The Consistency of Social and Individual Choices

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Abstract

Arrow famously shows that, under basic desiderata, the logical structure of social and individual choices may differ. They cannot simultaneously satisfy WARP. This paper shows simple aggregation rules such that, under Arrow's desiderata, individual and social choice can simultaneously satisfy Weak WARP. Thus, for decisions making processes characterized by Weak WARP, social and individual choice can have compatible logical structures.

1. Introduction

The idea that individual and social decisions may have incompatible logical structures is deeply ingrained in social science and can be traced back at least to Condorcet (1785). This idea was crystallized in Arrow's impossibility theorem (1950). Arrow considers mechanisms that aggregate individual choices into choice choices and satisfy basic desiderata: non-dictatorship, independence of irrelevant alternatives and Pareto optimality. He shows that if individuals rank alternatives with a single order, and so individual choices satisfy WARP (Samuelson (1938)), then social choices do not necessarily satisfy WARP. This result shows a fundamental discrepancy between individual and social choice that was confirmed in several extensions and related results. While the broad conclusion of a discrepancy between individual and groups decisions has rarely been challenged, empirical evidence suggests that individual choices may not satisfy WARP (see May (1954), Simonson and Tversky (1993), and Hubler et. al., (1982) for a few examples on a large literature). The difficulties with WARP was reinforced when specific models of ordinary procedures for individual decision making were shown to produce violations of WARP (see, Manzini and Mariotti (2007), (2012a)), Cherepanov et. al., (2013a), Lleras et. al., (2012) for a few examples on an also large literature). These models produce choices that violate WARP, but satisfy a natural generalization of WARP, called Weak WARP. Hence, Weak WARP provides the axiomatic foundation of individual decision making in a variety of models of seemingly unrelated procedures such as categorization, rationalization, limited attention and even mere compliance with the law.

The empirical evidence and modern decision-theoretic analysis suggests that individuals might have internal conflicts that may lead them to systematically violate WARP. However, individual choices are not arbitrary and may satisfy a weakening of WARP. This, by itself, does not imply that social and individual choices may have compatible logical structures. A priory, there is no reason to believe that if each individual decides as a dual-self then a group of such individuals may also decide as a dual- self (in the same way that a group of single-self individuals may not decide as a single-self).

Consider several individuals. Their choice functions are aggregated into a social choice function through a delegation process. That is, for each decision, one individual is selected to choose for the group. The identity of the decider may also depend on the profile of individual choice functions. The delegation rules must satisfy Arrow's desiderata. They must be non-dictatorial and satisfy independence of irrelevant alternatives (the Pareto principle is implicit in any delegation rule). Under this desiderata, simple delegation rules are constructed such that if individual choices satisfy Weak WARP then social choices must also satisfy Weak WARP. Therefore, when WARP is replaced by Weak WARP as the axiomatic foundation of choice, then the logical structure of social and individual choices may be consistent. In any model of decision making characterized by Weak WARP, social and individual choice can have similar logical structures.

The consistency between social and individual choice must, however, be qualified. It uses specific delegation rules and there are limits on the non-dictatorial qualities that can be achieved. Consider liberalism in the sense of Sen (1970). That is, for each individual there is a choice such that this individual is the always the social decider, regardless of the choices of anyone else. There are no liberal delegation rules that satisfy independence of irrelevant alternatives and such that social choices must satisfy Weak WARP, when individual choices do.

This paper is organized as follows: Section 1.1 shows a literature review. Main concepts and results are in section 2. Section 3 concludes. Proofs are in the appendix.

2. Related Literature

A growing literature focuses on conflicting motivations that may lead to violations of WARP in individual decision making. See, among many contributions, Ambrus and Rozen (2008), Apesteguia and Ballester (2008), Berheim and Rangel (2009), Chambers and Hayashi (2008), Cherepanov et. al., (2013a), (2013b), Clippel and Eliaz (2012), Clippel and Rozen (2012), Dietrich and List (2010), Eliaz, Richter and Rubinstein (2011), Eliaz and Spiegler (2011), Fudenberg and Levine (2006), Green and Hojman (2007), Heller (2009), Houy (2007), Houy and Tademuma (2009), Kalai, Rubinstein and Spiegler (2002), Katz and Sandroni (2014), Lleras et. al., (2012), Lehrer and Teper (2011), Masatlioglu and Nakajima (2007), Masatlioglu and Ok (2005), Manzini and Mariotti (2007), (2012a), (2012b), Masatlioglu, Nakajima and Ozbay (2012), Ok, Ortoleva, and Riella (2008), Salant and Rubinstein (2006), (2006a), (2011), Spiegler (2002), Tyson (2012).

Inconsistencies between social and individual decision making has been extensively explored in modern research. For example, Mongin (1995) shows difficulties in aggregating subjective probabilities, Jackson and Yariv (2011) shows difficulties with time consistent collective choices, and Zuber (2010) shows difficulties in aggregating preferences (see also Blackorby, Bossert, and Donaldson (2005), Browning (2000), Caplin and Leahy (2004), Gollier and Zeckhauser (2005), Hertzberg (2011), and Weiztman (2001)). Sobel (2014) studies the relationship between individual and group decisions in the context of information aggregation in a common values setting, see also Eliaz, Ray and Razin (2006).

If social preferences are not required to be transitive then there are aggregation rules that satisfy Arrow's other conditions. See, among other contributions, Brown (1975), Gibbard (2014) and Schick (1963) for an analysis. These results do not show a general form of consistency between social and individual decision making.

3. Basic Concepts

Let A be a finite set of alternatives. A non-empty subset $B \subseteq A$ of alternatives is called an *issue*. Let $\mathcal{B}$ be the set of all issues. A *choice function* is a mapping $C : \mathcal{B} \longrightarrow A$ such that $C(B) \in B$ for every $B \in \mathcal{B}$. So, a choice function is a mapping that takes an issue as input and returns a choice as output. Let $\mathcal{C}$ be the set of all choice functions.

Weak Axiom of Revealed Preference (WARP) A choice function C satisfies WARP iff

$$B \subseteq B^*,\ C(B^*) \in B \text{ then } C(B^*) = C(B).$$

The WARP axiom characterizes standard decision theory where agents optimize a single ordering of alternatives given the constraints B.

Weak WARP A choice function C satisfies Weak WARP iff

$$x \neq y,\ \{x, y\} \subseteq B \subseteq B^*,\ \text{if } C(\{x, y\}) = C(B^*) = x \text{ then } C(B) \neq y.$$

The Weak WARP axiom is a relaxation of WARP. It characterizes theories of constraint optimization, with additional constraints to the physical ones in B. These additional constraints can be psychological constraints (Cherepanov et. al., (2013a)), attention constraints (Lleras et. al., (2012)), or categorization constraints (Manzini and Mariotti (2011)); see Ehlers and Sprumont (2008) for related axioms.

Several ordinary decision procedures may violate WARP, but satisfy Weak WARP. In the case of categorization constraints, the decision maker decides in stages. For example, she may first decide to buy an American car and then choose the American car she likes best. In the case of attention constraints, she selects the alternative that she prefers among the options she noticed. In the case of psychological constraints, she selects the options that she wants, provided that she can justify her choice. Perhaps the simplest way to illustrate the broad point is to consider a law-abiding citizen, as in Katz and Sandroni (2014)). This citizen ranks alternatives by her order and, in addition, has a disutility for breaking the law (perhaps due to fear of punishment or to moral considerations). This is quite ordinary. There is nothing particularly irrational about ranking alternatives or fearing punishment. Now consider the following alternatives: option x is to endure physical injures that may result in permanent impairment, option y is react with lethal force, option z is to give up a large sum of money. Let's say that the law-abiding citizen ranks y over x over z. So, between y and x, the law-abiding citizen chooses y because this is her preferred choice and they are both legal (y may be taken in self-defense). But between x, y and z, she may decide for x because she prefers x over z and now y is no longer legal (and x is always legal). One cannot kill in self-defense if killing can be avoided by a monetary transfer. If the fear of punishment for breaking the law is sufficiently large, she chooses x on $\{x, y, z\}$, even though, combined with the choice of y on $\{x, y\}$, this violates WARP. In contrast, Weak WARP is not violated because no matter which other alternative w is made available, y is not legal if z is feasible.

This violation of WARP arises because the legality of an option depends on what was done and what could be done. In the example, y is legal in the choice between y and x, but not in the choice between x, y and z. Hence, the disutility from breaking the law depends on what is chosen and what is available. When a law-abiding citizen factors in her fear of punishment from the law, she cannot express her overall preferences with a single order. Hence, in what follows, each decision maker is associated with a choice function C. The profile of individual choice functions is then aggregated into a social choice function.

To ease the notation, consider first the case of two agents (the extension to n agents is straightforward and considered below). Agent i's individual choice function is given by $C_i, i = 1, 2$. A *delegation rule* is a mapping $\varphi : \mathcal{B}\text{x}\mathcal{C}^2 \longrightarrow \{1, 2\}$ that determines which agent decides for the group on each issue (depending on the profile of individual choice functions). Given a delegation rule φ and a pair of choice functions (C_1, C_2), the social choice function C_φ is such that $C_\varphi(B) = C_{\varphi(B,C_1,C_2))}(B)$ for every issue $B \in \mathcal{B}$.

Dictatorship A delegation rule φ *is* dictatorial if there exists an agent $i = 1, 2$ such that $\varphi(B, C_1, C_2) = i$, for every issue B and choice functions (C_1, C_2).

A delegation rule is dictatorial if the same agent always decides for the group. Given an issue B, the choice functions C and $\tilde{C}$ are $B-$*equivalent* if $C(B') = \tilde{C}(B')$ for every issue $B' \subseteq B$. So, two choice functions are $B-$equivalent if they produce the same choices in every issue contained in B, including B.

Independence of Irrelevant Alternatives (IIA) A delegation rule φ satisfies IIA if for every issue B, $\varphi(B, C_1, C_2) = \varphi(B, \tilde{C}_1, \tilde{C}_2)$ whenever C_i and $\tilde{C}_i$ are $B-$equivalent, $i = 1, 2$.

A delegation rule satisfies IIA if the social decider on issue B depends only on the individual choices involving alternatives in B. It does not involve individual choices on options outside B. One could relax the IIA requirement on delegation rules when WARP is relaxed on individual choice functions. However, as shown below, this may not be necessary.

Social and Individual Consistency A delegation rule φ is WARP-consistent if the social choice function C_φ satisfies WARP whenever the individual choice functions C_i satisfies WARP, $i = 1, 2$. A delegation rule φ is Weak WARP-consistent if the social choice function C_φ satisfies Weak WARP whenever the individual choice functions C_i satisfies Weak WARP, $i = 1, 2$.

So, if the delegation rule is consistent then individual and social choices have the same logical structure (based on either WARP or Weak WARP). Arrow's impossibility theorem implies that if a delegation rule satisfies IIA and is WARP-consistent then it is dictatorial. In contrast,

Theorem 1 There exist delegation rules that satisfy IIA, are Weak WARP-consistent and are non-dictatorial.

Theorem 1 shows that individual and social behavior may have the same logical structure. They can be Weak WARP consistent, even if the delegation rules satisfy the desiderata in Arrow's theorem. Thus, the WARP inconsistency between social and individual choice do not fully generalize to Weak WARP inconsistency. There are delegation rules such that if individual choose like law-abiding citizens then their social choices will also be like those of a law-abiding citizen. The same applies to choices made by agents who categorize and then choose, agents with psychological or moral constraints, agents who simply may not be attentive of their entire feasible set, and, in general, to agents who follow any procedure that satisfies Weak WARP.

A non-dictatorial and Weak WARP-consistent delegation rule that satisfies satisfy IIA is as follows: Let an alternative $y \in B$ *be* $B-$*anomalous for 1* if there exists an issue $B' \subset B$, $B' \neq B$, such that $y \in B'$ and $C_1(B') \neq y$. So, a $B-$anomalous choice produces a violation of WARP. In a sub-set of B, this choice was available and not taken by 1. Let $\bar{\varphi}$ be the delegation rule such that $\bar{\varphi}(B, C_1, C_2) =$

$$\begin{array}{ll} 2 & \text{if } C_1(B) \text{ is } B-\text{anomalous for 1 and } C_2(B) \text{ is not } B-\text{anomalous for 1;} \\ 1 & \text{otherwise.} \end{array}$$

So, 1 is the decider unless his choice produces a violation of WARP (with another choice made by 1 in a sub-issue) and 2's choice does not produce such violation. Then, 2 is the decider. The intuition in Theorem 1 is as follows: The rule $\bar{\varphi}$ is non-dictatorial because both agents may be deciders. It satisfies IIA because whether or not an alternative is $B-$anomalous depends only on choices about options in B. Finally, this rule is Weak WARP-consistent because it always produces less (or the same) violations of WARP that agent 1 produces. In fact, the example below shows a case in which all individual choice functions violate WARP and the social choice function do not.

Example 1. *There are 3 alternatives x, y and z. The individual choice functions of 1 and 2 are*

$$C_1(x, y) = x,\ C_1(y, z) = y,\ C_1(x, z) = x,\ C_1(x, y, z) = y;$$
$$C_2(x, y) = x,\ C_2(x, z) = z, C_2(y, z) = z \text{ and } C_2(x, y, z) = x.$$

So, the social choice function is:

$$C_{\bar{\varphi}}(x,y) = x; C_{\bar{\varphi}}(y,z) = y, C_{\bar{\varphi}}(x,z) = x \text{ and } C_{\bar{\varphi}}(x,y,z) = x$$

This example shows that $\bar{\varphi}$ is non-dictatorial because $\bar{\varphi}(\{y,z\}, C_1, C_2) = 1$ and $\bar{\varphi}(\{x,y,z\}, C_1, C_2) = 2$. The choices of 1 and 2 violate WARP, but their social choices do not. Instead, they follow the order x above y above z. Arrow's impossibility theorem shows that social choices may violate WARP even if individual choices do not. However, it may also be the case that social choices do not violate WARP, when individual choices do. The WARP inconsistency between social and individual choices can go either way. In contrast, Theorem 1 shows that some rules may make social and individual choices Weak WARP-consistent.

3.1. Extensions and Difficulties

Theorem 1 extends naturally to n agents. If both agents 1 and agent 2 may be deciders then the rule is non-dictatorial for any group size. However, a more meaningful form of non-dictatorship obtains when any of the n agents can be deciders in some circumstances. So, let C_i, $i = 1, ..., n$, be the individual choice function of i. A *delegation rule* is now a mapping $\varphi : \mathcal{B}\text{x}\mathcal{C}^n \longrightarrow \{1, ..., n\}$ that determines which agent decides for the group on each issue.

Minimum Participation for all A delegation rule φ satisfies Minimum Participation for all if for any agent i there is an issue B_i and a profile of individual choice functions $(C_1, ..., C_n)$ such that $\varphi(B_i, C_1, ..., C_n) = i$ and $C_j(B_i) \neq C_i(B_i)$, $j \neq i$.

A generalization of the delegation rule (with some abuse of notation also called $\bar{\varphi}$) for n agents is as follows: let $\bar{\varphi}(B, C_1, ..., C_n)$ be the smallest natural number k such that $C_k(B)$ is not $B-$anomalous for 1. If all individual choices on B are $B-$anomalous for 1 then $\bar{\varphi}(B, C_1, ..., C_n) = 1$. It is straightforward to show that $\bar{\varphi}$ satisfies Minimum Participation for all. Take example 1 and define arbitrarily choices involving any option other than x, y and z. Given agent $i \neq 1$, consider the profile of individual choice functions such that the choices of i are the choices of agent 2 in the example and the choices of all agents $j \neq i$ are the choices of agent 1 in the example. So, $i \neq 1$ is the decider on $\{x,y,z\}$. For a choice profile where 1 is the decider, consider the profile where the choices of agent 1 remain as in the example and the choices of all other agents are the choices of 2. Then, 1 is the decider on the issue $\{y, z\}$. The proof that $\bar{\varphi}$ satisfies IIA and is Weak WARP-consistent follow the same argument as in Theorem 1.

A stronger form of non-dictatorship occurs when each agent i has at least one issue such that i is the decider on that issue, no matter what are the choices of anyone else. Sen (1970) calls this property *liberalism*. It refers to the idea that there are some issues such as which side of the bed to sleep on that each person should determine, regardless of the sentiments of anyone else. Formally,

Liberalism A delegation rule φ is liberal if for each agent i there is an issue B_i such that $\varphi(B_i, C_1, ..., C_n) = i$ for any profile of choice functions $(C_1, ..., C_n)$.

The next result is an impossibility theorem showing that for delegation rules, there is a conflict between IIA, Liberalism and Weak WARP-consistency.

Theorem 2 Assume that there are at least four alternatives and two agents. There exist no delegation rule that satisfy IIA, is Weak WARP-consistent and liberal.

Theorem 2 is stronger when there are two agents (and shown only in this case). The proof first establishes that the same agent (say agent 1) is the decider on binary issues. This follows because if agents 1 and 2 disagree in two binary issues and different agents are deciders in different issues then, under IIA, this must still be so no matter what are their choices in non-binary issues. So, profiles of choice functions can be constructed so that the social choice function violate Weak WARP even when individual choice functions do not. Now if agent 2 is always the decider in an non-binary issue then it is possible to construct a profile of individual choice functions that satisfy Weak WARP and the social choice function does not. In conjunction, Theorems 1 and 2 show that there are non-dictatorial rules that satisfy IIA and are Weak WARP-consistent, but there are also limits on the non-dictatorial properties of such rules. They do not extend to liberalism.

4. Conclusion

Several ordinary procedures of individual decision making violate WARP and are characterized by Weak WARP. These procedures of individual decision making can be aggregated, by simple delegation rules, into a procedure for social decisions that also satisfy Weak WARP. This holds even if these rules are non-dictatorial and satisfy the Pareto principle and independence of irrelevant alternatives. It follows that, for models of decision characterized by Weak WARP, social and individual choices may have consistent logical structures. However, there are limitations on the non-dictatorial qualities that can be achieved.

5. Appendix:

Lemma 1. *Assume that there are at least four options different options x, y, z and w. There exists a choice function C that satisfies Weak WARP and any of the properties below*

$$C(x,y) = x,\ C(x,y,z) = y,\ C(x,z,w) = w \text{ and } C(A) = z; \tag{5.1}$$

$$C(x,y) = x,\ C(x,y,w) = y,\ C(w,y,z) = z \text{ and } C(A) = y; \tag{5.2}$$

$$C(x,y) = x,\ C(x,y,w) = y,\ C(w,x,z) = x, \text{ and } C(A) = z; \tag{5.3}$$

$$C(x,y) = x, C(x,y,z) = z, \text{ and } C(A) = x. \tag{5.4}$$

Proof: Assume, by contradiction, that there is a violation of Weak WARP on

$$\{x_1, y_1\} \subset B_1 \subset B_1^*.$$

So, $C(\{x_1, y_1\}) = C(B_1^*) = x_1$, and $C(B_1) = y_1 \neq x_1$.

For (5.1), consider an order that places z as the last alternative, y above x and w above x. Consider a choice function such that $C(x,y) = x$, $C(A) = z$ and for any other issue B, C maximizes this order on B. If $B_1^* = A$ then $x_1 = z$ and $C(\{x_1, y_1\}) = z$ but this is impossible because z is not chosen by C in any binary choice. So, $B_1^* \neq A$. Then, C chooses on B_1 and B_1^* by maximizing the same order. So, there can be no violation of WARP on $B_1 \subset B_1^*$. Thus, there is no violation of Weak WARP.

For (5.2), consider an order that places y as the last alternative, and w as the second last alternative and x as the third last alternative. Consider a choice function such that $C(x,y,w) = C(A) = y$ and for any other issue B, C maximizes this order on B. Now $B_1^* \neq A$ because y is not chosen by C in any binary choice. It follows that $B_1 = (x,y,w)$ because C maximizes the same order on $B_1 \neq \{x,y,w\}$ and $B_1^* \neq A$.

Thus, $y_1 = y$ and $x_1 \in \{x, w\}$. So, $C(B_1^*) \in \{x, w\}$, but this is impossible because B_1^* must have at least four alternatives.

For (5.3), consider an order that places z as the last alternative, y above x and x above w. The proof now follows the same argument as in the proof of (5.1).

For (5.4), consider an order that places x as the last alternative, and y as the second last option. Consider a choice function such that $C(x, y) = C(A) = x$ and for any other issue B, C maximizes this order on B. If $B_1^* = A$ then $\{x_1, y_1\} = \{x, y\}$ (because $C(\{x_1, y_1\}) = x$ and x is not chosen by C in any binary choice other than $\{x, y\}$). It follows that $x_1 = x$ and $y_1 = y$. But $C(B_1) = y$ is impossible because B_1 has at least three alternatives. So, $B_1^* \neq A$. Then, C chooses on B_1 and B_1^* by maximizing the same order. So, there can be no violation of WARP on $B_1 \subset B_1^*$. Thus, there is no violation of Weak WARP.

Lemma 2. *Assume that there are at least four options different options x, y, z and w. If φ is a delegation rule that satisfies IIA and Weak WARP-consistency then there is an agent $i \in \{1, 2\}$ such that $C_\varphi(B) = C_i(B)$ for every binary issue B.*

Proof: Assume, by contradiction, that there are two issues $\{x, y\}$ and $\{z, w\}$ and choice functions (C_1, C_2, C_φ) such that

$$C_\varphi(\{x, y\}) = C_1(\{x, y\}) \neq C_2(\{x, y\})$$

and

$$C_\varphi(\{z, w\}) = C_2(\{z, w\}) \neq C_1(\{z, w\}).$$

We can consider two cases. 1) There is no overlap between $\{x, y\}$ and $\{z, w\}$ and so all four options $\{x, y, z, w\}$ are different. 2) there is an overlap between $\{x, y\}$ and $\{z, w\}$ and so we can assume, without loss of generality that $z = y$ and all three options $\{x, y, z\}$ are different.

Case 1). We can assume, without loss of generality, that $C_1(\{x, y\}) = x$ and $C_2(\{z, w\}) = z$, (the proof remains exactly the same for any other choices). By Lemma 1, (5.1), there are choice functions $\tilde{C}_1$ and $\tilde{C}_2$ that satisfy Weak WARP such that

$$\tilde{C}_1(x, y) = x,\ \tilde{C}_1(x, y, z) = y,\ \tilde{C}_1(x, z, w) = w \text{ and } \tilde{C}_1(A) = z,$$

$$\tilde{C}_2(z, w) = z,\ \tilde{C}_2(x, w, x) = w,\ \tilde{C}_2(z, x, y) = y \text{ and } \tilde{C}_2(A) = x$$

(where the second equation obtains replacing x and z and y and w). By IIA,

$$\tilde{C}_\varphi(\{x,y\}) = C_\varphi(\{x,y\}) = C_1(\{x,y\}) = x,$$
$$\tilde{C}_\varphi(\{z,w\}) = C_\varphi(\{z,w\}) = C_2(\{z,w\}) = z.$$

Given that φ is a delegation rule,

$$\tilde{C}_\varphi(\{x,y,z\}) = y,$$
$$\tilde{C}_\varphi(\{x,z,w\}) = w,$$
$$\tilde{C}_\varphi(A) \in \{x,z\}.$$

If $\tilde{C}_\varphi(A) = x$ then $\tilde{C}_\varphi$ violates Weak WARP on $\{x,y\} \subset \{x,y,z\} \subset A$.

If $\tilde{C}_\varphi(A) = z$ then $\tilde{C}_\varphi$ violates Weak WARP on $\{z,w\} \subset \{x,z,w\} \subset A$.

Case 2 is now divided into three subcases. 2a) 1 chooses the non-overlapping option x and 2 chooses the overlapping option y. 2b) 1 chooses the non-overlapping option x and 2 also chooses the non-overlapping option z. 2c) 1 and 2 choose the overlapping option y. The case where 1 chooses the overlapping option y and 2 chooses the non-overlapping option z is analogous to 2a and, hence, omitted.

Case 2a) By Lemma 1, (5.2), there is a choice functions $\tilde{C}_1$ that satisfy Weak WARP such that

$$\tilde{C}_1(x,y) = x,\ \tilde{C}_1(x,y,w) = y,\ \tilde{C}_1(w,y,z) = z \text{ and } \tilde{C}_1(A) = y.$$

By Lemma 1, (5.3), there is a choice functions $\tilde{C}_2$ that satisfy Weak WARP such that

$$C(y,z) = y,\ C(y,z,w) = z,\ C(w,y,x) = y, \text{ and } C(A) = x.$$

(replacing x with y, y with z and z with x). By IIA,

$$\tilde{C}_\varphi(\{x,y\}) = C_\varphi(\{x,y\}) = C_1(\{x,y\}) = x,$$
$$\tilde{C}_\varphi(\{y,z\}) = C_\varphi(\{y,z\}) = C_2(\{y,z\}) = y.$$

Given that φ is a delegation rule,

$$\tilde{C}_\varphi(\{x,y,w\}) = y,$$
$$\tilde{C}_\varphi(\{w,y,z\}) = z,$$
$$\tilde{C}_\varphi(A) \in \{x,y\}.$$

If $\tilde{C}_\varphi(A) = x$ then $\tilde{C}_\varphi$ violates Weak WARP on $\{x, y\} \subset \{x, y, w\} \subset A$.

If $\tilde{C}_\varphi(A) = y$ then $\tilde{C}_\varphi$ violates Weak WARP on $\{y, z\} \subset \{w, y, z\} \subset A$.

Case 2b) By Lemma 1, (5.1), there are choice functions $\tilde{C}_1$ and $\tilde{C}_2$ that satisfy Weak WARP such that

$$C(x, y) = y, C(x, y, z) = z, \text{ and } C(A) = y.$$

$$\tilde{C}_2(y, z) = z,\ \tilde{C}_2(x, y, z) = y, \text{ and } \tilde{C}_2(A) = x$$

(where the second equation obtains replacing x and z). By IIA,

$$\begin{array}{c} \tilde{C}_\varphi(\{x, y\}) = C_\varphi(\{x, y\}) = C_1(\{x, y\}) = x, \\ \tilde{C}_\varphi(\{y, z\}) = C_\varphi(\{y, z\}) = C_2(\{y, z\}) = z. \end{array}$$

Given that φ is a delegation rule,

$$\begin{array}{c} \tilde{C}_\varphi(\{x, y, z\}) = y, \\ \tilde{C}_\varphi(A) \in \{x, z\}. \end{array}$$

If $\tilde{C}_\varphi(A) = x$ then $\tilde{C}_\varphi$ violates Weak WARP on $\{x, y\} \subset \{x, y, z\} \subset A$.

If $\tilde{C}_\varphi(A) = z$ then $\tilde{C}_\varphi$ violates Weak WARP on $\{y, z\} \subset \{x, y, z\} \subset A$.

Case 2c) By Lemma 1, (5.4), there are choice functions $\tilde{C}_1$ and $\tilde{C}_2$ that satisfy Weak WARP such that

$$\tilde{C}_1(x, y) = y, \tilde{C}_1(x, y, z) = z, \text{ and } \tilde{C}_1(A) = y,$$

$$\tilde{C}_2(y, z) = y,\ \tilde{C}_2(x, y, z) = x, \text{ and } \tilde{C}_2(A) = y$$

(where the first equation obtains replacing x and y and the second equation obtains replacing x with y, y with z and z with x). By IIA,

$$\begin{array}{c} \tilde{C}_\varphi(\{x, y\}) = C_\varphi(\{x, y\}) = C_1(\{x, y\}) = y, \\ \tilde{C}_\varphi(\{y, z\}) = C_\varphi(\{y, z\}) = C_2(\{y, z\}) = y. \end{array}$$

Given that φ is a delegation rule,

$$\begin{array}{c} \tilde{C}_\varphi(\{x, y, z\}) \in \{x, z\}, \\ \tilde{C}_\varphi(A) = y. \end{array}$$

If $\tilde{C}_\varphi(\{x,y,z\}) = x$ then $\tilde{C}_\varphi$ violates Weak WARP on $\{x,y\} \subset \{x,y,z\} \subset A$.
If $\tilde{C}_\varphi(\{x,y,z\}) = z$ then $\tilde{C}_\varphi$ violates Weak WARP on $\{y,z\} \subset \{x,y,z\} \subset A$.

5.1. Proof of Theorem 1

Fix an issue $B \in \mathcal{B}$. Assume that C_1 and $\tilde{C}_1$ are $B-$equivalent and $C_2(B) = \tilde{C}_2(B)$. By definition, whether a choice is $B-$anomalous for 1 depends only on the choices that 1 makes involving alternatives in B. So, $\bar{\varphi}(C_1, C_2, B) = 2$ if and only if $\bar{\varphi}(\tilde{C}_1, \tilde{C}_2, B) = 2$ and, hence, $\bar{\varphi}$ satisfies IIA.

Now assume that $C_{\bar{\varphi}}$ does not satisfy Weak WARP. Then, there are issues and alternatives such that $x \neq y$, $\{x,y\} \subseteq B_1 \subseteq B_2$, $C_{\bar{\varphi}}(\{x,y\}) = C_{\bar{\varphi}}(B_2) = x$ and $C_{\bar{\varphi}}(B_1) = y$. By definition, $C_1(\{x,y\}) = C_{\bar{\varphi}}(\{x,y\}) = x$ (because no alternative is $\{x,y\}-$anomalous for 1). So, $y \in B_1$ is B_1-anomalous for 1. Thus, if $C_2(B_1) = y$ then $\bar{\varphi}(B_1, C_1, C_2) = 1$ and so, $C_1(B_1) = C_{\bar{\varphi}}(B_1)$. If $C_2(B_1) \neq y$ then $C_1(B_1) = C_{\bar{\varphi}}(B_1)$. Thus, $C_1(B_1) = C_{\bar{\varphi}}(B_1) = y$. If follows that $x \in B_2$ is B_2-anomalous for 1. By the same argument, $C_1(B_2) = C_{\bar{\varphi}}(B_2) = x$. Hence, C_1 violates Weak WARP. So, $\bar{\varphi}$ satisfies Weak WARP consistency. The non-dictatorial property of $\bar{\varphi}$ is shown by example 1.

5.2. Proof of Theorem 2.

By Lemma 2, we can assume, without loss of generality that $C_\varphi(B) = C_1(B)$ for every binary issue B. Let $\bar{B}$ be an issue such that $\varphi(\bar{B}, C_1, C_2) = 2$ for every choice function C_1. The issue $\bar{B}$ is not binary and so, contains at least three options. Take any two options x and y in $\bar{B}$. First consider the case where $\bar{B} \neq A$ and choice functions (that satisfy Weak WARP) such that $C_1(x,y) = C_1(A) = x$ and $C_2(\bar{B}) = C_2(A) = y$ (see Lemma 1, (5.2), for existence of function C_1. For the existence of function C_2, just assume that 2 maximizes a single order that places y at the top.) So, $C_\varphi(x,y) = C_\varphi(A) = x$ and $C_\varphi(\bar{B}) = y$. Thus, φ is not Weak WARP consistent. Now consider the case where $\bar{B} = A$ and choice functions (that satisfy Weak WARP) such that $C_1(x,y) = x$, $C_1(x,y,z) = C_2(x,y,z) = y$ and $C_2(A) = x$ (see Lemma 1, (5.2), for the existence of C_1 and Lemma 1, (5.4), replacing z with x and w with z, for the existence of C_2). So, $C_\varphi(x,y) = C_\varphi(A) = x$ and $C_\varphi(x,y,z) = y$. Thus, φ is not Weak WARP consistent.

References

[1] Ambrus, A., and K. Rozen (2008) "Revealed Conflicting Preferences," mimeo.

[2] Apesteguia, J. and Ballester (2008) "A Characterization of Sequential Rationalizability," mimeo.

[3] Arrow, K. (1950), "A Difficulty in the Concept of Social Welfare," *The Journal of Political Economy*, 58(4), 328-346.

[4] Bernheim, D. and A. Rangel (2009) "Beyond Revealed Preference: Choice-Theoretic Foundations for Behavioral Welfare Economics," *The Quarterly Journal of Economics,* 124 (1), 51-104.

[5] Blackorby, C., W. Bossert, and D. Donaldson (2005) "Temporal Consistency," *Population Issues in Social Choice Theory, Welfare Economics, and Ethics*, Cambridge University Press, 272-285.

[6] Brown, J. (1975) "Aggregation of Preferences," *Quarterly Journal of Economics* 89, 456–469.

[7] Browning, M. (2000) "The Saving Behavior of a Two-Person Household," *Scandinavian Journal of Economics*, 102, 235-251.

[8] Caplin, A. and J. Leahy (2004) "The Social Discount Rate," *Journal of Political Economy*, 112, 1257-1268.

[9] Chambers, C. and T. Hayashi (2008) "Choice and Individual Welfare," mimeo, Caltech.

[10] Cherepanov, V., Feddersen, T, and A. Sandroni (2013a) "Rationalization" *Theoretical Economics* 8-3, 775–800.

[11] Cherepanov, V., Feddersen, T, and A. Sandroni (2013b) "Revealed Preferences and Aspirations in Warm Glow Theory," *Economic Theory* 54-3, 501-535.

[12] Clippel, G. and K. Eliaz (2012) "Reason-Based Choice: A Bargaining Rationale for the Attraction and Compromise Effects," *Theoretical Economics* 7(1), 125-162.

[13] Clippel, G. and K. Rozen (2012) "Bounded Rationality and Limited Datasets: Testable Implications Identification, and Out-of-Sample Predictions," mimeo.

[14] Condorcet, Marquis de. (1785), "Essai sur l'application de l'analyse a la probabilite des decisions rendues a la probabilite des voix," Paris: De l'imprimerie royale, translated in 1976 to "Essay on the Application of Mathematics to the Theory of Decision-Making." in Condorcet: Selected Writings, Indianapolis, IN: Bobbs-Merrill. K. M. Baker, editor.

[15] Dietrich, F. and C. List (2010) "A Reason-based Theory of Rational Choice," mimeo.

[16] Ehlers, L. and Y. Sprumont (2008) "Weakened Warp and Top-cycle Choice Rules," *Journal of Mathematical Economics*, 44, 87–94.

[17] Eliaz, K., M. Richter and A. Rubinstein (2011) "Choosing the Two Finalists," *Economic Theory*, 46, 211-219.

[18] Eliaz, K. and R. Spiegler (2011) "Consideration Sets and Competitive Marketing" *Review of Economic Studies*, 78-1, 235-262.

[19] Eliaz, K., D. Ray and R. Razin (2006) "Choice Shifts in Groups: A Decision-Theoretic Basis," *American Economic Review*, 96, 1321-1332.

[20] Fudenberg, D. and D. Levine (2006) "A Dual-Self Model of Impulse Control," *American Economic Review*, 96, 1449-1476.

[21] Gibbard, A. (2014) "Intransitive Social Indifference and the Arrow Dilemma," *Review of Economic Design*, 18, 3-10.

[22] Green, J., and D. Hojman (2007) "Choice, Rationality and Welfare Measurement," Harvard University, Working Paper.

[23] Heller, Y. (2009) "Justifiable Choice," mimeo, Tel Aviv University.

[24] Hertzberg, A. (2011) "Exponential Individuals, Hyperbolic Households" mimeo.

[25] Houy, N. (2007) "Rationality and Order-Dependent Sequential Rationality" *Theory and Decision*, 62, 119-134.

[26] Houy, N. and K. Tadenuma (2009), "Lexicographic Compositions of Multiple Criteria for Decision Making," *Journal of Economic Theory*, 144-4, 1770-1782.

[27] Hubler, J., J. Payne, and C. Puto (1982) "Adding Asymmetrically Dominated Alternatives; Violation of Regularity and the Similarity Hypothesis," *Journal of Consumer Research*, 9, 90-98.

[28] Jackson, M. and L. Yariv (2011) "Collective Dynamic Choice: The Necessity of Time Inconsistency," mimeo.

[29] Kalai, G., A. Rubinstein, and R. Spiegler (2002) "Rationalizing Choice Functions by Multiple Rationales," *Econometrica*, 70 (6), 2481-2488.

[30] Katz, L. and A. Sandroni (2014) "Why Laws Breeds Cycles," mimeo.

[31] Lehrer, E. and R. Teper (2011) "Justifiable Preferences," *Journal of Economic Theory*, 146, 762-774.

[32] Lleras, J., Masatlioglu, Y., D. Nakajima and E. Ozbay (2010) "When More is Less: Limited Consideration," mimeo.

[33] Manzini, P. and M. Mariotti (2007) "Sequentially Rationalizable Choice" *American Economic Review* **97**-5, 1824-1839.

[34] Manzini, P. and M. Mariotti (2012a) "Categorize then Choose: Boundedly Rational Choice and Welfare," *Journal of the European Economic Association,*10(5), 1141-1165.

[35] Manzini, P. and M. Mariotti (2012b) "Choice by Lexicographic Semiorders," *Theoretical Economics* 7(1), 1-23.

[36] Masatlioglu, Y. and D. Nakajima (2007) "A Theory of Choice by Elimination," mimeo.

[37] Masatlioglu, Y. and E. Ok (2005) "Rational Choice with Status Quo Bias," *Journal of Economic Theory*, 121, 1-29.

[38] Masatlioglu, Y., D. Nakajima and E. Ozbay (2012) "Revealed Attention," *American Economic Review,* 102(5), 2138-2205.

[39] May, K (1954) "Intransitivity, Utility, and the Aggregation of Preference Patterns," *Econometrica*, 22 (1), 1-13.

[40] Mongin (1995) "Consistent Bayesian Aggregation," *Journal of Economic Theory*, 66, 313-351.

[41] Moulin, H. (1985) "Choice Functions over a Finite Set: A Summary," *Social Choice and Welfare*, 2, 147-160.

[42] Ok, E., P. Ortoleva, and G. Riella (2008) "Rational Choice with Endogenous Reference Points," mimeo.

[43] Salant, Y. and A. Rubinstein (2006) "Two Comments on the Principle of Revealed Preferences," mimeo.

[44] Salant, Y. and A. Rubinstein (2006a) "A Model of Choice from Lists," *Theoretical Economics*, 1, 3-17.

[45] Salant, Y. and A. Rubinstein (2011) "Eliciting Welfare Preferences from Behavioral Data Sets," *Review of Economic Studies*, forthcoming.

[46] Samuelson, P. (1938) "The Empirical Implications of Utility Analysis," *Econometrica*, 6 (4), 344-356.

[47] Schick, F. (1963) "Arrow's Proof and the Logic of Preferences," *Philosophy of Science*, 36, 127-144.

[48] Sobel, J. (2014) "On the Relationship between Individual and Group Decisions," *Theoretical Economics*, 9, 163-185.

[49] Sen, A. (1970) "The impossibility of a Paretian Liberal," *Journal of Political Economy*, 78, 152-157.

[50] Simonson, I and A. Tversky (1993) "Context-Dependent Preferences," *Management Science*, 39 (10), 1179-1189.

[51] Sprumont, Y. (2000) "On the Testable Implications of Collective Choice Theories," *Journal of Economic Theory*, 93, 205-232.

[52] Spiegler, R. (2002) "Equilibrium in Justifiable Strategies: A Model of Reason-Based Choice in Extensive-Form Games" (2002), *Review of Economic Studies* 69, 691-706.

[53] Tversky, A. and E. Shafir (1992) "Choice Under Conflict: The Dynamics of Deferred Decision," *Psychological Science*, 3 (6), 358-361.

[54] Tyson, C. (2012) "Behavioral Implications of Shortlisting Procedures," *Social Choice and Welfare*, forthcoming.

[55] Weitzman, M. (2001) "Gamma Discouting," *American Economic Review*, 91, 260-271.

[56] Zuber, S. (2010) "The Aggregation of Preferences, Can we Ignore the Past?", mimeo.