Challengers, Democratic Contestation, and Electoral Accountability*

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Abstract

In most models of electoral accountability, the "challenger" is simply a passive replacement. We develop a model in which the challenger can actively criticize the incumbent's policy choices, and we use this model to analyze whether voters can use the challenger's critiques to enhance the incumbent's incentives to choose correct policies. If the challenger's information is soft, then challenger critiques are irrelevant, but if the challenger can obtain hard information the voter can use this information to discipline the incumbent. We extend our model to analyze selection concerns, the incumbency advantage, and incumbents' decisions about whether to focus on making policy or on generating public justifications for their policy choices.

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It is often argued that serious challengers are essential to the proper functioning of democratic accountability. For example, Ian Shapiro writes

> one of the principal reasons why opposition and political competition are essential to democratic politics is that they provide the mechanism through which democratic leaders are held to account. [*The Moral Foundations of Politics*, pp. 200-201]

Formal theories of accountability try to capture this idea, modeling elections in which the threat of removal may keep the incumbent in line. A viable challenger is crucial for this to work, because the voter's threat to remove the incumbent would not be credible if her outside option was unacceptable. In almost all existing models, however, the challenger is completely passive, so the models (by construction) overlook key tasks that challengers perform, such as criticizing the incumbent's performance.[1] The importance of analyzing challengers as active players, rather than passive replacements, is underscored by V.O. Key's frequently-cited passage:

> if a legislator is to worry about the attitude of his district, what he needs really to worry about is, not whether his performance pleases the constituency at the moment, but what the response of his constituency will be in the next campaign when persons aggrieved by his position attack his record. The constituency, thus, acquires a sanction largely through those political instruments that assure a challenge of the record. In the large, that function is an activity of the minority party. (*Public Opinion and American Democracy*, p. 499)

Several aspects of Key's argument are worth highlighting. First, the incumbent is making decisions in a forward-looking way, taking into account what voters will believe on election day as a function of what happens in the meantime. Second, voters' election day beliefs are affected by attacks made by interested parties, rather than just voters' direct observations. Third, these challenges of the

[1] In fact, the term "challenger" as used in many papers, including several of our own, is a misnomer, because the challengers don't actually challenge.

incumbent's record are what provide the voter with meaningful possibilities to sanction the incumbent for poor performance. And fourth, the opposition party is the main actor making the challenges.

Although Key's argument is an attractive account of how democratic contestation might work, it is not obvious that his claims always hang together logically. The main problem is that an electoral challenger's goal is not to provide neutral information about incumbent performance, but rather to win office. This drives a wedge between the interests of the challenger and those of the voter. So the question becomes: will a challenger transmit information to voters who know she has an incentive to misrepresent or otherwise unfairly criticize the incumbent's record in order to enjoy the benefits of office herself? What makes voters' problem particularly tricky is that even if they can design and commit to the optimal re-election rule to promote their interests, they have a very limited set of tools at their disposal – all that they can do is to retain the incumbent or replace him with the challenger. This one incentive must be used simultaneously to induce the incumbent to choose correct policies and to induce the challenger to supply information about the incumbent's record.

In this paper we develop a simple model of democratic contestation and electoral accountability to assess the intuition that voters benefit from the presence of a challenger who is an active, albeit self-interested, provider of information. The basic setup of our model is that the incumbent must exert effort to acquire information about what policy choices best promote voters' interests, and the challenger can subsequently gather information about whether the incumbent's policy choices had good effects. We analyze several variants of the model, employing different assumptions about the type of information that each actor can gather.

We begin by considering a soft information model, in which neither the incumbent nor the challenger can prove their claims to the voter. In this setting we show that voters face a tradeoff between (i) giving the incumbent a direct incentive to choose good policy (something they can do simply by using the challenger as a passive replacement rather than as an active provider of information) and (ii) giving the challenger an incentive to provide truthful information about the incumbent's performance.

Because voters cannot simultaneously accomplish both of these goals, so the fact that the challenger can gather information and report it to voters is electorally irrelevant. The voter is just as well off if he assesses the incumbent's performance directly, ignoring the challenger's statements and simply using her as a replacement in the event that the incumbent's performance is unsatisfactory.

We then ask the question of what happens if the challenger can gather hard, verifiable information about the incumbent's performance. Here we find that active challengers are electorally relevant – for example, the voter can give the incumbent an incentive to invest in policy expertise by adopting a re-election rule in which if the challenger presents compelling evidence that the incumbent's policy choice was incorrect then the incumbent is punished (removed) and the challenger is rewarded (elected).

We next extend the hard information model in three directions. First, we analyze a selection model in which voters cannot commit to a particular re-election rule, but rather play equilibrium strategies based on their updated beliefs about the incumbent's quality at the time of the election. We use this model to distinguish between different types of incumbency advantages and to analyze the link between selection and sanctioning (a la Fearon 1999).

Second, we consider the possibility that the incumbent can exert effort to generate verifiable information about optimal policy choices. The incumbent thus faces a tradeoff between learning about what policy is optimal versus making the case to the voters that this is indeed the correct policy. In a very literal sense, this is a model of accountability, because the incumbent has the capacity to provide an account for his actions To identify the effect of accountability, we solve the model under the assumption that the challenger is a passive player. The fact that the incumbent has the opportunity to account for his actions improves his policymaking relative to what would happen if he had no opportunity to justify his actions to voters. However, his effort allocation is distorted towards crafting the public case for his policy choices rather than choosing the correct policy. We then analyze the role of democratic contestation by seeing what happens if an active challenger can gather information to criticize the incumbent's policy record. We find that contestation moderates the accountability-induced distortion in incumbent effort and enables the voter to provide the incumbent with better incentives to choose appropriate policies. This happens because voters can use the information provided by the challenger as a substitute for justifications provided by the incumbent in support of his policy.

Finally, we consider possible foundations for the hard information assumption in our model. For example, voters may learn that a challenger falsified his claims because the media investigates to determine who is right when it observes disagreement between the incumbent and challenger.

Related literature

Most existing models of accountability fall into three categories: (i) theories in which all politicians are identical and the challenger is simply a replacement for the incumbent, (ii) theories in which politicians are heterogeneous and the challenger is a replacement drawn from the same pool as the incumbent, and (iii) theories in which politicians are heterogeneous and the challenger is drawn from a different pool than the incumbent.[2] Note that in all three canonical modeling setups the challenger is a completely passive player.

In a few models, challengers do take an action, typically by deciding whether to enter the race in the first place. Entry has been analyzed as something that incumbents seek to deter (Epstein and Zemsky 1995, Goodliffe 2005), or as a costly signal that the challenger knows herself to be high quality (Gordon, Huber, and Landa 2007). In a different vein, Kramer (1977) models challengers as adopting platforms in a sequence of elections between two parties. Note, however, that although these models provide valuable insights into some aspects of challenger behavior, challengers in these models only take the most minimal types of actions – entering the race or declaring platforms. Unlike the

[2] The literature is far to vast to review fully. The first category includes Ferejohn (1986) and Gordon and Huber (2002). The second includes Rogoff (1990), Maskin and Tirole (2004), Fox (2007), and Banks and Duggan (2008). The third includes Banks and Sundaram (1993), Zaller (1998), Canes-Wrone, Herron, and Shotts (2001), and Ashworth and Bueno de Mesquita (2008).

challenger in our model, they do not actively assess or criticize the incumbent's performance.

The first model we are aware of in which challengers make statements about what policies best serve voters' interests is Lemon (2005). In that model, the challenger makes an announcement either previous to or simultaneous with the incumbent's policy choice. This sequence of moves is contrary to the ordinary sequence of politics – in which incumbents take action and then opponents offer criticisms – and thus cannot be used to model challengers as attacking the incumbent's policy record.

The closest paper to ours is Warren (2009), who considers an opposition politician who can obtain a verifiable signal about the incumbent's private information. But his paper, which focuses on comparing journalistic and bureaucratic monitoring, does not have effort by the incumbent to learn what policies best promote voters' interests – instead incumbents might be non-congruent policy types. More fundamentally, the opposition in Warren's model is not motivated to win the next election, but rather to independently reveal bad information about the incumbent. This is a crucial distinction because a key tension that makes it difficult for voters to simultaneously provide incentives for challengers to provide information and for incumbents to choose good policies is the fact that both must be rewarded and punished via the same instrument, namely the outcome of the upcoming election.

The model

We analyze a single period model of policy choice followed by an election.

- The state of the world is $\omega \in \{A, B\}$. No one knows the state at the initial history; the common prior belief is that the states are equally likely.

- An incumbent must choose a policy $x \in \{A, B\}$.

- Before choosing policy, the incumbent gets a signal, $s \in \{A, B, \phi\}$. The signal is either perfect, i.e., $s \in \{A, B\}$ matches the true state of the world, or completely uninformative, $s = \phi$. The probability of receiving a perfectly informative signal is q, which is chosen by the incumbent at cost $c(q)$. This cost function is differentiable, strictly increasing and strictly convex, and satisfies $c(0) = 0$ and the usual Inada conditions.[3]

- The representative voter gets payoff 1 if the policy is correct ($x = \omega$) and payoff 0 if the policy is incorrect ($x \neq \omega$). These payoffs accrue after the election.

- After observing the incumbent's policy choice x, the challenger chooses the probability r that she gets a perfect signal of the state of the world. This effort costs $k(r)$, where k satisfies all of the usual properties.

- After observing her signal, the challenger gets to make an announcement about what the correct policy would have been. We analyze two different communication technologies, discussed below.

- The voter observes the policy implemented and the challenger's announcement and then votes for one of the two candidates.

- The incumbent and the challenger get payoff 1 from winning the election.

We consider two polar cases for the challenger's announcement.

Hard information The challenger's signal is verifiable by the voters, and cannot be faked. In this, case, a challenger who gets signal $s_c \in \{A, B, \phi\}$ must choose her report from $\{s_c, \phi\}$, where ϕ represents "no signal." Note that, if the challenger actually gets no signal, she can't report anything.

Soft information The challenger's signal itself cannot be communicated to the voter; instead, she makes a cheap talk announcement from the set $\{A, B, \phi\}$.

Because the incumbent and challenger are identical, the voter is indifferent between the candidates at any election history. As is typical in pure moral hazard models of electoral accountability, this

[3] We haven't checked to see what conditions are necessary.

creates multiple equilibria. As is also typical, we characterize the equilibrium that is best for the voter. In detail, we will fix an arbitrary voting rule for the voter, and find the incumbent's and challenger's best responses. Because the voter is indifferent at every history where he moves, each such pair are an equilibrium. We then optimize over voting rules, taking into account the political actors' best responses. One possible interpretation of a sanctioning model such as ours is that it characterizes the best possible outcome that voters could obtain in terms of accountability. We also extend our analysis below to determine whether selection concerns inhibit voters' ability to implement the optimal sanctioning rule.

It is also worthwhile to briefly discuss the interpretation of effort in our model. For an incumbent, the opportunity cost of effort exerted on policy making may be lost time to savor the perks of office, engage in graft, or promote the interests of favored constituencies. In the case of challengers it might be less time for fund-raising.

Baseline result with no challenger In this case, a reelection rule is a pair (p_A, p_B) where p_x is the probability the incumbent is reelected when she chooses policy x. Any such rule makes reelection independent of true state ω, so the incumbent has no incentive to exert any effort to identify the state. This highlights the fact that only when the voter gets some independent information about the state of the world can he control the incumbent. That's the role of the challenger in our model.

We will say that the challenger is *relevant* if the incumbent's effort choice is different than in the model without a challenger. The challenger is *irrelevant* if she is not relevant.

Soft information

We analyze two different versions of soft challenger information. In the first variant, the only information available to the voter at the time of the election is the incumbent's policy choice and the challenger's announcement about what policy is correct. In the second variant, the voter also has access to a noisy public signal about whether the policy choice was correct. In the appendix we show that a third variant, in which the voter either may learn the state of the world or learn nothing (as in Canes-Wrone, Herron, and Shotts 2001), yields results similar to those presented in the main text.

No public signal

The voter is indifferent between the two candidates at every election history, so he can choose any reelection rule as a best response to any conjecture about candidate play. He can distinguish 6 different histories h:

$$h \in \{A\phi, B\phi, AA, AB, BB, BA\}$$

where the first letter is the policy implemented by the incumbent and the second letter is the challenger's report. (ϕ signifies no report). Write p_h for the probability that the voter reelects the incumbent at history h.

Consider the case where the incumbent has implemented policy A. The challenger faces a menu of election probabilities, corresponding to the three possible reports she can make, A, B, and ϕ :

$$((1 - p_{AA}), (1 - p_{AB}), (1 - p_{A\phi}))\,.$$

Because her announcement is not tied to her signal, she can pick any of these at no cost and she will pick a report r that minimizes p_{Ar}. The only way the voter can extract accurate information is to make these three probabilities equal for each incumbent action, i.e., $p_{AA} = p_{AB} = p_{A\phi}$ and $p_{BB} = p_{BA} = p_{B\phi}$. But then the incumbent's reelection probability is independent of the true state ω, and we are back to the previous case – no control of the incumbent is possible. Thus we have our first result.

Claim 1 *Assume that the challenger's signal is soft information and there is no additional public information at the time of the election. Then the challenger is irrelevant.*

Challenger announcement plus public information

It may seem that the assumption of soft information is extreme, because the voter has no opportunity to see whether the challenger's statements are accurate. This raises a natural question: what happens if the challenger can make statements and then, before the election, the voter observes a public signal about the state of the world?

Clearly the voter could use this public information to give the challenger an incentive to provide accurate information, e.g., by electing the challenger if and only if the publicly-available information indicates that she made a correct announcement. Also, if the challenger makes accurate announcements, then the voter can use this information to discipline the incumbent, and give him an incentive to gather information that enables him to choose the correct policy.

The question, though, is whether these two goals – inducing challengers to gather and truthfully reveal information and inducing incumbents to gather information – can be achieved simultaneously. It turns out that they cannot, because the voter must use the same instrument to provide incentives for both politicians. We now develop this argument. [Not completely proved. Some gaps remain].

As before, the relevant baseline is what the voter can accomplish in the absence of a challenger. Suppose the public signal before the election is $s_p \in \{A, B\}$, and this signal is correct with probability $\psi \in (1/2, 1]$ Without a challenger, the best that the voter can do is to re-elect the incumbent if and only if $s_p = x$. Doing so maximizes the incumbent's difference between the incumbent's probability of winning when he picks the correct action and his probability of winning when he picks the incorrect action:

$$\psi - (1 - \psi) = 2\psi - 1.$$

Clearly, if $\psi = 1$, the challenger is irrelevant, because $2\psi - 1 = 1$, i.e., the incumbent is re-elected if and only if he chooses the correct action. Nothing could possibly give him stronger incentives to gather information.

The case where $\psi \in (1/2, 1)$ is a bit more complicated. Here, we show that there cannot exist a symmetric (with respect to A and B) set of re-election probabilities in which the challenger is relevant. We focus on the case where the challenger knows the true state of the world, even without exerting any information. This should be a best-case scenario for the challenger to be relevant, because we only need to worry about what the challenger will report, and not whether she is willing to work to gather information.

Let $p_{AAA} = p_{BBB}$ be the probability that the incumbent is re-elected when the challenger's statement and the public signal both indicate that the incumbent chose the correct policy. Let $p_{AAB} = p_{BBA}$ be the probability of re-election when the challenger agrees with the incumbent, but the public signal indicates that they are wrong. Let $p_{ABA} = p_{BAB}$ be the probability of re-election when the challenger criticizes the incumbent and the public signal indicates that the incumbent was correct. Finally, let $p_{ABB} = p_{BAA}$ be the probability of re-election when the challenger criticizes the incumbent and the public signal indicates that the challenger was correct.

For the challenger to be relevant, he cannot pool. Thus, because there are only two possible announcements he can make, the challenger must be weakly better off when revealing his information. So we focus on the case where the challenger perfectly reveals his signal, which maximizes the incumbent's incentives to choose the correct policy. Thus we're considering situations where the challenger will always praise correct incumbent policy choices and criticize incorrect ones.

For the incumbent to have better incentives to gather information with the challenger present than under the voter's optimal re-election scheme without the challenger requires that the difference between the incumbent's probability of winning when choosing the correct action and his probability of winning when choosing the incorrect action be strictly greater than $2\psi - 1$.

$$\begin{aligned}
[\psi p_{AAA} + (1-\psi)\, p_{AAB}] - [\psi p_{BAA} + (1-\psi)\, p_{BAB}] &> 2\psi - 1 \\
\psi p_{AAA} + (1-\psi)\, p_{AAB} - \psi p_{ABB} - (1-\psi)\, p_{ABA} &> 2\psi - 1. \qquad (1)
\end{aligned}$$

We also must ensure that the challenger is willing to report his signal truthfully, both when the incumbent's policy choice is correct, and when it is incorrect. The former requires

$$\begin{aligned}
\psi p_{AAA} + (1-\psi)\, p_{AAB} &\leq \psi p_{ABA} + (1-\psi)\, p_{ABB} \\
\psi p_{AAA} + (1-\psi)\, p_{AAB} - \psi p_{ABA} - (1-\psi)\, p_{ABB} &\leq 0. \qquad (2)
\end{aligned}$$

The latter requires

$$\begin{aligned}
\psi p_{ABB} + (1-\psi)\, p_{ABA} &\leq \psi p_{AAB} + (1-\psi)\, p_{AAA} \\
\psi p_{ABB} + (1-\psi)\, p_{ABA} - \psi p_{AAB} - (1-\psi)\, p_{AAA} &\leq 0. \qquad (3)
\end{aligned}$$

Subtracting the left hand side of Equation 2 from the left hand side of Equation 1 we see that a necessary condition for the challenger to be relevant (with symmetric incentives and truthful revelation by the challenger) is

$$\begin{aligned}
(2\psi - 1)\, p_{ABA} - (2\psi - 1)\, p_{ABB} &> 2\psi - 1. \\
p_{ABA} - p_{ABB} &> 1.
\end{aligned}$$

This is a contradiction, because both of these probabilities must be between 0 and 1. Thus we have our second result.

Claim 2 *Assume that the challenger's signal is soft information and that the public receives a (possibly noisy) public signal about the state of the world before the election. Then the challenger is irrelevant.*

Hard information

Having established results for irrelevance of challengers who can only obtain soft information, we now examine what happens if they can obtain hard information.[4] Our analysis of hard information provides a theoretical counterpart to Geer's (2006) empirical finding that negative campaign advertisements

[4] We conjecture that the results of a model in which the challenger is able to obtain either soft or hard information would be exactly like the hard information results we obtain here.

are generally fact based and his argument (in his conclusion) that challengers' negative advertisements play a key role in accountability and retrospective evaluation of incumbent performance.

As in our previous analysis the voter is indifferent between the two candidates at every election history, so he can choose any reelection rule as a best response to any conjecture about candidate play. He can distinguish 6 different histories h:

$$h \in \{A\phi, B\phi, AA, AB, BB, BA\}$$

From the symmetry of the problem, it seems that the voter will have an optimal rule in the class of symmetric rules:

$$p_{A\phi} = p_{B\phi}, \quad p_{AA} = p_{BB}, \quad p_{AB} = p_{BA}.$$

We'll proceed with this restriction in place, leaving for later the two tasks of proving that there is an optimal equilibrium in this class and determining whether there exist optimal equilibria outside this class.

It also seems obvious that the voter will want the incumbent to follow her signal. We'll assume that this is in fact what the incumbent does, and then check ex-post that it is optimal, both for the incumbent (within the equilibrium) and for the voter (across equilibria).

To find an equilibrium, fix any symmetric reelection rule $(p_{A\phi}, p_{AA}, p_{AB})$. If the incumbent chooses q (his probability of learning the state of the world) and the challenger chooses r (her probability of obtaining hard information), then the incumbent's expected utility is

$$q[rp_{AA} + (1-r)p_{A\phi}] + (1-q)[r(\frac{1}{2}p_{AA} + \frac{1}{2}p_{AB}) + (1-r)p_{A\phi}] - c(q)$$

and the first-order condition for his effort is

$$\begin{aligned} rp_{AA} + (1-r)p_{A\phi} - r(\frac{1}{2}p_{AA} + \frac{1}{2}p_{AB}) - (1-r)p_{A\phi} &= c'(q) \\ \frac{r}{2}(p_{AA} - p_{AB}) &= c'(q) \qquad (4) \end{aligned}$$

Similarly, the challenger's expected utility is

$$1 - q[rp_{AA} + (1-r)p_{A\phi}] - (1-q)[r(\frac{1}{2}p_{AA} + \frac{1}{2}p_{AB}) + (1-r)p_{A\phi}] - k(r).$$

and the first-order condition for her effort is

$$-\left(\frac{1+q}{2}\right)p_{AA} - \left(\frac{1-q}{2}\right)p_{AB} + p_{A\phi} = k'(r). \tag{5}$$

By convexity of costs, the objective functions are strictly concave so these first-order conditions are necessary and sufficient for equilibrium (modulo the reporting IC constraints, which we're still ignoring). Using either monotonicity of the marginal costs or the Edlin-Shannon monotonicity theorem, the incumbent's optimal q is strictly increasing in r so long as the voter is more likely to reelect the incumbent when the challenger reports hard information indicating that the policy was correct than when she reports that it was incorrect, i.e., $p_{AA} - p_{AB} > 0$. Likewise, the challenger's optimal r is decreasing in q for $p_{AA} - p_{AB} > 0$. It will be useful to think about this equilibrium in terms of a Cournot-style best-response diagram, crudely drawn as figure 1.

Two things are obvious from the picture (though they still need formal proof):

1. For a fixed reelection rule (and again, modulo reporting IC), there is a unique equilibrium in the induced game between the two candidates.

2. The equilibrium level of q is increasing in any variable that shifts both best responses up.

The optimal incentive scheme

Following up on the second point, Edlin-Shannon tells us that the challenger's best response is increasing in $p_{A\phi}$, and that both players' best responses are decreasing in p_{AB}. Thus the optimal equilibrium for the voter must involve $p_{A\phi} = 1$ and $p_{AB} = 0$. The optimal p_{AA} is trickier, because the incumbent's best response is increasing in that probability while the challenger's best response is decreasing. We now present two examples, to illustrate that the optimal p_{AA} may either be 1 or intermediate between 0 and 1.

Example 1. Consider a particular case: $c(q) = \frac{c}{2}q^2$ and $k(r) = \frac{k}{2}r^2$. Substituting these cost functions and $p_{A\phi} = 1$, $p_{AB} = 0$ into Equations 4 and 5 gives the equilibrium efforts as the solution to the system of linear equations:

$$\begin{aligned} \frac{r}{2}p_{AA} &= cq \qquad (6) \\ 1 - \frac{1+q}{2}p_{AA} &= kr. \qquad (7) \end{aligned}$$

Equation 6 implies $r = 2cq/p_{AA}$. We substitute this into Equation 7 to get

$$\begin{aligned} 1 - \frac{1+q}{2}p_{AA} &= 2kcq/p_{AA} \\ p_{AA} - \frac{p_{AA}^2}{2} - q\frac{p_{AA}^2}{2} &= 2kcq \\ 2p_{AA} - p_{AA}^2 &= q(4kc + p_{AA}^2) \end{aligned}$$

This yields the incumbent's equilibrium effort as a function of p_{AA}

$$q^*(p_{AA}) = \frac{2p_{AA} - p_{AA}^2}{4kc + p_{AA}^2}.$$

and the FOC for the optimal p_{AA} to maximize incumbent effort is

$$4kc\,(1 - p_{AA}) - p_{AA}^2 = 0$$

so the voter's optimal probability of re-electing the incumbent after the challenger produces information corroborating the incumbent's policy choice is $p_{AA} \in (0, 1)$.

However, as the next example shows it is also possible to have the optimal $p_{AA} = 1$.

Example 2. Let the cost functions for the incumbent and challenger be $c\,(q) = \alpha_1 q + \frac{\beta_1}{2}q^2$, and $k\,(r) = \alpha_2 r + \frac{\beta_2}{2}r^2$ respectively. For this example, set $\alpha_2 = 0$. Note that to have any incumbent effort requires $\alpha_1 < \frac{r}{2}$ because if he learns nothing his probability of re-election is $(1 - r)\,p_{A\phi} + \frac{r}{2}p_{AA} + \frac{r}{2}p_{AB}$ and thus by learning the true state of the world he can gain at most $1 - \left[(1 - r)\,p_{A\phi} + \frac{r}{2}p_{AA} + \frac{r}{2}p_{AB}\right] \leq 1 - \left[1 - r + \frac{r}{2}\right] = \frac{r}{2}$.

As via the derivations of Equations 6 and 7 from the incumbent and challenger cost functions we find FOC's for an interior solution:

$$\begin{aligned} \frac{r}{2}p_{AA} &= \alpha_1 + \beta_1 q \\ 1 - \frac{1+q}{2}p_{AA} &= \beta_2 r \end{aligned}$$

From the incumbent's FOC we obtain $r = \frac{2\alpha_1 + 2\beta_1 q}{p_{AA}}$ and plugging this into the challenger's FOC yields

$$1 - \frac{1+q}{2}p_{AA} = \beta_2 \frac{2\alpha_1 + 2\beta_1 q}{p_{AA}}$$

so for an interior optimum

$$q^*(p_{AA}) = \frac{p_{AA} - \frac{p_{AA}^2}{2} - 2\beta_2\alpha_1}{\frac{p_{AA}^2}{2} + 2\beta_1\beta_2}.$$

Differentiating with respect to p_{AA} yields

$$\frac{(1-p_{AA})\left(\frac{p_{AA}^2}{2} + 2\beta_1\beta_2\right) - p_{AA}\left(p_{AA} - \frac{p_{AA}^2}{2} - 2\beta_2\alpha_1\right)}{\left[\frac{p_{AA}^2}{2} + 2\beta_1\beta_2\right]^2}$$

Evaluated at $p_{AA} = 0$ this is strictly positive, i.e.,

$$\frac{2\beta_1\beta_2}{[2\beta_1\beta_2]^2} > 0.$$

Checking at $p_{AA} = 1$ yields

$$\frac{-\frac{1}{2} + 2\beta_2\alpha_1}{\left[\frac{p_{AA}^2}{2} + 2\beta_1\beta_2\right]^2}$$

which is strictly positive as long as $\beta_2\alpha_1 > \frac{1}{2}$, which we assume for the remainder of this example. We now show that given that the derivative is strictly positive at $p_{AA} = 0$ and $p_{AA} = 1$, it must be strictly positive for all $p_{AA} \in (0, 1)$. Suppose not, which would mean that there would be some p_{AA} such that the numerator of the derivative is equal to zero, i.e., such that

$$\begin{aligned} (1-p_{AA})\left(\frac{p_{AA}^2}{2} + 2\beta_1\beta_2\right) - p_{AA}\left(p_{AA} - \frac{p_{AA}^2}{2} - 2\beta_2\alpha_1\right) &= 0 \\ \frac{p_{AA}^2}{2} + 2\beta_1\beta_2 - \frac{p_{AA}^3}{2} - 2\beta_1\beta_2 p_{AA} - p_{AA}^2 + \frac{p_{AA}^3}{2} + 2\beta_2\alpha_1 p_{AA} &= 0 \\ -\frac{p_{AA}^2}{2} + 2\beta_1\beta_2(1-p_{AA}) + 2\beta_2\alpha_1 p_{AA} &= 0 \end{aligned}$$

Note that the derivative of this with respect to p_{AA} is

$$-p_{AA} - 2\beta_1\beta_2 + 2\beta_2\alpha_1$$

which is decreasing in p_{AA} so if the numerator is zero at some $p_{AA} \in (0,1)$ then it can't be zero at any higher value of p_{AA} and thus at $p_{AA} = 1$ we'd have $\frac{(1-p_{AA})\left(\frac{p_{AA}^2}{2}+2\beta_1\beta_2\right)-p_{AA}\left(p_{AA}-\frac{p_{AA}^2}{2}-2\beta_2\alpha_1\right)}{\left[\frac{p_{AA}^2}{2}+2\beta_1\beta_2\right]^2} < 0$, a contradiction.

Thus we have shown that for this parametrization, incumbent effort is increasing in p_{AA} for all $p_{AA} \in [0,1]$ and the optimal re-election rule for the voter to adopt is $p_{AA} = 1$, $p_{A\phi} = 1, p_{AB} = 0$.

Some implications of the analysis

1. Adding challenger who can gather hard information about incumbent performance makes the voter strictly better off.

2. The incumbent loses for sure if the challenger presents evidence that the policy choice was wrong, i.e., $p_{AB} = 0$.

3. Less obviously, the incumbent wins for sure if the challenger is silent, i.e., $p_{A\phi} = 1$. This is a kind of incumbency advantage. The reason the voter sets $p_{A\phi} = 1$ is to motivate the challenger to search for information. It is interesting, in light of traditional thinking on the incumbency advantage, that the advantage arises here as part of the optimal incentive scheme.

4. In Example 1, the voter mixes if the challenger says that the incumbent's policy choice was correct. Thus confirmation that her policy was correct hurts the incumbent relative to what would happen if the challenger uncovers no information.

5. In Example 2, the voter re-elects the incumbent any time the challenger produces information indicating that the incumbent's policy choice was correct. Note that this pattern of voter behavior with $p_{AA} = p_{A\phi} = 1$ is consistent with the interpretation that whenever the challenger

uncovers hard information indicating that the incumbent's policy choice was correct, she conceals this information. This fits with the intuition that challengers look for, and report, negative information, but don't ever report positive information about the incumbent.

Selection

Our first extension of our hard information model is to analyze selection. We assume that the incumbent's action can reveal something about his type, but the challenger's action reveals nothing about her type. This assumption is reasonable if the incumbent's ability to learn about the correct policy today is correlated to his ability to do so in the future, whereas the challenger's ability to dig up information about whether the incumbent chose the correct action is unrelated to her ability to choose correct policies if elected.

Specifically, assume that there are two types of politicians $\theta \in \{h, l\}$. A type h incumbent has the same technology for learning about policy as in our regular model. A type l incumbent cannot learn anything, regardless of how hard he works. The challenger's technology for generating hard information about the true state of the world is the same as in our previous model. Assume that κ is the prior probability that the incumbent is high quality and γ is the prior probability that the challenger is high quality.

An equilibrium consists of (among other things)

- The high-quality incumbent's choice of q
- The challenger's choice of r
- The voter's probability of re-electing the incumbent (for now assumed to be symmetric with respect to A and B) based on the challenger's announcement: $p_{AA}, p_{A\phi}, p_{AB}$, and
- The voter's beliefs about the probability that the incumbent is high quality at each history: $\mu_{AA}, \mu_{A\phi}, \mu_{AB}$.

We now discuss some equilibria for the model with selection. To parallel our sanctioning model, when multiple equilibria exist we should choose the one with the maximal q, but we haven't checked to confirm that the ones we present here indeed do that.

Incumbent ahead of challenger, $\kappa > \gamma$. There exists an equilibrium in which $q > 0$, $r > 0$, $p^*_{AA} = p^*_{A\phi} = 1$, $p^*_{AB} = 0$, the incumbent always follows his signal, and the challenger always reports any hard information that she obtains. Note that the re-election probabilities specified here induce strictly positive effort by both the incumbent and challenger. The reason $p^*_{A\phi} = 1$ is that $\mu^*_{A\phi} = \kappa > \gamma$. And $p^*_{AA} = 1$ because $\mu^*_{AA} > \mu^*_{A\phi} = \kappa > \gamma$. Note that to have $p^*_{AB} = 0$ we can't have κ be too much greater than γ, because if the difference between the two is too great then the voter won't update down far enough after observing hard information indicating that the incumbent chose the wrong policy.

This equilibrium may be just as good for the voter, in terms of inducing high q, as what the voter can do if he precommits to an electoral rule. This is the case if the optimal rule has $p_{AA} = 1$.

However it might be worse for the voter. This is the case if the optimal rule has $p_{AA} < 1$. Note that in this case the incumbent does better than he would have done in our sanctioning model, in the sense of having a higher probability of re-election, conditional on choosing the correct policy and having the challenger confirm that the policy choice was correct. This is a type of incumbency advantage, and a perverse one – due to selection concerns voters treat the incumbent too well and get worse accountability.

Incumbent and challenger tied, $\kappa = \gamma$. There exists an equilibrium in which $q > 0$, $r > 0$, $p^*_{AA} = p^*_{A\phi} = 1$, $p^*_{AB} = 0$, the incumbent always follows his signal and the challenger always reports any hard information that he obtains. Note that the re-election probabilities specified here induce strictly positive effort by both the incumbent and challenger. The reason $p^*_{A\phi} = 1$ is ok is that $\mu^*_{A\phi} = \kappa = \gamma$. And $p^*_{AA} = 1$ because $\mu^*_{AA} > \mu^*_{A\phi} = \kappa > \gamma$. Note also that any equilibrium in which the challenger exerts positive effort and reports (rather than concealing) the hard information she gathers requires $p^*_{AA} \leq p^*_{A\phi}$ and $p^*_{AB} \leq p^*_{A\phi}$ with at least one inequality strict.

As in the $\kappa > \gamma$ case this equilibria may be just as good for the voter as what he could do in a pure sanctioning model, or it may involve a perverse incumbency advantage.

The $\kappa = \gamma$ case can also be used to analyze a precise measure of the incumbency advantage. The term "incumbency advantage" is often used to refer simply to the fact that incumbents tend to beat challengers. A more precise definition is that an incumbency advantage occurs when the incumbent has a greater than 50% chance of winning the election, solely due to his status as the incumbent. Here, the incumbent and challenger are ex ante identical in terms of quality, $\kappa = \gamma$, but only differ in the types of actions they can take in their respective roles in the game.[5]

Specifically, note that the ex ante probability that the incumbent wins re-election is strictly greater than $\frac{1}{2}$. This is true averaging over incumbent types: the incumbent sometimes learns the state, and even if he doesn't learn the state he only loses office if he chooses the wrong policy (which occurs with probability $\frac{1}{2}$ conditional on having no information) *and* the challenger obtains hard information indicating that it was the wrong policy choice (which happens with probability r). In fact, even a low-quality incumbent is re-elected with probability strictly greater than $\frac{1}{2}$ in this model.

Incumbent behind challenger $\kappa < \gamma$ This, for reasons pointed out by Ashworth and Bueno de Mesquita (2008), is the least substantively likely case.

The concealment constraint $p^*_{AA} \leq p^*_{A\phi}$ and $p^*_{AB} \leq p^*_{A\phi}$ is important here. We'll look for an equilibrium with $p^*_{AB} = 0$ and $p^*_{AA} = p^*_{A\phi} = 1$. This will require that $\mu^*_{AA} \geq \gamma$ and $\mu^*_{A\phi} \geq \gamma$. This can be accomplished by having the challenger declare ϕ some of the time when he gets information that the incumbent chose the correct policy (probably easiest to say he does it all of the time).

We conjecture that this works for κ sufficiently close to γ but that for κ too low it will be impossible to get $\mu^*_{A\phi} \geq \gamma$ and thus there will be no equilibrium with strictly positive incumbent policy making

[5]The "incumbency advantage" we talked about above for the $\kappa > \gamma$ case is relative to the voter's optimal re-election rule with commitment, which is a different definition of incumbency advantage.

effort.

Incumbent policy justification

We now develop the second extension of our hard information model. To do this, we return to the sanctioning setup and enrich our model by allowing the incumbent to work on generating a justification for his policy choice. In particular, the incumbent can exert two types of effort. The first determines the probability that he will obtain a signal about which policy is correct. The second determines the probability that, conditional on obtaining a signal, this signal is hard information that he can present to voters as a justification for his policy choice. We first analyze this model without a challenger, and then determine how things change in the presence of a challenger.

Without a challenger

There are five possible signals the incumbent can receive: $s \in \left\{A, B, \bar{A}, \bar{B}, \phi\right\}$, where A and B are soft information perfectly revealing to the incumbent the state of the world, $\bar{A}$ and $\bar{B}$ are hard information revealing to the incumbent (and the voter, if the incumbent so chooses) the state of the world, and ϕ is no signal. The incumbent chooses $q = \Pr(s \neq \phi)$ and $\alpha = \Pr(s \in \left\{\bar{A}, \bar{B}\right\} \mid s \in \left\{A, B, \bar{A}, \bar{B}\right\})$. His cost function is $c(q + \alpha)$ for $c(\cdot)$ increasing and convex.

At the time of the election, the voter observes the incumbent's policy choice x, and, in the event that the incumbent produces hard information, this information $s \in \left\{\bar{A}, \bar{B}\right\}$ as well. Obviously, it is optimal for the voter to use a symmetric re-election scheme, re-electing the incumbent with probability 1 whenever he produces hard information that he chose the correct policy, and removing the incumbent otherwise.

This gives the following first order conditions for optimal incumbent effort:

$$\alpha = c'(q + \alpha) \tag{8}$$

$$q = c'(q + \alpha) \tag{9}$$

These imply that $q = \alpha$, i.e., the incumbent exerts as much effort on justification as he does on expertise. This is obviously better than in the absence of accountability, because, as we showed in our first baseline model, in that circumstance the incumbent exerts no effort whatsoever. However, the presence of accountability does distort the incumbent's effort allocation, because the voter would prefer that the incumbent devote his effort to choosing policy rather than generating a justification.

With a challenger

We now add in a challenger who has access to the same hard information technology as in our previous model. The relevant symmetric probabilities of election are $(p_{AA}, p_{A\phi A}, p_{A\phi\phi}, p_{A\phi B})$, corresponding to the case where the incumbent provides hard information in support of his policy, the case where the challenger provides hard information in support of the incumbent's policy, the case where neither actor provides any hard information, and the case where the challenger provides hard information that the incumbent's policy was incorrect.

The incumbent's payoff is:

$$q[\alpha p_{AA} + (1-\alpha)(rp_{A\phi A} + (1-r)p_{A\phi\phi})] + (1-q)[r(\frac{1}{2}p_{A\phi A} + \frac{1}{2}p_{A\phi B}) + (1-r)p_{A\phi\phi}] - c(q+\alpha)$$

and the first order conditions for his choice are

$$\begin{aligned} \alpha p_{AA} + r(1-\alpha-\frac{1}{2})p_{A\phi A} - \alpha(1-r)p_{A\phi\phi} - \frac{1}{2}rp_{A\phi B} &= c'(q+\alpha) && (10) \\ q[p_{AA} - (rp_{A\phi A} + (1-r)p_{A\phi\phi})] &= c'(q+\alpha). && (11) \end{aligned}$$

Similarly, the challenger's payoff is

$$1 - q[\alpha p_{AA} + (1-\alpha)(rp_{A\phi A} + (1-r)p_{A\phi\phi})] - (1-q)[r(\frac{1}{2}p_{A\phi A} + \frac{1}{2}p_{A\phi B}) + (1-r)p_{A\phi\phi}] - k(r).$$

and her first order condition is

$$-q(1-\alpha)(p_{A\phi A} - p_{A\phi\phi}) - (1-q)(\frac{1}{2}p_{A\phi A} + \frac{1}{2}p_{A\phi B} - p_{A\phi\phi}) = k'(r). \quad (12)$$

These are somewhat complicated, but we can already see that total incumbent effort may be lower in the presence of the challenger. Summing Equations 10 and 11 gives

$$(\alpha + q)p_{AA} - p_{A\phi A}r\left(\frac{1}{2} - \alpha\right) - p_{A\phi\phi}(1+\alpha)(1-r) - \frac{1}{2}rp_{A\phi B} = 2c'(q+\alpha)$$

whereas performing the analogous operation with Equations 8 and 9 gives

$$(\alpha + q) = 2c'(q+\alpha).$$

However, what the voter cares about is not the total effort but rather the effort exerted on policy expertise, i.e., q. One feasible voter strategy is to ignore the challenger and implement the optimal rule from before. To show that contestation makes the voter strictly better off, it suffices to show that, locally, rewarding the challenger for information improves incentives for the incumbent. To do so, we focus on changes along the manifold $p_{A\phi A} = p_{A\phi\phi} \equiv p$ (holding fixed $p_{AA} = 1$ and $p_{A\phi B} = 0$ and noting that $p = 0$ without a challenger). From that point, we show that, starting from the no challenger baseline, increasing p increases q.

We rewrite the first order conditions from Equations 10, 11, and 12 as the matrix equation

$$\begin{aligned}
\alpha + (\frac{r}{2} - \alpha)p - c'(q+\alpha) &= 0 \\
q(1-p) - c'(q+\alpha) &= 0 \\
\frac{1}{2}(1-q)p - k'(r) &= 0.
\end{aligned}$$

Totally differentiate with respect to p to get

$$\begin{aligned}
\alpha_p + (\frac{r_p}{2} - \alpha_p)p + (\frac{r}{2} - \alpha) - c''(\alpha+q)(\alpha_p + q_p) &= 0 \\
q_p(1-p) - q - c''(q+\alpha)(\alpha_p + q_p) &= 0 \\
-\frac{1}{2}q_p p + \frac{1}{2}(1-q) - k''(r)r_p &= 0.
\end{aligned}$$

Rewrite these as

$$\begin{pmatrix} 1-p-c'' & -c'' & \frac{1}{2}p \\ -c'' & 1-p-c'' & 0 \\ 0 & -\frac{1}{2}p & -k'' \end{pmatrix} \begin{pmatrix} \alpha_p \\ q_p \\ r_p \end{pmatrix} = \begin{pmatrix} \alpha - \frac{r}{2} \\ q \\ -\frac{1}{2}(1-q) \end{pmatrix}.$$

Cramer's Rule gives us

$$q_p = \frac{\begin{vmatrix} 1-p-c'' & \alpha - \frac{r}{2} & \frac{1}{2}p \\ -c'' & q & 0 \\ 0 & -\frac{1}{2}(1-q) & -k'' \end{vmatrix}}{\begin{vmatrix} 1-p-c'' & -c'' & \frac{1}{2}p \\ -c'' & 1-p-c'' & 0 \\ 0 & -\frac{1}{2}p & -k'' \end{vmatrix}}.$$

Evaluating the determinants gives

$$q_p = \frac{(1-p-c'')(-qk'') + c''(-k''(\alpha - r/2) + \frac{1}{4}(1-q)p)}{(1-p-c'')(-k''(1-p-c'')) + c''(c''k'' + p^2/4)}.$$

Next we impose $p = 0$, which implies $r = 0$ and $\alpha = q$, to get

$$q_p = -\frac{q}{2c'' - 1}$$

Finally, note that the second-order condition for the no-challenger condition is

$$\begin{vmatrix} 1-c'' & -c'' \\ -c'' & 1-c'' \end{vmatrix} = 1 - 2c'' > 0,$$

so we have

$$q_p > 0,$$

and thus adding the challenger allows the voter to induce greater incumbent effort on policy choice.

Hardening challenger's signal

We now develop the third extension of our hard information model. So far, in the hard information model, we have assumed that challengers cannot lie. One common justification for this type of assumption is that lying is not intrinsically impossible, but there's a chance of getting caught, and the actor, if caught, is subject to some punishment sufficient to deter lying. We now provide a brief, semi-formalized analysis of this intuition. It turns out that the nature of the punishment matters.

If the punishment is after the election (e.g., because the challenger cares about winning other offices or her standing in history, or because the challenger is a long-lived party that could be punished in the polls in the future) then if the punishment is sufficiently severe and detection is sufficiently likely, the model works exactly the way that we characterized it in the section on hard information. If the punishment is after election and the punishment is only moderately severe, and/or only somewhat likely to happen, then the challenger cannot be induced to behave truthfully in the rule that we characterized before. To induce accurate challenger statements the voter must adopt a re-election rule that provides incentives for the incumbent that are less powerful than the ones we previously characterized. However, the voter can at least give the incumbent *some* incentive to gather information, in contrast to the baseline model without a challenger.

Exogenous detection and punishment. Let ρ be the probability that the voter learns the challenger's true private signal, and that if the challenger is found out to have lied then he pays an exogenous cost R at the end of the game. Suppose that the optimal rule is such that $p_{AA} = p_{A\phi} = 1$, and $p_{AB} = 0$. Suppose the challenger's private information indicates that the incumbent's policy choice of A was correct. Then for the challenger to truthfully report A we need

$$1 - p_{AB} - \rho R > 0.$$

Recall that regardless of the values of p_{AA} and $p_{A\phi}$ the incumbent's incentive to invest, and hence the voter's utility, is decreasing in p_{AB}. Thus the optimal p_{AB} that makes it incentive-compatible for the challenger to truthfully report information when she learns that the incumbent's policy choice is correct is

$$p_{AB} = \max\{0, 1 - \rho R\}.$$

As an aside, note that the challenger still has a strictly positive incentive to invest in information-gathering here, because ex ante she doesn't know whether she'll learn that the incumbent's choice is correct, and if she does see that the incumbent's choice was incorrect then she gets utility strictly greater than zero by reporting this (because she wins with probability p_{AB}).

If ρR is sufficiently large then the challenger is fully deterred from lying. Also, for any $\rho > 0$ and $R > 0$ the voter is strictly better off with the challenger than without. This provides information for what sorts of challengers are most useful and least likely to lie – those who have strong reasons to care about their long run reputations.

Media detection and electoral punishment. Now suppose there's no ex post cost R for being caught lying, but rather that there exists a media that has interest in conflict, i.e., it investigates if and only if the challenger's announcement differs from the incumbent's policy choice. When the media investigates it learns the challenger's true signal with probability ρ.

First note that in our baseline hard information model there are IC constraints for reporting one's signal, e.g., if you see A you report A rather than ϕ. We solved that model without those constraints and showed afterwards that they were satisfied by the optimal rule for voters to use.

For reporting constraints in the presence of a media that investigates conflict, note that we must have $p_{AA} = p_{A\phi}$, because if not then given that the media doesn't investigate after either AA or $A\phi$, a challenger who sees A or ϕ will simply report whichever of those gives her the higher probability of winning.

Second, we need challenger to not claim that she sees B when seeing signal of A or ϕ. The relevant incentive constraint for truthful reporting is that the challenger's probability of winning if she reports that she saw A be higher than if she lies and claim to have seen B, i.e.,

$$1 - p_{AA} \geq (1-\rho)(1-p_{AB}) + \rho(1-p_{ABL})$$

where p_{ABL} is the probability that the incumbent wins after the challenger is caught lying.

There are two cases. In case 1, the optimal rule in the baseline hard information model was $p_{AA} = p_{A\phi} = 1$. In this case, the left hand side of the equation above would have to be zero, so clearly the fact that information isn't hard makes the voter worse off.

In case 2, the optimal rule in the baseline hard information case was $p_{A\phi} = 1, p_{AA} \in (0,1)$. Without hard information, though, these probabilities are constrained to be equal to each other so the voter is strictly worse off than in the baseline hard information model.

We now characterize the optimal rule for the voter in the model of soft information with media investigations. Clearly $p_{ABL} = 1$ is optimal because it maximally relaxes the truthful reporting constraint for the challenger and does not affect on the path incentives for the incumbent or challenger. The next step is to satisfy with equality

$$\begin{aligned} 1 - p_{AA} &\geq (1-\rho)(1-p_{AB}) + \rho(1-p_{ABL}) \\ 1 - p_{AA} &\geq (1-\rho)(1-p_{AB}) \\ p_{AA} &= \rho + (1-\rho)p_{AB} \end{aligned}$$

From Equations 4 and 5 we can show that for each actor's investment incentives we want to maximize $p_{AA} - p_{AB}$

$$\begin{aligned} & p_{AA} - p_{AB} \\ = {} & \rho + (1-\rho)p_{AB} - p_{AB} \\ = {} & \rho - \rho p_{AB} \end{aligned}$$

So the best that the voter can do when using the media as a check on challenger lying is to set $p_{AB} = 0$ and $p_{AA} = p_{A\phi} = \rho$.

Discussion and possible future work

One potentially interesting question for future papers concerns the optimal number of challengers. There are two interpretations of multiple challengers: multi-party competition or a primary contest in the challenging party. In either case, we would like to know how total effort and incumbent effort vary with the number of parties. It seems like there should be two effects. First, more challengers mean greater probability of finding information at fixed effort per challenger. But more challengers also means less effort from each of them.

Appendix: soft information with probabilistic revelation of state

In the main text we analyzed two polar cases. In one, the voter never receives a public signal, so the challenger's announcement cannot be credible. In the other, the voter always receives a public signal and thus can ignore the challenger's announcement. An intermediate possibility is that, as in Canes-Wrone, Herron and Shotts (2001), with some probability $\rho \in (0,1)$ the voter receives a public signal. An interesting question is whether in such a setup, the voter can use the public signal to harden the challenger's soft information. This requires that the voter discipline the challenger and also use the challenger's announcement to discipline the incumbent in the event that no public signal arrives before the election. It is non-obvious whether the voter can make himself better off by such an indirect scheme because the scheme would also have at least some bad direct effects. Specifically, relative to a scheme that ignores the challenger, it would diminish the incumbent's incentive to choose the correct policy in the event that a public signal arrives before the election.

We now develop a variant of our model to analyze this case. We look for an equilibrium that is symmetric with respect to A and B. Suppose the challenger has invested, and learns ω with probability r. We first deal with the special case of $r = 1$, then the more general case. We also assume that if uncertainty resolves, the voter learns the true value of ω. We may want to generalize this to having the public signal be correct with probability $\psi \in (1/2, 1]$.

To determine whether presence of a challenger can lead to higher q, we need to check two things.

1. Incumbent investment incentives. Specifically, the incumbent's level of investment will depend on $\Delta \equiv \Pr(win|s \in \{A,B\}) - \Pr(win|s = \phi)$, i.e., the difference that getting information makes in his probability of winning re-election. To check this, we first find the best that the voter can do without a challenger. We then see whether the voter can set up a greater Δ using the challenger.

2. Challenger announcement constraint. Specifically, conditional on learning ω, does the challenger

weakly prefer to tell the truth? Note that if, given the voter's re-election rule, the challenger weakly prefers to tell the truth then we can have her always tell the truth in an equilibrium, which should be better for setting up incumbent incentives.

We proceed in three steps. First, we determine the maximal incumbent investment incentive in the absence of a challenger. We then assume that the challenger is informed with probability 1 and show that if the challenger announcement constraint is satisfied, then incumbent investment incentives can be no stronger than in the absence of a challenger. Finally, we analyze the case of a challenger who is formed with probability $r \in (0,1)$.

No challenger. A symmetric, with respect to A and B, re-election rule for the voter consists of: p_ϕ (probability of re-election given no public signal), p_ω (probability of re-election given public information that $x = \omega$), and $p_{\tilde{}\omega}$ (probability of re-election given public information that $x \neq \omega$).

The incumbent's investment incentive is:

$$\begin{aligned} \Delta &= (1-\rho)\,p_\phi + \rho p_\omega - (1-\rho)\,p_\phi - \rho\left(\frac{1}{2}p_\omega + \frac{1}{2}p_{\tilde{}\omega}\right) \\ &= \frac{\rho}{2}\left(p_\omega - p_{\tilde{}\omega}\right) \end{aligned}$$

Not surprisingly, Δ is maximized when $p_\omega = 1$ and $p_{\tilde{}\omega} = 0$, i.e., when the voter maximally rewards correct policies and punishes incorrect ones. This leads to the greatest possible investment incentive for the incumbent in the baseline no-challenger case:

$$\Delta_0 \equiv \frac{\rho}{2}.$$

Informed challenger. To get some intuition on what happens with a challenger, we now suppose that the challenger is informed $(r = 1)$. We derive the incumbent's investment incentive under the assumption that the challenger truthfully reveals the state, and then later derive conditions for the challenger to weakly prefer to reveal the state. A voter's symmetric re-election rule now consists of 6 components. If there is no uncertainty resolution, then the incumbent wins with probability p_{AA} the challenger says incumbent's policy was correct and p_{AB} if the challenger says the incumbent's policy was wrong. If uncertainty does resolve, the re-election probabilities are p_{AAA} if the incumbent's policy and challenger's announcement were correct, p_{AAB} if they both were wrong, p_{ABA} if the incumbent was correct and the challenger was wrong, and p_{ABB} if the incumbent was wrong and the challenger was correct.

The incumbent's investment incentive, assuming truthful announcements by the challenger is:

$$\begin{aligned}
\Delta_C &\equiv (1-\rho)\,p_{AA} + \rho p_{AAA} - (1-\rho)\left(\frac{1}{2}p_{AA} + \frac{1}{2}p_{AB}\right) - \rho\left(\frac{1}{2}p_{AAA} + \frac{1}{2}p_{ABB}\right) \\
&= \frac{1-\rho}{2}\left(p_{AA} - p_{AB}\right) + \frac{\rho}{2}\left(p_{AAA} - p_{ABB}\right). \qquad (13)
\end{aligned}$$

For investment incentives to be greater than in the absence of the challenger requires this to be greater than Δ_0, i.e.,

$$\begin{aligned}
\frac{1-\rho}{2}\left(p_{AA} - p_{AB}\right) + \frac{\rho}{2}\left(p_{AAA} - p_{ABB}\right) &> \frac{\rho}{2} \\
(1-\rho)\left(p_{AA} - p_{AB}\right) + \rho\left(p_{AAA} - p_{ABB} - 1\right) &> 0 \\
p_{AA} - p_{AB} &> \frac{\rho}{1-\rho}\left(1 - p_{AAA} + p_{ABB}\right). \qquad (14)
\end{aligned}$$

We now turn to the challenger's announcement constraints. Note that because the voter never observes the incumbent's information (only his actions) and the challenger knows the true state of the world, she does not care about the probability q that the incumbent knows ω. There are two possible situations: either the incumbent's policy is correct or it is incorrect, and in either case the challenger needs to be willing to make truthful announcements.

When the incumbent's choice is correct, challenger honesty requires (note that in the p subscripts here ω matches the incumbent's policy choice)

$$\begin{aligned}
\Pr(\text{inc wins}|\text{challenger truthful}) - \Pr(\text{inc wins}|\text{challenger lies}) &\leq 0 \\
(1-\rho)\, p_{AA} + \rho p_{AAA} - (1-\rho)\, p_{AB} - \rho p_{ABA} &\leq 0 \\
p_{AA} - p_{AB} &\leq \frac{\rho}{1-\rho}\left(p_{ABA} - p_{AAA}\right) \qquad (15)
\end{aligned}$$

When the incumbent's choice is incorrect, challenger honesty requires (note that in the p subscripts here ω contradicts the incumbent's policy choice)

$$\begin{aligned}
\Pr(\text{inc wins}|\text{challenger truthful}) - \Pr(\text{inc wins}|\text{challenger lies}) &\leq 0 \\
(1-\rho)\, p_{AB} + \rho p_{ABB} - (1-\rho)\, p_{AA} - \rho p_{AAB} &\leq 0 \\
(1-\rho)\left(p_{AB} - p_{AA}\right) + \rho\left(p_{ABB} - p_{AAB}\right) &\leq 0. \qquad (16)
\end{aligned}$$

We set aside Equation 16, and focus on Equations 14 and 15.

From Equation 14 we see that the incumbent has to be rewarded sufficiently strongly for choosing the correct policy when uncertainty is not resolved. However, from Equation 15 we see that this reward can't be too large, because otherwise the challenger won't be willing to confirm that the incumbent's policy choice is correct. Combining the two inequalities leads to the following contradiction:

$$\begin{aligned}
\frac{\rho}{1-\rho}\left(p_{ABA} - p_{AAA}\right) &> \frac{\rho}{1-\rho}\left(1 - p_{AAA} + p_{ABB}\right) \\
p_{ABA} &> 1 + p_{ABB}
\end{aligned}$$

which cannot hold because both re-election probabilities must be between 0 and 1.

Challenger informed with probability $r \in (0,1)$. We now analyze the more general case in which the challenger is not always informed. Before starting, note a few things. First Δ_0 is the same as in the case where the challenger was informed. Second, the challenger announcement constraints for an informed challenger are unchanged, so we can still use Equations 15 and 16.

We do, however, need to analyze incentive constraints for an uninformed challenger. This can be broken down by whether $p_{AA} > (=) (<) p_{AB}$. Note that the case of equality is straightforward, because it means that the incumbent's incentives for investment must come solely from election probabilities after ω is realized, so we can apply the previously developed (and almost fully applicable to this case) argument for why when $\rho = 1$ the voter's welfare is not improved by adding a challenger.

For now, we'll assume that $p_{AA} > p_{AB}$ and that an uniformed challenger always announces that the incumbent chose the incorrect policy. If this is so, then we can write the incumbent's investment incentive as

$$\begin{aligned}
\Delta_{C1} \equiv\ & r(1-\rho)p_{AA} + r\rho p_{AAA} + (1-r)(1-\rho)p_{AB} + (1-r)\rho p_{ABA} \\
& -r(1-\rho)\left(\frac{1}{2}p_{AA} + \frac{1}{2}p_{AB}\right) - r\rho\left(\frac{1}{2}p_{AAA} + \frac{1}{2}p_{ABB}\right) \\
& -(1-r)(1-\rho)p_{AB} - (1-r)\rho\left(\frac{1}{2}p_{ABA} + \frac{1}{2}p_{ABB}\right) \\
=\ & \frac{r}{2}(1-\rho)(p_{AA} - p_{AB}) + \frac{r}{2}\rho p_{AAA} - \frac{1}{2}\rho p_{ABB} + \frac{1-r}{2}\rho p_{ABA}.
\end{aligned}$$

To have $\Delta_{C1} > \Delta_0$ requires

$$\begin{aligned}
\frac{r}{2}(1-\rho)(p_{AA} - p_{AB}) + \frac{r}{2}\rho p_{AAA} - \frac{1}{2}\rho p_{ABB} + \frac{1-r}{2}\rho p_{ABA} &> \frac{\rho}{2} \\
(1-\rho)(p_{AA} - p_{AB}) + \rho\left[p_{AAA} - \frac{p_{ABB}}{r} + \frac{1-r}{r}p_{ABA} - \frac{1}{r}\right] &> 0.
\end{aligned}$$

Along with the announcement constraint for informed challengers whose information confirms that the incumbent's policy choice was correct (Equation 15), this requires that

$$\begin{aligned}
(1-\rho)\frac{\rho}{1-\rho}(p_{ABA} - p_{AAA}) + \rho\left[p_{AAA} - \frac{p_{ABB}}{r} + \frac{1-r}{r}p_{ABA} - \frac{1}{r}\right] &> 0 \\
-\frac{p_{ABB}}{r} + \frac{1}{r}p_{ABA} - \frac{1}{r} &> 0 \\
p_{ABA} &> 1 + p_{ABB}
\end{aligned}$$

which is a contradiction.

The more general case is to not make assumptions about whether $p_{AA} > (=) (<) p_{AB}$, and to allow any arbitrary $\sigma \in [0, 1]$ for the probability that uninformed challengers agree with the incumbent. For now, we'll assume for now that this is the same σ for either incumbent policy choice. This gives the following expression for the incumbent's investment incentive

$$\begin{aligned}
\Delta(\sigma) &\equiv r\left[(1-\rho)\,p_{AA}+\rho p_{AAA}\right] \\
&\quad +(1-r)\left[(1-\rho)\left(\sigma p_{AA}+(1-\sigma)\,p_{AB}\right)+\rho\left(\sigma p_{AAA}+(1-\sigma)\,p_{ABA}\right)\right] \\
&\quad -r\left[(1-\rho)\left(\frac{1}{2}p_{AA}+\frac{1}{2}p_{AB}\right)+\rho\left(\frac{1}{2}p_{AAA}+\frac{1}{2}p_{ABB}\right)\right] \\
&\quad -(1-r)\left[(1-\rho)\left(\sigma p_{AA}+(1-\sigma)\,p_{AB}\right)+\rho\left[\sigma\left(\frac{1}{2}p_{AAA}+\frac{1}{2}p_{AAB}\right)+(1-\sigma)\left(\frac{1}{2}p_{ABA}+\frac{1}{2}p_{ABB}\right)\right]\right] \\
&= \frac{r}{2}(1-\rho)\left(p_{AA}-p_{AB}\right)+\frac{1}{2}\rho p_{AAA}\left[r+(1-r)\,\sigma\right]-\frac{1}{2}\rho\left[r+(1-r)(1-\sigma)\right]p_{ABB} \\
&\quad +\frac{1}{2}\rho(1-r)(1-\sigma)\,p_{ABA}-\frac{1}{2}(1-r)\,\rho\sigma p_{AAB}.
\end{aligned}$$

Because $\Delta(\sigma)$ is linear in σ, this means that for any set of $p's$, it will be maximized either at $\sigma = 0$ or at $\sigma = 1$. We checked the $\sigma = 0$ case above. Now we check $\sigma = 1$. To have incumbent investment incentives greater than in the absence of a challenger requires that

$$\Delta(1)=\frac{r}{2}(1-\rho)\left(p_{AA}-p_{AB}\right)+\frac{1}{2}\rho p_{AAA}-\frac{1}{2}\rho r p_{ABB}-\frac{1}{2}(1-r)\,\rho p_{AAB}>\frac{\rho}{2}.$$

Substituting in from the challenger announcement constraint (Equation 15) this becomes

$$\begin{aligned}
\frac{r}{2}(1-\rho)\frac{\rho}{1-\rho}\left(p_{ABA}-p_{AAA}\right)+\frac{1}{2}\rho p_{AAA}-\frac{1}{2}\rho r p_{ABB}-\frac{1}{2}(1-r)\,\rho p_{AAB} &> \frac{\rho}{2} \\
r\rho\left(p_{ABA}-p_{AAA}\right)+\rho p_{AAA}-\rho r p_{ABB}-(1-r)\,\rho p_{AAB} &> \rho \\
r\left(p_{ABA}-p_{AAA}\right)+p_{AAA}-r p_{ABB}-(1-r)\,p_{AAB}-1 &> 0 \\
r p_{ABA}+(1-r)\,p_{AAA}-r p_{ABB}-(1-r)\,p_{AAB}-1 &> 0 \\
r\left(p_{ABA}-p_{ABB}\right)+(1-r)\left(p_{AAA}-p_{AAB}\right) &> 1
\end{aligned}$$

which is a contradiction because r and all of the $p's$ are in $[0,1]$.

Note on interpretation: we may be able to show that the voter is strictly worse off if he pays attention to the challenger rather than ignoring her.

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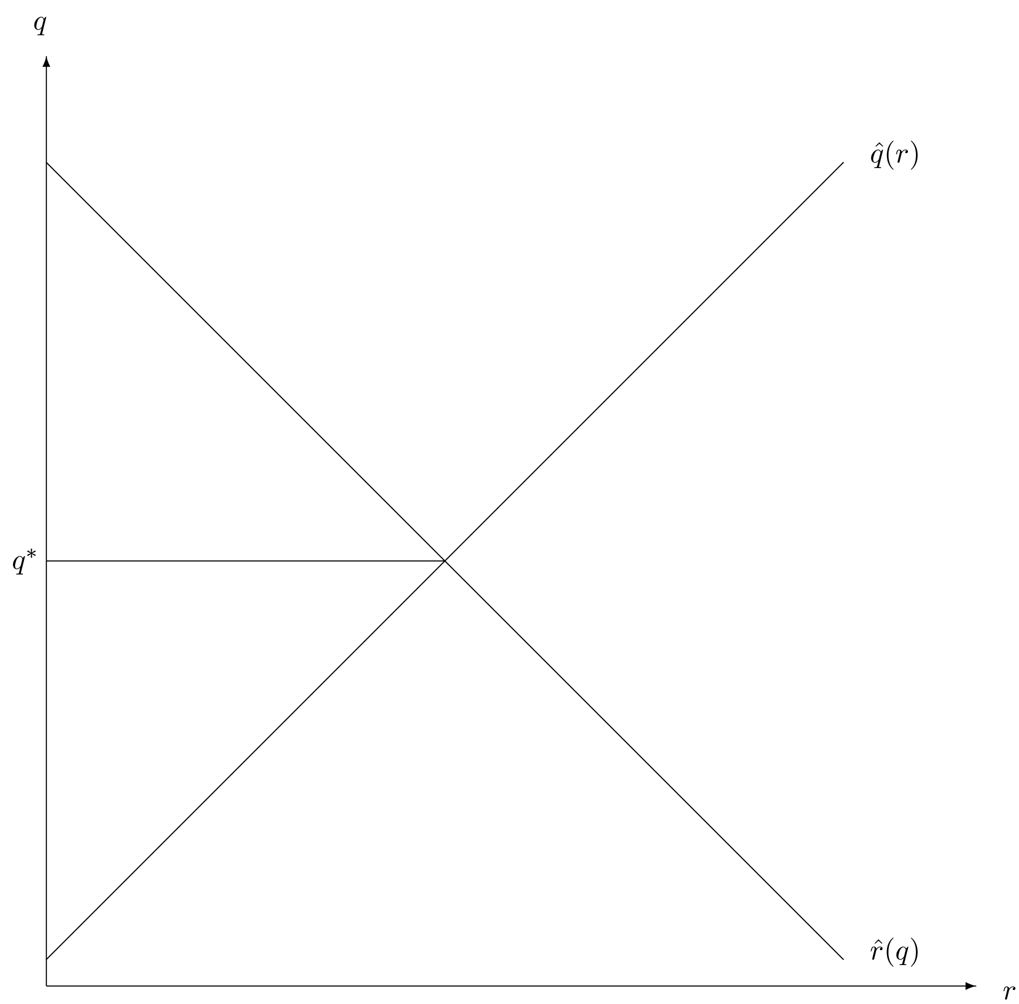

Figure 1: Best response diagram