Myopia and Discounting*

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Abstract

We assume that perfectly patient agents estimate the value of future events by generating noisy, unbiased simulations and combining those signals with priors to form posteriors. These posterior expectations exhibit *as-if discounting*: agents make choices as if they were maximizing a stream of known utils weighted by a discount function, $D(t)$. This as-if discount function reflects the fact that estimated utils are a combination of signals and priors, so average expectations are optimally shaded toward the mean of the prior distribution, generating behavior that partially mimics the properties of classical time preferences. When the simulation noise has variance that is linear in the event's horizon, the as-if discount function is hyperbolic, $D(t) = 1/(1 + \alpha t)$. Our agents exhibit systematic preference reversals, but have no taste for commitment because they suffer from imperfect foresight, which is not a self-control problem. In our framework, agents that are more skilled at forecasting (e.g., those with more intelligence) exhibit less discounting. Agents with more domain-relevant experience exhibit less discounting. Older agents exhibit less discounting (except those with cognitive decline). Agents who are encouraged to spend more time thinking about an intertemporal tradeoff exhibit less discounting. Agents who are unable to think carefully about an intertemporal tradeoff – e.g., due to cognitive load – exhibit more discounting. In our framework, patience is highly unstable, fluctuating with the accuracy of forecasting.

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1 Introduction

Most people appear to act *as if* they have a strong preference for earlier rewards over later rewards. For the last century economists have usually assumed that this type of behavior reflects (fundamental) time preferences, which economists model with discount factors that multiplicatively weight utils. If the one-period-ahead discount factor is δ, then δ utils experienced now are as valuable as one util experienced next period. If $\delta < 1$, economic agents prefer a current util to a delayed util.

However, such time preferences are only one of many ways to explain the empirical regularity that intertemporal choices are characterized by *declining sensitivity as utils are moved further away in time*. Diminishing sensitivity to future utils is also explained by imperfect information. For example, Böhm-Bawerk (1889) wrote that "we possess inadequate power to imagine and to abstract, or that we are not willing to put forth the necessary effort, but in any event we limn a more or less incomplete picture of our future wants and especially of the remotely distant ones. And then, there are all of those wants that never come to mind at all." Pigou (1920) similarly observed "that our telescopic faculty is defective, and that we, therefore, see future pleasures, as it were, on a diminished scale. That this is the right explanation is proved by the fact that exactly the same diminution is experienced when, apart from our tendency to forget ungratifying incidents, we contemplate the past."[1] Pigou believed that our imperfect ability to forecast the future mirrors our imperfect ability to recall the past.

To gain intuition for the role of imperfect forecasting, consider a driver who sees an upcoming pothole and estimates that it is small. A few moments later, he realizes that the pothole is large, swerves to avoid it, and crashes. This accident is likely a reflection of imperfect foresight, not procrastination or laziness. In this case, large delayed consequences are misperceived by an imperfectly farsighted driver. We probably wouldn't infer that the driver didn't care about the impending crash because it was in the "future." If the driver had foreseen the consequences, he would have braked earlier. In general, people will not respond optimally to future consequences that they do not anticipate (or only partially anticipate).

Likewise, consider a sailor who sees a few clouds forming on the horizon and doesn't immediately take the costly action of charting a new course. When her vessel is lashed by

[1] For a review of the history of theories of discounting see Loewenstein (1992).

a violent storm the next day, it is not clear whether she was lazy the previous night, or just mistaken in her forecast about the upcoming weather.

Decision-making is rife with situations in which a current action/inaction causes a stream of current and future consequences, many of which are hard to foresee. If delayed consequences are typically harder to foresee than immediate consequences, then decision-makers will appear to be impatient.

The role of imperfect information is also apparent in the seemingly impatient behavior of non-human animals. When monkeys are given an abstract intertemporal choice task on a computer, they act as if they discount delayed rewards at the rate of 10% *per second*. When the same monkeys are given a temporally analogous foraging task (also presented on a computer screen), the monkeys show very little discounting (Blanchard and Hayden 2015). Animal behavior appears to be impatient in completely novel domains and patient in domains that are evolutionarily relevant. As Blanchard and Hayden (2015) conclude, "Seemingly impulsive behavior in animals is an artifact of their difficulty understanding the structure of intertemporal choice tasks."

In the current paper, we argue that behavior that arises from imperfect foresight is hard to distinguish from behavior that arises from time preferences. We study a Bayesian decision-maker with *perfectly* patient time preferences who receives noisy signals about the future. The resulting signal-extraction problem leads the Bayesian agent to behave in a way that is easy to misinterpret as a time preference; we call this seemingly impatient behavior *as-if discounting*. Our analysis shows that lack of foresight generates behavior that has most of the same characteristics of behavior that arises from deep time preferences. In other words, a perfectly patient Bayesian decision-maker who receives noisy signals about the future will behave as if they have time preferences.

Opthalmic myopia arises when people can't clearly see distant objects. But myopia also means a "lack of foresight or discernment."[2] Such forecasting limitations matter when agents need to judge the value of events that will occur at a temporal distance. In this paper, we show that imperfect foresight – i.e., myopia – generates as-if discounting, even when the actors' true preferences are perfectly patient. More generally, we show that imperfect foresight makes agents appear to behave more impatiently than implied by their deep time preferences.

[2] Merriam-Webster.

Our formal model assumes that agents receive noisy, unbiased signals about future events and combine these signals with their priors to generate posterior beliefs about future events. Our key assumption is that the forecasting noise increases with the horizon of the forecast. We give special attention to the case in which the variance of the forecasting noise rises linearly with the forecasting horizon.

We provide an illustrative example of our framework in Section 2, where we study a binary choice problem: an actor chooses between an early reward and a mutually exclusive later reward. We show that when the variance of forecasting noise rises linearly with the event horizon, Bayesian agents will act as if they are hyperbolic discounters, even though their deep time preferences are perfectly patient.

In Section 3, we describe the broader implications of our framework, and identify predictions that distinguish our framework from time preference models. First, we show that our (perfectly patient) agents exhibit preference reversals of the same kind that are exhibited by agents with hyperbolic discount functions. However, these preference reversals do *not* reflect a self-control problem. The preference reversals arise because the agents obtain less noisy information with the passage of time. Accordingly, our agents do not wish to commit themselves; they act as-if they are naive hyperbolic discounters (Strotz 1957, Akerlof 1992, O'Donoghue and Rabin 1999) rather than sophisticated ones (Laibson 1997).

In the cross-section, our framework implies that agents with greater intelligence exhibit less as-if discounting – their superior forecasting ability enables them to make choices that are more responsive to future utility flows.

In addition, our agents exhibit as-if discounting that is domain specific. They exhibit less as-if discounting (i) when they have more overall life experience, (ii) when they are more experienced in the specific choice domain, (iii) when they have more time to think about an intertemporal choice (e.g., Imas, Khun, and Mironova, 2016), and (iv) when they have more cognitive bandwidth to think about their choice (e.g., Benjamin and Shapiro, 2015).

In Section 4, we generalize our example by making the action set continuous. We provide sufficient conditions that imply that perfectly patient agents who are imperfect forecasters will act as if they are naive hyperbolic discounters.

In Section 5, we discuss connections between our framework and related literatures on myopia, Bayesian cognition, risk, and discounting. Section 6 concludes.

2 A Basic Case: Binary Choice

Our approach can be explained with a simple example of a binary choice. Consider an agent at time zero, who must choose (irreversibly) between two mutually exclusive rewards: *Early* and *Late*. Reward *Early* would be experienced at date, $t \geq 0$. Reward *Late* would be experienced at date, $t + \tau > t$ (i.e., $\tau > 0$). The agent doesn't know the *true* value of *Early* and *Late*, respectively denoted, u_t and $u_{t+\tau}$. To simplify exposition and without loss of generality, we assume that these utility events are deterministic, though they were originally generated from a prior distribution that we will characterize below. (Note that any non-deterministic, zero-mean component is irrelevant because we are operating in utility space and we assume that our agents have classical expected utility preferences.)

Although the agent doesn't know the value of u_t and $u_{t+\tau}$, the agent can mentally simulate these deterministic rewards and thereby generate unbiased signals of their value:

$$s_t = u_t + \varepsilon_t$$

$$s_{t+\tau} = u_{t+\tau} + \varepsilon_{t+\tau}.$$

In the first equation, u_t is the true value of the *Early* utils and ε_t is the simulation noise. In the second equation, $u_{t+\tau}$ is the true value of the *Late* utils and $\varepsilon_{t+\tau}$ is the associated simulation noise. For tractability, we assume that the simulation noise is Gaussian. To simplify exposition, we assume that the correlation between ε_t and $\varepsilon_{t+\tau}$ is zero.

2.1 Simulation Noise

We assume that the longer the horizon, the greater the variance of the simulation noise. Intuitively, the further away the event, the harder it is to accurately simulate the event's utility. Because our set-up assumes that $t < t + \tau$, this assumption implies that

$$var(\varepsilon_t) < var(\varepsilon_{t+\tau}). \quad (1)$$

We will also sometimes assume that

$$\lim_{t \to \infty} var(\varepsilon_t) = \infty,$$

however this property is not important for our qualitative results.

We will pay particular attention to the special case of simulation noise that has a variance that is proportional to the simulation horizon:

$$var(\varepsilon_t) = \sigma^2_{\varepsilon_t} = \sigma^2_\varepsilon t \tag{2}$$

$$var(\varepsilon_{t+\tau}) = \sigma^2_{\varepsilon_{t+\tau}} = \sigma^2_\varepsilon (t+\tau). \tag{3}$$

This linearity assumption engenders a specific (hyperbolic) functional form in the analysis that follows. But this linearity assumption is not necessary for our qualitative results. We provide a complete characterization of noise functions below: i.e., necessary and sufficient =conditions for the noise function to generate as-if discounting with declining discount rates as the horizon increases. The case of linear variance is a special case in this larger class of noise functions.

2.2 Bayesian Priors and Posteriors

The agents in our model combine Bayesian priors with their signals (s_t and $s_{t+\tau}$) to generate a Bayesian posterior. We model the Bayesian prior over utility events (in whatever class of events we are studying) as a Gaussian density with mean μ and variance σ^2_u:

$$u \sim \mathcal{N}(\mu, \sigma^2_u). \tag{4}$$

Here μ is the average value in this class of utility events (e.g., visits to Philadelphia), whereas σ^2_u is the overall variance within the class (e.g., some trips are great – Philadelphia in June – and some trips are much less great – Philadelphia in January).

In the appendix, we derive the agent's Bayesian posterior distribution of u_t, which is generated by combining her prior (4) and her signal s_t:

$$u_t \sim \mathcal{N}\left(\mu + \frac{s_t - \mu}{1 + \frac{\sigma^2_{\varepsilon_t}}{\sigma^2_u}}, \left(1 - \frac{1}{1 + \frac{\sigma^2_{\varepsilon_t}}{\sigma^2_u}}\right)\sigma^2_u\right). \tag{5}$$

We summarize these results with the following propostion.

Proposition 1 *If the agent generates a mental simulation s_t, then her Bayesian posterior* *will be*

$$u_t \sim \mathcal{N}\left(\mu + D(t)(s_t - \mu),\, (1 - D(t))\,\sigma_u^2\right),$$

where

$$D(t) = \frac{1}{1 + \frac{\sigma_{\varepsilon_t}^2}{\sigma_u^2}}, \tag{6}$$

the variance of her simulation noise is $\sigma_{\varepsilon_t}^2$ *and her prior distribution is* $u \sim \mathcal{N}(\mu, \sigma_u^2)$.

For reasons that will become apparent below, we refer to $D(t)$ as the agent's as-if discount function. Because we assume that simulation noise, $\sigma_{\varepsilon_t}^2$, is increasing in t, $D(t)$ is decreasing in t, which is a standard property of a discount function. If $\lim_{t\to\infty} var(\varepsilon_t) = \infty$, then $\lim_{t\to\infty} D(t) = 0$, another common property of a discount function. In this case, the posterior expectation of u_t converges to the mean of the prior as the horizon increases. In notation,

$$\lim_{t\to\infty} \mathbb{E}_0[u_t \mid s_t] = \mu.$$

It is helpful to integrate posteriors over agents in the economy. We assume that the signals s_t are unbiased, so they are equal to u_t on average. Accordingly, the average forecast of u_t will be

$$\int_{s_t} \mathbb{E}_0[u_t \mid s_t]\, dF(s_t \mid u_t) = \mu + D(t)(u_t - \mu).$$

In general, the mean of the prior will be less extreme than the actual values of u_t. To model this statistical property, consider the illustrative case in which the prior is approximately equal to zero. (We will relax this restriction later.) Under this restriction, the average belief is

$$\int_{s_t} \mathbb{E}_0[u_t \mid s_t]\, dF(s_t \mid u_t) = D(t)u_t.$$

We now have an expression that looks like a discounted utility framework: $D(t)$ is a decreasing function and it multiplies the actual utility value u_t.

2.3 Hyperbolic As-if Discounting

We explore a benchmark case: noise that is linear in the horizon.

Lemma 1 *When we assume that* $var(\varepsilon_t) = \sigma^2_{\varepsilon_t} = \sigma^2_\varepsilon t$*, we obtain hyperbolic as-if discounting:*

$$D(t) = \frac{1}{1+\alpha t} \tag{7}$$

where

$$\alpha = \frac{\sigma^2_\varepsilon}{\sigma^2_u}, \tag{8}$$

which is the (one-period) noise-to-signal variance ratio.

The discount function, $D(t) = \frac{1}{1+\alpha t}$, implies an instantaneous discount rate

$$\text{discount rate} = -\frac{D'(t)}{D(t)} = \frac{\frac{\alpha}{(1+\alpha t)^2}}{\frac{1}{1+\alpha t}} = \frac{\alpha}{1+\alpha t}.$$

At horizon 0, the as-if discount rate is α. The as-if discount rate falls with t. As $t \to \infty$, the as-if discount rate converges to 0.

2.4 An Example When the Mean Priori Is not Zero ($\mu \neq 0$)

As we noted above, actual utility events will tend to be more extreme than priors. To capture this property, we previously set the mean of the prior distribution equal to zero: $\mu = 0$. We now relax this restriction and illustrate the general case with an example in which the mean of the prior distribution is $\mu = 1$. For this example, we assume that the simulation variance is linear in the time horizon and the variance ratio is $\frac{\sigma^2_\varepsilon}{\sigma^2_u} = 0.1$. Figure 1 plots the population level expectations of u_t for three values of u_t (holding the mean of the prior distribution fixed at $\mu = 1$):

$$u_t = \mu + 10 = 11$$
$$u_t = \mu - 1/2 = 1/2$$
$$u_t = \mu - 10 = -9.$$

When the three utility events are in the present ($t = 0$), the three expectations are equal to the true value of each utility event, respectively 11, 1/2, and -9. However, as the three utility events recede into the distant future, the three expectations revert to the mean of the prior, $\mu = 1$. This discounting towards the mean of the prior is hyperbolic because we are

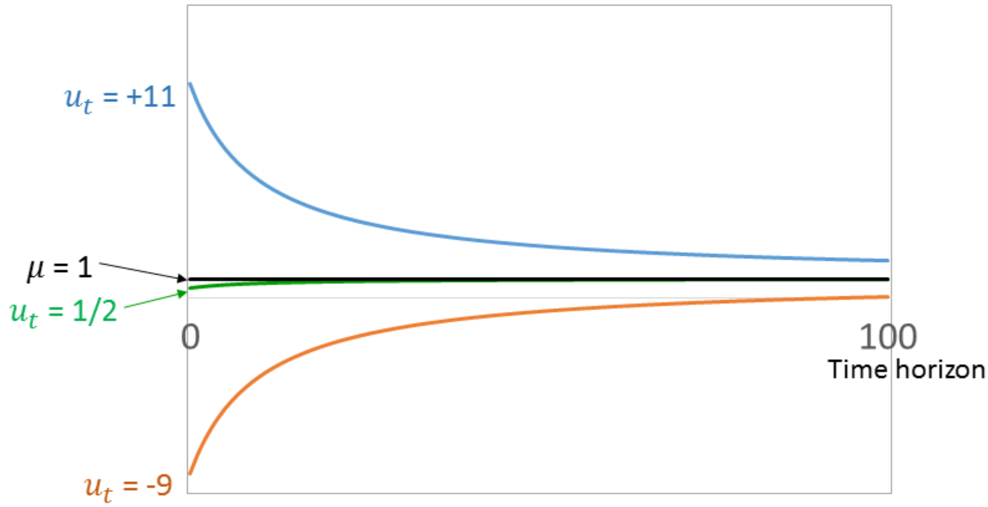

Figure 1: Plot of the average perceived value $\overline{u}_t$, given for three different true utilities u_t ($u_t \in \{-9, 1/2, 11\}$), as a function of the time horizon t. This average perceived value is: $\overline{u}_t = \mu + \frac{u_t - \mu}{1 + \frac{\sigma_\varepsilon^2}{\sigma_u^2} t}$. The figure uses $\sigma_\varepsilon^2 / \sigma_u^2 = 0.1$.

assuming linear variance (see subsection 2.3).

The $u_t = 11$ curve is characterized by standard discounting. The further ahead the utility event is shifted, the lower the perceived value of the event. The $u_t = -9$ curve is also characterized by standard discounting on most of its domain. As the event is moved further into the future, its value declines toward zero. However, at $t = 90$, the perceived value crosses the x-axis and continues asymptoting toward $\mu = -1$. Finally, the $u_t = 1/2$ line displays anti-discounting. The further the value is moved into the future, the higher its perceived value, as it asymptotes to the prior mean of $\mu = 1$.

These three lines illustrate the three types of cases that arise in our framework, including the special case of anti-discounting. Note that anti-discounting arises when the true value of u_t lies between 0 and the mean of the prior distribution, μ.

2.5 Probabilistic Choice

Our framework implies that choice is probablistic, because agents receive noisy signals about the value of future rewards. In our example, the agent chooses *Early* if and only if

$$D(t)s_t \geq D(t+\tau)s_{t+\tau},$$

where $D(t)$ is the as-if discount function introduced above and s_t and $s_{t+\tau}$ are the (unbiased) signals of the respective values of the *Early* and *Late* rewards.

From the perspective of an observer who knows the values of u_t and $u_{t+\tau}$, the probability that that agent chooses the *Early* reward is

$$\begin{aligned}\mathbb{P}(\text{choose } Early) &= \mathbb{P}\left[D\left(t\right)\left(u_t+\varepsilon_t\right) \geq D\left(t+\tau\right)\left(u_{t+\tau}+\varepsilon_{t+\tau}\right)\right] \\ &= \Phi\left(\frac{1}{\Sigma}\left[D(t)u_t - D(t+\tau)u_{t+\tau}\right]\right),\end{aligned}$$

where Φ is a Gaussian CDF and Σ is a scaling factor:

$$\Sigma = \sqrt{D(t)^2 var(\varepsilon_t) + D(t+\tau)^2 var(\varepsilon_{t+\tau})}.$$

This probabilistic choice function has natural properties. If $t = 0$ (i.e., the *Early* reward is an immediate reward), then,

$$\begin{aligned}\mathbb{P}(\text{choose } Early) &= \mathbb{P}\left[u_0 \geq D\left(\tau\right)\left(u_\tau+\varepsilon_\tau\right)\right] \\ &= \Phi\left(\frac{1}{\Sigma}\left[u_0 - D(\tau)u_\tau\right]\right).\end{aligned}$$

If we let the time delay between the *Early* reward and the *Late* reward go to infinity (i.e., $\tau \to \infty$), then

$$\lim_{\tau\to\infty} \mathbb{P}(\text{choose } Early) = 1_{u_0>0}.$$

This implies that the agent chooses the *Early* reward with probability one if three properties hold: (i) the *Early* reward is available immediately ($t = 0$), (ii) the *Late* reward is available arbitrarily far in the future ($\tau \to \infty$), and (iii) the *Early* reward is strictly positive ($u_0 > 0$). In other words, the agent behaves as if she places no value on the (infinitely) delayed *Late* reward.

Now assume that the *Early* reward is available with some delay, so that $t > 0$ (i.e., the *Early* reward is not immediate), then

$$\begin{aligned}\lim_{\tau\to\infty} \mathbb{P}(\text{choose } Early) &= \mathbb{P}\left[u_t + \varepsilon_t > 0\right] \\ &= \Phi\left(\frac{u_t}{\Sigma}\right).\end{aligned}$$

Accordingly, if the *Late* reward is available arbitrarily far in the future ($\tau \to \infty$), then the agent chooses the *Early* reward with the same probability that she perceives the *Early* reward to have positive value. Once again, the agent behaves as if she places no value on the (infinitely) delayed *Late* reward.

2.6 Preference Reversals without Commitment

In our setting, an observer who knows the values of u_t and $u_{t+\tau}$ will be able to predict (probabilistic) preference reversals. For example, consider the case of linear variances. In addition, assume that $u_{t+\tau} > u_t > 0$, and

$$u_t > D(\tau)u_{t+\tau}.$$

When the two options are sufficiently far in the future (large t), a majority of agents (if forced to choose) will prefer *Late* over *Early*, because

$$\mathbb{P}(\text{choose } Early) = \Phi\left(\frac{1}{\Sigma}\left[D(t)u_t - D(t+\tau)u_{t+\tau}\right]\right).$$

Note that

$$D(t)u_t - D(t+\tau)u_{t+\tau} = \frac{u_t}{1+\alpha t} - \frac{u_{t+\tau}}{1+\alpha\,(t+\tau)}.$$

For sufficiently large values of t, $u_{t+\tau} > u_t$ implies,

$$\frac{u_t}{1+\alpha t} - \frac{u_{t+\tau}}{1+\alpha\,(t+\tau)} < 0.$$

However, with the passage of time, all agents will eventually choose *Early* because $u_t > D(\tau)u_{t+\tau}$. More precisely, if agents were not forced to choose in advance, but were instead given the chance to choose at time t, all would choose *Early*.

In many economic models, such preference reversals are a sign of dynamic inconsistency in preferences.[3] That is not the case here. The agents in the current model have imperfect information, not dynamically inconsistent time preferences. Their externally predictable

[3] See McGuire and Kable (2012, 2013) for a setting in which preference reversals arise because of rational learning dynamics. If a delayed reward that was probabilitistically expected does not arrive after a period of waiting, the agent infers that the hazard rate of arrival is low and further waiting is not likely to pay off, and therefore reverts to choosing the immediate reward.

preference reversals are a result of their imperfect information. Accordingly, the agents in our model will not desire to limit their own choice sets. Preference reversals arise from their inference problems, not self-control problems.

2.7 More General Discounting Functions

We can provide necessary and sufficient conditions for the as-if discount function, $D(t)$, to exhibit decreasing impatience. In other words, we can derive necessary and sufficient conditions for the property that the instantaneous as-if discount rate

$$\rho(t) := -\frac{D'(t)}{D(t)}$$

is decreasing in the horizon t.

Proposition 2 *The as-if discount function $D(t)$ exhibits strictly decreasing impatience at time horizon t if and only if*

$$\frac{\frac{d^2\sigma_{\varepsilon_t}^2}{dt^2}}{\sigma_u^2} - \frac{\left(\frac{d\sigma_{\varepsilon_t}^2}{dt}/\sigma_u^2\right)^2}{\left(1+\frac{\sigma_{\varepsilon_t}^2}{\sigma_u^2}\right)} < 0.$$

This proposition is proved in the appendix. Because we assume that $\frac{d\sigma_{\varepsilon_t}^2}{dt} > 0$, this proposition yields an immediate corollary.

Lemma 2 *The as-if discount function $D(t)$ exhibits strictly decreasing impatience if the variance of simulation noise, $var(\varepsilon_t) = \sigma_{\varepsilon_t}^2$, is a weakly concave function of time.*

Accordingly, our model generates as-if discount rates that decrease as the horizon increases in many cases. We next study a boundary case.

Exponential As-if Discounting Our framework can also be reverse-engineered to generate exponential discounting as a special case. However, this requires assumptions on the variance function that we believe are heroic.

Lemma 3 *The as-if discount function $D(t)$ exhibits a constant discount rate, ρ, if and only if*

$$\sigma_{\varepsilon_t}^2 = [\exp(\rho t) - 1]\,\sigma_u^2.$$

Accordingly, the discount rate is exponential if and only if the simulation variance, $\sigma^2_{\varepsilon_t}$, rises exponentially. This Lemma is proven by setting

$$D(t) = \frac{1}{1 + \frac{\sigma^2_{\varepsilon_t}}{\sigma^2_u} t} = \exp(\rho t).$$

This sort of cognitive discounting is useful because of the tractability it induces (see for instance Gabaix 2016a,b).

3 Implications

We now discuss the key predictions of our model, emphasizing several ways that our model of myopia differs from other models in the intertemporal choice literature. As discussed above, our myopic agent acts as if she is maximizing a utility function with an as-if discount function, $D(\tau)$, where

$$D(\tau) = \frac{1}{1 + \frac{\sigma^2_{\varepsilon_\tau}}{\sigma^2_u}}.$$

When the variance of the forecasting noise is weakly concave in the simulation horizon, the discounting function is characterized by an instantaneous discount rate that falls with the horizon. When the forecasting noise is linear in the simulation horizon, so that $\sigma^2_{\varepsilon_\tau} = \sigma^2_\varepsilon \tau$ then the discount function is hyperbolic:

$$D(\tau) = \frac{1}{1 + \frac{\sigma^2_\varepsilon}{\sigma^2_u} \tau}.$$

These as-if discounting functions arise because of the imperfect information that the agent has when she generates forecasts. If she were asked to describe her preferences, she would say that she has no times preferences. In other words, she is *trying* to maximize

$$\sum_{\tau=0}^{T-t} u(a_{t+\tau}).$$

Her as-if discounting behavior arises because she doesn't have perfect foresight regarding the future utility flows $u(a_{t+\tau})$.

3.1 Absence of Commitment

The agents in this model have a forecasting problem, not a self-control problem. Therefore they are never willing to reduce their choice set (unless they are paid to do so). This absence of a willingness to pay for commitment may explain the lack of commitment technologies in markets. In real markets there is little commitment for commitment's sake.[4] Personal trainers and website blocking apps are frequently mentioned exceptions, but such technologies are not commonly used.

By contrast, economists have been able to elicit commitment in experiments (see Cohen, Ericson, Laibson, and White 2017, for a review). However, most of these experiments elicit only a weak taste for a commitment and little or no willingness to pay for commitment (e.g., Augenblick, Niederle and Sprenger 2015, Sadoff, Samek and Sprenger 2016).

Our myopia model predicts that agents will exhibit as-if hyperbolic discounting with preference reversals *and* no willingness to pay for commitment. In this sense, our model reproduces the predictions of the standard hyperbolic discounting model with naive beliefs (see O'Donoghue and Rabin 1999, 2001, Laibson 2015, and Ericson forthcoming). However, it also generate further implications, to which we now turn.

3.2 Intelligence Is Associated with Less As-if Discounting

Our model predicts that agents with less forecasting noise will exhibit less as-if discounting. Because of this mechanism, agents that are more intelligent will exhibit less as-if discounting.[5]

To see this formally, let H represent human capital and assume that the variance of forecasting noise is declining in human capital:

$$\frac{d\sigma_\varepsilon^2(H)}{dH} < 0.$$

The as-if discount rate is given by

$$-\frac{D'(t)}{D(t)} = \frac{\alpha}{1+\alpha t},$$

[4]However, there is a great deal of ancillary commitment, like mortgage contracts, which create a forced savings system as a by-product of a stream of loan/principal repayments.

[5]The underlying assumption is that more intelligent agents simulate the future with less noise—for instance because they generate more simulations. If they run n simulations, the variance of the average simulation will be $1/n$, so it will be lower.

where

$$\alpha = \frac{\sigma_\varepsilon^2(H)}{\sigma_u^2}.$$

The as-if discount rate is increasing in $\sigma_\varepsilon^2(H)$, so as-if discounting is decreasing in human capital, H.

The available evidence supports this prediction. Measured discount rates are negatively correlated with scores on IQ tests: see Benjamin, Brown, and Shapiro (2013), Burk et al. (2009) and Shamosh and Gray (2008). Indeed, such effects also arise across species. Tobin and Logue (1994) show that patience increases as the study population switches from pigeons, to rats, to humans.

3.3 Myopia Is Domain Specific

These comparative statics on cognitive function generate a wider set of predictions when forecasting ability varies across domains. For example, our framework predicts that agents with more domain-relevant experience, and hence better within-domain forecasting ability, will exhibit less discounting. Read, Frederick and Scholten (2013) report that people exhibit more patience when an intertemporal choice is posed as an investment rather than a (seemingly novel) money-now-vs-money-later decision. Relatedly, recall our earlier discussion of the monkey experiments reported by Blanchard and Hayden (2015): when an intertemporal choice is presented as a reward-now-vs-reward-later decision, monkeys choose far more impatiently then they do when a foraging problem is used to frame the intertemporal tradeoffs.

Likewise, our framework predicts that older agents – who generally have more life experience and consequently better forecasting skills – will exhibit less discounting. This prediction is supported by Green, Fry, and Myerson (1994). Relatedly, Addessi et al (2014) show that replacing one-for-one representations of future reward with more abstract one-for-many representations of the same future rewards, leads capuchin monkeys and (human) children to exhibit more impatience. In contrast, adults, who have more experience using abstract symbols, do *not* behave more impatiently when one-for-one representations of future reward are replaced with one-for-many representations. The Addessi et al (2014) experimental evidence implies that childhood impatience is due, at least in part, to children's less developed ability to cognitively represent (abstract) future rewards. Our framework also predicts that people who experience cognitive decline (e.g., due to normal aging) will exhibit more discounting;

see James, Boyle, Yu, Han, and Bennett 2015 for supporting evidence.

Our framework predicts that agents who are encouraged to spend more time thinking about a future tradeoff will exhibit less discounting. Imas, Kuhn, and Mironova (2016) robustly measure such an effect experimentally. In their experiment, some subjects decide at time 0 how to divide an effort task between time 0 and time t. Other subjects are given a preceding hour to decide how to divide the effort task between time 0 and time t. Subjects in the latter condition choose more patiently: their measured discount rate is 16 percentage points lower.

Our framework also predicts that rewards delivered in future periods that are cognitively well-simulated will exhibit less discounting. Peters and Büchel (2010) exogenously manipulate the salience of various future periods and find that higher salience/imagery of future reward periods increases the value of rewards delivered during those periods.

Our framework predicts that agents who are unable to think carefully about an intertemporal tradeoff – e.g., due to a cognitive load manipulation or the effects of alcohol – will exhibit more discounting. Steele and Josephs (1990), Shiv and Fedorikhin (1999), Hinson, Jameson, and Whitney (2003), and Benjamin, Brown and Shapiro (2013) document such effects. This prediction is closely related to the theory of cognitive scarcity: see Spears (2012), Eldar and Shafir (2013), and Schilbach et al (2016).

Finally, our framework predicts that discounting behavior will only be weakly correlated across domains because discounting is not a domain general preference, but rather the result of imperfect forecasting, which will naturally vary across domains. Chapman (1996) and Chabris et al (2008) document the low level of correlation in discount rates that are measured in different decision-making domains.

4 Extension to a Continuous Action

Until now we have studied the case in which the agent has two mutually exclusive actions: choose an *Early* reward or a *Late* reward. We now generalize the action space to a continuum. We then provide sufficient conditions that enable us to apply our framework to a general, multi-period intertemporal choice problem.

The upshot of this section is that the economics of the binary action case still goes through, though with more complex mathematics.

4.1 Modelling How Agents Observe with Noise a Whole Utility Function

Suppose that an action a leads to a true payoff $u(a)$. However, the agent observes this noisily: we suppose that the agent observes the "noised up" version of the utility function:

$$\mathbf{s} = (s(a))_{a \in \mathcal{A}} \tag{9}$$

of the whole function $u = (u(a))_{a \in \mathcal{A}}$, where $\mathcal{A} = [\underline{a}, \overline{a}]$ is the support of the action, which is assumed to contain 0 (this is just a normalization). This noised-up version is assumed to take the form:

$$s(a) = u(a) + \sigma_{\varepsilon_t} W(a) + \chi \sigma_{\varepsilon_t} \eta_0 \tag{10}$$

for all $a \in \mathcal{A}$. There is a continuous noise $W(a)$, modelled as standard Brownian motion with $W(0) = 0$ except that W is "two-sided", i.e. runs to the left and right of 0.[6,7] The noise is modelled as proportional to σ_{ε_t} when the utility is seen from a distance t. For instance, the linear case is $\sigma_{\varepsilon_t} = \sigma_\varepsilon \sqrt{t}$. The term $\chi \sigma_{\varepsilon_t} \eta_0$ ensures that the value at $a = 0$ is also perceived with noise ($\sigma_{\eta_0} = 1$, χ is a parameter).

Given this perceived curve $\mathbf{s}$, what's his posterior about $u(a)$? We will see the under the "right" assumptions (to be specified soon), we simply have

$$\mathbb{E}[u(a) \mid \mathbf{s}] = D(t)\, s(a)$$

with the same $D(t)$ as in the binary case. This means that the representative agent just dampens the true function.

4.2 Assumptions for Our Result

Here we detail the assumptions we use for the results. But the reader may wish to skip to the result itself, in the next subsection 4.3.

Assumption 1 (Wiener decomposition) *We suppose that function* $v(\cdot) := \frac{u(\cdot) - u(0)}{\sigma_u}$ *is per-*

[6] Formally, $W(x)_{x \geq 0}$ and $W(-x)_{x \geq 0}$ are independent Brownian motions.

[7] See Callender and Hummel (2014) for a recent model using inference on Brownian paths, though with a signal structure different from ours.

ceived as drawn from the Wiener measure, and $u(0)$ is drawn as $u(0) \sim N(0, \chi^2 \sigma_u^2)$ independent of v. We call

$$D(t) = \frac{1}{1 + \frac{\sigma_{\varepsilon_t}^2}{\sigma_u^2}} \tag{11}$$

where σ_{ε_t} is as in (10).

Let us state this assumption in more user-friendly language. The value of $u(0)$ is also seen as random—and we index its randomness by χ. The rest of the function u, outside the intercept, is also random. To specify this, we $v(a) := \frac{u(a)-u(0)}{\sigma_u}$, which is u normalized to have 0 intercept, and with standardized size (so $\mathbb{E}\left[v(1)^2\right] = 1$). We view v a a "random function" drawn from a distribution. For simplicity, we consider that it's drawn from the simplest distribution of random functions – the so-called Wiener measure (Brownian motions are typical instances of such functions).[8] Basically, the assumption is that the component of $du(a)$ are drawn as i.i.d. normal increments, like a Brownian motion, with square width $\sigma_u^2 da$. Note that this refers to the distribution assumed by the agent when he performs his Bayesian inference, not necessarily the true distribution.

The appendix (section 7.2) proposes a variant, Assumption 2, with polynomial utility, that uses more elementary mathematics, at the cost of heavier notations and proofs.

4.3 Perceived Utility Function Given the True Utility

We can now derive the utility perceived by the agent, given she agent sees the whole noised-up function $\boldsymbol{s}$ (equation (10)).

Proposition 3 (Perceived utility for a continuous utility function) *Make Assumption 1 or 2. Then, the perceived utility is proportional to the signal:*

$$\mathbb{E}[u(a) \mid \mathbf{s}] = D(t)\, s(a)$$

where $D(t) = \frac{1}{1+\frac{\sigma_{\varepsilon_t}^2}{\sigma_u^2}}$. As a result, the average perceived utility $\bar{u}(a)$, defined as:

$$\bar{u}(a) := \mathbb{E}[\mathbb{E}[u(a) \mid \mathbf{s}] \mid \mathbf{u}]$$

[8] We could imagine a number of variants, e.g. u'' would be drawn from this distribution; or, to keep u concave, we could have $\ln(-u'')$ drawn from that distribution. This becomes quickly more mathematically involved, so we leave this to a separate investigation, and focus on what we view as the simplest case.

satisfies:

$$\bar{u}(a) = D(t)\, u(a) \tag{12}$$

This means that the average perceived utility is $D(t)\, u(a)$ rather than plainly $u(a)$, exactly like in the simple two-action (consume / don't consume) case.

4.4 The Representative Agent Perspective

4.4.1 Assumptions for a Tractable Generalization

To cleanly study dynamic problem, we assume the following (in addition to the assumptions of Proposition 3).[9]

A1. The agent treats the noise at all simulation horizons as uncorrelated.

A2. The agent has Gaussian priors with 0 mean (and no correlation between $u_t, u_{t+\tau}$).

A3. The agent acts as if she won't learn new simulation information in the future.[10]

The notion of "average behavior" is potentially messy with non-linear utilities. Hence, we find it useful to define the following form of "representative agent" version of the model. We study the equilibrium path in which all simulation noise happens to be realized as zero (but the agent doesn't know this). In our illustrative example, this corresponds to $\varepsilon_t = 0$. For instance, we had $s_t = u_t + \varepsilon_t$ and $\mathbb{E}[u_t \mid s_t] = D(t)s_t$. The representative agent draws noise $\varepsilon_t = 0$, so for the representative agent, $\mathbb{E}[u_t \mid s_t] = D(t)\, u_t$.

Proposition 4 (Dynamic choices of the representative agent) *Assume that the agent has dynamically consistent preferences*

$$\sum_{t=0}^{T} u(a_t).$$

Then A1-A3 imply that at each time period $t \in \{0, ..., T\}$ the representative agent acts as if she is trying to maximize

$$\sum_{\tau=0}^{T-t} D(\tau)u(a_{t+\tau})$$

[9] There are many alternative ways to generate variances that are linear in the forecasting horizon, including new signals that contain all of the information of the old signals.

[10] This is the assumption of the "anticipated utility" framework of Kreps (1998) used also by Cogley and Sargent (2008).

where

$$D(\tau) = \frac{1}{1 + \frac{\sigma^2_{\varepsilon_\tau}}{\sigma^2_u}}.$$

Corollary 1 *Assume that simulation variance is linear in the horizon of the simulation: $\sigma^2_{\varepsilon_\tau} = \tau\sigma^2_\varepsilon$. Then, at each time period $t \in \{0, ..., T\}$ the representative agent acts as if she is trying to maximize*

$$\sum_{\tau=0}^{T-t} D(\tau)u(a_{t+\tau}),$$

where

$$D(\tau) = \frac{1}{1 + \alpha\tau}$$
$$\alpha = \frac{\sigma^2_\varepsilon}{\sigma^2_u}.$$

Proposition 4 shows that our basic results extend to arbitrary utility functions with continuous actions.

5 Literatures on Related Mechanisms

We now review other lines of research that are related to this paper and on which this paper builds. We review three literatures: models of myopia, Bayesian foundations of imperfect and costly cognition, risk-based models of as-if discounting (including risk-based models with probability distortions).

5.1 Myopia

Political economists, psychologists, and other social scientists have long posited that impatient behavior was due in part to imperfect foresight. These ideas were informally described by political economists, including Böhm-Bawerk (1889), and economists, including Pigou (1921), both of whom are quoted in the introduction of this paper.

These informal explanations have been joined by formal, mathematical definitions, models, and analyses of that incorporate various formulations of myopia. For example, Brown and Lewis (1981) provide an axiomatic definition of myopia. Feldstein (1985) evaluates the optimality of social security under the assumptions of myopia and partial myopia (modeled as a low discount factor in a two-period decision problem). Phillippe Jéhiel (1995) studies two-player games in which players have limited forecasting horizons. Spears (2012) generate a forecasting horizon that is endogenous because forward-looking calculations are costly. Gabaix, Laibson, Moloche, and Weinberg (2006) report experimental evidence that supports a model in which agents choose an endogenous forecasting horizon at which the cognitive cost and estimated utility benefit of marginally increasing the forecasting horizon are equalized. This optimal forecasting framework generates a complex option value problem with respect to information acquisition (see also Fudenberg, Strack, and Strzalecki 2016).

In the current paper, we assume that the agent has noisy signals about the future, which engenders Bayesian forecasts that have "myopic" properties: i.e., declining sensitivity to future events. When the noise is linear in the forecasting horizon, the as-if discounting takes a simple hyperbolic form. Accordingly, our paper introduces a tractable microfoundation for myopia.

5.2 Bayesian Models of Attention and Cognition

The current paper assumes that agents are Bayesian, which adopts the approach of early decision-theory pioneers like Raiffa and Schlaifer (1961). There is a growing body of literature (in economics, cognitive psychology, and neuroscience) that studies the effects of noisy perception and Bayesian inference, and uses this combination to explain seemingly suboptimal behaviors. One of the pioneering examples is the work of Commons, Woodford and Ducheny (1982), and Commons, Woodford and Trudeau (1991) who use this approach to generate a theory of hyperbolic memory recall – in their framework, the noisy signals are memories, whereas the noisy signals in our model are simulations of the future. The literature on attention allocation assumes that agents have limited information, which is mathematically equivalent to the assumption that agents have noisy signals about the state of the world. Geanakoplos and Milgrom (1991), Sims (2003), Kamenica (2008), Woodford (2009), Gabaix (2014), Schwartzstein (2014), Hanna, Mullainathan and Schwartzstein (2014), Allcott and Taubinsky (2015), Matejka, Steiner and Stewart (forth.), Taubinsky and Rees-Jones (2016a,b), study agents who allocate their limited attentional bandwidth to the activities that they believe are the most valuable.[11] Steiner and Stewart (2016) and Khaw, Li, and

[11] Another strand of the literature uses non-Bayesian rules to govern attention and salience (e.g. Bordalo, Gennaioli and Shleifer 2012, 2013), though it might be probably be given some quasi-Bayesian interpretation.

Woodford (2017) study an environment in which agents react to the noise in their probability perceptions by (optimally) distorting their perceived probabilities in a way that mimics the probability mapping in prospect theory (Kahneman and Tversky 1979).

Our paper adopts the approach that unifies the work above: noisy signals plus Bayesian inference jointly produce as-if behavior that appears to be imperfectly rational. Specifically, in our case, this combination generates as-if hyperbolic discounting.

5.3 Risk-Based Models of As-if Discounting

It has long been recognized that time preferences engender the same kind of behavior that is associated with risk or mortality (e.g., Yaari 1965). For example, if promised future rewards may be permanently withdrawn/lost at a constant hazard rate, ρ, then a perfectly patient decision-maker should be indifferent between 1 util at time zero and $\exp(\rho\tau)$ utils a time τ. In this example, risk induces a perfectly patient agent to appear to discount the future with exponential discount rate ρ.

This type of risk-based discounting can also produce hyperboloid discount functions under specific assumptions about a non-constant hazard rate (see Sozou 1998, Azfar 1999, Weitzman 2001, Halevy 2004, and Dasgupta and Maskin 2005). For instance, Azfar, Sozou and Weitzman assume that the hazard rate that governs the withdrawal of rewards is itself drawn from a distribution and has a value that can only be inferred from the observed data. This assumption produces preferences that are characterized by a declining discount rate as the horizon increases – the more time passes without a withdrawal, the more likely that one of the low hazard rates is the hazard rate that was drawn from the distribution at the start of time, implying a lower effective discount rate at longer horizons. Risk can also produce hyperboloid discount functions because of probability transformations that are characterized by a certainty effect, whereby a certain present reward is discretely more valuable than an even slightly uncertain delayed reward (see the non-expected utility frameworks in Prelec and Loewenstein 1991, Quiggin and Horowitz 1995, Keren and Roelofsma 1995, Weber and Chapman 2005, Halevy 2008, Epper, Fehr-Duda and Bruhin 2011, Baucells and Heukamp 2012, Andreoni and Sprenger 2012, and Fehr-Duda and Epper 2015).

Our model works off a related but different risk mechanism than those listed above. The uncertainty in our model is due to noise that is generated by the forecaster herself. For example, our mechanism predicts that an expert would exhibit little as-if discounting in her domain of expertise (she forecasts the future with little or no noise) while a non-expert would exhibit substantial as-if discounting in the *same* domain (she forecasts the future with relatively more noise than the expert). Likewise, our framework predicts that cognitive load should increase as-if discounting because it reduces an agent's ability to forecast accurately. Accordingly, our noise-based discounting mechanism is not propagated by external risk (like mortality or the likelihood of default), but rather by noise associated with the limited forecasting ability of the decision maker.

Finally, our framework is consistent with Bayesian decision-making and expected utility theory. Accordingly, our agent will not be dynamically inconsistent and will not pay for commitment. In our framework, preference reversals reflect classical information acquisition, not weakness of will.

Our key assumption is that the agent has (unbiased) noise in her signals about the future. This noise leads our agent to optimally down-weight her simulations of the future and therefore place more weight on her priors. Consequently, she ends up being (rationally) imperfectly responsive to future contingencies and therefore behaves as if she discounts the future. As her expertise and experience improves (over her lifetime, or as she gains domain-specific knowledge), she shifts her behavior and acts as-if she has become more patient.

6 Conclusion

We assume that *perfectly patient* agents estimate the value of future events by generating noisy, unbiased simulations of those events. Our agents combine these noisy signals with their priors, thereby forming posterior utility expectations. We show that these expectations exhibit a property that we call *as-if discounting*. Specifically, the agent makes choices as if she were maximizing a stream of known utils weighted by an as-if discount function, $D(t)$. This as-if discount function adjusts for the fact that future utils are not actually known by the agent and must be estimated with noisy signals and priors. This estimation shades the estimated utils toward the mean of the prior distribution, creating behavior that largely mimics the effect of classical time preferences.

When the simulation noise has a variance that is linear in the event's horizon, the as-if discount function is hyperbolic:

$$D(t) = \frac{1}{1 + \alpha t},$$

where α is the ratio of the variance of (per-period) simulation noise to the variance of events in the agent's prior distribution.

Our model generates several predictions that match the known empirical evidence. Our agents exhibit systematic preference reversals. Our agents have no intrinsic taste for commitment, because they suffer from an imperfect forecasting problem, not a self-control problem. Our agents will exhibit comparative statics with respect to cognitive function: people who are more skilled at forecasting (e.g., those with greater intelligence) will exhibit less discounting.

Our framework predicts many domain-specific discounting effects. Agents with more domain-relevant experience will exhibit less discounting. Older agents will exhibit less discounting (except those with cognitive decline, who will exhibit more discounting). Agents who are encouraged to spend more time thinking about a future tradeoff will exhibit less discounting. Finally, agents who are unable to think carefully about an intertemporal tradeoff – e.g., due to a cognitive load manipulation – will exhibit more discounting.

Our framework predicts that discounting is a highly variable/plastic phenomenon that arises from imperfect forecasting of future rewards/costs. Our model provides a complementary alternative to the classical assumption that discounting arises from a deep preference for known rewards (costs) to be moved earlier (later) in time.

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7 Appendix: Proofs and Complements

7.1 Omitted Proofs

Proof of Proposition 1 This proof is very elementary, but for completeness we provide its calculations. We normalize $\mu = 0$ without loss of generality (for instance, by considering $u'_t = u_t - \mu$ and $s'_t = s_t - \mu$). It is well-known that $u_t \mid s_t$ is Gaussian distributed, and can be represented:

$$u_t = \lambda s_t + \eta_t \tag{13}$$

for some λ, and some Gaussian variable η_t independent of s_t, so that $\mathbb{E}[s_t \eta_t] = 0$. Multiplying (13) by s_t on both sides and taking the expectations gives: $\mathbb{E}[u_t s_t] = \lambda \mathbb{E}[s_t^2]$, i.e.

$$\begin{aligned} \lambda &= \frac{\mathbb{E}[u_t s_t]}{\mathbb{E}[s_t^2]} = \frac{\mathbb{E}[u_t (u_t + \varepsilon_t)]}{\mathbb{E}\left[(u_t + \varepsilon_t)^2\right]} = \frac{\mathbb{E}[u_t^2]}{\mathbb{E}[u_t^2 + \varepsilon_t^2]} \text{ as } \mathbb{E}[u_t \varepsilon_t] = 0 \\ &= \frac{\sigma_u^2}{\sigma_u^2 + \sigma_{\varepsilon_t}^2} = \frac{1}{1 + \frac{\sigma_{\varepsilon_t}^2}{\sigma_u^2}} = D(t). \end{aligned}$$

Next, taking the variance of both sides of (13), we have

$$\sigma_u^2 = \lambda^2 \sigma_s^2 + var(\eta_t)$$

as $cov(s_t, \eta_t) = 0$ and with $\sigma_s^2 = \sigma_u^2 + \sigma_{\varepsilon_t}^2$. So, using $\lambda \sigma_s^2 = \sigma_u^2$,

$$var(\eta_t) = \sigma_u^2 - \lambda^2 \sigma_s^2 = \sigma_u^2 - \lambda \sigma_u^2 = (1 - \lambda) \sigma_u^2.$$

Hence, $u_t \mid s_t \sim \mathcal{N}(\lambda s_t, (1 - \lambda) \sigma_u^2)$, as announced.

Proof of Proposition 3 It is a corollary of Proposition 5 (for Assumption 1) and Proposition 6 (for Assumption 2) below.

Proof of Proposition 4 Given our assumptions, the agent at time t will want to maximize

$$\max_{(a_{t+\tau})_{\tau \geq 0}} \mathbb{E}\left[\sum_{\tau=0}^{T-t} u(a_{t+\tau}) \mid \mathbf{s}\right] = \max_{(a_{t+\tau})_{\tau \geq 0}} \sum_{\tau=0}^{T-t} D(\tau)\, s_{t+\tau}(a_{t+\tau})$$

where $\mathbf{s} = (s_t(y), ..., s_{t+\tau}(y))_{y \in \mathcal{A}}$. Assumption A1-A3 allows us to remove expected values. For our representative agent, we have $s_{t+\tau}(a) = u_{t+\tau}(a)$. Hence, this representative agent maximizes at time t:

$$\max_{(a_{t+\tau})_{\tau \geq 0}} \sum_{\tau=0}^{T-t} D(\tau) u(a_{t+\tau}).$$

7.2 Complements to the Continuous Actions Case

Here are some complements to Section 4. To simplify the notations, we set $\sigma = \sigma_{\varepsilon_t}$.

7.2.1 Result for the Wiener case

Proposition 5 (Bayesian updating with functions) *Under Assumption 1, we have*

$$\mathbb{E}[u(a) \mid \mathbf{s}] = \lambda s(a)$$

with $\lambda = \frac{1}{1+\sigma_{\varepsilon_t}^2/\sigma_u^2}$. *This means that we can do Bayesian updating on this space of functions.*

Proof. Take the increments:

$$ds(a) = du(a) + \sigma dW(a)$$

The key observation is that the $ds(a)$'s are all Gaussians innovations, independent of the value of the functions are other points $y \neq a$. So, by the formulation for Gaussian updating we used before:

$$\mathbb{E}[du(a) \mid ds(a)] = \lambda ds(a)$$

with $\lambda = \frac{1}{1+\sigma^2/\sigma_u^2}$. Next, because the $du(a)$ and $dW(a)$ are independent,

$$\mathbb{E}[du(a) \mid \mathbf{s}] = \mathbb{E}[du(a) \mid ds(a)] = \lambda ds(a). \tag{14}$$

Next, the behavior at 0 needs a special treatment. Because $s(0) = u(0) + \chi\sigma\eta_0$, $\mathbb{E}[u(0) \mid s(0)] = \lambda_0 s(0)$, with $\lambda_0 = \frac{1}{1+\frac{var(\chi\sigma\eta_0)}{var(u_0)}} = \frac{1}{1+\frac{\chi^2\sigma^2}{\chi^2\sigma_u^2}} = \lambda$. Then, $\mathbb{E}[u(0) \mid s(0)] = \lambda s(0)$, and by independence:

$$\mathbb{E}[u(0) \mid \mathbf{s}] = \lambda s(0). \tag{15}$$

Hence, integrating from 0 to a, we get

$$\begin{aligned}\mathbb{E}\left[u(a) \mid \mathbf{s}\right] &= \mathbb{E}\left[u(0) + \int_{y=0}^{a} du(y) \mid \mathbf{s}\right] = \mathbb{E}\left[u(0) \mid \mathbf{s}\right] + \int_{y=0}^{a} \mathbb{E}\left[du(y) \mid \mathbf{s}\right] \\ &= \lambda s(0) + \int_{0}^{a} \lambda ds(y) \\ &= \lambda s(a).\end{aligned}$$

7.2.2 Polynomial utility

Here we provide assumptions that are a little more elementary, but apply only when the utility function $u(a)$ is a polynomial in a. For instance, we want to capture that $u(a) = b_0 + b_1 a + b_2 a^2$ with unknown coefficients b_i, that the agent wants to learn from noisy signals.

Assumptions for the polynomial utility case We shall use the Legendre $P_i(a)$ polynomials as a basis, as they are more convenient than the plain monomials a^i. We have for instance:[12]

$$P_0(a) = 1, \qquad P_1(a) = a, \qquad P_2(a) = \frac{1}{2}\left(3a^2 - 1\right), \qquad P_3(a) = \frac{1}{2}\left(5a^3 - 3a\right).$$

Using the product

$$\langle f \mid g \rangle := \int_{-1}^{1} f(a)\, g(a)\, da, \tag{16}$$

we have the standard result: $\langle P_i \mid P_j \rangle = \frac{1}{i+\frac{1}{2}} 1_{i=j}$. So we define q_i to be a rescaled version of the standard Legendre polynomial:

$$q_i(a) := \sqrt{i + \frac{1}{2}}\, P_i(a), \tag{17}$$

so that

$$\langle q_i \mid q_j \rangle = 1_{i=j}. \tag{18}$$

Polynomial q_i has degree i, and the q_i's form an orthogonal basis for polynomial functions.

We can now state our assumption.

[12] More generally we have $P_i(a) = \frac{1}{2^i i!} \frac{d^i}{da^i}\left[\left(a^2 - 1\right)^i\right]$ by Rodrigues' formula.

Assumption 2 (Utility function as drawn from a random distribution on polynomial basis) *We decompose the true utility function $u(a)$ as:*

$$u(a) = \sum_{i=-1}^{\infty} f_i Q_i(a) \tag{19}$$

where $Q_{-1}(a) \equiv 1$ and for $i \geq 0$, $Q_i(a) = \int_0^a q_i(y)\,dy$, where $q_i(y)$ is the i-th normalized Legrendre polynomial (17). We assume that a finite subset I such that coefficients $\{f_i\}_{i \in I}$ are nonzero and that the f_i for $i \in I$ are i.i.d. and follow a $N(0, \sigma_u^2)$ distribution. Also assume $\sigma_{f_{-1}}^2 = \chi^2 \sigma_u^2$.

We note that, in the limit where all coefficients are non-zero, we get the "Wiener" case.

Result We prove a more general proposition.

Proposition 6 *Suppose that coefficients f_i are drawn from the Gaussian $N\left(0, \sigma_{f_i}^2\right)$, and jointly Gaussian and uncorrelated. Then, the posterior $\mathbb{E}[u(a) \mid \mathbf{s}] := \mathbb{E}\left[u(a) \mid (s(y))_{y \in [-1,1]}\right]$ is:*

$$\mathbb{E}[u(a) \mid \mathbf{s}] = \sum_{i=-1}^{\infty} \mathbb{E}[f_i \mid \mathbf{s}]\, Q_i(a)$$

where, for $i \geq 1$

$$\mathbb{E}[f_i \mid \mathbf{s}] = \lambda_i \langle q_i \mid d\mathbf{s} \rangle = \lambda_i \int_{a=-1}^{1} q_i(a)\, ds(a)$$

$$\lambda_i = 1/\left(1 + \sigma^2/var(f_i)\right)$$

while $\mathbb{E}[f_{-1} \mid s] = \lambda_{-1} s(0)$ with $\lambda_{-1} = 1/\left(1 + \chi^2\sigma^2/var(f_{-1})\right)$. This implies that the average posterior is:

$$\overline{u}(a) := \mathbb{E}[[u(a) \mid \mathbf{s}] \mid f] = \sum_{i=-1}^{\infty} \lambda_i f_i Q_i(a). \tag{20}$$

In particular, take the case of flat priors of Assumption 2, and call $\lambda = 1/\left(1 + \frac{\sigma^2}{\sigma_u^2}\right)$. Then,

$$\overline{u}(a) = \lambda u(a) \tag{21}$$

i.e. we obtain uniform dampening.

Proof of Proposition 6 Suppose that we have a function $u(a)$, and we observe, as in (10),

$$s(a) = u(a) + \sigma W(a) + \chi\sigma\eta_0 \tag{22}$$

where $W(a)$ is a Brownian motion and $\tilde{\eta}_0 = \chi\sigma\eta_0$ is a Gaussian variable of mean zero. Differentiate:

$$\begin{aligned} ds(a) &= u'(a)\,da + \sigma dW(a) \\ u'(a) &= \sum_{j=-1}^{\infty} f_j Q_j'(a) = \sum_{j=0}^{\infty} f_j q_j(a)\,. \end{aligned}$$

Hence:

$$ds(a) = \sum_{j=0}^{\infty} f_j q_j(a)\,da + \sigma dW(a).$$

The agent wants to infer u given s, i.e. f given $d\mathbf{s}$ (we consider the intercept $u(0)$ at the end). Multiplying the previous equation by $q_i(a)$ and integrating between -1 and 1 gives:

$$\begin{aligned} S_i &:= \langle q_i \mid d\mathbf{s}\rangle \\ &= \sum_j f_j \langle q_i \mid q_j\rangle + \sigma \langle q_i \mid dW\rangle \\ &= f_i + \sigma \langle q_i \mid dW\rangle \end{aligned} \tag{23}$$

because of (18).

Hence we can write the signal $S_i := \langle q_i \mid d\mathbf{s}\rangle$ as

$$S_i = f_i + \sigma\varepsilon_i \tag{24}$$

with $\varepsilon_i := \langle q_i \mid dW\rangle = \int_{-1}^{1} q_i(a)\,dW_a$ satisfies $\mathbb{E}[\varepsilon_i] = 0$. In addition

$$\begin{aligned} \mathbb{E}[\varepsilon_i\varepsilon_j] &= \mathbb{E}\left[\left(\int q_i(a)\,dW_a\right)\left(\int q_j(a)\,dW_a\right)\right] = \int q_i(a)\,q_j(a)\,da \\ &= 1_{i=j}. \end{aligned}$$

Hence, the signal-extraction problem $\mathbb{E}[f_i \mid \mathbf{s}]$ is quite simple, as only S_i is informative about f_i: $\mathbb{E}[f_i \mid \mathbf{s}] = \mathbb{E}[f_i \mid S_i]$. Given (24),

$$\mathbb{E}[f_i \mid \mathbf{s}] = \lambda_i S_i \tag{25}$$

$$\lambda_i = 1/\left(1 + \sigma^2 / var(f_i)\right). \tag{26}$$

Hence, we have

$$\begin{aligned}\mathbb{E}[u'(a) \mid \mathbf{s}] &= \sum_{i=0}^{\infty} \mathbb{E}[f_i \mid \mathbf{s}]\, q_i(a) \\ &= \sum_{i=0}^{\infty} \mathbb{E}[f_i \mid \mathbf{s}]\, Q_i'(a)\end{aligned}$$

We next study the intercept in (22), $u(0)$. Given $s(0) = u(0) + \chi\sigma\eta_0$ and $u(0) = f_{-1}$,

$$\mathbb{E}[u(0) \mid \mathbf{s}] = \mathbb{E}[f_{-1} \mid s(0)] = \lambda_{-1} s(0) = \lambda_{-1} S_{-1}$$

where $S_{-1} := s(0)$ and $\lambda_{-1} = 1/\left(1 + \chi^2\sigma^2 / var(f_{-1})\right)$. Integrating,

$$\begin{aligned}\mathbb{E}[u(a) \mid s] &= \mathbb{E}[u(0) \mid s] + \mathbb{E}\left[\int_0^a u'(b)\, db \mid s\right] \\ &= \lambda_{-1} S_{-1} + \sum_{i=0}^{\infty} \lambda_i S_i Q_i(a) = \sum_{i=-1}^{\infty} \lambda_i S_i Q_i(a).\end{aligned}$$

In addition, the average perception is:

$$\begin{aligned}\overline{u}(a) := \mathbb{E}[[u(a) \mid \mathbf{s}] \mid \mathbf{u}] &= \sum_{i=-1}^{\infty} \lambda_i \mathbb{E}[S_i \mid f]\, Q_i(a) \\ &= \sum_{i=-1}^{\infty} \lambda_i f_i Q_i(a) \end{aligned} \tag{27}$$

If we assume a "flat" prior of Assumption 2, where $var(f_i)$ is independent of i (if $var(f_i) > 0$), we have for $i \geq 0$

$$\lambda_i = \lambda = \frac{1}{1 + \frac{\sigma^2}{var(f_i)}} = \frac{1}{1 + \frac{\sigma^2}{\sigma_u^2}}.$$

Furthermore, as $\sigma_{f_{-1}}^2 = \chi^2 \sigma_u^2$,

$$\lambda_{-1} = \frac{1}{1 + \frac{\chi^2 \sigma^2}{var(f_{-1})}} = \lambda.$$

Hence, $\lambda_i = \lambda$ for all $i \geq -1$, and (27) implies:

$$\overline{u}\,(a) = \sum_{i=-1}^{\infty} \lambda_i f_i Q_i\,(a) = \lambda \sum_{i=-1}^{\infty} f_i Q_i\,(a) = \lambda u\,(a)\,.$$