Electoral Accountability and Responsive Democracy*

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Abstract

We consider in depth the canonical two-period model of elections with adverse selection (hidden preferences) and moral hazard (hidden actions), in which neither voters nor politicians can commit to future choices. We prove existence of electoral equilibria, and show that it involves office holders mixing in the first period. Increasing office motivation leads above average politicians to exert increasing effort, while below average politicians split between exerting increasing effort or taking it easy. Thus, electoral accountability achieves a compromise between providing incentives for good behavior in the first period and selecting better than average politicians in the second period.

1 Introduction

Representative democracy, by definition, entails the delegation of power by society to elected officials. A main concern for representative democracy is then to devise means to discipline politicians in office to achieve desirable policy outcomes for citizens. Political thinkers since Madison, if not earlier, have considered the possibility of re-election to be an essential device in this regard,[1] and the goal of this paper is to apply the tools of formal political theory to study the incentives provided by the mechanism of democratic elections and the implied linkage between voter preferences and policy outcomes. In doing so, we must move beyond the basic Downsian model of static elections, the stalwart of formal work on electoral

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[1] *The Federalist* 57, in particular, offers a discussion of the role of "frequent elections" in the selection of politicians and the control of politicians while in office.

competition, to explicitly incorporate a temporal dimension within the analysis. An active and growing literature on electoral accountability, starting with the seminal work of Barro (1973) and Ferejohn (1986), has undertaken this line of inquiry, with the goal of improving our understanding of the operation of real-world political systems and the conditions under which democracies succeed or fail. This, in turn, has the potential to facilitate the design of political institutions that produce socially desirable policy outcomes.

Nevertheless, our understanding of the fundamental interplay between disciplining incentives provided by the possibility of re-election and the temptation of opportunistic behavior in the present remains incomplete. With few exceptions, such as Fearon (1999), Ashworth (2005), and Ashworth and Bueno de Mesquita (2008), the literature on electoral accountability has paid relatively little attention to the situation in which the preferences *and* actions of politicians are unobserved by voters. Such settings combine salient aspects of real-world elections, but they present obstacles to the application of game-theoretic tools, and as a consequence, research has been conducted under special modeling assumptions about the type space, the action space, or the information held by politicians.

In this article, we present a formal model of elections that allows us to study the dynamic incentives facing politicians—and the policy choices emerging from those incentives—in environments with realistically sparse information. We prove existence of equilibrium under general conditions, and we provide a characterization of equilibrium behavior in the model. In spite of the sparseness of information available to voters, we establish the possibility of *responsive democracy* when politicians are highly office motivated. In particular, the incentives of re-election induce above average politicians (i.e., politicians who are better in expectation for voters than the challenger) to exert high levels of effort, while below average politicians necessarily split between some who exert high levels of effort and some who take it easy in the first period. This implies that in a two-type version of the model, low ability politicians must play a mixed strategy. In general, then, the electoral mechanism achieves a compromise between providing incentives for good behavior from current office holders and selecting better than average politicians for reelection, with some below average politicians mimicking above average politicians.

We conduct our research in the framework of a two-period model in which the incumbent politician in office in the first period faces a randomly chosen challenger in the second period. Variations of the two-period period model have been employed in the graduate textbooks of Persson and Tabellini (2000) and Besley (2006), providing a minimal setting to study intertemporal incentives; in this sense the two-period model can be regarded as a "workhorse" model for the study of elections. We assume that politicians' preferences are private information, i.e., *adverse selection* is present, and that political choices are observed by voters only with some noise, i.e., they are subject to imperfect monitoring, or *moral hazard.* We consider the rent-seeking environment studied in the public choice tradition of Barro (1973) and Ferejohn (1986), in which politicians have a short-run incentive to shirk from exerting effort while in office, or equivalently to engage in rent-seeking activities that hurt other citizens. Politicians differ with regard to their preference for rent-seeking (or equivalently, they differ in their cost of effort). We maintain the key assumption of the electoral accountability literature that neither politicians nor voters can commit to future actions, in the spirit of the citizen-candidate tradition of Osborne and Slivinski (1996) and Besley and Coate (1997). An implication is that in equilibrium, both the policy choices of politicians and the re-election standard used by voters must be time consistent, in the sense that first-period choices of politicians and voters must be optimal in light of expected behavior in the future.

We develop a notion of *electoral equilibrium*, a refinement of perfect Bayesian equilibrium that imposes structure suitable to the formal study of elections, namely, voters defer to the incumbent when indifferent, and voters are more willing to reelect incumbents after observed better policy outcomes. Put simply, electoral equilibrium requires that voters follow a straightforward retrospective rule: re-elect the incumbent if and only if the policy outcome is equal to or better than some standard. We note that electoral equilibria must solve a non-trivial fixed point problem: the first period office holder's choice must take account of the re-election standard of voters, and the updating of voter beliefs (and thus their re-election standard) depends on choices of the first period office holder via Bayes' rule. We impose sufficient structure (satisfied in special cases of interest) that in the first period, a politician can have at most two optimal policy choices—"taking it easy" and "going for broke"—and we use this to establish existence of electoral equilibrium. We then show that when politicians are highly office motivated, the re-election standard used by voters in equilibrium becomes arbitrarily demanding, and all "above average" politician types exert arbitrarily high effort in their first term of office. The increasing standard used by voters, on the other hand, leads the worst politician types to abandon the pursuit of re-election and shirk in the first term with positive probability.

This responsive democracy result may look superficially similar to the median voter theorem in the traditional Downsian framework, but the logic underlying it is very different: candidates cannot make binding campaign promises, and they do not compete for votes in the Downsian sense; rather, they are citizen candidates whose policy choices must maximize their payoffs in equilibrium, and the responsiveness result is driven by politicians' concern for reputation. Specifically, the desire to be re-elected can induce politicians to mimic types whose preferences are more closely aligned with voters, and if the reward for political office is large enough, then this incentive leads above average types to exert arbitrarily high effort. Thus, electoral accountability engenders the possibility of responsive democracy, despite the paucity of instruments that the voters can wield.

We emphasize that the equilibrium standard used by voters is optimal given politicians' choices, but it is not optimally set ex ante: voters do not set the standard before the election in order to elicit maximal effort. Because voters, like politicians, face a commitment problem, they "best respond" in equilibrium by re-electing the incumbent when the expected payoff from doing so, conditional on the observed policy outcome, exceeds the prospects of a challenger; in other words, the equilibrium standard is time consistent. This facet of the equilibrium analysis stacks the deck against our responsive democracy result, and it means that responsiveness does not rely on any assumption that voters can commit ex ante to a socially optimal standard of re-election.

In Section 2, we present the two-period electoral accountability model. In Section 3, we define the electoral equilibria that are the focus of our analysis. In Section 4, we impose added structure on the model and take preliminary steps toward the main results, namely, showing the existence of at most two local maximizers to the problem faced by each type of politician in equilibrium. In Section 5, we prove existence and provide a characterization of electoral equilibria, and we discuss the difficulties in achieving existence due to non-convexity of the first-term office holder's optimization problem. In Section 6, we present our result on responsive democracy as politicians become office motivated. In Section 7, we discuss in some detail the relationship of our paper with the electoral accountability literature. In Section 8, we gather final remarks.

2 Electoral accountability model

We analyze a two-period model of elections involving a representative voter, an incumbent politician, and a challenger. Prior to the game, nature chooses the types of the incumbent and challenger from the finite set $T = \{1, \dots, n\}$, with $n \geqslant 2$. These types are private information–in particular, they are unobserved by the voter–and are drawn identically and independently. We let $p_j > 0$ denote the prior probability that a politician is type j. In period 1, the incumbent makes a policy choice $x_1 \in X = \mathbb{R}_+$, which is unobserved by the voter, and a policy outcome y_1 is drawn from $Y = \mathbb{R}$ according to the distribution $F(\cdot|x_1)$. In contrast to the choice x_1, the outcome y_1 is observed by the voter. Then the voter chooses between the incumbent politician and the challenger. In period 2, the winner of the election makes a policy choice $x_2 \in X$, a policy outcome y_2 is drawn from $F(\cdot|x_2)$, and the game ends. Figure 1 illustrates the timeline of events in the model.

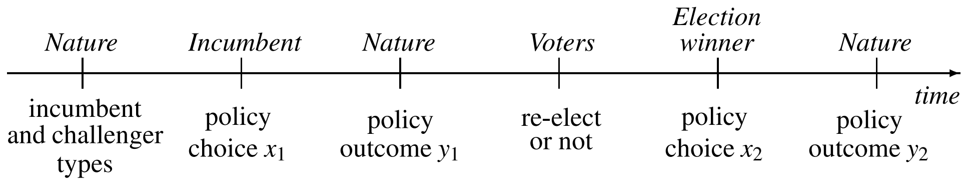

Figure 1: Timeline

Given policy choice x and outcome y in either period, each player obtains a payoff of $u(y)$ if not in office, while an office holder of type j receives a payoff of $w_j(x)+\beta$, where $\beta \geqslant 0$ represents the benefits of holding office. Total payoffs for the voter and politicians are the sum of per-period payoffs. We assume that $u\colon Y \to \mathbb{R}$ is continuous and strictly increasing, and $w_j\colon X \to \mathbb{R}$ are twice differentiable and concave. We also assume that marginal utilities are ordered by type: for all $j<n$, we have

(A1) $w_j'(x) < w_{j+1}'(x)$ for all x, $w_1'(0) \geqslant 0$, and $w_n'(x) < 0$ for large enough x.

Our assumptions imply that each politician type j has an optimal policy $\hat{x}_j$, and the ideal policies are strictly ordered according to type:

$$0 \leqslant \hat{x}_1 < \hat{x}_2 < \cdots < \hat{x}_n.$$

Moreover, we assume that the ideal politician payoffs are ordered by type: for all $j<n$, we have

(A2) $$w_j(\hat{x}_j) \quad \leqslant \quad w_{j+1}(\hat{x}_{j+1}).$$

Intuitively, the voter has increasing preferences over policy outcomes, while a politician who holds office incurs a cost for higher policy choices, and the cost is lower for higher politician types.

Our assumptions admit two simple specifications that are worthy of note. One common specification is *quadratic utility,* in which case $w_j(x) = -(x-\hat{x}_j)^2$. Another specification of interest is is *exponential utility,* whereby $w_j(x) = -e^{x-\hat{x}_j}+x$.

We assume that the outcome distribution $F(\cdot|x)$ has a jointly differentiable density $f(y|x)$ and that for all $x \in X$, $F(\cdot|x)$ has full support on $Y=\mathbb{R}$. For simplicity we take the policy choice x to be a shift parameter on the density of outcomes, so, abusing notation slightly, the density can be written $f(y|x) = f(y-x)$ for some fixed density $f(\cdot)$, and the probability that the realized outcome is less than y given policy x is simply $F(y-x)$. We assume that f satisfies the standard monotone likelihood ratio property (MLRP), i.e.,

$$\text{(A3)} \qquad \frac{f(y-x)}{f(y-x')} > \frac{f(y'-x)}{f(y'-x')}$$

for all $x > x'$ and all $y > y'$. This implies that greater policy outcomes induce the voter to update favorably their beliefs about the policy adopted by the incumbent in the first period. As is well-known, the MLRP implies that the density function is unimodal, and that both the density and the distribution functions are strictly log-concave.[2] Moreover, we assume

$$\text{(A4)} \qquad \lim_{y\to-\infty} \frac{f(y-x)}{f(y-x')} = \lim_{y\to+\infty} \frac{f(y-x')}{f(y-x)} = 0$$

when $x > x'$, so that arbitrarily extreme signals become arbitrarily informative. In particular, we capture the benchmark case in which $f(\cdot)$ is a mean-zero normal density, which implies that conditional on the policy choice x, the outcome is normally distributed with mean x.

Note that we can assume politicians share the voter's preferences over policy outcomes by setting

$$w_j(x) = \mathbb{E}[u(y)|x] - \zeta_j c(x)$$

for some strictly increasing, continuously differentiable, convex function $c{:}X \to \mathbb{R}_+$ and parameters ordered by type: for all $j < n$, $\zeta_j > \zeta_{j+1}$, and by letting $\mathbb{E}[u(y)|x] = x$. This last equation follows if $u(y) = y$ and f is symmetric around 0. In this version of the model, it is natural to view policy outcomes as a level of public good or (the inverse of) corruption, and politician types then reflect different abilities to provide the public good or a distaste for corruption while in office.

3 Electoral equilibrium

As in the citizen-candidate model, we assume that neither the incumbent nor the challenger can make binding promises before an election. We also assume that the voter cannot commit her vote, so that voting as well as policy making must be time consistent. Thus, our analysis focusses on perfect Bayesian equilibria of the electoral accountability model, under additional refinements to preclude implausible behavior on the part of the voter and politicians.

[2]See Bagnoli and Bergstrom (2005) for an in-depth analysis of log concavity and related conditions.

A *strategy for the type j incumbent* is a pair (π_j^1, π_j^2), where

$$\pi_j^1 \in \triangle(X) \quad \text{and} \quad \pi_j^2 : X \times Y \to \triangle(X),$$

specifying mixtures over policy choices in period 1 and policy choices in period 2 for each possible previous policy choice and observed outcome.[3]

A *strategy for the type j challenger* is a mapping

$$\gamma_j : Y \to \triangle(X),$$

specifying mixtures over policy choices in period 2 for each policy type and observed outcome. A *strategy for the voter* is a mapping

$$\rho : Y \to [0,1],$$

where $\rho(y)$ is the probability of a vote for the incumbent given outcome y. A *belief system for the voter* is a probability distribution $\mu(\cdot|y_1)$ on $T \times X$ as a function of the observed outcome.

A strategy profile $\sigma = ((\pi_j, \gamma_j)_{j \in T}, \rho)$ is *sequentially rational* given belief system μ if neither the incumbent nor the challenger can gain by deviating from the proposed strategies at any decision node, and if the voter votes for the candidate that makes her best off in expectation following all possible realizations of y_1.[4] Beliefs μ are *consistent* with the strategy profile σ if for every y_1, the distribution $\mu(j, x|y_1)$ is derived from $(\pi_j^1)_{j \in T}$ via Bayes' rule. A *perfect Bayesian equilibrium* is a pair (σ, μ) such that the strategy profile σ is sequentially rational given the beliefs μ, and μ is consistent with σ.

Sequential rationality implies that challengers will choose their ideal policies since they cannot hope to be re-elected, so that γ_j assigns probability one to $\hat{x}_j$ for all y_1. This implies that the expected payoff of electing the challenger for the voter is

$$V^C = \sum_k p_k \mathbb{E}[u(y)|\hat{x}_k].$$

Similarly, sequential rationality implies that π_j^2 assigns probability one to $\hat{x}_j$ for all x_1 and all y_1, so henceforth we simplify notation by dropping the superscript from π_j^1 for the mixture over policies used by the type j politician in the first period. It follows that the expected payoff to the voter from re-electing the incumbent is

$$V^I(y_1) = \sum_k \mu_T(k|y_1) \mathbb{E}[u(y)|\hat{x}_k],$$

[3]The notation $\triangle(\cdot)$ indicates the set of probability distributions over a given set. Measurability of strategies or subsets of policies will be assumed implicitly, as needed, without further mention.

[4]In the terminology of Fearon (1999), voters focus on the problem of "selection," rather than "sanctioning." See his essay for arguments in support of this behavioral postulate.

where $\mu_T(j|y_1)$ is the marginal distribution of the incumbent's type given policy outcome y_1. Thus, the incumbent is re-elected if $V^I(y_1) > V^C$ and only if $V^I(y_1) \geqslant V^C$. Sequential rationality does not pin down the ballot of the voter when she is indifferent between the incumbent and challenger; we say the equilibrium is *deferential* if the voter favors the incumbent when indifferent, so that the incumbent is re-elected if and only if $V^I(y_1) \geqslant V^C$.

This general formulation of deferential equilibrium implies that there is an *acceptance set* of policy outcomes such that the incumbent is re-elected with probability one after realizations in this set and loses for sure after realizations outside the set:

$$A = \{y_1 \in Y : V^I(y_1) \geqslant V^C\}.$$

We say an equilibrium is *monotonic* if the acceptance set is closed, and if for every policy outcome belonging to the acceptance set, every greater outcome is also acceptable, i.e., for all $y \in A$ and all $y' \geqslant y$, we have $y' \in A$. Put differently, the voter follows a simple retrospective rule given by $\bar{y} \in \mathbb{R} \cup \{-\infty, \infty\}$ such that she re-elects the incumbent if and only if $y \geqslant \bar{y}$. The monotonicity condition imposes a natural linkage between the voter's utility over policy outcomes and the informational content of those outcomes in the first period.

Finally, an *electoral equilibrium* is a perfect Bayesian equilibrium that is deferential and monotonic. Electoral equilibria are then characterized by three conditions. First, the threshold $\bar{y}$ must be such that, anticipating that politicians choose their ideal policies in the second period, the expected utility of re-electing the incumbent after observing y, given the voters' belief system, is greater than or equal to $\sum_k p_k \mathbb{E}[u(y)|\hat{x}_k]$ if and only if $y \geqslant \bar{y}$. Second, each politician type j, knowing that she is re-elected if and only if $y \geqslant \bar{y}$, mixes over optimal actions in the first period, i.e., the type j incumbent's policy strategy π_j places probability one on maximizers of

$$w_j(x) + (1 - F(\bar{y} - x))[w_j(\hat{x}_j) + \beta] + F(\bar{y} - x)V^C. \tag{1}$$

Third, updating of voter beliefs follows Bayes rule, after observing outcome y. In particular, when the policy mixtures π_j have discrete supports, we can write

$$\mu_T(j|y) = \frac{p_j \sum_x f(y-x)\pi_j(x)}{\sum_k p_k \sum_x f(y-x)\pi_k(x)}.$$

Since the outcome density is positive, every outcome is on the path of play, so Bayes' rule pins down the voter's beliefs. We henceforth summarize an electoral equilibrium by the strategy profile σ, leaving beliefs implicit.

4 Preliminary analysis

To facilitate the analysis, we henceforth assume that all incumbent types are in principle interested in re-election, i.e.,

(A5) $$w_1(\hat{x}_1) + \beta > V^C,$$

so that if re-election is assured by choosing their ideal policies in the first period, then the benefits of re-election outweigh the costs. Note that the incumbent can always choose her ideal policy, so it is never optimal for the politician to choose large policies x for which $w_j(x) + \beta < V^C$. By (A5) and concavity, it is never optimal to choose a policy below the politician's ideal policy, so there is at least one solution to the incumbent's problem in the first period. Denoting by x_j^* such a solution, note that $x_j^* \geqslant \hat{x}_j$ and hence $x_j^* > 0$ for $j > 1$. In particular, the necessary first order condition for a solution of the incumbent's maximization problem (1) is

$$w_j'(x_j^*) \quad = \quad -f(\overline{y} - x_j^*)[w_j(\hat{x}_j) + \beta - V^C]. \tag{2}$$

That is, the marginal disutility in the current period from increasing the policy choice is just offset by the marginal utility in the second period, owing to the politician's increased chance of re-election. By (A2) and (A5), the right-hand side of (2) is negative, and we see that for an arbitrary cutoff $\overline{y}$, the politician optimally exerts a positive amount of effort, i.e., chooses $x_j^* > \hat{x}_j$, in the first term of office.

We can gain some insight into the incumbent's problem and the importance of mixing in equilibrium by reformulating it in terms of optimization subject to an inequality constraint. Define a new objective function

$$U_j(x,r) \quad = \quad w_j(x) + r[w_j(\hat{x}_j) + \beta - V^C],$$

which is the expected utility if the politician chooses policy x and is re-elected with probability r, minus a constant term corresponding to the current enjoyment of office. Note that U_j is concave in (x,r) and quasi-linear in r. Of course, given x, the re-election probability is in fact pinned down as $1 - F(\overline{y} - x)$. Defining the constraint function

$$g(x,r) \quad = \quad 1 - F(\overline{y} - x) - r,$$

we can then formulate the politician's optimization problem as

$$\begin{aligned} &\max\nolimits_{(x,r)} U_j(x,r) \\ &\text{s.t. } g(x,r) \leqslant 0, \end{aligned}$$

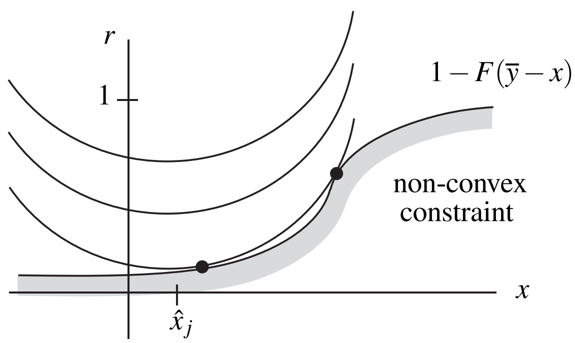

Figure 2: Politician's optimization problem

which has the general form depicted in Figure 2. Here, the objective function is well-behaved, but the constraint set inherits the natural non-convexity of the distribution function F, leading to the possibility of multiple solutions. This, in turn, can lead to multiple optimal policies and the necessity of mixing in equilibrium; see Figure 2 for an illustration of this multiplicity.

One of our contributions is a condition that is satisfied in environments of interest and limits the need for mixing to at most two policy choices for each type.[5] Assume that for all j, all finite $\overline{y}$, and all $x, \tilde{x}, z$ with $\hat{x}_j < x < \tilde{x} < z$, we have the following condition:

(A6) $$\text{if } \frac{w_j''(x)}{w_j'(x)} \leqslant -\frac{f'(\overline{y}-x)}{f(\overline{y}-x)} \text{ and } \frac{w_j''(z)}{w_j'(z)} \leqslant -\frac{f'(\overline{y}-z)}{f(\overline{y}-z)}, \text{ then } \frac{w_j''(\tilde{x})}{w_j'(\tilde{x})} < -\frac{f'(\overline{y}-\tilde{x})}{f(\overline{y}-\tilde{x})}.$$

That is, the set of $x > \hat{x}_j$ such that $w_j''(x)/w_j'(x) \leqslant -f'(\overline{y}-x)/f(\overline{y}-x)$ is convex, and if x and z satisfy the inequality, then every policy between them satisfies it strictly. To understand this condition, note that by log concavity of $f(\cdot)$, the term $-f'(\overline{y}-x)/f(\overline{y}-x)$ is strictly decreasing in x, and thus (A6) is satisfied if the coefficient of absolute risk aversion, $w_j''(x)/w_j'(x)$, does not decrease too fast to the right of the type j politicians' ideal policy. To illustrate, when the utility function w_j is quadratic, the coefficient of absolute risk aversion is $1/(x-\hat{x}_j)$, and when the density f is standard normal, the likelihood ratio $-f'(\overline{y}-x)/f(\overline{y}-x)$ simplifies to $\overline{y}-x$. Thus, (A6) is satisfied in the quadratic-normal special case, depicted in

[5] The possibility of multiple optimizers has a counterpart in static models of elections with probabilistic voting, where log concavity is sufficient to ensure existence of equilibria in pure strategies (cf. Roemer (1997) and Bernhardt et al. (2009)).

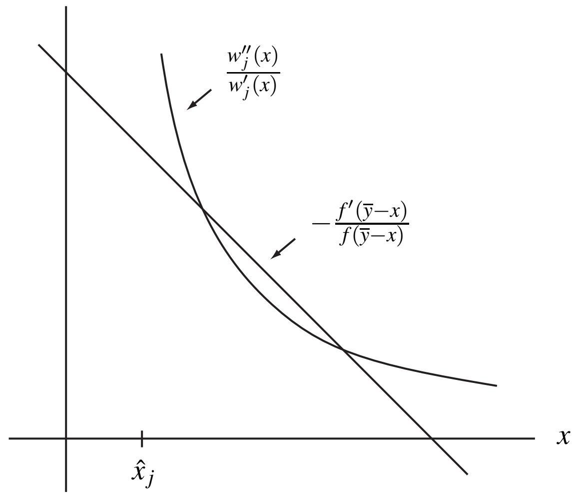

Figure 3: Quadratic-normal special case

Figure 3. Likewise, in the case of exponential utility, the coefficient of risk aversion is $1/(1-\exp(\hat{x}_j - x))$, which is decreasing and convex, and again (A6) is satisfied.

The usefulness of (A6) is delineated in the next result, which implies that for arbitrary cutoffs, each type of incumbent has at most two optimal policies as a function of the cutoff. The greater solution to the incumbent's optimization problem, which is denoted $x_j^*(\overline{y})$, corresponds to "going for broke," while the least solution, denoted $x_{*,j}(\overline{y})$, corresponds to "taking it easy." When these two policy choices coincide, the politician has a unique optimal policy; a gap between the two choices reflects the possibility that the increase in effort involved in going for broke is just offset by the increase in probability of being re-elected.

Proposition 1. *Assume (A1), (A2), (A5) and (A6). For every cutoff $\overline{y} \in Y$ and every type j, there are at most two optimal policies, i.e. two maximizers of the objective function (1). For every type j, the greatest and least optimal policies for type j, $x_j^*(\cdot)$ and $x_{*,j}(\cdot)$ are, respectively, upper and lower semi-continuous functions of $\overline{y}$.*

Proof. Suppose toward a contradiction that there are three distinct local maximizers of the type j politicians' objective function, say x', x'', and x''' with $x' < x'' < x'''$. Thus, there are local minimizers z' and z'' such that $x' < z' < x'' < z'' < x'''$. With (A2) and (A5), inspection of the first order condition (2) at $x = z', z''$ reveals that $w_j'(z') < 0$ and $w_j'(z'') < 0$, and we can rewrite the first order condition at z' and z''

as

$$w_j(\hat{x}_j)+\beta-V^C \;=\; -\frac{w_j'(z')}{f(\overline{y}-z')} \;=\; -\frac{w_j'(z'')}{f(\overline{y}-z'')}.$$

By the necessary second order condition for a local minimizer, the second derivative at z' satisfies

$$0 \;\leqslant\; w_j''(z')-f'(\overline{y}-z')[w_j(\hat{x}_j)+\beta-V^C] \;=\; w_j''(z')-f'(\overline{y}-z')\left[-\frac{w_j'(z')}{f(\overline{y}-z')}\right],$$

or equivalently,

$$\frac{w_j''(z')}{w_j'(z')} \;\leqslant\; -\frac{f'(\overline{y}-z')}{f(\overline{y}-z')}.$$

Similarly, we have

$$\frac{w_j''(z'')}{w_j'(z'')} \;\leqslant\; -\frac{f'(\overline{y}-z'')}{f(\overline{y}-z'')}.$$

Since x'' is a local maximizer, the first order condition holds at x'', and the second derivative at x'' is non-positive, but then we have

$$\frac{w_j''(x'')}{w_j'(x'')} \;\geqslant\; -\frac{f'(y-x'')}{f(y-x'')},$$

contradicting (A6). We conclude that the objective function has at most two local maximizers, and therefore there are at most two optimal policies for type j.

From previous arguments and (A1), optimal policies for type j are bounded below by $\hat{x}_j \geqslant 0$ and above by $\overline{x}_j > \hat{x}_j$ such that $w_j(\overline{x}_j)+\beta = V^C$. A standard application of Berge's theorem of the maximum (see e.g. Border (1985), Theorem 12.1) implies that the correspondence of optimal best responses is nonempty-valued and is upper hemi-continuous in $\overline{y}$. Since the correspondence of optimal best-responses includes at most two policies for each cutoff, upper hemi-continuity of the best response correspondence is equivalent to the greatest and least optimal policies for type j, $x_j^*(\cdot)$ and $x_{*,j}(\cdot)$ being, respectively, upper and lower semi-continuous functions of $\overline{y}$. □

The idea of the proof is that, at any local minimizer of the politician's objective function 1, the likelihood ratio must exceed the coefficient of absolute risk aversion. (A6) then implies that there is at most one local minimizer of the politician's objective function, so that the objective function has either one or two maximizers.

In terms of Figure 3, there is at most one maximizer below the region of policy choices such that the likelihood ratio exceeds the coefficient of absolute risk aversion, and at most one maximizer above that region.

We can visualize the effects of changes in the cutoff in the objective function of politicians using Figure 3. Recall that the likelihood ratio in the figure is equal to $\bar{y}-x$. If the cutoff $\bar{y}$ is small enough, the likelihood ratio is always below the coefficient of absolute risk aversion so the objective function is concave and has a unique maximizer. If the cutoff is larger, so that the two curves cross as drawn in the figure, then two local maximizers are possible. Moreover, as the cutoff increases, one local maximizer must get closer to the ideal policy, while the other–if it exists–must grow large.

In Figure 4, we have illustrated the politician's objective function in the simple quadratic-normal example, for $\hat{x}_j = 1$, $\beta - V^C = 20$, and $\bar{y}$ taking values in 3.4, 4.21 and 5. In the first case, there is a unique local maximum–and hence the global optimum–near 4.11, in the second case there are two local maxima near 1 and 4.51, while in the third case there is again a unique local maximum–and hence the global optimum–near 1. In Figure 5, we have illustrated the maximizers of the politician's objective function for the same example. Note that the greatest and least optimal policies, $x_j^*(\bar{y})$ and $x_{*,j}(\bar{y})$, coincide for all values of the cutoff except at unique cutoff near 4.21. While, looking at the problem of one politician type, a value of the cutoff such that the objective function has two global optima seems a knife-edge possibility, equilibrium existence may require precisely a value such that a politician type randomizes between "taking it easy" and "going for broke."

The next proposition establishes that the incumbent's objective function satisfies the important property that differences in payoffs are monotone in type. We say that $U_j(x,1-F(\bar{y}-x))$ is *supermodular* in (j,x) if for all (j,x) and all (k,z) with $j>k$ and $x>z$, we have

$$\begin{aligned} & U_j(x,1-F(\bar{y}-x)) - U_j(z,1-F(\bar{y}-z)) \\ > \quad & U_k(x,1-F(\bar{y}-x)) - U_k(z,1-F(\bar{y}-z)). \end{aligned}$$

Proposition 2. *Assume (A1), (A2) and (A5). For every cutoff $\bar{y}$, the incumbent's objective function, $U_j(x,1-F(\bar{y}-x))$, is super modular in (j,x).*

Proof. Consider $j>k$ and $x>z$. We must show that

$$\begin{aligned} & w_j(x) - w_j(z) + (F(\bar{y}-z) - F(\bar{y}-x))[w_j(\hat{x}_j) + \beta - V^C] \\ > \quad & w_k(x) - w_k(z) + (F(\bar{y}-z) - F(\bar{y}-x))[w_k(\hat{x}_k) + \beta - V^C]. \end{aligned}$$

Since $x>z$, we have $F(\bar{y}-z) - F(\bar{y}-x) > 0$. Using (A2) and (A5),

$$\begin{aligned} & (F(\bar{y}-z) - F(\bar{y}-x))[w_j(\hat{x}_j) + \beta - V^C] \\ > \quad & (F(\bar{y}-z) - F(\bar{y}-x))[w_k(\hat{x}_k) + \beta - V^C]. \end{aligned}$$

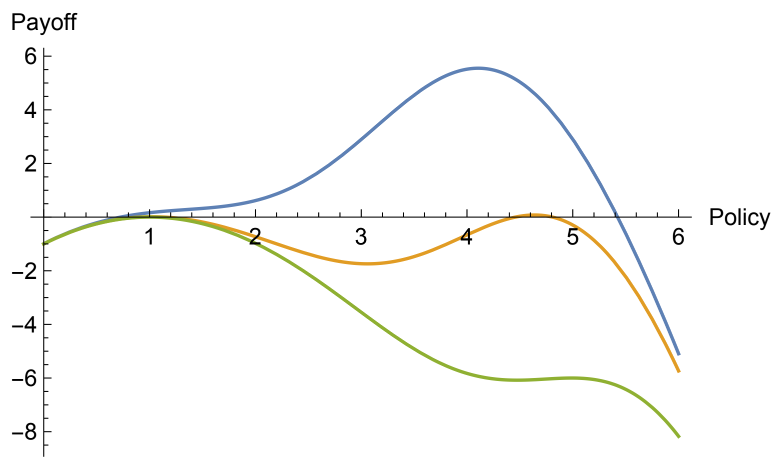

Figure 4: Politician's payoff function for different voter cutoffs

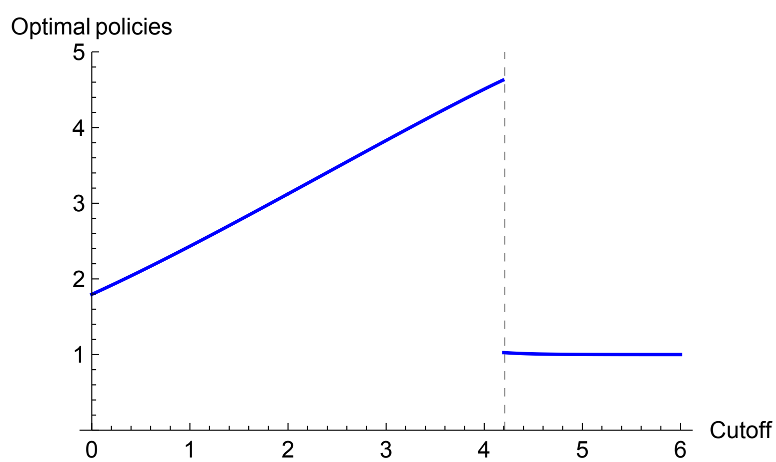

Figure 5: Politician's optimal policies as function of voter cutoff

It then remains to be shown that $w_j(x) - w_j(z) > w_k(x) - w_k(z)$; this follows from continuous differentiability of w_j and from (A1). □

An implication of Propositions 1 and 2 is that given an arbitrary value $\overline{y}$ of the cutoff, the optimal policy choices of the types are strictly ordered by type, i.e.,

$$\text{for all } j < n, \quad x_j^*(\overline{y}) \ < \ x_{*,j+1}(\overline{y}).$$

This ordering property will, in turn, be key for establishing existence of equilibrium.

The above ordering property is very useful in combination with the fact that, as shown below, given arbitrary policy choices $x_1 < x_2 < \cdots < x_n$ of the politician types in the first period, there is a unique outcome, which we denote $y^*(x_1,\ldots,x_n)$, such that conditional on realizing this value, the voter is indifferent between reelecting the incumbent and electing a challenger. Moreover, this extends to the case of mixed policy strategies $\pi_1,\ldots,\pi_n$ with discrete supports that are strictly ordered by type, i.e., for all $j < n$,

$$\max\{x : \pi_j(x) > 0\} \quad < \quad \min\{x : \pi_{j+1}(x) > 0\}.$$

That is, there is a unique solution in $\overline{y}$ to the equation $V^I(\overline{y}) = V^C$, or more explicitly,

$$\sum_k \mu_T(k|\overline{y})\mathbb{E}[u(y)|\hat{x}_k] \quad = \quad \sum_k p_k \mathbb{E}[u(y)|\hat{x}_k]. \tag{3}$$

We let $y^*(\pi_1,\ldots,\pi_n)$ denote the solution to the voter's indifference condition as a function of policy choices.

In addition to uniqueness, the next proposition establishes that the cutoff is continuous in policy strategies and lies between the choices of the type 1 and type n politicians, shifted by the mode of the density of $f(\cdot)$, which we denote by $\hat{z}$.

Proposition 3. *Assume (A3) and (A4). For all mixed policy strategies* $\pi_1,\ldots,\pi_n$ *with discrete supports that are strictly ordered by type and for all belief systems* μ *derived via Bayes rule, there is a unique solution to the voter's indifference condition (3), and the solution* $y^*(\pi_1,\ldots,\pi_n)$ *is continuous as a function of mixed policies. Moreover, this solution lies between the extreme policy choices shifted by the mode of the outcome density, i.e.,*

$$\min\{x : \pi_1(x) > 0\} + \hat{z} \ < \ y^*(\pi_1,\ldots,\pi_n) \ < \ \max\{x : \pi_n(x) > 0\} + \hat{z}.$$

Proof. For existence of a solution to the indifference condition, fix $\pi_1,\ldots,\pi_n$ with supports that are strictly ordered by type, and note that the left-hand side of (3) is continuous in $\overline{y}$. Let $x_n = \min\{x : \pi_n(x) > 0\}$ be the lowest policy chosen with positive probability by the type n politicians. For all $j < n$ and all x with $\pi_j(x) > 0$, (A4) implies that $f(\overline{y}-x)/f(\overline{y}-x_n) \to 0$ as $\overline{y} \to \infty$. Thus, using (A4),

$$\mu_T(j|\overline{y}) \;=\; \frac{p_j \sum_x f(\overline{y}-x)\pi_j(x)}{\sum_k p_k \sum_x f(\overline{y}-x)\pi_k(x)} \;\leqslant\; \frac{p_j}{p_n}\sum_x \frac{f(\overline{y}-x)}{f(\overline{y}-x_n)}\frac{\pi_k(x)}{\pi_n(x_n)} \;\to\; 0$$

as $\overline{y} \to \infty$, which implies that $\mu_T(n|\overline{y})$ goes to one as the cutoff increases. In words, when the policies of the politicians are ordered by type, high realizations of the outcome become arbitrarily strong evidence that the incumbent is the best possible type. Similarly, $\mu_T(1|\overline{y})$ goes to one as $\overline{y}$ decreases without bound. Thus, the left-hand side of (3) approaches $\mathbb{E}[u(y)|\hat{x}_n]$ when the cutoff is large, and it approaches $\mathbb{E}[u(y)|\hat{x}_1]$ when the cutoff is small, and existence of a solution follows from the intermediate value theorem.

To show uniqueness, we claim that the left-hand side of (3) is strictly increasing in $\overline{y}$. Since higher types choose better policies for the voter, to prove the claim it is enough to show that $\mu_T(\cdot|\overline{y})$ exhibits first order stochastic dominance over $\mu_T(\cdot|\overline{y}')$ for $\overline{y} > \overline{y}'$; we claim the slightly stronger condition that for each $1 \leqslant j \leqslant n$, $\overline{y} > \overline{y}'$ implies

$$\sum_{k\geqslant j} \mu_T(j|\overline{y}) > \sum_{k\geqslant j} \mu_T(j|\overline{y}').$$

This is the case if

$$\frac{\sum_{k=j}^{n} p_k \sum_x f(\overline{y}-x)\pi_k(x)}{\sum_{m=1}^{n} p_m \sum_{x'} f(\overline{y}-x')\pi_m(x')} \;>\; \frac{\sum_{k=j}^{n} p_k \sum_x f(\overline{y}'-x)\pi_k(x)}{\sum_{m=1}^{n} p_m \sum_x f(\overline{y}'-x')\pi_m(x')},$$

or equivalently, cancelling repeated terms,

$$\sum_{m=1}^{j-1}\sum_{k=j}^{n} p_m p_k \sum_{x'}\sum_{x} \pi_m(x')\pi_k(x) f(\overline{y}'-x') f(\overline{y}-x)$$
$$> \sum_{m=1}^{j-1}\sum_{k=j}^{n} p_m p_k \sum_{x'}\sum_{x} \pi_m(x')\pi_k(x) f(\overline{y}-x') f(\overline{y}'-x).$$

Since supports are strictly ordered by type, $m < k$ and $\pi_m(x')\pi_k(x) > 0$ imply $x > x'$. From (A3), then,

$$f(\overline{y}'-x')f(\overline{y}-x) > f(\overline{y}-x')f(\overline{y}'-x)$$

for $\pi_m(x')\pi_k(x) > 0$ and $\overline{y} > \overline{y}'$, and the desired inequality follows.[6]

[6]Banks and Sundaram (1998) develop a similar argument in a related problem (Lemma A.6).

Standard continuity arguments imply that $y^*(\pi_1,\ldots,\pi_n)$ is continuous as a function of mixed policy strategies with discrete supports.

To obtain the upper bound on the cutoff, consider any $\overline{y} \geqslant \max\{x : \pi_n(x) > 0\} + \hat{z}$. Recall that the posterior probability that the politician is type j, conditional on observing $\overline{y}$, is

$$\mu_T(j|\overline{y}) \quad = \quad \frac{p_j \sum_x f(\overline{y}-x)\pi_j(x)}{\sum_k p_k \sum_x f(\overline{y}-x)\pi_k(x)}.$$

Note that for all $k > j$ and all policies x_j with $\pi_j(x_j) > 0$ and x_k with $\pi_k(x_k) > 0$, we have $\hat{z} \leqslant \overline{y} - x_k < \overline{y} - x_j$. Since $f(\cdot)$ is single-peaked by (A3), we see that for all $x_1,\ldots,x_n$ such that each x_k is in the support of π_k, we have

$$f(\overline{y}-x_1) \ < \ f(\overline{y}-x_2) \ < \ \cdots \ < \ f(\overline{y}-x_n).$$

Therefore, the coefficients on prior beliefs are ordered by type, i.e.,

$$\frac{\sum_x f(\overline{y}-x)\pi_1(x)}{\sum_k p_k \sum_x f(\overline{y}-x)\pi_k(x)} \ < \ \cdots < \ \frac{\sum_x f(\overline{y}-x)\pi_n(x)}{\sum_k p_k \sum_x f(\overline{y}-x_k)\pi_k(x)},$$

and we conclude that the posterior distribution $\mu_T(\cdot|\overline{y})$ first order stochastically dominates the prior, contradicting the indifference condition. An analogous argument derives a contradiction for the case $\overline{y} \leqslant \min\{x : \pi_1(x) > 0\} + \hat{z}$. $\square$

To see the structure of $y^*(\pi_1,\ldots,\pi_n)$ for the special case of two types using pure policy strategies, the voter's cutoff is simply the solution to $\mu_T(2|y) = p_2$, so that conditional on the cutoff, the probability that the incumbent is the high type is just equal to the prior probability. Letting x_1 and x_2 be the policies chosen by the two types, this means that $y^*(x_1,x_2)$ solves the equation

$$p_2 \quad = \quad \frac{p_2 f(y-x_2)}{p_1 f(y-x_1) + p_2 f(y-x_2)},$$

or after manipulating, it means that the likelihood of y is the same given the policy choices of the politician types, i.e., $f(y-x_1) = f(y-x_2)$. Adding the assumption that the density $f(\cdot)$ is normal with mean zero, the cutoff is simply the midpoint of the politicians' choices, i.e.,

$$y^*(x_1,x_2) \quad = \quad \frac{x_1+x_2}{2}.$$

Indeed, this characterization as the midpoint of policy choices extends to any density $f(\cdot)$ that is symmetric around zero.

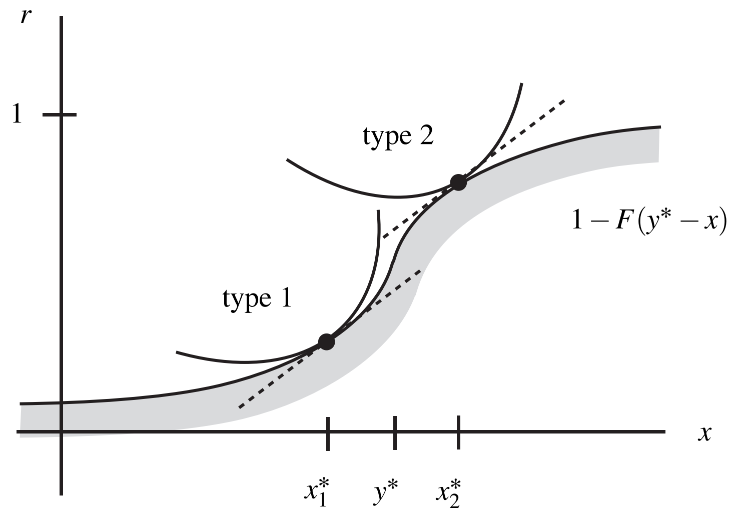

Figure 6: Electoral equilibrium in pure strategies with two types

The preceding observations allow us to graphically depict an electoral equilibrium in pure policy strategies for the case of two types with equal prior probability and density $f(\cdot)$ symmetric around zero. In Figure 6, we draw the indifference curves of the type 1 and 2 politicians through their optimal policies, x_1^* and x_2^* given the constraint set determined by the cutoff y^*. This is reflected in the tangency condition at each optimal policy. Moreover, the voter's indifference condition implies that the likelihood of outcome y^* is equal given either optimal policy, and this implies that the two tangent lines have equal slopes. Indeed, using the first order condition for office holders of types 1 and 2, we have

$$\frac{w_1'(x_1^*)}{w_1(\hat{x}_1)+\beta-V^C} \;=\; -f(y^*-x_1^*) \;=\; -f(y^*-x_2^*) \;=\; \frac{w_2'(x_2^*)}{w_2(\hat{x}_2)+\beta-V^C},$$

as claimed. Note that when the office benefit β increases and the cutoff $\overline{y}$ is held fixed, the indifference curves of the politician types become flatter, and optimal policies will move to the right, suggesting that higher office benefit leads to greater policy responsiveness. Of course, the voters' cutoff is itself endogenous and will respond to variation of parameters. If voters become more demanding when office benefit increases, so that $\overline{y}$ increases, it may be that the overall effect is that some politician types reduce effort when office motivation is higher.

As an illustration, consider again the simple quadratic-normal example with two types, e.g. $w_j(x) = -(x-\hat{x}_j)^2$ for $j = 1,2$, with $0 \leqslant \hat{x}_1 < \hat{x}_2$, f equal to the standard normal density, and $\beta > V^C$. From the necessary first order condition, we get that an equilibrium in pure policies must satisfy

$$x_j^* = \hat{x}_j + \varepsilon(\beta) \text{ for } j = 1,2, \quad \text{and} \quad y^* = \frac{\hat{x}_1 + \hat{x}_2}{2} + \varepsilon(\beta),$$

where

$$\varepsilon(\beta) = \left(\frac{\beta - V^C}{2}\right) f\left(\frac{\hat{x}_2 - \hat{x}_1}{2}\right).$$

In terms of Figure 6, in an equilibrium in pure strategies, as β increases, the curve $1 - F(y^* - x)$ moves to the right in parallel with the points x_1^*, x_2^* and y^*.

Note that for β near V^C, $\varepsilon(\beta)$ is close to zero, so that y^* as defined above is close to $(\hat{x}_1 + \hat{x}_2)/2$. It is simple to check that if $\hat{x}_2 - \hat{x}_1 < 4$ and $\overline{y}$ is near $(\hat{x}_1 + \hat{x}_2)/2$, the likelihood ratio is below the coefficient of absolute risk aversion for $x > \hat{x}_1$, implying that the objective function of either politician type is concave and has a unique maximizer. Thus, there is indeed an equilibrium in pure strategies as described above when the value of the office benefit is small and the favorite policies of the politicians are not too far apart. However, the necessary second order condition fails for the type 1 politician at $\hat{x}_1 + \varepsilon(\beta)$ whenever

$$\beta > V^C + \frac{2}{-f'(\frac{\hat{x}_2 - \hat{x}_1}{2})}. \tag{4}$$

That is, for large values of the office benefit, an electoral equilibrium in the quadratic-normal example necessarily involves mixing.

Note that, in the quadratic-normal example, the coefficient of absolute risk aversion is larger for type 2 than for type 1 for all values of $x > \hat{x}_2$. In terms of Figure 3, the curve $w_2''(x)/w_2'(x)$ lies above the curve $w_1''(x)/w_1'(x)$. If the cutoff is such that the likelihood ratio curve cuts the risk aversion curves for both politician types, the lowest and highest policy such that the coefficient of risk aversion of type 1 is equal to the likelihood ratio are more extreme than those for type 2. Recall that local maximizers of the objective function of a politician must be such that the coefficient of risk aversion is above the likelihood ratio. Since optimal policies are strictly ordered by type, it follows that in a mixed strategy equilibrium of the quadratic-normal example, only one of the two politician types is randomizing. Results in the next two sections imply that there is in indeed always an equilibrium in the quadratic-normal example, and that for large values of the office, type 2 politicians exert unboundedly large effort, while type 1 politicians randomize between arbitrarily large effort and policies close to their ideal policy.

5 Existence and characterization of electoral equilibria

Our first main result establishes existence of electoral equilibrium, along with a partial characterization of equilibria. Importantly, electoral equilibria must solve a complicated fixed point problem: optimal policy choices of politician types depend on the cutoff used by the voter, and the cutoff used by the voter depends, via Bayes rule, on the policy choices of politician types. Nevertheless, we rely on Propositions 1–3 to provide a fixed point argument and overcome the existence problem.

Theorem 5.1. *Assume (A1)–(A6). Then there is an electoral equilibrium, and every electoral equilibrium is given by mixed policy strategies* $\pi_1^*, \ldots, \pi_n^*$ *and a finite cutoff* y^* *such that:*

(i) each type j *politician mixes over policies using* π_j^**, which places positive probability on at most two policies, say* x_j^* *and* $x_{*,j}$*, where* $\hat{x}_j < x_{*,j} \leqslant x_j^*$*,*

(ii) the supports of policy strategies are strictly ordered by type, i.e., for all $j < n$*, we have* $x_j^* < x_{*,j+1}$*,*

(iii) the voter re-elects the incumbent if and only if $y \geqslant y^*$*, where the cutoff lies between the extreme policies shifted by the mode of the outcome density, i.e.,* $x_{*,1} + \hat{z} < y^* < x_n^* + \hat{z}$*.*

Proof. In proving the proposition, we must address three technical subtleties. The first is that when supports of mixed policy choices are only weakly ordered, the left-hand side of (3) is only weakly increasing, so that the equality has a closed, convex (not necessarily singleton) set of solutions. In fact, if all politician types choose the same policy with probability one, then updating does not occur and incumbents are always re-elected, so that the voter's cutoff is negatively infinite. As policy choices of politician types converge to the same policy, this means that the cutoff either jumps discontinuously (from a bounded, finite level) or diverges to negative infinity. We circumvent this problem by deriving a positive lower bound on the distance between optimal policy choices of the different types. Indeed, we first observe that equilibrium policy choices are bounded above by $\overline{x} > \hat{x}_n$ such that $V^C = w_n(\overline{x}) + \beta$. Indeed, from (A1) and (A2), $\overline{x} > \overline{x}_j$ for $j < n$, where $\overline{x}_j > \hat{x}_j$ such that $V^C = w_j(\overline{x}_j) + \beta$. That is, if the type n politician is indifferent between choosing her ideal policy with no chance of re-election and choosing $\overline{x}$ and win with certainty, then no policy above $\overline{x}$ can be optimal for any type given any cutoff.

Next, given any cutoff $\overline{y}$ and any type j politician, there are at most two optimal policies, by Proposition 1, and each satisfies the first order condition (2). Note that $f(\overline{y} - x) \to 0$ uniformly on $[0, \overline{x}]$ as $|\overline{y}| \to \infty$, and from the first order condition, this implies that the optimal policies of the type j politician converge to the ideal policy, i.e., $x^*_j(\overline{y}) \to \hat{x}_j$ and $x_{*,j}(\overline{y}) \to \hat{x}_j$. Thus, we can choose a sufficiently large interval $[y_L, y_H]$ and $\varepsilon' > 0$ such that for all $\overline{y}$ outside the interval, optimal policies differ across types by at least ε', i.e., for all $j < n$, we have $|x_{*,j+1}(\overline{y}) - x^*_j(\overline{y})| > \varepsilon'$. By upper semi-continuity of $x^*_j(\cdot)$ and lower semi-continuity of $x_{*,j+1}(\cdot)$, the function $|x_{*,j+1}(\overline{y}) - x^*_j(\overline{y})|$ is lower semi-continuous and therefore attains its minimum on the (nonempty, compact) interval $[y_L, y_H]$. Since, from Propositions 1 and 2, $x_{*,j+1}(\overline{y}) > x^*_j(\overline{y})$ for all $\overline{y}$, this minimum is positive. Thus, there exists $\varepsilon'' > 0$ such that for all $\overline{y} \in [y_L, y_H]$, optimal policies differ by at least ε''. Finally, we set $\varepsilon = \min\{\varepsilon', \varepsilon''\}$ to establish the desired lower bound.

We are interested in the profiles $(\pi_1, \ldots, \pi_n)$ such that for all politician types j, π_j places positive probability on at most two alternatives, and the supports of mixed policy strategies are strictly ordered by type and separated by a distance of at least ε, i.e., for all $j < n$ and all policies x_j with $\pi_j(x_j) > 0$ and x_{j+1} with $\pi_{j+1}(x_{j+1}) > 0$, we have $x_j + \varepsilon \leqslant x_{j+1}$. It is convenient to represent such a profile by a $3n$-tuple (x,z,r), where $x = (x_1, \ldots, x_n) \in [0, \overline{x}]^n$, $z = (z_1, \ldots, z_n) \in [0, \overline{x}]^n$, and $r = (r_1, \ldots, r_n) \in [0,1]^n$. In addition, we require that for all j, we have $x_j \leqslant z_j$, and that for all $j < n$, we have $z_j + \varepsilon \leqslant x_{j+1}$. We then associate (x,z,r) with the profile of mixed policy strategies such that the type j politician places probability r_j on x_j and the remaining probability $1 - r_j$ on z_j. Letting D^ε consist of all such $3n$-tuples (x,z,r), we see that D^ε is nonempty, convex, and compact. Using this representation, we can define (abusing notation slightly) the induced cutoff $y^*(x,z,r)$, which is continuous as a function of its arguments.

The second difficulty is that the set Y of policy outcomes is not compact, so that the voter's cutoff is, in principle, unbounded. To circumvent this problem, we note that by continuity of the function $y^*(\cdot)$ the image $y^*(D^\varepsilon)$ is compact, and we can let $\overline{Y}$ be a closed interval containing this image. The existence proof then proceeds with an application of a fixed point theorem that relaxes Kakutani's conditions. We define the correspondence $\Phi\colon D^\varepsilon \times \overline{Y} \rightrightarrows D^\varepsilon \times \overline{Y}$ so that for each $(x,z,r,\overline{y})$, the value of Φ consists of $(3n+1)$-tuples $(\tilde{x}, \tilde{z}, \tilde{r}, \tilde{y})$ such that for every politician type j, the mixed policy strategy represented by $(\tilde{x}_j, \tilde{z}_j, \tilde{r}_j)$ is optimal given $\overline{y}$, and $\tilde{y}$ is the unique cutoff induced by the indifference condition:

$$\Phi(x,z,r,\overline{y}) \quad = \quad \left\{ (\tilde{x}, \tilde{z}, \tilde{r}, \tilde{y}) \in D^\varepsilon \times \overline{Y} \;\middle|\; \begin{array}{c} \text{for all } j,\ \tilde{x}_j \leqslant \tilde{z}_j, \\ \tilde{r}_j > 0 \Rightarrow \tilde{x}_j \in \{x_{*,j}(\overline{y}), x^*_j(\overline{y})\}, \\ \tilde{r}_j < 1 \Rightarrow \tilde{z}_j \in \{x_{*,j}(\overline{y}), x^*_j(\overline{y})\}, \\ \text{and } \tilde{y} = y^*(x,z,r) \end{array} \right\}.$$

Of note, we require that the first policy coordinate $\tilde{x}_j$ is less than or equal to the second, $\tilde{z}_j$, and we require that these are optimal when chosen with positive probability.

To deduce the existence of a fixed point of Φ, we first verify that the correspondence is upper hemi-continuous with closed values, i.e., it has closed graph. This property is not immediately obvious, because optimal policies are not unique, and the functions $x_j^*(\cdot)$ and $x_{*,j}(\cdot)$ are not continuous. It is important that we allow for the possibility that $\tilde{x}_j = \tilde{z}_j$, in which case both policies are equal to either the least optimal policy $x_{*,j}(\overline{y})$ or to the greatest optimal policy $x_j^*(\overline{y})$. Of course, these policies can coincide as well. Let $\{(x^m, z^m, r^m, \overline{y}^m)\}$ be any sequence converging to $(x, z, r, \overline{y})$ in $D^\varepsilon \times \overline{Y}$, and consider a corresponding sequence $\{(\tilde{x}^m, \tilde{z}^m, \tilde{r}^m, \tilde{y}^m)\}$ such that $(\tilde{x}^m, \tilde{z}^m, \tilde{r}^m, \tilde{y}^m)$ belongs to $\Phi((x^m, z^m, r^m, \overline{y}^m)$ for all m and $(\tilde{x}^m, \tilde{z}^m, \tilde{r}^m, \tilde{y}^m) \to (\tilde{x}, \tilde{z}, \tilde{r}, \tilde{y})$. We must show that $(\tilde{x}, \tilde{z}, \tilde{r}, \tilde{y}) \in \Phi(x, z, r, \overline{y})$. Since limits preserve weak inequalities, it is immediate that for all j, we have $\tilde{x}_j \leqslant \tilde{z}_j$, and continuity of $y^*(\cdot)$ implies $\tilde{y} = y^*(x, z, r)$. It remains to establish optimality of policies. It remains to establish optimality of policies adopted with positive probability, and we consider x_j, as the argument for z_j is analogous. To this end, suppose $\tilde{r}_j > 0$, so that for sufficiently high m, we also have $\tilde{r}_j^m > 0$, implying $\tilde{x}_j^m \in \{x_{*,j}(\overline{y}^m), x_j^*(\overline{y}^m)\}$. Since the best response correspondence is upper hemi-continuous (Proposition 1), $\tilde{x}_j$ is an optimal policy for the type j politician given cutoff $\overline{y}$. If $\tilde{x}_j \notin \{x_{*,j}(\overline{y}), x_j^*(\overline{y})\}$, then this implies the politician has at least three optimal policies, contradicting Proposition 1. Thus, $\tilde{x}_j$ is either the least or greatest optimal policy given $\overline{y}$, as desired.

This formulation yields a correspondence that is defined on a convex and compact domain and that is upper hemi-continuous and has nonempty, closed values. The typical application of Kakutani's fixed point theorem also proceeds by verifying convex values of the correspondence, and this leads to the third difficulty: Φ does not have this property. In particular, this property fails if $(x, z, r, \overline{y})$ is such that $x_j^*(\overline{y}) \neq x_{*,j}(\overline{y})$ for some j. Nevertheless, the values of the correspondence are contractible, and this is sufficient for existence of a fixed point. A subset $C \subseteq \Re^d$ of Euclidean space is *contractible* if there is an element $\overline{c} \in C$ and a continuous mapping $h: C \times [0,1] \to C$ such that for all $c \in C$, $h(c, 0) = c$ and $h(c, 1) = \overline{c}$. That is, the set can be continuously deformed to a single element. Convex sets are contractible, but convexity is not necessary for contractibility. It is straightforward to see that $\Phi(x, z, r, \overline{y})$ is contractible to the element $(\hat{x}, \hat{z}, \hat{r}, \hat{y})$ such that: for all j,

- $\hat{x}_j = x_{*,j}(\overline{y})$,
- $\hat{z}_j = x_j^*(\overline{y})$,
- $\hat{r}_j = 1$,

where of course $\hat{y} = y^*(x, z, r)$ is fixed by construction. To reduce notation, we provide an informal description of the mapping h, breaking the unit interval into four components. Consider any $(\tilde{x},\tilde{z},\tilde{r},\tilde{y}) \in \Phi(x,z,r,\overline{y})$. For $t \in [0,.25]$, we continuously adjust each $\tilde{r}_j$ by dropping these values to zero. For $t \in (.25,.5]$, we continuously adjust each $\tilde{x}_j$ to $x_{*,j}(\overline{y})$. This adjustment requires that $\tilde{x}_j$ take sub-optimal values, but because the probability on $\tilde{x}_j$ is zero, we remain in the value of the correspondence. For $t \in (.5,.75]$, we continuously adjust each $\tilde{r}_j$ to one. And for $t \in (.75,1]$, we continuously adjust each $\tilde{z}_j$ to $x^*_j(\overline{y})$. This completes the construction, and we conclude that the values of Φ are contractible.

The correspondence Φ is upper hemi-continuous with nonempty, closed, contractible values, and the domain $D^\varepsilon \times \overline{Y}$ is nonempty, compact, and convex. Therefore, the Eilenberg-Montgomery fixed point theorem (see McLennan (2014), Theorem 14.1.5) implies that Φ has a fixed point,[7] (x^*,z^*,r^*,y^*), which yields an electoral equilibrium. Finally, the characterization results in (i)–(iii) follow directly from Propositions 1–3. □

6 Responsive democracy

We have not yet touched on the possibility of responsive democracy, meaning that incumbents choose high levels of policy, despite the short run temptation to choose their ideal policies. Given the short time horizon (and limited ability of the voter to sanction politicians), and given the divergence in preferences between the voter and politicians, the prospects for well-functioning democratic elections may seem dim. Nevertheless, when β is large, so that politicians are substantially office-motivated, we obtain a form of responsive democracy. We now make use of a standard Inada-type condition: for all j,

(A7) $$\lim_{x\to\infty} w'_j(x) = -\infty.$$

We also slightly strengthen (A1): for all $j > 1$,

(A8) $w'_j(x)$ is uniformly continuous, and there is some $\varepsilon > 0$ such that $w'_j(x) - w'_{j-1}(x) < \varepsilon$ for all x.

Intuitively, we require the marginal cost of effort to increase without bound with effort, and we require that the differences in marginal cost between higher and lower types do not vanish as effort increases. Both assumptions are satisfied in the quadratic and exponential cases and many other cases of interest. Let $G = \{j : \mathbb{E}[u(y)|\hat{x}_j] > V^C\}$ denote the set of "above average types," which are such

[7] The Eilenberg-Montgomery fixed point theorem holds for a domain that is a nonempty compact absolute retract. Every compact, convex set is an absolute retract (McLennan, 2014), so this assumption is satisfied automatically.

that the expected utility from their ideal policy exceeds the expected utility from a challenger. Let $\ell = \min G$ be the smallest above average type.

Our second main result provides a characterization of electoral equilibria when office benefit is high. We find that the voter becomes arbitrarily demanding, in the sense that the equilibrium cutoff diverges to infinity, that the policy choices of all politician types become close to their ideal policy or arbitrarily large, and that all above average types in fact exert unbounded effort. Moreover, when the office benefit becomes arbitrarily large, the best politician type below average puts positive probability on a policy that becomes arbitrarily large with the office benefit, while the worse politician type puts positive probability on a policy close the politician's ideal policy.

If the voter's utility function is unbounded, then an immediate implication, since type n is above average and $p_n > 0$, is that the voter's expected utility from politicians' choices in the first period increases without bound as office benefit becomes large. If instead the voter's payoff is bounded, i.e.

$$\lim_{y\to\infty} u(y) = \overline{u} \quad \text{for some } \overline{u} \in \Re_+,$$

then the voter's expected utility from politicians' choices in the first period is bounded below by

$$\sum_{j\notin G} p_j \mathbb{E}[u(y)|\hat{x}_j] \quad + \quad \sum_{j\in G} p_j \overline{u}.$$

Note that condition (A7) is used only for parts (iii) and (iv) of the result, and condition (A8) only for part (iv). As in the previous section, we use x_j^* and $x_{*,j}$ to denote, respectively, the highest and the lowest policy choices of politicians of type j in a given equilibrium.

Theorem 6.1. *Assume (A1)–(A8). Let the office benefit* β *be arbitrarily large. Then for every selection of electoral equilibria* σ*, the voter's cutoff diverges to infinity; the policy choices of each politician type either accumulate at their ideal policy or increase without bound; the policy choices of all above average types increase without bound; and the lowest policy of the lowest type accumulates at that type's ideal policy:*

(i) $y^* \to \infty$,

(ii) for all j*, all* $\varepsilon > 0$*, and sufficiently large* β*, we have either*

$$x_{*,j} = x_j^* \in (\hat{x}_j, \hat{x}_j + \varepsilon) \cup (\tfrac{1}{\varepsilon}, \infty) \quad \textit{or} \quad x_{*,j} \in (\hat{x}_j, \hat{x}_j + \varepsilon) \ \textit{and} \ x_j^* \in (\tfrac{1}{\varepsilon}, \infty),$$

(iii) $x^*_{\ell-1} \to \infty$ *and thus for all* $j \geqslant \ell$, *and sufficiently large* β, *we have* $x_{*,j} = x^*_j \to \infty$,

(iv) $x_{*,1} \to \hat{x}_1$.

Proof. Consider an electoral equilibrium as β becomes large. By Theorem 5.1, each politician type j mixes between two policies, x^*_j and $x_{*,j}$, and the voter uses a finite cutoff y^*. Suppose there is a subsequence such that y^* is bounded above, say $y^* \leqslant \bar{y}$. By Theorem 5.1, the equilibrium cutoff lies in the compact set $[\hat{x}_1 + \hat{z}, \bar{y}]$. Then the first order condition for the type 1 politician in (2) implies that $x_{*,1} \to \infty$, and in particular, we have $x_{*,1} > \bar{y} - \hat{z}$ for large enough β, but this contradicts $x_{*,1} + \hat{z} \leqslant y^* \leqslant \bar{y}$. We conclude that y^* diverges to infinity, which proves (i).

To prove (ii), suppose there is a type j, an $\varepsilon > 0$, and a subsequence of office benefit levels such that $\hat{x}_j + \varepsilon \leqslant x^*_j \leqslant \frac{1}{\varepsilon}$. Going to a subsequence, we can assume $x^*_j \to \tilde{x}_j$ such that $\hat{x}_j < \tilde{x}_j < \infty$. Then for sufficiently large β, we have $\hat{x}_j < x^*_j$. For these parameters, the payoff to the type j politician from choosing $\hat{x}_j$ instead of x^*_j is non-positive, and thus we note that

$$(F(y^* - \hat{x}_j) - F(y^* - x^*_j))[w_j(\hat{x}_j) + \beta - V^C] \quad \geqslant \quad w_j(\hat{x}_j) - w_j(x^*_j).$$

That is, the current gains from choosing the ideal policy are offset by future losses. Since $y^* \to \infty$, the limit of

$$\frac{F(y^* - x^*_j) - F(y^* - \tilde{x}_j - 1)}{F(y^* - \hat{x}_j) - F(y^* - x^*_j)}$$

as β becomes large is indeterminate, and by L'Hôpital's rule, the limit is equal to

$$\lim \frac{f(y^* - x^*_j) - f(y^* - \tilde{x}_j - 1)}{f(y^* - \hat{x}_j) - f(y^* - x^*_j)} \;=\; \lim \frac{f(y^* - \tilde{x}_j - 1)\left(\frac{f(y^* - x^*_j)}{f(y^* - \tilde{x}_j - 1)} - 1\right)}{f(y^* - x^*_j)\left(\frac{f(y^* - \hat{x}_j)}{f(y^* - x^*_j)} - 1\right)} \;=\; \infty,$$

where we use (A3) and (A4). Then, however, the future gain from choosing $\tilde{x}_j + 1$ instead of x^*_j strictly exceeds current losses, i.e.,

$$(F(y^* - x^*_j) - F(y^* - \tilde{x}_j - 1))[w_j(\hat{x}_j) + \beta - V^C] \quad > \quad w_j(x^*_j) - w_j(\tilde{x}_j + 1), \quad (5)$$

for high enough β. To be specific, let

$$\begin{aligned} A &= w_j(\hat{x}_j) + \beta - V^C, \\ B &= w_j(\hat{x}_j) - w_j(x^*_j), \text{ and} \\ C &= w_j(x^*_j) - w_j(\tilde{x}_j + 1), \end{aligned}$$

where A is evaluated at sufficiently large β. Note that since $\hat{x}_j < \tilde{x}_j < \infty$, we have $\lim B > 0$ and $\lim C < \infty$. We have noted that $(F(y^* - \hat{x}_j) - F(y^* - x^*_j))A \geqslant B$ for sufficiently large β, and we have shown that as β becomes large, we have

$$\frac{F(y^* - x^*_j) - F(y^* - \tilde{x} - 1)}{F(y^* - \hat{x}_j) - F(y^* - x^*_j)} \quad > \quad \frac{C}{B}.$$

Combining these facts, we have

$$(F(y^* - \hat{x}_j) - F(y^* - x^*_j))A\left(\frac{F(y^* - x^*_j) - F(y^* - \tilde{x} - 1)}{F(y^* - \hat{x}_j) - F(y^* - x^*_j)}\right) \quad > \quad B\left(\frac{C}{B}\right),$$

which yields (5) for large β. This gives the type j politician a profitable deviation from x^*_j, a contradiction. A similar argument holds for $x_{*,j}$. It follows that for all j, all $\varepsilon > 0$, and sufficiently large β, we have $\{x_{*,j}, x^*_j\} \subset (\hat{x}_j, \hat{x}_j + \varepsilon) \cup (\frac{1}{\varepsilon}, \infty)$,

To establish that $\{x_{*,j}, x^*_j\} \subset (\hat{x}_j, \hat{x}_j + \varepsilon)$ for all $\varepsilon > 0$, and sufficiently large β, implies $x_{*,j} = x^*_j$ for sufficiently large β, suppose otherwise. Then there must be a a sequence of equilibria such that along that sequence $x_{*,j} \neq x^*_j$ and $\{x_{*,j}, x^*_j\} \subset (\hat{x}_j, \hat{x}_j + \varepsilon)$ for some type j for all ε and for sufficiently large β. Using part (i), we can find a subsequence of equilibria for increasing values of the office benefit such that along that subsequence the voter cutoff is strictly increasing in β. In each equilibrium in the sequence there must be a local minimizer located in between $x_{*,j}$ and x^*_j. Note that a local minimizer must satisfy the necessary second order condition:

$$\frac{w''_j(x)}{w'_j(x)} \leqslant -\frac{f'(y-x)}{f(y-x)}$$

for $y = y^*$. Let $\tilde{X}(y) \subseteq X$ denote the set of policies satisfying the inequality above for a given voter cutoff. By (A6), $\tilde{X}(y)$ is convex for any y. From the necessary second order condition for a maximizer, we get

$$\frac{w''_j(x_{*,j})}{w'_j(x_{*,j})} \geqslant -\frac{f'(y^* - x_{*,j})}{f(y^* - x_{*,j})} \quad \text{and} \quad \frac{w''_j(x^*_j)}{w'_j(x^*_j)} \geqslant -\frac{f'(y^* - x^*_j)}{f(y^* - x^*_j)}.$$

Since there must be a minimizer in the interval $[x_{*,j}, x^*_j]$, these two inequalities imply $\varnothing \neq \tilde{X}(y^*) \subseteq [x_{*,j}, x^*_j]$. Now fix one value of β, say β', and let x' denote the minimizer in between $x'_{*,j}$ and x'^*_j, so that $x' > \hat{x}_j$ and $x' \in \tilde{X}(y'^*)$. Since f is log-concave by (A3) and y^* is increasing in the value of the office, for any $\beta > \beta'$ we get $x' \in \tilde{X}(y^*)$. But then $x' \in [x_{*,j}, x^*_j]$ for any $\beta > \beta'$, which implies $x_{*,j} \leqslant x' \leqslant x^*_j$ for all $\beta > \beta'$, so that $x^*_j > \hat{x}_j + \varepsilon$ for small enough ε, a contradiction. A similar argument establishes that $\{x_{*,j}, x^*_j\} \subset (\frac{1}{\varepsilon}, \infty)$ implies $x_{*,j} = x^*_j$ for sufficiently large β.

To prove (iii), suppose that $x^*_{\ell-1}$ does not diverge to infinity. By (ii), there is a subsequence such that $x^*_{\ell-1} \to \hat{x}_{\ell-1}$. Now fix politician type $j \leqslant \ell - 1$, and note that since equilibrium policy choices are ordered by type, we have $x^*_j \to \hat{x}_j$. Using the expression for Bayes rule, the posterior probability of type j conditional on observing y^* satisfies

$$\mu_T(j|y^*) \;\; = \;\; \frac{p_j \sum_x f(y^* - x)\pi_j(x)}{\sum_k p_k \sum_x f(y^* - x)\pi_k(x)} \;\leqslant\; \frac{p_j f(y^* - x^*_j)}{\sum_{k \geqslant \ell} p_k \sum_x f(y^* - x)\pi_k(x)},$$

where the inequality uses $y^* \to \infty$ with (A3) and (A4). Note that

$$\sum_{k \geqslant \ell} p_k \sum_x f(y^* - x)\pi_k(x) \;\; = \;\; \sum_{k \geqslant \ell} p_k [f(y^* - x^*_k)\pi_k(x^*_k) + f(y^* - x_{*,k})\pi_k(x_{*,k})].$$

Dividing by $f(y^* - x^*_j)$, we obtain the expression

$$\sum_{k \geqslant \ell} p_k \left[\frac{f(y^* - x^*_k)}{f(y^* - x^*_j)} \pi_k(x^*_k) + \frac{f(y^* - x_{*,k})}{f(y^* - x^*_j)} \pi_k(x_{*,k}) \right].$$

By (A3) and (A4), if $x^*_k \to \hat{x}_k$, then we have $\frac{f(y^* - x^*_k)}{f(y^* - x^*_j)} \to \infty$. By (ii), the remaining case is $x^*_k \to \infty$. Note that in this case, (A7) implies $w'_k(x^*_k) \to -\infty$, and thus the first order condition in (2) implies that $f(y^* - x^*_k)\beta \to \infty$. The first order condition for type j implies $f(y^* - x^*_j)\beta \to 0$, and we infer that $\frac{f(y^* - x^*_k)}{f(y^* - x^*_j)} \to \infty$. Similarly, $\frac{f(y^* - x_{*,k})}{f(y^* - x^*_j)} \to \infty$ for all $k \geqslant \ell$. Thus, we have

$$\mu_T(j|y^*) \;\leqslant\; \frac{p_j}{\sum_{k \geqslant \ell} p_k \sum_x \frac{f(y^* - x)}{f(y^* - x^*_j)} \pi_k(x)} \;\to\; 0.$$

We conclude that the voter's posterior beliefs conditional on y^* place probability arbitrarily close to one on above average types $j \geqslant \ell$, contradicting the indifference condition in (3). Therefore, we have $x^*_{\ell-1} \to \infty$, and since policy choices are ordered by type, this implies that $x_{*,j} \to \infty$ for all $j \geqslant \ell$.

For part (iv), note that, from part (ii), either $x_{*,1} \to \hat{x}_1$ or $x_{*,1}$ increases without bound. Suppose, by way of contradiction, that $x_{*,1}$ increases without bound. From the necessary first and second order conditions of type 1's problem, we must have

$$\frac{w''(x_{*,1})}{w'(x_{*,1})} \geqslant -\frac{f'(y^* - x_{*,1})}{f(y^* - x_{*,1})}.$$

From (A7), we have that the left-hand side of the inequality above converges uniformly to zero from above as $x_{*,1} \to \infty$. Since f is log-concave by (A3), it follows that $-f'(z)/f(z)$ is strictly increasing in z, and moreover $-f'(z)/f(z) > 0$ for $z > \hat{z}$. From Proposition 5.1(iii), $y^* - x_{*,1} > \hat{z}$, where $\hat{z}$ is the mode of the outcome density, so that the right-hand side of the inequality above is strictly positive. We conclude that $y^* - x_{*,1}$ must converge to $\hat{z}$ from above as $x_{*,1}$ increases without bound. We claim that there must be some $\kappa > 0$ such that $|x_{*,2} - y^*| > \kappa + \hat{z}$ for large enough β. For suppose not; then we can find a subsequence of values of $x_{*,2}$ such that $|x_{*,2} - y^*|$ converges to $\hat{z}$. From the first order condition of politician's types 1 and 2 we have

$$(w_2(\hat{x}_2) + \beta - V^C) f(y^* - x_{*,2}) - (w_1(\hat{x}_1) + \beta - V^C) f(y^* - x_{*,1}) \\ = -w_2'(x_{*,2}) + w_1'(x_{*,1}).$$

Along the proposed subsequence, the left-hand side in the equation above converges to $(w_2(\hat{x}_2) - w_1(\hat{x}_1)) f(\hat{z}) \geqslant 0$, but the right-hand side is negative for large enough β, since, from (A8),

$$-w_2'(x_{*,2}) + w_1'(x_{*,1}) \leqslant -\varepsilon - w_1'(x_{*,2}) + w_1'(x_{*,1})$$

for some $\varepsilon > 0$, and, from uniform continuity, there is some $\delta > 0$ such that $|x_{*,2} - x_{*,1}| < \delta$ implies $|w_1'(x_{*,1}) - w_1'(x_{*,2})| < \varepsilon$. Since policies are ordered by type, from the previous argument we have that

$$x_{*,1} < \hat{z} < \hat{z} + \kappa < x_{*,2} \leqslant x_2^* < x_{*,3} \leqslant x_3^* < \cdots,$$

and moreover $x_{*,1} \to \hat{z}$. Since f is single-peaked at $\hat{z}$, we see that for all $x_1, \ldots, x_n$ such that each x_k is in the support of π_k, and for large enough β we have

$$f(\bar{y} - x_1) \;<\; f(\bar{y} - x_2) \;<\; \cdots \;<\; f(\bar{y} - x_n).$$

A contradiction can be obtained following the same argument in the proof of proposition 3, which proves part (iv). □

Part (ii) of Theorem 6.1 implies that at most one type of politician chooses a proper mixed strategy for large values of the office benefit. Part (iii) implies that such type, if it exists, must be below average. If there are only two types, then type 1 a fortiori is below average and type 2 above average; that is, $\ell - 1 = 1$. Moreover, from part (iii) and (iv) of the theorem, type 1 must be playing a mixed strategy:

Corollary 1. *Assume (A1)–(A8) and $n = 2$. Let the office benefit β be arbitrarily large. Then for every selection of electoral equilibria, and for large enough β, a politicians of type 1 adopt a mixed strategy, with $x_{*,1} \to \hat{x}_1$ and $x_1^* \to \infty$ and politicians of type 2 adopt a pure strategy, with $x_2^* \to \infty$.*

That is, in a two-type model, for large enough value of the office, the low type plays a mixed strategy between "taking it easy" and "going for broke," while the high type "goes for broke" with probability one. As an illustration, consider again the simple quadratic-normal example with two types, that is $w_j(x) = -(x-\hat{x}_j)^2$ for $j = 1,2$ and $F = \Phi$, where Φ is the standard normal distribution, and let $\hat{x}_j = j$ for $j = 1,2$ and $\beta - V^C = 20$. Note that the problem of politician's type 1 is depicted in Figures 4 and 5. Inequality 4 fails, so an equilibrium is necessarily mixed. We can construct a mixed strategy equilibrium recursively. First, we find out the cutoff that makes type 1 indifferent; in this example, for $y^* \approx 4.21$, type 1 is indifferent between $x_{*,1} \approx 1$ and $x_1^* \approx 4.51$. Solving the first-order condition of the problem of type 2, we get $x_2^* \approx 4.98$. Finally, we can obtain $\pi_1(x_{*,1})$ and $\pi_1(x_1^*)$ from the indifference condition for the voter. Assuming the voter's payoff is linear in the policy outcome, we get $\pi_1(x_{*,1}) \approx 0.21$ and $\pi_1(x_1^*) \approx 0.79$.

7 Literature review

Closely related to our model is Fearon's (1999) seminal work on "selection and sanctioning," with some important differences. First, Fearon assumes that there are just two types, that utility is quadratic, and that incentive constraints bind for only one type. Second, he assumes that a random shock is added directly to the voter's utility, and not to the underlying policy outcome; thus, Fearon's model cannot generally be interpreted as capturing an uncertain linkage between policy and observable variables, such as employment status, inflation, etc., on which voters might base their decisions.[8] Third, Fearon focus his analysis on pure strategy equilibria, but he does not prove existence of such an equilibrium. At issue is the possibility of non-convexities in the first-term office holder's objective function; this is exemplified in our Figure 2, and it arises in Fearon's model through non-concavity of the objective function in his equation (1). In our model, as in his, a policy satisfying first order conditions need not be a global maximizer of the politician's problem, even if it satisfies the second order condition. It is well-known that in non-convex games, existence of pure strategy equilibria is problematic, and this is true, in particular, of the electoral accountability model: it may be that none of the candidates for equilibrium identified by Fearon are, in fact, equilibria. In contrast, we impose sufficient structure on the model, in the form of (A1)–(A7), to establish existence of electoral equilibria and that mixing is limited to at most two policy choices for each politician type.

[8] The distinction between utility and policy outcomes disappears when the voter is risk neutral, i.e., u is affine linear, but the approaches are not equivalent in the general case, when the voter may be risk averse or risk loving.

Chapter 3 of Besley (2006) presents a two-period, two-type model in which the incumbent politician observes the values of a binary state of the world and preference shock, and then makes a binary policy choice. Closer to the model of our paper, Chapter 4 (coauthored with Smart) of the book investigates a two-type model in which an incumbent essentially chooses a level x of shirking, and voters observe this with noise, $x+\varepsilon$. Besley and Smart assume, however, that the incumbent politician observes the policy shock ε before her choice of x; in addition, the policy choice of the good type of politician is fixed exogenously. In these models, the politician's choice is either explicitly between two possible policies, or it reduces to a finite number of policies, so that equilibria in mixed strategies are assured to exist.

Chapter 4 of Persson and Tabellini (2000) contains a simplified, two-period model of symmetric learning, in which politicians are parameterized by a skill level that is unobserved by voters and politicians themselves. In this setting, voters and politicians update their beliefs symmetrically along the equilibrium path, and signaling cannot occur. Moreover, voters are assumed to be risk neutral. Ashworth (2005) considers a three-period model of symmetric learning that further differs from ours in that the skill level of a politician evolves over time according to a random walk. Although the model assumes three periods, the first-term office holder has private information about her ability only in the second and third terms, as her action in office are hidden from voters. Ashworth assumes that office benefit is small relative to the variance of policy outcomes in order to guarantee existence of equilibrium in pure strategies. Ashworth and Bueno de Mesquita (2008) use a variant of the model, one in which the voter has quadratic policy utility and a stochastic partisan preference, to establish existence and comparative statics of incumbency advantage.

Other work, including Barganza (2000) and Canes-Wrone et al. (2001), studies a two-type model in which politicians differ in ability. In the latter paper, the voter's desired policy depends on the realization of a state of the world, about which politicians are better informed. Politicians may have an incentive to pander to voters by knowingly choosing policies that are not in the voter's best interest. Maskin and Tirole (2004) study pandering in a two-type model in which politicians differ in preferences. Austen-Smith and Banks (1989) investigate the voter's ability to discipline politicians when all politicians have the same preferences, so that the model is one of pure moral hazard. In a two-period model of pure adverse selection, where politicians' policy choices are directly observed by voters, Duggan and Martinelli ((forthcoming) show that responsive democracy can arise due to the incentive of all politician types to "imitate" the median type. Theorem 6.1 in the current paper establishes that a form of this incentive holds for all above average types, delivering the responsiveness result even when policy choices are observed by voters with noise.

Due to difficult theoretical issues, the literature going beyond two periods is small. In an infinite-horizon version of our model, Banks and Sundaram (1993) find existence of (a continuum of) perfect Bayesian equilibria in which the voter uses a trigger strategy: if the observed policy outcome ever falls below a given level, then the voter resolves to replace the incumbent with a challenger, and the incumbent shirks for all remaining terms of office. The difficulty with such equilibria is that even if the incumbent is a good type with very high probability, there is a chance that the "trigger" is pulled, and then equilibrium strategies dictate that the otherwise attractive incumbent politician is replaced. Banks and Sundaram (1998) study the infinite-horizon model with a two-period term limit give an initial argument for existence of equilibrium. Duggan (2015) corrects an error in the argument of Banks and Sundaram and establishes limits on the possibility of responsive democracy in the infinite-horizon model: because voters cannot commit to replacing a politician after her first term of office, the voter's expected payoff from a first-term office holder is bounded strictly above by the expected utility from the ideal policy of the best politician type. Thus, the commitment problem of voters implies a qualitative difference between the two-period model and the infinite-horizon model with a two-period term limit. Another recent related work on term limits is Kartik and Van Weelden (2015).

Recently, Acemoglu et al. (2013) have analyzed an electoral model with policy outcomes in an ideological spectrum and two citizen types: a median type and a right-wing type, with ideal point to the right of the median. Other than this spatial structure, the modeling framework is similar to ours: the first period politician's type is unobserved by the median, she makes a policy choice that is observed with noise, and the median voter then decides between re-electing the incumbent or installing an unknown challenger in the second period. Assuming quadratic utility and that the noise density is symmetric around zero and sufficiently dispersed relative to office benefit,[9] the authors show that there is a unique equilibrium in pure strategies, and in equilibrium the median politician type chooses policies to the left of the median. Although this choice hurts median citizens (including the politician) in the first period, the cost is outweighed by the potential benefit of re-election in the second period: the median voter's equilibrium strategy is to re-elect the incumbent if the realized policy outcome lies to the left of a given cutoff, and thus the median politician's probability of re-election increases as her policy choice moves to the left. Furthermore, the extent of this populist bias increases in office benefit, and if office benefit is sufficiently great, then even the right-wing candidate chooses

[9]The assumption that the density is dispersed allows the authors to focus on pure strategy equilibria; in general, the illustration in Figure 2 points to the need for mixing in equilibrium.

leftist policies. The focus of Acemoglu et al. (2013) on the ideological model may appear to remove their analysis from the framework of the current paper, but our results have direct implications for the effects of populism in the one-dimensional spatial model. In particular, the techniques developed in this article allow to study situations in which their model has no pure strategy equilibria.

8 Conclusion

The two-period model of elections provides a tractable setting for analysis of the interplay between short-term opportunistic incentives and long-term re-election incentives in determining politicians' behavior. We consider a natural, but analytically challenging, environment in which voters are imperfectly informed about both the preferences and the actions of politicians, and we allow for an arbitrary finite set of politician types and general preferences. In line with the extant literature on electoral accountability, we assume that politicians and voters cannot commit to future actions, opening the scope for opportunistic behavior and creating potential difficulties for the success of democratic electoral mechanisms. We believe the two-period accountability model provides a canonical framework in which to approach these issues, but despite this, foundational questions of equilibrium existence and responsiveness of policy to voter preferences have remained open. We address these questions by showing that office holder mix over at most two policy choices—"taking it easy" and "going for broke"—and establishing existence of electoral equilibrium. We then establish the possibility of responsive democracy: as politicians become more office motivated, the re-election standard used by voters becomes arbitrarily demanding, and the equilibrium level of expected effort in the first period by above average politicians becomes arbitrarily large. We conclude that incentives present in democratic elections have the potential to discipline the actions of elected representatives, mitigating the difficulties inherent in voters' sparse information and limited ability to sanction politicians.

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