Leasing and Secondary Markets:
Theory and Evidence from Commercial Aircraft[*]

Job Market Paper

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Abstract

The number of transactions in used aircraft is three times larger than the number of purchases of new aircraft. Furthermore, aircraft lessors own one third of all aircraft operated by world carriers. This paper develops a dynamic model of transactions in used capital to illustrate the role of lessors as intermediaries that enhance the efficiency with which aircraft are redeployed across carriers and tests the empirical implications using a dataset on aircraft and carriers fleets.

Carriers trade aircraft either to replace their fleet or to reduce excess capacity. Trading frictions hinder the efficiency of the allocation of capital and lessors reduce frictions by centralizing the exchange. Thus, leased aircraft trade more frequently and produce a higher output than owned aircraft, as lower barriers to trade imply that lessees are more productive than owners and allow a finer pairing between quality of capital and productivity of carriers on leased aircraft.

In the empirical section, consistent with the model, we find that 1) leased aircraft have a holding duration 40% shorter than owned aircraft; 2) leased aircraft have 8% higher output than owned aircraft. The estimates imply that most of the gain in output arises because lower barriers to trade increase the average productivity of carriers, and finer pairing contributes only to 0.14% of the gain. The estimates also imply that the gain from perfect pairing between quality and productivity would raise current output only by 0.56%.

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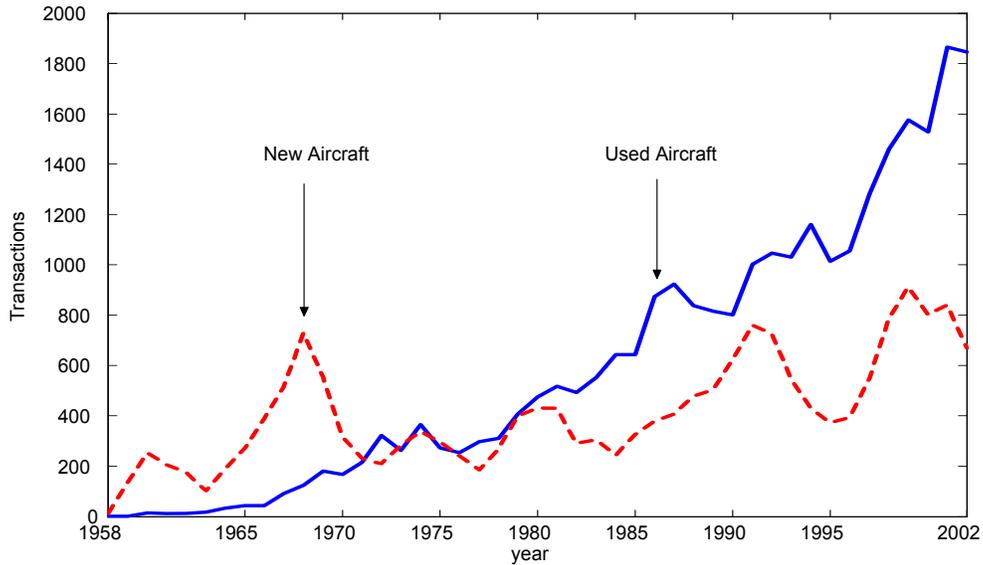
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FIG. 1 - TRANSACTIONS IN THE PRIMARY (DOTTED LINE) AND SECONDARY (SOLID LINE) MARKETS FOR NARROWBODY AND WIDEBODY AIRCRAFT.

1   Introduction

This paper studies the link between the efficiency of secondary markets for firms' inputs and the efficiency of production of final output. I develop a theoretical model of transactions in used capital when trading is subject to frictions. I show that lessors act as intermediaries that enhance the efficiency with which capital is redeployed across firms. The model has several empirical predictions that I test using a dataset on aircraft and carriers fleets. The estimates imply a considerable gain in output for leased aircraft (8% in 2002-2003).

Markets for used capital equipment are very active. For example, more than two thirds of all machine tools sold in the United States in 1960 were used (Waterson (1964)); more than half of the total number of trucks traded in the United States in 1977 were traded in secondary markets (Bond (1983)) and active markets exists for used medical equipment, construction equipment and aircraft. Figure 1 plots the number of transactions in the primary and the secondary markets for Narrowbody and Widebody aircraft. Since the mid 80's, trade in the secondary market for aircraft has grown steadily and today the number of transactions on the used market is about three times the number of purchases of new aircraft.

A large share of these transactions is due to leasing. A substantial number of the aircraft currently operated by major carriers is under an operating lease, a pure rental contract between a
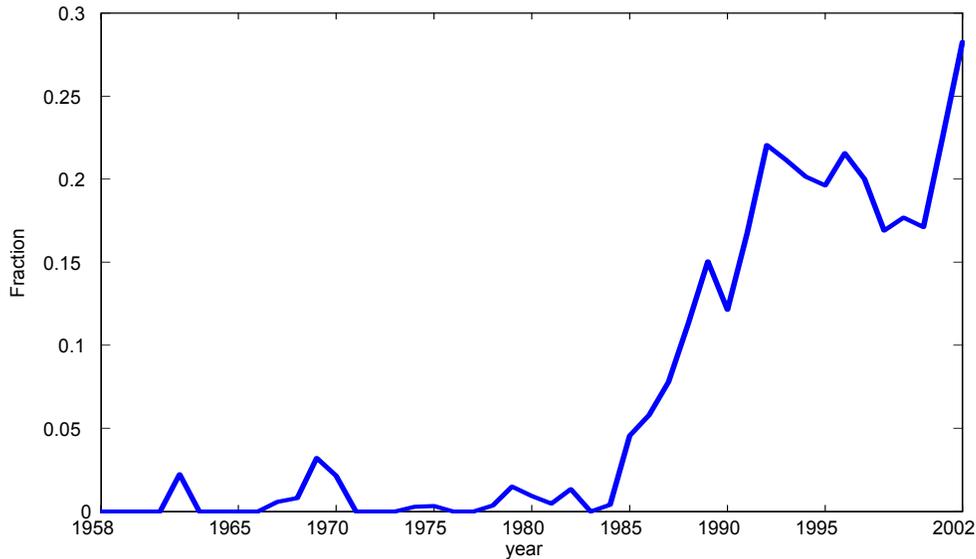
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FIG. 2 - FRACTION OF NEW NARROWBODY AND WIDEBODY AIRCRAFT DELIVERED TO LESSORS BY YEAR.

lessor and an airline for the use of the aircraft for a short period of time (4-5 years).[1] Figure 2 plots the fraction of new Narrowbody and Widebody aircraft delivered to lessors to total aircraft delivered by year. The figure shows that lessors are actively engaged in the purchase of new aircraft and their acquisitions have increased rapidly in recent years. But lessors are also very active participants in secondary markets, because they frequently buy used aircraft and, more importantly, because they lease out each aircraft several times during its useful lifetime.

In this paper, I construct a model of transactions in used aircraft to understand the role of lessors when trading is subject to frictions. The model combines four key ingredients: 1) firms have heterogeneous stochastic efficiency; 2) aircraft are produced every period and depreciate; 3) aircraft can be bought or leased; 4) used aircraft sales are subject to frictions. In this world, secondary markets play a fundamental allocative role. Because the quality of capital and the efficiency of the firm are complements, firms self select and acquire different quality of capital according to their efficiency. Thus, firms trade used capital for two reasons. The first is the replacement of old aircraft. When the quality of capital depreciates over time, firms sell their old aircraft to acquire more productive ones. The second is the adjustment of productive capacity. When either cost or demand shocks adversely affect profitability, firms shrink and sell aircraft. Viceversa, when shocks positively affect profitability, firms expand and acquire aircraft.

If there is no leasing, I show that trading frictions and stochastic productivity hinder the

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[1] See Section 3 for more details on the Operating Lease.

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efficiency of the allocation of capital and the production of output. Efficiency requires perfect matching between quality of the capital and productivity of the firm. However, transaction costs create a wedge between the price paid by the buyer and the price received by the seller that acts as a barrier to trade. This implies that matching between quality and productivity is coarse. In particular, some firms operating new aircraft are less productive than some firms operating old aircraft; and some firms operating an aircraft are less productive than some firms not operating any aircraft.

I argue that lessors act as intermediaries that reduce transaction costs.[2] When carriers can buy or lease aircraft, I show that leased aircraft trade more frequently. This improves the matching process between productive firms and capital and, as a result, leased aircraft produce a higher output than owned aircraft. This gain can be decomposed into three distinct effects. The first is a *parking effect*: leased aircraft are parked less frequently than owned aircraft. The second is a *productivity effect*: the productivity of lessees is higher than the productivity of owners, conditional on the aircraft being in use. The third is a *pairing effect*: the covariance between quality of the aircraft and productivity of the carrier is higher for leased aircraft than for owned aircraft, conditional on the aircraft being in use.

In the empirical section, I find evidence consistent with the implications of the model using data on aircraft and carriers. Data on airlines' fleets have been provided to me by AVSOFT, an aviation software company. AVSOFT produces a detailed information system database for the aviation market. For each Western built fixed wing civil aircraft, the database contains information on aircraft characteristics (such as age, engine, total number of hours flown, the total number of landings), the history of the aircraft (including the list of operators) and the current ownership status (Owned vs. Leased).

The data reveals that leased aircraft have a 40% shorter average holding duration, 8% more flying hours and are parked 60% less frequently than owned aircraft.

I estimate the depreciation pattern of aircraft in order to obtain measures of quality of aircraft and productivity of the carriers. The estimates reveal that the distribution of productivity associated with leased aircraft stochastically dominates the distribution of productivity associated with owned ones. Based on the estimates, I can decompose the gain in output due to leasing into the empirical counterparts of the three components identified in the theoretical model: 1.8% of the difference in output is explained by the *parking effect;* 6.2% is explained by the *productivity effect* and .14% is due to the *pairing effect.* The estimates also imply that perfect pairing between quality and productivity would increase output only by .71%. Thus, leasing captures 20% of all possible gains from pairing.

I argue that the growth of trade in the secondary markets for aircraft since the mid 1980s is consistent with my model. The Airline Deregulation Act of 1978 dramatically reduced entry

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[2]Transaction costs are meant to include search costs for a potential buyer and other trading frictions. Since the primary activity of lessors is to rent the fleet they own, it seems natural to consider they have a cost advantage in the transactions.

costs, thereby increasing the competitiveness of airline markets.[3] This increase in competitiveness amplified the volatility of firm level output, implying that carriers need to adjust their fleets more frequently. The volume of trade on secondary market increased substantially due to higher inter-firm reallocation of inputs.[4] The entry of lessors in the mid 1980s, as documented in Figure 2, therefore exactly coincides with a period of expansion of trade in secondary markets, when the need for market intermediaries to coordinate sellers and buyers becomes stronger.[5]

This paper identifies lessors as intermediaries who provide liquidity and reduce frictions in secondary markets. Thus, I highlight a novel role for leasing in capital equipment that has been ignored in the literature.[6] I believe that the mechanisms identified in these paper are not unique to the aircraft markets: they may help understand the role of leasing for a wide range of capital equipment. Moreover, this paper is one of the few papers that tries to empirically measure the gain due to intermediation and to institutions that enhance the efficiency of trading. This is of particular importance because the efficiency of secondary markets for capital equipment is a key factor in any industry's speed of adjustment after a shock or a policy intervention.

The paper is organized as follows. Section 2 discusses the related literature. Section 3 illustrates some institutional characteristics of secondary markets for commercial aircraft and of aircraft lessors. Section 4 lays out the model economy. I first analyze the benchmark case of no transaction costs, then I present the case when aircraft can only be owned but trading is subject to transaction costs. Finally, I introduce a stylized lessor that offers aircraft for lease. I then extend the model to capture factor hoarding, a pervasive feature of the data. The empirical analysis is performed in Section 5. Section 6 concludes. Mathematical derivations and proofs of most propositions are collected in the Appendixes B and C.

2 Related Literature

The paper is related to several strands of the literature. The first important strand is the literature on durable goods. This literature has generally analyzed consumer durables, with Bond (1983)

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[3] The airline industry was governed by the Civil Aereonautics Board (CAB) from 1938 to 1984. Under the Airline Deregulation Act of 1978, the industry was deregulated in stages. From January 1, 1982, all controls on entry and exit were removed, while airfares were deregulated from January 1, 1983. The actual changes were implemented rather more rapidly. Finally, on January 1, 1985 the governance of the airline industry was transferred from the Civil Aereonautics Board to the Department of Transportation.

[4] Ramey and Shapiro (1998) analyze Compustat data and also find a significant increase in capital reallocation across firms and industries in the 80s and 90s.

[5] Steven F. Udvar-Hazy, Chairman and CEO of ILFC, one of the largest aircraft lessors, declares: "The inevitability of change creates a constant flow of upswings and downturns in air transportation. But one thing does not change – the continuous need for rapid, economical deployment of high performance aircraft. ILFC understood this reality as early as 1973 when we pioneered the world's first aircraft operating lease." Available at `http://www.ilfc.com/ceo.htm`.

[6] Smith and Wakeman (1985) analyze the determinants of corporate leasing policies and notice an incentive to lease if the lessor has a comparative advantage in disposing of the asset.

being a notable exception. Rust (1985), Anderson and Ginsburgh (1994), Hendel and Lizzeri (1999a), Porter and Sattler (1999), Stolyarov (2002) and Esteban and Shum (2003) concentrate on the interactions between primary and secondary markets for consumer durables such as cars. In these models, all gains from trade arise from the depreciation of the durable.[7] In this context, Waldman (1997) and Hendel and Lizzeri (1999b, 2002) have analyzed the incentive to lease for a manufacturer. These papers show that the manufacturer might prefer to lease because used units traded on secondary markets compete with the new units produced by the manufacturer.[8] In the aircraft market, however, as explained in more detail in the next Section 3, the largest lessors are not the manufacturers of the aircraft. Thus, the explanations provided by the literature do not apply. In contrast, in my paper lessors are intermediaries that specialize in trading.

The second is the literature on firms' investment. More relevant to my work are Cooper and Haltiwanger (1993), Cooper, Haltiwanger and Power (1998), who focus on the replacement problem, and Ramey and Shapiro (2001) and Rampini and Eisfeldt (2003) who study capital reallocations. These papers analyze a single firm's problem without explicitly solving the equilibrium in the market for capital.

A strand of the literature in corporate finance examine the corporate decisions to lease. These studies generally focus on the financial lease. Smith and Wakeman (1985) provide an exhaustive but informal overview and Sharpe and Nguyen (1995) present a detailed empirical analysis. This literature focuses primarily on financial reasons to lease, while I focus on the *effect* of leasing on secondary markets for aircraft and on the output produced by the carriers.

Further, this paper is related to the literature on intermediaries. Spulber (1999) presents a thorough analysis and surveys the literature. Here, I present one of the first empirical analysis that quantifies the gain due to intermediary firms specializing in transactions.

Lastly, a long series of papers have analyzed the airline industry. Most literature has analyzed the entry or the pricing decision of carriers, and few papers have discussed transactions of aircraft, with Pulvino (1998,1999) and Gilligan (2004) being notable exceptions. Pulvino (1998,1999) uses transaction level data and finds that airlines under financial pressure sells aircraft at a discount. He further shows that distressed airlines experience higher rates of asset sales than no distressed airlines. As long as economic and financial forces are linked, my paper provides a rationale for his findings. Gilligan (2004) uses data from the market for business jets and finds evidence consistent with the fact that leasing ameliorates the consequences of information asymmetries about the quality of used aircraft. Hence, both my paper and Gilligan's work show empirically how leasing mitigates market imperfections.

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[7]See also House and Leahy (2004), where, in contrast, gains from trade arise because in each period indivduals draw a match parameter that describes how they like the car they own, or how well it suits their needs.

[8]Bulow (1986) shows how a monopolist prefers to lease to solve the time inconsistency problem that gives rise to the Coase conjecture.

3   Background

The market for used commercial aircraft has three main institutional characteristics.[9] First, it is a worldwide single market, where aircraft are often transferred from an operator in one country to an operator in another country. Second, the market is dominated by privately negotiated transactions. Most major carriers have staff devoted to the acquisition and disposition of aircraft and sometimes independent brokers are used to match buyers and sellers. Third, sometimes aircraft are purchased by governments and air cargo companies, but the major players are airlines and lessors.

The operating lease business was essentially founded by International Lease Finance Corporation (ILFC) in the early 1970s and today many companies are in the field. The biggest one is General Electric Capital Aviation Services (GECAS), a unit of General Electric Company. GECAS today owns approximately 1100 aircraft, manages approximately 300 aircraft for others and has more than 230 airline customers. Interestingly, the largest lessors in the field are not the manufacturers of aircraft - Boeing and Airbus - even though both have recently established Aircraft Trading/Leasing divisions.

In its Annual Report, ILFC describes its business as follows:

> "*International Lease Finance Corporation is primarily engaged in the acquisition of new commercial jet aircraft and the leasing of those aircraft to airlines throughout the world. In addition to its leasing activity, the Company regularly sells aircraft from its leased aircraft fleet to third party lessors and airlines. In some cases the Company provides fleet management services to companies with aircraft portfolios for a management fee. The Company also remarkets and sells aircraft owned by others for a fee.*"[10]

An operating lease means that the lessor retains ownership of the asset and rents it to the airline for a period of time that in general varies between four to eight years. Many variations of the operating lease have evolved, but for the airlines the key point is when they want to replace or reduce capacity much of the job of finding a buyer for its used capital has been taken over by another party, the lessor. Most leases are on a "net" basis with the lessee responsible for all operating expenses. In addition, normal maintenance and repairs, airframe and engine overhauls, and compliance with return conditions of flight equipment on lease are paid for by the lessee. Under the provisions of some leases, the lessor contributes to the cost of certain airframe and engine overhauls. Lessors require their lessees to comply with the standards of either the United States Federal Aviation Administration or its foreign equivalent. Lessors make periodic inspections of the condition of their leased aircraft.

Table 1 presents the share of aircraft for passenger transportation under an operating lease contract as of April 2003. Approximately one third of all Narrowbody and Widebody aircraft

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[9]See Pulvino (1998).

[10]Form 10-K, International Lease Finance Corporation - ILFC. Filed on March 11, 2004.

Table 1

	Leased	Owned
Total	33.41%	66.59%
Narrowbody	37.16%	62.84%
Widebody	23.07%	76.93%

Notes: as of April 2003.

worldwide is leased. The share is larger for Narrowbody (37.16%) than for Widebody Aircraft (23.07%).

Another type of leasing contract already existed with an older history, the financial lease. In a financial lease, the lease terms are longer, usually fifteen years to twenty years, and the airline has an option to buy the airplane at the end of the lease term, so it is, in effect, a way of financing the purchase of the equipment.

4 Model

In this Section I lay out the model. I first present the case of no leasing where all aircraft are owned by carriers. In subsection 4.5 I introduce a stylized lessor that offers aircraft for lease and carriers choose between leasing and owning aircraft.

Firms - There is a unit mass of firms. Firms differ in their efficiency parameter $z \in [\underline{z}, \overline{z}]$. Efficiency is stochastic and it evolves according to

$$z_t \sim F(z) \text{ with probability } \alpha$$
$$z_t = z_{t-1} \text{ with probability } 1 - \alpha$$

where $\alpha \in [0, 1)$ measures the volatility of a firm's efficiency.[11] I assume that $F(z)$ is uniform on $[0, 1]$.[12] Each firm can operate at most one aircraft. The per-period revenue and output of a firm of efficiency z operating an aircraft of quality q_i are

$$\pi(z, q_i) = zq_i$$

I refer to the firms collectively as the *industry*.

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[11] The results derived in the paper depend only on the stochastic nature of efficiency and not on the particular process assumed, as it will become clear. The specific process makes later derivations more tractable.

[12] This assumption greatly simplifies the calculations and allows closed form solution for several quantities of interest. I have checked numerically the robustness of my results to different distributional assumptions and all qualitative features of the model remain unchanged.

Capital Goods - In each period an exogenous flow x of aircraft enters the economy. A new aircraft produces a flow of service equal to q_1. At the end of each period, a quality q_1 aircraft depreciates to $q_2 < q_1$ with probability $\gamma_1 > 0$. A quality q_2 aircraft does not depreciate but dies with probability γ_2 with $0 < \gamma_1 \leq \gamma_2 < 1$. The total number of aircraft of quality q_i is equal to $X_i = \frac{x}{\gamma_i}$ and I require that

$$X = X_1 + X_2 = \frac{x}{\gamma_1} + \frac{x}{\gamma_2} < 1$$

so that some firms do not operate any aircraft.

Trade - At the beginning of each period, firms can trade aircraft. The buyer pays price p_i for an aircraft, the seller receives $p_i - T$, where T represents transaction costs and $i = 1, 2$. Transaction costs are lump-sum and are not as large as to prevent trade. This requires that transaction costs are lower than the (endogenous) prices. As it will become clear, a sufficient condition is that

$$T < \frac{(1 - X) q_2}{1 - \beta (1 - \gamma_2)}$$

For simplicity, I assume also that the change in the efficiency parameter z and the depreciation of the aircraft are mutually exclusive events, so that at most one of them can happen in each period to a firm.

For notational convenience, let $q_3 = 0$ denote the quality of an aircraft that just died, whose prices is equal to $p_3 = 0$. Note that a firm with no aircraft has a q_3 aircraft.

4.1 Firm's problem

Let $V_i(z)$, $i = 1, 2$, be the value of a firm of efficiency z that operates an aircraft q_i and $V_3(z)$ the value of a firm that does not operate any aircraft (operates an aircraft $q_3 = 0$). $V_1(z)$ is equal to

$$
\begin{aligned}
V_1(z) =\ & \frac{z q_1}{1 - \beta (1 - \alpha - \gamma_1)} + \\
& \frac{\beta \alpha \int_{\underline{z}}^{\overline{z}} \max \{V_1(z), V_2(z) + p_1 - T - p_2, V_3(z) + p_1 - T\}\, dF(z)}{1 - \beta (1 - \alpha - \gamma_1)} + \\
& \frac{\beta \gamma_1 \max \{V_1(z) - p_1 + p_2 - T, V_2(z), V_3(z) + p_2 - T\}}{1 - \beta (1 - \alpha - \gamma_1)}
\end{aligned}
$$

Similarly, $V_2(z)$ is given by

$$
\begin{aligned}
V_2(z) =\ & \frac{z q_2}{1 - \beta (1 - \alpha - \gamma_2)} + \\
& \frac{\beta \alpha \int_{\underline{z}}^{\overline{z}} \max \{V_1(z) - p_1 + p_2 - T, V_2(z), V_3(z) + p_2 - T\}\, dF(z)}{1 - \beta (1 - \alpha - \gamma_2)} + \\
& \frac{\beta \gamma_2 \max \{V_1(z) - p_1, V_2(z) - p_2, V_3(z)\}}{1 - \beta (1 - \alpha - \gamma_2)}
\end{aligned}
$$

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where β is the discount factor common to all firms. The first term in $V_i(z)$ is current revenue of the firm. Then with probability α the firm gets a new draw of z, so the firm takes expectation over its optimal future actions, conditioning on the aircraft it owns. With probability γ_i the aircraft depreciates and the firm chooses between replacing it (the first options in the curly brackets), or keeping the current q_{i+1} aircraft.

Similarly, $V_3(z)$ is given by

$$V_3(z) = \frac{\beta\alpha \int_{\underline{z}}^{\overline{z}} \max\{V_1(z) - p_1, V_2(z) - p_2, V_3(z)\}\, dF(z)}{1 - \beta(1 - \alpha)}$$

Let

$$W_j(z) = \max\{V_j(z), V_i(z) - p_i + p_j - T*1\{q_j \neq q_3\}, V_3(z) + p_j - T*1\{q_j \neq q_3\}\}$$

denote the value of a firm who owns an aircraft of quality q_j for $j = 1, 2, 3$, where $1\{q_j \neq q_3\}$ is the indicator function equal to 1 if $q_j \neq q_3$ and 0 otherwise. The firm chooses between continuing to operate the aircraft it owns and selling it to acquire and operate an aircraft of quality q_i, $i \neq j$. Clearly, for firms with a q_j aircraft, there exist cutoff values $z_i(j)$, $i = 1, 2$ and $j = 1, 2, 3$, such that firms with efficiency $z \geq z_1(j)$ operate q_1 aircraft, trading a q_j aircraft if they were previously owning it $(j \neq 1)$; firms with efficiency $z \in [z_2(j), z_1(j))$ operate q_2 aircraft, again after trading a q_j aircraft if they were previously owning it $(j \neq 2)$, while firms $z < z_2(j)$ sell their q_j aircraft and exit (keep staying out) if $j = 1, 2$ $(j = 3)$.

I denote throughout the exposition the $z_j(3)$ values as the *buying/entry thresholds*, the $z_2(j)$ values as the *exit thresholds*, the $z_1(j)$ values as the *trading thresholds* and the $z_j(j)$ values as the *selling thresholds* for $j = 1, 2$.

4.2 Benchmark: no transaction costs. $T = 0$

As a useful benchmark, I first consider how these different thresholds are determined in equilibrium when there are no transaction costs.

Proposition 1 *When there are no transaction cost* $z_1(1) = z_1(2) = z_1(3) = z_1$ *and* $z_2(1) = z_2(2) = z_2(3) = z_2$. *Moreover, equilibrium on the market for each quality of aircraft* q_i *requires that* z_1 *and* z_2 *satisfy*

$$\begin{aligned} X_1 &= 1 - F(z_1) \\ X_2 &= F(z_1) - F(z_2) \end{aligned}$$

Proof. See Appendix. ∎

Prices of the aircraft must be such that the marginal firms z_i are indifferent between quality q_i and quality q_{i+1}

$$V_i(z_i) - p_i = V_{i+1}(z_i) - p_{i+1}$$

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The resulting prices are

$$p_2 = \frac{z_2 q_2}{1 - \beta(1 - \gamma_2)}$$

$$p_1 = p_2\left(\frac{1 - \beta(1 - \gamma_1 - \gamma_2)}{1 - \beta(1 - \gamma_1)}\right) + \frac{z_1(q_1 - q_2)}{1 - \beta(1 - \gamma_1)}$$

When there are no transaction costs, previous ownership of a specific aircraft is irrelevant and the set of firms is partitioned such that the highest efficiency firms choose the high quality q_1 aircraft, the intermediate efficiency firms choose the low quality q_2 aircraft and the lowest efficiency firms do not operate any aircraft. Three features of this equilibrium are worth noting. First, only the most efficient firms operate an aircraft, as all the firms above the z_2 threshold own an aircraft. Second, we have perfect pairing between quality of the aircraft and efficiency of the carrier, as the set of firms operating the q_1 aircraft is disjoint from the set of firms operating a q_2 aircraft. These two considerations together imply that the equilibrium allocation maximizes the total industry output. Lastly, the equilibrium allocation of aircraft and prices is independent of the volatility parameter α, as it can be verified from inspection of the equations determining it. The equilibrium is exactly the same in the case of deterministic productivity ($\alpha = 0$) and in the case of iid productivity ($\alpha = 1$), even though the volume of trade obviously increases as α increases.

In this setting, the introduction of leasing contracts has no effects on the equilibrium allocations. More specifically, if a lessor offers aircraft for lease at a rental rate r_i per period, leased aircraft and owned aircraft would have exactly the same probability of being traded, the same distribution of holding durations and would produce the same output. When there are no transaction costs, the equilibrium is clearly equal to a perfect rental market.

4.3 The Effects of Transaction Costs

The presence of a transaction cost modifies the previous benchmark in a significant way. Now the quality of the aircraft that a firm owns becomes an additional state variable and determines jointly with the efficiency z the optimal policy of the firm. The transaction cost acts in two different ways. Logically, it acts as a barrier to sell when a firm owns an aircraft, but it also acts as a barrier to buy when a carrier does not own an aircraft. Compared to the benchmark of no transaction cost, this implies that firms now require a lower draw of z before selling their aircraft to downgrade it and a higher draw of z before upgrading it.

Thus the thresholds are modified in the following way:

Proposition 2 *If the transaction cost is strictly positive we have:* $z_1(2) > z_1(3) > z_1(1)$ *and* $z_2(1) = z_2(3) > z_2(2)$.

Proof. $z_1(2)$ satisfies

$$V_1(z_1(2)) - p_1 + p_2 - T = V_2(z_1(2)) \tag{1}$$

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$z_1(3)$ satisfies

$$V_1(z_1(3)) - p_1 = V_2(z_1(3)) - p_2 \tag{2}$$

while $z_1(1)$ satisfies

$$V_1(z_1(1)) = V_2(z_1(1)) + p_1 - T - p_2 \tag{3}$$

Combining equations (1), (2) and (3)

$$V_1(z_1(2)) - V_2(z_1(2)) - T = V_1(z_1(3)) - V_2(z_1(3)) = V_1(z_1(1)) - V_2(z_1(1)) + T$$

and the result follows since $V_1(z) - V_2(z)$ is increasing in z.

$z_2(1)$ satisfies

$$V_2(z_2(1)) + p_1 - T - p_2 = V_3(z_2(1)) + p_1 - T \tag{4}$$

$z_2(3)$ satisfies

$$V_2(z_2(3)) - p_2 = V_3(z_2(3)) \tag{5}$$

$z_2(2)$ satisfies

$$V_2(z_2(2)) = V_3(z_2(2)) + p_2 - T \tag{6}$$

Combining equations (4), (5) and (6)

$$V_2(z_2(1)) - V_3(z_2(1)) = V_2(z_2(3)) - V_3(z_2(3)) = V_2(z_2(2)) - V_3(z_2(2)) + T$$

and the result follows since $V_2(z) - V_3(z) = V_2(z) - V_3$ is increasing in z. ∎

To completely characterize the equilibrium allocation, we need to understand the relationship between the two thresholds $z_1(1)$ and $z_2(1) = z_2(3)$. Intuitively, when the transaction cost is very high, no firm owning a q_1 aircraft is willing to pay it to downgrade to a q_2 aircraft, while if the cost is moderate, downgrading might be beneficial. The equilibrium and the qualitative features of the model in the two alternative scenarios are very similar, but unfortunately most of the equations that characterize it are not. In the sequel, I restrict the analysis to the case of moderate transaction costs, the case in which $z_1(1) > z_2(1) = z_2(3)$ and some firms always downgrade in equilibrium. As it will become clear in the analysis, the restriction has no effect on the empirical predictions of the model.

4.3.1 The Effect on Prices

Although I am not modelling the production of new aircraft, an analysis of the effects of transaction costs on prices of new aircraft might shed some lights on the incentive of the producers to alleviate trading frictions on the secondary markets. Transaction costs are usually not controlled by the producers, but there are several instances where the producer can affect them.

The prices p_i must be such that a firm with no aircraft and efficiency exactly equal to the threshold values is indifferent between a q_i aircraft and a q_{i+1} aircraft. In particular, prices p_i satisfy

$$V_i(z_i(3)) - p_i = V_{i+1}(z_i(3)) - p_{i+1}$$

For analytical convenience, let

$$J_i = \int_{\underline{z}}^{\overline{z}} W_i(z) \, dF(z)$$

denote the expected value of a firm with aircraft q_i when it receives a new draw of productivity. For firm $z_2(3)$ we have

$$V_2(z_2(3)) = \frac{z_2(3) q_2 + \beta \alpha J_2 - \beta \gamma_2 p_2}{1 - \beta(1 - \alpha)}$$

$$V_3 = \frac{\beta \alpha J_3}{1 - \beta(1 - \alpha)}$$

Hence, p_2 must satisfy

$$p_2 = \frac{z_2(3) q_2 + \beta \alpha (J_2 - J_3)}{1 - \beta(1 - \alpha - \gamma_2)} \tag{7}$$

Similarly, using the indifference condition $V_1(z_1(3)) - p_1 = V_2(z_1(3)) - p_2$, we obtain

$$V_1(z_1(3)) = \frac{z_1(3) q_1 + \beta \alpha J_1 + \beta \gamma_1 V_2(z_1(3))}{1 - \beta(1 - \alpha - \gamma_1)}$$

$$V_2(z_1(3)) = \frac{z_1(3) q_2 + \beta \alpha J_2 - \beta \gamma_2 p_2}{1 - \beta(1 - \alpha)}$$

from which we can derive the price of a q_1 aircraft

$$p_1 = p_2 \left(1 + \frac{\beta \gamma_2}{1 - \beta(1 - \alpha - \gamma_1)}\right) + \frac{z_1(3)(q_1 - q_2) + \beta \alpha (J_1 - J_2)}{1 - \beta(1 - \alpha - \gamma_1)} \tag{8}$$

$J_1 - J_2$ and $J_2 - J_3$ have complicated analytical solutions that are derived in Appendix B.1. In the case no transaction costs, clearly $J_i - J_{i+1}$ is simply given by the difference in prices $p_i - p_{i+1}$, while when the transaction cost is positive $J_i - J_{i+1}$ satisfies

$$p_i - p_{i+1} - T < J_i - J_{i+1} < p_i - p_{i+1} + T$$

Figure 3 illustrates the results of a numerical analysis. The left panel depicts the price p_1 while the right panel depicts the price p_2 as a function of the volatility parameter α. While p_2 is not monotonic in α and is everywhere below the value that would arise without transaction cost, it is interesting to note that for high values of α, the price p_1 of new aircraft when trading is subject to frictions are higher than the case without frictions.[13] The intuition is that the profits of each carrier are given by the product of its efficiency and the quality of the aircraft it operates. For high values of the volatility parameter α, the contribution of the current efficiency z to the firm's

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[13]Note that the case discussed here corresponds to a generalization of the case considered by Anderson and Gins-burgh. They analyze the case of $\alpha = 0$ and $\gamma_1 = \gamma_2 = 1$. See also Hendel and Lizzeri (1999, a) for the effect of the efficiency of the secondary market on the profits of a producer.
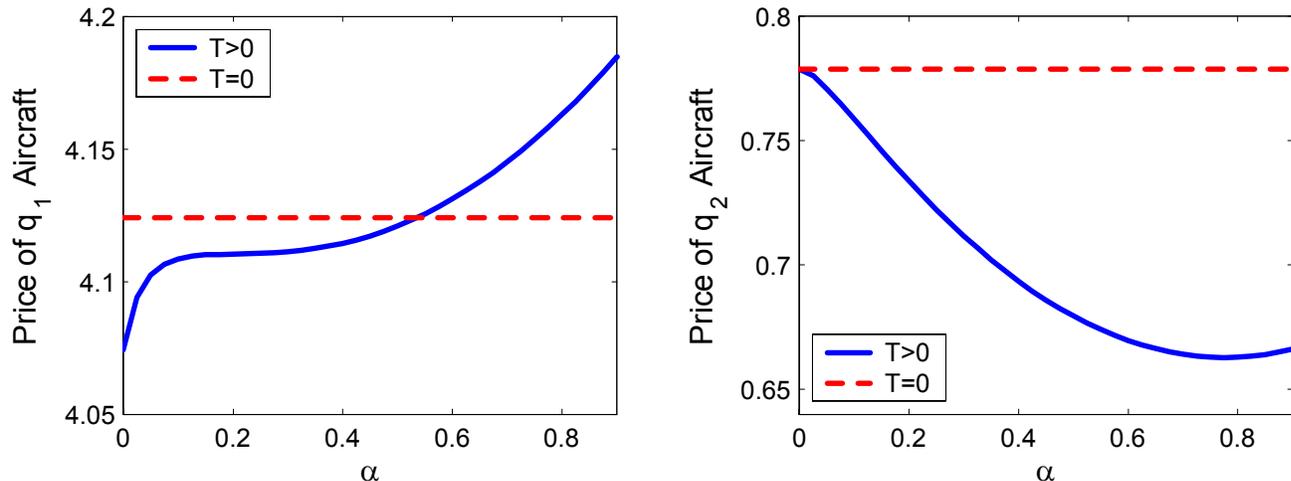
value is small, as productivity is highly volatile. In contrast, the contribution of the quality of capital becomes more relevant and hence the price p_1 increases as the volatility rises. Moreover, the transaction cost "locks" firms to the quality of capital they own and therefore q_1 aircraft are more valuable as α increases when $T > 0$.

4.3.2   The Effect on Carriers' Output

In this section I study how the transaction cost affects the carriers' output. In particular, I derive the demand of each quality q_i of aircraft, which I use to obtain the steady state distribution of owners. Each firm's output is zq_i, hence integrating over the steady state probability distribution $h_i(z)$ I obtain the total output produced by the industry. Thus, I can then evaluate the effect of trading frictions on industry output.

Each firm's demand for a specific quality of aircraft is determined jointly by its current productivity and by the aircraft it owns. Two motives push firms to trade aircraft. The first is the depreciation of the aircraft: for any level of z, a firm obtains higher profits if it operates a q_1 aircraft, and, due to complementarity between quality and efficiency, this force is stronger for the highest efficiency firms. Hence, highest efficiency firms upgrade their aircraft when it depreciates. The second is the stochastic nature of productivity: a firm receiving a bad draw of z might find optimal to sell the aircraft it owns and exit. Viceversa, a high enough new draw of z induces a firm

to replace its current aircraft with a higher quality one.

The derivation of demand for each quality q_i is a bit tedious, but I report it because it helps understand the differential role that depreciation and stochastic productivity have on the equilibrium. Moreover, it is also instrumental to the derivation of the steady state probability distribution $h_i(z)$ of firms' efficiencies z.

Owners of q_1 aircraft - A mass X_1 owns a q_1 aircraft. Of these:

- a fraction α receive a new efficiency draw. A fraction $F(z_2(1))$ of them has a draw of the efficiency parameter z below the selling threshold $z_2(1)$ so they sell their q_1 aircraft and exit. A fraction $F(z_1(1)) - F(z_2(1))$ sell their q_1 aircraft and buy a q_2 aircraft. The complementary fraction $1 - F(z_1(1))$ keep the q_1 aircraft;

- a fraction γ_1 has the same efficiency as the previous period and their aircraft does depreciate. Firms $z \in [z_2(2), z_1(1)]$ keep the q_2 aircraft (since no firms below $z_1(1)$ has a q_1 aircraft) and firms $z \in [z_1(2), \overline{z}]$ replace the q_1 aircraft. Under the assumptions on transaction costs, no firm exits. Weighted by their relative frequency, the fraction $H_1(z_1(2))$ keep the q_2 aircraft and the fraction $1 - H_1(z_1(2))$ buys a q_1 aircraft, where $H_i(z)$ is the steady state cumulative distribution of firms with q_i aircraft, which will be derived below;

- a fraction $(1 - \alpha - \gamma_1)$ has the same efficiency as before and the aircraft does not depreciate. By optimality of the previous decision, they keep the aircraft.

Owners of q_2 aircraft - A mass X_2 owns a q_2 aircraft. Of these:

- a fraction α receives a new efficiency draw and their aircraft does not depreciate. A fraction $F(z_2(2))$ of them has a draw of the efficiency parameter z below the selling threshold $z_2(2)$ so they sell their q_2 aircraft and exit. A fraction $F(z_1(2)) - F(z_2(2))$ keeps the q_2 aircraft. The complementary fraction $1 - F(z_1(2))$ sells the q_2 aircraft and buys a q_1 aircraft;

- a fraction γ_2 has the same efficiency as the previous period and their aircraft dies. Firms $z < z_2(3)$ exit; firms $z \in [z_2(3), z_1(3)]$ buy a q_2 aircraft and firms $z \in [z_1(3), \overline{z}]$ buy a q_1 aircraft. Weighted by their relative frequency, a fraction $H_2(z_2(3))$ exits; the fraction $H_2(z_1(3)) - H_2(z_2(3))$ buys a q_2 aircraft and the fraction $1 - H_2(z_1(3))$ buys a q_1 aircraft, where $H_2(z)$ is the steady state distribution of firms with q_2 aircraft, which I derive below;

- a fraction $(1 - \alpha - \gamma_2)$ has the same efficiency as before and the aircraft does not depreciate. By optimality of the previous decision, they keep the aircraft.

Firms with no aircraft - A mass $1 - X$ of firms has no aircraft. Of these:

- a fraction α receives a new efficiency draw. A fraction $F(z_2(3))$ does not buy any aircraft, a fraction $F(z_1(3)) - F(z_2(3))$ buys a q_2 aircraft, a fraction $1 - F(z_1(3))$ buys a q_1 aircraft;

- the complementary fraction $1 - \alpha$ has the same efficiency as before. By optimality of the previous decision, they keep the aircraft.

In equilibrium, the demand for each quality q_i aircraft must equal to the total supply X_i. For q_1 aircraft, this requires that

$$
\begin{aligned}
X_1 \;=\; & X_1 \left(\alpha \left(1 - F \left(z_1 \left(1 \right) \right) \right) + \gamma_1 \left(1 - H_1 \left(z_1 \left(2 \right) \right) \right) + 1 - \alpha - \gamma_1 \right) + \\
& X_2 \left(\alpha \left(1 - F \left(z_1 \left(2 \right) \right) \right) + \gamma_2 \left(1 - H_2 \left(z_1 \left(3 \right) \right) \right) \right) + \\
& \left(1 - X \right) \alpha \left(1 - F \left(z_1 \left(3 \right) \right) \right)
\end{aligned} \tag{9}
$$

For q_2 aircraft, this requires that

$$
\begin{aligned}
X_2 \;=\; & X_1 \left(\alpha \left(F \left(z_1 \left(1 \right) \right) - F \left(z_2 \left(3 \right) \right) \right) + \gamma_1 H_1 \left(z_1 \left(2 \right) \right) \right) + \\
& X_2 \left(\alpha \left(F \left(z_1 \left(2 \right) \right) - F \left(z_2 \left(2 \right) \right) \right) + \gamma_2 \left(H_2 \left(z_1 \left(3 \right) \right) - H_2 \left(z_2 \left(3 \right) \right) \right) + 1 - \alpha - \gamma_2 \right) + \\
& \left(1 - X \right) \alpha \left(F \left(z_1 \left(3 \right) \right) - F \left(z_2 \left(3 \right) \right) \right)
\end{aligned} \tag{10}
$$

while the mass of firms with no aircraft must satisfy

$$
1 - X = \left(1 - X \right) \left(\alpha F \left(z_2 \left(3 \right) \right) + 1 - \alpha \right) + X_1 \alpha F \left(z_2 \left(3 \right) \right) + X_2 \left(\alpha F \left(z_2 \left(2 \right) \right) + \gamma_2 H_2 \left(z_2 \left(3 \right) \right) \right) \tag{11}
$$

Definition 3 *An equilibrium is a pair of prices* $\left(p_1^*, p_2^* \right)$ *such that*

1. *Firms maximize their value, i.e. equations (1), (2), (3), (4), (5) and (6) are satisfied.*

2. *Markets for each quality* q_i *clear, i.e. equations (9) and (10) are satisfied.*

From equations (10) and (9), I can construct the steady state probability distribution function of firms with a q_i aircraft. It corresponds to

$$
h_2 \left(z \right) = \begin{cases}
\alpha f \left(z \right) + \left(1 - \alpha - \gamma_2 \right) h_2 \left(z \right) & \text{for } z_2 \left(2 \right) \leq z < z_2 \left(3 \right) = z_2 \left(1 \right) \\
\alpha f \left(z \right) + \left(1 - \alpha - \gamma_2 \right) h_2 \left(z \right) + \frac{1-X}{X_2} \alpha f \left(z \right) + \frac{X_1}{X_2} \alpha f \left(z \right) + \gamma_2 h_2 \left(z \right) & \text{for } z_2 \left(3 \right) \leq z < z_1 \left(1 \right) \\
\alpha f \left(z \right) + \left(1 - \alpha - \gamma_2 \right) h_2 \left(z \right) + \frac{1-X}{X_2} \alpha f \left(z \right) + \frac{X_1}{X_2} \gamma_1 h_1 \left(z \right) + \gamma_2 h_2 \left(z \right) & \text{for } z_1 \left(1 \right) \leq z < z_1 \left(3 \right) \\
\alpha f \left(z \right) + \left(1 - \alpha - \gamma_2 \right) h_2 \left(z \right) + \frac{X_1}{X_2} \gamma_1 h_1 \left(z \right) & \text{for } z_1 \left(3 \right) \leq z \leq z_1 \left(2 \right)
\end{cases}
$$

for a q_2 aircraft and

$$
h_1 \left(z \right) = \begin{cases}
\alpha f \left(z \right) + \left(1 - \alpha - \gamma_1 \right) h_1 \left(z \right) & \text{for } z_1 \left(1 \right) \leq z < z_1 \left(3 \right) \\
\alpha f \left(z \right) + \left(1 - \alpha - \gamma_1 \right) h_1 \left(z \right) + \frac{X_2}{X_1} \gamma_2 h_2 \left(z \right) + \frac{1-X}{X_1} \alpha f \left(z \right) & \text{for } z_1 \left(3 \right) \leq z < z_1 \left(2 \right) \\
\alpha f \left(z \right) + \left(1 - \alpha - \gamma_1 \right) h_1 \left(z \right) + \frac{1-X}{X_1} \alpha f \left(z \right) + \frac{X_2}{X_1} \alpha f \left(z \right) + \gamma_1 h_1 \left(z \right) & \text{for } z \geq z_1 \left(2 \right)
\end{cases}
$$

for a q_1 aircraft. Combining the two distributions and using $\frac{X_2}{X_1} = \frac{\gamma_1}{\gamma_2}$, we obtain

$$
h_2(z) = \begin{cases} \frac{\alpha f(z)}{\alpha+\gamma_2} & \text{for } z_2(2) \leq z < z_2(3) = z_2(1) \\ \frac{f(z)}{X_2} & \text{for } z_2(3) \leq z < z_1(1) \\ f(z)\left(\frac{1-X_1}{X_2}\right) + \frac{\gamma_2 f(z)}{\alpha+\gamma_1} & \text{for } z_1(1) \leq z < z_1(3) \\ f(z) + \frac{\frac{1-X}{X_1}f(z)\gamma_2}{(\alpha+\gamma_2+\gamma_1)} & \text{for } z_1(3) \leq z \leq z_1(2) \end{cases}
\tag{12}
$$

and

$$
h_1(z) = \begin{cases} \frac{\alpha f(z)}{\alpha+\gamma_1} & \text{for } z_1(1) \leq z < z_1(3) \\ f(z) + \frac{\frac{1-X}{X_1}f(z)(\alpha+\gamma_2)}{\alpha+\gamma_2+\gamma_1} & \text{for } z_1(3) \leq z < z_1(2) \\ \frac{f(z)}{X_1} & \text{for } z \geq z_1(2) \end{cases}
\tag{13}
$$

I have already highlighted that the transaction cost acts as a barrier. As a result of this barrier, the transaction cost leads to a misallocation of capital. Efficiency requires that aircraft are operated by the most productive firms only. But since $z_1(2) > z_1(3) > z_1(1)$ and $z_2(1) = z_2(3) > z_2(2)$, this implies that some firms operating a q_2 aircraft are less productive than some firms that are not operating any aircraft, and some firms operating a q_1 aircraft are less productive than some firms operating a q_2 aircraft. The total industry output loss due to the transaction costs is

$$
\int_{z_1}^{\overline{z}} sq_1 dF(s) + \int_{z_2}^{z_1} sq_2 dF(s) - X_1 \int_{z_1(1)}^{\overline{z}} sq_1 dH_1(s) - X_2 \int_{z_2(2)}^{z_1(2)} sq_2 dH_2(s)
$$

which is always larger than zero when $T > 0$.

The next proposition shows that the loss in output can be attributed entirely to the misallocation of q_1 aircraft and, even though the distribution $h_2(z)$ has a smaller lower bound when transaction costs are positive, an average q_2 aircraft produces more output when trading is subject to frictions, provided that the difference $q_1 - q_2$ is not too large.

Proposition 4 *The average output produced by q_1 aircraft is lower when there are transaction costs. The average output produced by a q_2 aircraft is higher when there are transaction costs, provided $q_1 > q_2/2$.*

Proof. See Appendix. ∎

4.3.3 The Effect on Volume of Trade and Turnover of Carriers

The previous derivation of the demand function can be used to construct the steady state fraction of firms who trade their aircraft and the industry exit ad entry rate. Aggregating the behavior of each firm, the total volume of trade - the fraction of units traded - is given by the sum of the firms

replacing their aircraft and the firms with no aircraft that buy an aircraft, normalized by the total mass of aircraft. It corresponds to

$$\frac{X_1\gamma_1\left(1 - H_1\left(z_1\left(2\right)\right)\right) + X_2\alpha\left(1 - F\left(z_1\left(2\right)\right)\right) + X_2\gamma_2\left(1 - H_2\left(z_1\left(3\right)\right)\right)}{X} +$$

$$\frac{X_2\alpha F\left(z_2\left(2\right)\right) + \left(1 - X\right)\alpha\left(1 - F\left(z_1\left(3\right)\right)\right)}{X} \qquad (14)$$

The first term is given by firms who replace the q_1 aircraft they were operating in the previous period who just depreciated. The second term is given by firms who owned a q_2 aircraft who now upgrade to a q_1 aircraft because they had a high draw of productivity. The third term corresponds to highly efficient firms whose q_2 aircraft just died. The forth term is given by firms with a q_2 aircraft that exit after a bad z draw. The last term is given by the firms with no aircraft that now acquire a q_1 aircraft following a good z draw. Note that some firms trading an aircraft are not reported, as including them would double count trades. For example, a firm with no aircraft that acquire a q_2 aircraft is the buying side of the firms selling a q_1 aircraft included in equation (14).

Similarly, we obtain the exit rate

$$\frac{X_1\alpha F\left(z_2\left(3\right)\right) + X_2\alpha F\left(z_2\left(2\right)\right) + X_2\gamma_2 H_2\left(z_2\left(3\right)\right)}{X} \qquad (15)$$

and the entry rate

$$\frac{\left(1 - X\right)\alpha\left(1 - F\left(z_2\left(3\right)\right)\right)}{X} \qquad (16)$$

In steady state the entry and exit rate are equal and they measure the turnover of firms. The larger the transaction cost the lower the volume of trade and the turnover of carriers.

4.4 Comparative Statics

In this section I analyze how the equilibrium is affected by changes in the primitive parameters of the model. I argued in the Introduction that the Airline Deregulation Act of 1978 increased the volatility of carriers' output through an increase in competition and this section explore what are the effects of this increase in volatility - as captured by the parameter α - on the equilibrium.

The next proposition illustrates how the behavior of the thresholds is influenced by the primitives of the model:

Proposition 5 *The difference between the buying $z_i\left(3\right)$ thresholds and the selling $z_i\left(i\right)$ thresholds is*

(i) *higher, the higher is the transaction cost:* $\frac{\partial\left(z_i\left(3\right) - z_i\left(i\right)\right)}{\partial T} > 0$;

(ii) *higher, the higher is the probability that the aircraft depreciates:* $\frac{\partial\left(z_i\left(3\right) - z_i\left(i\right)\right)}{\partial\gamma_i} > 0$;

(iii) *higher, the higher is the volatility of firms' efficiency:* $\frac{\partial\left(z_i\left(3\right) - z_i\left(i\right)\right)}{\partial\alpha} > 0$.

Proof. Using the indifference condition

$$V_2\left(z_2\left(3\right)\right) - p_2 = V_3 = V_2\left(z_2\left(2\right)\right) - p_2 + T$$

we obtain

$$\frac{\left(z_2\left(3\right) - z_2\left(2\right)\right)q_2}{1 - \beta\left(1 - \alpha - \gamma_2\right)} = T \tag{17}$$

Using the indifference conditions

$$
\begin{aligned}
V_1\left(z_1\left(3\right)\right) - p_1 &= V_2\left(z_1\left(3\right)\right) - p_2 \\
V_1\left(z_1\left(1\right)\right) &= V_2\left(z_1\left(1\right)\right) - p_2 + p_1 - T
\end{aligned}
$$

we have

$$\frac{\left(z_1\left(3\right) - z_1\left(1\right)\right)\left(q_1 - q_2\right)}{1 - \beta\left(1 - \alpha - \gamma_1\right)} = T \tag{18}$$

Therefore, the three claims of the Proposition follow trivially by inspection of equations (17) and (18). ∎

Identical results also holds for the difference between the trading threshold $z_1\left(2\right)$ and the buying threshold $z_1\left(3\right)$. The results are largely intuitive. The effect of a larger transaction cost is to increase the barrier to entry and the barrier to exit. Thus, the difference $z_i\left(3\right) - z_i\left(i\right)$ increases.

The effect of an increase in the volatility α can be easily understood making reference to the literature of investment under uncertainty (e.g. Dixit and Pindyck, 1994). The transaction cost T is a sunk investment. Raising α raises the volatility of this investment. This increases the option value of inaction and it translates into wider bands $z_i\left(3\right) - z_i\left(i\right)$. The next Corollary shows more precisely that the larger is α, the larger is the z draw before a firm acquire an aircraft and the smaller is the z draw before selling it, for a sufficiently high discount factor β.[14]

Corollary 6 *For any $\left(T, \alpha, \gamma_1, \gamma_2\right)$, there exists a β^* such that for $\beta > \beta^*$ the buying threshold is higher and the selling threshold is lower, the higher is the volatility of firms' efficiency: $\frac{\partial z_i\left(3\right)}{\partial \alpha} > 0 > \frac{\partial z_i\left(i\right)}{\partial \alpha}$.*

Proof. See Appendix. ∎

The inaction due to higher volatility has direct consequences on the total output of the industry. Since the band $z_1\left(3\right) - z_1\left(1\right)$ widens as volatility increases, this implies that the marginal and average productivity of carriers that operate a q_1 aircraft decreases. Thus, the output produced by a q_1 aircraft decreases as α increases. The effect on the total output produced by a quality q_2 aircraft is ambiguous, as two opposite forces operates. As volatility increases, the lower bound $z_2\left(2\right)$ of the $H_2\left(z\right)$ distribution decreases and the upper bound $z_2\left(1\right)$ increases, so the effect on the efficiency of the average operator of a q_2 carrier is ambiguous. Intuitively, when the quality difference $q_1 - q_2$

---

[14]Extensive numerical analysis shows that for any $\left(T, \alpha, \gamma_1, \gamma_2\right)$, the result holds for β below .5

is high, high efficiency carriers replace their aircraft more frequently, lowering the $z_2(1)$ threshold. As a result, the average efficiency of a carrier operating a q_2 aircraft is low. On the other side, when the difference $q_1 - q_2$ is small, the advantage of replacing the q_1 aircraft that just depreciated to q_2 is small too, so the $z_2(1)$ threshold is high and the average efficiency of a carrier operating a q_2 aircraft is high too.

The next proposition summarizes this discussion, highlighting that the composite effect on total industry output is unambiguous: total output decreases as volatility increases.

Proposition 7 *There exists a $\widetilde{\beta}$ such that for $\beta > \widetilde{\beta}$ a higher volatility affects industry output in the following way:*

(i) *it decreases the total output produced by firms with q_1 aircraft;*

(ii) *it increases the total output produced by firms with q_2 aircraft when $q_2 > \frac{q_1}{2}$;*

(iii) *it decreases total industry output.*

Proof. See Appendix ∎

4.5   Leasing

Figure 2 documents the importance of leasing in the market for aircraft. In this section I introduce a stylized lessor that offers aircraft for lease to understand the way leasing affects the trade of aircraft and industry output. Moreover, this section constitutes a fundamental guide for the empirical analysis performed later in the paper.

The analysis is based on a fundamental assumption, namely that the transaction cost on leased aircraft is lower than the transaction cost on owned units. In Section 3 I reported a large lessor's description of its business activity. It highlighted its role as an intermediary. It seems then natural to assume that lessors have a cost advantage in trading aircraft compared to a carrier whose main activity is to provide passenger transportation. Moreover, all results derived in this section depend only on transaction costs being lower on leased units, so I assume without loss of generality that transaction costs on leased units are zero.

Assume now that a lessor acquires aircraft at the market price p_i and offers them at a rental rate r_i per period. I start by assuming that the lessor offers quantities $0 < X_{L_1} < X_1$ and $0 < X_{L_2} < X_2$ of aircraft to derive the demand for leased aircraft. I then determine the quantities X_{L_1} and X_{L_2} that maximizes the lessor's profits.

Let X_{L_1} and X_{L_2} denote the quantity of q_1 and q_2 aircraft available for lease, respectively. For firms that choose to lease, the problem becomes static and a necessary condition for leasing is

$$zq_i \geq r_i$$

Consider the problem of a firm choosing a q_2 aircraft. The value of following the policy of leasing is

$$V_{L_2}(z) = \frac{zq_2 - r_2 + \beta\alpha J_L}{1 - \beta(1 - \alpha)}$$

where

$$J_L = \int \max\{V_{L_1}(z), V_{L_2}(z), V_1(z) - p_1, V_2(z) - p_2, V_3(z)\} dF(z)$$

Hence leasing is preferred to owning if and only if

$$V_{L_2}(z) \geq V_2(z) - p_2 = \frac{zq_2 + \beta\alpha J_2 - \beta\gamma_2 p_2}{1 - \beta(1 - \alpha)} - p_2 \qquad (19)$$

where now J_2 is

$$J_2 = \int \max\{V_{L_1}(z) + p_2 - T, V_{L_2}(z) + p_2 - T, V_1(z) - p_1 + p_2 - T, V_2(z), V_3(z) + p_2 - T\} dF(z)$$

Substituting for $V_{L_2}(z)$, equation (19) reads as

$$\frac{zq_2 - r_2 + \beta\alpha J_L}{1 - \beta(1 - \alpha)} \geq \frac{zq_2 + \beta\alpha J_2 - \beta\gamma_2 p_2}{1 - \beta(1 - \alpha)} - p_2$$

which is independent of z. Hence either all firms prefer leasing versus selling, or prefer selling versus leasing or are exactly indifferent between the two alternatives. Clearly, if $0 < X_{L_2} < X_2$ then in equilibrium all these firms are indifferent between leasing and owning. This implies that the entry threshold $z_2(3)$ is the same for firms that in equilibrium lease and for firms that buy an aircraft.

The problem for the firms that choose a q_1 aircraft is slightly more complicated, since these firms are partitioned and some firms prefer to replace their owned aircraft when it depreciates, while other firms prefer to keep a q_2 aircraft.

The value of following the policy of leasing a q_1 aircraft is

$$V_{L_1}(z) = \frac{zq_1 - r_1 + \beta\alpha J_L}{1 - \beta(1 - \alpha)}$$

Hence leasing is preferred to owning if and only if

$$V_{L_1}(z) \geq V_1(z) - p_1 \qquad (20)$$

Consider firms that would choose to replace the aircraft, firms for whom $\max\{V_1(z) - p_1 + p_2 - T, V_2(z)\} = V_1(z) - p_1 + p_2 - T$. Equation (20) corresponds to

$$\frac{zq_1 - r_1 + \beta\alpha J_L}{1 - \beta(1 - \alpha)} \geq \frac{zq_1 + \beta\alpha J_1 + \beta\gamma_1(-p_1 + p_2 - T)}{1 - \beta(1 - \alpha)} - p_1$$

which is independent of z. Hence either these firms prefer leasing versus selling, or prefer selling versus leasing or are exactly indifferent between the two alternatives.

Consider now firms that would choose to keep the owned aircraft that just depreciated from q_1 to q_2. These are firms for which $\max\{V_1(z) - p_1 + p_2 - T, V_2(z)\} = V_2(z)$. Equation (20) corresponds to

$$\frac{zq_1 - r_1 + \beta\alpha J_L}{1 - \beta(1 - \alpha)} \geq \frac{zq_1 + \beta\alpha J_1 + \beta\gamma_1 V_2(z)}{1 - \beta(1 - \alpha - \gamma_1)} - p_1$$

where

$$V_2(z) = \frac{zq_2 + \beta\alpha J_2 + \beta\gamma_2(V_1(z) - p_1)}{1 - \beta(1 - \alpha - \gamma_2)}$$

Note that if the firms that would choose to replace the owned aircraft prefer owning to leasing (or are indifferent between the two), then firms that would keep the depreciated aircraft strictly prefer to own, as can be seen by

$$\frac{zq_1 - r_1 + \beta\alpha J_L}{1 - \beta(1 - \alpha)}$$
$$\leq \frac{zq_1 + \beta\alpha J_1 + \beta\gamma_1(V_1(z) - p_1 + p_2 - T)}{1 - \beta(1 - \alpha - \gamma_1)} - p_1$$
$$< \frac{zq_1 + \beta\alpha J_1 + \beta\gamma_1 V_2(z)}{1 - \beta(1 - \alpha - \gamma_1)} - p_1$$

Therefore, there exists a firm $z_1^*(3)$ such that all firms $z \geq z_1^*(3)$ lease a q_1 aircraft and firms $z < z_1^*(3)$ prefer to own. The marginal firm is always indifferent between the two options. Hence, firms choosing q_1 aircraft self select, with only the most efficient choosing leased aircraft. A lower transaction cost allows them to replace the aircraft at no cost. Moreover, due to the complementarity between quality q_i and efficiency, the gain from a q_1 aircraft is increasing in z and therefore only the most productive firms lease q_1 aircraft.

Equilibrium thus requires that

1. All firms acquiring old q_2 aircraft are indifferent between leasing and owning

$$V_2(z) - p_2 = V_{L_2}(z) \tag{21}$$

   and the marginal firms satisfies

$$V_2(z_2^L(3)) - p_2 = V_{L_2}(z_2^L(3)) = V_3$$

2. The marginal firms selling old q_2 aircraft satisfy

$$V_2(z_2^L(2)) = V_3 + p_2 - T$$

3. The marginal firm acquiring a new q_1 aircraft satisfies

$$V_1(z_1^L(3)) - p_1 = V_2(z_1^L(3)) - p_2$$

22

4. The marginal firms selling new q_1 aircraft satisfy

$$V_1\left(z_1^L\left(1\right)\right) = V_2\left(z_1^L\left(1\right)\right) - p_2 + p_1 - T$$

5. The marginal firms selling old q_2 aircraft and buying new q_1 aircraft satisfy

$$V_1\left(z_1^L\left(2\right)\right) - p_1 + p_2 - T = V_2\left(z_1^L\left(2\right)\right)$$

6. There exists a firm $z_1^*\left(3\right)$ which is exactly indifferent between buying a new q_1 aircraft or leasing it. All firms with efficiency higher than $z_1^*\left(3\right)$ lease and the firms with efficiency lower than $z_1^*\left(3\right)$ own.

$$V_1\left(z_1^*\left(3\right)\right) - p_1 = V_{L_1}\left(z_1^*\left(3\right)\right) \tag{22}$$

7. Demand of new q_1 and old q_2 aircraft equates supply[15]

$$
\begin{aligned}
X_1 \ = \ & \left(X_1 - X_{L_1}\right)\left(\alpha\left(1 - F\left(z_1^L\left(1\right)\right)\right) + \gamma_1\left(1 - H_1\left(z_1^L\left(2\right)\right)\right) + 1 - \alpha - \gamma_1\right) && (23) \\
& + X_{L_1}\left(\alpha\left(1 - F\left(z_1^*\left(3\right)\right)\right) + 1 - \alpha\right) \\
& + \left(X_2 - X_{L_2}\right)\left(\alpha\left(1 - F\left(z_1^L\left(2\right)\right)\right) + \gamma_2\left(1 - H_2\left(z_1^L\left(3\right)\right)\right)\right) + X_{L_2}\left(\alpha\left(1 - F\left(z_1^L\left(3\right)\right)\right)\right) \\
& + \left(1 - X_1 - X_2\right)\alpha\left(1 - F\left(z_1^L\left(3\right)\right)\right)
\end{aligned}
$$

and demand of old q_2 aircraft equates supply

$$
\begin{aligned}
X_2 \ = \ & \left(X_1 - X_{L_1}\right)\left(\alpha\left(F\left(z_1^L\left(1\right)\right) - F\left(z_2^L\left(3\right)\right)\right) + \gamma_1 H_1\left(z_1^L\left(2\right)\right)\right) && (24) \\
& + X_{L_1}\left(\alpha\left(F\left(z_1^L\left(3\right)\right) - F\left(z_2^L\left(3\right)\right)\right)\right) \\
& + \left(X_2 - X_{L_2}\right)\left(\alpha\left(F\left(z_1^L\left(2\right)\right) - F\left(z_2^L\left(2\right)\right)\right) + \gamma_2\left(H_2\left(z_1^L\left(3\right)\right) - H_2\left(z_2\left(3\right)\right)\right) + 1 - \alpha - \gamma_2\right) \\
& + X_{L_2}\left(\alpha\left(F\left(z_1^L\left(3\right)\right) - F\left(z_2^L\left(2\right)\right)\right) + 1 - \alpha\right) \\
& + \left(1 - X_1 - X_2\right)\alpha\left(F\left(z_1^L\left(3\right)\right) - F\left(z_2^L\left(3\right)\right)\right)
\end{aligned}
$$

where $H_i\left(z\right)$ is the steady state distribution of firms owning an aircraft, which can be derived in an analogous way as it was derived in Section 4.3.2.

I can now evaluate the effect of leasing on the volume of trade and output.

Proposition 8 *Leased aircraft trade more frequently and have an average holding duration shorter than owned aircraft.*

---

[15]I have implicitly excluded the possibility that a firm that owns a new q_1 aircraft and receives a new draw of productivity high enough, sells the new q_1 aircraft in order to lease a new q_1 aircraft. This is obviously not the case when all firms leasing q_1 aircraft are indifferent between leasing and owning, but could in principle arise if T is very small and X_{L_1} is very close to X_1.

Proof. See Appendix ∎

The intuition for the proposition is straightforward. I highlighted throughout the exposition that the transaction cost is a barrier to sell and it is then intuitive that units with lower transaction cost - leased aircraft - trade more frequently and have a shorter holding duration than owned units.

The most important consequence of lower transaction costs is its effect on output. The next proposition and its corollary show that leasing has a positive effect on output. Moreover, the proposition constitutes an important guide in the empirical analysis as it shows that

Proposition 9 *The average output of a new q_1 leased aircraft is higher than the average output of a new q_1 owned aircraft. The average output of an old q_2 leased aircraft is lower than the average output of an old q_2 owned aircraft, provided $q_1 > q_2/2$*

Proof. See Appendix ∎

However, the gain obtained on new units dominates:

Corollary 10 *Total industry output increases with the share of leased aircraft.*

Proof. See Appendix. ∎

Leasing improves the matching between the quality of the aircraft and the productivity of the firm and this results in higher output. The effect of matching can be decomposed in two distinct forces. The first is a *productivity effect.* Lessees are on average more productive than owners. This is the sum of two distinct effects. The first is a selection effect, as very productive firms choose to lease because leasing allows them to replace their q_1 aircraft at lower costs when it depreciates. The second effect is given by a lower barrier to sell, which implies that the least efficient firm that owns a q_2 aircraft is less efficient than the least efficient firm that lease a q_2 aircraft, since the option value of not selling the aircraft increases as the transaction cost rises. In this sense, the transaction cost acts here as, for example, the cost of firing labor acts in the general equilibrium model of Hopenhayn and Rogerson (1993). Empirically, the distribution of efficiency of lessees of aircraft stochastically dominates the distribution of efficiency of owners.

The second effect is a *pairing effect.* Conditional on operating an aircraft, lower transaction costs allow firms to select more precisely the quality of the aircraft based on their productivity. This increases output since the most productive firms always operate the best aircraft. Note that Proposition 9 tells us that this gain arises entirely from quality q_1 aircraft, as pairing implies that leased q_2 aircraft produce a lower output than owned q_2 aircraft. Empirically, the covariance between the quality of the aircraft and the productivity of the carrier is higher on leased aircraft than for owned ones.

4.5.1 Lessor's Optimal Quantity

I now discuss briefly the lessor's choice of quantity X_L. The analysis is mainly performed through a numerical analysis, but it highlights different forces at work.

The lessor acquires aircraft at the market price p_i and rents them at a rental rate r_i. His per-period profits, net of any fixed cost of operation (if any), are

$$\pi_L = \sum_{i=1,2} (r_i - (p_i - \beta (1 - \gamma_i) p_i - \beta \gamma_i p_{i+1})) X_{L_i} \qquad (25)$$

where $p_3 = 0$. On each leased unit X_{L_i}, the lessor's revenue is equal to the rental rate r_i. His cost is given by the implicit rental rate on ownership - where the discount factor is adjusted to take into account that with probability γ_i each aircraft depreciates.

The optimal quantity of aircraft X_{L_i} offered by the lessor must maximize his per-period profits because the decision to lease is static. Hence the lessor solves

$$\max_{r_1, r_2, X_{L_1}, X_{L_2}} \sum_{i=1,2} (r_i - (p_i - \beta (1 - \gamma_i) p_i - \beta \gamma_i p_{i+1})) X_{L_i} \qquad (26)$$

subject to the demands (23) and (24) and the price conditions (21) and (22). Moreover, we also need to impose the condition that

$$X_2 - X_{L_2} \geq (X_1 - X_{L_1}) \gamma_1 H_1 (z_1 (2)) + (X_2 - X_{L_2}) (1 - \alpha - \gamma_2) + (X_2 - X_{L_2}) \alpha (z_1 (2) - z_2 (2))$$

which simply says that the mass of old q_2 aircraft that are owned must be at least as large as the mass of new q_1 aircraft that depreciate and whose owner prefer not to replace, plus the q_2 aircraft owned by carriers that either are in the same condition as the previous period, or simply received a new draw of efficiency such that it is optimal to keep the q_2 aircraft.

The problem cannot be solved analytically. As any monopolist, the lessor trades off a higher markup and a higher quantity. I present in Figure 4 the results of a numerical simulation and a comparative static with respect to the parameter α. We observe that profits and rental rates r_i increase monotonically in α, while total aircraft for lease X_{L_i} and the resulting prices p_i decreases monotonically in α.

I believe two facts are worth highlighting. The first is the change in profits due to an increase in volatility. Figure 2 showed that until the early 80s, leasing was almost nonexistent. Suddenly, around 1984 lessors started to acquire a large number of aircraft in the primary markets and the leasing business started. Note that this is just a few years after the Airline Deregulation Act of 1978 and it seems natural to associate the increase in competition with an increase in volatility and an increase in the turnover of firms. Hence, in terms of my model, the increase in competition makes the leasing business more profitable and this might help explain why it started exactly after the Deregulation.

The second important aspect I want to stress is the effect of leasing on prices of q_1 aircraft. One particular feature of the aircraft leasing business is that the largest firms involved are not the manufacturers of the aircraft, even though in recent years Boeing and Airbus have established their trading units. Note however that the bottom left panel of Figure 4 shows that the prices of
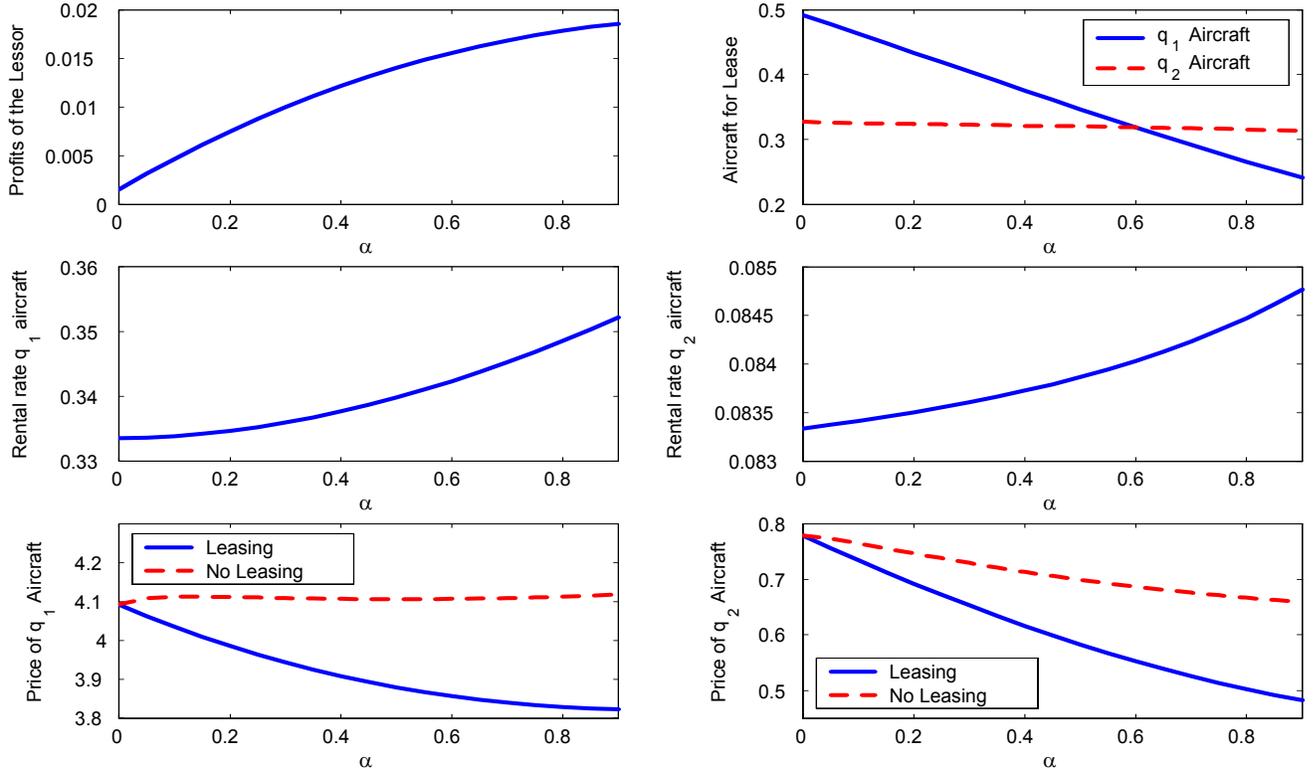
FIG. 4 - PROFITS OF THE LESSOR, AIRCRAFT FOR LEASE, RENTAL RATES AND PRICES OF AIR-CRAFT. BASELINE PARAMETERS ARE $x = .02$, $\gamma_1 = .04$, $\gamma_2 = .06$, $\beta = .95$, $q_1 = 1$, $q_2 = .5$, $T = .08$.

q_1 aircraft when a lessor offers aircraft for lease are everywhere below the level that would ensue absent the lessor. Although I am not modelling the production of aircraft, this might shed some light on the (non) incentive for the manufacturers to offer aircraft for lease.

4.6 Factor Hoarding

One important feature of the airline market is that frequently aircraft are parked in the desert. The purpose of this section is to show how the model can easily be extended to capture this relevant feature of the data. For simplicity, I consider the case in which $q_2 = 0$, so that there are only q_1 aircraft available.

I add two features to the model. The first is a fixed cost of operation f that the firm pays if it operates the aircraft. In the specific case of airlines, this might be the rental cost of an airport gate, for example. The second is a temporary shock that each firm receives with probability μ.

The analysis shows that, when hit by a temporary shock, some firms prefer not to sell the aircraft but keep it inactive to operate it in the future. The relevance of this phenomenon is stronger the higher are transaction costs. In particular, the analysis shows that leased aircraft are parked less frequently than owned aircraft.

Assume that temporary shocks are i.i.d. across time and firms. When hit by a temporary shock the firm can produce only

$$y = bzq_1$$

where $0 \leq b < 1$ measures the fraction of potential output that a firm can produce when hit by the temporary shock. Since firms have a fixed cost if producing, now a firm might prefer not to produce and still keep the aircraft. Hence, for $a \in \{b, 1\}$ the firm per period revenue are

$$\pi = \max\{azq_1 - f, 0\}$$

Moreover, in order to guarantee that in equilibrium some aircraft are parked, I need to impose that the transaction cost is such that firms hit by the temporary shock do not sell the aircraft and buy it in the following period if they do not receive a temporary shock again. So assume that the transaction cost satisfies

$$T > p_1 - \beta p_1$$

For a firm with efficiency z and temporary shock $a \in \{b, 1\}$, the value of owning an aircraft reads

$$
V_1(z, a) = \frac{\max\{azq_1 - f, 0\} + \beta\alpha \int_{\underline{z}}^{\overline{z}} \max\{V_1(z, 1), V_3(z, 1) + p_1 - T\} dF(z)}{1 - \beta(1 - \alpha - \mu - \gamma_1)} +
$$
$$
\frac{\beta\gamma_1 \max\{V_1(z, 1) - p_1, V_3(z, 1)\} + \beta\mu \max\{V_1(z, b), V_3(z, b) + p_1 - T\}}{1 - \beta(1 - \alpha - \mu - \gamma_1)}
$$

so that

$$
\begin{aligned}
V_1(z, b) &= V_1(z, 1) - (zq_1 - f - \max\{bzq_1 - f, 0\}) \\
&= V_1(z, 1) - zq_1 + \max\{bzq_1, f\}
\end{aligned}
$$

The value of having no aircraft is

$$
\begin{aligned}
V_3(z, b) &= V_3(z, 1) = V_3(z) = V_3 \\
&= \frac{\beta\alpha \int_{\underline{z}}^{\overline{z}} \max\{V_1(z, 1) - p_1, V_3(z)\} dF(z) + \beta(1 - \alpha - \mu) \max\{V_1(z, 1) - p_1, V_3(z)\}}{1 - \beta(1 - \alpha)}
\end{aligned}
$$

Let $z_1(a, j)$, $a = b, 1$, be the efficiency of firms that are indifferent between acquiring (selling) an aircraft if $j = 3$ ($j = 1$) and not. As in the previous sections, we clearly still have that a firm that

is just indifferent between acquiring an aircraft and not is more efficient than a firm that is just indifferent between just selling the aircraft and not:

$$z_1(1,3) > z_1(1,1)$$

Furthermore, we also have that a firm hit by a temporary shock that is just indifferent between selling the aircraft and not is more efficient than a firm not hit by a temporary shock:

$$z_1(b,1) > z_1(1,1)$$

The indifferent firm satisfies

$$V_3(z_1(b,1),b) + p_1 - T = V_1(z_1(b,1),b) = V_1(z_1(b,1),1) - z_1(b,1)q_1 + \max\left\{bz^1(b,1)q_1, f\right\}$$

and since

$$T > p_1 - \beta p_1 > z_1(b,1)q_1 - \max\left\{bz^1(b,1)q_1, f\right\}$$

we have

$$z_1(1,3) > z_1(b,1) > z_1(1,1)$$

Moreover, in order for the model to simultaneously exhibit factor hoarding, positive aircraft prices and entry and exit of firms, we must have

$$z_1(1,1)q_1 > f > bz^1(b,1)q_1$$

In Appendix B.2 I derive the distribution of "long-run" efficiency z and the distribution of output. They are obtained in a similar fashion as those of Subsection 4.3.2, but now the two distributions differ because of the temporary shock. In particular, a positive mass of aircraft equal to $\frac{\alpha\mu\left(F\left(\frac{f}{bq_1}\right) - F(z_1(1,b))\right)}{\alpha+\gamma_1+\mu}$ is parked in every period.

When $0 < X_L < X$ aircraft are available for lease, the value of leasing is

$$V_L = \frac{zq - f - r_1 + \beta\alpha J_L + \beta\mu \max\{V_L, V_3, V_1 - p_1\}}{1 - \beta(1 - \alpha - \mu)}$$

while the value of owning is

$$
\begin{aligned}
V_1(z,1) &= \frac{zq_1 - f + \beta\alpha J_1}{1 - \beta(1 - \alpha - \mu - \gamma_1)} + \frac{\beta\gamma_1(V_1(z,1) - p_1) + \beta\mu V_1(z,b)}{1 - \beta(1 - \alpha - \mu - \gamma_1)} \\
&= \frac{zq_1 - f + \beta\alpha J_1}{1 - \beta(1 - \alpha)} - \frac{\beta\gamma_1 p_1 + \beta\mu(zq_1 - \max\{bzq_1, f\})}{1 - \beta(1 - \alpha)}
\end{aligned}
$$

The next proposition shows that this extension of the model highlights a new force for why firms might strictly prefer to lease. In this scenario, leasing allows the lowest productivity firms to avoid to park the aircraft when hit by a negative shock.

Proposition 11 *Assume $f > bz^1(1,3)q_1$. If the highest productivity firms lease, than lower productivity firms leases too.*

Proof. See Appendix ∎

Note that the proposition highlights a force towards selection that is exactly opposite to the force highlighted in Subsection 4.5. While in Subsection 4.5 high efficiency firms were leasing to replace the aircraft more easily, now low efficiency firms prefer to lease because they are more likely to park the aircraft when hit by a negative shock. Which force dominates depends critically on the relative magnitudes of the forces at work. If q_1 aircraft depreciate fast - high γ_1- or if the difference $q_1 - q_2$ is large with respect to the probability μ of receiving a negative shock, then we should expect high efficiency firms to lease more than low productivity firms. In the opposite case of μ very small and either large γ_1 or large $q_1 - q_2$ (or both), we should observe high efficiency firms more likely to own their aircraft than low efficiency firms. However, note that the observation on the efficiency holds only at the time of the acquisition of the aircraft and lower transaction costs on leased units always imply that firms at the bottom of the efficiency distribution are owners.

The next proposition shows an additional way to understand the impact of transaction costs on output. Lower transaction costs imply that fewer leased aircraft are parked.

Proposition 12 *Only owned aircraft are parked.*

Proof. Obviously, no firm leasing an aircraft keeps it parked, as it would increase profits (decrease losses) by returning the aircraft for one period and renting it the next period. However, some firms owning an aircraft might park it if $zq_1 - f < 0$. ∎

Hence, the proposition highlights a third force that pushes for leased aircraft to have a higher output than owned aircraft. I call this force the *parking effect*: a higher fraction of leased aircraft are utilized at any moment, while a higher fraction of owned aircraft is parked inactive.

4.7 Summary of Predictions

The model shows how the (in)efficiency of transactions translates into (in)efficiency of production. It illustrates the effects of leasing's lower trading frictions on the production of final output. In the next section, I present empirical evidence consistent with the implications of the model. The model predicts a direct link between the holding duration of each aircraft and the productivity of its operator. In particular, the model predicts the following. 1) Leased aircraft have shorter holding durations and trade more frequently than owned aircraft. 2) Leased aircraft produce a higher output than owned aircraft. This is the result of three effects. 2a) The *parking effect*: leased aircraft are parked less frequently. 2b) The *productivity effect*: conditional on the aircraft being in use, the efficiency of lessees is higher than the efficiency of owners. 2c) The *pairing effect*: conditional on the aircraft being in use, the covariance between quality of the aircraft and efficiency of the carrier is higher for leased aircraft.

The model highlights some forces that push carriers to self select into leasing and owning when not all units are leased. In particular, the model shows that low efficiency carriers prefer to lease because they are more likely to park the aircraft and high efficiency carriers prefer to lease in order to replace their aircraft when it depreciates. For reasons of tractability, I assumed that the stochastic process governing the evolution of efficiency and the transaction cost are common to all carriers. However, it is easy to extend the model to accommodate heterogeneous volatility and heterogeneous transaction costs. Similar comparative statics as those obtained in Subsection 4.4 hold, namely higher volatility firms and/or higher transaction cost firms have wider inaction bands, are on average less efficient and more willing to lease.[16] A higher volatility increases the probability of having to sell the aircraft and therefore of paying the transaction cost. Hence, higher volatility carriers are willing to pay more than lower volatility carriers to lease their aircraft.

In conclusion, several different forces point toward the selection of carriers into leasing. Examining carefully the importance of each factor is an interesting research question. In the empirical section, I also provide evidence showing that selection of high efficiency carriers into leasing being a very small factor, if not absent at all.[17] Hence, the empirical evidence suggests that the observed differences in output between leased and owned aircraft arise *because* lower barriers to trade improve the matching process between productive capital and efficient firms.

5   Empirical Evidence: Commercial Aircraft

In this Section I present empirical evidence of the main implications of the model using data from commercial aircraft. I first present the data used in the analysis. I then present some preliminary evidence consistent with the fact that transaction costs are significantly different between leased and owned aircraft. In particular, I compare holding durations, probability of parking the aircraft, probability of trading the aircraft and hours flown[18] between leased and owned aircraft. I next estimate the depreciation pattern of aircraft to obtain estimates of quality of capital and productivity of carriers. Based on these estimates, I quantify the relevance of the *parking,* the *productivity* and the *pairing effects.* I also evaluate the output gain that would ensue if pairing between quality of capital and productivity were perfect. In the concluding part of the Section, I discuss alternative explanations presenting additional evidence against them.

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[16]Note that in Subsection 4.4 all firms have the same volatility α and I examine the effect of the change in α for all carriers. Here I allow firms with different α to coexist. In particular, in this last case all carriers face the same prices p_i, while in the first case changing α changes the prices p_i too.

[17]Evidence from consumer durables such as cars indicates that high income individuals lease more than lower income individuals. However, the volatility of income does not appear to be an important determinant of consumers' decision to lease. See Hendel and Lizzeri (2002).

[18]I use hours flown to measure the output produced by a carrier operating an aircraft.

5.1 Data

The source of information used in this paper is ACAS, a detailed database for the aviation market compiled by AVSOFT, a producer of computer based aviation market information systems and safety management software. ACAS is organized in several different files that classify aircraft and operators according to different characteristics of interest. In my analysis I use one main file:

1. *Current Aircraft Datafile.* This file contains informations on the characteristics of each aircraft, such as the type (e.g. Boeing 747, Airbus A-340), the model (e.g. Boeing 747-200), the engine, the noise stage. It also contains information related to the utilization of the aircraft by the current operator, such as the delivery date, the annual flying hours and landings, the cumulative flying hours and landings with the current operator, the operational role (ex: per passenger transportation, freighter, corporate aircraft...). It specifies if the aircraft is owned by the current operator or leased, and in this case, the type of lease (Financial vs. Operating) and length of the contract. The reference date for this file is April 2003.

   I occasionally complement this main file with a second datafile:

2. *Time-series Utilization Datafile.* This file contains the flying hours and landings of each aircraft for each month since January 1990.

I focus my analysis on Widebody aircraft used for passenger transportation.[19,20] I keep in my sample only those aircraft operated by the same carrier in the last 12 months.[21]

5.2 Preliminary Evidence

Table 2 presents simple patterns. Leased aircraft represent approximately 20% of the observations in my sample. The top panel indicates that leased aircraft have higher output and a shorter holding

---

[19] The choice of Widebody aircraft is motivated by the choice of instruments in the estimation of the quality of the aircraft, as it will become clear in Subsection 5.3. I performed similar analysis on Narrowbody aircraft obtaining similar qualitative results. In particular, I found evidence consistent with the *parking, productivity* and *sorting effects.*

[20] The database classifies a number of aircraft as "for lease", meaning that they currently with the lessor. These aircraft are not included in my analysis, since lessors lease a large number of freighters too and the data do not allow me to distinguish between the two when the aircraft is with the lesssor. See Subsection 5.5 for a robustness check that takes into account this fact.

[21] This is because I measure each aircraft' output as the total hours flown in one year. The data report hours flown also for the last month and the last 3 months. The choice of annual output however reduces the impact of different seasonality for different carriers. One concern is that this selection might cause a bias similar to the one analyzed in the literature on the estimation of production function (e.g. Olley and Pakes, 1996) as a carrier might be less likely to trade a young aircraft, since the carrier could expect larger future returns for any level of the current productivity. However, in my case this concern seems minor for two reasons: 1) results reported in Table 4 show that age of the aircraft does not significantly predict the probability of sale; 2) I am employing instruments that are uncorrelated with productivity.

TABLE 2

SUMMARY STATISTICS

	TOTAL AIRCRAFT	HOURS FLOWN	DURATION	AGE
PANEL A:	ALL AIRCRAFT			
LEASED	579	3709 (1293)	6.19 (4.75)	8.90 (6.80)
OWNED	2269	3255 (1383)	9.46 (6.34)	10.31 (7.29)
PANEL B:	AIRCRAFT	WITH	HOURS	FLOWN>0
LEASED	565	3801 (1167)	6.16 (4.68)	8.61 (6.51)
OWNED	2124	3477 (1128)	9.07 (5.91)	9.51 (6.67)

Notes: Standard errors in parenthesis.

duration than owned ones, as predicted by the model. On average, leased aircraft fly 14% more hours, have a holding duration 36% shorter and are 14% younger than owned aircraft.

The bottom panel reports similar figures, averaging over aircraft that have positive flying hours and were not parked for the entire 12 months considered. Hours flown are 9.3% within this subsample - somewhat less than the 14% of the full sample, but still a considerable increase. Average duration and average age are both lower in this subsample compared to the full sample, indicating that carriers tend to park older aircraft when necessary. Moreover, note that the fraction of leased aircraft that have been parked ($(579 - 565)/579 = 2.4\%$) for the entire sample period is approximately 2.5 times lower than the fraction of owned aircraft parked ($(2269 - 2124)/2269 = 6.3\%$), as predicted by Proposition 12.

Holding durations - Figure 5 presents the empirical distribution of the current holding durations.[22] The dotted line represents owned aircraft, while the solid line represents leased aircraft. It is readily apparent that on average the holding duration of owned aircraft is higher than the duration of leased aircraft, consistent with theoretical predictions of Proposition 8. A standard Kolmogorov-Smirnov test of the equality of distributions rejects the null hypothesis of equal distributions at the 1% level (the asymptotic p-value is equal to $3.2465 * 10^{-27}$). Moreover, I also test for first order stochastic dominance applying the non-parametric procedures proposed by Davidson and Duclos (2000) and Barrett and Donald (2003). Both tests accept the null hypothesis that the distribution of holding durations of owned aircraft first order stochastically dominates the distri-

---

[22]Hence, all durations are right-censored.
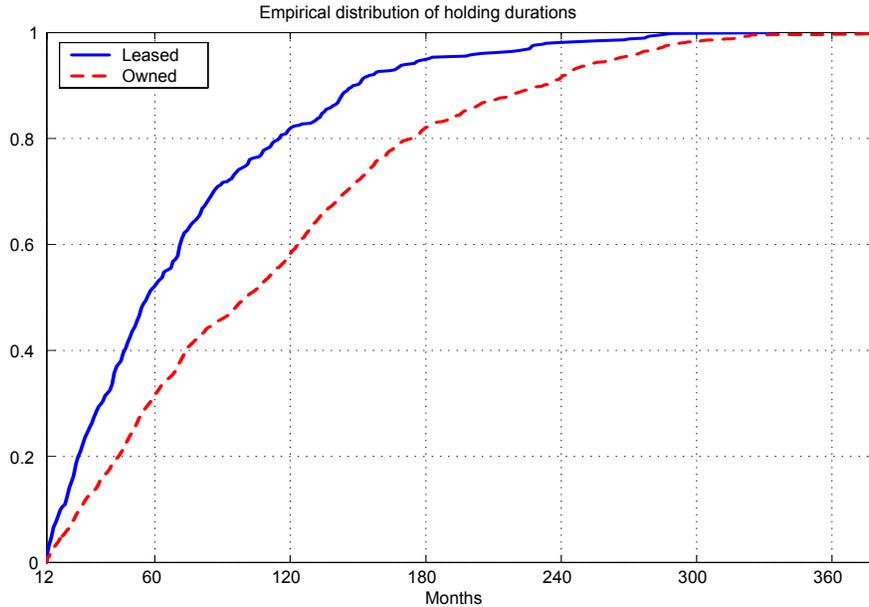
Empirical distribution of holding durations

Fig. 5 - Empirical cumulative distribution functions of holding durations, owned aircraft (dotted line) vs. leased aircraft (solid line).

bution of holding durations of leased aircraft at the 1% level. Appendix A presents the details of the procedures and the formal results of the tests.[23]

Output - The model highlights three forces that explain the higher output of leased aircraft versus owned ones. The first is the *parking effect:* lower barrier to sell on leased aircraft imply that a lower fraction of leased planes are ever parked. An operator hit by a negative shock is more likely to park the aircraft if it is owned than if it is leased. The second and third effects arise when aircraft are utilized. The second effect is a *productivity effect:* on average firms are more productive if they lease then if they own aircraft. This is the result of two distinct forces. The first is that a lower barrier to sell increases the efficiency of the marginal seller and therefore of the average operator. The second is a force toward selection, as very productive firms might want to lease in order to replace more frequently their aircraft. Note that there is a fundamental difference in causality between the two. In the first case, lessees are more productive *because* they lease. In the second case, firms lease *because* they are more productive. I later show that the effect of selection is likely to be negligible. The third is the *pairing effect*: lower transaction costs on leased units

—————————————

[23] As holding durations might be correlated within carriers, I also compare the distributions of the median holding duration for each carrier and again I accept the hypothesis that the distribution of holding durations of owned aircraft first order stochastic dominates the distribution of holding durations of leased aircraft.

TABLE 3

LEASING AND PARKED AIRCRAFT

PROBABILITY OF PARKING	(1)	(2)
CONSTANT	-.0660	.0583
	(.006)	(.045)
AGE	.0125	.0071
	(.0009)	(.0011)
LEASED	$-.0219$	$-.0181$
	(.0077)	(.009)
MODEL DUMMIES	NO	YES
R^2	.160	.278
OBSERVATIONS	2848	2848

Notes: Robust standard errors in parenthesis.

imply that the covariance between quality of the aircraft and productivity of the operator is higher for leased units than for owned units. I now present evidence that all three effects capture salient features of the data.

To investigate the *parking effect,* Proposition 12 suggests that we compare the probability of being parked inactive of two observationally equivalent aircraft, one leased and one owned. Table 3 presents the results of a linear probability model[24] where the dependent variable is equal to one if the aircraft has been parked for the entire 12 months between May 2002-April 2003. The results in Column (1) show that, conditional on age, the probability that a leased aircraft is parked is approximately 33% lower than for an owned aircraft. Since the fraction of owned aircraft parked is 6.3%, the coefficient of -.0219 on the indicator variable for leased aircraft represent a sizable decrease of approximately one third in the probability of parking the aircraft. Moreover, the result in Column (2) shows that result is robust to conditioning on aircraft model.

One way to understand the barriers to trade and to investigate the relevance of the *productivity effect* is to compare the probability of trade between leased and owned aircraft as a function of the utilization of the aircraft in the year prior to trade. The analysis of subsection 4.5 suggests that leased aircraft should a higher utilization before being traded than leased aircraft. For this reason, I merged the *Current Aircraft Datafile* with the *Time-series Utilization Datafile* and obtained the hours flown in the period between May 2001-April 2002.

---

[24]I use a linear probability model instead of a probit or logit because the aircraft model fixed effects create the well known problem of perfect classification (Ruud (2000), pag.754) in the maximum likelihood estimation of probit or logit, as some aircraft model are never parked. When aircraft fixed effects are removed, the results of probit/logit and linear probability model were very similar.

TABLE 4

LEASING AND TRADE

PROBABILITY OF TRADE	(1)	(2)
CONSTANT	.1056	.0405
	(.046)	(.041)
AGE	−.00008	−.00003
	(.0012)	(.0012)
HOURS FLOWN IN T-1	−.0413	−.0214
	(.0056)	(.0050)
HOURS FLOWN IN T-1*LEASED		−.0662
		(.012)
LEASED	.1382	.3538
	(.0145)	(.048)
MODEL DUMMIES	YES	YES
R^2	.156	.171
OBSERVATIONS	2888	2888

Notes: Robust standard errors in parenthesis.

Table 4 reports the coefficients of a linear probability model where the dependent variable is equal to one if the aircraft has been traded in the period May 2002-April 2003. The results in Column (1) indicate that, conditional on age and utilization in the previous year, leased aircraft are 13% more likely to be traded. This is consistent with the result of Proposition 8 and reinforces the previous analysis of holding durations. In Column (2), the utilization of the aircraft is interacted with the indicator variable for leased aircraft to analyze the impact of previous utilization differently for leased and owned aircraft. Figure 6 depicts the fitted values from the results of Column (2). For any level of hours flown, the probability of trading a leased aircraft is always higher than the probability of trading an owned one. The difference is particularly stark when the aircraft was parked in the previous period. The difference of probabilities reduces as utilization increases and it disappears completely for the aircraft with more than 5000 hours flown.

To investigate the presence of the *pairing effect*, I construct a test motivated by Proposition 9. Proposition 9 shows that the output gain due to leasing unfolds in a subtle way. In particular, conditional on the aircraft being in use, we should observe high quality leased aircraft produce a higher output than high quality owned aircraft, but old leased aircraft produce a lower output than old owned aircraft. Assuming that quality and age are inversely related, I can explore the *pairing*
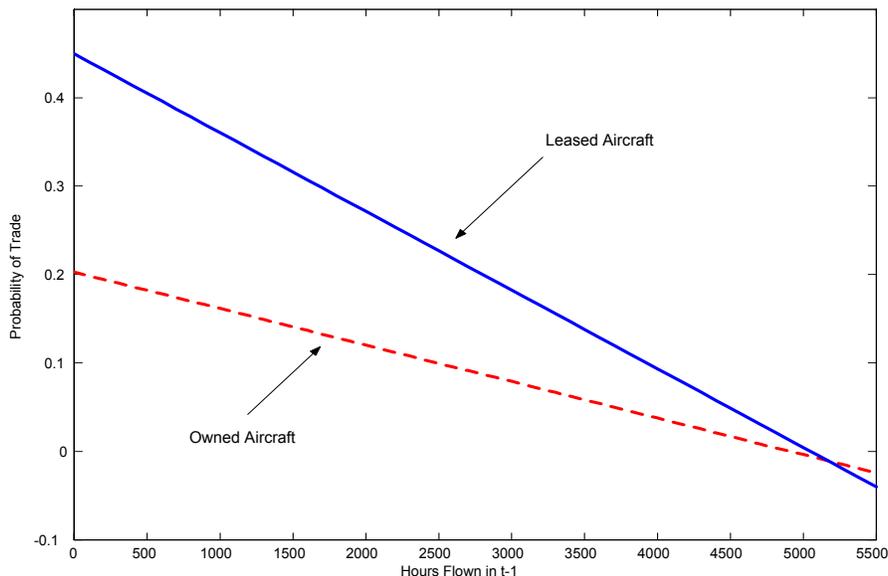
FIG. 6 - PROBABILITY OF TRADE AS A FUNCTION OF UTILIZATION IN PREVIOUS YEAR, LEASED
AIRCRAFT (SOLID LINE) VS. OWNED AIRCRAFT (DOTTED LINE). BASED ON COEFFICIENTS OF
COLUMN (2) OF TABLE 4.

effect estimating the following equation

$$
\begin{aligned}
y_{ijl} = & \ \beta_0 + \beta_1 Age + \beta_2 Age^2 + \beta_3 Leased + \beta_4 Age * Leased \\
& + \beta_5 Age^2 * Leased + \epsilon_l + \epsilon_l * Leased + \epsilon_{ijl}
\end{aligned}
\tag{27}
$$

on aircraft that were not parked only. The dependent variable y_{ijl} is the log of hours flown in
the last 12 months and *Leased* is an indicator variable equal to 1 if the aircraft is leased and
zero otherwise. As in the literature on the estimation of production functions (e.g.: Olley and
Pakes, 1996), estimation of equation (27) is plagued by a simultaneity problem generated by the
relationship between efficiency and quality of capital. However it is exactly this bias that informs
about the *pairing effect*. The coefficients on the age variables for leased aircraft should be more
correlated with the residual than the coefficient of the age variables for owned aircraft and therefore
exhibit a larger downward bias. The specification used in equation (27) allows to explore the
correlations between efficiency and quality of capital differently between leased and owned units.

The estimates of the coefficient are reported in Table 5. Column (1) reports the result of
a truncated regression that corrects for the selection criterion, while Column (2) the results of
an OLS regression that does not correct for selection. The coefficients and standard errors are
almost identical. Note that the coefficients of the variables *Age*, *Leased* and *Age * Leased* are not
significantly different from zero at conventional levels. However, the F-test reveals that $\beta_2 + \beta_5$ is

36

TABLE 5

LEASING AND HOURS FLOWN

LOG(HOURS FLOWN)	(1)	(2)
AGE	−.0163	−.0163
	(.008)	(.008)
AGE^2	−.0004	−.0004
	.0004	.0004
AGE*LEASED	.0142	.0142
	(.013)	(.013)
AGE^2*LEASED	−.0008	−.0008
	(.007)	(.007)
LEASED	.0601	.0601
	(.407)	(.411)
CONSTANT	8.224	8.224
	(.079)	(.079)
MODEL DUMMIES	YES	YES
MODEL DUMMIES*LEASED	YES	YES
LOG LIKELIHOOD	−2118.3	
R^2		.2867
OBSERVATIONS	2689	2689

Notes: Robust Standard errors in parenthesis. Column (1) reports the parameter estimated using a truncated regression, while Column (2) reports the parameter estimated using Ordinary Least Squares.

significantly different form zero at the 5% level, so that the coefficient on the quadratic term for leased aircraft is significantly different from zero.

Figure 7 plots the fitted output based on the estimated parameters.[25] The solid line represents leased units while the dotted line represents owned units. We clearly observe the pattern predicted by Proposition 9 and by the *pairing effect*: leased units have higher output on young units, while owned units have higher output on old units, indicating that the correlation between efficiency and quality of aircraft is higher for leased aircraft than for owned aircraft.

Combined, this evidence is consistent with the hypothesis that leasing increases capacity utilization reducing the barriers to trade between carriers. The data reveal patterns consistent with the three distinct effects highlighted by the theoretical model. The goal of the next subsection is to estimate the depreciation pattern of aircraft and obtain measures of the quality of aircraft and

---

[25]The intercept terms are calculated as the weighted average of the constant and the aircraft model fixed effects, where the weigths are given by the frequency of each model.
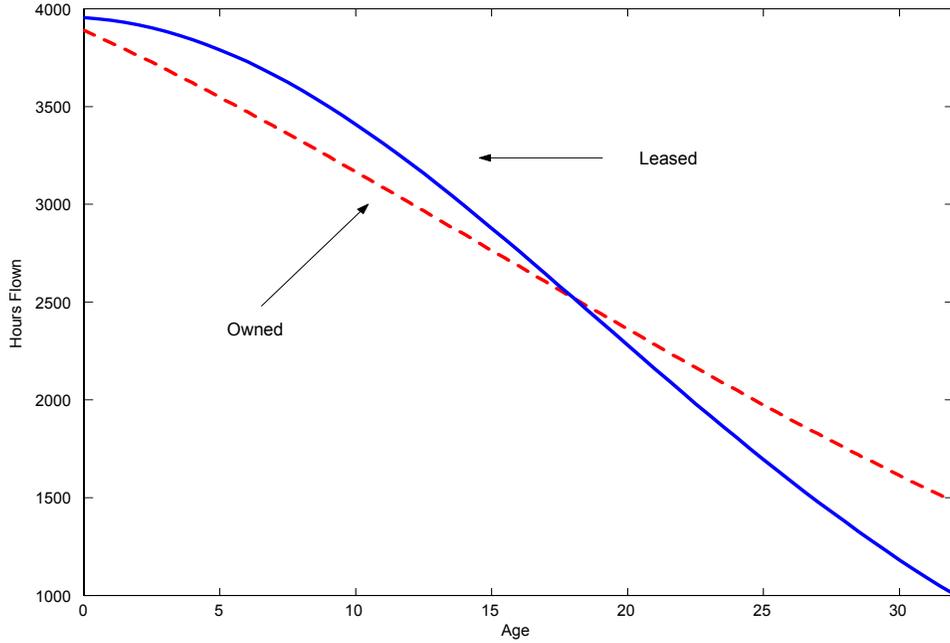
FIG. 7

HOURS FLOWN, LEASED (SOLID LINE) VS. OWNED AIRCRAFT (DOTTED LINE).

productivity of carriers to precisely quantify the magnitude of each of the three effects.

5.3 Quantifying the effects

The extension of subsection 4.6 provides a natural empirical framework to estimate the depreciation patterns of aircraft and hence the productivity of each carrier. Let

$$q_{ijl} = \exp\left(\beta_0 + \beta_1 Age_{ijl}\right)$$

be the quality of aircraft ijl of model l operated by carrier j and let

$$z_{ijl} = \exp\epsilon_{ijl}$$

be the efficiency of carrier j when operating aircraft ijl. I assume that the log of potential output y^*_{ijl} and operating cost f_{ijl} of aircraft ijl are given by the following two equations

$$
\begin{align}
y^*_{ijl} &= \log q_{ijl} + \log z_{ijl} + \epsilon_l \tag{28}\\
f_{ijl} &= \gamma Z_{ijl} + \eta_{ijl} \tag{29}
\end{align}
$$

38

where $\gamma Z_{ijl} = \gamma_0 + \gamma_1 Age_{ijl}$ and ϵ_l are aircraft model fixed effects. Hence we observe

$$
\begin{aligned}
y_{ijl} &= y_{ijl}^* \text{ if } y_{ijl}^* \geq f_i \\
y_{ijl} &= 0 \quad \text{if } y_{ijl}^* < f_i
\end{aligned}
$$

so observed output is equal to potential output when potential output is higher than the fixed cost of operation, while the aircraft is parked otherwise.[26] I assume that $\left(\epsilon_{ijl}, \eta_{ijl}\right)$ are normal random variables, with mean zero and covariance matrix

$$
\Sigma = \left(\begin{array}{cc} \sigma_\epsilon^2 & \rho \sigma_\epsilon \sigma_\eta \\ \rho \sigma_\epsilon \sigma_\eta & \sigma_\eta^2 \end{array}\right)
$$

This is a censored regression model with unobserved stochastic threshold. This model was introduced by Nelson (1977) to study the individual labor supply decision. Maddala (1983, pag. 174-178) and Ridder and Van Monfort (2001) discuss the identification of the model.[27]

Let $\beta X_{ijl} = \log q_{ijl} + \epsilon_l$ and let $\zeta = (\beta, \gamma, \sigma_\epsilon, \sigma_\eta, \rho)$ be the parameters' vector. Using standard results for bivariate normal random variables, the log-likelihood function is

$$
\begin{aligned}
L\left(\zeta\right) = {} & -N_1 \log \sigma_\epsilon - \frac{1}{2\sigma_\epsilon^2} \sum_{y_{ijl}^* \geq f_i} (y_{ijl} - \beta X_{ijl})^2 + \\
& \sum_{y_{ijl}^* \geq f_i} \log \Phi \left(\frac{y_{ijl} - \gamma Z_{ijl} - \frac{\rho \sigma_\eta}{\sigma_\epsilon} (y_{ijl} - \beta X_{ijl})}{\sqrt{\sigma_\eta^2 (1 - \rho^2)}} \right) + \sum_{y_{ijl}^* < f_i} \log \Phi \left(\frac{\gamma Z_{ijl} - \beta X_{ijl}}{\sqrt{\sigma_\eta^2 + \sigma_\epsilon^2 - 2\rho \sigma_\epsilon \sigma_\eta}} \right)
\end{aligned} \tag{30}
$$

As pointed out in the previous section, an endogeneity problem arises as productivity and age of aircraft are likely to be correlated. To solve this issue, I use instruments that are correlated with the age of the aircraft, but arguably uncorrelated with the error term. The choice of instruments is motivated by the recent literature that estimates demand and the degree of product differentiation among Widebody aircraft (Benkard, 2004 and Irwin and Pavcnik, 2004). More specifically, I exploit the fact that different aircraft models are not perfect substitutes for one another, but fundamental characteristics such as the flying range, the number of seats and the engine make each type of aircraft a different type of capital suited to serve specific routes. This is clearly seen by inspecting Figure 2 of Benkard (2004), which shows in a Range-Seats diagram the degree of product differentiation for Widebody aircraft. I then use instruments similar to those used by Berry, Levinsohn and Pakes (1995) to estimate utility function with unobservable quality. I use as instruments for the age of aircraft ijl own aircraft attributes (dummies for the maker of the engine, dummies for seats

---

[26] As some aircraft are parked and have zero hours flown, I modify as $\log$ (Hours Flown)=0 when this is the case.

[27] In particular, Ridder and Van Monfort (2001) show that the assumption of normality of the disturbances is sufficient for identification and no exclusion restrictions is needed. As equations (28) and (29) show, I used aircraft model fixed effects only in the output equation (28). I have also estimated the model with including aircraft model fixed effects in the cost equation, obtaining almost identical results.

TABLE 6

MAXIMUM LIKELIHOOD ESTIMATES OF EQUATION (30)

		OUTPUT EQUATION		COST EQUATION
CONSTANT	β_0	8.009 (.195)	γ_0	5.2229 (.3895)
AGE	β_1	−.0114 (.006)	γ_1	.0643 (.0155)
σ_ϵ		.5412 (.099)		
σ_η		1.1385 (.105)		
ρ		0.4863 (.056)		
LOG LIKELIHOOD		−86.494		
OBSERVATIONS		2848		

Notes: Standard errors in parenthesis. Aircraft model fixed effects not reported.

categories, manufacturers dummies) and summary measures (average fuselage number) of aircraft of the same type as model l operated by carriers different than carrier j.[28]

The properties of this type of two-step estimator are well understood. See Newey and McFadden (1994) for a discussion of the asymptotic properties of this type of two-step estimator.

Table 6 reports the estimates of the parameters. The coefficient β_1 on age is estimated to be equal to −.0114, indicating that the potential output an aircraft can produce decreases very slowly with age. The estimates of γ_1 equal to .0643 indicates that the cost of operating an aircraft increases faster with age than the decrease in potential output.[29]

I now use the estimated coefficients to obtain measures of quality of the aircraft and productivity of the carrier. Quality is

$$\widehat{q}_{ijl} = \exp\left(\widehat{\beta}_0 + \widehat{\beta}_1 Age_{ijl}\right)$$

while productivity is the residual of the output function, i.e.

$$\theta_{ij} = \exp\left(y_{ijl} - \log\widehat{q}_{ijl} - \epsilon_l\right) = \frac{\exp(y_{ijl})}{\widehat{q}_{ijl}\exp\left(\epsilon_l\right)}.$$

---

[28]Note that the model partition is finer than the type partition, as described in the Data Subsection 5.1. For example, aircraft models Boeing 747-100, 747-200, 747-300, 747-400 and 747-S all correspond to the aircraft type Boeing 747. Hence, I am using as instrument for the age of aircraft Boeing 747-100 operated by carrier l the average fuselage number of all Boeing 747 operated by carriers different than l.

[29]I have also estimated the model with a full set of carriers fixed effects in the output equation and aircraft model fixed effects in the cost equation. The results were almost identical and, hence, omitted

Empirical distribution of Productivity
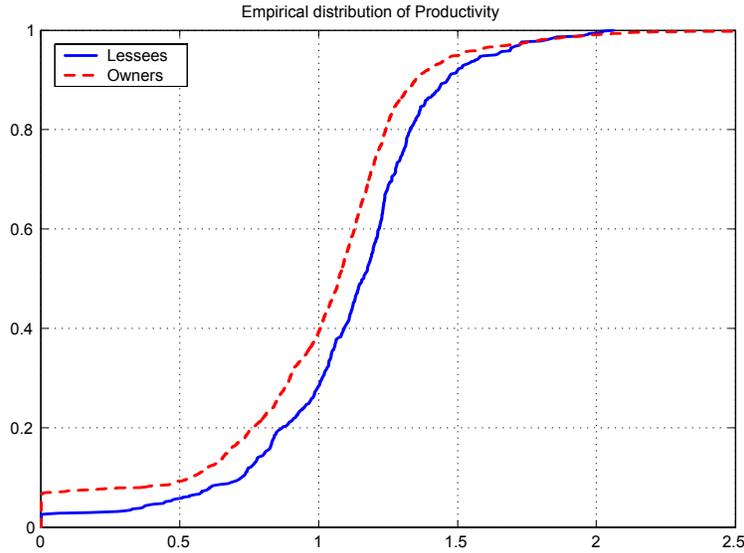
FIG. 8 - EMPIRICAL CUMULATIVE DISTRIBUTION FUNCTIONS OF PRODUCTIVITY, OWNERS (DOT-TED LINE) VS. LESSEES (SOLID LINE).

The empirical distributions of owners and lessees productivities are shown in Figure 8. The dotted line represents the productivity of carriers when operating owned aircraft (owners), while the solid line represents the productivity of carriers when operating leased aircraft (lessees). Simple visual inspection shows that lessees' productivity is higher than owners'. If productivity was directly observed, I could use the same tests already employed in subsection 5.2 to compare the distribution of productivity of lessees and owners. However the productivity is not directly observed but rather estimated and the sampling variability of the estimated parameters must be taken into account when constructing the distributions of the test statistics. Hence, I follow Abadie (2001) and use a bootstrap procedure to compute the p-values of the test statistics. The Kolmogorov-Smirnov test of the equality of distributions rejects the null hypothesis of equal distributions (the bootstrapped p-value is equal to 0). Moreover, the tests for first order stochastic dominance proposed by Davidson and Duclos (2000) and Barrett and Donald (2003) accept the null hypothesis that the distribution of productivity of lessees first order stochastically dominates the distribution of productivity of owners (the bootstrapped p-values are equal to 0.98 and 1, respectively). Practically speaking, the problem of sampling variability does not seem a major concern because of the large sample size of the dataset. Appendix A presents the details of the procedures and the formal results of the tests.

Table 7 tabulates some statistics of the two measures. All differences between the tabulated statistics are significantly different form 0 at a 5-percent level. On average, productivity is 11% higher on leased aircraft. The table also shows that the difference in productivity between leased

41

TABLE 7
STATISTICS OF ESTIMATED PRODUCTIVITY

	θ_{ij} LEASED	θ_{ij} OWNED	DIFFERENCE
MEAN	1.109	.999	.1089*
10TH PERCENTILE	.724	.543	.1806*
50TH PERCENTILE	1.162	1.072	.0898*
90TH PERCENTILE	1.463	1.348	.1152*
OBSERVATIONS	579	2269	

Notes: *Significantly different from 0 at the 5-percent level or higher.

and owned units is larger at lower percentiles of the distribution, which is consistent with the theoretical model.

I now use the estimates of productivity in order to decompose the total gain due to leasing in the *parking, productivity and pairing effects*.

5.3.1 Decomposing the gains

Let average output Y be

$$
\begin{aligned}
Y &= \Pr(Y > 0) * E(Y|Y > 0) = \Pr(zq > 0) * E(zq|zq > 0) \qquad (31)\\
&= \Pr(Y > 0) * (E(z|Y > 0) * E(q|Y > 0) + Cov(z, q|Y > 0))
\end{aligned}
$$

The first term in equation (31) measures the fraction of aircraft not parked and the second measures the average output of each aircraft not parked. This second term is then given by the sum of the product of the average quality of the aircraft and the average productivity of the carrier and the covariance between these two terms.

The percentage difference between leased and owned units can be expressed as

$$
\begin{aligned}
\log Y_L - \log Y_O &= \log\left(\Pr(Y_L > 0)\left(E(z_L|Y_L > 0) E(q_L|Y_L > 0) + Cov(z_L, q_L|Y_L > 0)\right)\right) - \\
&\quad \log\left(\Pr(Y_O > 0)\left(E(z_O|Y_O > 0) E(q_O|Y_O > 0) + Cov(z_O, q_O|Y_O > 0)\right)\right)
\end{aligned}
$$

Rearranging, we obtain

$$
\begin{aligned}
\log Y_L - \log Y_O &\approx \log \Pr(Y_L > 0) - \log \Pr(Y_O > 0) + \\
&\quad \log\left(E(z_L|Y_L > 0) E(q_L|Y_L > 0)\right) - \log\left(E(z_O|Y_O > 0) E(q_L|Y_L > 0)\right) + \\
&\quad \frac{Cov(z_L, q_L|Y_L > 0)}{E(z_L|Y_L > 0) E(q_L|Y_L > 0) + Cov(z_L, q_L|Y_L > 0)} - \\
&\quad \frac{Cov(z_O, q_O|Y_O > 0)}{E(z_O|Y_O > 0) E(q_L|Y_L > 0) + Cov(z_O, q_O|Y_O > 0)}
\end{aligned}
$$

where the last passage follows from a Taylor expansion of $\log \left(E\left(z_i|Y_i > 0 \right) E\left(q_i|Y_i > 0 \right) + Cov\left(z_i, q_i|Y_i > 0 \right) \right)$ around $E\left(z_i|Y_i > 0 \right) E\left(q_i|Y_i > 0 \right).$ [30]

As I desire to obtain the effect of leasing conditional on the quality of aircraft, I further approximate the expression above as

$$
\begin{aligned}
\log Y_L - \log Y_O \;\approx\; & \log \Pr\left(Y_L > 0|q_L \right) - \log \Pr\left(Y_O > 0|q_O \right) + \\
& \log E\left(z_L|Y_L > 0 \right) - \log E\left(z_O|Y_O > 0 \right) + \\
& \frac{Cov\left(z_L, q_L|Y_L > 0 \right) - Cov\left(z_O, q_O|Y_O > 0 \right)}{E\left(z|Y > 0 \right) E\left(q|Y > 0 \right) + Cov\left(z, q|Y > 0 \right)}
\end{aligned}
$$

Therefore we obtain the total effect of leasing as the sum of the empirical counterparts of the three effects outlined in the theoretical model. The term $\log \Pr\left(Y_L > 0|q_L \right) - \log \Pr\left(Y_O > 0|q_O \right)$ measures the *parking effect*. The term $\log E\left(z_L|Y_L > 0 \right) - \log E\left(z_O|Y_O > 0 \right)$ measures the *productivity effect*. The term $\frac{Cov(z_L, q_L|Y_L > 0) - Cov(z_O, q_O|Y_O > 0)}{E(z|Y > 0)E(q|Y > 0) + Cov(z, q|Y > 0)}$ measures the *pairing effect*.

Based on the estimates reported above, I can then decompose the total effect of leasing as follows

$$
\log \Pr\left(Y_L > 0|q_L \right) - \log \Pr\left(Y_O > 0|q_O \right) \;=\; .0181 \tag{32}
$$

$$
\log E\left(z_L|Y_L > 0 \right) - \log E\left(z_O|Y_O > 0 \right) \;=\; .0618 \tag{33}
$$

$$
\frac{Cov\left(z_L, q_L|Y_L > 0 \right) - Cov\left(z_O, q_O|Y_O > 0 \right)}{E\left(z|Y > 0 \right) E\left(q|Y > 0 \right) + Cov\left(z, q|Y > 0 \right)} \;=\; .0014 \tag{34}
$$

The estimates reveal that the biggest effect (6.18%, approximately 80% of the total gain) is due to difference in productivity for aircraft that are in use. 1.8% can be attributed to the difference with which aircraft are parked and .14% can be attributed to differences in the covariances between the estimated productivity and the estimated quality of capital for aircraft in use.

5.4 Perfect pairing

In this section, I evaluate the gain that would arise if there was perfect pairing between quality and productivity. This is an important benchmark to compare the results of previous sections. Thus, aircraft model by model, I assign the highest quality of capital to the highest productivity firm, the second highest quality of capital to the second highest productivity firm and so on. I then compute the total output that would ensue and compare it with different alternative scenarios.

Table 8 presents the result of this comparison. In the first line I evaluate the gain of perfect pairing with respect to actual output, keeping the current mix of owned and leased aircraft. In the second line, I calculate the output gain of perfect pairing compared to the case when all aircraft are owned, while in the third line I compute the gain compared to the case when all aircraft are leased.

---

[30] As it will become clear later, the approximation is justified since $Cov\left(z_i, q_i|Y_i > 0 \right)$ is small compared to $E\left(z_i|Y_i > 0 \right) E\left(q_i|Y_i > 0 \right).$

TABLE 8

	PERFECT PAIRING
ACTUAL OUTPUT	+0.67%
ALL OWNED	+0.71%
ALL LEASED	+0.56%

Notes: see text for explanations.

This table shows, then, that the .14% gain in pairing when moving from ownership to leasing is remarkable, as it represents $\frac{0.14}{0.71} = 20\%$ of all possible gains due to pairing. All three lines of the table show very small gains, ranging from a minimum of .56% to a maximum of .71% only.

The results are indicative of the relative magnitudes of the effects at work. The output gains due to perfect pairing are far smaller than the gains obtained from a higher average productivity. This might seems surprising at first, but a careful evaluation of the data explains the reason. Assume for simplicity that the correlation between quality and productivity is zero on owned units while it is equal to one in the case of leased aircraft. Assume further that there is no difference in average productivity and average quality between leased and owned aircraft. The percentage gain in output due to pairing is

$$\frac{Y_L - Y_O}{Y_O} = \frac{E(z_L q_L)}{E(z_O q_O)} - 1 = \frac{E(z_L) E(q_L) + Cov(z_L, q_L)}{E(z_O) E(q_O)} - 1$$

$$= \frac{Cov(z_L, q_L)}{E(z_L) E(q_L)} = \frac{\rho \sigma_{z_L} \sigma_{q_L}}{E(z_L) E(q_L)} = \frac{\sigma_{z_L}}{E(z_L)} \frac{\sigma_{q_L}}{E(q_L)}$$

The data show that $\frac{\sigma_{z_L}}{E(z_L)} \approx .1$ and $\frac{\sigma_{q_L}}{E(q_L)} \approx .3$ so that the gain of moving form no pairing at all to perfect pairing is around 3%, so very small. This figure represents an extreme upper bound of the total gains form pairing. From it, we need to substract the effect of the imperfect pairing of ownership, and more importantly the fact that different aircraft models are differentiated products. As I explained when I motivated the choice of instruments, the quality/age of the aircraft likely enters in the second stage of a carrier's decision to acquire an aircraft, after the choice of the model of the aircraft. The small gain of .71% are then explained.[31]

5.5 Alternative Hypothesis and Additional Evidence

The evidence and the estimates obtained in the previous subsections all points towards a positive causal effect of leasing on productivity due to a reduction in the transaction costs. One major concern before establishing causality running from leasing to productivity is rejecting the opposite

---

[31]McAfee (2002) shows that using two classes obtains at most 50% of all gains due to matching.

TABLE 9

AVERAGE HOURS FLOWN, LEASED VS. OWNED AIRCRAFT

	AIRCRAFT	HOURS FLOWN
LEASED	302	3771
OWNED	719	3561

Notes: Averages calculated on aircraft acquired by their current operator in the last 5 years only.

direction of causality from productivity to leasing. In other words, the results could have been caused entirely by the selection of the most productive carriers into leasing. In this case, it would be hard to precisely assess the output gain due to leasing. However, several arguments point against the results being mainly driven by selection.

The first argument arises by a careful inspection of the Figure 7. The patterns seem to be inconsistent with the selection hypothesis. Note that the largest difference between leased and owned aircraft is found at the age 7 of the aircraft, not at age 1 or 2, as we would expect under pure selection. Moreover, Table 9 reports the raw averages of the flying hours for aircraft that have been acquired by their current operator in the last 5 years only. The Table shows that in this subsample leased aircraft have only 5.8% more hours flown than owned aircraft, while the datum for the entire sample in Table 2 was 14%. This indicates that the gap between the output produced by leased and owned units widens as the holding duration increases, consistent with the hypothesis of lower barriers to trade and inconsistent with the pure selection hypothesis[32].

The second argument comes from inspection of Figure 6. The depicted patterns are inconsistent with selection. If selection was important, we should observe the difference between leased and owned aircraft to increase with previous utilization, as high productivity implies high utilization. This is clearly not the case.

An additional argument comes from analysis of the distribution functions of lessees' and owners' productivities in Figure 8. The two distributions move almost parallel after the initial difference at low levels of productivities and the difference does not grow larger as productivity increases. More formally, Appendix A shows that, when restricting the analysis to the top 15% of carriers' productivities, a Kolmogorov-Smirnov test of the equality of distributions does not reject the null hypothesis of equal distributions (the bootstrapped p-value is equal to 0.608). Also the Davidson and Duclos (2000) and Barrett and Donald (2003) tests for first order stochastic dominance reject the null hypothesis of first order stochastic dominance (the bootstrapped p-values are equal to 0.302 and 0, respectively).[33]

One further concern is that the gain is estimated on aircraft that have been with their current

---

[32]This fact is also confirmed by a regression of the estimated productivity on the holding durations. The slope is found to be negative and significantly different from zero, indicating again that productivity deteriorates over time.

[33]Similar results were obtained also for the top 20% and 25% of the distribution.

TABLE 10

CUMULATIVE HOURS FLOWN AND LEASING

LOG $\left(\frac{\text{CUMULATIVE HOURS FLOWN}}{\text{AGE}}\right)$	(1)	(2)
CONSTANT	7.835	7.739
	(.099)	(.063)
AGE	−.0061	−.0062
	(.001)	(.002)
LEASED	.0599	.0816
	(.010)	(.018)
MODEL DUMMIES	YES	YES
R^2	.2075	.3282
OBSERVATIONS	2847	1183

Notes: Robust standard errors in parenthesis.

operator for an entire year. If there is a sizable loss in output during the trading period, the estimated gain could be overestimated. The literature on investment (e.g. Cooper and Haltiwanger, 1993 and Caballero and Engel, 1999) traditionally assumes that firms must shut down operations for a fixed period of time when trading capital. As I documented that leased units are traded more frequently, the associated loss in output could potentially be substantial and invalidate my results. In the same spirit, it could also be that aircraft remain a long time with the lessor once a carrier returns it, so that the productivity gains while in use are offset by a loss of output when returned. To investigate these issues, I calculate for each aircraft the average annual hours flown since the delivery date and compare the values of leased aircraft with the values for owned aircraft. This measure is not perfect since lessors buy several aircraft on secondary markets and therefore for those aircraft the average annual hours flown mixes periods of ownership with periods of leasing. However, the measure would cast doubt on the estimated gains if owned aircraft were found to have a higher average annual hours flown than leased aircraft. Table 10 shows that this is not the case. Column (1) shows that leased aircraft have 6% more average hours flown than owned ones.[34] Moreover, in order to try to partially control for the incomplete historical status of the aircraft (leased vs. owned) I resort to one institutional characteristic of the leasing business. Specifically, in Column (2) I restrict the sample to aircraft with General Electric engine only. Since General Electric is both a large lessor and a large engine maker, it is more likely that leased aircraft with General Electric engine have been owned by a lessor (GECAS) and leased out since their original delivery date. Column (2) shows that in this subsample leased aircraft have 8.16% more average hours flown than owned ones. Hence, the results in Table 10 indicate that the output gain for leased aircraft holds also for different periods of time. Both columns show that the gain of leasing

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[34]One observation was dropped since it was such a large outlier to change completely the estimated coefficients.

46

TABLE 11

PRODUCTIVITY θ	(1)
CONSTANT	.7617
	(.095)
%LEASED	.9533
	(.342)
TOTAL AIRCRAFT	.0007
	(.00024)
TOTAL AIRCRAFT FOR LEASE	$-.0030$
	(.0009)
R^2	.2675
OBSERVATIONS	27

Notes: Weighted Least Squares Regression. Weights are the total number of aircraft for each model. Robust Standard errors in parenthesis.

is only slightly attenuated and still rather large.[35]

An interesting additional robustness check of my results is to compare the estimated difference in productivity across different aircraft models. It seems logical to expect that transaction costs are higher for less popular models that have been produced in fewer units. In terms of my model, the productivity of operators should be higher when they operate more popular aircraft. To verify these claims, I calculate the average operators' productivity for each aircraft model and then regress it on the percentage of leased aircraft, the total number of aircraft and the product of the two, which corresponds to the total number of aircraft for lease. Table 11 presents the results. The signs of the coefficients are exactly as predicted. The more aircraft are available for each model and the higher the percentage of aircraft available for lease, the larger is the productivity of the carriers. The sign of the coefficient on the cross product shows also that the positive impact of leasing decreases the more popular a model is. Note also that all coefficients are significantly different from zero at the 1% level despite the fact the number of observations is very small, as there are only 27 models of Widebody aircraft.

Combined, these results constitute a powerful robustness check to my analysis. All evidences point toward the fact that lessors increase the efficiency of the airline industry facilitating the trade of aircraft among carriers.

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[35]One fact to take into account is that the period of observation (April 2002-March 2003) coincides with an extremely unfavorable period for the airline industry, in which, for example, the number of aircraft parked was larger than usual.

6 Extensions and Conclusion

The purpose of this paper has been to examine the link between the efficiency of transactions of used capital in secondary markets and the efficiency of production of final output. I argued that lessors act as intermediaries that reduce the transaction costs and I constructed a model of trade in used capital to understand the role of lessors when trading is subject to frictions. In the empirical section, I used a dataset on aircraft and carriers' fleets to offer empirical evidence of the main implications of the model concerning the differences in holding duration and output between leased and owned aircraft.

The results support the notion that when transaction costs prevent the efficiency of allocation via decentralized trade, firms have incentive to develop institutions and adopt contractual arrangements that reduce the inefficiencies resulting from transaction costs. The empirical analysis reveals a considerable gain in output (8% in 2002-2003) due to the particular institution analyzed - aircraft leasing.

This paper has focused on the *effect* of leasing on the efficiency of secondary markets and the analysis can be extended along several dimensions. Consistent with the observed market structure in the market for aircraft, I assumed that lessors have more market power than carriers. It is a feature common to most market intermediaries to be large and to have some degree of market power. However, the exact forces that shape this structure are not well understood. Two motivations jointly seem most prominent for aircraft. The first is that in a decentralized market with random meeting among firms, a large intermediary increases the probability of a successful match between a buyer and a seller.[36] In this sense, the results in Table 11 seems suggestive that this force can potentially play a relevant role, as they are consistent with lower transaction costs on more popular aircraft. The second reason in the aircraft market are the large financial resources involved. Aircraft cost billions of dollars and if lessors need a certain critical size before turning profitable, financial constraints may constitute a barrier to entry and reinforce market power.[37]

Furthermore, it seems interesting to study more carefully the relationship between manufacturers and lessors and between lessors and carriers. In the stylized setting of Subsection 4.5.1, the analysis reveals that prices of new aircraft decrease when a lessors offer aircraft for lease, thus reducing the revenues of the manufacturer. This may give manufacturers the incentive not to sell new aircraft to lessors. The paper also assumed away any contracting or bargaining issues over the terms of the lease. However, the data reveal cross-sectional differences in the durations of leasing contracts. A more careful study of the incentives of manufacturers, lessors and carriers might shed new light on how leasing affects the transactions of capital equipment and the efficiency of

---

[36] In the model of Rubinstein and Wolinsky (1985), middlemen are assumed to trade one unit per period of time. Allowing the size of middlemen to be endogenous could explain why intermediaries are often larger than buyers and sellers.

[37] In the leasing business, the largest three firms belong to vast conglomerates: GECAS is a unit of General Electric; ILFC is a unit of AIG, the world's largest insurer; Ansett Aviation is a unit of Morgan Stanley, the investment bank.

production.

Moreover, in this paper I focused on the steady state where all variables are constant over time, ignoring any aggregate uncertainty. The airline business, however, is subject to wide fluctuations that influence the trade of aircraft in secondary markets. In future research, I plan to examine how economic fluctuations affect the reallocation of aircraft across carriers. More specifically, I desire to study how the two forces generating trade highlighted in this paper - depreciation of aircraft and shocks to carriers' efficiency - influence reallocation in an economy subject to stochastic fluctuations.[38] Cooper and Haltiwanger (1993) have documented how the automobile industry concentrate the replacement of machines during *seasonal* downturns. However, how the gains from trade vary when cycles are not deterministic is an open question. The analysis could also shed light on how the industry' cross-sectional distribution of aircraft vintages respond to fluctuations, i.e. if recession are cleansing, in the language of Caballero and Hammour (1994).

A   Tests of First Order Stochastic Dominance (FOSD)

Let X_Z and X_W be random variables with corresponding cdfs $G_Z(\cdot)$ and $G_W(\cdot)$. $G_Z(\cdot)$ first order stochastically dominates $G_W(\cdot)$ if

$$G_Z(x) \leq G_W(x) \text{ for all } x \text{ and}$$
$$G_Z(x) < G_W(x) \text{ for some } x$$

Let the empirical distributions be defined by

$$\widehat{G}_i(x) = N_i^{-1} \sum_{j=1}^{N_i} 1\{X_i \leq x\} \text{ for } i = Z, W$$

where $1\{\cdot\}$ denotes the indicator function and N_i are the number of observations from distribution G_i. Using the empirical cdfs, I perform test of the hypothesis:

$$G_Z(x) = G_W(x), \forall x \in R \tag{35}$$

and

$$G_Z(x) \leq G_W(x), \forall x \in R \tag{36}$$

The test of (35) is conducted using the familiar Kolmogorov-Smirnov test statistics

$$S_1 = \left(\frac{N_Z N_W}{N_Z + N_W} \right)^{1/2} \sup_a \left| \widehat{G}_Z(a) - \widehat{G}_W(a) \right|$$

The test of (36) is conducted using the procedure introduced by Davidson and Duclos (2000). They show that we can make use of a predetermined grid of points a_j for $j = 1, ..., m$ and construct the t statistics

$$t(a_j) = \frac{\widehat{G}_Z(a_j) - \widehat{G}_W(a_j)}{\sqrt{\frac{\left(\widehat{G}_Z(a_j) - \widehat{G}_W(a_j)^2 \right)}{N_Z} + \frac{\left(\widehat{G}_Z(a_j) - \widehat{G}_W(a_j)^2 \right)}{N_W}}}$$

---

[38] Goolsbee (1997) documents that reallocations of Boeing 707 increased during periods of high fuel cost and/or worst economic conditions. More recently, Rampini and Eisfeldt (2003) use Compustat data and find that reallocation is procyclical.

TABLE 12

TEST OF FOSD FOR HOLDING DURATIONS

HOLDING DURATIONS

MONTHS	13	29	44	60	76	92	108	124	139	155
$t\left(a_j\right)$	-.57	-1.79	-2.58	-3.31	-4.04	-4.53	-5.01	-5.46	-5.87	-7.09
MONTHS	171	187	203	218	234	250	266	281	297	313
$t\left(a_j\right)$	-7.40	-7.67	-7.06	-6.62	-6.86	-5.83	-4.76	-5.14	-5.10	-3.87

to test H_1 (dominance) against H_2 (no restriction). The hypothesis H_1 is rejected against the unconstrained alternative H_2 if any of the t statistics is significant with the positive sign, where significance is determined asymptotically by the critical values $d_{\alpha,m,\infty}$ of the Studentized Modulus (SMM) distribution with m and infinite number of degrees of freedom at the $\alpha\%$ confidence level. In practice, this implies that we accept the hypothesis of $G_Z\left(\cdot\right)$ first order stochastically dominating $G_W\left(\cdot\right)$ if

$$-t\left(a_j\right) \quad > \quad d_{\alpha,m,\infty} \text{ for some } j \text{ and}$$
$$t\left(a_j\right) \quad < \quad d_{\alpha,m,\infty} \text{ for all } j$$

An undesirable feature of the test proposed by Davidson and Duclos is that the comparisons made at a fixed number of arbitrary chosen points introduce the possibility of test inconsistency. Barrett and Donald (2003) follow McFadden (1989) and modify the Kolmogorov Smirnov test to construct the test statistics

$$\hat{S}_1 = \left(\frac{N_Z N_W}{N_Z + N_W}\right)^{1/2} \sup_a \left(\widehat{G}_Z\left(a\right) - \widehat{G}_W\left(a\right)\right)$$

Barrett and Donald show that we can compute p-values by $\exp\left(-2\left(\hat{S}_1\right)^2\right)$, so that if $\widehat{G}_Z\left(a\right) - \widehat{G}_W\left(a\right) \leq 0$ for all a, then asymptotically the probability of rejection of the null hypothesis of stochastic dominance is zero.

I apply these tests to the distributions of holding durations of leased and owned aircraft, to the distributions of productivity of lessees and owners and to the distributions of productivity of lessees and owners in the top 15% of the pooled productivity distribution. I now describe the details for each case.

Holding durations - The Kolmogorov Smirnov test rejects the null hypothesis of equality of distributions between owned aircraft and leased aircraft. The asymptotic p-value is equal to $3.2465 * 10^{-27}$.

As for the Davidson and Duclos' test, I choose a grid with $m = 20$ equally spaced points between the first and the 99th percentile of the distribution of pooled holding durations. The critical values, tabulated in Stoline and Ury (1979), are $d_{\alpha,m,\infty} = 4.043$ for $\alpha = 1$, $d_{\alpha,m,\infty} = 3.643$ for $\alpha = 5$ and $d_{\alpha,m,\infty} = 3.453$ for $\alpha = 10$. Table 12 presents values of the t statistics. Results clearly show that the distribution of holding durations of owned aircraft first order stochastically dominates the distribution of holding durations of leased aircraft as all t statistics are negative and the absolute value of the largest one is $7.67 > 4.043$.

As for the Barrett and Donald' test, the distribution of holding durations of owned aircraft is everywhere below the distribution of holding durations of leased aircraft. Hence, the probability of rejection of the null hypothesis of stochastic dominance is zero.

TABLE 13

TEST OF FOSD FOR PRODUCTIVITY DISTRIBUTIONS

					PRODUCTIVITY					
PRODUCTIVITY	.094	.19	.28	.37	.46	.56	.65	.75	.84	.94
$t\left(a_j\right)$	-5.17	-5.19	-5.26	-3.67	-3.24	-3.33	-4.00	-4.75	-3.52	-5.03
PRODUCTIVITY	1.03	1.13	1.22	1.31	1.41	1.50	1.59	1.69	1.78	1.87
$t\left(a_j\right)$	-5.32	-6.13	-7.22	-5.76	-3.92	-2.45	-1.52	-1.44	-0.30	0.22

TABLE 14

TEST OF FOSD FOR PRODUCTIVITY DISTRIBUTIONS, UPPER TAIL

					PRODUCTIVITY, TOP 15%					
PRODUCTIVITY	1.37	1.43	1.49	1.55	1.61	1.67	1.73	1.79	1.85	1.91
$t\left(a_j\right)$	.81	.07	.22	.73	.45	.87	2.27	.93	1.29	.23

Productivity - Since productivity is estimated rather than observed, the sampling variability of the estimated parameters must be taken into account when constructing the distributions of the test statistics. Hence, I bootstrap the p-values of the test statistics, following the procedure described in Abadie (2001). Abadie also provides a set of weak regularity conditions to imply consistency. These assumptions do not require continuity of the distributions and, in particular, are satisfied by distributions with probability mass at zero.

The Kolmogorov-Smirnov test of the equality of distributions rejects the null hypothesis of equal distributions (the bootstrapped p-value is equal to 0). As for the Davidson and Duclos' test, I choose a grid with $m = 20$ equally spaced points between the first and the 99th percentile of the distribution of pooled productivities. Table 13 presents values of the t statistics. The bootstrapped p-value is .98, so the test accepts the null hypothesis that the distribution of productivity of lessees first order stochastically dominates the distribution of productivity of owners. The Barrett and Donald' test also accepts the null hypothesis of stochastic dominance. The bootstrapped p-value is 1.

Productivity, upper 15th percentile - In this case, productivity is restricted to be in the top 15% of the pooled productivity distribution, which correspond to all values of productivity above 1.309. The procedures are exactly as described above. The Kolmogorov-Smirnov test of the equality of distributions does not reject the null hypothesis of equal distributions (the bootstrapped p-value is equal to .608). As for the Davidson and Duclos' test, I choose a grid with $m = 10$ equally spaced points between the 85th and the 99th percentile of the distribution of pooled productivities. Table 14 presents values of the t statistics. The test rejects the null hypothesis that the distribution of productivity of lessees first order stochastically dominates the distribution of productivity of owners in the top 15% of the pooled productivity distribution (the bootstrapped p-value is .303). Also the Barrett and Donald' test rejects the null of stochastic dominance (the bootstrapped p-value is 0).

B Mathematical derivations

B.1 Derivation of $J_1 - J_2$ and $J_2 - J_3$

We have

$$
\begin{aligned}
J_2 - J_3 &= \int_{z_1(2)}^{\overline{z}} \left(V_1\left(z\right) - p_1 + p_2 - T\right) dF\left(z\right) + \int_{z_2(2)}^{z_1(2)} V_2\left(z\right) dF\left(z\right) + \int_{\underline{z}}^{z_2(2)} \left(V_3\left(z\right) + p_2 - T\right) dF\left(z\right) \\
&\quad - \int_{z_1(3)}^{\overline{z}} \left(V_1\left(z\right) - p_1\right) dF\left(z\right) - \int_{z_2(3)}^{z_1(3)} \left(V_2\left(z\right) - p_2\right) dF\left(z\right) - \int_{\underline{z}}^{z_2(3)} V_3\left(z\right) dF\left(z\right) \\
&= p_1 \left(1 - F\left(z_1\left(3\right)\right)\right) + p_2 \left(F\left(z_1\left(3\right)\right) - F\left(z_2\left(3\right)\right)\right) + \left(p_2 - p_1 - T\right)\left(1 - F\left(z_1\left(2\right)\right)\right) \\
&\quad + \left(p_2 - T\right) F\left(z_2\left(2\right)\right) - \int_{z_1(3)}^{z_1(2)} \left(V_1\left(z\right) - V_2\left(z\right)\right) dF\left(z\right) + \int_{z_2(2)}^{z_2(3)} \left(V_2\left(z\right) - V_3\left(z\right)\right) dF\left(z\right)
\end{aligned}
$$

For $z \in \left[z_1\left(3\right), z_1\left(2\right)\right]$

$$
\begin{aligned}
V_1\left(z\right) &= \frac{zq_1 + \beta\alpha J_1 + \beta\gamma_1 V_2\left(z\right)}{1 - \beta\left(1 - \alpha - \gamma_1\right)} \\
V_2\left(z\right) &= \frac{zq_2 + \beta\alpha J_2 + \beta\gamma_2\left(V_1\left(z\right) - p_1\right)}{1 - \beta\left(1 - \alpha - \gamma_2\right)} \\
V_1\left(z\right) - V_2\left(z\right) &= \frac{z\left(q_1 - q_2\right) + \beta\alpha\left(J_1 - J_2\right) + \beta\gamma_2 p_1}{\alpha\beta - \beta + \beta\gamma_1 + \beta\gamma_2 + 1}
\end{aligned}
$$

Integrating

$$
\begin{aligned}
&\int_{z_1(3)}^{z_1(2)} \left(V_1\left(s\right) - V_2\left(s\right)\right) dF\left(s\right) \\
&= \frac{\left(\beta\alpha\left(J_1 - J_2\right) + \beta\gamma_2 p_1\right)\left(F\left(z_1\left(2\right)\right) - F\left(z_1\left(3\right)\right)\right)}{\alpha\beta - \beta + \beta\gamma_1 + \beta\gamma_2 + 1} + \int_{z_1(3)}^{z_1(2)} \frac{s\left(q_1 - q_2\right)}{\alpha\beta - \beta + \beta\gamma_1 + \beta\gamma_2 + 1} dF\left(s\right)
\end{aligned}
$$

Similarly,

$$
\begin{aligned}
&\int_{z_2(2)}^{z_2(3)} \left(V_2\left(z\right) - V_3\left(z\right)\right) dF\left(z\right) \\
&= \int_{z_2(2)}^{z_2(3)} \left(\frac{zq_2 + \beta\alpha J_2 + \beta\gamma_2 V_3\left(z\right)}{1 - \beta\left(1 - \alpha - \gamma_2\right)} - V_3\left(z\right)\right) dF\left(z\right) = \int_{z_2(2)}^{z_2(3)} \frac{zq_2 + \beta\alpha\left(J_2 - J_3\right)}{1 - \beta\left(1 - \alpha - \gamma_2\right)} dF\left(z\right) \\
&= \frac{\beta\alpha\left(J_2 - J_3\right)\left(F\left(z_2\left(3\right)\right) - F\left(z_2\left(2\right)\right)\right)}{1 - \beta\left(1 - \alpha - \gamma_2\right)} + \int_{z_2(2)}^{z_2(3)} \frac{zq_2}{1 - \beta\left(1 - \alpha - \gamma_2\right)} dF\left(z\right)
\end{aligned}
$$

Combining

$$
\begin{aligned}
J_2 - J_3 &= \left(p_1\left(1 - F\left(z_1\left(3\right)\right)\right) + p_2\left(F\left(z_1\left(3\right)\right) - F\left(z_2\left(3\right)\right)\right)\right) \quad\quad\quad (37) \\
&\quad + \left(p_2 - p_1 - T\right)\left(1 - F\left(z_1\left(2\right)\right)\right) + \left(p_2 - T\right) F\left(z_2\left(2\right)\right) \\
&\quad - \frac{\left(\beta\alpha\left(J_1 - J_2\right) + \beta\gamma_2 p_1\right)\left(F\left(z_1\left(2\right)\right) - F\left(z_1\left(3\right)\right)\right)}{\alpha\beta - \beta + \beta\gamma_1 + \beta\gamma_2 + 1} - \int_{z_1(3)}^{z_1(2)} \frac{s\left(q_1 - q_2\right)}{\alpha\beta - \beta + \beta\gamma_1 + \beta\gamma_2 + 1} dF\left(s\right) \\
&\quad + \left. \int_{z_2(2)}^{z_2(3)} \frac{zq_2}{1 - \beta\left(1 - \alpha - \gamma_2\right)} dF\left(z\right)\right)\left(1 - \frac{\beta\alpha\left(F\left(z_2\left(3\right)\right) - F\left(z_2\left(2\right)\right)\right)}{1 - \beta\left(1 - \alpha - \gamma_2\right)}\right)^{-1}
\end{aligned}
$$

Moreover,

$$
\begin{aligned}
J_1 - J_2 &= \int_{\underline{z}}^{\overline{z}} \max\{V_1(z), V_2(z) - p_2 + p_1 - T, V_3(z) + p_1 - T\} dF(z) \\
&\quad - \int_{\underline{z}}^{\overline{z}} \max\{V_1(z) - p_1 + p_2 - T, V_2(z), V_3(z) + p_2 - T\} dF(z) \\
&= \int_{z_1(1)}^{\overline{z}} V_1(z) dF(z) + \int_{z_2(3)}^{z_1(1)} (V_2(z) - p_2 + p_1 - T) dF(z) + \int_{\underline{z}}^{z_2(3)} (V_3(z) + p_1 - T) dF(z) \\
&\quad - \int_{z_1(2)}^{\overline{z}} (V_1(z) - p_1 + p_2 - T) dF(z) - \int_{z_2(2)}^{z_1(2)} V_2(z) dF(z) - \int_{\underline{z}}^{z_2(2)} (V_3(z) + p_2 - T) dF(z) \\
&= (p_1 - p_2 - T)(F(z_1(1)) - F(z_2(3))) + (p_1 - T) F(z_2(3)) - (p_2 - p_1 - T)(1 - F(z_1(2))) \\
&\quad - (p_2 - T) F(z_2(2)) + \int_{z_1(1)}^{z_1(2)} (V_1(z) - V_2(z)) dF(z) - \int_{z_2(2)}^{z_2(3)} (V_2(z) - V_3(z)) dF(z)
\end{aligned}
$$

For $z \in [z_1(1), z_1(3)]$

$$
\begin{aligned}
V_1(z) &= \frac{zq_1 + \beta\alpha J_1 + \beta\gamma_1 V_2(z)}{1 - \beta(1 - \alpha - \gamma_1)} \\
V_2(z) &= \frac{zq_2 + \beta\alpha J_2 + \beta\gamma_2(V_2(z) - p_2)}{1 - \beta(1 - \alpha - \gamma_2)} = \frac{zq_2 + \beta\alpha J_2 - \beta\gamma_2 p_2}{1 - \beta(1 - \alpha)} \\
V_1(z) - V_2(z) &= \frac{zq_1 + \beta\alpha J_1 + \beta\gamma_1 V_2(z)}{1 - \beta(1 - \alpha - \gamma_1)} - V_2(z) = \frac{z(q_1 - q_2) + \beta\alpha(J_1 - J_2) + \beta\gamma_2 p_2}{1 - \beta(1 - \alpha - \gamma_1)}
\end{aligned}
$$

so that

$$
\begin{aligned}
&\int_{z_1(1)}^{z_1(2)} (V_1(s) - V_2(s)) dF(s) \\
&= \frac{(\beta\alpha(J_1 - J_2) + \beta\gamma_2 p_2)(F(z_1(3)) - F(z_1(1)))}{1 - \beta(1 - \alpha - \gamma_1)} + \int_{z_1(1)}^{z_1(3)} \frac{s(q_1 - q_2)}{1 - \beta(1 - \alpha - \gamma_1)} dF(s) \\
&\quad + \frac{(\beta\alpha(J_1 - J_2) + \beta\gamma_2 p_1)(F(z_1(2)) - F(z_1(3)))}{\alpha\beta - \beta + \beta\gamma_1 + \beta\gamma_2 + 1} + \int_{z_1(3)}^{z_1(2)} \frac{s(q_1 - q_2)}{\alpha\beta - \beta + \beta\gamma_1 + \beta\gamma_2 + 1} dF(s)
\end{aligned}
$$

Also

$$
\begin{aligned}
&\int_{z_2(2)}^{z_2(3)} (V_2(s) - V_3(s)) dF(s) \\
&= \frac{\beta\alpha(J_2 - J_3)(F(z_2(3)) - F(z_2(2)))}{1 - \beta(1 - \alpha - \gamma_2)} + \int_{z_2(2)}^{z_2(3)} \frac{sq_2}{1 - \beta(1 - \alpha - \gamma_2)} dF(s)
\end{aligned}
$$

Combining and rearranging, we obtain

$$
\begin{aligned}
J_1 - J_2 &= \Big((p_1 - p_2 - T)(F(z_1(1)) - F(z_2(3))) + (p_1 - T) F(z_2(3)) \tag{38} \\
&\quad - (p_2 - p_1 - T)(1 - F(z_1(2))) - (p_2 - T) F(z_2(2)) \\
&\quad + \frac{\beta\gamma_2 p_2(F(z_1(3)) - F(z_1(1)))}{1 - \beta(1 - \alpha - \gamma_1)} + \frac{\beta\gamma_2 p_1(F(z_1(2)) - F(z_1(3)))}{1 - \beta(1 - \alpha - \gamma_1 - \gamma_2)} \\
&\quad - \frac{\beta\alpha(J_2 - J_3)(F(z_2(3)) - F(z_2(2)))}{1 - \beta(1 - \alpha - \gamma_2)} + \int_{z_1(1)}^{z^{b_1}} \frac{s(q_1 - q_2)}{1 - \beta(1 - \alpha - \gamma_1)} dF(s) + \int_{z_1(3)}^{z_1(2)} \frac{s(q_1 - q_2)}{1 - \beta(1 - \alpha - \gamma_1 - \gamma_2)} dF(s) \\
&\quad - \int_{z_2(2)}^{z_2(3)} \frac{sq_2}{1 - \beta(1 - \alpha - \gamma_2)} dF(s) \Big) \left(1 - \frac{\beta\alpha(F(z_1(3)) - F(z_1(1)))}{1 - \beta(1 - \alpha - \gamma_1)} - \frac{\beta\alpha(F(z_1(2)) - F(z_1(3)))}{1 - \beta(1 - \alpha - \gamma_1 - \gamma_2)}\right)^{-1}
\end{aligned}
$$

Hence, equations (7), (8), (38) and (37) form 4 equation in 4 unknowns $(p_1, p_2, J_1 - J_2, J_2 - J_3)$.

B.2 Derivation of distribution of efficiency z and distribution of output

The distribution of efficiency z is

$$h_1(z) = \begin{cases} \frac{\alpha f(z)}{\alpha+\gamma_1+\mu} & \text{for } z_1(1,1) \leq z < z_1(b,1) \\ \frac{\alpha f(z)}{\alpha+\gamma_1} & \text{for } z_1(b,1) \leq z < z_1(1,3) \\ \frac{f(x)}{X} & \text{for } z \geq z_1(1,3) \end{cases}$$

while the distribution of output depends on weather

$$f > bz^1(1,3)q_1$$

or not. In the case $f < bz^1(1,3)q_1$ the firm that is just indifferent between buying an aircraft or not does not park it when hit by a temporary shock and therefore the distribution of output is

$$g_1(y) = \begin{cases} \frac{\alpha\mu\left(F\left(\frac{f}{bq_1}\right)-F(z_1(1,b))\right)}{\alpha+\gamma_1+\mu} & \text{for } y=0 \\ \frac{(1-\mu)\alpha f\left(\frac{y}{q_1}\right)}{\alpha+\gamma_1+\mu} & \text{for } z_1(1,1)q_1 \leq y < z_1(b,1)q_1 \\ \frac{(1-\mu)\alpha f\left(\frac{y}{q_1}\right)}{\alpha+\gamma_1} & \text{for } z_1(b,1)q_1 \leq y < f \\ \frac{(1-\mu)\alpha f\left(\frac{y}{q_1}\right)}{\alpha+\gamma_1}+\frac{\mu\alpha f\left(\frac{y}{bq_1}\right)}{\alpha+\gamma_1} & \text{for } f \leq y < z_1(1,3)q_1 \\ \frac{(1-\mu)f\left(\frac{y}{q_1}\right)}{X}+\frac{\mu f\left(\frac{y}{bq_1}\right)}{X} & \text{for } z_1(1,3)q_1 \leq y < b\overline{z}q_1 \\ \frac{(1-\mu)f\left(\frac{y}{q_1}\right)}{X} & \text{for } y \geq b\overline{z}q_1 \end{cases}$$

In the opposite case of $f > bz^1(1,3)q_1$, the $z_1(1,3)$ firm prefers to park the aircraft when hit by a temporary shock. The distribution of output is then

$$g_1(y) = \begin{cases} \frac{\alpha\mu\left(F\left(\frac{f}{bq_1}\right)-F(z_1(1,b))\right)}{\alpha+\gamma_1+\mu} & \text{for } y=0 \\ \frac{(1-\mu)\alpha f\left(\frac{y}{q_1}\right)}{\alpha+\gamma_1+\mu} & \text{for } z_1(1,1)q_1 \leq y < z_1(b,1)q_1 \\ \frac{(1-\mu)\alpha f\left(\frac{y}{q_1}\right)}{\alpha+\gamma_1} & \text{for } z_1(b,1)q_1 \leq y < z_1(1,3)q_1 \\ \frac{(1-\mu)\alpha f\left(\frac{y}{q_1}\right)}{X} & \text{for } z_1(1,3)q_1 \leq y < f \\ \frac{(1-\mu)f\left(\frac{y}{q_1}\right)}{X}+\frac{\mu f\left(\frac{y}{bq_1}\right)}{X} & \text{for } z_1(1,3)q_1 \leq y < b\overline{z}q_1 \\ \frac{(1-\mu)f\left(\frac{y}{q_1}\right)}{X} & \text{for } y \geq b\overline{z}q_1 \end{cases}$$

C Proof of Propositions

C.1 Proof of Proposition 1

$z_1(2)$ satisfies

$$V_1(z_1(2)) - p_1 + p_2 = V_2(z_1(2)) \tag{39}$$

$z_1(3)$ satisfies

$$V_1(z_1(3)) - p_1 = V_2(z_1(3)) - p_2 \tag{40}$$

while $z_1(1)$ satisfies

$$V_1(z_1(1)) = V_2(z_1(1)) + p_1 - p_2 \tag{41}$$

Combining equations (1), (2) and (3)

$$V_1\left(z_1\left(2\right)\right) - V_2\left(z_1\left(2\right)\right) = V_1\left(z_1\left(3\right)\right) - V_2\left(z_1\left(3\right)\right) = V_1\left(z_1\left(1\right)\right) - V_2\left(z_1\left(1\right)\right)$$

Since $V_1\left(z\right) - V_2\left(z\right)$ is strictly increasing in z, we have that $z_1\left(1\right) = z_1\left(2\right) = z_1\left(3\right) = z_1$

$z_2\left(1\right)$ satisfies

$$V_2\left(z_2\left(1\right)\right) + p_1 - p_2 = V_3\left(z_2\left(1\right)\right) + p_1 \tag{42}$$

$z_2\left(3\right)$ satisfies

$$V_2\left(z_2\left(3\right)\right) - p_2 = V_3\left(z_2\left(3\right)\right) \tag{43}$$

$z_2\left(2\right)$ satisfies

$$V_2\left(z_2\left(2\right)\right) = V_3\left(z_2\left(2\right)\right) + p_2 \tag{44}$$

Combining equations (4), (5) and (6)

$$V_2\left(z_2\left(1\right)\right) - V_3\left(z_2\left(1\right)\right) = V_2\left(z_2\left(3\right)\right) - V_3\left(z_2\left(3\right)\right) = V_2\left(z_2\left(2\right)\right) - V_3\left(z_2\left(2\right)\right)$$

Since $V_2\left(z\right) - V_3\left(z\right) = V_2\left(z\right) - V_3$ is increasing in z. we have that $z_2\left(1\right) = z_2\left(2\right) = z_2\left(3\right) = z_2$

C.2   Proof of Proposition 4

Assume $\alpha = 0$. Using the distributions (13) and (12), we have that $z_2\left(2\right) = z_2\left(3\right) = z_2\left(1\right)$ and $z_1\left(1\right) = z_1\left(3\right)$.

Let us compare the output of q_1 aircraft. We obtain

$$
\begin{aligned}
\int_{z_1}^{\overline{z}} sq_1 dF\left(s\right) - X_1 \int_{z_1\left(3\right)}^{\overline{z}} sq_1 dH_1\left(s\right) \quad &\propto \quad 1 - \left(z_1\right)^2 - \left(\left(\left(z_1\left(2\right)\right)^2 - \left(z_1\left(3\right)\right)^2\right)\left(X_1 + \frac{\left(1 - X_1 - X_2\right)\gamma_2}{\gamma_2 + \gamma_1}\right) + 1 - \left(z_1\left(2\right)\right)^2\right) \\
&= \quad -\left(z_1\right)^2 + \frac{\gamma_1\left(z_1\left(2\right)\right)^2 + \gamma_2\left(z_1\left(3\right)\right)^2}{\gamma_2 + \gamma_1} \\
&= \quad -\left(1 - X_1\right)^2 + \frac{\gamma_1\left(z_1\left(2\right)\right)^2 + \gamma_2\left(z_1\left(3\right)\right)^2}{\gamma_2 + \gamma_1}
\end{aligned} \tag{45}
$$

From $\int_0^1 h_1\left(z\right) = 1$, we obtain

$$
\begin{aligned}
1 &= \int_{z_1\left(2\right)}^1 \frac{1}{X_1} dz + \frac{1}{X_1}\int_{z_1\left(3\right)}^{z_1\left(2\right)}\left(X_1 + \frac{\left(1 - X_1 - X_2\right)\gamma_2}{\gamma_2 + \gamma_1}\right) dz = \frac{1 - z_1\left(2\right)}{X_1} + \frac{\gamma_2\left(z_1\left(2\right) - z_1\left(3\right)\right)}{X_1\left(\gamma_2 + \gamma_1\right)} \\
&= \frac{\left(1 - z_1\left(2\right)\right)\left(\gamma_2 + \gamma_1\right) + \gamma_2\left(z_1\left(2\right) - z_1\left(3\right)\right)}{X_1\left(\gamma_2 + \gamma_1\right)} = \frac{\gamma_2 + \gamma_1 - \gamma_1 z_1\left(2\right) - \gamma_2 z_1\left(3\right)}{X_1\left(\gamma_2 + \gamma_1\right)}
\end{aligned}
$$

Hence, we can substitute $X_1 = \frac{\gamma_2 + \gamma_1 - \gamma_1 z_1\left(2\right) - \gamma_2 z_1\left(3\right)}{\gamma_2 + \gamma_1}$ in equation (45) to obtain

$$-\left(\frac{\gamma_1 + \gamma_2}{\gamma_2 + \gamma_1}\right)^2 + \frac{\gamma_1\left(z_1\left(2\right)\right)^2 + \gamma_2\left(z_1\left(3\right)\right)^2}{\gamma_2 + \gamma_1} = \frac{\left(z_1\left(3\right) - z_1\left(2\right)\right)^2 \gamma_1 \gamma_2}{\left(\gamma_1 + \gamma_2\right)^2} > 0$$

Moreover, since $\alpha = 0$, we obtain that $z_2\left(2\right) = z_2\left(3\right) = z_2\left(1\right) = z_2$ and

$$
\begin{aligned}
&\int_{z_2}^{z_1} sq_2 dF\left(s\right) - X_2 \int_{z_2\left(2\right)}^{z_1\left(2\right)} sq_2 dH_2\left(s\right) \\
\propto \quad &\left(z_1\right)^2 - \left(z_2\right)^2 - \left(X_2 + \frac{\frac{1 - X}{X_1}\gamma_2 X_2}{\gamma_2 + \gamma_1}\right)\left(\left(z_1\left(2\right)\right)^2 - \left(z_1\left(3\right)\right)^2\right) - \left(z_1\left(3\right)\right)^2 + \left(z_2\left(3\right)\right)^2 \\
= \quad &\left(1 - X_1\right)^2 - \left(\frac{\gamma_1}{\gamma_2 + \gamma_1}\right)\left(z_1\left(2\right)\right)^2 - \left(\frac{\gamma_2}{\gamma_2 + \gamma_1}\right)\left(z_1\left(3\right)\right)^2 \\
= \quad &\left(1 - \frac{\gamma_2 + \gamma_1 - \gamma_1 z_1\left(2\right) - \gamma_2 z_1\left(3\right)}{\gamma_2 + \gamma_1}\right)^2 - \left(\frac{\gamma_1}{\gamma_2 + \gamma_1}\right)\left(z_1\left(2\right)\right)^2 - \left(\frac{\gamma_2}{\gamma_2 + \gamma_1}\right)\left(z_1\left(3\right)\right)^2 < 0
\end{aligned}
$$

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The proof then follows by showing that the derivatives of $\int_{z_1}^{\overline{z}} sq_1 dF(s) - X_1 \int_{z_1(3)}^{\overline{z}} sq_1 dH_1(s)$ and $\int_{z_2}^{z_1} sq_2 dF(s) - X_2 \int_{z_2(2)}^{z_1(2)} sq_2 dH_2(s)$ with respect to α are negative and positive, respectively. This is formally stated in the comparative statics results of Subsection 4.4. See the Proof of Proposition 7.

C.3 Proof of Proposition 6

An important step in the proof is the following lemma

Lemma 13 $\frac{\partial z_2(3)}{\partial \alpha} > 0$

Proof. Suppose not, $\frac{\partial z_2(3)}{\partial \alpha} < 0$. Differentiating equation (17) with respect to α, we obtain

$$\frac{\partial z_2(3)}{\partial \alpha} - \frac{\partial z_2(2)}{\partial \alpha} = \frac{T\beta}{q_2} > 0$$

we have that

$$\frac{\partial z_2(2)}{\partial \alpha} < 0$$

Using equation (11), we have that

$$
\begin{aligned}
1 - X &= (1 - X_2)\alpha F(z_2(3)) + (1 - X)(1 - \alpha) + X_2(\alpha F(z_2(2)) + \gamma_2 H_2(z_2(3))) \\
&= (1 - X_2)\alpha F(z_2(3)) + (1 - X)(1 - \alpha) + X_2\left(\alpha F(z_2(2)) + \frac{\alpha\gamma_2(F(z_2(3)) - F(z_2(2)))}{\alpha + \gamma_2}\right) \\
&= (1 - X_2)F(z_2(3)) + X_2\left(\frac{\gamma_2 F(z_2(3)) + \alpha F(z_2(2))}{\alpha + \gamma_2}\right)
\end{aligned}
$$

Differentiating with respect to α, we obtain

$$(1 - X_2)\left(f(z_2(3))\frac{\partial z_2(3)}{\partial \alpha}\right) + X_2\left(\frac{(\alpha + \gamma_2)\left(\gamma_2 f(z_2(3))\frac{\partial z_2(3)}{\partial \alpha} + \alpha f(z_2(2))\frac{\partial z_2(2)}{\partial \alpha}\right) - \gamma_2(F(z_2(3)) - F(z_2(2)))}{(\alpha + \gamma_2)^2}\right)$$

but all terms are negative, which is impossible. Hence $\frac{\partial z_2(3)}{\partial \alpha} > 0$ ∎

We next state

Lemma 14 *There exist a β^* such that if $\beta > \beta^*$, then $\frac{\partial z_2(2)}{\partial \alpha} < 0$*

Proof. Using

$$F(z) = X_1 H_1(z) + X_2 H_2(z) + (1 - X_1 - X_2)H_3(z)$$

evaluated at $z_1(1)$, we obtain

$$
\begin{aligned}
F(z_1(1)) &= X_2 H_2(z_1(1)) + (1 - X_1 - X_2) \\
&= X_2\left(\frac{\alpha(F(z_2(3)) - F(z_2(2)))}{\alpha + \gamma_2} + \frac{(F(z_1(1)) - F(z_2(3)))}{X_2}\right) + (1 - X_1 - X_2)
\end{aligned}
$$

Hence

$$F(z_2(3))\left(1 - \frac{\alpha X_2}{\alpha + \gamma_2}\right) = -\frac{\alpha X_2 F(z_2(2))}{\alpha + \gamma_2} + (1 - X_1 - X_2)$$

Using $F(z) = z$

$$z_2(3)\left(1 - \frac{\alpha X_2}{\alpha + \gamma_2}\right) = -\frac{\alpha X_2 z_2(2)}{\alpha + \gamma_2} + (1 - X_1 - X_2)$$

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Differentiating

$$\frac{\partial z_2\left(3\right)}{\partial \alpha}\left(1-\frac{\alpha X_2}{\alpha +\gamma_2}\right)-\frac{\gamma_2 X_2}{\left(\alpha +\gamma_2\right)^2}z_2\left(3\right)=-\frac{\partial z_2\left(2\right)}{\partial \alpha}\left(\frac{\alpha X_2}{\alpha +\gamma_2}\right)-\frac{\gamma_2 X_2}{\left(\alpha +\gamma_2\right)^2}z_2\left(2\right)$$

Using $\frac{\partial z_2(3)}{\partial \alpha}-\frac{\partial z_2(2)}{\partial \alpha}=\frac{\beta T}{q_2}$ and $\frac{(z_2(3)-z_2(2))q_2}{(1-\beta(1-\alpha-\gamma_2))}=T$ and rearranging

$$\frac{\partial z_2\left(2\right)}{\partial \alpha}=\frac{T\left(1-\beta\left(1-\alpha -\gamma_2\right)\right)}{q_2}\frac{\gamma_2 X_2}{\left(\alpha +\gamma_2\right)^2}-\left(1-\frac{\alpha X_2}{\alpha +\gamma_2}\right)\frac{\beta T}{q_2}=\frac{\left(X_2\gamma_2\left(1-\beta\right)-\beta\left(1-X_2\right)\left(\alpha +\gamma_2\right)^2\right)T}{\left(\alpha +\gamma_2\right)^2 q_2}$$

which is the sum of two terms, the first positive, the second negative. Note that at $\beta =1$

$$\frac{\partial z_2\left(2\right)}{\partial \alpha}=\frac{\left(-\left(1-X_2\right)\left(\alpha +\gamma_2\right)^2\right)T}{\left(\alpha +\gamma_2\right)^2 q_2}=\frac{-\left(1-X_2\right)T}{q_2}<0 \tag{46}$$

By continuity, there is a value β^* such that if $\beta >\beta^*$, then $\frac{\partial z_2(2)}{\partial \alpha}<0$ ∎

We next study the bands of the $H_1\left(z\right)$ distribution.

Lemma 15 $\frac{\partial z_1(2)}{\partial \alpha}>0$

Proof. Using

$$H_1\left(z_1\left(2\right)\right)=\int_{z_1(1)}^{z_1(2)}h_1\left(s\right)ds=1-\int_{z_1(2)}^{z}h_1\left(s\right)ds$$

we have

$$H_1\left(z_1\left(2\right)\right)=\frac{\alpha\left(z_1\left(3\right)-z_1\left(1\right)\right)}{\left(\alpha +\gamma_1\right)}+\left(z_1\left(2\right)-z_1\left(3\right)\right)\left(1+\frac{\frac{1-X}{X_1}\left(\alpha +\gamma_2\right)}{\alpha +\gamma_2+\gamma_1}\right)=1-\frac{1-z_1\left(2\right)}{X_1}$$

Using $\frac{(z_1(3)-z_1(1))(q_1-q_2)}{(1-\beta(1-\alpha-\gamma_1))}=T=\frac{(z_1(2)-z_1(3))(q_1-q_2)}{(1-\beta(1-\alpha-\gamma_1))}$ we obtain

$$\frac{T\left(1-\beta\left(1-\alpha -\gamma_1\right)\right)}{\left(q_1-q_2\right)}\left(1+\frac{\alpha}{\alpha +\gamma_1}+\frac{\frac{1-X_1-X_2}{X_1}\left(\alpha +\gamma_2\right)}{\alpha +\gamma_2+\gamma_1}\right)=1-\frac{1-z_1\left(2\right)}{X_1}$$

Differentiating with respect to α

$$\frac{\partial z_1\left(2\right)}{\partial \alpha}\frac{1}{X_1}=\frac{T\beta}{\left(q_1-q_2\right)}\left(1+\frac{\alpha}{\alpha +\gamma_1}+\frac{\frac{1-X_1-X_2}{X_1}\left(\alpha +\gamma_2\right)}{\alpha +\gamma_2+\gamma_1}\right)+\frac{T\left(1-\beta\left(1-\alpha -\gamma_1\right)\right)}{\left(q_1-q_2\right)}\left(\frac{\gamma_1}{\left(\alpha +\gamma_1\right)^2}+\frac{\left(1-X_1-X_2\right)\gamma_1}{\left(\alpha +\gamma_1+\gamma_2\right)^2 X_1}\right)>0$$

Hence

$$\frac{\partial z_1\left(2\right)}{\partial \alpha}>0$$

∎

Moreover

Lemma 16 *There exist a β^{**} such that if $\beta >\beta^{**}$, then $\frac{\partial z_1(1)}{\partial \alpha}<0$*

Proof. Use the cdf $h_2\left(z\right)$ to obtain

$$\begin{aligned}H_2\left(z_1\left(1\right)\right)&=\frac{\alpha\left(z_2\left(3\right)-z_2\left(2\right)\right)}{\alpha +\gamma_2}+\frac{z_1\left(1\right)-z_2\left(3\right)}{X_2}\\&=1-\left(z_1\left(3\right)-z_1\left(1\right)\right)\left(\frac{1-X_1}{X_2}+\frac{\gamma_2}{\alpha +\gamma_1}\right)-\left(z_1\left(2\right)-z_1\left(3\right)\right)\left(1+\frac{\frac{1-X}{X_1}\gamma_2}{\alpha +\gamma_2+\gamma_1}\right)\end{aligned}$$

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Substituting $\frac{(z_1(3)-z_1(1))(q_1-q_2)}{(1-\beta(1-\alpha-\gamma_1))} = T = \frac{(z_1(2)-z_1(3))(q_1-q_2)}{(1-\beta(1-\alpha-\gamma_1))} = \frac{(z_2(3)-z_2(2))q_2}{(1-\beta(1-\alpha-\gamma_2))}$

$$\frac{\alpha T\left(1-\beta\left(1-\alpha-\gamma_2\right)\right)}{\left(\alpha+\gamma_2\right)q_2}+\frac{z_1\left(1\right)-z_2\left(3\right)}{X_2}=1-\left(\frac{T\left(1-\beta\left(1-\alpha-\gamma_1\right)\right)}{\left(q_1-q_2\right)}\left(\frac{1-X_1}{X_2}+\frac{\gamma_2}{\alpha+\gamma_1}+1+\frac{\frac{1-X}{X_1}\gamma_2}{\alpha+\gamma_2+\gamma_1}\right)\right)$$

Differentiating with respect to α we obtain

$$\frac{\left(\gamma_2\left(1-\beta\right)+\left(\alpha+\gamma_2\right)^2\beta\right)}{\left(\alpha+\gamma_2\right)^2 q_2}T+\frac{1}{X_2}\left(\frac{\partial z_1\left(1\right)}{\partial\alpha}-\frac{\partial z_2\left(3\right)}{\partial\alpha}\right)=$$

$$\frac{T\left(1-\beta\left(1-\alpha-\gamma_1\right)\right)}{\left(q_1-q_2\right)}\left(\frac{\gamma_2}{\left(\alpha+\gamma_1\right)^2}+\frac{\left(1-X\right)\gamma_2}{\left(\alpha+\gamma_1+\gamma_2\right)^2 X_1}\right)\qquad -\frac{\beta T}{\left(q_1-q_2\right)}\left(\frac{1-X_1}{X_2}+\frac{\gamma_2}{\alpha+\gamma_1}+1+\frac{\frac{1-X}{X_1}\gamma_2}{\alpha+\gamma_2+\gamma_1}\right)$$

Rearrange as

$$\frac{1}{X_2}\left(\frac{\partial z_1\left(1\right)}{\partial\alpha}-\frac{\partial z_2\left(3\right)}{\partial\alpha}\right)=\frac{T\left(1-\beta\left(1-\alpha-\gamma_1\right)\right)}{\left(q_1-q_2\right)}\left(\frac{\gamma_2}{\left(\alpha+\gamma_1\right)^2}+\frac{\left(1-X\right)\gamma_2}{\left(\alpha+\gamma_1+\gamma_2\right)^2 X_1}\right)$$

$$-\frac{\beta T}{\left(q_1-q_2\right)}\left(\frac{1-X_1}{X_2}+\frac{\gamma_2}{\alpha+\gamma_1}+1+\frac{\frac{1-X}{X_1}\gamma_2}{\alpha+\gamma_2+\gamma_1}\right)-\frac{\left(\gamma_2\left(1-\beta\right)+\left(\alpha+\gamma_2\right)^2\beta\right)}{\left(\alpha+\gamma_2\right)^2 q_2}T$$

Using $\frac{\partial z_2(3)}{\partial\alpha}-\frac{\partial z_2(2)}{\partial\alpha}=\frac{T\beta}{q_2}$ in the above expression and rearranging, we obtain

$$\frac{1}{X_2}\frac{\partial z_1\left(1\right)}{\partial\alpha}=\frac{T}{\left(q_1-q_2\right)}\left(\frac{-\left(1-X_1-X_2\right)\gamma_2^2}{\left(\alpha+\gamma_1+\gamma_2\right)^2 X_1}\right)-\frac{T}{\left(q_1-q_2\right)}\left(\frac{1-X_1+X_2}{X_2}\right)-\frac{\left(\gamma_2\left(1-\beta\right)+\left(\alpha+\gamma_2\right)^2\beta\right)}{\left(\alpha+\gamma_2\right)^2 q_2}T+\left(\frac{\partial z_2\left(2\right)}{\partial\alpha}+\frac{\beta T}{q_2}\right)\frac{1}{X_2}$$

Now take $\beta=1$, use equation (46) and rearrange to obtain

$$\frac{\partial z_1\left(1\right)}{\partial\alpha}=\frac{T}{\left(q_1-q_2\right)}\left(\frac{-\left(1-X_1-X_2\right)\gamma_2^2 X_2}{\left(\alpha+\gamma_1+\gamma_2\right)^2 X_1}\right)-\frac{T}{\left(q_1-q_2\right)}\left(1-X_1+X_2\right)-\frac{T}{q_2}\left(X_2-1\right)+\frac{\partial z_2\left(2\right)}{\partial\alpha}1$$

$$=-\frac{T}{\left(q_1-q_2\right)}\frac{\left(1-X_1-X_2\right)\gamma_2^2 X_2}{\left(\alpha+\gamma_1+\gamma_2\right)^2 X_1}-\frac{T}{\left(q_1-q_2\right)}\left(1-X_1+X_2\right)<0$$

∎

C.4 Proof of Proposition 7

1. Using the cdf $h_1\left(z\right)$, we have

$$X_1\int_{z_1(1)}^{z}sh_1\left(s\right)ds=X_1\left(\int_{z_1(1)}^{z_1(3)}s\frac{\alpha f\left(s\right)}{\alpha+\gamma_1}ds+\int_{z_1(3)}^{z_1(2)}sf\left(s\right)\left(1+\frac{\frac{1-X}{X_1}\left(\alpha+\gamma_2\right)}{\alpha+\gamma_2+\gamma_1}\right)ds+\int_{z_1(2)}^{1}\frac{sf\left(s\right)}{X_1}ds\right)=$$

$$\frac{\alpha X_1}{\alpha+\gamma_1}\frac{\left(z_1\left(3\right)\right)^2-\left(z_1\left(1\right)\right)^2}{2}+\left(X_1+\frac{\left(1-X_1-X_2\right)\left(\alpha+\gamma_2\right)}{\alpha+\gamma_2+\gamma_1}\right)\frac{\left(z_1\left(2\right)\right)^2-\left(z_1\left(3\right)\right)^2}{2}+\frac{1-\left(z_1\left(2\right)\right)^2}{2}\qquad(47)$$

Substituting

$$z_1\left(3\right)-z_1\left(1\right)=z_1\left(2\right)-z_1\left(3\right)=\frac{T\left(1-\beta\left(1-\alpha-\gamma_1\right)\right)}{\left(q_1-q_2\right)}$$

$$z_1\left(3\right)+z_1\left(1\right)=2z^1\left(1\right)+\frac{T\left(1-\beta\left(1-\alpha-\gamma_1\right)\right)}{\left(q_1-q_2\right)}$$

$$z_1\left(2\right)+z_1\left(3\right)=2z^1\left(2\right)-\frac{T\left(1-\beta\left(1-\alpha-\gamma_1\right)\right)}{\left(q_1-q_2\right)}$$

in equation (47), we obtain

$$\frac{\alpha X_1}{\alpha + \gamma_1}\left(\frac{1}{2}\left(\frac{T\left(1 - \beta\left(1 - \alpha - \gamma_1\right)\right)}{\left(q_1 - q_2\right)}\right)^2 + z_1\left(1\right)\frac{T\left(1 - \beta\left(1 - \alpha - \gamma_1\right)\right)}{\left(q_1 - q_2\right)}\right) +$$

$$\left(X_1 + \frac{\left(1 - X_1 - X_2\right)\left(\alpha + \gamma_2\right)}{\alpha + \gamma_2 + \gamma_1}\right)\left(z_1\left(2\right)\frac{T\left(1 - \beta\left(1 - \alpha - \gamma_1\right)\right)}{q_1 - q_2} - \frac{1}{2}\left(\frac{T\left(1 - \beta\left(1 - \alpha - \gamma_1\right)\right)}{\left(q_1 - q_2\right)}\right)^2\right) + \frac{1 - \left(z_1\left(2\right)\right)^2}{2}$$

$$= \quad \frac{1}{2}\left(-X_1 - \frac{\left(1 - X_1 - X_2\right)\left(\alpha + \gamma_2\right)}{\alpha + \gamma_2 + \gamma_1} + \frac{\alpha X_1}{\alpha + \gamma_1}\right)\left(\frac{T\left(1 - \beta\left(1 - \alpha - \gamma_1\right)\right)}{q_1 - q_2}\right)^2 +$$

$$z_1\left(1\right)\frac{T\left(1 - \beta\left(1 - \alpha - \gamma_1\right)\right)}{\left(q_1 - q_2\right)}\frac{\alpha X_1}{\alpha + \gamma_1} + z_1\left(2\right)\frac{T\left(1 - \beta\left(1 - \alpha - \gamma_1\right)\right)}{q_1 - q_2}\left(X_1 + \frac{\left(1 - X_1 - X_2\right)\left(\alpha + \gamma_2\right)}{\alpha + \gamma_2 + \gamma_1}\right) + \frac{1 - \left(z_1\left(2\right)\right)^2}{2}$$

Differentiating with respect to α and evaluating at $\beta = 1$

$$\frac{1}{2}\left(\frac{X_1}{\alpha + \gamma_1} - \frac{1 - X_2 - X_1}{\alpha + \gamma_1 + \gamma_2} - \frac{X_1\alpha}{\left(\alpha + \gamma_1\right)^2} + \frac{\left(1 - X_2 - X_1\right)\left(\alpha + \gamma_2\right)}{\left(\alpha + \gamma_1 + \gamma_2\right)^2}\right)\left(\frac{T\left(\alpha + \gamma_1\right)}{\left(q_1 - q_2\right)}\right)^2$$

$$+ \left(-X_1 - \frac{\left(1 - X_1 - X_2\right)\left(\alpha + \gamma_2\right)}{\alpha + \gamma_2 + \gamma_1} + \frac{\alpha X_1}{\alpha + \gamma_1}\right)\left(\frac{\left(\alpha + \gamma_1\right)T^2}{\left(q_2 - q_1\right)^2}\right)$$

$$+ \frac{\partial z_1\left(1\right)}{\partial \alpha}\frac{T\left(\alpha + \gamma_1\right)}{\left(q_1 - q_2\right)}\frac{\alpha X_1}{\alpha + \gamma_2} + \frac{\partial z_1\left(2\right)}{\partial \alpha}\frac{T\left(\alpha + \gamma_1\right)}{\left(q_1 - q_2\right)}\left(X_1 + \frac{\left(1 - X_1 - X_2\right)\left(\alpha + \gamma_2\right)}{\alpha + \gamma_2 + \gamma_1}\right)$$

$$+ \frac{z_1\left(1\right)\left(2\alpha\gamma_2 + \gamma_1\gamma_2 + \alpha^2\right)TX_1}{\left(q_1 - q_2\right)\left(\alpha + \gamma_2\right)^2} - z_1\left(2\right)\frac{\partial z_1\left(2\right)}{\partial \alpha}$$

$$+ z_1\left(2\right)\left(\frac{\left(\alpha + \gamma_1\right)\left(1 - X_1 - X_2\right)T\gamma_1}{\left(q_1 - q_2\right)\left(\alpha + \gamma_1 + \gamma_2\right)^2} - \frac{\left(\alpha + \gamma_2 - \alpha X_2\right)T}{\left(q_1 - q_2\right)\left(\alpha + \gamma_1 + \gamma_2\right)}\right)$$

Substituting for $\frac{\partial z_1(1)}{\partial \alpha}$, $\frac{\partial z_1(2)}{\partial \alpha}$, $z_1\left(2\right)$ and rearranging, we obtain

$$\frac{1}{2}\left(-\frac{\left(1 - X_2 - X_1\right)\gamma_1}{\left(\alpha + \gamma_1 + \gamma_2\right)^2}\right)\left(\frac{T\left(\alpha + \gamma_1\right)}{\left(q_1 - q_2\right)}\right)^2 - \left(\frac{\gamma_1 X_1}{2\left(\alpha + \gamma_1\right)} + \frac{\left(1 - X_1 - X_2\right)\left(\alpha + \gamma_2\right)}{\alpha + \gamma_2 + \gamma_1}\right)\frac{\left(\alpha + \gamma_1\right)T^2}{\left(q_2 - q_1\right)^2} \quad (48)$$

$$- \left(\frac{\left(1 - X_1 - X_2\right)\gamma_2^2 X_2}{\left(\alpha + \gamma_1 + \gamma_2\right)^2 X_1} + \left(1 - X_1 + X_2\right)\right)\frac{\left(\alpha + \gamma_1\right)T^2}{\left(q_2 - q_1\right)^2}\frac{\alpha X_1}{\alpha + \gamma_2}$$

$$- \left(2X_1 + \frac{\left(1 - X_1 - X_2\right)\left(\alpha + \gamma_2\right)}{\alpha + \gamma_2 + \gamma_1} + \frac{\left(1 - X_1 - X_2\right)\left(\alpha + \gamma_1\right)\gamma_1}{\left(\alpha + \gamma_1 + \gamma_2\right)^2}\right)\frac{\left(\alpha + \gamma_1\right)T^2}{\left(q_2 - q_1\right)^2}\left(\frac{\alpha + \gamma_2 + \alpha X_2 + 2\gamma_1}{\alpha + \gamma_1 + \gamma_2}\right)$$

$$+ \frac{2\left(\alpha + \gamma_1\right)T^2}{\left(q_2 - q_1\right)^2}\left(\frac{\left(\alpha + \gamma_1\right)\left(1 - X_1 - X_2\right)\gamma_1}{\left(\alpha + \gamma_1 + \gamma_2\right)^2} - \frac{\left(\alpha + \gamma_2 - \alpha X_2\right)}{\left(\alpha + \gamma_1 + \gamma_2\right)}\right)$$

$$- z_1\left(1\right)\frac{T}{\left(q_1 - q_2\right)}\left(\frac{\left(\alpha + \gamma_2 - \alpha X_2\right)}{\left(\alpha + \gamma_1 + \gamma_2\right)} + 2X_1 + \frac{\left(1 - X_1 - X_2\right)\left(\alpha + \gamma_2\right)}{\alpha + \gamma_2 + \gamma_1} - \frac{\left(2\alpha\gamma_2 + \gamma_1\gamma_2 + \alpha^2\right)X_1}{\left(\alpha + \gamma_2\right)^2}\right)$$

Note that

$$\frac{\left(\alpha + \gamma_1\right)\left(1 - X_1 - X_2\right)\gamma_1}{\left(\alpha + \gamma_1 + \gamma_2\right)^2} - \frac{\left(\alpha + \gamma_2 - \alpha X_2\right)}{\left(\alpha + \gamma_1 + \gamma_2\right)} = \frac{-\gamma_1\gamma_2 - 2\alpha\gamma_2 - \alpha^2\left(1 - X_2\right) + \gamma_1^2 - \gamma_2^2 - X_1\gamma_1^2 - X_2\gamma_1^2}{\left(\alpha + \gamma_1 + \gamma_2\right)^2} < 0$$

since $\gamma_2 > \gamma_1$. Hence all terms are negative and the result follows.

2. Using the steady state distribution (12) we obtain

$$X_2 \int_{z_2(2)}^{z_1(2)} sh_2(s) \, ds$$

$$= X_2 \int_{z_2(2)}^{z_2(3)} sh_2(s) \, ds + X_2 \int_{z_2(3)}^{z_1(1)} sh_2(s) \, ds + X_2 \int_{z_1(1)}^{z_1(3)} sh_2(s) \, ds + X_2 \int_{z_1(3)}^{z_1(2)} sh_2(s) \, ds$$

$$= X_2 \left(\int_{z_2(2)}^{z_2(3)} \frac{s\alpha}{\alpha + \gamma_2} ds + \int_{z_2(3)}^{z_1(1)} \frac{s}{X_2} ds + \int_{z_1(1)}^{z_1(3)} s\left(\frac{1 - X_1}{X_2} + \frac{\gamma_2}{\alpha + \gamma_1} \right) ds + \int_{z_1(3)}^{z_1(2)} s\left(1 + \frac{\frac{1-X}{X_1}\gamma_2}{\alpha + \gamma_2 + \gamma_1} \right) ds \right)$$

$$= \left(\frac{\alpha X_2}{\alpha + \gamma_2} \right) \left(\frac{(z_2(3))^2 - (z_2(2))^2}{2} \right) + \left(\frac{(z_1(1))^2 - (z_2(3))^2}{2} \right) + \tag{49}$$

$$\left(1 - X_1 + \frac{\gamma_2 X_2}{\alpha + \gamma_1} \right) \left(\frac{(z_1(3))^2 - (z_1(1))^2}{2} \right) + \left(X_2 + \frac{\frac{1-X_1-X_2}{X_1}\gamma_2 X_2}{\alpha + \gamma_2 + \gamma_1} \right) \left(\frac{(z_1(2))^2 - (z_1(3))^2}{2} \right)$$

Substituting

$$(z_2(3))^2 - (z_2(2))^2 = (z_2(3) - z_2(2))(z_2(3) + z_2(2)) = \frac{T(1 - \beta(1 - \alpha - \gamma_2))}{q_2} \left(2z^2(3) - \frac{T(1 - \beta(1 - \alpha - \gamma_2))}{q_2} \right)$$

$$(z_1(3))^2 - (z_1(1))^2 = \frac{T(1 - \beta(1 - \alpha - \gamma_1))}{q_1 - q_2} \left(2z^1(1) + \frac{T(1 - \beta(1 - \alpha - \gamma_1))}{q_1 - q_2} \right)$$

$$(z_1(2))^2 - (z_1(3))^2 = \frac{T(1 - \beta(1 - \alpha - \gamma_1))}{q_1 - q_2} \left(2z^1(2) - \frac{T(1 - \beta(1 - \alpha - \gamma_1))}{q_1 - q_2} \right)$$

in equation (49), we obtain

$$= \frac{1}{2} \left(\frac{\alpha X_2}{\alpha + \gamma_2} \right) \frac{T(1 - \beta(1 - \alpha - \gamma_2))}{q_2} \left(2z^2(3) - \frac{T(1 - \beta(1 - \alpha - \gamma_2))}{q_2} \right) + \left(\frac{(z_1(1))^2 - (z_2(3))^2}{2} \right)$$

$$+ \frac{1}{2} \left(1 - X_1 + \frac{\gamma_2 X_2}{\alpha + \gamma_1} \right) \frac{T(1 - \beta(1 - \alpha - \gamma_1))}{q_1 - q_2} \left(2z^1(1) + \frac{T(1 - \beta(1 - \alpha - \gamma_1))}{q_1 - q_2} \right)$$

$$+ \frac{1}{2} \left(X_2 + \frac{\frac{1-X_1-X_2}{X_1}\gamma_2 X_2}{\alpha + \gamma_2 + \gamma_1} \right) \frac{T(1 - \beta(1 - \alpha - \gamma_1))}{q_1 - q_2} \left(2z^1(2) - \frac{T(1 - \beta(1 - \alpha - \gamma_1))}{q_1 - q_2} \right)$$

Take $\beta = 1$, and differentiate with respect to α

$$= \frac{1}{2} (\alpha X_2) \frac{T}{q_2} \left(2z^2(3) - \frac{T(\alpha + \gamma_2)}{q_2} \right) + \frac{1}{2} \left(1 - X_1 + \frac{\gamma_2 X_2}{\alpha + \gamma_1} \right) \frac{T(\alpha + \gamma_1)}{q_1 - q_2} \left(2z^1(1) + \frac{T(\alpha + \gamma_1)}{q_1 - q_2} \right)$$

$$+ \frac{1}{2} \left(X_2 + \frac{\frac{1-X_1-X_2}{X_1}\gamma_2 X_2}{\alpha + \gamma_2 + \gamma_1} \right) \frac{T(\alpha + \gamma_1)}{q_1 - q_2} \left(2z^1(2) - \frac{T(\alpha + \gamma_1)}{q_1 - q_2} \right) + \left(\frac{(z_1(1))^2 - (z_2(3))^2}{2} \right)$$

After repeated substitutions and some tedious algebra, the above term simplifies to

$$-\frac{(2\alpha + \gamma_2 - 2\alpha X_2) T^2 X_2}{2q_2^2} + \frac{T^2}{2(q_1 - q_2)^2} \left(4X_1 X_2 + X_1 \gamma_1 + 2\alpha X_1 + 2\alpha X_1 X_2 - 2\alpha X_1^2 \right) + \tag{50}$$

$$\frac{T^2(1 - X_1 - X_2)\gamma_1}{2(\alpha + \gamma_2 + \gamma_1)^2 (q_1 - q_2)^2} \left(2\alpha + 2\gamma_2 + 2\alpha X_1 + 4X_1 \gamma_1 + 2X_1 \gamma_2 + 2X_2 \gamma_1 - 2X_2 \gamma_2 + 2\alpha X_1 \gamma_2 - (\alpha + \gamma_1)^2 \right) +$$

$$\frac{(1 - X_1 - X_2)\gamma_1}{\alpha + \gamma_2 + \gamma_1} \frac{T^2}{(q_1 - q_2)^2} \left(\frac{(1 - X_1 - X_2)(\alpha + \gamma_1)\gamma_1}{(\alpha + \gamma_1 + \gamma_2)^2} + \frac{(\alpha + \gamma_2) X_2}{\gamma_1} \right)$$

Note that the first term is negative while the second, third and forth are positive. Moreover, at $q_1 - q_2 = q_2$

$$\frac{T^2}{2(q_1 - q_2)^2}\left(4X_1 X_2 + 2\alpha(X_1 - X_2)(1 - X_1 - X_2) + 2\alpha X_1 X_2\right)$$

$$+\frac{T^2(1 - X_1 - X_2)\gamma_1\left(2\alpha + 2\gamma_2 + 2\alpha X_1 + 4X_1\gamma_1 + 2X_1\gamma_2 + 2X_2\gamma_1 - 2X_2\gamma_2 + 2\alpha X_1\gamma_2 - (\alpha + \gamma_1)^2\right)}{2(\alpha + \gamma_2 + \gamma_1)^2(q_1 - q_2)^2}$$

$$+\frac{(1 - X_1 - X_2)\gamma_1}{\alpha + \gamma_2 + \gamma_1}\frac{T^2}{(q_1 - q_2)^2}\left(\frac{(1 - X_1 - X_2)(\alpha + \gamma_1)\gamma_1}{(\alpha + \gamma_1 + \gamma_2)^2} + \frac{(\alpha + \gamma_2)X_2}{\gamma_1}\right)$$

all terms are positive. Hence, at $\beta = 1$, there exists a $q_2^* < q_1/2$ such that for all $q_2 > q_2^*$ the derivative is positive. Similarly at $q_2 = 0$ we have that the first term in equation (50) dominates and the derivative is negative.

3. Note that

$$Y = \sum_{i=1,2}\int_0^1 q_i X_i sh_i(s)\,ds$$

$$\frac{\partial Y}{\partial\alpha} = \sum_{i=1,2} q_i X_i \frac{\partial}{\partial\alpha}\int_0^1 sh_i(s)\,ds$$

We will prove the extreme case of $q_1 = q_2$. All cases of $q_1 < q_2$ follows. Summing the derivatives (48) and (50), we obtain

$$-\left(2X_1 + \frac{(1 - X_1 - X_2)(\alpha + \gamma_2)}{\alpha + \gamma_2 + \gamma_1} + \frac{(1 - X_1 - X_2)(\alpha + \gamma_1)\gamma_1}{(\alpha + \gamma_1 + \gamma_2)^2}\right)\frac{(\alpha + \gamma_1)T^2}{(q_2 - q_1)^2}\left(\frac{\alpha X_2 + \gamma_1}{\alpha + \gamma_1 + \gamma_2}\right) \tag{51}$$

$$+(z_1(1))\frac{T}{(q_1 - q_2)}\left(-\frac{(\alpha + \gamma_2 - \alpha X_2)}{(\alpha + \gamma_1 + \gamma_2)} - 2X_1 - \frac{(1 - X_1 - X_2)(\alpha + \gamma_2)}{\alpha + \gamma_2 + \gamma_1} + \frac{(2\alpha\gamma_2 + \gamma_1\gamma_2 + \alpha^2)X_1}{(\alpha + \gamma_2)^2}\right)$$

$$-\frac{(2\alpha + \gamma_2 - 2\alpha X_2)T^2 X_2}{2q_2^2} + \frac{T^2 X_1}{(q_1 - q_2)^2}\left(2X_2 - \alpha + \alpha X_2 - 2\gamma_1 - \alpha X_1 - (1 - X_1 + X_2)\frac{\alpha(\alpha + \gamma_1)}{\alpha + \gamma_2}\right)$$

$$+\frac{T^2(1 - X_1 - X_2)\gamma_1}{(\alpha + \gamma_2 + \gamma_1)^2(q_1 - q_2)^2}\left(\alpha + \gamma_2 + \alpha X_1 + X_1\gamma_1 + X_1\gamma_2 + X_2\gamma_1 + \alpha X_1\gamma_2 - 2(\alpha + \gamma_1)^2 - \frac{(\alpha + \gamma_1)\alpha\gamma_2 X_1}{\alpha + \gamma_2}\right)$$

$$-\frac{2(\alpha + \gamma_1)T^2}{(q_2 - q_1)^2}\left(\frac{(\alpha + \gamma_1)(1 - X_1 - X_2)\gamma_1}{(\alpha + \gamma_1 + \gamma_2)^2} - \frac{(\alpha + \gamma_2 - \alpha X_2)}{(\alpha + \gamma_1 + \gamma_2)}\right)$$

$$+\frac{(1 - X_1 - X_2)\gamma_1}{\alpha + \gamma_2 + \gamma_1}\frac{T^2}{(q_1 - q_2)^2}\left(\frac{(1 - X_1 - X_2)(\alpha + \gamma_1)\gamma_1}{(\alpha + \gamma_1 + \gamma_2)^2} + \frac{(\alpha + \gamma_2)X_2}{\gamma_1} - \frac{(\alpha + \gamma_1)(\alpha + \gamma_2)}{\gamma_1}\right)$$

note that

$$z_1(1) > (1 - X_1 - X_2) \text{ and } \frac{T}{(q_1 - q_2)} < 1$$

where the second inequality follows from

$$X_2 > z_1(3) - z_1(1) = \frac{T(\alpha + \gamma_1)}{(q_1 - q_2)} = \frac{T}{(q_1 - q_2)}$$

evaluated at $\alpha + \gamma_1 = 1$. Hence

$$z_1(1)\frac{T}{(q_1 - q_2)} > \frac{(1 - X_1 - X_2)}{X_2}\frac{T^2}{(q_1 - q_2)^2}$$

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Therefore, we have that equation (51) is striclty less than

$$\frac{(1-X_1-X_2)}{X_2}\frac{T^2}{(q_1-q_2)^2}\left(-\frac{(\alpha+\gamma_2-\alpha X_2)}{(\alpha+\gamma_1+\gamma_2)}-X_1\right)-\left(\frac{(1-X_1-X_2)\gamma_2^2 X_2}{(\alpha+\gamma_1+\gamma_2)^2 X_1}\right)\frac{(\alpha+\gamma_1)T^2}{(q_2-q_1)^2}\frac{\alpha X_1}{\alpha+\gamma_2}$$

$$-\left(\frac{(1-X_1-X_2)(\alpha+\gamma_2)}{\alpha+\gamma_2+\gamma_1}+\frac{(1-X_1-X_2)(\alpha+\gamma_1)\gamma_1}{(\alpha+\gamma_1+\gamma_2)^2}\right)\frac{(\alpha+\gamma_1)T^2}{(q_2-q_1)^2}\left(\frac{\alpha X_2+\gamma_1}{\alpha+\gamma_1+\gamma_2}\right)$$

$$+\frac{2(\alpha+\gamma_1)T^2}{(q_2-q_1)^2}\left(\frac{(\alpha+\gamma_1)(1-X_1-X_2)\gamma_1}{(\alpha+\gamma_1+\gamma_2)^2}-\frac{(\alpha+\gamma_2-\alpha X_2)}{(\alpha+\gamma_1+\gamma_2)}\right)+\frac{(1-X_1-X_2)(\alpha+\gamma_2)X_2}{\alpha+\gamma_2+\gamma_1}\frac{T^2}{(q_1-q_2)^2}$$

$$-\left(\frac{2(1-X_1-X_2)(\alpha+\gamma_2)(\alpha+\gamma_1)+2X_1(\alpha X_2+\gamma_1)(\alpha+\gamma_1)}{\alpha+\gamma_2+\gamma_1}+\frac{(1-X_1+X_2)(\alpha+\gamma_1)\alpha X_1}{\alpha+\gamma_2}\right)\frac{T^2}{(q_1-q_2)^2}$$

$$+\frac{(1-X_1-X_2)}{X_2}\frac{T^2}{(q_1-q_2)^2}\left(-X_1+\frac{(2\alpha\gamma_2+\gamma_1\gamma_2+\alpha^2)X_1}{(\alpha+\gamma_2)^2}\right)-\frac{(2\alpha+\gamma_2-2\alpha X_2)T^2 X_2}{2q_2^2}$$

$$-\frac{T^2}{(q_1-q_2)^2}\alpha X_1(X_1-X_2)-\frac{(1-X_1-X_2)^2}{\alpha+\gamma_2+\gamma_1}\frac{T^2}{(q_1-q_2)^2}\left(\frac{\gamma_1^2\alpha-\gamma_1^2 X_2\alpha+\gamma_1^2\gamma_2-\gamma_1^3 X_2}{(\gamma_1+\gamma_2+\alpha)^2 X_2}\right)$$

$$-\frac{(1-X_1-X_2)^2}{\alpha+\gamma_2+\gamma_1}\frac{T^2}{(q_1-q_2)^2}\left(\frac{4\alpha\gamma_1\gamma_2+2\alpha^2\gamma_1+2\gamma_2^2\gamma_1+(\alpha+\gamma_2)^3}{(\gamma_1+\gamma_2+\alpha)^2 X_2}\right)$$

$$+\frac{T^2(1-X_1-X_2)\gamma_1}{2(\alpha+\gamma_2+\gamma_1)^2(q_1-q_2)^2}(2\alpha+2\gamma_2+2X_2\gamma_2+2X_1\gamma_2+2\alpha X_1+2X_2\gamma_1+2\alpha X_1\gamma_2)$$

$$+\left(-2X_1(-2X_2+\alpha+2\gamma_1)+\frac{(1-X_1-X_2)(\alpha+\gamma_2)X_2}{\alpha+\gamma_2+\gamma_1}\right)\frac{T^2}{(q_1-q_2)^2}$$

Note that the only positive terms are the last two. Moreover, all terms are of order higher that the first one, which is negative. Hence the result follows. I omit the details of the calculations.

C.5 Proof of Proposition 8

Consider a q_1 aircraft. The depreciations probabilities are the same for leased and owned aircraft, so I focus on the effects of the new draws of the efficiency parameter z. Conditional on receiving a new draw of efficiency, a carrier with a leased aircraft keeps it if $z > z^{1^*}(3)$. A carrier with an owned aircraft keeps it if $z > z_1(1)$. Since the current level of efficiency has no effect on the next new draw of efficiency, the result follows for q_1 aircraft since $z^{1^*}(3) > z_1(1)$.

Consider a q_2 aircraft. Conditional on receiving a new draw of efficiency, a carrier with a leased aircraft keeps it if $z_1(3) < z < z_2(3)$. A carrier with an owned aircraft keeps it if $z_1(2) \leq z \leq z_2(2)$. Since the current level of efficiency has no effect on the next new draw of efficiency, the result follows aircraft since $z_1(3) < z_1(2)$ and $z_2(3) < z_2(2)$.

Hence leased aircraft trade more frequently have an average holding duration lower than owned aircraft.

C.6 Proof of Proposition 9 and Proof of Corollary 10

The Proofs are almost identical to the Proof of Proposition 7 and are therefore omitted.

C.7　Proof of Proposition 11

A z high enough firm keeps operating the aircraft when hit by a temporary shock. Hence

$$
\begin{aligned}
V_L &= \frac{zq - f - r_1 + \beta\alpha J_L + \beta\mu \max\{V_L, V_3, V_1 - p_1\}}{1 - \beta(1 - \alpha - \mu)} \\
&= \frac{zq - f - r_1 + \beta\alpha J_L - \beta\mu(1 - b)zq_1}{1 - \beta(1 - \alpha)} \\
V_1(z, 1) &= \frac{zq_1(1 - \beta\mu(1 - b)) - f + \beta\alpha J_1 - \beta\gamma_1 p_1}{1 - \beta(1 - \alpha)}
\end{aligned}
$$

Thus, high productivity firms lease if and only if

$$
\begin{aligned}
\frac{zq - f - r_1 + \beta\alpha J_L - \beta\mu(1 - b)zq_1}{1 - \beta(1 - \alpha)} &\geq \frac{zq_1(1 - \beta\mu(1 - b)) - f + \beta\alpha J_1 - \beta\gamma_1 p_1}{1 - \beta(1 - \alpha)} - p_1 \\
\frac{r_1 + \beta\alpha(J_1 - J_L)}{1 - \beta(1 - \alpha - \gamma_1)} &\leq p_1
\end{aligned}
$$

For the firm whose z is just above $z_1(1, 3)$ we have

$$
\begin{aligned}
V_L &= \frac{zq_1 - f - r_1 + \beta\alpha J_L + \beta\mu V_3}{1 - \beta(1 - \alpha - \mu)} \\
&= \frac{zq_1 - f - r_1 + \beta\alpha J_L + \beta\mu\left(\frac{\beta\alpha J_L}{1 - \beta\mu}\right)}{1 - \beta(1 - \alpha)}(1 - \beta\mu) \\
V_1(z, 1) &= \frac{(zq_1 - f)(1 - \beta\mu) + \beta\alpha J_1 - \beta\gamma_1 p_1}{1 - \beta(1 - \alpha)}
\end{aligned}
$$

Hence they lease if and only if

$$
\begin{aligned}
\frac{zq - f - r_1 + \beta\alpha J_L + \beta\mu\left(\frac{\beta\alpha J_L}{1 - \beta\mu}\right)}{1 - \beta(1 - \alpha)}(1 - \beta\mu) &\geq \frac{(zq_1 - f)(1 - \beta\mu) + \beta\alpha J_1 - \beta\gamma_1 p_1}{1 - \beta(1 - \alpha)} - p_1 \\
-\frac{\left(-r_1 + \beta\alpha J_L + \beta\mu\left(\frac{\beta\alpha J_L}{1 - \beta\mu}\right)\right)(1 - \beta\mu) + \beta\alpha J_1}{1 - \beta(1 - \alpha - \gamma_1)} &\leq p_1
\end{aligned}
$$

So suppose that the proposition is false and only the lower productivity firm leases. Then it must be that

$$
r_1 + \beta\alpha(J_1 - J_L) < r_1(1 - \beta\mu) + \beta\alpha(J_1 - J_L)
$$

Impossible

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