < 1$. Moreover, $\tilde{\beta}^k(s_A, c_k) > 1$ when k is sufficiently large. Hence, there is a $\tilde{s}_k \geq s_A$ such that $\tilde{s}_k$ is an equilibrium cutoff of the voting game with the organizer. The proof that when $\tilde{s}_k > s_A$, as $k \to \infty$, in state A the probability that policy a is implemented follows identical reasoning as Lemma 9, so we skip it. Clearly, in state B, the organizer recruits no new voter, and since $m_k \to \infty$, in state B , policy b is implemented with a probability that approaches to 1, and hence information is aggregated in both states.

A.5 Proof of Theorem 5 (Unanimity)

Proof. Let $\rho(s) := \frac{f(s|A)}{f(s|B)}$ We start by noting the identity:

$$\frac{F(s_m|A)}{F(s_m|B)} = (1 + \frac{F(s_m|A) - F(s_m|B)}{F(s_m|B)})$$

The indifference condition of the cutoff signal delivers that:

$$\left(\frac{F(s_m|A)}{F(s_m|B)}\right)^{m-1} = \left(1 + \frac{F(s_m|A) - F(s_m|B)}{F(s_m|B)}\right)^{(m-1)} = \frac{(1-\pi)}{\pi\rho(s_m)}$$

The next calculation follows from the Lemma in the main text which showed that $s_m \to 1$, and hence that $F(s_m|B) \to 1$, and that $F(s_m|A) - F(s_m|B) \to 0$:

$$\lim_{m\to\infty} \left(1 + \frac{(F(s_m|A) - F(s_m|B))}{F(s_m|B))}\right)^{(m-1)} = \frac{(1-\pi)}{\pi\rho(1)}$$
$$= \lim_{m\to\infty} \left(1 + \frac{(F(s_m|A) - F(s_m|B))}{F(s_m|B))}\right)^{m}$$

Note that the probability that a is chosen in state A is given by the expression $p_0^m := (F(s_m|A))^m$. As $m \to \infty$, this calculation yields

$$\lim_{m\to\infty} p_0^m = \lim_{m\to\infty} (1 - (1 - F(s_m|A)))^m$$
$$= \left(\frac{(1-\pi)}{\pi\rho(1)}\right)^{-\frac{f(1|A)}{f(1|A)-f(1|B)}}.$$

The last line follows, because if $\lim_n (1 + x_n)^n = a$ and if $\lim_n \frac{y_n}{x_n} = z$, then $\lim_n (1 + y_n)^n = a^z$. Now if we apply this by defining $x_n = \frac{F(s_m|A)-F(s_m|B)}{F(s_m|B)}$, and $y_n = -(1 - F(s_m|A))$, we get the expression for $\lim p_0^m$. Similarly, the probability that policy b is chosen in state B, $p_1^m = 1 - (F(s_m|B))^m$. As $m \to \infty$, this calculation yields:

$$\lim_{m\to\infty} 1 - p_1^m = \lim_{m\to\infty} \left(1 - \frac{(1-F(s_m|B))m}{m}\right)^m$$
$$= \left(\frac{1-\pi}{\pi\rho(1)}\right)^{-\frac{f(1|B)}{f(1|A)-f(1|B)}}.$$

This completes the first part of the theorem. To show that $\left(\frac{(1-\pi)}{\pi\rho(1)}\right)^{-\frac{f(1|A)}{f(1|A)-f(1|B)}}$ goes to 1 if $\frac{f(1|A)}{f(1|B)}$ goes to 0 is by taking the log of the expression $\left(\frac{(1-\pi)}{\pi\rho(1)}\right)^{-\frac{f(1|A)}{f(1|A)-f(1|B)}}$, and using l'hopital rule to show that this ln expression goes to zero. The other part similarly follows.

$\square$

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