Time to Decide: Information Search and Revelation in Groups*
PRELIMINARY AND INCOMPLETE

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Abstract

We analyze costly information acquisition and information revelation in groups evaluating different decision options in a dynamic setting. Even when group members have perfectly aligned interests the group may inefficiently delay decisions. When deadlines are absent or far, uninformed group members freeride on each others' efforts to acquire information. When deadlines come close, successful group members stop revealing their information in an attempt to incentivize others to continue searching for information. Surprisingly, setting a tighter deadline may increase the expected decision time and increase the expected accuracy of the decision in the unique equilibrium. As long as the deadline is set optimally, welfare is higher when information is only privately observable to the agent who obtained information rather than to the entire group.

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1 Introduction

Many important decisions are made by committees. This paper studies joint decision-making in teams in a world where agents must be motivated to acquire information prior to making a decision. These situations are very common. For example, members of an executive board must gain information about a project's profitability and likelihood of success in order to evaluate which corporate strategy to pursue. In trial juries it is important that jurors pay attention to the evidence in order to make an informed judgement. Parents must investigate the quality of schools before deciding where to send their offspring. Important national policy decisions are made by a council of ministers who must gather information about the possible courses of action, and, of course, faculty members must gather and communicate information about potential candidates when deciding to recruit new colleagues.

While group members search individually for information, information about the alternatives at hand is a public good in the absence of conflicting preferences. The analysis identifies the communication between decision-makers about their findings as a key determinant of their willingness to expend effort to gain information. In their decision-making process committees face two important challenges in choosing the most valuable option, even in the absence of conflicting interests. First, team members must be willing to invest time and effort to search for information. This involves costly decisions such as reading and compiling market forecasts, evaluating judicial evidence or reading school quality assessment reports. The standard model of collective decisions under uncertainty does not speak to this issue because it assumes that decision makers are endowed with (costless) information. Second, team members should share information efficiently and in a timely manner. When some expert members that previously succeeded in finding information about the best possible course of action fail to communicate their information, the committee may unduly delay its decision or make the wrong choices.

These two considerations are often in conflict and lead to a complex trade-off between incentives for private information gathering and intra-committee communication. This trade-off is critical to the understanding of why groups often fail to make decisions in a timely manner. First, in their seminal research on group decision-making Stasser and Titus (1985) and Stasser (1999) show that groups do not share information effectively when members possess private information. When members of a group have different pieces of information, people tend to discuss the information that they all posses in common, and they do not always share or emphasize the information privately held by each group member. The lack of proper information sharing and integration inhibits group problem-solving effectiveness. Management scholars have long stressed that while group decision-making tends to lead to more information and knowledge being available when decisions are made, the decision-making process often takes longer and is costlier. In his popular textbook on management practices Griffin (2006, page 250) notes that "perhaps the biggest drawback from group and decision making is the additional time and hence the greater expense entailed. [...] Assuming the group or team decision is better, the additional expense may be justified." In particular, the desire to maintain the motivation of other committee members to search for information can lead experts to keep mum about their own discoveries. Conversely, the reluctance to share information in a timely manner may undermine the group members' incentives to search for information. To help promote the effectiveness of group and team decision making Griffin (2006) advocates the careful use of deadlines: "Time and cost can be managed by setting a deadline by which the decision must be made final." Our analysis of the complex interaction between incentives for information acquisition and information sharing shows how standard team practices to incentivize group members, like the imposition of deadlines and disclosure rules, while beneficial can also backfire when used incorrectly. While deadlines are expected to increase the cost of freeriding, the resulting increase in search efforts reduces the incentives to reveal information when this information discourages group members to search intensively. This view is in line with Carrison's (2003, page 122) case study analysis of how organizations manage to meet critical time challenges. He argues that "whenever the workplace is charged with the electricity of a race against time, clear communication can suffer." In contrast, disclosure rules which inhibit group members to conceal their information, reduce the incentives to become informed.

We formally analyze these competing forces in a (continuous-time) model where each team member of a group of two can choose to gather information and (unilaterally) call a decision before a finite time horizon T. More information helps the team members to make better decisions, but collecting this information is privately costly. Since the group members equally share the benefits of a more accurate decision, the costly acquisition of information becomes a public good and members of the group will attempt to free-ride on the information acquisition efforts of other members. Thus, one might think that the group will always be underinformed when making its decision. However, this intuition ignores that team members may withhold information from the group when obtained information is not readily observable to all partners. Although team members will divulge all the information they have as soon as a decision is called to ensure that the team takes the best possible decision, they have strong incentives not to communicate what they know before that. The reason for such behavior is that with decreasing returns to information, a team member is less willing to search for information if another team member is already informed. An informed team member would always like to pretend he is not informed to induce other members to gather information and then reveal his information when a decision is called. Of course, in equilibrium team members will anticipate such behavior and accordingly update their belief that their team member is informed. A higher belief that a team member is informed reduces the incentive to search for additional information, since this additional piece will contribute less to the decision. A higher belief also reduces the incentive to delay a decision, as more informed team members search less intensively for information.

In particular, we provide a characterization of the symmetric equilibrium strategies employed by the members of the team. Most notably, the behavior of the players is significantly affected by the length of the time that is still available until the final deadline at time T and thus agent behavior displays a number of interesting characteristics. When T is relatively small, that is to say there is little time between the start of the game and the deadline to become informed, players exert maximum effort to become informed and reveal no information when successful. Players have little time to become informed and thus have strong incentives from the start of the game to exert effort themselves rather than to count on their partners' effort. In response to this high effort choice any informed agent prefers not to disclose her information and delay a decision since she benefits from the potential acquisition of an additional signal by a hard-working uninformed agent. However, as T increases, this equilibrium is no longer sustainable. The belief of uninformed agents that the other team member is informed would be too high and an uninformed agent would no longer be willing to exert such a high level of effort. As a result, uninformed agents choose lower effort levels to compensate for the fact that the other team member is likely to be informed and just delaying the decision. The total effort exerted during the game remains unchanged. As T increases even further, informed agents may no longer prefer to delay their decision since the total cost of delay δT increases as T increases. As a result, when T is large an informed agent will initially prefer to forego any delay costs and instead choose to immediately call a decision upon acquiring a signal. Uninformed agents initially free-ride on each others' efforts to acquire this signal. The unique equilibrium for long games thus has two phases: a first phase of low effort intensity and full information disclosure, a second stage of high effort intensity and no information disclosure.

The unique equilibrium of the game suggest that inefficient delay is due to the lack of information search far from the deadline and the lack of information revelation close to the deadline. As no information is revealed close to the deadline, committees are expected to take decisions early on or to wait until the deadline. Relatedly, our model suggests an explanation for why committees may delay decisions without its members actually looking for more information. That is to say, when deadlines are very strict our model provides a formal characterization of "Parkinson's Law" (Parkinson 1955) which posits that "work fills the time available" as all decisions will be delayed until the final deadline.

We also investigate the optimal length of the deadline and show that there is a unique finite deadline that maximizes agents' ex ante welfare. This optimal deadline is set in such a way that it maximizes beneficial search efforts while minimizing unnecessary decision delays. Although tight deadlines are expected to reduce the decision time, but also the expected decision precision, the opposite may happen in this setting. The reason is that a longer deadline increases the probability that information is still acquired before team members stop revealing information and delay decisions until the deadline. Hence, this increases the probability of an early decision taken with potentially less information in expectation.

Our model also highlights the importance of observability of successful information acquisition. When information is immediately observable to all team members, free-riding on information acquisition will lead to a severe underprovision of search effort, but at least decisions will always be taken immediately once information has been obtained. In contrast, when information is only privately observable team members have stronger incentives to search and may even collectively overinvest in information acquisition in expectation, but decisions will be taken unnecessarily late in an attempt to benefit from another team member's information acquisition. We provide a clean comparison of the polar cases of private and public information. Surprisingly, we show that as long as the agents can optimally set the deadline ex ante, welfare is always higher when information is only privately observable. This is because in the private information case the agents can reap informational rents from successful information acquisition and thus have stronger incentives to invest in effort that outweigh any costs resulting from delayed decisions. However, we also show that when the deadline is set inefficiently short and the returns to additional information are decreasing rapidly, the team members may instead benefit from making information publicly observable.

Finally, we consider a number of extensions of our baseline model such as the use of explicit contracts for the team members as well as third-party information aggregators such as a committee chairperson who can call a decision when both team members are informed and are unnecessarily delaying making a decision until the deadline.

1.1 Related Literature

Our model is related to the large and growing literature on decision-making in groups. Several previous contributions have focused on the distorted incentives to reveal private information in the presence of conflicting preferences (Li, Rosen and Suen 2001; Dessein 2007, Gerardi and Yariv 2007)[1], of reputation or career concerns (Ottaviani and Sorensen 2001; Levy 2007; Visser and Swank 2007) and of different voting rules (Feddersen and Pesendorfer 1998). In our model, preferences are perfectly aligned, conditional on the available information, and individuals strictly prefer to reveal their private information when a decision is taken. Another strand of the literature analyzes how incentives for individual information acquisition in committees can be optimally provided by structuring the decision procedure (Persico 2004) , the size of the committee (Cai 2009), the voting rules (Li 2001, Gerardi and Yariv 2008), or restricting the action space (Szalay 2005). Gershkov and Szentes (2009) are one notable exception in this literature as they also focus on influencing the prior information of the members acquiring information. They find that a social planner who accounts for members' efforts would leave any member as much in the dark as possible. Blanes-I-Vidal and Moeller (2010) also study the impact on team members' incentives from communicating private information, but their focus is on incentives to exert effort on a common project rather. We study the incentives to acquire and reveal information in a dynamic setting. Agents can not only choose whether to disclose information or not, but also when to disclose information. In terms of modeling approach our paper is most closely related to Bonatti and Hörner (2010) who study effort incentives in teams in a continuous-time framework, but they abstract away from the incentive problems related to information sharing that are central to our analysis. Finally, delay and optimal deadlines in group decision-making are also the focus of Damiano, Li and Suen (2009, 2010) who study repeated voting games between team members with differing interests.

The remainder of the paper is organized as follows. In Section 2 we introduce the model in its

[1] Another strand of literature studies how incentives for information acquisition arise if decision-makers have different preferences (Aghion and Tirole 1997) or beliefs (Che and Kartik 2009) and thus take different decisions conditional on holding the same information.

most general form. In order to build intuition for our continuous-time results, we first focus on a simple two-period case in Section 3 which highlights the main driving forces of our analysis. We then introduce the more general continuous-time model and characterize the equilibrium strategies in Section 4. In Section 5, we derive several comparative statics results regarding welfare and expected decision time and decision precision as a fuction of the length of the game. In Section 6, we compare the equilibrium strategies and welfare when the acquired information is private and public. Finally, in Section 7 we discuss a number of extensions. Section 8 concludes.

2 Setup

A team of two agents choose a decision a to match a state of the world θ. Both agents have identical preferences regarding the decision to be taken for a given state of the world, but the state of the world is unknown. The decision utility is given by a quadratic loss function $-(\theta - a)^2$. Agents can exert costly effort e to acquire information about the state of the world to make a more informed decision. However, information acquisition costs are borne privately and an agent's effort is unobservable to the other agent. This results in a standard moral hazard problem within the team; both agents would like to freeride on each others' effort to acquire information. In addition, information about the state of the world is only observable to the agent who acquired it. Hence, agents may choose whether or not to disclose the information they obtained to induce their team partner to acquire more information.

By exerting effort an agent increases the probability of acquiring an additional signal about the state of the world. We call an agent *informed* if she acquired a signal and *uninformed* if she did not obtain a signal before. A better informed decision reduces the expected quadratic utility loss. We assume that the returns to additional information are decreasing. Denoting the *additional* value of the n-th signal by α_n, this implies $\alpha_n \geq \alpha_{n+1}$ for any n. In particular, we assume that each agent starts with an identical normal prior $\theta \sim N\left(0, \frac{1}{\varepsilon}\right)$ with the precision denoted by ε. Agents can acquire additional signals s. Each signal is independent and normally distributed with precision τ, $s \sim N\left(\theta, \frac{1}{\tau}\right)$. The expected loss when taking a decision with n signals simplifies to $\frac{1}{\varepsilon + \tau n}$. Hence, the returns to an additional signal are decreasing and at a faster rate when the signal is more precise. Notice that as the signal becomes infinitely precise, the marginal value of the first and any following signal are given by

$$\lim_{\tau \to \infty} \alpha_1 = \frac{1}{\varepsilon} \text{ and } \lim_{\tau \to \infty} \alpha_n = 0 \text{ for } n \geq 2.$$

By revealing information an agent reduces her partner's incentives to acquire more information as the expected value of the additional information is lower. We assume that agents cannot credibly reveal that they have no information. We also assume that agents may choose whether or not to call a decision. Calling a decision is irreversible so once it is called by either agent a decision must be made. Since the decision preferences are aligned, agents reveal any information they have when a decision is called before deciding on an action. The game ends after an action a is taken. As long as a decision is not called, agents incur an additive delay cost δ.

3 A Simple Model

We first consider a simple model to highlight some of the forces that govern the agents' decisions to exert effort and to disclose information. In this simple two-period model, we assume that each agent starts uninformed with probability ϕ. The first agent moves in the first period, the second agent moves in the second period.

The first agent chooses the probability that a decision is called in the first period. We denote this decision probability by $d(\phi, n) : [0,1] \times \{0,1\} \to [0,1]$ which depends on the probability ϕ that the second agent is uninformed and the number of signals n the agent has obtained. If a decision is called, both agents reveal their information and take the decision $a = E[\theta|\Omega_1]$ given all the information Ω_1 known in the first period. If no decision is called in the first period, both agents bear a cost δ from delaying the decision until the second period. However, delaying the decision to the second period allows the second agent to acquire more information.

The agent who moves in the second period, chooses effort according to the effort function $e\left(\tilde{\phi}, n\right) : [0,1] \times \{0,1\} \to [0, e_{\max}]$ given her updated belief $\tilde{\phi}$ that the other agent is uninformed. When exerting effort at cost ce, this agent obtains a signal with probability λe, where c and λ measure the marginal cost and return to effort. If the first agent has not called a decision in the first period, a decision is called in the second period after the effort choice of the second agent. Both agents reveal their information and take the decision $a = E[\theta|\Omega_2]$ given all the information Ω_2 available in the second period.

Incentives to exert effort When the second agent is uninformed, her marginal gain from exerting additional effort equals

$$\lambda\left[\tilde{\phi}\alpha_1 + (1-\tilde{\phi})\alpha_2\right] - c,$$

where $\tilde{\phi}$ is the second agent's belief that the first agent is uninformed given that she has not called a decision in the first period. The marginal return to effort depends on the increase in probability of obtaining an additional signal and the expected value of that signal. The marginal return is thus higher the more likely it is that the first agent is uninformed as this increases the expected value of an additional signal. If the marginal gain is positive, the uninformed agent will exert maximum effort ($e\left(\tilde{\phi}, 0\right) = e_{\max}$). If the marginal gain is negative, the uninformed agent will exert no effort ($e\left(\tilde{\phi}, 0\right) = 0$). Define $\bar{\phi}_e$ as the belief for which the uninformed agent is indifferent about how much effort to exert,

$$\lambda\left[\bar{\phi}_e\alpha_1 + (1-\bar{\phi}_e)\alpha_2\right] = c.$$

We assume $\lambda\alpha_1 > c > \lambda\alpha_2$. Hence, an agent who is informed or knows that the first agent is informed will exert no effort, i.e., $e\left(\tilde{\phi}, 1\right) = 0$ for any $\tilde{\phi}$. An agent who is uninformed ($n = 0$) and knows that the other agent is uninformed ($\tilde{\phi} = 1$) will exert maximal effort, i.e., $e(1,0) = e_{\max}$.

Incentives to delay a decision When the first agent is informed, her gain from delaying a decision until the second period equals

$$\lambda e\left(\tilde{\phi},0\right)\phi\alpha_2-\delta.$$

By incurring a delay cost δ, the first agent allows the second agent to acquire an additional signal. The first agent anticipates that the second agent will exert effort only if she is uninformed and does not know that the other agent is informed. The gain from delaying a decision is higher the higher the expected effort level exerted by the second agent. This depends on the probability ϕ that the other agent is uninformed and the effort level $e\left(\tilde{\phi},0\right)$ exerted by the agent when uninformed. Define $\bar{e}(\phi)$ as the effort cut-off level for which the informed agent is indifferent between delaying and calling a decision,

$$\lambda\bar{e}(\phi)\phi\alpha_2=\delta.$$

Note that the expected return to delaying a decision in the first period is at least as high for an uninformed agent as it is for an informed agent, since $\alpha_1>\alpha_2$. If the informed agent is indifferent and calls a decision with probability $d(\phi,1)\geq 0$, the uninformed agent strictly prefers to delay a decision $d(\phi,0)=0$. We assume that δ is sufficiently small for an uniformed agent not to call a decision, i.e., $d(\phi,0)=0$ for any $\phi>0$. As a result, the second agent revises his belief that the first agent is uninformed upward such that the posterior is

$$\tilde{\phi}=\frac{\phi}{\phi+(1-\phi)(1-d(1,\phi))}\geq\phi.$$

3.1 Equilibrium

If the cost of delay is too high, the first player never chooses to conceal her information when informed and immediately calls a decision, regardless of the effort an uninformed player would exert in the second period. However, if the cost of delay is sufficiently low, the effort level that the second player is expected to exert determines whether the first player finds it worthwile to delay a decision. The incentives for the second player to exert effort depend on her updated belief $\tilde{\phi}$ that the first player is uninformed. If the incentives for exerting effort are *large*, the unique equilibrium involves uninformed players exerting maximum effort and informed players delaying a decision. If the incentives for exerting effort are *small*, the informed agent will call a decision with positive probability. This increases the incentives of the second agent who now believes that it is more likely that the first agent is uninformed when she did not call a decision. The second agent will exert the lower effort level $\bar{e}(\phi)$ such that the first player is indifferent between call a decision immediately and delaying when informed.

Proposition 1 *For $\delta>\lambda e_{\max}\phi\alpha_2$, an informed first player calls a decision, while an uninformed second player exerts $e(1,0)=e_{\max}$. Otherwise, the unique equilibrium strategies and beliefs are as follows:*

(i) if $\phi \geq \bar{\phi}_e$, an informed first player does not call a decision, $d(\phi, 1) = 0$. An uninformed second player chooses to exert maximum effort $e\left(\tilde{\phi}, 0\right) = e_{\max}$. Her belief remains unchanged $\tilde{\phi} = \phi$.

(ii) if $\phi < \bar{\phi}_e$, an informed first player calls a decision with positive probability, $d(\phi, 1) = \frac{\bar{\phi}_e - \phi}{\bar{\phi}_e(1-\phi)}$. An uninformed second player chooses to exert effort $e\left(\tilde{\phi}, 0\right) = \bar{e}(\phi)$. Her belief increases to $\tilde{\phi} = \bar{\phi}_e$.

Proof. If $\delta > \lambda e_{\max} \phi \alpha_2$, the informed player calls a decision for any $e \in [0, e_{\max}]$. If no decision is called, the second player updates her prior belief to $\tilde{\phi} = 1$. She exerts maximal effort $e(1, 0) = e_{\max}$, since $\lambda \alpha_1 > c$. This equilibrium is unique.
If $\delta \leq \lambda e_{\max} \phi \alpha_2$, the informed player is willing to delay a decision only if $e \geq \bar{e}(\phi)$. The second player exerts maximum effort only if $\tilde{\phi} \geq \bar{\phi}_e$. If the first player never calls a decision, the second player's belief remains unchanged $\tilde{\phi} = \phi$. We distinguish between two cases.
In the first case with $\phi > \bar{\phi}_e$, the equilibrium strategies are as described in (i). The uninformed second player exerts maximum effort as $\tilde{\phi} = \phi > \bar{\phi}_e$. The informed first player delays as $e_{\max} \geq \bar{e}(\phi)$. Moreover, the equilibrium is unique. If the informed player calls a decision with $d(\phi, 1) > 0$, the second player would update his belief to $\tilde{\phi} > \phi\left(\geq \bar{\phi}_e\right)$ and exert maximum effort $e_{\max} \geq \bar{e}(\phi)$. Hence, the informed player is not willing to call a decision. This constitutes a contradiction. The strategy is thus unique except when $\bar{e}(\phi) = e_{\max}$. In this case, the first agent is also willing to call a decision, despite the maximum effort of the second agent.
In the second case with $\phi \leq \bar{\phi}_e$, the equilibrium strategies are as described in (ii). The uninformed second player is willing to exert effort $e\left(\tilde{\phi}, 0\right) = \bar{e}(\phi)$ as his updated belief equals $\tilde{\phi} = \bar{\phi}_e$ when $d(\phi, 1) = \frac{\bar{\phi}_e - \phi}{\bar{\phi}_e(1-\phi)} \geq 0$. The informed first player is willing to call a decision with positive probability as $e\left(\tilde{\phi}, 1\right) = \bar{e}(\phi)$. Also this equilibrium is unique. If the second player would exert a lower effort, the informed first player would call a decision. Hence, the second player would update his belief to $\tilde{\phi} = 1$, and thus be unwilling to exert low effort. If the first player would exert a higher effort level, the informed first player would delay a decision. Hence, the second player would keep the same belief $\tilde{\phi} = \phi < \bar{\phi}_e$, and thus be unwilling to exert high effort. ■

The proposition shows that the private nature of the acquired information is essential. First, a player may use the option to conceal her information and thus delay a decision in equilibrium even though he is informed. Second, under private information the gains from being informed are higher. We formalize these claims in the following two corollaries.

Corollary 1 *No equilibrium exists in which informed players always disclose their information when the cost of delay is small.*

Proof. When $\delta \leq \lambda e_{\max} \phi \alpha_2$, the equilibrium described in the Proposition involves the informed agent delaying the decision and thus not disclosing information with positive probability d. ■

Consider an equilibrium in which informed players always disclose their information immediately. As long as no decision has been called, uninformed players will exert maximum effort when given the last chance to obtain the valuable first signal. As a result, players who are informed, will delay calling a decision as they know with certainty that the other player is still uninformed and thus exerts maximum effort. Hence, an equilibrium with no delay cannot exist. This intuition generalizes for multiple rounds of information acquisition and disclosure.[2]

Second, the option to conceal information affects the efficiency of the equilibrium. The private nature of information leads to inefficient delay, but also increases the value of becoming informed. The reason is that a player is not willing to search for information if she knows that her partner is already informed. Hence, if information is observable, an informed player cannot gain from delaying a decision. If information is unobservable, an informed player can gain from delaying a decision if that induces the second player to acquire more information. However, the option to conceal information makes it more valuable to become informed as one can still induce the other player to acquire information.

Corollary 2 *The first player's gain from being informed is higher if her information is private rather than public.*

Proof. When information is public, the values of being informed and uninformed are

$$\begin{aligned} V^{I,Pub}(\phi) &= -\frac{1}{\varepsilon} + \alpha_1 + (1-\phi)\,\alpha_2 \\ V^{U,Pub}(\phi) &= -\frac{1}{\varepsilon} - \delta + \lambda e_{\max}\phi\alpha_1 + (1-\phi)\,\alpha_1. \end{aligned}$$

When information is private and $\phi > \bar{\phi}_e$, the value of being informed increases to

$$V^{I,Priv}(\phi) = -\frac{1}{\varepsilon} + \alpha_1 + (1-\phi)\,\alpha_2 + \lambda e_{\max}\phi\alpha_2 - \delta,$$

while the value of being uinformed remains unchanged. Hence, $V^{I,Pub}(\phi) - V^{U,Pub}(\phi) \leq V^{I,Priv}(\phi) - V^{U,Priv}(\phi)$ for $\phi > \bar{\phi}_e$, since $\lambda e_{\max}\phi\alpha_2 - \delta \geq 0$. When information is private and $\phi \leq \bar{\phi}_e$, the value of being informed remains unchanged, but the value of being uninformed decreases to

$$V^{U,Priv} = -\frac{1}{\varepsilon} - \delta + \lambda\bar{e}(\phi)\,\phi\alpha_1 + (1-\phi)\,\alpha_1.$$

Hence, $V^{I,Pub}(\phi) - V^{U,Pub}(\phi) \leq V^{I,Priv}(\phi) - V^{U,Priv}(\phi)$ for $\phi \leq \bar{\phi}_e$, since $\lambda\left[e_{\max} - \bar{e}(\phi)\right]\phi\alpha_1 \geq 0$. ■

The corollary implies that private information increases the incentives of the first player to become informed. If the first player could exert effort, this would mitigate the inefficiency due to the moral hazard in teams problem. Note, however, that it may be socially efficient not to search for a second signal. In this case, the private nature of information will lead to overacquisition of information.

[2] Notice that in a model where all players start with the same information as in the next section, no player will ever conceal information up to the point that the other player becomes discouraged to exert effort. Hence, in equilibrium, ϕ never drops below $\bar{\phi}_e$.

4 A Continuous-Time Model

We now consider a continuous-time setup where t denotes the time of the game. The game ends at a (possibly in)finite horizon at time T or before if a decision has been called by an agent. Both agents start the game uninformed.

As long as a decision has not been called, each agent chooses how much effort to exert. Effort is denoted by the function $e(t,n) : [0,T] \times N \to [0, e_{\max}]$ where n is the number of signals the agent has acquired up to time t. The effort function is piecewise continuous over time. An agent's effort level e determines the exponential rate λe at which an additional signal is acquired. The exponential arrival of signals is independent for both players, conditional on their respective efforts. The agent incurs a linear effort cost ce.

At each point in time, an agent chooses to call a decision or not, denoted by $\bar{d}(t,n) : [0,T] \times N \to \{\textit{not call}, \textit{call}\}$. Each agent incurs an additive delay cost δ as long as no decision has been called. When one agent calls a decision at time t, the two agents agree to take the decision $a = E[\theta|\Omega_t]$ given all the information Ω_t known at time t. At that point the quadratic loss $-(\theta - a)^2$ is realized and the game ends. When no decision has been called before the deadline is reached at time T, a decision is called with certainty.

The analysis in this paper focuses on equilibria in which uninformed agents search for information and informed agents call decisions. As in our analysis of the simple model, we assume that $c > \lambda\alpha_2$. Hence, the number of signals acquired by one agent will only ever be 0 or 1 in equilibrium.[3] We therefore drop the second argument of the effort function and write $e(t)$ for the effort strategy of the uninformed agent. We also assume that $[\lambda\alpha_1 - c]\, e_{\max} > \delta$ such that an uninformed agent acting alone would be prepared to search for information in order to make a decision. In Section 4.2 we also show that no symmetric equilibria involve uninformed agents calling a decision in equilibrium unless the equilibrium is one in which both types of agents call a decision with certainty at a point in time, which is in effect a deadline supported by off-equilibrium beliefs. To provide a concise description of the important elements of our model we will proceed by imposing $\bar{d}(t,0) = \textit{not call}$ for all t. We therefore drop the second argument of the decision function and write $d(t)$ for the decision strategy of the informed agent.

At this point it is important to note that while making specific assumptions about the decision-making process our analysis is sufficiently general to incorporate voluntary verifiable disclosure of signals, communication between agents as well as different decision protocols. Notice that the verifiable disclosure of a signal implies that a decision is called immediately, since agents always stop searching when they know that a signal was acquired. Hence, giving the agents the option to verifiably disclose their information would not change the equilibrium. Similarly, allowing them to communicate through cheap talk would not affect our analysis since an informed agent would never want to report that he has obtained a signal. Second, since only informed agents call a decision, any player would agree to call a decision once it is called by her partner. Hence, the assumption

[3] In a more general setup one could assume that $\lambda\alpha_n > c > \lambda\alpha_{n+1}$ such that it is optimal to search for n signals if one's partner is uninformed. Here, for the sake of simplicity and tractability, we have chosen $n = 1$.

that a decision is called unilaterally is without loss of generality. We refer any interested reader to a more general treatment of the model in the appendix.

The probability that an agent does not acquire a signal by time t provided a decision has not yet been called by the other agent, equals

$$\sigma(t) = \exp\left(-\int_0^t \lambda e(s)\, ds\right)$$

We allow for mixing strategies regarding the decision to call at any given instance. From an agent's perspective the probability that the other agent will call a decision by time t may be written as a weakly increasing function of time $\tilde{\rho}(t)$. Since agents may decide not to call a decision after acquiring a signal, an agent holds a belief that her partner is still uninformed when no decision has been called. We denote this belief by

$$\phi(t) = \frac{1-\tilde{\sigma}(t)}{1-\tilde{\rho}(t)}.$$

In Section 4.2 we show that in all symmetric equilibria, subject to the earlier caveat, the equilibrium decision strategy is described by a continuous $\tilde{\rho}(t)$. In the interest of clarity we will restrict our attention to mixing strategies which result in a continuous $\tilde{\rho}(t)$ in the main body of the paper and refer the reader to the appendix for the general specification. We describe an agent's mixed strategy at different points in time by $d(t) : [0,T] \to \{call\} \times [0,\infty)$. If $d(t) = call$ and $\phi(t) = 1$, the hazard rate at which decisions are being made is the rate at which uniformed agents are becoming informed. Hence,

$$\frac{\frac{d\tilde{\rho}(t)}{dt}}{(1-\tilde{\rho}(t))} = \lambda e(t) \text{ if } d(t) = call \text{ and } \phi(t) = 1.$$

Otherwise, for $\phi(t) < 1$, the hazard rate at which decisions are being made is described by $d(t) \in [0,\infty)$ and $\phi(t)$ in the following way,

$$\frac{\frac{d\tilde{\rho}(t)}{dt}}{(1-\tilde{\rho}(t))} = d(t)(1-\phi(t)).$$

Bayesian updating implies that an agent's belief evolves in the following way,

$$\frac{d\phi(t)}{dt} = \begin{cases} 0 \text{ if } d(t) = call \text{ and } \phi(t) = 1, \\ [d(t)(1-\phi(t)) - \lambda e(t)]\phi(t) \text{ otherwise.} \end{cases} \tag{1}$$

Hence, if $d(t)(1-\phi(t)) = \lambda e(t)$ and $\phi(t) < 1$ or $d(t) = call$ and $\phi(t) = 1$ the belief $\phi(t)$ remains constant over time.

Sequential Equilibrium We consider symmetric sequential equilibria of the continuous game with deadline T. The equilibrium strategy is the same for any continuation game starting at t and denoted by $\{e^*(t), d^*(t) | t \in [0,T]\}$. Any off-equilibrium strategy either ends the game or is without consequence for the optimal strategy. The posterior belief $\phi^*(t)$ is formed according to Bayesian updating for a given strategy profile as in (1).

To characterize the equilibrium strategies, we use the continuation value of the game at time t for the informed and uninformed player, denoted by $V^I(t)$ and $V^U(t)$ respectively. The sufficient conditions for $e^*(t), d^*(t)$ and $\phi^*(t)$ to constitute a sequential equilibrium are as follows. For any t, the continuation value for the informed player of the sequential equilibrium equals

$$V^I(t) = \max_{\hat{t}\in[t,T]} -\frac{1}{\varepsilon+\tau} + \int_t^{\hat{t}} [\alpha_2 - \delta(s-t)] \frac{\tilde{\rho}^{*\prime}(s)}{1-\tilde{\rho}^*(t)} ds + \frac{1-\tilde{\rho}^*(\hat{t})}{1-\tilde{\rho}^*(t)} \left\{ \left(1-\phi^*(\hat{t})\right)\alpha_2 - \delta\left[\hat{t}-t\right] \right\}. \tag{2}$$

where $\tilde{\rho}^*(t), \theta^*(t)$ are consistent with $d^*(t)$ and $e^*(t)$ as defined earlier. Defining $\hat{t}(t)$ as the set of maximizers $\hat{t} \subset [t,T]$ of the maximization in (2), the calling decisions satisfy

$$\begin{array}{ll} d^*(t) = call & \text{if } \hat{t}(t) = \{t\}, \\ d^*(t) = 0 & \text{if } \min \hat{t}(t) > t, \\ d^*(t) \in \{call\} \times [0,\infty) & \text{otherwise.} \end{array} \tag{3}$$

For any t, the continuation value for the uninformed player of the sequential equilibrium equals

$$\begin{aligned} V^U(t) = -\frac{1}{\varepsilon} &+ \int_t^T \left[\alpha_1 - \delta(s-t) - c\int_t^s e^*(r)\,dr\right] \frac{\sigma^*(s)\,\tilde{\rho}^{*\prime}(s)}{\sigma^*(t)\,(1-\tilde{\rho}^*(t))} ds \\ &+ \int_t^T \left[\left[V^I(s) - \left(-\frac{1}{\varepsilon}\right)\right] - \delta(s-t) - c\int_t^s e^*(r)\,dr\right] \frac{\sigma'^*(s)\,(1-\tilde{\rho}^*(s))}{\sigma^*(t)\,(1-\tilde{\rho}^*(t))} ds \\ &+ \frac{\sigma^*(T)\,(1-\tilde{\rho}^*(T))}{\sigma^*(t)\,(1-\tilde{\rho}^*(t))} \left\{ (1-\phi^*(T))\,\alpha_1 - \delta(T-t) - c\int_t^T e^*(s)\,ds \right\}. \end{aligned}$$

The effort decisions satisfy

$$e^*(t) \in \arg\max_{e\in[0,1]} \lambda e \left(V^I(t) - V^U(t)\right) - ce. \tag{4}$$

To ensure that uninformed agents do not have a strict incentive to call a decision in the sequential equilibrium, we have

$$V^U(t) \geq -\frac{1}{\varepsilon} + (1-\phi^*(t))\,\alpha_1.$$

Incentives to exert effort An uninformed agent is more willing to exert effort the more valuable it is to become informed. The return to effort does depend on the difference in the continuation values when informed and uninformed as in (4). When an agent is uninformed at time t, her marginal gain from exerting additional effort equals

$$\lambda\left[V^I(t) - V^U(t)\right] - c.$$

Before the deadline, the value of becoming informed also depends on the foregone expected cost of effort and delay when the agent were still uninformed. At the deadline, the value of becoming informed only depends on the gained accuracy of the decision. Clearly, an informed agent can make a more accurate decision, but the value of having acquired a signal is lower when the partner has acquired a signal as well. Evaluated at the deadline T, the marginal gain of effort equals

$$\lambda \left[\phi\left(T\right)\alpha_1 + \left(1-\phi\left(T\right)\right)\alpha_2\right] - c.$$

This mirrors the expression in the simple model of the previous section. At the deadline, an uninformed player is unwilling to exert any effort if the probability that her partner is uninformed $\phi\left(T\right) < \bar{\phi}_e$, where the threshold $\bar{\phi}_e$ is defined by the equation

$$\lambda \left[\bar{\phi}_e\alpha_1 + \left(1-\bar{\phi}_e\right)\alpha_2\right] = c.$$

Incentives to delay a decision An informed agent is willing to delay a decision to give the opportunity to her partner to become informed as well. When deciding how long to delay a decision, the agent trades off the potential increase in the accuracy of the decision if an uninformed partner becomes informed with the expected cost from delaying the decision. The agent takes into account that her partner may or may not call a decision when becoming informed or may be informed already as is clear from the maximization in (2). However, just before the deadline, the incentive to delay a decision is approximately equal to

$$\lambda e\left(T\right)\phi\left(T\right)\alpha_2 - \delta.$$

The return to delaying only depends on the expected increase in accuracy, which happens with the probability that a still uninformed partner acquires a signal. This mirrors the expression in the simple model. We define $\bar{\phi}_d$ as the belief for which an informed agent is unwilling to delay a decision at the deadline when the other agent exerts maximum effort, i.e.,

$$\lambda e_{\max}\bar{\phi}_d\alpha_2 = \delta.$$

4.1 Equilibrium

We now characterize the symmetric equilibrium strategies of the continuous game with deadline T. Equilibrium strategies change as the agents approach the deadline at T, but they also depend on how large T is, that is to say how tight the deadline is set at the start of the game. When the deadline is sufficiently tight (T is sufficiently small), the unique equilibrium involves delay coupled with maximum effort throughout the game. When the deadline is sufficiently loose (T is sufficiently large), any equilibrium involves no delay coupled with low effort at the start of the game and full delay coupled with high effort close to the deadline. We first assume that at the deadline the incentives to search are small relative to the incentives to delay, i.e. $\bar{\phi}_e > \bar{\phi}_d$, implying that uninformed agents would stop exerting effort, before informed agents stop delaying as the equilibrium belief $\phi(T)$ decreases. In this case, there will be at most two different equilibrium regions: no delay coupled with low effort, followed by delay and high effort. We then consider the opposite case, i.e. $\bar{\phi}_e \leq \bar{\phi}_d$, in which case there will be at most three different regions: no delay and low effort, followed by delay and high effort, and finally, mixing delay coupled with maximum effort.

4.1.1 Small Incentives for Search ($\bar{\phi}_e > \bar{\phi}_d$)

The equilibrium strategies are characterized in Proposition 2. There are three distinct cases to consider depending on the length of the game denoted by T. We define the thresholds X_e and Y_e. The first threshold X_e equals the amount of time after which the belief $\phi(t)$ reaches $\bar{\phi}_e$ when an uninformed partner equals maximum effort but discloses no information upon becoming informed,

$$\exp\left(-\lambda e_{\max} X_e\right) = \bar{\phi}_e.$$

This threshold thus determines the maximum amount of time that uninformed players can be induced to exert maximum effort while no informed player is disclosing information. The second threshold Y_e equals the amount of time for which the total delay cost equals the expected value for an informed player of having a partner who is informed with probability $1 - \bar{\phi}_e$,

$$\delta Y_e = \left(1 - \bar{\phi}_e\right) \alpha_2.$$

This threshold thus determines the maximum amount of time for which an informed player is willing to delay a decision if the probability ϕ that her partner is uninformed falls from 1 to $\bar{\phi}_e$. Note that $X_e < Y_e$ as $\bar{\phi}_e > \bar{\phi}_d$.[4]

Proposition 2 *If $\bar{\phi}_e > \bar{\phi}_d$, then the equilibrium strategies and beliefs are as follows:*
i) If $T < X_e$, any informed player chooses not to call a decision, $d(t) = 0$, for all t, while any uninformed player chooses to exert maximum effort, $e(t) = e_{\max}$. The agents' beliefs evolve according to $\phi(t) = \exp(-\lambda e_{\max} t)$.
ii) If $X_e < T < Y_e$, any informed player chooses not to call a decision, $d(t) = 0$, for all t, while any uninformed player chooses $e(t)$ which is not uniquely determined, but the effort choice must satisfy the conditions

$$\exp\left(-\lambda \int_0^t e(s)\, ds\right) \geq 1 - [t - t_0] \frac{\delta}{\alpha_2} \text{ for all } t \in [t_0, T] \tag{5}$$

[4] Since $\bar{\phi}_e > \bar{\phi}_d$, an informed partner is willing to delay locally when $\phi(t) \geq \bar{\phi}_e$ and an uninformed partner exerts $e = e_{\max}$. That is, for $T = X_e$,

$$\delta < \lambda e_{\max} \phi(t) \alpha_2 \text{ for all } t \leq T.$$

Hence,

$$\delta X_e < \int_0^{X_e} \lambda e_{\max} \phi(t)\, dt \alpha_2 = \left(1 - \bar{\phi}_e\right) \alpha_2.$$

and

$$\exp\left(-\lambda\int_{t_0}^{T} e\left(s\right)ds\right)=\bar{\phi}_e, \tag{6}$$

for $t_0 = 0$*. The agents' beliefs evolve according to* $\phi(t) = \exp\left(-\lambda\int_0^t e(s)\,ds\right)$ *for all* t*.*
iii) If $T > Y_e$*, the informed agent calls an immediate decision for* $t < t_e \equiv T - Y_e$ *and no decision* $d(t) = 0$ *for* $t \geq t_e$ *while the uninformed agent chooses* $e(t) = \frac{\delta}{c}$ *for* $t < t_e$ *and chooses* $e(t)$ *for* $t \geq t_e$ *which is not uniquely determined but the effort choice must satisfy the conditions (5) and (6) for* $t_0 = t_e$ *The agents' beliefs evolve according to* $\phi(t) = 1$ *for* $t < t_e$*,* $\phi(t) = \exp\left(-\lambda\int_{t_e}^t e(s)\,ds\right)$ *for* $t \geq t_e$ *and* $\phi(T) = \bar{\phi}_e$*.*

Proof. See appendix. ■

The theoretical results in the preceding proposition have an intuitive interpretation which builds on the features of our simple model of the previous section. The results illustrate the dynamics that emerge from the trade-off between the conflicting objectives of information acquisition and information sharing. Consider the first case where T is relatively small, that is to say there is little time between the start of the game and the deadline to become informed. In particular, the incentives to provide effort originate from the marginal value of information when a decision is taken. Since informed individuals always delay until the deadline, this is entirely determined by the marginal value of an extra signal at the deadline, that is

$$\phi\left(T\right)\alpha_1+\left(1-\phi\left(T\right)\right)\alpha_2.$$

As long as the pursued signal is likely to be the first signal, the incentives for effort are sufficiently high to support maximal effort. Since T is smaller than X_e, the belief ϕ cannot fall below the threshold $\bar{\phi}_e$ and the agent thus chooses to exert maximum effort $e_{\max}$. In response to this high effort choice any informed agent prefers not to call a decision and to delay since she benefits from the potential acquisition of an additional signal by a hard-working uninformed agent. Note that since $e(t) = e_{\max}$ and $d(t) = 0$, any agent correctly believes that as time passes it is more and more likely that the other agent is informed, but that she shies away from calling a decision.

As T increases above X_e, the equilibrium outlined in the previous case is no longer sustainable. As the belief ϕ falls below the threshold $\bar{\phi}_e$, uninformed agents would no longer be willing to exert such a high level of effort. In equilibrium, uninformed agents now choose lower effort levels in a way that ensures that at time T the belief ϕ is exactly at the threshold $\bar{\phi}_e$, i.e., $\exp\left(-\lambda\int_0^T e(s)\,ds\right) = \bar{\phi}_e$. This belief at the deadline makes uninformed agents indifferent with respect to the level of effort they choose throughout the game. By becoming informed at t, an agent avoids the expected cost of exerting additional effort to become informed and the risk of ending up uninformed at the deadline. Hence,

$$V^I\left(t\right)-V^U\left(t\right)=\left\{1-\exp\left[-\lambda e\left(T-t\right)\right]\right\}\frac{c}{\lambda}+\exp\left[-\lambda e\left(T-t\right)\right]\left(V^I\left(T\right)-V^U\left(T\right)\right)$$

The incentives to exert effort "reverberate back" from the incentives to exert effort at the deadline. The marginal net gain (loss) of effort at time t is a share of the marginal net gain (loss) at the deadline at T, which is smaller when t is further away from the deadline at T. To see this, rewrite the previous equation to obtain

$$\begin{aligned} V^{I}(t)-V^{U}(t) &= \frac{c}{\lambda}+\exp\left[-\lambda e(T-t)\right]\left(V^{I}(T)-V^{U}(T)-\frac{c}{\lambda}\right) \\ &= \frac{c}{\lambda}+\exp\left[-\lambda e(T-t)\right]\left[\phi(T)\alpha_{1}+(1-\phi(T))\alpha_{2}-\frac{c}{\lambda}\right]. \end{aligned} \tag{7}$$

Incentives to exert effort exist at t, that is $V^{I}(t)-V^{U}(t)\geq\frac{c}{\lambda}$, provided that they exist at time T, that is $V^{I}(T)-V^{U}(T)\geq\frac{c}{\lambda}$. As a result, when an uninformed agent is indifferent with regards to her effort choice at the deadline $\phi(T)=\bar{\phi}_{e}$, she is also indifferent at any time t before.

The equilibrium path of effort is not unique, but to ensure that informed agents are willing to defer a decision until the deadline at any point during the game, uninformed agents need to backload their effort sufficiently. One possible equilibrium is that uninformed agents choose effort levels $e(t)=0$ for $t<T-X_{e}$ and $e(t)=e_{\max}$ for $t\geq T-X_{e}$. Notice also that as T increases, the aggregate effort exerted by uninformed agents remains the same, but their average effort intensity decreases.

As T increases further, informed agents may no longer prefer to delay their decision. While the aggregate benefit of delaying a decision through the potential information acquisition by the uninformed partner remains constant at $\left(1-\bar{\phi}_{e}\right)\alpha_{2}$, the aggregate cost of delay δT increases as T increases. As a result, when T exceeds Y_{e}, an informed agent will initially, that is as long as $t<t_{e}$, prefer to forego any delay costs and instead choose to immediately call a decision upon acquiring a signal. Hence, in contrast to the two previous cases the incentives for effort are now composed of the incentive to bring forward the time at which a decision is taken, thereby avoiding delay costs, and of the incentive to free ride on the effort of the other agent, thereby avoiding effort costs. In equilibrium, these two effects exactly balance each other when the other agent exerts effort $e_{-i}=\frac{\delta}{c}$. To see this, note that if an agent i shift effort by Δe_{i} to the next instant, this allows her to avoid the expected effort costs $\lambda e_{-i}c\Delta e_{i}$, since the rate at which the other agent acquires information is λe_{-i}. On the other hand, the shift of effort in time increases delay costs δ at the rate $\lambda\Delta e_{i}$, hence the additional delay cost is $\lambda\delta\Delta e_{i}$. These two effects exactly offset one another when

$$\lambda e_{-i}c\Delta e_{i}=\lambda\delta\Delta e_{i}\Leftrightarrow e_{-i}=\frac{\delta}{c}.$$

Hence, an uninformed agent is indifferent with regards to her effort choice. The effort level exerted during this phase of full disclosure is lower than the average effort level in the phase of no disclosure, $\frac{\delta}{c}<\frac{X_{e}}{Y_{e}}e_{\max}$, reflecting that a close deadline overcomes the temptation to free-ride.[5] Note also that when $e_{-i}=\frac{\delta}{c}$, an informed agent is not willing to delay a decision since $\lambda\frac{\delta}{c}\alpha_{2}<\delta$, since $\lambda\alpha_{2}<c$.

[5] This follows as $\lambda\frac{\delta}{c}\alpha_{2}Y_{e}<\delta Y_{e}=\left(1-\bar{\phi}_{e}\right)\alpha_{2}=\int_{0}^{X_{e}}\lambda e_{\max}\phi(t)\,dt\alpha_{2}<\int_{0}^{X_{e}}\lambda e_{\max}dt\alpha_{2}=\lambda e_{\max}X_{e}\alpha_{2}$.

If no decision has been called up to $t_e = T - Y_e$, the equilibrium is identical to case ii) from t_e onwards.

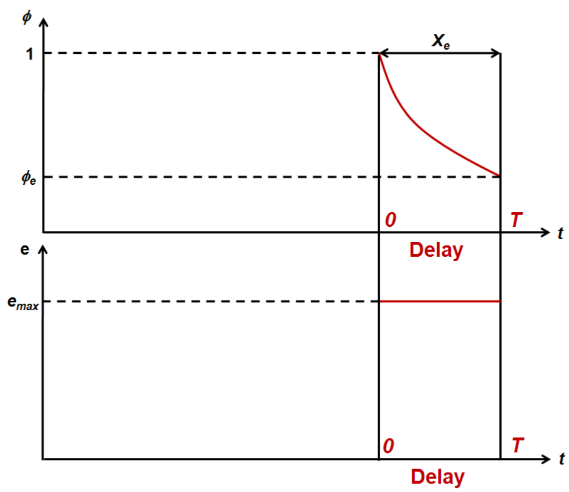

Figure 1.A: ϕ and e for $T < X_e$.

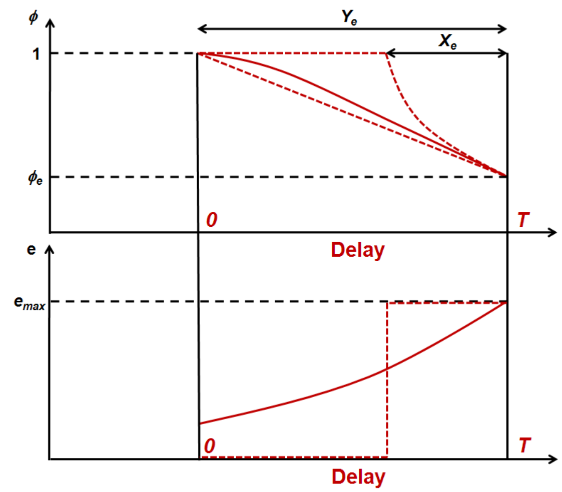

Figure 1.B: ϕ and e for $Y_e > T > X_e$.

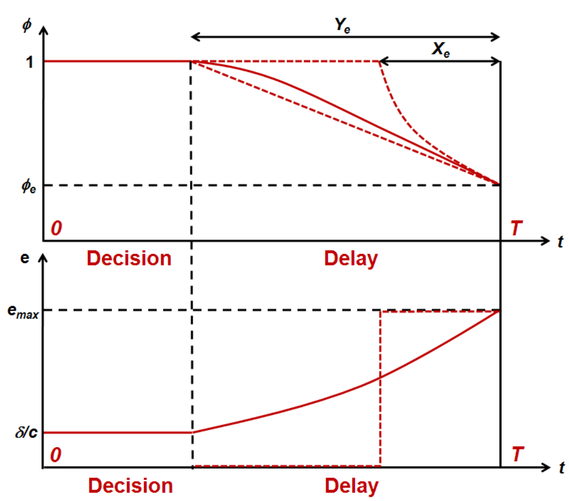

Figure 1.C: ϕ and e for $T > Y_e$.

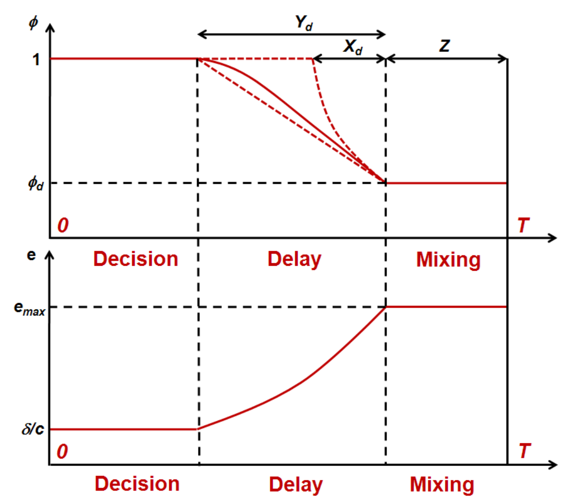

Figure 1.D: ϕ and e for $T > Y_d$ when $\bar{\phi}_e < \bar{\phi}_d$.

The three first panels of Figure 1 illustrate the evolution over time t of the belief ϕ and the equilibrium effort and decision choice for different lengths of the deadline T. In Panel 1.A, the deadline T is relatively tight, that is to say, $T < X_e$. As a result, uninformed agents exert maximum effort $e_{\max}$ over the entire course of the game and informed agents delay making a decision. As a result, the belief ϕ declines from the complete certainty that the other agent is uninformed at $t = 0$ to ϕ_e by the end of the game at T. Next, in Panel 1.B the length of the deadline T is longer, specifically $Y_e > T > X_e$, and hence in response to the delay decision of informed team members, uninformed agents no longer exert maximum effort during the entire game. Instead, they choose to exert lower effort in such a way that the belief ϕ is equal to $\bar{\phi}_e$ at the end of the game. Note that since effort is not fully tied down in equlibrium, there are several ways in which uninformed agents can spread their effort. The solid and the dotted red lines depict two different equilibrium paths for effort e and the evolutions of the belief ϕ that are associated with the different equilibrium effort paths. Finally, in Panel 1.C we depict the equilibrium paths for loose deadlines where $T > Y_e$. As discussed before, at the beginning of the game uninformed agents exert effort $e = \frac{\delta}{c}$ and informed agents call a decision immediately. Thus, during this initial decision phase the belief ϕ remains constant at 1. However, once enough time has elapsed the delay phase begins and the game proceeds as in Panel 1.B.

4.1.2 Large Incentives for Search ($\bar{\phi}_d \geq \bar{\phi}_e$)

We now briefly consider the case where the incentives to exert effort for the uninformed agent exceed the incentives to delay for the informed agent at the deadline. In equilibrium, the belief $\phi(t)$ cannot drop below $\bar{\phi}_d$, since informed player would strictly prefer to call a decision as it is too likely that her partner is already informed. As before, we will proceed by considering deadlines of different length T. There are four distinct cases to consider. We define two thresholds X_d and Y_d, similar to X_e and Y_e, and an additional threshold Z. The threshold X_d solves

$$\exp\left(-\lambda e_{\max} X_d\right) = \bar{\phi}_d.$$

The threshold Y_d solves

$$\left(1 - \bar{\phi}_d\right)\alpha_2 = \delta Y_d.$$

The characterization of the equilibrium is very similar as before, with the exception of a final stage which lasts up to Z for games with length exceeding X_d. Once the length of the game exceeds X_d and uninformed agents have exerted maximum effort until $t = X_d$, informed players will call decisions at a rate $d(t)$ such that $\left(1 - \bar{\phi}_d\right) d(t) = \lambda e_{\max}$ keeping the belief constant at $\bar{\phi}_d$. The threshold Z is the maximum length of this mixing stage which maintains maximum incentives to exert effort throughout the game,

$$Z = \frac{1}{2\lambda} \ln \frac{\left(2\phi^* \alpha_1 + (1 - \phi^*)\alpha_2 - \frac{c+\delta}{\lambda}\right)}{(1 - \phi^*)\alpha_2 + \frac{\delta - c}{\lambda}}.$$

Proposition 3 *If $\bar{\phi}_d \geq \bar{\phi}_e$, then the equilibrium strategies and beliefs are as follows:*
i) If $T < X_d$, any informed player chooses not to call a decision, $d(t) = 0$, for all t while any uninformed player chooses to exert maximum effort, $e(t) = e_{\max}$, for all t. The agents' beliefs evolve according to $\phi(t) = \exp(-\lambda t)$.
ii) If $X_d < T < X_d + Z$, any informed player chooses not to call a decision up for $t < X_d$ and to call for a decision at the mixing rate $d(t) = \frac{\lambda^2 \alpha_2}{\lambda \alpha_2 - \delta}$ for $t \geq X_d$. Any uninformed player chooses to exert maximum effort, $e(t) = e_{\max}$, for all t. The agents' beliefs evolve according to $\phi(t) = \exp(-\lambda t)$ for $t \leq X_d$ and $\phi(t) = \bar{\phi}_d$ for $t > X_d$.

iii) If $X_d + Z < T < Y_d + Z$, any informed agent chooses not to call a decision for $t < t_d \equiv T - Z$, and to call for a decision at the mixing rate $d(t) = \frac{\lambda^2 \alpha_2}{\lambda \alpha_2 - \delta}$ for $t \geq t_d$. Any uninformed player chooses to exert effort $e(t)$ for $0 \leq t < t_d$ which is not uniquely determined, but the effort choice must satisfy the following conditions:

$$\phi(t) = \exp\left(-\lambda \int_0^t e(s)\, ds\right) \leq \frac{\delta}{\alpha_2}[t - t_0] \text{ for } t \in [t_0, t_d] \tag{8}$$

and

$$\phi(t_d) = \exp\left(-\lambda \int_{t_0}^{t_d} e(s)\, ds\right) = \frac{\delta}{\lambda \alpha_2}, \tag{9}$$

for $t_0 = 0$. For $t \geq t_d$ the uninformed agent exerts maximal effort $e(t) = e_{\max}$. The agents' beliefs evolve according to $\phi(t) = \exp\left(-\lambda \int_0^t e(s)\, ds\right)$ for $0 \leq t \leq t_d$ and $\phi(t) = \bar{\phi}_d$ for $t > t_d$.
iv) If $T > Y_d + Z$, any informed player chooses to call for an immediate decision for $t < t_d - Y_d$, not to call a decision, $d(t) = 0$ for $t_d - Y_d \leq t < t_d$, and to call for a decision at the mixing rate $d(t) = \frac{\lambda^2 \alpha_2}{\lambda \alpha_2 - \delta}$ for $t \geq t_d$. Any uninformed agent chooses to exert effort $e(t) = \frac{\delta}{c}$ for $t < t_d - Y_d$, and to exert effort $e(t)$ for $t_d - Y_d \leq t < t_d$ which is not uniquely determined but must satisfy the following conditions (8) and (6) for $t_0 = t_d - Y_d$, and to exert maximal effort $e(t) = e_{\max}$ for $t \geq t_d$.

Proof. See appendix. ■

For $T < X_d$, the equilibrium strategies are exactly like before. For $T \geq X_d$, the marginal value of information at and close to the deadline is strictly greater than $\frac{c}{\lambda}$, unlike in the small incentives case. This also continues to be the case for all longer deadlines. As the length of the game T increases, however, the incentives for effort at a given time decrease. To see this, consider the incentives for effort at $t = 0$ which are given by

$$V^I(0) - V^U(0) = \frac{c}{\lambda} + \bar{\phi}_d \left(V^I(X_d) - V^U(X_d) - \frac{c}{\lambda}\right).$$

The only part of the expression which changes with T, is $V^U(X_d)$ since all the other terms above are constants and

$$V^I(X_d) = V^I(T) = -\frac{1}{\varepsilon + \tau} + \left(1 - \bar{\phi}_d\right)\alpha_2.$$

$V^U(X_d)$ can be rewritten as

$$\begin{aligned} V^U(X_d) \quad &= \quad \left[-\frac{1}{\varepsilon + \tau} + \frac{(1 - \phi^*)\alpha_2}{2} - (ce + \delta)\frac{1}{2\lambda e}\right] \times \\ &\quad \left(1 - e^{-2\lambda(T - X_d)}\right) + e^{-2\lambda(T - X_d)} V^{UN}(T). \end{aligned}$$

This is a weighted sum of the expected payoff conditional on either finding information or the other agent calling a decision prior to the deadline and the payoff from being uninformed at the deadline. Both of these payoffs are independent of T and it is only the relative likelihood of each which is affected by T. The likelihood of being uninformed at the deadline $e^{-2\lambda(T-X_d)}$ decreases in T. Hence, the continuation value of being uninformed at X_d is increasing in T. There exists a deadline $T = X_d + Z$ where $V^I(X_d) - V^U(X_d) = \frac{c}{\lambda}$ and an agent is indifferent about exerting effort at $t = 0$, so $V^I(0) - V^U(0) = \frac{c}{\lambda}$. For T larger than $X_d + Z$, that is case iii) above, maximal effort by uninformed agents can no longer be sustained throughout the entire game. In equilibrium, an uninformed agent reduces her average effort intensity before $t_d = T - Z$ such that $\phi(t_d) = \bar{\phi}_d$, while the informed agent fully delays. For T larger than $Y_d + Z$, informed agents will no longer prefer to delay their decision at the beginning of the game, exactly as in the case with small incentives.

Figure 1.D graphically illustrates the evolution of ϕ and e over the course of the game for the case of large incentives. As discussed before, equilibrium behavior of informed agents is divided into three distinct phases. At the beginning of the game during the decision phase, agents immediately call a decision upon becoming informed. Thus, agents know for sure that their partner must be uninformed whenever no decision has been called in the past. However, once enough time has elapsed for the decision phase to be over, informed agents prefer to delay calling a decision and thus the equlibrium belief ϕ falls until it reaches ϕ_d. At that point, informed agents are indifferent between calling and delaying the decision and thus probabilistically choose one or the other until the conclusion of the game in such a way that ϕ remains constant at ϕ_d. Note again, that the dotted red lines for ϕ illustrate different equilibrium paths associated with different equilibrium paths for e. The evolution of effort in Figure 1.D is similar to the evolution of effort in Figure 1.C. Uninformed agents choose $e = \frac{\delta}{c}$ during the initial decision phase and then start increasing their effort until they exert effort $e_{\max}$ at the end of the game.

4.1.3 Discussion

Our propositions formally establish that even when committee members have perfectly aligned interests decisions may be significantly delayed. This is due to two factors. First, team members do not search very intensely when the deadline is very far away and hence without the adequate information available to the group no decision can be taken by the group. Second, when the deadline is close delay occurs due to a lack of information sharing. Although uninformed agents search intensely for information and hence the group is likely to have valuable information at its disposal, no decision will be taken until the deadline since an informed agent will prefer not to divulge their information in order to keep any uniformed team member highly motivated to search for additional information.

As we showed above the timing of delay crucially depends on how far in the future the final deadline. When the deadline is very close, the team will always delay decisions until the final deadline. This delay is strongly reminiscient of a widely accepted behavioral law called Parkinson's Law. This law, as stated in its original source (Parkinson 1957), posits that "work expands so as to fill the time available for its completion." In our context, this means that the amount of time in which the team has to make a decision is exactly the amount of time it will take to make said decision. In our model this occurs for any deadline length $T < Y_e$ when decisions are only made exactly at the deadline. The relationship between delay, performance and deadlines has been extensively studied both in laboratory and field settings (Locke 1966, 1967; Bryan and Locke 1967; Locke et al. 1981; Schonberger 1981; Latham et al. 1982; Peters et al. 1984; Gutierrez and Guvelis 1991). A loose deadline then leads to a decline of the workers' performance and to a delay of the activity. For example, Brian and Locke (1967) presented individual and groups of college students with a fixed number of simple arithmetic tasks and varied the amount of time allowed to work on them. Their results indicated that subjects who were given twice the amount of time to complete the tasks worked significantly longer than subjects who were given just enough time to complete them. In light of our findings on decision delay, one important implication of our model is that a tight deadline choice may not always lead to faster decision-making. More specifically, while increasing the deadline initially leads to better performance, extending the length of the deadline above X_e will only bring about delay. Team members will only search just as hard in aggregate as if they had a shorter deadline X_e and will incur costly delay to fill the time before a decision is finally made at the deadline. We study the relationship between performance, delay and deadline choice in detail when we investigate the choice of the optimal length of a deadline in Section 5.

As is apparent from the above propositions, a team does not always delay its decisions once it has obtained some information. When the deadline is far away, the group will make a decision as soon as one team member has successfully gathered information. When decisions are taken early on in this manner, the agents effectively avoid the delay costs that result from a lack of information sharing and low information acquisition effort. This is a pattern commonly found in investment committees in private equity partnerships or on executive boards that face a deadline by which funds have to be invested or returned to investors or corporate headquarters. Although we are not aware of any particular study that examines the timing of corporate investment decisions, our model predicts that committees should either take a decision relatively early on in the process or right at the deadline. However, our prediction that the effort exerted by uninformed committee members rises as the deadline draws nearer, chimes well with both anecdotal and empirical evidence.

4.2 Uniqueness

In this subsection we discuss the uniqueness of the equilibrium described in Propositions 2 and 3 among the set of symmetric equilibria. We find that subject to excluding equilibria which involve strategies whereby both uninformed and informed agents call a decision at the same instant of time with probability 1 conditional on reaching that time, the unique set of symmetric equilibria are those described in the proposition. We argue that our exclusion is justified as equilibria where individuals are calling a decision with certainty at a point in time are in effect deadlines which are enforced by appropriately specified off-equilibrium beliefs.

We proceed by first increasing the action space for agents compared to that considered in Section.4.2. We allow uninformed agents to call decisions which we will denote by a function $\mu(t)$. We still refer to the probability that an informed calls a decision by $\rho(t)$. We also allow agents to adopt piecewise continuous decision functions for $\rho(t)$ and $\mu(t)$. To this end define

$$D_{\rho}(t) = \lim_{s \to t^{+}} \rho(s) - \rho(t)$$

and

$$D_{\mu}(t) = \lim_{s \to t^{+}} \mu(s) - \mu(t)$$

to describe the probability mass of decisions at any points of discontinuity.

Otherwise the model is the same as earlier. We give a full specification of the model in the appendix for the interested reader. As before, we denote equilibrium strategies by a superscript *. A symmetric perfect bayesian equilibrium may be described by a tuple $(e^{*}(t), \rho^{*}(t), \mu^{*}(t), \phi^{*}(t))$ if $\rho^{*}(t) + \mu^{*}(t) < 1$ for all $t < T$, where $\phi^{*}(t)$ is the bayesian belief an agent has at time t that the other agent is uninformed conditional on no decision being called prior to that time. If $\exists t' < T : \rho^{*}(t') + \mu^{*}(t') = 1$ then it must also include off-equilibrium strategies and beliefs $(e^{*}(r|t), \rho^{*}(r|t), \mu^{*}(r|t), \phi^{*}(r|t))$ $f$or all times t where $\rho^{*}(t) + \mu^{*}(t) = 1$ which themselves are sequential equilibria of those subgames, where $\phi^{*}(r|t)$ is the bayesian belief an agent has at time r that the other agent is uninformed conditional on no decision being called prior to that time in a subgame starting at time t. We now rule out some types of decision strategies at on-equilibrium times by the uninformed agent. The following lemma rules out a continuously increasing $\mu^{*}(t)$.

Lemma 1 $\nexists \mu^{*}(t), r > 0, \varepsilon > 0 : \frac{d\mu^{*}(t)}{dt} > 0$ *for* $t \in [r - \varepsilon, r]$.

Proof. See appendix. ■

Lemma 1 shows that in the set of symmetric equilibria there is no mixing in the decision strategy by an uninformed player during on-equilibrium times. The following lemma rules out a jump in the decision function $\mu(t)$ at on-equilibrium times if that jump does not occur when both types informed and uninformed call a decision with certainty at that instant.

Lemma 2 $\nexists \mu^{*}(t), 0 < s < T : D_{\mu^{*}}(s) > 0$ *and* $\mu^{*}(s) + \rho^{*}(s) < 1$.

Proof. See appendix. ■

Hence, the only equilibria involving $D_{\mu^{*}}(s) > 0$ also have $\mu^{*}(s) + \rho^{*}(s) = 1$ whereby beliefs at times later than s are off-equilbrium. In this case it may be possible to support uninformed agents calling a decision with appropriately specified off-equilibrium beliefs. However we will exclude this type of equilibrium as we feel for all intents and purposes it is equivalent to imposing a deadline at that time. We thus continue the analysis under the assumption that $\mu^{*}(t) = 0$ for all t. This implies that all $t \leq T$ are reached with some non-zero probability in equilibrium thus there are no off-equilibrium times at which strategies and beliefs must be specified. An individual may find himself at an on-equilibrium time but where his own decision history is inconsistent with equilibrium. All costs are sunk and so the subgame is identical to the on-equilibrium subgame so in these instances the off-equilibrium actions are the on-equilibrium actions at the corresponding time. The following proposition shows that the unique set of symmetric perfect Bayesian equilibria are those specified in Propositions 2 and 3.

Proposition 4 *Suppose $\mu^*(t) = 0$ then the set of equilibria described in Propositions 2 and 3 are the unique sets of symmetric perfect Bayesian equilibria under small and large incentives respectively.*

Proof. See appendix. ■

The above proposition establishes that the information withholding through delay in the lead-up to the deadline which we discussed in the previous subsections, is a characteristic of all symmetric equilibria. It also adds weight to the consideration of the welfare implications of the equilibria, optimal deadlines to maximize welfare and comparative statics with equilibria with observable signals and alternative decision making structures. We take up these questions in the next section.

5 Setting Deadlines

In this section, we analyze the trade-offs committees face when setting deadlines. With a tight deadline a committee risks making a decision without the desirable information. However, such a tight deadline incentivizes the members to work hard to acquire the desirable information in time. With a loose deadline the committee members procrastinate and only begin gathering information in earnest when the deadline is close. The expected decision time and the expected accuracy of the decision jointly determine the expected welfare for the committee members at the start of the process. Altering the deadline changes the expected decision time and the expected accuracy of the decision in different directions. The private nature of information, however, affects this trade-off in two significant ways. First, as a tight deadline increases the incentives to acquire information, it also increases the incentives to conceal information and thus to delay a decision. As a consequence, the expected decision time may actually be larger when a closer deadline is set. Second, when informed team members are concealing their information in the hope that other team members may acquire more information, a tighter deadline will reduce this inefficient delay.

5.1 Decision Time

In this subsection we examine the effect of the deadline on the expected time until a decision is made. The natural intuition is that tighter deadlines lead to shorter decision times. We show that this need not be the case and that instead the expected decision time may be non-monotonic in the length of the deadline. For loose deadlines, $T > Y_e$, the equilibrium is characterized by two phases: a phase of low effort and immediate decisions followed by a phase of pure delay and higher effort. In this case increasing the deadline T decreases the probability that agents reach the later period where decisions are delayed until the deadline. The overall effect of increasing the deadline is ambiguous as the combination of immediate decisions despite slow information acquisition may be a slower or faster process than incurring the fixed delay of Y_e upon reaching the later period. In contrast, for tight deadlines, $T \leq Y_e$, there is no initial period of low effort and immediate decisions, but instead informed partners never disclose their information and the team always delays its decision until the deadline. The expected decision time equals T. Hence, a closer deadline will always reduce the expected decision time. The following proposition formalizes these ideas.

Proposition 5 *For $T \leq Y_e$, the expected decision time is increasing in T. For $T > Y_e$, the expected decision time is decreasing in T if and only if $\frac{\alpha_2}{\alpha_1} > \frac{\bar{\phi}_e}{1-\bar{\phi}_e}$ and increasing otherwise.*

Proof. For $T \leq Y_e$, $Et_c = T$. Hence, $\frac{dEt_c}{dT} > 0$. For $T > Y_e$,

$$\begin{aligned} Et_c &= \int_0^{T-Y_e} t 2\lambda \frac{\delta}{c} \exp\left(-2\lambda \frac{\delta}{c} t\right) dt + \exp\left(-2\lambda \frac{\delta}{c} T_{nd}\right) T \\ &= -\left(T - Y_e\right) \exp\left[-2\lambda e \left(T - Y_e\right)\right] + \frac{1}{2\lambda \frac{\delta}{c}} \left\{1 - \exp\left[-2\lambda \frac{\delta}{c}\left(T - Y_e\right)\right]\right\} + \exp\left[-2\lambda \frac{\delta}{c}\left(T - Y_e\right)\right] T \\ &= \frac{1}{2\lambda \frac{\delta}{c}} \left\{1 - \exp\left[-2\lambda \frac{\delta}{c}\left(T - Y_e\right)\right]\right\} + \exp\left[-2\lambda \frac{\delta}{c}\left(T - Y_e\right)\right] Y_e \end{aligned}$$

Hence,

$$\begin{aligned} \frac{dEt_c}{dT} &= 2\lambda \frac{\delta}{c} \exp\left[-2\lambda \frac{\delta}{c}\left(T - Y_e\right)\right] \left(\frac{1}{2\lambda \frac{\delta}{c}} - Y_e\right) \\ &= 2\lambda \frac{\delta}{c} \exp\left(-2\lambda \frac{\delta}{c} T_{nd}\right) \left[\frac{c}{2\lambda\delta} - \frac{\left(1 - \bar{\phi}_e\right)\alpha_2}{\delta}\right] \\ &= 2\lambda \frac{\delta}{c} \exp\left(-2\lambda \frac{\delta}{c} T_{nd}\right) \frac{1}{\delta} \left[\frac{c}{2\lambda} - \frac{\left(\alpha_1 - \frac{c}{\lambda}\right)\alpha_2}{\alpha_1 - \alpha_2}\right]. \end{aligned}$$

It follows that $\frac{dEt_c}{dT} < 0$ if and only if

$$\frac{c}{2\lambda} < \frac{\left(\alpha_1 - \frac{c}{\lambda}\right)\alpha_2}{\alpha_1 - \alpha_2}$$

$\Leftrightarrow$

$$\alpha_1 - \frac{c}{\lambda} > \frac{\alpha_1}{\alpha_2}\left(\frac{c}{\lambda} - \alpha_2\right)$$

Since, $\alpha_1 > \frac{c}{\lambda} > \alpha_2$ by definition, both sides of the inequality are positive and the relationship is satisfied for $\frac{c}{\lambda}$ close to α_2 and violated for $\frac{c}{\lambda}$ close to α_1. Finally, note that

$$\bar{\phi}_e \alpha_1 + \left(1 - \bar{\phi}_e\right)\alpha_2 = \frac{c}{\lambda}$$

and hence $\frac{dEt_c}{dT} < 0$ if and only if

$$\frac{\alpha_2}{\alpha_1} > \frac{\bar{\phi}_e}{1 - \bar{\phi}_e}.$$

■

The intuition for the case of tight deadlines where $T \leq Y_e$ is straightforward. Closer deadlines always reduce delay by shortening the amount of time during which the agents would only wait until the deadline to make a decision. In contrast, for loose deadlines, $T > Y_e$, shortening the deadline can reduce or increase the expected decision time. To see this, note that the expected decision time equals

$$Et_c = \int_0^{T-Y_e} t_c f(t_c)\, dt_c + [1 - F(T - Y_e)]\, T,$$

where $f(t_c) = 2\lambda\frac{\delta}{c}\exp\left(-2\lambda\frac{\delta}{c}t_c\right)$ is the probability that a signal is acquired at time t_c, when the two agents are exerting the low equilibrium effort level $e^*(t_c) = \frac{\delta}{c}$. The derivative of the expected decision time with respect to the deadline is then given by

$$\frac{dEt_c}{dT} = f(T - Y_e)\left(\frac{1}{2\lambda\frac{\delta}{c}} - Y_e\right).$$

Hence, the expected decision time Et_c is decreasing in the length of the deadline T, $\frac{dEt_c}{dT} < 0$, if $\frac{1}{2\lambda\frac{\delta}{c}} < Y_e$. In other words, if the fixed delay Y_e that agents are willing to incur once the deadline begins to affect their behavior, is greater than the expected decision time without a deadline $\frac{1}{2\lambda e^*(T)}$ then lenghtening the deadline will have adverse effects on the expected decision time. The necessary and sufficient condition for this to be the case is $\frac{c}{\lambda} - \alpha_2 < \frac{\alpha_2}{\alpha_1}\left(\alpha_1 - \frac{c}{\lambda}\right)$. This particular relationship is satisfied when the difference between the marginal benefit of a second signal α_2 and the adjusted marginal cost of effort $\frac{c}{\lambda}$ is small relative to the difference between the marginal benefit of the first signal α_1 and the adjusted marginal cost of effort. The intuition for this result is that a large difference between α_1 and $\frac{c}{\lambda}$ provides uninformed workers with strong incentives to become informed. As a result, they are willing to exert maximum effort for a long period of time (Y_e is large) during which no decision will be taken by the team. If this delay phase is sufficiently large, increasing the deadline and avoiding the delay phase altogether can actually lead to a faster decision on average. Conversely, when $\alpha_1 - \frac{c}{\lambda}$, and consequently Y_e, is not very large, increasing the deadline will only lead to more delay until a decision is made.

5.2 Decision Precision

In addition to influencing how long it will take a group to make a decision, the choice of deadline also affects the expected precision that is available to the agents when a decision is made. When more signals are acquired, the agents have a more precise posterior distribution and thus incur a lower expected loss when making a decision. One would expect that a longer deadline would always allow for more information to be accumulated by the agents and hence lead to a more precise decision-making process. However, as we will show below, this intuition is only partly correct. It misses an important feature of our analysis, namely that agents may choose to call a decision before the deadline.

Proposition 6 *The expected precision is increasing in T for $T \leq X_e$ and it is constant for $X_e <$*

$T \leq Y_e$. For $T > Y_e$, it is decreasing in T if and only if $\frac{\alpha_2}{\alpha_1} > \left(\frac{\phi_e}{1-\phi_e}\right)^2$ and increasing otherwise.

Proof. If $T \leq X_e$, agents only call a decision at the deadline and uninformed agents are exerting maximum effort until the deadline. Hence, the expected number of acquired signals is strictly increasing in T. If $X_e < T \leq Y_e$, agents still only call a decision at the deadline and uninformed agents are exerting the same aggregate amount of effort until the deadline. Thus, the expected number of acquired signals is the same for any T in this range. Finally, if $T > Y_e$ agents may call a decision before the deadline. They exert effort $\frac{\delta}{c}$ and immediately call a decision when informed. Thus, for large T the expected number of acquired signals when a decision is made is approximately equal to 1 and the expected utility loss when a decision is called is equal to $-\frac{1}{\varepsilon+\tau}$. In contrast for $X_e < T \leq Y_e$ the expected utility loss when a decision is called is equal to

$$(1-\phi_e)^2\left(-\frac{1}{\varepsilon+2\tau}\right)+2(1-\phi_e)\,\phi_e\left(-\frac{1}{\varepsilon+\tau}\right)+\phi_e^2\left(-\frac{1}{\varepsilon}\right)$$

Thus, whether the expected precision is increasing (or decreasing) in T for $T > Y_e$ depends on whether the following condition holds

$$-\frac{1}{\varepsilon+\tau} < (1-\phi_e)^2\left(-\frac{1}{\varepsilon+2\tau}\right)+2(1-\phi_e)\,\phi_e\left(-\frac{1}{\varepsilon+\tau}\right)+\phi_e^2\left(-\frac{1}{\varepsilon}\right) < -\frac{1}{\varepsilon+\tau}$$

$\Leftrightarrow$

$$\frac{\alpha_2}{\alpha_1} < \left(\frac{\phi_e}{1-\phi_e}\right)^2$$

Let $\alpha_1 = \frac{2c}{\lambda}$ and $\alpha_2 = \frac{c}{2\lambda}$, hence $\frac{\alpha_2}{\alpha_1} = \frac{1}{4}$ and

$$\phi_e = \frac{\frac{c}{\lambda}-\alpha_2}{\alpha_1-\alpha_2} = \frac{1}{3}$$

and so

$$\frac{\phi_e^2}{(1-\phi_e)^2} = \frac{1}{4}$$

Finally, note that $\phi_d = \frac{\delta}{\lambda e \alpha_2} = \frac{\delta}{ec}$ and hence let δ be small such that $\phi_d < \phi_e$ is satisfied. Thus, for slightly smaller or larger values of α_1 the expected value of information when a decision is made may be increasing or decreasing for $T > Y_e$. ■

The previous proposition showed that the length of the deadline T has unambiguously positive effects on the expected precision of the eventual decision made by the agents. However, as T increases agents may call a decision before the deadline is reached because they prefer to forego the delay that comes with waiting in the hope that the other agent may find a signal. As T grows large it becomes more likely that a decision is immediately called by an informed agent who can rely only on one piece of information. The key comparison is therefore whether the expected precision for $T \leq Y_e$ is greater or smaller than the expected precision of a single signal. For $T \leq Y_e$, both agents may be informed, either one of the agents is informed or neither of the agents is informed. The first and the third case are absent when T is very large (and greater that Y_e). Thus, when α_2 is small compared to α_1 the second signal is of relatively little informational value and for $T \leq Y_e$ the agents run the risk of ending up with no signal at all. Thus, the expected precision is increasing in T. Conversely, when the second signal is quite valuable, the expected precision is higher for $T \leq Y_e$ and hence the expected precision is decreasing in T for $T > Y_e$.

Interestingly, when $\frac{\alpha_2}{\alpha_1} > \frac{\phi_e^2}{(1-\phi_e)^2}$ and $X_e < T \leq Y_e$ the expected precision of the decision can be inefficiently high. That is to say, agents may acquire too many signals than is efficient even from an ex-ante point of view. In particular, if it is socially efficient for both agents to acquire only a single signal, the agents will overacquire information in expectation. While it is not too surprising that agents may end up acquiring too much information ex-post given our assumption of private information acquisition, it is quite surprising that despite the free-riding problem with respect to effort provision that the agents face even ex-ante overacquisition of information can occur in our model. As we shall show in the next section, ex-ante overacquisition of information may occur even when the deadline T is set optimally.

5.3 Welfare

In the previous subsections we analyzed how the length of the deadline may increase or decrease the expected decision time and the expected precision of the decision taken by the agents. In this subsection we turn our attention to the effect of the deadline on the welfare of the agents. We characterize the welfare of each agent as a function of the deadline and find that there exists a finite and unique welfare maximizing deadline for the group of agents. In the case of small incentives the optimal deadline is set such that agents always delay their decision until the deadline, but still engage in welfare-improving information acquisition. The following proposition formally characterizes the effect of the deadline on welfare.

Proposition 7 *The expected utility of the game is maximized for $T^* = X_e$. The expected utility is strictly increasing in the length of the deadline T for $0 \leq T \leq X_e$, stricly decreasing in T for $X_e < T \leq Y_e$ and independent of T for $T \geq Y_e$.*

Proof. *Notice that the expected utility at the start $t = 0$ of the game with a deadline at T equals*

$$V_T^U(0) = \begin{cases} -\frac{1}{\varepsilon+\tau} - \frac{c}{\lambda} \text{ for } T \geq Y_e \\ -\frac{1}{\varepsilon+\tau} + \left(1-\bar{\phi}_e\right)\alpha_2 - \delta T - \frac{c}{\lambda} \text{ for } Y_e \geq T > X_e \\ -\frac{1}{\varepsilon+\tau} + \alpha_1\left(1-2\phi(T)\right) - (\alpha_1-\alpha_2)\left(1-\phi(T)\right)^2 + \frac{c}{\lambda}\phi(T) - \delta T - \frac{c}{\lambda} \text{ for } 0 \leq T \leq X_e \\ \text{where } \phi(T) = \exp(-\lambda e_{\max} T) \text{ and } \bar{\phi}_e = \exp(-\lambda e_{\max} X_e) \end{cases}$$

The change in welfare when changing T for $X_e < T \leq Y_e$ is immediate since the aggregate effort and hence expected precision of the decision are independent of the deadline, but the delay increases with the length of the deadline. For $T \geq Y_e$, welfare is $\frac{1}{\varepsilon+\tau} - \frac{c}{\lambda}$ which is independent of T hence *the result is immediate. Now, for* $0 \leq T \leq X_e$ *consider* $\frac{dV_T^U(0)}{dT}$:

$$\begin{aligned}\frac{dV_T^U(0)}{dT} &= -2\alpha_1\phi'(T) + 2(\alpha_1-\alpha_2)(1-\phi(T))\phi'(T) + \frac{c}{\lambda}\phi'(T) - \delta \\ &= \left[2(\alpha_1-\alpha_2)(1-\phi(T)) - 2\alpha_1 + \frac{c}{\lambda}\right]\phi'(T) - \delta \\ &= 2\lambda e_{\max}\left[\alpha_1\phi(T) + \alpha_2(1-\phi(T)) - \frac{c}{2\lambda}\right]\exp(-\lambda e_{\max}T) - \delta\end{aligned}$$

At $T = X_e$*, we have* $\phi(T) = \bar{\phi}_e$*. Hence,*

$$\begin{aligned}\frac{dV_T^U(0)}{dT} &= 2\lambda e_{\max}\left(\frac{c}{2\lambda}\right)\bar{\phi}_e - \delta \\ &= e_{\max}c\bar{\phi}_e - \delta\end{aligned}$$

In the case of small incentives, we have $\bar{\phi}_e > \bar{\phi}_d$ *and*

$$e_{\max}\bar{\phi}_d\alpha_2 > \frac{\delta}{\lambda} > \frac{\delta}{c}\alpha_2.$$

Hence, we have

$$e_{\max}c\bar{\phi}_e > \delta$$

which in turn implies

$$\frac{dV_T^U(0)}{dT} > 0.$$

Note further that

$$\frac{\partial}{\partial T}\left(\frac{dV_T^U(0)}{dT}\right) = 2\lambda e_{\max} \left\{-\lambda e_{\max}\phi(T)\left[\alpha_1\phi(T) + \alpha_2(1-\phi(T)) - \frac{c}{2\lambda}\right] - (\lambda e_{\max}\phi(T))^2(\alpha_1-\alpha_2)\right\} < 0$$

■

As the proposition shows, there is a unique finite welfare-maximizing deadline which is to set $T = X_e$. The intuition for this result is as follows. For very tight deadlines, $T \leq X^e$, agents exert maximum effort and make a decision only when the deadline arrives. Thus, increasing the deadline improves welfare because even though it increases the time until a decision is made, it also allows the agents more time to intensely search for and acquire valuable information prior to the deadline. However, once T is larger than X_e the aggregate effort exerted and the expected information acquired prior to the deadline is the same for all $X_e < T \leq Y_e$. That is to say, over the course of the game the team members simply choose their effort in such a way that they are equally well-informed at the deadline, no matter whether the deadline occurs at X_e or Y_e or at anytime in between. Consequently, any increase in the deadline over and above X_e only introduces additional costly delay before a decision is made at the deadline. Finally, for loose deadlines, $T \geq Y_e$, the welfare of the agents is independent of the deadline. The reason for this is that both agents are indifferent with respect to the effort level they exert during the delay phase of the equilibrium, that is,

$$\lambda \left[V_T^I (0) - V_T^U (0) \right] = c \text{ for any } T > Y^e.$$

To see this note that since a decision is immediately called when one agent is informed, the value of being informed, $V_T^I(0) = -\frac{1}{\varepsilon + \tau}$, does not depend on the deadline in this stage. Furthermore, the value of being uninformed, $V_T^U(0) = V_T^I(0) - \frac{c}{\lambda}$, does not depend on the deadline either. In fact, the effort level chosen during the phase of no delay is such that the lost opportunity to acquire a signal for free, $\lambda e^*(0) \left[V_T^I(0) - V_T^U(0) \right]$, is exactly offset by the foregone delay δ. Thus, the welfare-maximizing deadline is to set $T = X_e$. Extending the deadline any further does not affect the expected amount of information acquired by the deadline and only introduces costly delay.

Figure 3 graphically illustrates how expected welfare at the start of the game varies with the choice of the deadline T. For $T < X_e$ welfare is increasing in T since agents exert maximum effort and are given more time to acquire valuable information. However, once T is extended beyond X_e welfare declines since aggregate effort remains constant and only unnecessary delay costs are incurred. Welfare is constant for $T > Y_e$.

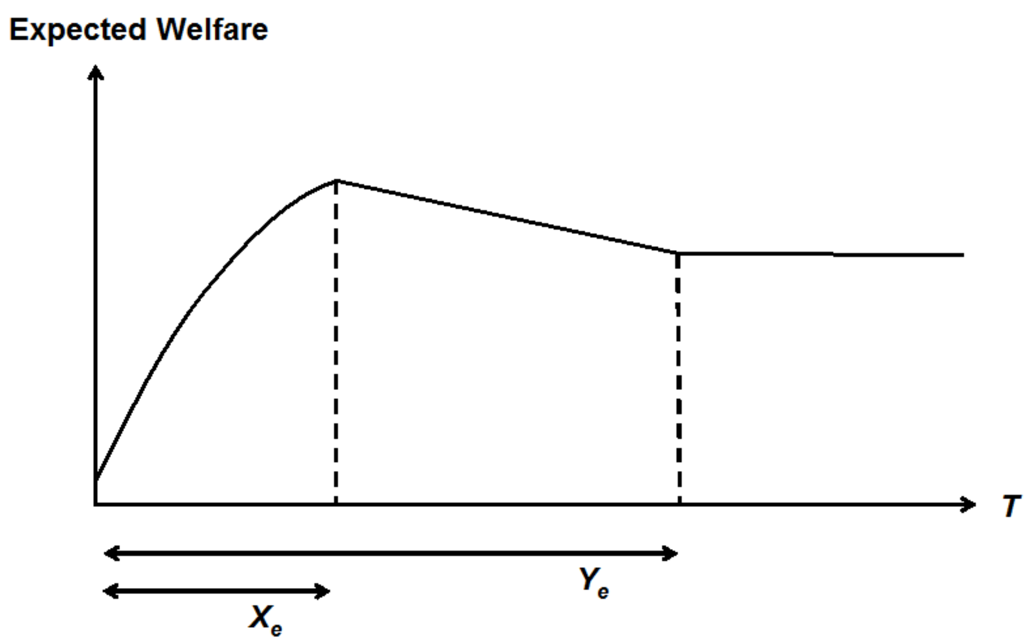

Figure 2: Expected Welfare at $t = 0$ for different lengths of the deadline T

The optimal deadline thus has a length X_e and is affected by changes in the underlying model parameters. Remember that the threshold X_e solves

$$\exp\left(-\lambda e_{\max} X_e\right) = \bar{\phi}_e = \frac{\frac{c}{\lambda} - \alpha_2}{\alpha_1 - \alpha_2}.$$

Corollary 3 *The optimal length of the deadline $T^* = X_e$ is decreasing in $e_{\max}$, c and increasing in α_1, α_2 and ambiguous with respect to λ.*

The interpretation for many of these comparative statics is intuitive. When the maximal effort $e_{\max}$ increases, agents exert a higher level of effort as the deadline approaches. Thus, the belief ϕ falls faster towards $\bar{\phi}_e$ and hence maximal effort can only be sustained for a shorter period of time. In other words, with a higher $e_{\max}$ it takes less time to reach the critical level of aggregate effort above which agents begin to shade effort and thus the optimal deadline is shorter. An increase in the marginal benefit of effort for finding a signal λ has a similar effect on the optimal deadline in that it reduces the time needed for ϕ to reach $\bar{\phi}_e$ when maximum effort is exerted. However, an increase in λ also has a second countervailing effect because it reduces $\bar{\phi}_e$ by making it more beneficial to exert effort. The aggregate effect on the optimal deadline of a change in λ is therefore ambiguous. In contrast, an increase in the marginal cost c unambiguously shortens the optimal deadline by increasing $\bar{\phi}_e$. When it is more costly to exert effort, an uninformed agent will be indifferent between exerting and not exerting effort even when it is less likely that the other agent is informed and hence maximal effort can be sustained for a shorter period of time. Conversely, increases in the value of the first α_1 and the second signal α_2 both raise the marginal benefit of effort and thus decrease $\bar{\phi}_e$ with the eventual effect of lengthening the optimal deadline. Finally, note that the optimal deadline is independent of δ. This independence result is due to our initial assumption that δ is sufficiently small for informed agents to be willing to delay. The maximum level of effort exerted by uninformed agents is always sufficient to outweigh any costs resulting from delay.

For the large incentives case, $\bar{\phi}_d \geq \bar{\phi}_e$, most of the results generalize immediately with one notable difference. There are now large incentives for effort such that uninformed agents will exert maximum effort even when informed agents are mixing between calling and not calling a decision. There is still a unique finite deadline T^* that maximizes the agents' welfare, but it is $T^* = X_d + Z$. As in the case of small incentives it is again optimal to set the deadline exactly at the greatest length of time during which uninformed agents will exert maximum effort at any point in time.

In this section we have characterized the welfare of agents as a function of the deadline and shown that there is a unique finite deadline which maximizes agents' welfare. In the next section we contrast this result with a setting where the information acquired by an agent is observable to the other agent. This allows us to highlight the importance of the private nature of information acquisition in the above result.

6 Private Information: Incentives vs. Delay

In this section, we analyze how the private nature of acquired information affects welfare. When information is public, the acquisition of information discourages all partners from searching and thus leads to an immediate decision. When information is private, informed players can hide their information and delay calling a decision. The option to keep information private increases the returns to becoming informed relative to the case when information is public, which in turn increases a partner's incentives to search. We show that when the acquired information is private and the deadline is set optimally - trading off search incentives and inefficient delay - the expected welfare at the start of the game is higher than when the acquired information is public.

6.1 Public Information

We first characterize the equilibrium of the game when the information obtained by individual group members is immediately visible to the entire group. The equilibrium behavior of agents turns out to be much less complex than when information is private. First, whenever an agent succeeds in finding a signal, a decision is called immediately. The reason is simply that no player would find it privately optimal to search for a second signal. Second, both agents choose low effort when the deadline is still far away and they switch to exerting maximal effort when the deadline is close. We formally characterize equilibrium behavior in the following proposition.

Proposition 8 *When information is public, the equilibrium strategies are as follows:*
i) If $T < \Delta$, any uninformed player chooses to exert maximum effort, $e(t) = e_{\max}$, for all t.
ii) If $T \geq \Delta$, any informed player chooses to exert $e(t) = \frac{\delta}{c}$ for $t < T - \Delta$ and chooses $e(t) = e_{\max}$ for $t \geq T - \Delta$, where

$$\exp\left(-2\lambda e_{\max}\Delta\right) = \frac{c - \frac{\delta}{e_{\max}}}{2\lambda\alpha_1 - \left(c + \frac{\delta}{e_{\max}}\right)}.$$

Proof. *Since $\lambda\alpha_2 - c < 0$, no player would find it privately optimal to search for a second signal and hence the game ends as soon as one player becomes informed. At time t, the value of being informed equals*

$$V^I(t) = -\frac{1}{\varepsilon + \tau}$$

The value of being uninformed at t, when all agents exert maximum effort until the deadline, equals

$$V^U(t) = -\frac{1}{\varepsilon} + \left[\alpha_1 - \left(\frac{c}{2\lambda} + \frac{\delta}{2\lambda e_{\max}}\right)\right]\left\{1 - \exp\left[-2\lambda e_{\max}(T - t)\right]\right\}.$$

Since $\alpha_1 > \left(\frac{c}{2\lambda} + \frac{\delta}{2\lambda e_{\max}}\right)$, this value is decreasing over time, $\frac{dV^U(t)}{dt} < 0$. Agents thus are exert maximum effort for any $t > T - \Delta$, where Δ is defined by

$$V^I(T - \Delta) - V^U(T - \Delta) = \frac{c}{\lambda}.$$

At any time t before this threshold $T - \Delta$, each agents exerts $e(t) = \frac{\delta}{c}$, which makes every agent indifferent about how much effort to exert. This equilibrium strategy corresponds to the first stage in the private information case for long deadlines. ∎

The public nature of information affects the agents' incentives to search for information. The value of being informed is constant throughout the course of the game, since agents can be sure that a decision is called as soon as one signal is obtained. For tight deadlines, $T < \Delta$, the value of being uninformed decreases over time because the likelihood of having to take an uninformed decision at the deadline increases. As a result, the incentive to exert effort increases as the deadline approaches. When the time left until the deadline is exactly Δ, a player is indifferent about how much effort to exert if his partner exerts maximum effort until the deadline. Now, consider increasing the deadline T above the threshold Δ. At any time t before $T - \Delta$ is reached, a player is indifferent about how much effort to exert only if his partner exerts $e(t) = \frac{\delta}{c}$ and the value of being uninformed is independent of the remaining time until the deadline. Not surprisingly, this corresponds to the initial phase of low effort and full disclosure for long deadlines in the case of private information.

6.2 Welfare Comparison

As long as the deadline is set sufficiently far away at the beginning of the game, welfare does not depend on the deadline, nor does it depend on whether the acquired information is public or private. However, in contrast to the private information case, a decrease in the deadline can only decrease the expected welfare in the public information case. This has the important implication that a higher welfare level can be attained by making the acquired information private. In particular, for intermediate deadlines the welfare with private information strictly exceeds the welfare with public information.

Proposition 9 *The highest welfare achieved by optimally setting the deadline when information is private exceeds the highest welfare when information is public.*

Proof. *For the case of public information the continuation values at the beginning of the game are given by*

$$V_T^{U,pub}(0) = \begin{cases} -\frac{1}{\varepsilon+\tau} - \frac{c}{\lambda} \text{ for } T \geq \Delta \\ -\frac{1}{\varepsilon} + \left[\alpha_1 - \left(\frac{c}{2\lambda} + \frac{\delta}{2\lambda e}\right)\right]\left[1 - \exp\left(-2\lambda e_{\max} T\right)\right] \text{ for } T < \Delta \end{cases}$$

Together with our previous welfare results this implies the following relations for any $T' > \max\{Y_e, \Delta\}$*:*

$$\max_T V_T^{U,priv}(0) > V_{T'}^{U,priv}(0) = V_{T'}^{U,pub}(0) = \max_T V_T^{U,pub}(0)$$

Hence,

$$\max_T V_T^{U,priv}(0) > \max_T V_T^{U,pub}(0).$$

■

The incentives to acquire information are different when information is public so that players do not have the option to conceal information after becoming informed. A priori, it may seem that the change in the observability of information can increase or decrease the search incentives. The private nature of information provides more incentives for uninformed players by allowing informed players to 'rest on their laurels'. However, the fact that other players may be already informed, decreases the value of additional information and thus the incentives to search. The first effect dominates the second effect in equilibrium. When information is private, maximum incentives for search can be sustained throughout for games with longer deadlines than when information is public, i.e., $X_e > \Delta$. This also implies that for games with long deadlines, the initial stage of low effort $e(t) = \frac{\delta}{c}$ lasts longer when information is public, since $Y_e > X_e$.[6]

Corollary 4 *Incentives for maximal search can be sustained for longer when information is private than when it is public.*

Proof. *For $T > X_e$, an equilibrium with private information exists in which $e(t) = 0$ for $t < T - X_e$ and $e(t) = e_{\max}$ for $t \geq T - X_e$. In the unique equilibrium with public information $e(t) = e_{\max}$ iff $t \geq T - \Delta$. We show that $X_e > \Delta$ by contradiction. We derive a necessary condition for $X_e < \Delta$ which is also sufficient for $X_e > \Delta$. By definition, X_e is the deadline T solving*

$$\begin{aligned} V_T^{I,priv}(0) - V_T^{U,priv}(0) &= \tfrac{c}{\lambda} \\ \Leftrightarrow \quad \alpha_1 \exp(-\lambda e_{\max} T) + \alpha_2 [1 - \exp(-\lambda e_{\max} T)] &= \tfrac{c}{\lambda} \\ \Leftrightarrow \quad \alpha_1 \phi(T) + \alpha_2 (1 - \phi(T)) &= \tfrac{c}{\lambda}, \end{aligned}$$

where $\phi(T) = \exp(-\lambda e_{\max} T)$. Also by definition, Δ is the deadline T solving

$$\begin{aligned} V_T^{I,pub}(0) - V_T^{U,pub}(0) &= \tfrac{c}{\lambda} \\ \Leftrightarrow \quad \alpha_1 \exp(-2\lambda e_{\max} T) + \left(\tfrac{c}{2\lambda} + \tfrac{\delta}{2\lambda e_{\max}}\right) \{1 - \exp[-2\lambda e_{\max} T]\} &= \tfrac{c}{\lambda} \\ \Leftrightarrow \quad \alpha_1 \phi(T)^2 + \left(\tfrac{c}{2\lambda} + \tfrac{\delta}{2\lambda e_{\max}}\right) \left[1 - \phi(T)^2\right] &= \tfrac{c}{\lambda}, \end{aligned}$$

where we use $\exp(-2\lambda e_{\max} T) = \phi(T)^2$. Since $\phi(T)^2 \leq \phi(T)$, a necessary condition for $X_e < T$ is

$$\alpha_2 < \frac{c}{2\lambda} + \frac{\delta}{2\lambda e_{\max}}, \tag{10}$$

implying that searching for a second signal is socially inefficient. We now show that when this inequality holds, welfare under public information exceeds the welfare under private information for small deadlines,

$$V_T^{U,pub}(0) - V_T^{U,priv}(0) > 0 \text{ for } T \leq \min\{X_e, \Delta\}. \tag{11}$$

However, from Proposition 9, we have

$$V_{X_e}^{U,priv}(0) \geq V_{\Delta}^{U,pub}(0) = \max_T V_T^{U,pub}(0) \geq V_{X_e}^{U,pub}(0).$$

Hence, this implies that $X_e > \Delta$. We found a contradiction. To establish the inequality (11), we use that for $T \leq \Delta$,

$$V_T^{U,pub}(0) = -\frac{1}{\varepsilon} + \alpha_1 \left(1 - \phi(T)^2\right) - \frac{c}{2\lambda}\left(1 - \phi(T)^2\right) - \frac{\delta}{2\lambda e_{\max}}\left(1 - \phi(T)^2\right),$$

[6] Notice that equilibria exist for which $e^{priv}(t) \geq e^{pub}(t)$ for any t. However, $e^{priv}(t) = 0$ ($< e^{pub}(t)$) for some t may well be part of an equilibrium strategy for an uninformed player.

and for $T < X_e$,

$$V_T^{U,priv}(0) = -\frac{1}{\varepsilon} + \alpha_1 (1 - \phi(T)) - \frac{c}{\lambda}(1 - \phi(T)) + [\alpha_1 \phi(T) + \alpha_2 (1 - \phi(T))](1 - \phi(T)) - \delta T.$$

Hence, to achieve higher welfare in the private information case, we need

$$\alpha_1 \left[(1 - \phi(T)) - \left(1 - \phi(T)^2\right)\right] - \left[\frac{c}{\lambda}(1 - \phi(T)) - \frac{c}{2\lambda}\left(1 - \phi(T)^2\right)\right]$$
$$+ [\alpha_1 \phi(T) + \alpha_2 (1 - \phi(T))](1 - \phi(T)) - \left[\delta T - \frac{\delta}{2\lambda e_{\max}}\left(1 - \phi(T)^2\right)\right] > 0.$$

$\Leftrightarrow$

$$-\alpha_1 [1 - \phi(T)] \phi(T) - \frac{c}{2\lambda}(1 - \phi(T))^2$$
$$+ \alpha_1 \phi(T)(1 - \phi(T)) + \alpha_2 (1 - \phi(T))^2 - \left[\delta T - \frac{\delta}{2\lambda e_{\max}}\left(1 - \phi(T)^2\right)\right] > 0.$$

$\Leftrightarrow$

$$\left[\frac{c}{2\lambda} - \alpha_2\right](1 - \phi(T))^2 > -\delta T + \frac{\delta}{2\lambda e_{\max}}\left(1 - \phi(T)^2\right)$$

Using $-\delta T + \frac{\delta}{2\lambda e_{\max}}\left(1 - \phi(T)^2\right) + \frac{\delta}{2\lambda e_{\max}}(1 - \phi(T))^2$

$$= -\delta T + \frac{\delta}{\lambda e_{\max}}(1 - \phi(T))$$
$$= \delta \frac{(1 - \exp(-\lambda e_{\max} T)) - \lambda e_{\max} T}{\lambda e_{\max}} < 0,$$

we see that this is implied by

$$\left[\frac{c}{2\lambda} - \alpha_2\right](1 - \phi(T))^2 > -\frac{\delta}{2\lambda e_{\max}}(1 - \phi(T))^2,$$

which follows from inequality (10). ■

Clearly, the increased search incentives due to the private nature of information increase the team partners' welfare by mitigating the inefficiency due to free-riding. However, for short deadlines, $T < \Delta$, incentives are sufficiently strong for players to exert maximum effort, regardless of whether information is public or private.

The option to hold back information also affects welfare through the delay of decisions. Informed players stop searching and may delay a decision in the hope that their partner becomes informed. Hence, both players may have stopped searching, but still delay a decision not knowing that their partner is already informed. This is clearly inefficient ex post. However, as any uninformed player would stop searching if the information were disclosed, any opportunity to acquire a second signal is lost when information is public. This opportunity can only be valuable if it is socially efficient for one player to search for a second signal, $(2\lambda\alpha_2 - c)\, e_{\max} > \delta$. Hence, when the deadline is short and it is socially inefficient to acquire a second signal, the team partners' welfare is increased by committing to disclose any information they acquire.

Corollary 5 *Welfare under public information exceeds welfare under private information if* $T \leq \Delta$ *and* $\alpha_2 < \frac{c}{2\lambda} + \frac{\delta}{2\lambda}$.
Proof. *See the proof of the previous Corollary.* ■

The Proposition shows that keeping information private is always preferrable for the team when the deadline can be set optimally. That is, the stronger search incentives due to the private nature information can be exploited at a relatively low cost of inefficient delay by setting the appropriate deadline. In Figure 4, we compare the expected welfare at $t = 0$ for different lengths of the deadline T under public and private information. When the deadline is set inefficiently short, welfare under public information can be higher than under private information as shown by the dotted red line.

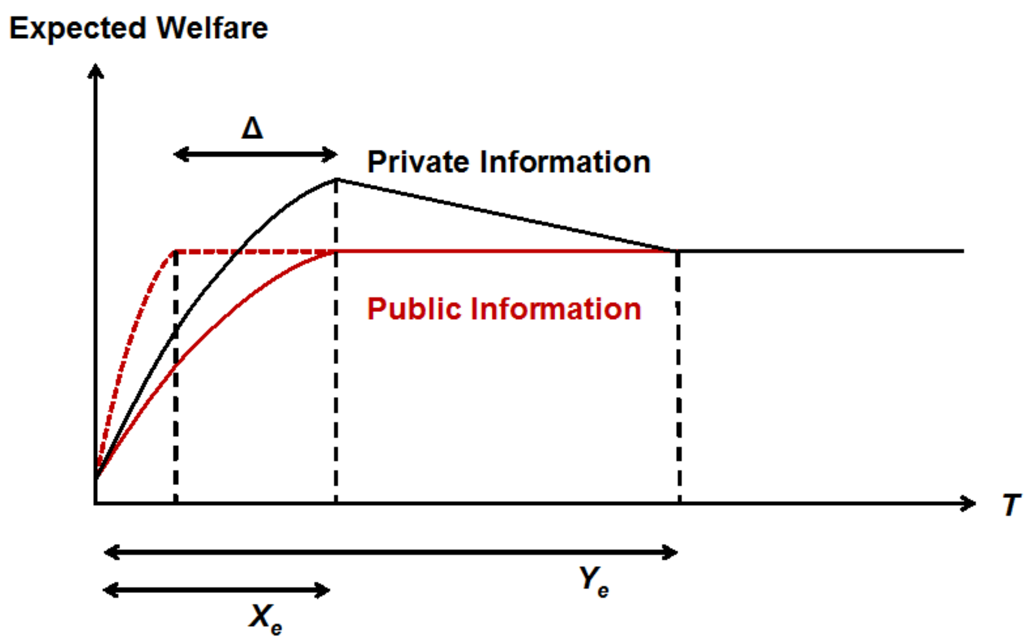

Figure 3: Expected Welfare at $t = 0$ for different lengths of the deadline T under private and public information

7 Extensions (preliminary)

In this section we consider extensions such as modifications in the signal structure, expanding the number of players, different decision protocols that require both players to agree on calling a decision as well as the introduction of explicit contracts. We also consider the role of a third party intermediary such as a committee chairperson who aggregates information by the different group members. We discuss how these extensions qualitatively affect the predictions of our model.

7.1 Signal structure

We first extend our model to consider a setting where more than one signal in total is needed for it to be individually worthwhile to stop searching and take a decision. From a modelling standpoint it would be necessary to introduce an additional cheap talk action so that agents can reveal information to one another without this implying a decision is called. In the model above this distinction was unnecessary as common knowledge of an agent holding a signal was sufficient for both agents to prefer to call a decision.

This extension has the potentially appealing characteristic that an agent with insufficient information to call a decision alone may nevertheless induce a team decision if her information combined with the information of the other agent is enough to induce an agent to call a decision. We conjecture that in this setting the equilibrium can be analyzed in a similar fashion to the earlier model. The basic intuition here is that if there is a number of signals at which an individual finds it at least weakly optimal to exert zero effort and delay, then an individual with fewer signals must strictly prefer to delay. If this is the case then there is a number of signals n such that the behavior is analogous to an informed individual in the earlier model and individuals with fewer signals continue to exert effort and delay the decision. The analysis of this extension becomes more complicated as we extend the deadline and there is a region where individuals reduce their effort in order to free ride at the start of the game.

The simplest model to capture this type of interaction would be a similar setup to the original model where we attach a greater value to obtaining a second signal. In particular, let $\alpha_2 > \frac{c}{\lambda} + \frac{\delta}{\lambda e_{\max}}$ whereby an agent with one signal will prefer to continue searching provided his belief that the other agent remains uninformed is high enough and $\alpha_3 < \frac{c}{\lambda}$ such that an agent with two signals will not exert effort. In the earlier model we considered two cases. In one equilibrium beliefs evolve such that an informed individual exerts zero effort and strictly prefers to delay a decision until the deadline. In the other beliefs are such that an informed individual is mixing between delay and calling a decision as the deadline is approached and is thus indifferent. In a setting where individuals can have zero, one or two signals one potentially feasible equilibrium, similar to that case of small incentives, may involve individuals with two signals strictly preferring to delay a decision until the deadline in which case an individual with 1 or 0 signals ought to also strictly prefer to delay. In the other equilibrium, analogous to the large incentives case, an individual with two signals may begin mixing between delay and calling a decision. Again in this setting individuals who are better informed have a greater incentive to call a decision, thus agents with no or only one signal will not call a decision.

7.2 More than two players

One can think of the effects of adding more players as affecting X_e, the longest deadline which can sustain maximum effort by all players as an equilibrium, and Y_e the maximum deadline whereby no decision is taken until the deadline. For a given length of deadline T both the value of X_e and Y_e become shorter as a third player is added. In the two person case incentives are driven by the belief a player holds about the other player obtaining a signal prior to the deadline. The relationship which determines X_e is given by

$$\lambda \left[\exp\left(-\lambda e_{\max} X_e\right)\alpha_1 + \left(1-\exp\left(-\lambda e_{\max} X_e\right)\right)\alpha_2\right] = c.$$

With an additional player the relevant threshold $X_e^{'}$ is defined by the following equation

$$\lambda \left\{ \left[\exp\left(-\lambda e_{\max} X_e'\right)\right]^2 \alpha_1 + 2\exp\left(-\lambda e_{\max} X_e'\right)\left[1-\exp\left(-\lambda e_{\max} X_e'\right)\right]\alpha_2 + \left[1-\exp\left(-\lambda e_{\max} X_e'\right)\right]^2 \alpha_3 \right\} = c.$$

It is now relatively straightforward to show that $X_e' < X_e$. The relationship for Y_e and Y_e' are

$$Y_e = \frac{\left(1-\exp\left(-\lambda e_{\max} X_e\right)\right)\alpha_2}{\delta}$$

and

$$Y_e' = \frac{2\exp\left(-\lambda e_{\max} X_e'\right)\left(1-\exp\left(-\lambda e_{\max} X_e'\right)\right)\alpha_2 + \left(1-\exp\left(-\lambda e_{\max} X_e'\right)\right)^2 \alpha_3}{\delta}.$$

Using the equality of the terms inside the brackets we have

$$\begin{aligned}\left(\exp\left(-\lambda e_{\max} X_e'\right)\right)^2 \alpha_1 + 2\exp\left(-\lambda e_{\max} X_e'\right)\left(1-\exp\left(-\lambda e_{\max} X_e'\right)\right)\alpha_2 + \left(1-\exp\left(-\lambda e_{\max} X_e'\right)\right)^2 \alpha_3 \\ = \exp\left(-\lambda e_{\max} X_e\right)\alpha_1 + \left(1-\exp\left(-\lambda e_{\max} X_e\right)\right)\alpha_2\end{aligned}$$

thus we can conclude that

$$\begin{aligned}2\exp\left(-\lambda e_{\max} X_e'\right)\left(1-\exp\left(-\lambda e_{\max} X_e'\right)\right)\alpha_2 + \left(1-\exp\left(-\lambda e_{\max} X_e'\right)\right)^2 \alpha_3 \\ < \left(1-\exp\left(-\lambda e_{\max} X_e\right)\right)\alpha_2\end{aligned}$$

and hence $Y_e' < Y_e$.

7.3 Alternative mechanisms

The incentives for an agent to reveal information in our model are determined by the trade-off between search benefit of waiting and the associated costs of delay, or more formally

$$\lambda \widetilde{e}\left(t\right)\phi\left(t\right)\alpha_2 \gtrless \delta$$

This equation reveals the tension between effort provision and incentives for revealing information that we investigate in the model. All else equal the higher equilibrium effort $\widetilde{e}\left(t\right)$ of an uninformed agent, the greater are the incentives to withhold information for an informed agent. A lesson we draw from our analysis is that providing incentives for effort does not necessarily result in faster decisions, but that it may simply shift the source of inefficiency from one of limited effort provision to limited information sharing. A second lesson is that when a second signal is socially efficient the ability of agents to delay may be beneficial for welfare.

We consider our setting to be one in which there are severe restrictions on the types of mechansims and contracts that are feasible. One type of contract or mechanism that may be reasonable in our setting is a payment that depends on time. An example is a decreasing payment (increasing punishment) to the agents depending on when the decision is taken. Essentially, this type of scheme would simply increase the delay cost δ. If we take the case of a long deadline then the free-riding effort level is $\frac{\delta}{c}$. A relatively simple way to obtain maximum effort and immediate information revelation would be to set the rate of decrease (increase) of a reward (punishment) for making a decision at a particular time equal to $ce_{\max} - \delta$. The effective discount rate an agent then faces is $ce_{\max}$. Hence, this contract fixes the free-riding effort at $e_{\max}$ and information is revealed immediately as $ce_{\max} > \lambda e_{\max}\alpha_2$. However, in implementing this type of scheme one must also check that agents still have an incentive to search for information at all. This is satisfied in our original model by the assumption that $\alpha_1 > \frac{c}{\lambda} + \frac{\delta}{\lambda e_{\max}}$. In adjusting the effective delay cost this constraint may no longer hold and agents will simply call an immediate decision. If $\alpha_1 > \frac{2c}{\lambda}$, this will not occur.

There is a broad range of assumptions one can make about what can be contracted on which can help in full or part fix the incentive issues of our model. Other types of mechanisms may augment the decision making protocol to limit the ability of agents to include their information that they hold if they are not calling the decision themselves. These mechanisms require commitment at the decision making stage not to include potentially valuable information in the decision-making process. We also note that to the extent that these changes simply affect the incentive for an agent to reveal information immediately, the analysis is identical to the public information case analyzed earlier. As we showed before this is not necessarily welfare-improving and with an optimally set deadline is is welfare-decreasing.

7.4 Mediator

We previously showed that there is a unique optimal deadline which maximizes the ex-ante welfare of the players. In this case there are clear inefficiencies when both players acquire a signal prior to the deadline, but the decision is nonetheless delayed as neither player shares their information. Thus, there are potential gains from the introduction of a mediator who cannot exert effort (or whose effort decision is unaffected by any information found by the other agents), but is able to facilitate communication. The role of the mediator would be to avoid situations in which both players acquired a signal, but neither is willing to reveal it to the other agent. Thus, the decision is unnecessarily being delayed without any additional information acquisition.

The presence of a mediator changes the agents' effort incentives in the following important way. When the deadline is far away the option of an agent to free-ride on the information acquisition efforts of the other agent is effectively eliminated if a mediator is commited not to take a decision until both agents have each found a piece of information. Hence, agents exert maximum effort and it is in fact an equilibrium action for the mediator to delay a decision while uninformed agents exert maximum effort. What is surprising is that as the deadline draws closer the option to exert little effort and wait until the deadline for a decision is a less costly option and so the deadline undermines the incentives for effort. Note though that this is only one potential equilibrium in this setting. A mediator's strategy to call a decision after receiving one signal from either agent results in lower effort from the agents due to free-riding and hence makes the strategy to call a decision after one signal also an equilibrium. In this case an approaching deadline will increase the incentives for effort.

8 Conclusion

In private and public organizations, teams are often allocated the dual task of finding and taking a decision. In this paper we have investigated the link between the incentive to search and the incentive to share decision-relevant information in this type of team setting. One clear lesson that emerges is that team members are reluctant to disclose information that undermines the incentives of fellow team members to search for further information. As a result, although a strict deadline provides strong incentives for agents to gather information, it also mutes the incentives to reveal information and to make a fast decision. In this light, it may therefore not be too surprising that strict deadlines may sometimes be counterproductive when immediate decisions are required, in particular in settings where "Parkison's Law" applies. Furthermore, we have shown that mutual monitoring in teams is not a panacea to solving incentive problems. In fact, in most cases the non-observability of information is precisely what allows agents to circumvent the moral hazard in teams problem. However, the interested reader may ask how differing opinions or preferences in addition to conflicts about effort provision and information sharing among the team members may influence our findings. We leave these interesting questions to future research.

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A Omitted Proofs

A.1 Proofs for Equilibrium Properties

Proof of Proposition 2. The proof proceeds in several steps to establish the result for the relevant regions.

i) Case 1: $T < X_e$

Informed strategy

$d(t) = 0$ for all t.

Uninformed strategy

$d(t) = 0$ for all t.

$e(t) = 1$ for all t.

Beliefs

Beliefs evolve according to $\phi(t) = \exp\left(-\lambda \int_0^t e(s)\, ds\right)$ for all t.

We proceed with the proof by writing out the implied continuation values of informed and uninformed agents. At the deadline the continuation values are simply the expected value of a decision at that time.

$$V^I(T) = -\frac{1}{\varepsilon + \tau} + (1 - \exp(-\lambda T))\, \alpha_2$$

and

$$V^U(T) = -\frac{1}{\varepsilon} + (1 - \exp(-\lambda T))\, \alpha_1$$

also

$$V^I(T) - V^U(T) = \exp(-\lambda T)\,\alpha_1 + (1-\exp(-\lambda T))\,\alpha_2$$

For general $0 \leq t \leq T$ the continuation values are

$$V^I(t) = V^I(T) - \delta(T-t)$$

and

$$V^U(t) = (1-\exp(-\lambda(T-t)))\,V^I(T) + \exp(-\lambda(T-t))\,V^U(T) \\ - \frac{c}{\lambda}(1-\exp(-\lambda(T-t))) - \delta(T-t)$$

Informed strategy

Check the informed individual's decision strategy $d(t) = 0$ is optimal by noting:

$$V^I(t) > -\frac{1}{\varepsilon+\tau} + (1-\exp(-\lambda t))\,\alpha_2 \text{ for all } t < X_e.$$

Where the right hand side is the value of a decision at a time t. Hence it is optimal to wait for the informed agent.

Uninformed strategy

Checking an uninformed agent's choice of effort

$$V^I(t) - V^U(t) = \frac{c}{\lambda} + \exp(-\lambda[T-t])\left[V^I(T) - V^U(T) - \frac{c}{\lambda}\right]$$

As $\phi(T) > \bar{\phi}_e$, the uninformed player wants to exert maximum effort for all $0 \leq t \leq T$, since

$$\begin{aligned} V^I(t) - V^U(t) - \frac{c}{\lambda} &= \exp(-\lambda[T-t])\left[V^I(T) - V^U(T) - \frac{c}{\lambda}\right] \\ &= \exp(-\lambda[T-t])\left[\exp(-\lambda T)\,\alpha_1 + (1-\exp(-\lambda T))\,\alpha_2 - \frac{c}{\lambda}\right] \\ &> 0 \text{ if } \phi(T) > \bar{\phi}_e \end{aligned}$$

Finally, we check that the uninformed individual will not call a decision by noting that

$$V^U(t) = (1-\exp(-\lambda(T-t)))\,V^I(T) + \exp(-\lambda(T-t))\,V^U(T) \\ - \frac{c}{\lambda}(1-\exp(-\lambda(T-t))) - \delta(T-t)$$

$$V^U(t) > -\frac{1}{\varepsilon} + (1-\exp(-\lambda t))\,\alpha_1 \text{ provided } t < \frac{1}{\lambda}\ln\frac{1}{\bar{\phi}_e}$$

ii) Case 2: $X_e < T < Y_e$

Informed strategy

$d(t) = 0$ for all t.

Uninformed strategy

$d(t) = 0$ for all t.

$e(t)$ satisfies

$$\exp\left(-\lambda \int_0^t e(s)\, ds\right) \geq \phi_e + (T-t)\frac{\delta}{\alpha_2}$$

and

$$\exp\left(-\lambda \int_0^T e(s)\, ds\right) = \bar{\phi}_e.$$

Beliefs

Beliefs evolve according to $\phi(t) = \exp\left(-\lambda \int_0^t e(s)\, ds\right)$ for all t.

At the deadline the continuation values are simply the expected value of a decision at that time.

$$V^I(T) = -\frac{1}{\varepsilon+\tau} + \left(1-\bar{\phi}_e\right)\alpha_2$$

and

$$V^U(T) = -\frac{1}{\varepsilon} + \left(1-\bar{\phi}_e\right)\alpha_1$$

also

$$V^I(T) - V^U(T) = \bar{\phi}_e\alpha_1 + \left(1-\bar{\phi}_e\right)\alpha_2$$

For general $0 \leq t \leq T$ the continuation values are

$$V^I(t) = V^I(T) - \delta(T-t)$$

and

$$\begin{aligned} V^U(t) = \left(1-\exp\left(-\lambda\int_t^T e(s)\,ds\right)\right)V^I(T) + \exp\left(-\lambda\int_t^T e(s)\,ds\right)V^U(T) \\ -\frac{c}{\lambda}\left(1-\exp\left(-\lambda\int_t^T e(s)\,ds\right)\right) - \delta(T-t) \end{aligned}$$

Informed strategy

Informed individual's decision strategy

$$\begin{aligned} V^I(t) &\geq -\frac{1}{\varepsilon+\tau} + (1-\phi(t))\alpha_2 \\ \phi(t) &\geq \frac{\delta}{\alpha_2}(T-t) + \bar{\phi}_e \\ \exp\left(-\lambda\int_0^t e(s)\,ds\right) &\geq \frac{\delta}{\alpha_2}(T-t) + \bar{\phi}_e \end{aligned}$$

This is true given the effort strategy specified. Hence it is weakly optimal to wait for the informed agent.

Uninformed strategy

Checking an uninformed agent's choice of effort

$$V^I(t) - V^U(t) = \frac{c}{\lambda} + \exp\left(-\lambda \int_t^T e(s)\,ds\right)\left[V^I(T) - V^U(T) - \frac{c}{\lambda}\right]$$

As $\phi(T) = \bar{\phi}_e$, the uninformed player is indifferent about the level of effort she exerts for all $0 \le t \le T$ since

$$\begin{aligned} V^I(t) - V^U(t) - \frac{c}{\lambda} &= \exp\left(-\lambda \int_t^T e(t)\,dt\right)\left[V^I(T) - V^U(T) - \frac{c}{\lambda}\right] \\ &= \exp\left(-\lambda \int_t^T e(t)\,dt\right)\left[\bar{\phi}_e\alpha_1 + \left(1 - \bar{\phi}_e\right)\alpha_2 - \frac{c}{\lambda}\right] \\ &= 0 \text{ if } \phi(T) = \bar{\phi}_e \end{aligned}$$

Finally, we check that the uninformed individual will not call a decision by noting that

$$\begin{aligned} V^U(t) = \left(1 - \exp\left(-\lambda \int_t^T e(s)\,ds\right)\right) V^I(T) + \exp\left(-\lambda \int_t^T e(s)\,ds\right) V^U(T) \\ - \frac{c}{\lambda}\left(1 - \exp\left(-\lambda \int_t^T e(s)\,ds\right)\right) - \delta(T - t) \end{aligned}$$

$$V^U(t) > -\frac{1}{\varepsilon} + \left(1 - \exp\left(-\lambda \int_t^T e(t)\,dt\right)\right)\alpha_1$$

iii) Case 3: $T > \frac{1}{\delta}(1 - \bar{\phi}_e)\alpha_2$

Informed strategy

$$d(t) = \begin{cases} \textit{call} \text{ for } t < T - Y_e \\ 0 \text{ for } t \ge T - Y_e \end{cases}$$

Uninformed strategy

$d(t) = 0$ for all t.

$e(t) = \frac{\delta}{c}$ for $t < T - Y_e$

$e(t)$ satisfies

$$\exp\left(-\lambda \int_{T-Y_e}^t e(s)\,ds\right) \ge \phi_e + (T - t)\frac{\delta}{\alpha_2}$$

and

$$\exp\left(-\lambda \int_{T-Y_e}^T e(s)\,ds\right) = \bar{\phi}_e.$$

for $t \ge T - Y_e$

Beliefs

$\phi(t) = 1$ for $t < T - Y_e$.

$\phi(t) = \exp\left(-\lambda \int_0^t e(s)\,ds\right)$ for $t \ge T - Y_e$.

For $0 \le t < T - \frac{1}{\delta}(1 - \bar{\phi}_e)\alpha_2$ immediate decisions are called then the belief is $\phi(t) = 0$ for the subgame from $t = T - \frac{1}{\delta}(1 - \bar{\phi}_e)\alpha_2$ hence the proof for the strategies being equilibria of that subgame are encompassed in case 2. It remains to show that the strategies specified for $t < T - \frac{1}{\delta}(1-\bar{\phi}_e)\alpha_2$ also constitute an equilibrium. The continuation values for an informed individual is given by

$$\begin{aligned} V^{IN}(t) &= -\frac{1}{\varepsilon+\tau} \text{ for } t < T - \frac{1}{\delta}(1-\bar{\phi}_e)\alpha_2 \\ &= -\frac{1}{\varepsilon+\tau} + \delta\left(t - T + \frac{1}{\delta}(1-\bar{\phi}_e)\alpha_2\right) \text{ for } t < T - \frac{1}{\delta}(1-\bar{\phi}_e)\alpha_2 \end{aligned}$$

and for an uninformed individual

$$V^{UN}(t) = \int_t^{T-\frac{1}{\delta}(1-\bar{\phi}_e)\alpha_2} \left(V^{IN}(t) - c\int_t^s e(r)\,dr - \delta(s-t)\right) 2\lambda e(s)\, e^{-2\lambda\int_t^s e(r)dr} ds$$
$$+ e^{-2\lambda\int_t^{T-\frac{1}{\delta}(1-\bar{\phi}_e)\alpha_2} e(r)dr}\left(V^{UN}\left(T - \frac{1}{\delta}(1-\bar{\phi}_e)\alpha_2\right) - c\int_t^{T-\frac{1}{\delta}(1-\bar{\phi}_e)\alpha_2} e(r)\,dr - \delta\left(T - \frac{1}{\delta}(1-\bar{\phi}_e)\alpha_2 - t\right)\right)$$

$$\begin{aligned} V^{UN}(t) = &\int_t^{T-\frac{1}{\delta}(1-\bar{\phi}_e)\alpha_2} \left(V^{IN}(t) - c\int_t^s \frac{\delta}{c}dr - \delta(s-t)\right) 2\lambda\frac{\delta}{c} e^{-2\lambda\int_t^s \frac{\delta}{c}dr} ds \\ &+ e^{-2\lambda\int_t^{T-\frac{1}{\delta}(1-\bar{\phi}_e)\alpha_2} \frac{\delta}{c}dr}\left(V^{UN}(t') - c\int_t^{T-\frac{1}{\delta}(1-\bar{\phi}_e)\alpha_2} \frac{\delta}{c}dr - \delta\left(T - \frac{1}{\delta}(1-\bar{\phi}_e)\alpha_2 - t\right)\right) \end{aligned}$$

$$V^{UN}(t) = \int_t^{T-\frac{1}{\delta}(1-\bar{\phi}_e)\alpha_2} \left(V^{IN}(t) - (2\delta)(s-t)\right) 2\lambda\frac{\delta}{c} e^{-2\lambda\int_t^s \frac{\delta}{c}dr} ds$$
$$+ e^{-2\lambda\frac{\delta}{c}(t'-t)}\left(V^{UN}\left(T - \frac{1}{\delta}(1-\bar{\phi}_e)\alpha_2\right) - c\frac{\delta}{c}\left(T - \frac{1}{\delta}(1-\bar{\phi}_e)\alpha_2 - t\right) - \delta\left(T - \frac{1}{\delta}(1-\bar{\phi}_e)\alpha_2 - t\right)\right)$$

$$\begin{aligned} V^{UN}(t) = &\left(V^{IN}(t) - \frac{2\delta}{2\lambda\frac{\delta}{c}}\right)\left(1 - e^{-2\lambda\frac{\delta}{c}\left(T-\frac{1}{\delta}(1-\bar{\phi}_e)\alpha_2 - t\right)}\right) \\ &+ e^{-2\lambda\frac{\delta}{c}\left(T-\frac{1}{\delta}(1-\bar{\phi}_e)\alpha_2 - t\right)} V^{UN}\left(T - \frac{1}{\delta}(1-\bar{\phi}_e)\alpha_2\right) \end{aligned}$$

where $V^{UN}(T - Y_e)$ is $V^{UN}(0)$ from case 2 for $T = Y_e$ hence it is:

$$V^{UN}(T - Y_e) = -\frac{1}{\varepsilon+\tau} - \frac{c}{\lambda}$$

and we can further simplify the above to

$$V^{UN}(t) = V^{IN}(t) - \frac{c}{\lambda}$$

Hence this holds for all t.

Informed strategy

The payoff from waiting until $\widehat{t} < T - \frac{1}{\delta}(1-\bar{\phi}_e)\alpha_2$ upon becoming informed at time t which we denote $V^I\left(\widehat{t}|t\right)$ is:

$$V^I\left(\widehat{t}|t\right) = \int_t^{\widehat{t}} \left(-\frac{1}{\varepsilon+\tau} + \alpha_2 - \delta\left(s-t\right)\right)\lambda\frac{\delta}{c}\exp\left(-\lambda\frac{\delta}{c}\left(s-t\right)\right)ds + \exp\left(-\lambda\frac{\delta}{c}\left(\widehat{t}-t\right)\right)\left(-\frac{1}{\varepsilon+\tau} - \delta\left(\widehat{t}-t\right)\right)$$

this can be simplified to

$$V^I\left(\widehat{t}|t\right) = \left(-\frac{1}{\varepsilon+\tau} + \alpha_2 - \frac{c}{\lambda}\right)\left(1-\exp\left(-\lambda\frac{\delta}{c}\left(\widehat{t}-t\right)\right)\right) + \exp\left(-\lambda\frac{\delta}{c}\left(\widehat{t}-t\right)\right)\left(-\frac{1}{\varepsilon+\tau}\right)$$

where we have used

$$\int_t^{\widehat{t}} \lambda\frac{\delta}{c}\left(s-t\right)\exp\left(-\lambda\frac{\delta}{c}\left(s-t\right)\right)ds = -\left[\left(s-t\right)\exp\left(-\lambda\frac{\delta}{c}\left(s-t\right)\right)\right]_t^{\widehat{t}} + \int_t^{\widehat{t}} \exp\left(-\lambda\frac{\delta}{c}\left(s-t\right)\right)ds$$

$$\int_t^{\widehat{t}} \lambda\frac{\delta}{c}\left(s-t\right)\exp\left(-\lambda\frac{\delta}{c}\left(s-t\right)\right)ds = -\left(\widehat{t}-t\right)\exp\left(-\lambda\frac{\delta}{c}\left(\widehat{t}-t\right)\right) + \frac{c}{\lambda\delta}\left(1-\exp\left(-\lambda\frac{\delta}{c}\left(\widehat{t}-t\right)\right)\right)$$

The optimal decision time is therefore

$$\arg\max_{\widehat{t}\geq 0} V^I\left(\widehat{t}|t\right) = 0$$

since $\frac{c}{\lambda} > \alpha_2$ and given the results in case 2 the agent will not prefer to wait beyond $T - \frac{1}{\delta}(1-\bar{\phi}_e)\alpha_2$. Hence an informed agent optimally makes an immediate decision for $t < T - \frac{1}{\delta}(1-\bar{\phi}_e)\alpha_2$.

Uninformed strategy

We immediately find that the uninformed agent is indifferent about putting in effort at all times t because $V^{UN}(t) = V^{IN}(t) - \frac{c}{\lambda}$ so her effort strategy is an equilibrium. Furthermore $V^{IN}(t) - \frac{c}{\lambda} > -\frac{1}{\varepsilon}$ so it is never optimal for the uninformed agent to call a decision. ■

Proof of Proposition 3. This is the proof for the case $T > \frac{(1-\phi^*)\alpha_2}{\delta} + \varsigma$.

i) Case 1: $T < \frac{1}{\lambda}\ln\frac{1}{\bar{\phi}_d} = X_d$

Informed strategy

$d(t) = 0$ for all t.

Uninformed strategy

$d(t) = 0$ for all t.

$e(t) = 1$ for all t.

Beliefs

Beliefs evolve according to $\phi(t) = \exp\left(-\lambda \int_0^t e(s)\, ds\right)$ for all t.

We proceed with the proof by writing out the implied continuation values of informed and uninformed agents. At the deadline the continuation values are simply the expected value of a decision at that time.

$$V^I(T) = -\frac{1}{\varepsilon + \tau} + (1 - \exp(-\lambda T))\, \alpha_2$$

and

$$V^U(T) = -\frac{1}{\varepsilon} + (1 - \exp(-\lambda T))\, \alpha_1$$

also

$$V^I(T) - V^U(T) = \exp(-\lambda T)\, \alpha_1 + (1 - \exp(-\lambda T))\, \alpha_2$$

For general $0 \leq t \leq T$ the continuation values are

$$V^I(t) = V^I(T) - \delta(T - t)$$

and

$$\begin{aligned} V^U(t) = {} & (1 - \exp(-\lambda(T - t)))\, V^I(T) + \exp(-\lambda(T - t))\, V^U(T) \\ & - \frac{c}{\lambda}(1 - \exp(-\lambda(T - t))) - \delta(T - t) \end{aligned}$$

Informed strategy

Checking the informed individual's decision strategy is optimal by noting:

$$V^I(t) > -\frac{1}{\varepsilon + \tau} + (1 - \exp(-\lambda t))\, \alpha_2 \text{ for all } t < \frac{1}{\lambda} \ln \frac{1}{\bar{\phi}_d}.$$

Here the right hand side is the value of a decision at a time t. Hence it is optimal to wait for the informed agent.

Uninformed strategy

Checking an uninformed agent's choice of effort

$$V^I(t) - V^U(t) = \frac{c}{\lambda} + \exp(-\lambda e_{\max}[T - t])\left[V^I(T) - V^U(T) - \frac{c}{\lambda}\right]$$

As $\phi(T) > \bar{\phi}_d$, the uninformed player wants to exert maximum effort for all $0 \leq t \leq T$, since

$$\begin{aligned} V^I(t) - V^U(t) - \frac{c}{\lambda} &= \exp(-\lambda e_{\max}[T - t])\left[V^I(T) - V^U(T) - \frac{c}{\lambda}\right] \\ &= \exp(-\lambda e_{\max}[T - t])\left[\exp(-\lambda T)\, \alpha_1 + (1 - \exp(-\lambda T))\, \alpha_2 - \frac{c}{\lambda}\right] \end{aligned}$$

$$V^{I}(t) - V^{U}(t) - \frac{c}{\lambda} > 0 \text{ for } \phi(T) > \bar{\phi}_d > \bar{\phi}_e$$

Finally, we check that the uninformed individual will not call a decision by noting that

$$V^{U}(t) = (1 - \exp(-\lambda(T-t)))\, V^{I}(T) + \exp(-\lambda(T-t))\, V^{U}(T) - \frac{c}{\lambda}(1 - \exp(-\lambda(T-t))) - \delta(T-t)$$

$$V^{U}(t) > -\frac{1}{\varepsilon} + (1 - \exp(-\lambda t))\,\alpha_1 \text{ provided } t < \frac{1}{\lambda}\ln\frac{1}{\bar{\phi}_e}$$

ii) **Case 2** $X_d \leq T < X_d + Z$

Informed strategy

$d(t) = 0$ for $t < X_d$.

$d(t) = \frac{\lambda^2 \alpha_2}{\lambda\alpha_2 - \delta}$ for $t > X_d$.

Uninformed strategy

$d(t) = 0$ for all t.

$e(t) = 1$ for all t.

Beliefs

$\phi(t) = \exp\left(-\lambda \int_0^t e(s)\, ds\right)$ for $t < X_d$.

$\phi(t) = \bar{\phi}_d$ for $t \geq X_d$.

The continuation value at the deadline at time T for an informed agents is:

$$V^{IN}(T) = -\frac{1}{\varepsilon + \tau} + \left(1 - \bar{\phi}_d\right)\alpha_2$$

and for an uninformed agent is

$$V^{UN}(T) = -\frac{1}{\varepsilon} + \left(1 - \bar{\phi}_d\right)\alpha_1.$$

Hence, the difference between being informed and uninformed is

$$V^{IN}(T) - V^{UN}(T) = \bar{\phi}_d \alpha_1 + \left(1 - \bar{\phi}_d\right)\alpha_2.$$

Working backwards consider the continuation value for an agent at time $X_d < t < T$. The informed agent has payoff

$$V^{IN}(t) = V^{IN}(T) = -\frac{1}{\varepsilon + \tau} + \left(1 - \bar{\phi}_d\right)\alpha_2$$

The uninformed agent has a payoff

$$V^{UN}(t) = \int_t^T \left(-\frac{1}{\varepsilon + \tau} + \frac{\left(1 - \bar{\phi}_d\right)\alpha_2}{2} - (c + \delta)(s - t)\right) 2\lambda e^{-2\lambda(s-t)} ds + e^{-2\lambda(T-t)}\left(V^{UN}(T) - (c + \delta)(T - t)\right)$$

$$V^{UN}(t) = \left(-\frac{1}{\varepsilon+\tau} + \frac{\left(1-\bar{\phi}_d\right)\alpha_2}{2}\right)\left(1-e^{-2\lambda(T-t)}\right)$$
$$-(c+\delta)\left[-(T-t)\,e^{-2\lambda(T-t)} + \frac{1}{2\lambda}\left(1-e^{-2\lambda(T-t)}\right)\right]$$
$$+e^{-2\lambda(T-t)}\left(V^{UN}(T)-(c+\delta)(T-t)\right)$$

$$V^{UN}(t) = \left[-\frac{1}{\varepsilon+\tau} + \frac{\left(1-\bar{\phi}_d\right)\alpha_2}{2} - (c+\delta)\frac{1}{2\lambda}\right]\times$$
$$\left(1-e^{-2\lambda(T-t)}\right) + e^{-2\lambda(T-t)}\left(V^{UN}(T)\right)$$

$$V^{UN}(t) = -\frac{1}{\varepsilon+\tau} + \frac{\left(1-\bar{\phi}_d\right)\alpha_2}{2} - (c+\delta)\frac{1}{2\lambda}$$
$$-e^{-2\lambda(T-t)}\left[\bar{\phi}_d\alpha_1 + \frac{\left(1-\bar{\phi}_d\right)\alpha_2}{2} - (c+\delta)\frac{1}{2\lambda}\right]$$

Note that the 2 in the pdf comes from the probability of one of two events occuring, “I find information” or “other agent calls a decision,” in this time interval the rate of information acquisition is the same as the rate at which the other agent is calling decisions thus the pdf has a 2. The payoff contains the term $\frac{\left(1-\bar{\phi}_d\right)\alpha_2}{2}$ because with 50% probability I will find information before the other agent calls a decision in which case the payoff is increased by $\left(1-\bar{\phi}_d\right)\alpha_2$. The difference between being informed and uninformed is given by

$$V^{IN}(t) - V^{UN}(t) = \frac{\left(1-\bar{\phi}_d\right)\alpha_2}{2} + (c+\delta)\frac{1}{2\lambda}$$
$$+e^{-2\lambda(T-t)}\left[\bar{\phi}_d\alpha_1 + \frac{\left(1-\bar{\phi}_d\right)\alpha_2}{2} - (c+\delta)\frac{1}{2\lambda}\right]$$

which satisfies $V^{IN}(t) - V^{UN}(t) = \frac{c}{\lambda}$ if $T-t=Z$ by definition of Z and $\frac{d\left(V^{IN}(t)-V^{UN}(t)\right)}{dt} > 0$ because $\frac{dV^{IN}}{dt} = 0$ and $\frac{dV^{UN}}{dt} < 0$.

For $0 \leq t < X_d$ the continuation value for the informed agent is:

$$V^{IN}(t) = -\frac{1}{\varepsilon+\tau} + (1-\phi^*)\,\alpha_2 - \delta\,(X_d - t)$$

the uninformed agent

$$V^{UN}(t) = \int_t^{X_d}\left(V^{IN}(X_d) - c\,(s-t)\right)\lambda e^{-\lambda(s-t)}ds - \delta\,(X_d-t)$$
$$+e^{-\lambda(X_d-t)}\left(V^{UN}(X_d) - c\,(X_d-t)\right)$$

$$V^{UN}(t) = \left(V^{IN}(X_d) - \frac{c}{\lambda}\right)\left(1 - e^{-\lambda(X_d - t)}\right) + V^{UN}(X_d)\, e^{-\lambda(X_d - t)} - \delta\left(\widehat{t} - t\right)$$

and the difference between the two is

$$V^{IN}(t) - V^{UN}(t) = \frac{c}{\lambda} + e^{-\lambda(X_d - t)}\left(V^{IN}(X_d) - V^{UN}(X_d) - \frac{c}{\lambda}\right)$$

Informed strategy

An informed agent delays decisions for $t < X_d$, in this case the payoff from an immediate decision would be $-\frac{1}{\varepsilon+\tau} + (1 - \phi(t))\,\alpha_2$. The condition for the informed agent to delay is hence

$$-\frac{1}{\varepsilon+\tau} + \left(1 - \bar{\phi}_d\right)\alpha_2 - \delta\,(X_d - t) \geq -\frac{1}{\varepsilon+\tau} + (1 - \phi(t))\,\alpha_2$$

rearranging and substituting in for $\bar{\phi}_d$ we get

$$\exp(-\lambda t) - \bar{\phi}_d \geq \frac{\delta}{\alpha_2}(X_d - t)$$

which holds with equality for $t = X_d$, furthermore the derivative of the LHS is strictly less than the RHS $-\lambda \exp(-\lambda t) < -\frac{\delta}{\alpha_2}$ for $t < X_d$ hence the relation holds for $t < X_d$. For $t \geq X_d$ an informed individual must be indifferent about a decision now versus delaying a decision any amount Δt into the future. Value of a decision at t is

$$-\frac{1}{\varepsilon+\tau} + \left(1 - \bar{\phi}_d\right)\alpha_2$$

the value of waiting and making a decision at $t + \Delta t$ is $V(t + \Delta t|t)$ and is calculated as

$$V(t+\Delta t|t) = \int_t^{t+\Delta t}\left(-\frac{1}{\varepsilon+\tau} + \alpha_2 - \delta(s-t)\right)\frac{\lambda^2\alpha_2}{\lambda\alpha_2 - \delta}(1-\phi(s))\exp\left(-\int_t^s \frac{\lambda^2\alpha_2}{\lambda\alpha_2-\delta}(1-\phi(r))\,dr\right)ds + \exp\left(-\int_t^{t+\Delta t}\frac{\lambda^2\alpha_2}{\lambda\alpha_2-\delta}(1-\phi(r))\,dr\right)$$

$$V(t+\Delta t|t) = \int_t^{t+\Delta t}\left(-\frac{1}{\varepsilon+\tau} + \alpha_2 - \delta(s-t)\right)\frac{\lambda^2\alpha_2}{\lambda\alpha_2 - \delta}\left(1-\bar{\phi}_d\right)\exp\left(-\frac{\lambda^2\alpha_2}{\lambda\alpha_2-\delta}\left(1-\bar{\phi}_d\right)(s-t)\right)ds + \exp\left(-\frac{\lambda^2\alpha_2}{\lambda\alpha_2-\delta}\left(1-\bar{\phi}_d\right)\Delta t\right)\left(-\frac{1}{\varepsilon+\tau} + \left(1-\bar{\phi}_d\right)\alpha_2 - \delta\Delta t\right)$$

$$V(t+\Delta t|t) = \left(-\frac{1}{\varepsilon+\tau} + \alpha_2 - \delta\left(\frac{\lambda\alpha_2 - \delta}{\lambda^2\alpha_2\left(1-\bar{\phi}_d\right)}\right)\right)\left(1 - \exp\left(-\frac{\lambda^2\alpha_2}{\lambda\alpha_2-\delta}\left(1-\bar{\phi}_d\right)\Delta t\right)\right)$$
$$+\left(-\frac{1}{\varepsilon+\tau} + \left(1-\bar{\phi}_d\right)\alpha_2\right)\exp\left(-\frac{\lambda^2\alpha_2}{\lambda\alpha_2-\delta}\left(1-\bar{\phi}_d\right)\Delta t\right)$$

$$V(t+\Delta t|t) = -\frac{1}{\varepsilon+\tau} + \left(1-\bar{\phi}_d\right)\alpha_2$$

since

$$\delta\left(\frac{\lambda\alpha_2-\delta}{\lambda^2\alpha_2\left(1-\bar{\phi}_d\right)}\right) = \delta\left(\frac{\lambda\alpha_2-\delta}{\lambda\left(\lambda\alpha_2-\delta\right)}\right) = \frac{\delta}{\lambda\alpha_2}\alpha_2 = \bar{\phi}_d\alpha_2$$

hence informed individuals are indifferent about calling an immediate decision and delaying the decision and a mixing strategy is an equilibrium strategy.

Uninformed strategy

An uninformed agent will exert maximum effort because $V^{IN}(t) - V^{UN}(t) > \frac{c}{\lambda}$ for all t provided $T < X_d + Z$. An uninformed agent will not call a decision since the payoff from calling a decision is $-\frac{1}{\varepsilon} + (1-\phi(t))\alpha_1$ which is increasing in t, equal to the continuation value $V^{UN}(T)$ at the deadline and $\frac{dV^{UN}(t)}{dt} < 0$ hence $V^{UN}(t) > -\frac{1}{\varepsilon} + (1-\phi(t))\alpha_1$ for all $t < T$.

iii) **Case 3** $X_d + Z < T < Y_d + Z$

Informed strategy

$d(t) = 0$ for $t < T - Z$.

$d(t) = \frac{\lambda^2\alpha_2}{\lambda\alpha_2-\delta}$ for $t \geq T - Z$.

Uninformed strategy

$d(t) = 0$ for all t.

$e(t)$ satisfies

$$\exp\left(-\lambda\int_0^t e(s)\,ds\right) \geq \bar{\phi}_d + (T-Z-t)\frac{\delta}{\alpha_2}$$

and

$$\exp\left(-\lambda\int_0^{T-Z} e(s)\,ds\right) = \bar{\phi}_d.$$

for $t < T - Z$.

$e(t) = 1$ for all $t \geq T - Z$.

Beliefs

$\phi(t) = \exp\left(-\lambda\int_0^t e(s)\,ds\right)$ for $t < T - Z$

$\phi(t) = \bar{\phi}_d$ for $t \geq T - Z$.

The continuation value at the deadline at time T for an informed agents is:

$$V^{IN}(T) = -\frac{1}{\varepsilon+\tau} + \left(1-\bar{\phi}_d\right)\alpha_2$$

and for an uninformed agent is

$$V^{UN}(T) = -\frac{1}{\varepsilon} + \left(1 - \bar{\phi}_d\right)\alpha_1.$$

Hence, the difference between being informed and uninformed is

$$V^{IN}(T) - V^{UN}(T) = \bar{\phi}_d\alpha_1 + \left(1 - \bar{\phi}_d\right)\alpha_2.$$

Working backwards consider the continuation value for an agent at time $T - Z < t < T$. The informed agent has payoff

$$V^{IN}(t) = V^{IN}(T) = -\frac{1}{\varepsilon + \tau} + \left(1 - \bar{\phi}_d\right)\alpha_2$$

and the uninformed agent has payoff

$$V^{UN}(t) = -\frac{1}{\varepsilon + \tau} + \frac{\left(1 - \bar{\phi}_d\right)\alpha_2}{2} - (c + \delta)\frac{1}{2\lambda} - e^{-2\lambda(T-t)}\left[\phi^*\alpha_1 + \frac{\left(1 - \bar{\phi}_d\right)\alpha_2}{2} - (c + \delta)\frac{1}{2\lambda}\right]$$

the difference between the two is

$$V^{IN}(t) - V^{UN}(t) = \frac{\left(1 - \bar{\phi}_d\right)\alpha_2}{2} + (c + \delta)\frac{1}{2\lambda} + e^{-2\lambda(T-t)}\left[\bar{\phi}_d\alpha_1 + \frac{\left(1 - \bar{\phi}_d\right)\alpha_2}{2} - (c + \delta)\frac{1}{2\lambda}\right]$$

furthermore at $t = T - Z$

$$V^{IN}(t) - V^{UN}(t) = \frac{c}{\lambda}$$

and for $t > T - Z$

$$V^{IN}(t) - V^{UN}(t) > \frac{c}{\lambda}$$

For $0 \leq t \leq T - Z$ the continuation value for the informed individual is

$$V^{IN}(t) = -\frac{1}{\varepsilon + \tau} + \left(1 - \bar{\phi}_d\right)\alpha_2 - \delta(T - Z - t)$$

and the uninformed agent

$$V^{U}(t) = \left(1 - \exp\left(-\lambda\int_t^{T-Z} e(s)\,ds\right)\right)V^{I}(T - Z) + \exp\left(-\lambda\int_t^{T-Z} e(s)\,ds\right)V^{U}(T - Z) - \frac{c}{\lambda}\left(1 - \exp\left(-\lambda\int_t^{T-Z} e(s)\,ds\right)\right) - \delta(T - Z - t)$$

hence

$$V^{IN}(t) - V^{UN}(t) = \frac{c}{\lambda} \text{ for } 0 \leq t \leq T - Z$$

Now considering the equilibrium strategies. The subgames for $t \geq T - Z$ are identical to those described above in case 2 for $t \geq X_d$ and the proof is identical. Turning to $t < T - Z$.

Informed strategy

The condition for the informed agent to delay is

$$-\frac{1}{\varepsilon + \tau} + \left(1 - \bar{\phi}_d\right)\alpha_2 - \delta\left(X_d - t\right) \geq -\frac{1}{\varepsilon + \tau} + \left(1 - \phi(t)\right)\alpha_2$$

rearranging and substituting in for $\phi(t)$ we get

$$\exp\left(-\lambda\int_0^t e(s)\,ds\right) \geq \bar{\phi}_d + \frac{\delta}{\alpha_2}\left(X_d - t\right)$$

which is the condition on the equilibrium effort strategy of the uninformed agent.

Uninformed strategy

An uninformed agent is indifferent about exerting effort for $0 \leq t \leq T - Z$ so any effort level is a potentially an equilibrium strategy and for $t > T - Z$ the agent has strict incentives to exert effort up to the maximum effort level. An uninformed agent will not call a decision provided that

$$\begin{aligned}
-\frac{1}{\varepsilon} + \left(1 - \phi(t)\right)\alpha_1 &< V^{UN}(t) \\
-\frac{1}{\varepsilon} + \left(1 - \phi(t)\right)\alpha_1 &< -\frac{1}{\varepsilon + \tau} + \left(1 - \bar{\phi}_d\right)\alpha_2 - \frac{c}{\lambda} - \delta\left(X_d - t\right) \\
-\frac{1}{\varepsilon + \tau} + \left(1 - \phi(t)\right)\alpha_2 - \left(\phi(t)\alpha_1 + \left(1 - \phi(t)\right)\alpha_2\right) &< -\frac{1}{\varepsilon + \tau} + \left(1 - \bar{\phi}_d\right)\alpha_2 - \frac{c}{\lambda} - \delta\left(X_d - t\right)
\end{aligned}$$

we know from above that

$$-\frac{1}{\varepsilon + \tau} + \left(1 - \phi(t)\right)\alpha_2 \leq -\frac{1}{\varepsilon + \tau} + \left(1 - \bar{\phi}_d\right)\alpha_2 - \delta\left(X_d - t\right)$$

hence remains to show that

$$\phi(t)\alpha_1 + \left(1 - \phi(t)\right)\alpha_2 > \frac{c}{\lambda}$$

which is true because $\bar{\phi}_e\alpha_1 + \left(1 - \bar{\phi}_e\right)\alpha_2 = \frac{c}{\lambda}$ and $\phi(t) > \bar{\phi}_d > \bar{\phi}_e$.

iv) **Case 4** $T > Y_d + Z$

Informed strategy

$d(t) = \mathit{call}$ for $0 \leq t \leq T - Y_d - Z$

$d(t) = 0$ for $T - Y_d - Z < t < T - Z$.

$d(t) = \frac{\lambda^2\alpha_2}{\lambda\alpha_2 - \delta}$ for $t \geq T - Z$.

Uninformed strategy

$d(t) = 0$ for all t.

$e(t) = \frac{\delta}{c}$ for $0 \leq t \leq T - Y_d - Z$

$e(t)$ satisfies

$$\exp\left(-\lambda\int_{T-Y_d-Z}^{t} e(s)\,ds\right) \geq \bar{\phi}_d + (T-Z-t)\frac{\delta}{\alpha_2}$$

and

$$\exp\left(-\lambda\int_{T-Y_d-Z}^{T-Z} e(s)\,ds\right) = \bar{\phi}_d.$$

for $T-Y_d-Z<t<T-Z$.

$e(t)=1$ for $t\geq T-Z$.

Beliefs

$\phi(t)=1$ for $0\leq t\leq T-Y_d-Z$

$\phi(t)=\exp\left(-\lambda\int_{T-Y_d-Z}^{t} e(s)\,ds\right)$ for $T-Y_d-Z<t<T-Z$.

$\phi(t)=\bar{\phi}_d$ for $t\geq T-Z$.

Here again we note that all subgames starting from $t=T-Y_d+Z$ are encompassed by the proof of Case 3 above, and the continuation values at $t=T-Y_d+Z$ are

$$\begin{aligned} V^{IN}(t) &= -\frac{1}{\varepsilon+\tau} \\ V^{UN}(t) &= -\frac{1}{\varepsilon+\tau}-\frac{c}{\lambda} \end{aligned}$$

which are exactly the same continuation values as in **Case 3** of the small incentives earlier for $t=T-Y_e$. In this case the strategies and the proof for the subgames $t<Y_d+Z$ are identical to that for **Case 3** of the small incentives earlier for $t=T-Y_e$. ■

A.2 Proofs for Uniqueness

A.2.1 General Model Description

Define $\theta(t)$ as the conditional probability of being uninformed by time t given that the other agent doesn't call a decision prior to t. Hence $1-\theta(t)$ is the conditional probability of being informed by that time. Define $\mu(t)$ conditional as the probability of calling an uninformed decision by time t given that the other agent doesn't call a decision prior to t. Define $\rho(t)$ conditional probability of calling an informed decision by time t given that the other agent doesn't call a decision prior to t. $\theta(t)$ changes over time according to

$$\frac{d\theta}{dt}=-\lambda e(t)\left(\theta(t)-\mu(t)\right)$$

Effort strategy of the uninformed agent influences $\frac{d\theta}{dt}$. The decision strategy of an uninformed agent influences both $\frac{d\theta}{dt}$ and $\mu(t)$. The decision strategy of the informed agent controls $\rho(t)$. The following relationships between these functions hold

$$\mu(t)+\rho(t)\leq 1$$

also

$$\mu(t) \le \theta(t)$$

and

$$\rho(t) \le 1 - \theta(t)$$

The bayesian belief $\phi(t)$ at a time t that the other agent is uninformed conditional on no decision prior to that time is

$$\phi(t) = \frac{\theta(t) - \mu(t)}{1 - \mu(t) - \rho(t)}$$

A strategy for an agent maps into a path for $\theta(t), \mu(t), \rho(t)$. We restrict our attention to strategies which result in piecewise continuously differential functions of $\theta(t), \mu(t), \rho(t)$. Clearly given the nature of the model $\frac{d\theta}{dt} \le 0$ agents don't lose/forget signals, $\frac{d\mu}{dt}, \frac{d\rho}{dt} \ge 0$ decisions and calling a decision are irreversible. The upper bound on $e(t)$ also insures that $\theta(t)$ is continuous.

We will use $\tilde{}$ to denote the strategies of the other player. We have assumed that $\rho(t)$ and $\mu(t)$ are continuous and differentiable at all but a finite number of points. Denote the set of points where the strategy is discontinuous by $\chi_\rho = \{t_1^\rho, ...t_\nu^\rho\}, \chi_\mu = \{t_1^\mu, ...t_\nu^\mu\}, \tilde{\chi}_\rho = \{\tilde{t}_1^\rho, ...\tilde{t}_{\tilde{\nu}}^\rho\}, \tilde{\chi}_\mu = \{\tilde{t}_1^\mu, ...\tilde{t}_{\tilde{\nu}}^\mu\}$ and define $\chi = \chi_\rho \cup \chi_\mu$ and $\tilde{\chi} = \tilde{\chi}_\rho \cup \tilde{\chi}_\mu$. Also define

$$D_\rho(t) = \lim_{s \to t^+} \rho(s) - \rho(t)$$

and

$$D_\mu(t) = \lim_{s \to t^+} \mu(s) - \mu(t)$$

Clearly these are non-zero only at points in χ_ρ and χ_μ respectively and represent the probability a decision is called at that moment conditional on the other agent not calling a decision prior to that time. The objective function of the agent is:

$$
\begin{aligned}
& \max_{e(t),\mu(t),\rho(t)} -\frac{1}{\varepsilon}+\int_0^T \left(\alpha_1+\left(\frac{1-\tilde{\theta}(t)-\tilde{\rho}(t)}{1-\tilde{\mu}(t)-\tilde{\rho}(t)}\right)\alpha_2\right)\frac{d\rho}{dt}\left[1-\tilde{\mu}(t)-\tilde{\rho}(t)\right]dt \\
& +\int_0^T \left(\frac{1-\tilde{\theta}(t)-\tilde{\rho}(t)}{1-\tilde{\mu}(t)-\tilde{\rho}(t)}\right)\alpha_1\frac{d\mu}{dt}\left[1-\tilde{\mu}(t)-\tilde{\rho}(t)\right]dt \\
& +\int_0^T \left(\alpha_1+\left(\frac{1-\theta(t)-\rho(t)}{1-\mu(t)-\rho(t)}\right)\alpha_2\right)\frac{d\tilde{\rho}}{dt}\left[1-\mu(t)-\rho(t)\right]dt \\
& +\int_0^T \left(\frac{1-\theta(t)-\rho(t)}{1-\mu(t)-\rho(t)}\right)\alpha_1\frac{d\tilde{\mu}}{dt}\left[1-\mu(t)-\rho(t)\right]dt \\
& +\sum_{t\in\chi\cap\tilde{\chi}}\left[1-\tilde{\mu}(t)-\tilde{\rho}(t)\right]\left\{\begin{array}{c} D_\rho(t)\left(\alpha_1+\left(\frac{1-\tilde{\theta}(t)-\tilde{\rho}(t)}{1-\tilde{\mu}(t)-\tilde{\rho}(t)}\right)\alpha_2\right) \\ +D_\mu(t)\left(\frac{1-\tilde{\theta}(t)-\tilde{\rho}(t)}{1-\tilde{\mu}(t)-\tilde{\rho}(t)}\right)\alpha_1 \end{array}\right\} \\
& +\sum_{t\in\chi\cap\tilde{\chi}}\left[1-\mu(t)-\rho(t)-D_\rho(t)-D_\mu(t)\right]\left\{\begin{array}{c} D_{\tilde{\rho}}(t)\left(\alpha_1+\left(\frac{1-\theta(t)-\rho(t)-D_\rho(t)}{1-\mu(t)-\rho(t)-D_\rho(t)-D_\mu(t)}\right)\alpha_2\right) \\ D_{\tilde{\mu}}(t)\left(\frac{1-\theta(t)-\rho(t)-D_\rho(t)}{1-\mu(t)-\rho(t)-D_\rho(t)-D_\mu(t)}\right)\alpha_1 \end{array}\right\} \\
& +\sum_{t\in\chi\backslash\tilde{\chi}}\left[1-\tilde{\mu}(t)-\tilde{\rho}(t)\right]\left[D_\rho(t)\left(\alpha_1+\left(\frac{1-\tilde{\theta}(t)-\tilde{\rho}(t)}{1-\tilde{\mu}(t)-\tilde{\rho}(t)}\right)\alpha_2\right)+D_\mu(t)\left(\frac{1-\tilde{\theta}(t)-\tilde{\rho}(t)}{1-\tilde{\mu}(t)-\tilde{\rho}(t)}\right)\alpha_1\right] \\
& +\sum_{t\in\tilde{\chi}\backslash\chi}\left[1-\mu(t)-\rho(t)\right]\left[D_{\tilde{\rho}}(t)\left(\alpha_1+\left(\frac{1-\theta(t)-\rho(t)}{1-\mu(t)-\rho(t)}\right)\alpha_2\right)+D_{\tilde{\mu}}(t)\left(\frac{1-\theta(t)-\rho(t)}{1-\mu(t)-\rho(t)}\right)\alpha_1\right] \\
& +\left(1-\mu(T)-\rho(T)\right)\left(1-\tilde{\mu}(T)-\tilde{\rho}(T)\right)\left(\alpha_1+\left(\frac{1-\tilde{\theta}(T)-\tilde{\rho}(T)}{1-\tilde{\mu}(T)-\tilde{\rho}(T)}\right)\alpha_2\right) \\
& -\delta T\left(1-\mu(T)-\rho(T)\right)\left(1-\tilde{\mu}(T)-\tilde{\rho}(T)\right) \\
& -\int_0^T \delta\left(1-\mu(t)-\rho(t)\right)\left(1-\tilde{\mu}(t)-\tilde{\rho}(t)\right)dt \\
& -c\left(1-\tilde{\mu}(T)-\tilde{\rho}(T)\right)\left(\theta(T)-\mu(T)\right)\int_0^T e(t)\,dt \\
& -c\int_0^T e(t)\left(1-\tilde{\mu}(t)-\tilde{\rho}(t)\right)\left(\theta(t)-\mu(t)\right)dt \\
& st
\end{aligned}
$$

$$
\begin{aligned}
\rho(t) &\leq 1-\theta(t) \\
\mu(t) &\leq \theta(t) \\
\theta(0) &= 1 \\
\frac{d\theta}{dt} &= -\lambda e(t)\left(\theta(t)-\mu(t)\right)
\end{aligned}
$$

where $\tilde{\mu}(t), \tilde{\rho}(t), \tilde{\theta}(t)$ capture the strategy of the other player.

We are interested in Perfect Bayesian Equilibria of the model so strategies must be an equilibrium for all subgames starting at each time t. We describe these by writing out the problem in terms of continuation value functions for the informed $V^I(t)$ and uninformed agent $V^U(t)$ upon reaching a time t. Also write $\tilde{\rho}(s|t)$ $\tilde{\mu}(s|t)$for the perceived probabilities that an informed and uninformed agent calls a decision at $s \geq t$ given the agent is at t. At on-equilibrium times these are $\tilde{\rho}(s|t) = \frac{\tilde{\rho}(s)-\tilde{\rho}(t)}{1-\tilde{\rho}(t)}$ and $\tilde{\mu}(s|t) = \frac{\tilde{\mu}(s)-\tilde{\mu}(t)}{1-\tilde{\mu}(t)}$ however if an off-equilibrium time is reached then this is no longer the case. Write $\widehat{t}_{\rho}(t) = \inf\{r \geq t|\rho^*(r|t) > 0\}$ and $\widehat{t}_{\mu}(t) = \inf\{r \geq t|\mu^*(r|t) > 0\}$ as describing the next time at which an agents calls a decision according to the strategies $\rho^*(r|t), \mu^*(r|t)$ upon reaching a time t. The continuation value from being informed is just the payoff from implementing the optimal stopping policy $\widehat{t}$ from that moment forward:

$$\begin{aligned} V^I(t) &= \max_{\widehat{t}\in[t,T]} -\frac{1}{\varepsilon+\tau} + \int_t^{\widehat{t}} (\alpha_2 - \delta(r-t))\frac{d\tilde{\rho}(r|t)}{dt}dr + \int_t^{\widehat{t}} (-\delta(r-t))\frac{d\tilde{\mu}(r|t)}{dt}dr \qquad (12) \\ &+ \sum_{\substack{r\in\tilde{\chi} \\ t<r<\widehat{t}}} D_{\tilde{\rho}}(r|t)(\alpha_2 - \delta(r-t)) + D_{\tilde{\mu}}(r|t)(-\delta(r-t)) \\ &+ \left(1-\tilde{\mu}\left(\widehat{t}|t\right) - \tilde{\rho}\left(\widehat{t}|t\right)\right)\left(\left(1-\phi^*\left(\widehat{t}|t\right)\right)\alpha_2 - \delta\left[\widehat{t}-t\right]\right) \end{aligned}$$

where $\widehat{t}(t)$ may be set valued if the optimizer of equation 12 is not unique. The decision strategy is optimal provided that:

$$\begin{aligned} \lim_{s\to r^+} \rho^*(s|t) &= \theta(r) \text{ if } r = \hat{t}^*(t) \\ \frac{d\rho^*(r|t)}{dt} &\geq 0 \text{ or } D_{\rho^*}(r|t) \geq 0 \text{ if } r \in \hat{t}^*(t) \\ \frac{d\rho^*(r|t)}{dt} &= 0 \text{ if } r \notin \hat{t}^*(t) \end{aligned}$$

and these conditions insure ρ satisfies the adding up constraint:

$$\int_{\hat{t}^*(t)} \frac{d\rho^*(t)}{dt}dt = \theta\left(\max\left(\hat{t}^*(t)\right)\right) - \rho(t) + (\theta(T) - \rho(T)) \times \mathbf{1}\left(\max\left(\hat{t}^*(t)\right) = T\right)$$

The payoff from being uninformed is an effort and stopping problem given by:

$$
\begin{aligned}
V^U(t) = \max_{\substack{\widehat{t}\in[t,T] \\ e(r|t)\text{ for } r\in[t,\widehat{t}]}} & -\frac{1}{\varepsilon} \\
& + \int_t^{\widehat{t}} \left(\alpha_1 - \delta(r-t) - c\int_t^r e(w|t)\,dw\right) \frac{d\tilde{\rho}(r|t)}{dt} \left[1 - \exp\left(-\lambda\int_t^r e(w|t)\,dw\right)\right] dr \\
& + \int_t^{\widehat{t}} \left(-\delta(r-t) - c\int_t^r e(w|t)\,dw\right) \frac{d\tilde{\mu}(r)}{dt} \left[1 - \exp\left(-\lambda\int_t^r e(w|t)\,dw\right)\right] dr \\
& + \int_t^{\widehat{t}} \left(V^I(t) - \delta(r-t) - c\int_t^r e(w|t)\,dw\right) \lambda e(r) \exp\left(-\lambda\int_t^r e(w|t)\,dw\right) [1 - \tilde{\mu}(r|t) - \tilde{\rho}(r|t)]\, dr \\
& + \sum_{\substack{r\in\tilde{\chi}\setminus\chi \\ t<r<\widehat{t}_\mu(t)}} \frac{1-\mu(r)-\theta(r)}{1-\mu(t)-\theta(t)} \left[\begin{array}{c} D_{\tilde{\rho}}(r|t)\left(\alpha_1 - \delta(r-t) - c\int_t^r e(w|t)\,dw\right) \\ +D_{\tilde{\mu}}(r|t)\left(-\delta(r-t) - c\int_t^r e(w|t)\,dw\right) \end{array}\right] \\
& + \left(1 - \exp\left(-\int_t^{\widehat{t}} e(r)\,dr\right)\right) \left(1 - \tilde{\mu}(\widehat{t}|t) - \tilde{\rho}(\widehat{t}|t)\right) \left[\left(1 - \phi^*(\widehat{t}|t)\right)\alpha_1 - \delta(\widehat{t}-t) - c\int_t^{\widehat{t}} e(r|t)\,dr\right]
\end{aligned}
$$

In this case the condition for the effort strategy profile to be an equilibrium satisfies:

$$
e^*(t) = \arg\max_{e\in[0,1]} \lambda e\left(V^I(t) - V^U(t)\right) - ce
$$

the decision strategy is an equilibrium provided that:

$$
\begin{aligned}
\lim_{s\to r^+} \mu^*(s|t) &= 1 - \theta(r) \text{ if } r = \hat{t}^*(t) \\
\frac{d\mu^*(r|t)}{dt} &\geq 0 \text{ or } D_{\rho^*}(r|t) \geq 0 \text{ if } r \in \hat{t}^*(t) \\
\frac{d\mu^*(r|t)}{dt} &= 0 \text{ if } r \notin \hat{t}^*(t)
\end{aligned}
$$

where $\hat{t}^*(t)$ solves (**??**) the uninformed agent's effort and stopping problem. These conditions also insure it satisfies the adding up constraint

$$
\int_{\hat{t}^*(t)} \frac{d\mu^*(t)}{dt}dt = 1 - \theta\left(\max\left(\hat{t}^*(t)\right)\right) - \mu(t) + (1 - \theta(T) - \mu(T)) \times \mathbf{1}\left(\max\left(\hat{t}^*(t)\right) = T\right)
$$

A perfect bayesian equilibrium may be described by a tuple $(e^*(t), \rho^*(t), \mu^*(t), \phi^*(t))$ if $\rho^*(t) + \mu^*(t) < 1$ for all $t < T$, where $\phi^*(t)$ is the bayesian belief an agent has at time t that the other agent is uninformed conditional on no decision being called prior to that time. If $\exists t' < T : \rho^*(t') + \mu^*(t') = 1$ then it must also include off-equilibrium strategies and beliefs $(e^*(r|t), \rho^*(r|t), \mu^*(r|t), \phi^*(r|t))$ for all times t where $\rho^*(t) + \mu^*(t) = 1$ which themselves are equilibria of those subgames, where $\phi^*(r|t)$ is the bayesian belief an agent has at time r that the other agent is uninformed conditional on no decision being called prior to that time in a subgame starting at time t. We now rule out some types of decision strategies at on-equilibrium times by the uninformed agent. The following lemma rules out a continuously increasing $\mu^*(t)$.

Lemma 3 $\nexists \mu^*(t), r > 0, \varepsilon > 0 : \frac{d\mu^*(t)}{dt} > 0$ *for* $t \in [r-\varepsilon, r]$.

Proof. Suppose not and $\exists \mu^*(t) : \frac{d\mu^*(t)}{dt} > 0$ for $t \in [r-\varepsilon, r]$. Then

$$\rho^*(t) = \theta^*(t) \text{ for } t \in (r-\varepsilon, r)$$

and hence

$$\phi^*(t) = 1 \text{ for } t \in (r-\varepsilon, r)$$

if not then $\exists r' > t$ such that

$$\begin{aligned}
-\frac{1}{\varepsilon+\tau} + (1-\phi^*(t))\alpha_2 \quad \leq \quad & -\frac{1}{\varepsilon+\tau} + \left(1-\mu\left(r'|t\right)\right)\left[\left(1-\phi^*\left(r'\right)\alpha_2 - \delta\left(r'-t\right)\right)\right] \\
& -\delta\left[\int_t^{r'} (y-t)\frac{d\mu^*(y|t)}{dt}dy + \sum_{\substack{y\in\tilde{\chi} \\ t<y<r'}} D_{\tilde{\mu}}(y|t)\left(-\delta(y-t)\right)\right]
\end{aligned}$$

this can be rewritten as

$$\begin{aligned}
& \delta\left[\int_t^{r'} (y-t)\frac{d\mu^*(y|t)}{dt}dy + \sum_{\substack{y\in\tilde{\chi} \\ t<y<r'}} D_{\tilde{\mu}}(y|t)\left(-\delta(y-t)\right) + \left(r'-t\right)\left(1-\mu\left(r'|t\right)\right)\right] \\
\leq \quad & \alpha_2\left[\ (1-\mu(r'|t))(1-\phi^*(r')) - (1-\phi^*(t))\ \right]
\end{aligned}$$

but if that is the case then an uninformed agent could do strictly better by delaying until r' since the comparison of payoffs would result in the same expression except with α_1 replacing α_2. The inequality would then be strict and this would be a contradiction of the uninformed agent mixing at t. Hence $\phi^*(t) = 1$. However if $\phi^*(t) = 1$ then an uniformed agent can do strictly better by delaying and putting in effort over a period of time since

$$\lambda e_{\max}\alpha_1 > ce_{\max} + \delta$$

by assumption. ■

The following lemma rules out a jump in the decision function $\mu(t)$ at on-equilibrium times if that jump doesn't occur when both types informed and uninformed call a decision with certainty at that instant.

Lemma 4 $\nexists \mu^*(t), 0 < s < T : D_{\mu^*}(s) > 0$ *and* $\mu^*(s) + \rho^*(s) < 1$

Proof. Suppose not and $\exists s : D_{\mu^*}(s) > 0$ and $\mu^*(s) + \rho^*(s) < 1$. As above this implies $\phi^*(t) = 1$ by the same reasoning and hence an uninformed agent can do better than an immediate decision by delaying and putting in effort since

$$\lambda e_{\max}\alpha_1 > ce_{\max} + \delta$$

by assumption. ■

Hence the only equilibria involving $D_{\mu^*}(s) > 0$ also have $\mu^*(s) + \rho^*(s) = 1$ whereby beliefs at times later than s are off-equilbrium. In this case it may be possible to suppport uninformed agents calling a decision with appropriately specified off-equilibrium beliefs. However we will exclude this type of equilibrium as we feel for all intents and purposes it is equivalent to imposing a deadline at time s. We thus continue the analysis under the assumption that $\mu^*(t) = 0$ for all t. The following lemma rules out jumps in the decision function of the informed type $\rho(t)$.

Lemma 5 $\nexists \rho^*(t), 0 < t^\wedge < T : D_{\rho^*}(t^\wedge) > 0.$

Proof. Proceed with a proof by contradiction. Say there is an equilibrium with a mass point at a time $t^\wedge$ where a mass of $D_{\rho^*}(t^\wedge) = (1-\phi)\beta > 0$ decisions are called. For this to be the case then $\phi < 1$. If $\phi = 1$ $\rho^*(t) = \theta^*(t)$ informed agents may only call decision at the rate at which uninformed agents are becoming informed. Consider $\lim_{t\to t^{\wedge-}} e(t)$ and $\lim_{t\to t^{\wedge+}} e(t)$. For there to be a mass point the following conditions need to hold for an agent not to call an earlier or later decision.

$$\lim_{t\to t^{\wedge-}} e^*(t) \geq \frac{\delta}{\lambda\alpha_2\left(1-\lim_{t\to t^{\wedge-}}\phi(t^\wedge)\right)} - \varepsilon \text{ for any } \varepsilon > 0$$

and

$$\lim_{t\to t^{\wedge+}} e^*(t) \leq \frac{\delta}{\lambda\alpha_2\left(1-\lim_{t\to t^{\wedge+}}\phi(t^\wedge)\right)} + \varepsilon \text{ for any } \varepsilon > 0$$

the first of these implies that an uninformed agent will be strictly willing to wait in the neighborhood of $t^{\wedge-}$ hence will not be prepared to call a decision prior to $t^\wedge$. Also note that $\lim_{t\to t^{\wedge+}}\phi(t^\wedge) > \lim_{t\to t^{\wedge-}}\phi(t^\wedge)$ due to the mass point. This implies there is a discontinuous change in the effort level at $t^\wedge$. We can write the continuation value from being uninformed at time $t = t^\wedge - \Delta t$ as:

$$\begin{aligned}
V^U(t^\wedge - \Delta t) \;=\; & \int_{t^\wedge-\Delta t}^{t^{\wedge-}} \left(\frac{\frac{d\theta(s|t^\wedge-\Delta t)}{ds}}{1-\theta(s|t^\wedge-\Delta t)} V^I(t) + \left(-\frac{1}{\varepsilon+\tau}\right)\frac{\frac{d\rho(s|t^\wedge-\Delta t)}{ds}}{1-\rho(s|t^\wedge-\Delta t)} - ce^*(s) - \delta \right) \times \\
& (1-\theta(s|t^\wedge-\Delta t))(1-\rho(s|t^\wedge-\Delta t))\,ds \\
& + (1-\theta(t^\wedge|t^\wedge-\Delta t))\,D_\rho(t^\wedge|t^\wedge-\Delta t)\left(-\frac{1}{\varepsilon+\tau} - \delta\Delta t - c\int_{t^\wedge-\Delta t}^{t^\wedge} e^*(s)\,ds\right) \\
& \left(\begin{array}{c} \int_{t^{\wedge+}}^{t^\wedge+\Delta t} \left(\frac{\frac{d\theta(s|t^\wedge-\Delta t)}{ds}}{1-\theta(s|t^\wedge-\Delta t)} V^I(t) + \left(-\frac{1}{\varepsilon+\tau}\right)\frac{\frac{d\rho(s|t^\wedge-\Delta t)}{ds}}{1-\rho(s|t^\wedge-\Delta t)} - ce^*(s) - \delta \right) \times \\ (1-\theta(s|t^\wedge-\Delta t))(1-\rho(s|t^\wedge-\Delta t))\,ds \end{array} \right) \\
& + (1-\theta(t^\wedge+\Delta t|t^\wedge-\Delta t))(1-\rho(t^\wedge+\Delta t|t^\wedge-\Delta t))\,V^U(t^\wedge+\Delta t)
\end{aligned}$$

Where Δt may always be chosen small enough such that there are no other points of discontinuity of $\rho^*(t)$ for $t \in [t^\wedge - \Delta t, t^\wedge + \Delta t]$ other than at $t = t^\wedge$. Now consider moving a unit of effort from

$t\hat{} - \varepsilon$ to $t\hat{} + \varepsilon$ by augmenting the strategy $e^*(t)$ as follows:

$$\begin{aligned} e^{**}(t) &= e^*(t) - \varepsilon \text{ for } t \in [t\hat{} - \Delta t, t\hat{}) \\ e^{**}(t) &= e^*(t) + \varepsilon \text{ for } t \in [t\hat{}, t\hat{} + \Delta t] \end{aligned}$$

The strategies are piecewise continuous so we can always find a Δt such that they are continuous over the intervals $[t - \Delta t, t)$ and $(t, t + \Delta t]$. Using a Taylor series expansion

$$\begin{aligned} \lim_{\Delta t \to 0} \frac{V^U(t\hat{} - \Delta t | e^{**}) - V^U(t\hat{} - \Delta t | e^*)}{\Delta t} &= -\varepsilon\left(\lambda\left(\lim_{t \to t\hat{}-} V^I(t)\right) - c\right) + \lambda\varepsilon D_\rho(t\hat{} | t\hat{} - \Delta t)\left(-\frac{1}{\varepsilon + \tau}\right) \\ &\quad + \varepsilon(1 - D_\rho(t\hat{} | t\hat{} - \Delta t))\left(\lambda\left(\lim_{t \to t\hat{}+} V^I(t)\right) - c\right) - O(\Delta t) \end{aligned}$$

$$\begin{aligned} \lim_{\Delta t \to 0} \frac{V^U(t\hat{} - \Delta t | e^{**}) - V^U(t\hat{} - \Delta t | e^*)}{\Delta t} &\geq -\varepsilon\left(\lambda\left(-\frac{1}{\varepsilon + \tau} + (1 - \phi(t\hat{}))\alpha_2\right) - c\right) \\ &+ \lambda\varepsilon D_\rho(t\hat{} | t\hat{} - \Delta t)\left(-\frac{1}{\varepsilon + \tau}\right) \\ &+ \varepsilon(1 - D_\rho(t\hat{} | t\hat{} - \Delta t))\left(\lambda\left(-\frac{1}{\varepsilon + \tau} + (1 - \phi(t\hat{}) - D_\rho(t\hat{} | t\hat{} - \Delta t))\alpha_2\right) - c\right) - O(\Delta t) \end{aligned}$$

Define the right-hand side by R then

$$\begin{aligned} R &= -D_\rho(t\hat{} | t\hat{} - \Delta t)\lambda\alpha_2\varepsilon + D_\rho(t\hat{} | t\hat{} - \Delta t)c\varepsilon - O(\Delta t) \\ &= \varepsilon D_\rho(t\hat{} | t\hat{} - \Delta t)(c - \lambda\alpha_2) - O(\Delta t) \\ &> 0 \end{aligned}$$

Hence, there exists $\Delta t > 0$ such that this change in strategy is profitable which is a contradiction that the original effort $e^*(t)$ is optimal and can be part of an equilibrium. ■

This along with the earlier lemmas that uninformed individuals do not call decisions implies that $\phi(t), \rho(t), \theta(t)$ are all continuous

Lemma 6 $V^I(t)$ *is continuous.*

Proof. The continuity of $\phi(t), \rho(t), \theta(t)$ insure that

$$\begin{aligned} f\left(t, \widehat{t}\right) = -\frac{1}{\varepsilon + \tau} + \int_t^{\widehat{t}} (\alpha_2 - \delta(r - t))\frac{d\tilde{\rho}(r|t)}{dt}dr \\ + \left(1 - \tilde{\rho}\left(\widehat{t}|t\right)\right)\left(\left(1 - \phi^*\left(\widehat{t}|t\right)\right)\alpha_2 - \delta\left[\widehat{t} - t\right]\right) \end{aligned}$$

is continuous in $\hat{t}$. Hence

$$V^I(t) = \max_{\hat{t} \in [t, T]} f(t)$$

is continuous in t by Theorem of the Maximum (Berge 1963). ■

Lemma 7 *$V^U(t)$ is continuous.*

Proof. The continuity of $\phi(t), \rho(t), \theta(t)$ insure that

$$f(t, e(r|t)) = -\frac{1}{\varepsilon} + \int_t^T \left(\alpha_1 - \delta(r-t) - c\int_t^r e(w|t)\,dw\right)\frac{d\tilde{\rho}(r|t)}{dt}\left[1 - \exp\left(-\lambda\int_t^r e(w|t)\,dw\right)\right]dr$$
$$+\int_t^T \left(V^I(t) - \delta(r-t) - c\int_t^r e(w|t)\,dw\right)\lambda e(r)\exp\left(-\lambda\int_t^r e(w|t)\,dw\right)[1-\tilde{\rho}(r|t)]\,dr$$
$$+\left(1 - \exp\left(-\int_t^T e(r)\,dr\right)\right)(1-\tilde{\rho}(T|t))\left[(1-\phi^*(T|t))\,\alpha_1 - \delta(T-t) - c\int_t^T e(r|t)\,dr\right]$$

is continuous in $e(r|t)$. Hence

$$V^I(t) = \max_{e(r|t)\in C_1([t,T],[0,e_{\max}])} f(t, e(r|t))$$

where $C_1([t,T],[0,e_{\max}])$ are piecewise continuous functions with domain $[t,T]$ and range $[0,e_{\max}]$, is continuous in t by Theorem of the Maximum (Berge 1963). ■

Lemma 8 *Suppose $\rho^*(t)$ and $e^*(t)$ constitute equilibrium strategies and $\exists s, \Delta s > 0 : \frac{d\rho^*(t)}{dt} > 0$ and $0 < e^*(t) < e_{\max}$ for $t \in [s, s+\Delta s]$, then $e^*(t) = \frac{\delta}{\lambda\phi(t)\alpha_2}$ and $\lambda e^*(t) = \frac{\frac{d\rho^*(t)}{dt}}{1-\rho^*(t)}$ for $t \in [s+\Delta s]$.*

Proof. Proceed using $d(t)(1-\phi(t)) = \frac{\frac{d\rho^*(t)}{dt}}{1-\rho^*(t)}$. The incentives for delaying vs taking a decision are equal if the agent is mixing

$$-\frac{1}{\varepsilon+\tau} + (1-\phi^*(t))\,\alpha_2 = \int_t^{t+\Delta t} -\delta(r-t)\,d(r)(1-\phi(r))\exp\left(-\int_t^r d(s)(1-\phi(s))\,ds\right)dr$$
$$+\left(-\frac{1}{\varepsilon+\tau} + \alpha_2\right)\left(1 - \exp\left(-\int_t^{t+\Delta t} d(s)(1-\phi(s))\,ds\right)\right)$$
$$+\exp\left(-\int_t^r d(s)(1-\phi(s))\,ds\right)\left(-\frac{1}{\varepsilon+\tau} + (1-\phi(t+\Delta t))\,\alpha_2 - \Delta t\delta\right)$$

this may be rearranged to get

$$\delta\int_t^{t+\Delta t}(r-t)\,d(r)(1-\phi(r))\exp\left(-\int_t^r d(s)(1-\phi(s))\,ds\right)dr$$
$$+\delta\Delta t\exp\left(-\int_t^{t+\Delta t} d(s)(1-\phi(s))\,ds\right) = \alpha_2\left(\phi(t) - \phi(t+\Delta t)\exp\left(-\int_t^{t+\Delta t} d(s)(1-\phi(s))\,ds\right)\right)$$

Apply a Taylor series expansion to $\phi(t+\Delta t)$:

$$\begin{aligned}\phi(t+\Delta t) = \phi(t) - \int_t^{t+\Delta t} (\lambda e(s) - d(s)(1-\phi(s)))\,\phi(s)\,ds \\ = \phi(t)(1-\Delta t(\lambda e(t)\phi(t) - d(t)(1-\phi(t)))) \\ - \frac{d(\phi(t)(\lambda e(t) - d(t)(1-\phi(t))))}{dt}\frac{(\Delta t^2)}{2} + O(\Delta t^3)\end{aligned}$$

apply it also to $\exp\left(-\int_t^{t+\Delta t} d(s)(1-\phi(s))\,ds\right)$:

$$\begin{aligned}\exp\left(-\int_t^{t+\Delta t} d(s)(1-\phi(s))\,ds\right) = 1 - \Delta t d(t)(1-\phi(t)) - \frac{d(d(t)(1-\phi(t)))}{dt}\frac{(\Delta t)^2}{2} \\ + \frac{(d(t)(1-\phi(t)))^2(\Delta t)^2}{2} + O(\Delta t^3)\end{aligned}$$

putting these together, the inside of the brackets on the right hand side becomes

$$\begin{aligned}\phi(t) - \phi(t+\Delta t)\exp\left(-\int_t^{t+\Delta t} d(s)(1-\phi(s))\,ds\right) = -\phi(t)(\lambda e(t)\phi(t) - d(t)(1-\phi(t))) \\ + \frac{d(\phi(t)(\lambda e(t) - d(t)(1-\phi(t))))}{dt}\frac{(\Delta t^2)}{2} \\ - \phi(t)\left(\begin{array}{c}\Delta t d(t)(1-\phi(t)) - \frac{d(d(t)(1-\phi(t)))}{dt}\frac{(\Delta t)^2}{2} \\ + \frac{(d(t)(1-\phi(t)))^2(\Delta t)^2}{2}\end{array}\right) \\ - (\Delta t)^2\phi(t)(\lambda e(t)\phi(t) - d(t)(1-\phi(t)))\,d(t)(1-\phi(t)) + O(\Delta t^3)\end{aligned}$$

simplifying

$$\begin{aligned}\phi(t) - \phi(t+\Delta t)\exp\left(-\int_t^{t+\Delta t} d(s)(1-\phi(s))\,ds\right) = \Delta t\lambda e(t)\phi(t) \\ + \frac{(\Delta t)^2}{2}\frac{d(\phi(t)(\lambda e(t) - d(t)(1-\phi(t))))}{dt} \\ + \phi(t)\frac{(\Delta t)^2}{2}\left(\frac{d(d(t)(1-\phi(t)))}{dt} - (d(t)(1-\phi(t)))^2\right) \\ - (\Delta t)^2\phi(t)(\lambda e(t)\phi(t) - d(t)(1-\phi(t)))\,d(t)(1-\phi(t)) + O(\Delta t^3)\end{aligned}$$

and further

$$\begin{aligned}\phi(t) - \phi(t+\Delta t)\exp\left(-\int_t^{t+\Delta t} d(s)(1-\phi(s))\,ds\right) = \Delta t\lambda e(t)\phi(t) \\ + \frac{(\Delta t)^2}{2}\left(\frac{d(\lambda e(t)\phi(t))}{dt} - \phi(t)\lambda e(t)\,d(t)(1-\phi(t))\right) \\ + (\Delta t)^2\phi(t)(d(t)(1-\phi(t)))(d(t)(1-\phi(t)) - \lambda e(t)) + O(\Delta t^3)\end{aligned}$$

Now perform a Taylor series expansion on the left-hand side:

$$\delta \int_t^{t+\Delta t} (r-t)\, d(r)\, (1-\phi(r)) \exp\left(-\int_t^r d(s)\,(1-\phi(s))\, ds\right) dr$$
$$+ \delta \Delta t \exp\left(-\int_t^{t+\Delta t} d(s)\,(1-\phi(s))\, ds\right) = \delta \frac{(\Delta t)^2}{2} d(t)\,(1-\phi(t))$$
$$+ \delta \Delta t\,(1-\Delta t d(t)\,(1-\phi(t))) + O\left(\Delta t^3\right)$$

$$= \delta \Delta t - \delta \frac{(\Delta t)^2}{2} d(t)\,(1-\phi(t)) + O\left(\Delta t^3\right)$$

Equating the Δt terms from the left and right hand sides leads one to conclude $\delta = \lambda e(t)\,\phi(t)\,\alpha_2$, substituting this into the remaing terms of the indifference condition and moving all terms to the left-hand side yields

$$-\delta \frac{(\Delta t)^2}{2} d(t)\,(1-\phi(t)) + \frac{(\Delta t)^2}{2} \phi(t)\,\lambda e(t)\, d(t)\,(1-\phi(t))$$
$$-(\Delta t)^2 \phi(t)\,(d(t)\,(1-\phi(t)))\,(d(t)\,(1-\phi(t)) - \lambda e(t)) + O\left(\Delta t^3\right)$$
$$= -(\Delta t)^2 \phi(t)\,(d(t)\,(1-\phi(t)))\,(d(t)\,(1-\phi(t)) - \lambda e(t)) + O\left(\Delta t^3\right)$$

$$= 0$$

hence if $d(t) > 0$ and $0 < \phi(t) < 1$ then the indifference condition implies $d(t)\,(1-\phi(t)) = \lambda e(t)$ and hence $\frac{d\phi}{dt} = 0$ when $d(t) > 0$ and $\frac{d\phi}{dt} \leq 0$ for all t. ■

Lemma 9 $\phi^*(t) \geq \max\left\{\bar{\phi}_d, \bar{\phi}_e\right\}$

Proof. Suppose $\exists t : \phi^*(t) < \bar{\phi}_d$ then $\exists \Delta t > 0 : \phi^*(s) < \bar{\phi}_d$ for $s \in [t-\Delta t, t]$, however this is a contradiction as an informed player will strictly prefer to call a decision for all $s \in [t-\Delta t, t]$ and hence $\phi^*(t) > \bar{\phi}_d$ whixh is a contradiction that $\exists t : \phi^*(t) < \bar{\phi}_d$. Suppose $\exists t : \phi^*(t) < \bar{\phi}_e$ then $\exists \Delta t > 0 : \phi^*(t-\Delta t) < \bar{\phi}_e$, however this implies an upper bound on the value of information is $(1-\phi(t))\,\alpha_2 + \phi(t)\,\alpha_1$ hence for any time s such that $\phi^*(s) < \bar{\phi}_e \Rightarrow e^*(s) = 0$ therefore $\frac{d\phi^*(t)}{dt} \geq 0$ so $\phi^*(t) \geq \bar{\phi}_e$. ■

Lemma 10 *Suppose there exists an equilibrium* $\{e^*(t), d^*(t), \phi^*(t)\}$ *and times* $t'' > t'$ *such that* $(\phi^*(t') - \phi^*(t''))\,\alpha_2 > \delta(t''-t')$, *then* $d^*(t) = 0$ *for* $t \in (t', \bar{t}'')$ *where* $\bar{t}'' = \sup\{t | \phi^*(t) = \phi^*(t'')\}$

Proof. Suppose that there exists $[t_l, t_u] \subset [t', t'']$ where $d^*(t) > 0$ and $\phi^*(t) < \phi(t'')$ for $t \in [t_l, t_u]$, note from lemma 8 we also have $\frac{d\phi^*}{dt} = 0$ over this domain. If there is more than one instance of this take the last instance. Now at least one of the following two conditions hold

$$\left(\phi^*(t_u) - \phi^*\left(t''\right)\right)\alpha_2 > \delta\left(t''-t_u\right)$$

$$\left(\phi^*(t_u) - \phi^*\left(t''\right)\right)\alpha_2 = \delta\left(t''-t_u\right)$$

or

$$\left(\phi^*(t_u) - \phi^*\left(t''\right)\right)\alpha_2 < \delta\left(t'' - t_u\right)$$

If the first holds then $\exists\varepsilon < 0$ such that for $t \in [t_u - \varepsilon, t_u]$ the informed individual strictly prefers to wait until t'' to call a decision. If the third holds then $\exists\varepsilon < 0 : (\phi^*(t_u) - \phi^*(t'' + \varepsilon))\alpha_2 < \delta(t'' - t_u - \varepsilon)$ and an informed agent strictly prefers to make an immediate decision at $t = t_u + \varepsilon$ than wait until t_u which contradicts $d(t) = 0$ for $t_u < t < t''$. If $(\phi^*(t_u) - \phi^*(t''))\alpha_2 = \delta(t'' - t_u)$ then the informed agent is indifferent between making a decision at t_u and t'' however the agent must also be indifferent between making a decision at any $t \in [t_l, t_u]$ since $d(t) > 0$. This is a contradiction since a decision at $t_l \leq t < t_u$ has payoff

$$\begin{aligned}
-\frac{1}{\varepsilon+\tau} + (1 - \phi^*(t))\alpha_2 &= -\frac{1}{\varepsilon+\tau} + (1 - \phi^*(t_u))\alpha_2 \\
&= -\frac{1}{\varepsilon+\tau} + \left(1 - \phi^*\left(t''\right)\right)\alpha_2 - \delta\left(t'' - t_u\right) \\
&> -\frac{1}{\varepsilon+\tau} + \left(1 - \phi^*\left(t''\right)\right)\alpha_2 - \delta\left(t'' - t\right) \text{ for } t < t_u
\end{aligned}$$

■

Lemma 11 *Suppose $e^*(t) < e_{\max}$ for some t then under large incentives $\phi(T) = \bar{\phi}_d$ and under small incentives $\phi(T) = \bar{\phi}_e$*

Proof. Suppose not $\phi(T) > \max\{\bar{\phi}_d, \bar{\phi}_e\}$. Let $\hat{t} = \inf\{t|e^*(t) < e_{\max}\}$Now being informed at time $\hat{t}$ has continuation value given by

$$V^I\left(\hat{t}\right) = -\frac{1}{\varepsilon+\tau} + (1 - \phi(T))\alpha_2 - \delta\left(T - \hat{t}\right)$$

since the optimal strategy for an informed individual is to delay until the deadline since $e(t) = e_{\max}$ and $\phi(t) > \bar{\phi}_d$ for $t \geq \hat{t}$. Now checking the continuation value for the uninformed individual.

$$\begin{aligned}
V^U\left(\hat{t}\right) = &\left(-\frac{1}{\varepsilon+\tau} + (1 - \phi(T))\alpha_2 - \frac{c}{\lambda}\right)\left(1 - \exp\left(-\lambda e_{\max}\left(T - \hat{t}\right)\right)\right) \\
&+ \exp\left(-\lambda e_{\max}\left(T - \hat{t}\right)\right)\left(-\frac{1}{\varepsilon} + (1 - \phi(T))\alpha_1\right) - \delta\left(T - \hat{t}\right)
\end{aligned}$$

hence incentives for effort are

$$V^I\left(\hat{t}\right) - V^U\left(\hat{t}\right) = \frac{c}{\lambda} + \exp\left(-\lambda e_{\max}\left(T - \hat{t}\right)\right)\left((1 - \phi(T))\alpha_2 + \phi(T)\alpha_1 - \frac{c}{\lambda}\right)$$

also

$$
\begin{aligned}
(1-\phi(T))\,\alpha_2+\phi(T)\,\alpha_1 &> \left(\left(1-\bar{\phi}_d\right)\alpha_2+\bar{\phi}_d\alpha_1\right) \\
&> \left(\left(1-\bar{\phi}_e\right)\alpha_2+\bar{\phi}_e\alpha_1\right) \\
&= \frac{c}{\lambda}
\end{aligned}
$$

by definition of $\bar{\phi}_d$ and $\bar{\phi}_e$. Hence $V^I\left(\hat{t}\right)-V^U\left(\hat{t}\right)>\frac{c}{\lambda}$ and $\exists\delta>0: V^I\left(\hat{t}-\delta\right)-V^U\left(\hat{t}-\delta\right)>\frac{c}{\lambda}$ hence $e^*\left(\hat{t}-\delta\right)=e_{\max}$ which is a contradiction of $\hat{t}=\inf\{t|e^*(t)<e_{\max}\}$. ■

The previous lemmas restrict the set of potential equilibria to those where $\phi(t)$ is continuous, decreasing and bounded below by $\max\left\{\bar{\phi}_d,\bar{\phi}_e\right\}$. Furthermore if decisions are taken prior to the deadline then $\frac{d\phi}{dt}=0$ during those times.

A.2.2 Proof for uniqueness of symmetric equilibria set under large incentives

Define $V^{I*}(t)$ and $V^{U*}(t)$

$$
\begin{aligned}
V^{I*}(t) &= -\frac{1}{\varepsilon+\tau}+\left(1-\bar{\phi}_d\right)\alpha_2 \\
V^{U*}(t) &= -\frac{1}{\varepsilon+\tau}+\frac{\left(1-\bar{\phi}_d\right)\alpha_2}{2}-\left(c+\frac{\delta}{e_{\max}}\right)\frac{1}{2\lambda} \\
&\quad -\exp\left(-2\lambda e_{\max}(T-t)\right)\left[\bar{\phi}_d\alpha_1+\frac{\left(1-\bar{\phi}_d\right)\alpha_2}{2}-\left(c+\frac{\delta}{e_{\max}}\right)\frac{1}{2\lambda}\right]
\end{aligned}
$$

also note that

$$
\begin{aligned}
V^{I*}(t)-V^{U*}(t)=\frac{\left(1-\bar{\phi}_d\right)\alpha_2}{2}+\left(c+\frac{\delta}{e_{\max}}\right)\frac{1}{2\lambda} \\
+e^{-2\lambda(T-t)}\left[\bar{\phi}_d\alpha_1+\frac{\left(1-\bar{\phi}_d\right)\alpha_2}{2}-\left(c+\frac{\delta}{e_{\max}}\right)\frac{1}{2\lambda}\right]
\end{aligned}
$$

and provided $T-t<Z$ we have

$$
V^{I*}(t)-V^{U*}(t)>\frac{c}{\lambda}
$$

Also define $\tilde{t}_d(\phi), V^{I*}(t,\phi)$ and $V^{U*}(t,\phi)$:

$$
\tilde{t}_d(\phi)=\frac{1}{\lambda}\ln\frac{\phi(t)}{\bar{\phi}_d}
$$

$$V^{I*}(t,\phi) = \begin{cases} -\frac{1}{\varepsilon+\tau} + (1-\phi(t)\exp(-\lambda e_{\max}(T-t)))\alpha_2 - \delta(T-t) & \text{for } T-\tilde{t}_d(\phi) < t \leq T \\ V^{I*}\left(t+\tilde{t}\right) - \delta\tilde{t} & \text{for } T-\tilde{t}_d(\phi) - Z \leq t \leq T-\tilde{t}_d(\phi) \end{cases}$$

$$V^{U*}(t,\phi) = \begin{cases} (1-\exp(-\lambda e_{\max}(T-t)))\left(-\frac{1}{\varepsilon+\tau} + (1-\phi(t)\exp(-\lambda e_{\max}(T-t)))\alpha_2 - \frac{c}{\lambda}\right) & \\ +\exp(-\lambda e_{\max}(T-t))\left(-\frac{1}{\varepsilon} + (1-\phi(t)\exp(-\lambda e_{\max}(T-t)))\alpha_1\right) - \delta(T-t) & \text{for } T - \\ \left(1-\exp\left(-\lambda e_{\max}\tilde{t}\right)\right)\left(V^{I*}\left(t+\tilde{t}\right) - \frac{c}{\lambda}\right) & \\ +\exp\left(-\lambda e_{\max}\tilde{t}\right)V^{U*}\left(t+\tilde{t}\right) - \delta\tilde{t} & \text{for } T-\tilde{t}_d(\phi) \end{cases}$$

note that

$$V^{I*}(t,\phi) - V^{U*}(t,\phi) = \begin{cases} \frac{c}{\lambda} + \exp(-\lambda e_{\max}(T-t))\begin{pmatrix} \phi(t)\exp(-\lambda e_{\max}(T-t))\alpha_1 \\ +(1-\phi(t)\exp(-\lambda e_{\max}(T-t)))\alpha_2 - \frac{c}{\lambda} \end{pmatrix} & \text{for } T - \\ \frac{c}{\lambda} + \exp\left(-\lambda e_{\max}\tilde{t}\right)\left(V^{I*}\left(t+\tilde{t}\right) - V^{U*}\left(t+\tilde{t}\right) - \frac{c}{\lambda}\right) & \text{for } T-\tilde{t}_d(\phi) \end{cases}$$

hence

$$V^{I*}(t,\phi) - V^{U*}(t,\phi) > \frac{c}{\lambda} \text{ for } T-\tilde{t}_d(\phi) - Z < t \leq T$$

Lemma 12 *The unique equilbrium strategies in any subgame starting at t with beliefs $\phi(t)$ such that $t \geq T - Z - \tilde{t}_d(\phi)$ is $e^*(s) = e_{\max}$ for $t < s \leq T$* $d^*(s) = \begin{cases} 0 \text{ for } t \leq s \leq \tilde{t}_d(\phi) \\ \frac{\lambda^2\alpha_2}{\lambda\alpha_2 - \delta} \text{ for } \tilde{t}_d(\phi) < s \leq T \end{cases}$

Proof. Proceed by supposing $\exists s \geq t, \varepsilon > 0$ such that $e^*(r) < e_{\max}$ for $r \in [s-\varepsilon, s)$. If this is the case we can check the continuation values at $\widehat{s}$ where

$$\widehat{s} = \sup\{r | e^*(r) < 1\}.$$

Given that $e^*(s) = e_{\max}$ for $r \geq \widehat{s}$ then the unique decision strategy is

$$d^*(s) = \begin{cases} 0 \text{ for } \widehat{s} \leq s \leq \min\left\{T, \widehat{s} + \tilde{t}_d(\phi(\widehat{s}))\right\} \\ \frac{\lambda^2\alpha_2}{\lambda\alpha_2-\delta} \text{ for } \widehat{s} + \tilde{t}_d(\phi(\widehat{s})) < s \leq T \text{ if } \widehat{s} + \tilde{t}(\phi(\widehat{s})) < T \end{cases}$$

since the only belief at which an informed individual will call a decision is $\phi = \bar{\phi}_d$ when the uninformed agent is exerting maximum effort. Hence we can write the continuation values as $V^{I*}(\widehat{s},\phi)$, $V^{U*}(\widehat{s},\phi)$. The contradiction now comes from noting that for $t > T - Z - \tilde{t}_d(\phi)$ $V^{I*}(t,\phi) - V^{U*}(t,\phi) > \frac{c}{\lambda}$ hence $\exists\zeta : V^{I*}(r,\phi(r)) - V^{U*}(r,\phi(r)) > \frac{c}{\lambda}$ and $e^*(r) < e_{\max}$ for $r \in [\widehat{s}-\zeta, \widehat{s})$ which means $e^*(r)$ is not an equilibrium strategy. ■

Lemma 13 *Suppose $T \geq X_d + Z$ then an upper bound on $\phi^*(t)$ is given by*

$$\phi^*(t) \leq \begin{matrix} \bar{\phi}_d \exp(\lambda(T-Z-t)) & \text{for } T - X_d - Z \leq t < T - Z \\ \bar{\phi}_d & \text{for } T - Z \leq t \leq T \end{matrix}$$

Proof. Suppose $\exists t' \phi^* (t') > \bar{\phi}_d \exp (\lambda (T - Z - t'))$ for $T - X_d - Z \leq t' < T - Z$ or $\phi^* (t) > \bar{\phi}_d$ for $T - Z \leq t' \leq T$ then

$$\exists (s, \phi^* (s)) \in \left\{ \begin{array}{l} (r (\phi), \phi) \,|r (\phi) = T - X_d - Z + \frac{1}{\lambda e_{\max}} \ln \frac{1}{\phi} \\ +\gamma \left(t' - \left(T - X_d - Z + \frac{1}{\lambda e_{\max}} \ln \frac{1}{\phi}\right)\right), \gamma \in (0,1), \phi \in [\phi (t), 1] \end{array} \right\}.$$

Now $s > T - Z - \tilde{t}_d (\phi^* (s))$ hence the unique equilibrium of the subgame starting from $(s, \phi^* (s))$ is given by Lemma 12. However bayesian beliefs $\widehat{\phi}^* (r)$ in this subgame reach

$$\widehat{\phi}^* (r) = \phi^* \left(t'\right) atr = (T - X_d - Z) + \frac{1}{\lambda e_{\max}} \ln \frac{1}{\phi^* (t')} + \gamma (s) \left(t' - (T - X_d - Z) + \frac{1}{\lambda e_{\max}} \ln \frac{1}{\phi^* (t')}\right)$$

where $\gamma (s) = \frac{s - (T - X_d - Z) + \frac{1}{\lambda e_{\max}} \ln \frac{1}{\phi^* (s)}}{t' - (T - X_d - Z) + \frac{1}{\lambda e_{\max}} \ln \frac{1}{\phi^* (t')}} < 1$ hence $r < t$ and $\widehat{\phi}^* (t') > \phi^* (t')$ hence $\phi^* (t)$ is not part of a Perfect Bayesian equilibrium. ■

Together with lemma 9 this uniquely determines $\phi (t) = \bar{\phi}_d$ for $t \geq T - Z$ if $T \geq X_d + Z$

Lemma 14 *Suppose $T \geq X_d + Z$ then a lower bound on $\phi^* (t)$ is given by*

$$\phi^* (t) \geq \begin{array}{cc} \bar{\phi}_d & \text{for } t \geq T - Z \\ \bar{\phi}_d + \delta (T - Z - t) & \text{for } T - Y_d - Z \leq t < T - Z \\ 1 & \text{for } t \leq T - Y_d - Z \end{array}$$

Proof. As noted above $\phi (t) = \bar{\phi}_d$ for $t \geq T - Z$ if $T \geq X_d + Z$ is uniquely determined. Now suppose $\exists s : \phi (s) < 1$ for a $s < T - Y_d - Z$ or $\phi (s) < \bar{\phi}_d + \delta (T - Z - s)$ for $T - Y_d - Z \leq t < T - Z$. If $d (t) = 0$ for $s \leq t < T - Z$ there is an immediate contradiction as informed individuals would strictly prefer to call an immediate decision. Hence the only way this could be part of an equilibrium is if $d (t) \neq 0$ for a subset of the interval $[s, T - Z]$. Suppose that there exists $[t_l, t_u] \subset [s, T - Z]$ where $d^* (t) > 0$ and $\phi^* (t) < \bar{\phi}_d$ for $t \in [t_l, t_u]$, note from lemma 8 we also have $\frac{d\phi^*}{dt} = 0$ over this domain. If there is more than one instance of this take the last instance. Given this interval one of the following two conditions hold

$$\left(\phi^* (t_u) - \bar{\phi}_d\right) \alpha_2 > \delta (T - Z - t_u)$$

$$\left(\phi^* (t_u) - \bar{\phi}_d\right) \alpha_2 = \delta (T - Z - t_u)$$

or

$$\left(\phi^* (t_u) - \bar{\phi}_d\right) \alpha_2 < \delta (T - Z - t_u)$$

If the first holds then $\exists \varepsilon < 0$ such that for $t \in [t_u - \varepsilon, t_u]$ the informed individual strictly prefers to wait until $T - Z$ to call a decision. If the third holds then $\left(\phi^* (t_u) - \bar{\phi}_d\right) \alpha_2 < \delta (T - Z - t_u)$ and an informed agent strictly prefers to make an immediate decision at $t = t_u$ than wait until $T - Z$ which contradicts $d (t) = 0$ for $t_u < t < T - Z$. If $\left(\phi^* (t_u) - \bar{\phi}_d\right) \alpha_2 = \delta (T - Z - t_u)$ then the informed agent is indifferent between making a decision at t_u and $T-Z$ however the agent must also be indifferent between making a decision at any $t \in [t_l, t_u]$ since $d(t) > 0$, this is a contradiction since a decision at $t_l \leq t < t_u$ has payoff

$$\begin{aligned} & -\frac{1}{\varepsilon+\tau} + (1-\phi^*(t))\,\alpha_2 \\ = & -\frac{1}{\varepsilon+\tau} + (1-\phi^*(t_u))\,\alpha_2 \\ = & -\frac{1}{\varepsilon+\tau} + \left(1-\bar{\phi}_d\right)\alpha_2 - \delta(T-Z-t_u) \\ > & -\frac{1}{\varepsilon+\tau} + \left(1-\bar{\phi}_d\right)\alpha_2 - \delta(T-Z-t) \text{ for } t < t_u \end{aligned}$$

Finally $\nexists s < T-Z : \phi^*(s) = \bar{\phi}_d$ as continuation values would be given by $V^{I*}(t), V^{U*}(t)$ for the subgame starting at s which would need to have the unique equilibrium effort strategy $e^*(t) = 1$ however as shown above $V^{I*}(t) - V^{U*}(t) < \frac{c}{\lambda}$ if $t < T-Z$ hence this can not be an equilibrium. ■

These two lemmas provide an upper and lower bound on the values of $\phi^*(t)$ in equilibrium. The proof for uniqueness now proceeds by showing that the only equilibrium strategies which support values of ϕ between these bounds are the ones given in the propositions.

Proof. i) Case 1: $T < X_d$

Informed strategy

$d(t) = 0$ for all t.

Uninformed strategy

$e(t) = e_{\max}$ for all t.

Beliefs

$\phi(t) = \exp(-\lambda e_{\max} t)$ for all t.

ii) Case 2: $X_d \leq T < X_d + Z$

Informed strategy

$d(t) = 0$ for $t < X_d$.

$d(t) = \frac{\lambda^2 \alpha_2}{\lambda \alpha_2 - \delta}$ for $t \geq X_d$.

Uninformed strategy

$e(t) = e_{\max}$ for all t.

Beliefs

$\phi(t) = \exp(-\lambda e_{\max} t)$ for $t < X_d$.

$\phi(t) = \bar{\phi}_d$ for $t \geq X_d$.

Lemma 12 covers these first two cases.

iii) **Case 3** $X_d + Z < T < Y_d + Z$

Informed strategy

$d(t) = 0$ for $t < T-Z$.

$d(t) = \frac{\lambda^2 \alpha_2}{\lambda \alpha_2 - \delta}$ for $t \geq T-Z$.

Uninformed strategy

$d(t) = 0$ for all t.

$e(t)$ satisfies

$$\exp\left(-\lambda\int_0^t e(s)\,ds\right) \geq \bar{\phi}_d + (T - Z - t)\frac{\delta}{\alpha_2}$$

and

$$\exp\left(-\lambda\int_0^{T-Z} e(s)\,ds\right) = \bar{\phi}_d.$$

for $t < T - Z$.

$e(t) = 1$ for all $t \geq T - Z$.

Beliefs

$\phi(t) = \exp\left(-\lambda\int_0^t e(s)\,ds\right)$ for $t < T - Z$

$\phi(t) = \bar{\phi}_d$ for $t \geq T - Z$.

Lemmas 12, 14 and 13 determine the bounds on $\phi(t)$ and that $\phi^*(t) = \bar{\phi}_d, e^*(t) = e_{\max}$ for $t \geq T - Z$. Lemma 10 implies that $d(t) = 0$ for $t < T - Z$ which suffices along with the earlier lemmas the equilibrium strategy set.

iv) **Case 4** $T > Y_d + Z$

Informed strategy

$d(t) = call$ for $0 \leq t \leq T - Y_d - Z$

$d(t) = 0$ for $T - Y_d - Z < t < T - Z$.

$d(t) = \frac{\lambda^2\alpha_2}{\lambda\alpha_2 - \delta}$ for $t \geq T - Z$.

Uninformed strategy

$d(t) = 0$ for all t.

$e(t) = \frac{\delta}{c}$ for $0 \leq t \leq T - Y_d - Z$

$e(t)$ satisfies

$$\exp\left(-\lambda\int_{T-Y_d-Z}^t e(s)\,ds\right) \geq \bar{\phi}_d + (T - Z - t)\frac{\delta}{\alpha_2}$$

and

$$\exp\left(-\lambda\int_{T-Y_d-Z}^{T-Z} e(s)\,ds\right) = \bar{\phi}_d.$$

for $T - Y_d - Z < t < T - Z$.

$e(t) = 1$ for $t \geq T - Z$.

Beliefs

$\phi(t) = 1$ for $0 \leq t \leq T - Y_d - Z$

$\phi(t) = \exp\left(-\lambda\int_{T-Y_d-Z}^t e(s)\,ds\right)$ for $T - Y_d - Z < t < T - Z$.

$\phi(t) = \bar{\phi}_d$ for $t \geq T - Z$.

Case 3 above covers the subgames for $t > T - Z - \widehat{T}$ remains to show that the above strategies are unique for $t \leq T - Z - \widehat{T}$. For $t \leq T - Z - \widehat{T}$ we have shown that $\phi(t) = 1$. First rule out that $e(t) = 0$, if this were the case the continuation payoffs would be $V^I(t) = -\frac{1}{\varepsilon+\tau}$ and $V^U(t) = -\frac{1}{\varepsilon+\tau} - \frac{c}{\lambda} - \left(\tilde{t} - t\right)\delta$ where $\tilde{t}$ is the next time $e(t) > 0$, therefore the strategy $e(t) = 0$ is not optimal as $V^I - V^U > \frac{c}{\lambda}$. Implying that $0 < e(t) \leq \frac{\delta}{\lambda\alpha_2}$ and individuals make decision immediately $V^I(t) = -\frac{1}{\varepsilon+\tau}$ so $V^U(t) = -\frac{1}{\varepsilon+\tau} - \frac{c}{\lambda}$ for $t \leq T - Z - \widehat{T}$.

$$\begin{aligned}
V^U(t) &= \int_t^{t+\Delta t} \left(-\frac{1}{\varepsilon+\tau} - c\int_t^s e(r)\,dr - \delta(s-t)\right) 2\lambda e(s)\exp\left(-2\lambda\int_t^s e(r)\,dr\right) ds \\
&\quad + \exp\left(-2\lambda\int_t^{t+\Delta t} e(r)\,dr\right)\left(V^U(t+\Delta t) - \delta\Delta t - c\int_t^{t+\Delta t} e(r)\,dr\right) \\
&= \left(-\frac{1}{\varepsilon+\tau} - \frac{c}{2\lambda}\right)\left(1 - \exp\left(-2\lambda\int_t^{t+\Delta t} e(r)\,dr\right)\right) \\
&\quad - \int_t^{t+\Delta t} \delta(s-t)\,2\lambda e(s)\exp\left(-2\lambda\int_t^s e(r)\,dr\right) ds \\
&\quad + \exp\left(-2\lambda\int_t^{t+\Delta t} e(r)\,dr\right)\left(V^U(t+\Delta t) - \delta\Delta t\right) \\
&= -\frac{1}{\varepsilon+\tau} - \frac{c}{\lambda} + \frac{c}{2\lambda}\left(1 - \exp\left(-2\lambda\int_t^{t+\Delta t} e(r)\,dr\right)\right) \\
&\quad - \int_t^{t+\Delta t} \delta(s-t)\,2\lambda e(s)\exp\left(-2\lambda\int_t^s e(r)\,dr\right) ds \\
&\quad - \exp\left(-2\lambda\int_t^{t+\Delta t} e(r)\,dr\right)\delta\Delta t
\end{aligned}$$

hence

$$\begin{aligned}
\frac{c}{2\lambda}\left(1 - \exp\left(-2\lambda\int_t^{t+\Delta t} e(r)\,dr\right)\right) &= \int_t^{t+\Delta t} \delta(s-t)\,2\lambda e(s)\exp\left(-2\lambda\int_t^s e(r)\,dr\right) ds \\
&\quad + \exp\left(-2\lambda\int_t^{t+\Delta t} e(r)\,dr\right)\delta\Delta t \\
\frac{c}{2\lambda}2\lambda e(t)\,\Delta t + O\left(\Delta t^2\right) &= \delta\Delta t + O\left(\Delta t^2\right)
\end{aligned}$$

hence we require that $e(t) = \frac{\delta}{c}$. ■

A.2.3 Proof for uniqueness of symmetric equilibria set under small incentives

Define

$$\tilde{t}_e(\phi) = \frac{1}{\lambda}\ln\frac{\phi}{\bar{\phi}_e}$$

$$V_e^{I*}(t,\phi) = -\frac{1}{\varepsilon+\tau} + (1 - \phi\exp[-\lambda e_{\max}(T-t)])\,\alpha_2 - \delta\tilde{t}_e(\phi)$$

$$\begin{aligned}
V_e^{U*}(t,\phi) = (1 - \exp[-\lambda e_{\max}(T-t)])&\left(-\frac{1}{\varepsilon+\tau} + (1 - \phi\exp[-\lambda e_{\max}(T-t)])\,\alpha_2 - \frac{c}{\lambda}\right) \\
&+ \exp[-\lambda e_{\max}(T-t)]\left(-\frac{1}{\varepsilon} + (1 - \phi\exp[-\lambda e_{\max}(T-t)])\,\alpha_1\right) - \delta\tilde{t}_e(\phi)
\end{aligned}$$

Also

$$V_e^{I*}(t,\phi) - V_e^{U*}(t,\phi) = \frac{c}{\lambda}$$
$$+\exp\left[-\lambda e_{\max}(T-t)\right]\left(\phi\exp\left[-\lambda e_{\max}(T-t)\right]\alpha_1 + \left(1-\phi\exp\left[-\lambda e_{\max}(T-t)\right]\right)\alpha_2 - \frac{c}{\lambda}\right)$$

$$V_e^{I*}(t,\phi) - V_e^{U*}(t,\phi) > \frac{c}{\lambda}$$

provided

$$t < \tilde{t}_e(\phi)$$

Lemma 15 *The unique equilbrium strategies in any subgame starting at* t *with beliefs* $\phi(t)$ *such that* $t \geq T - \tilde{t}_e(\phi)$ *is* $e^*(s) = e_{\max}$ *for* $t \leq s \leq T$ $d^*(s) = 0$.

Proof. Proceed by supposing $\exists s \geq t, \varepsilon > 0$ such that $e^*(r) < e_{\max}$ for $r \in [s-\varepsilon, s)$. If this is the case we can check the continuation values at $\widehat{s}$ where

$$\widehat{s} = \sup\{r | e^*(r) < 1\}.$$

Given that $e^*(r) = e_{\max}$ for $r \geq \widehat{s}$ then the unique decision strategy is

$$d^*(r) = 0$$

since the only belief at which an informed individual will call a decision is $\bar{\phi}_d <$ when the uninformed agent is exerting maximum effort. Hence we can write the continuation values as $V_e^{I*}(\widehat{s}, \phi(\widehat{s}))$, $V_e^{U*}(\widehat{s}, \phi(\widehat{s}))$. The contradiction now comes from noting that $t > T - \tilde{t}_e(\phi) \Rightarrow \widehat{s} > T - -\tilde{t}_e(\phi(\widehat{s}))$ hence $V_e^{I*}(\widehat{s}, \phi(\widehat{s})) - V_e^{U*}(\widehat{s}, \phi(\widehat{s})) > \frac{c}{\lambda}$ thus $\exists \zeta : V_e^{I*}(r, \phi(r)) - V_e^{U*}(r, \phi(r)) > \frac{c}{\lambda}$ and $e^*(r) < e_{\max}$ for $r \in [\widehat{s} - \zeta, \widehat{s})$ which means $e^*(r)$ is not an equilibrium strategy. ■

Lemma 16 *Suppose* $T \geq X_e$ *then an upper bound on* $\phi^*(t)$ *is given by*

$$\phi^*(t) \leq \begin{array}{cc} 1 & \text{for } t < T - X_e \\ \bar{\phi}_e \exp(\lambda(T-t)) & \text{for } T - X_e \leq t < T \end{array}$$

Proof. Suppose $\exists t', \phi^*(t') > \bar{\phi}_e \exp(\lambda(T-t))$ for $T - X_e \leq t' < T$ then

$$\exists(s, \phi^*(s)) \in \left\{ \begin{array}{c} (r(\phi), \phi) \,|\, r(\phi) = T - X_e + \frac{1}{\lambda e_{\max}} \ln\frac{1}{\phi} \\ +\gamma\left(t' - \left(T - X_e + \frac{1}{\lambda e_{\max}} \ln\frac{1}{\phi}\right)\right), \gamma \in (0,1), \phi \in [\phi(t'), 1] \end{array} \right\}$$

Now $s > T - \tilde{t}_e(\phi^*(s))$ hence the unique equilibrium of the subgame starting from $(s, \phi^*(s))$ is given by Lemma 16. However bayesian beliefs $\widehat{\phi}^*(r)$ in this subgame reach $\widehat{\phi}^*(r) = \phi^*(t')$ at

$r = (T - X_e) + \frac{1}{\lambda e_{\max}} \ln \frac{1}{\phi^*(t')} + \gamma(s) \left(t' - (T - X_e) + \frac{1}{\lambda e_{\max}} \ln \frac{1}{\phi^*(t')} \right)$ where

$$\gamma(s) = \frac{s - (T - X_e) + \frac{1}{\lambda e_{\max}} \ln \frac{1}{\phi^*(s)}}{t' - (T - X_e) + \frac{1}{\lambda e_{\max}} \ln \frac{1}{\phi^*(t')}} < 1$$

hence $r < t'$ and $\widehat{\phi}^*(t') > \phi^*(t')$ hence $\phi^*(t')$ is not part of Perfect Bayesian Equilibrium. ■

This uniquely determines $\phi(T) = \bar{\phi}_e$ for $T \geq X_e$.

Lemma 17 *Suppose $T \geq X_e$ then a lower bound on $\phi^*(t)$ is given by*

$$\phi^*(t) \geq \begin{array}{ll} \bar{\phi}_e + \delta(T - t) & \text{for } T - Y_e \leq t < T \\ 1 & \text{for } t \leq T - Y_e \end{array}$$

Proof. As noted above $\phi(T) = \bar{\phi}_e$ if $T \geq X_e$. Now suppose $\exists s : \phi(s) < 1$ for a $s < T - Y_e$ or $\phi(s) < \bar{\phi}_e + \delta(T - s)$ for $T - Y_e \leq t < T$. If $d(t) = 0$ for $s \leq t < T$ there is an immediate contradiction as informed individuals would strictly prefer to call an immediate decision than wait until T. Hence the only way this could be part of an equilibrium is if $d(t) \neq 0$ for a subset of the interval $[s, T]$. Suppose that there exists $[t_l, t_u] \subset [s, T]$ where $d^*(t) > 0$ and $\phi^*(t) < \phi(T)$ for $t \in [t_l, t_u]$, note from lemma 8 we also have $\frac{d\phi^*}{dt} = 0$ over this domain. If there is more than one instance of this take the last instance. Given this interval one of the following two conditions hold

$$\left(\phi^*(t_u) - \bar{\phi}_e\right) \alpha_2 > \delta(T - t_u)$$

$$\left(\phi^*(t_u) - \bar{\phi}_e\right) \alpha_2 = \delta(T - t_u)$$

or

$$\left(\phi^*(t_u) - \bar{\phi}_e\right) \alpha_2 < \delta(T - t_u)$$

If the first holds then $\exists \varepsilon < 0$ such that for $t \in [t_u - \varepsilon, t_u]$ the informed individual strictly prefers to wait until T to call a decision. If the third holds then $\exists \varepsilon' < 0 : \left(\phi^*(t_u + \varepsilon') - \bar{\phi}_e\right) \alpha_2 < \delta(T - t_u - \varepsilon)$ and an informed agent strictly prefers to make an immediate decision at $t = t_u + \varepsilon'$ than wait until t_u which contradicts $d(t) = 0$ for $t_u < t < T$. If $(\phi^*(t_u) - \phi^*(T)) \alpha_2 = \delta(T - t_u)$ then the informed agent is indifferent between making a decision at t_u and T however the agent is also be indifferent between making a decision at any $t \in [t_l, t_u]$ since $d(t) > 0$, this is a contradiction since a decision at $t_l \leq t < t_u$ has payoff

$$\begin{aligned} & -\frac{1}{\varepsilon + \tau} + (1 - \phi^*(t)) \alpha_2 \\ = & -\frac{1}{\varepsilon + \tau} + (1 - \phi^*(t_u)) \alpha_2 \\ = & -\frac{1}{\varepsilon + \tau} + (1 - \phi^*(T)) \alpha_2 - \delta(T - t_u) \\ > & -\frac{1}{\varepsilon + \tau} + (1 - \phi^*(T)) \alpha_2 - \delta(T - t) \text{ for } t < t_u \end{aligned}$$

Where the final line is the payoff from waiting at time t until T which is strictly worse than an immediate decision. Finally we need to check that $\phi(t) < \bar{\phi}_e = \phi(T)$ for $t < T$. If not $\exists s < T : \phi^*(s) < \bar{\phi}_e$ and $\frac{d\phi}{dt} = 0$ for $t \geq s$. This immediately implies that the effort and decision strategies are $e^*(t) = \frac{\delta}{\lambda \bar{\phi}_e \alpha_2}$ and $d^*(t) = \frac{\delta}{(1-\bar{\phi}_e)\bar{\phi}_e \alpha_2}$ for $t \geq s$. Now calculating the continuation values at s

$$\begin{aligned} V^{I*}(t) &= -\frac{1}{\varepsilon+\tau} + \left(1-\bar{\phi}_e\right)\alpha_2 \\ V^{U*}(t) &= -\frac{1}{\varepsilon+\tau} + \frac{\left(1-\bar{\phi}_e\right)\alpha_2}{2} - \left(c + \frac{\delta}{e^*(t)}\right)\frac{1}{2\lambda} \\ &\quad - \exp\left(-2\lambda e^*(t)(T-t)\right)\left[\bar{\phi}_e \alpha_1 + \frac{\left(1-\bar{\phi}_e\right)\alpha_2}{2} - \left(c + \frac{\delta}{e^*(t)}\right)\frac{1}{2\lambda}\right] \end{aligned}$$

also note that

$$\begin{aligned} V^{I*}(t) - V^{U*}(t) &= \frac{\left(1-\bar{\phi}_e\right)\alpha_2}{2} + \left(c + \frac{\delta}{e^*(t)}\right)\frac{1}{2\lambda} \\ &\quad + \exp\left(-2\lambda e^*(t)(T-t)\right)\left[\bar{\phi}_e \alpha_1 + \frac{\left(1-\bar{\phi}_e\right)\alpha_2}{2} - \left(c + \frac{\delta}{e^*(t)}\right)\frac{1}{2\lambda}\right] \end{aligned}$$

$$V^{I*}(t) - V^{U*}(t) = \frac{\alpha_2 + \frac{c}{\lambda}}{2} + \exp\left(-2\lambda e^*(t)(T-t)\right)\left[\frac{\bar{\phi}_e \alpha_1}{2} - \frac{\bar{\phi}_e \alpha_2}{2}\right]$$

$$\lim_{t \to T} V^{I*}(t) - V^{U*}(t) = \frac{\bar{\phi}_e \alpha_1 + \frac{c}{\lambda}}{2} < \frac{c}{\lambda}$$

where the final inequality follows from the definition of $\bar{\phi}_e$. Hence $e^*(s) > 0$ is not an equilibrium strategy and hence $\nexists s < T : \phi^*(s) < \bar{\phi}_e$ ■

Proof. i) Case 1: $T < X_e$

Informed strategy

$d(t) = 0$ for all t.

Uninformed strategy

$d(t) = 0$ for all t.

$e(t) = 1$ for all t.

Beliefs

Beliefs evolve according to $\phi(t) = \exp\left(-\lambda \int_0^t e(s)\,ds\right)$ for all t.

Follows immediately from Lemma 15.

ii) Case 2: $X_e < T < Y_e$

Informed strategy

$d(t) = 0$ for all t.

Uninformed strategy

$d(t) = 0$ for all t.

$e(t)$ satisfies

$$\exp\left(-\lambda\int_0^t e(s)\,ds\right) \geq \phi_e + (T-t)\frac{\delta}{\alpha_2}$$

and

$$\exp\left(-\lambda\int_0^T e(s)\,ds\right) = \bar{\phi}_e.$$

Beliefs

Beliefs evolve according to $\phi(t) = \exp\left(-\lambda\int_0^t e(s)\,ds\right)$ for all t.

Lemma 10 and $\phi(T) = \bar{\phi}_e$ (as shown above from Lemmas 15 and 16) imply that $d(t) = 0$ for all t. The restriction on $e^*(t)$ comes from Lemma 17. Beliefs are then given by Bayesian updating.

iii) Case 3: $T > \frac{1}{\delta}(1-\bar{\phi}_e)\alpha_2$

Informed strategy

$$d(t) = \begin{cases} \textit{call} \text{ for } t < T - Y_e \\ 0 \text{ for } t \geq T - Y_e \end{cases}$$

Uninformed strategy

$d(t) = 0$ for all t.

$e(t) = \frac{\delta}{c}$ for $t < T - Y_e$

$e(t)$ satisfies

$$\exp\left(-\lambda\int_{T-Y_e}^t e(s)\,ds\right) \geq \phi_e + (T-t)\frac{\delta}{\alpha_2}$$

and

$$\exp\left(-\lambda\int_{T-Y_e}^T e(s)\,ds\right) = \bar{\phi}_e.$$

for $t \geq T - Y_e$

Beliefs

$\phi(t) = 1$ for $t < T - Y_e$.

$\phi(t) = \exp\left(-\lambda\int_0^t e(s)\,ds\right)$ for $t \geq T - Y_e$.

The proof for Case 2 encompasses the subgames for $t \geq T - Y_e$. The uniqueness for $t < T - Y_e$ is completely analogous to the proof in case 4 for large incentives of the uniqueness of equilibrium strategies for $t < T - Y_d - Z$. ■